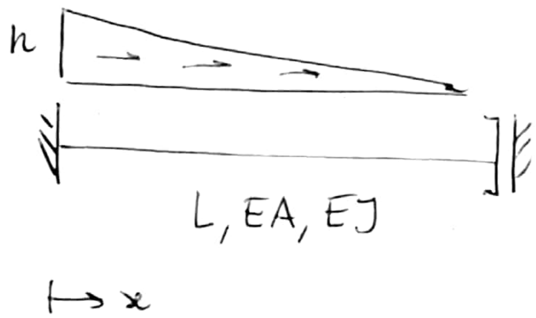


Find the displacement at $x = L/3$

Data

$L = 2000$ mm

$EA = 1.8 \cdot 10^6$ N

$EJ = 1.8 \cdot 10^8$ N mm^2

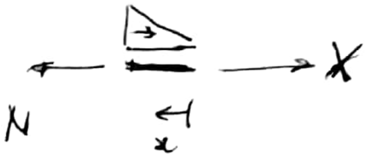
$n = 2\,(1 + F/10)$ N/mm

Solution ($F = 0$)

Evaluate first the unknown reaction force X

Real

Dummy



$$N = X + \frac{n x^2}{2L}$$

$$\delta N = 1$$

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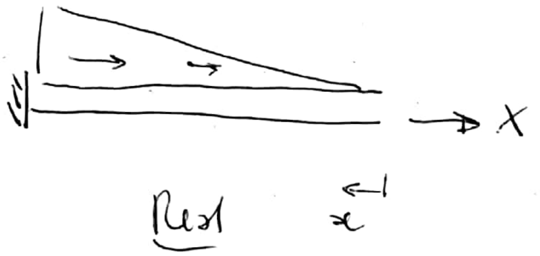
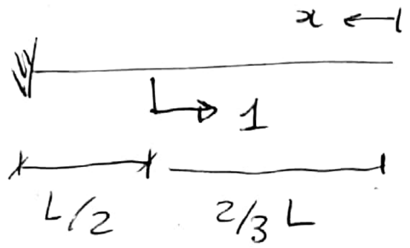
$$\int_0^L \frac{N\,\delta N}{EA}\,dx = 0 \quad \Rightarrow \quad X = -\frac{nL}{6}$$

Evaluate now the displacement

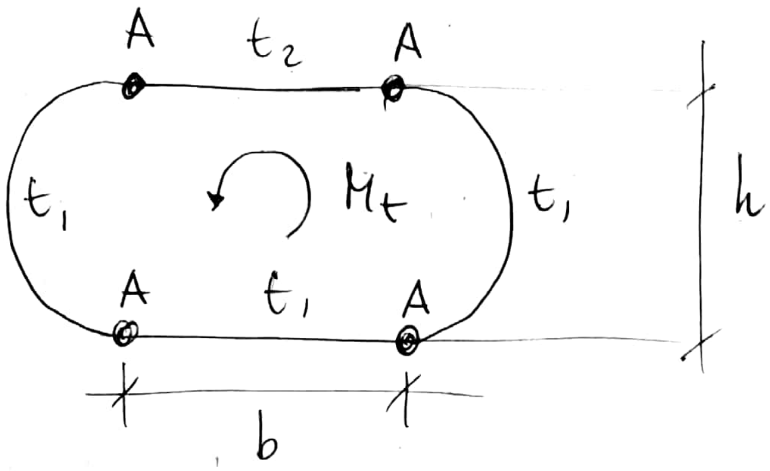




As before $\qquad \delta N = +1 \quad x \geq 2/3\,L$

$$\int_{2/3\,L}^{L} \frac{N\,\delta N}{EA}\,dx = s$$, from which:

$$s = \frac{54X + 19Ln}{162\,EA}\cdot L = 0.274\ \text{mm}$$



Determine the shear stress τ in the panel of thickness t_2..

Data

$A = 500\ mm^2$

$h = 200\ (1 + E/10)\ mm$

$b = 150\ mm$

$t_1 = 1.3\ mm$

$t_2 = 1.6\ mm$

$M_t = 10^7\ Nmm$

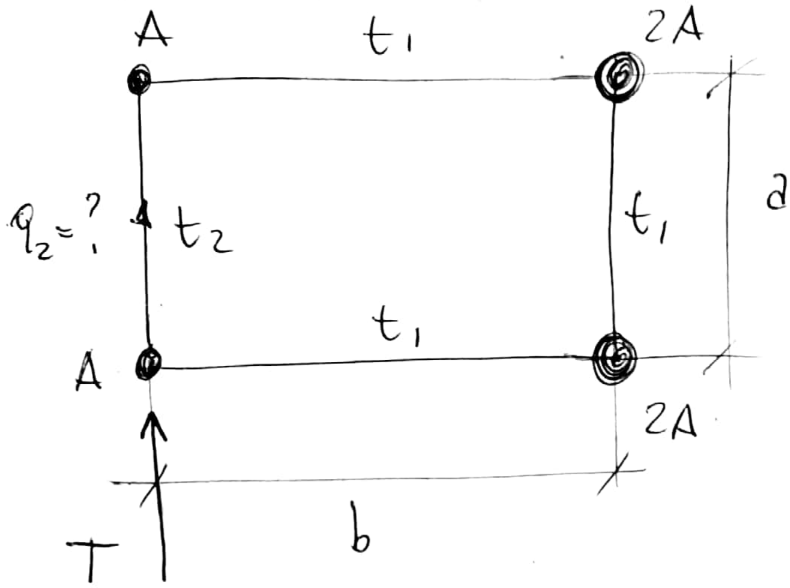
Solution ($E = 0$)

The shear flow in the panels is obtained as

$2q\Omega = M_t$ with $\Omega = bh + \pi h^2/4$

and

$\tau = q/t_2 = 50.88\ MPa$



Calculate the shear flow q_2

Data

$A = 800\ (1 + A/10)\ mm^2 \quad T = 7600\ N$

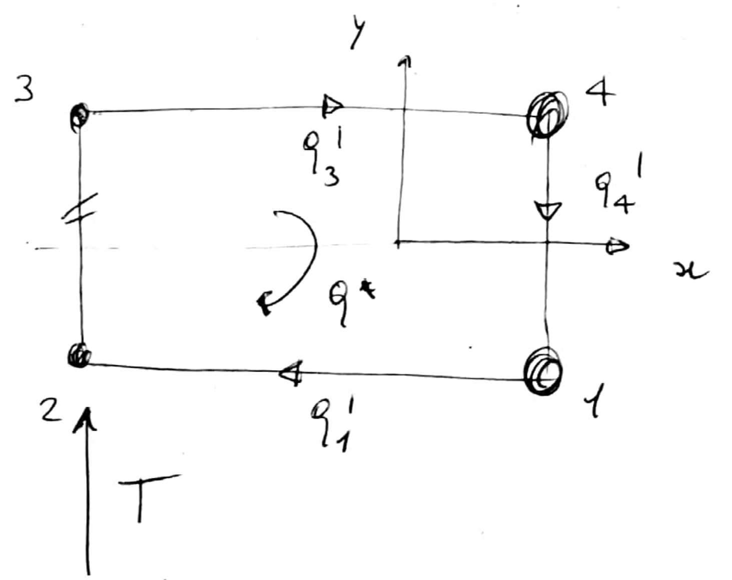
$a = 250\ mm$

$b = 600\ mm$

$t_1 = 1\ mm$

$t_2 = 2\ (1 + C/10)\ mm$

Solution $(A = C = 0)$



$J_{xx} = \frac{3}{2} A a^2$

$S_{x_3}' = A a/2$

$S_{x_4}' = \frac{3}{2} A a$

$S_{x_1}' = S_{x_3}'$

Apply the shear flow equation:

$$q_3' = -T\frac{S_{x3}'}{J_{xx}} = -T/3a\,; \quad q_1' = q_3'$$

$$q_4' = -T\frac{S_{x4}'}{J_{xx}} = -T/a\,;$$

Apply the equivalence wrt z:

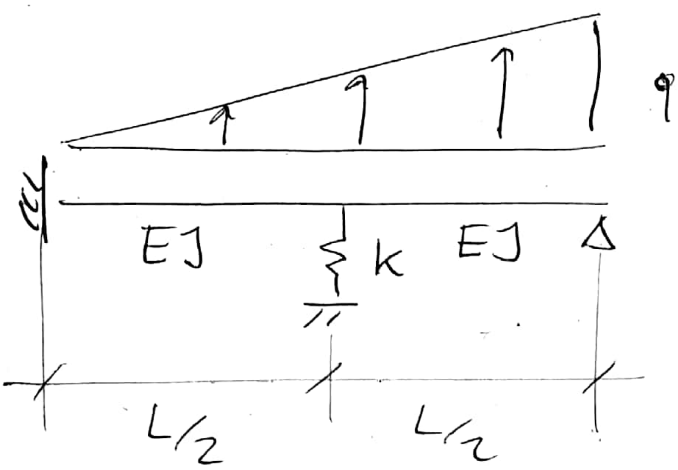
$$2\Omega_c\, q^* + 2q_3'\,\Omega_3 + 2q_4'\,\Omega_4 = 0$$

$$\Omega_c = ab\,; \quad \Omega_3 = \Omega_4 = ab/2 \quad , \text{ so:}$$

$$2q^* + q_3' + q_4' = 0$$

$$\Rightarrow \quad q^* = \frac{2}{3}\,T/a = 20.27 \text{ N/mm}$$

$$q_2 = q^* = 20.27 \text{ N/mm}$$



Using the Ritz method and a 1-dof polynomial approximation, determine the vertical displacement in correspondence of the spring

Data

$L = 750$ mm

$q = 2$

$EJ = 1.25 \cdot 10^8$ N mm^2

$K = 50(1 + A/10)$ N/mm

Solution $(A=0)$

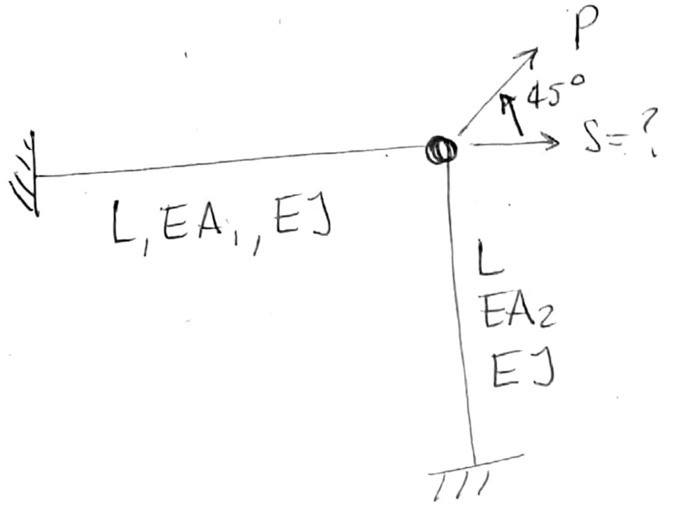
$$W = (x^3 - L x^2) q_1$$

$$\int_0^L \delta W_{/xx} \, EJ \, W_{/xx} \, dx + \delta w(L/2) \, k \, w(L/2)$$

$$= \int_0^L \delta w \, q \frac{x}{L} \, dx$$, from which

$$(4EJL^3 + K/_{64} L^6) q_1 = - 1/_{20} \, q L^4$$, so:

$$w(L/2) = - L^3/_8 \, q_1 = 4.768 \text{ mm}$$



Use the Ritz method and the simplest polynomial approximation for evaluating s.

Data

$L = 730$ mm

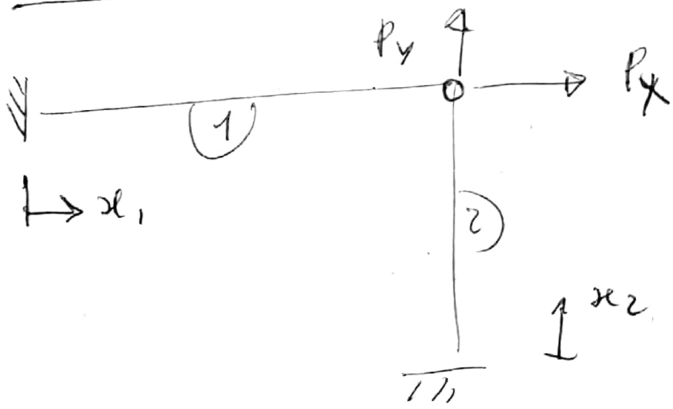
$EA_1 = 1.15 \cdot 10^6 \, (1 + F/10)$ N

$EA_2 = 3 \cdot 10^5$ N

$EJ = 1.8 \cdot 10^7$ Nmm^2

$P = 2500 \, (1 + A/10)$ N

Solution $(A = F = 0)$



Due to uncoupling between axial and bending we can consider only the axial response of beam 1 and the bending one of beam 2

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$u_1 = q_1\, x$

$w_2 = q_2\, x^2$

but $u_1(L) = w_2(L) \Rightarrow q_1 = q_2 L$, so:

$u_1 = q_2\, x_1 L$

$w_2 = q_2\, x_2^2$

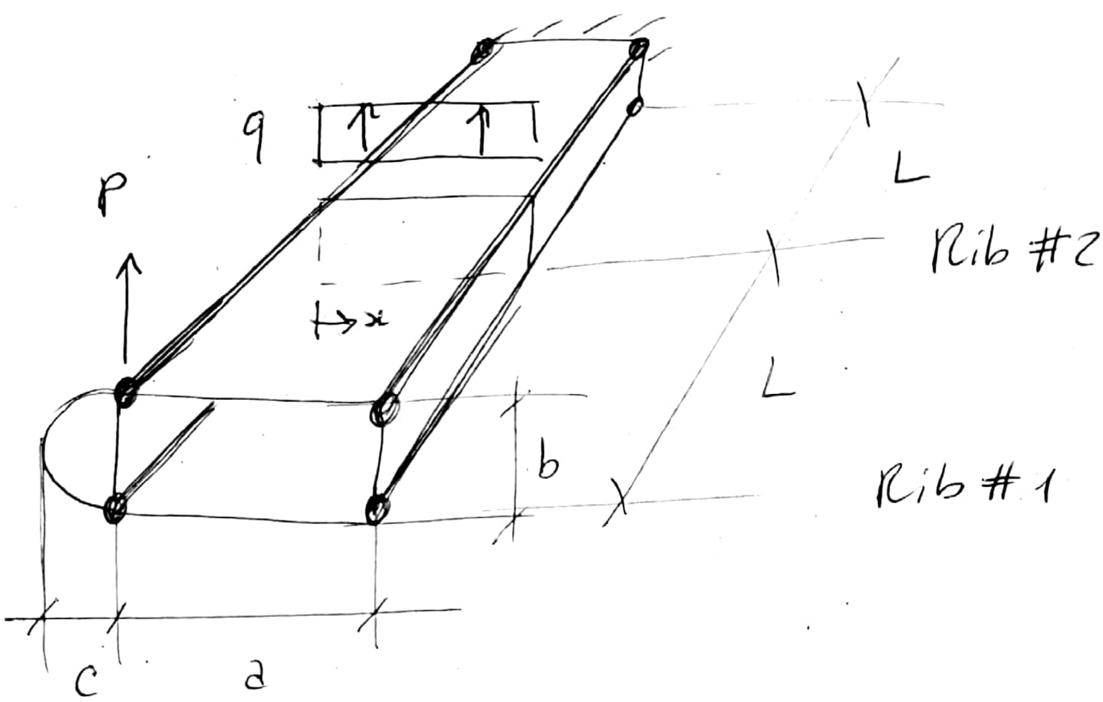
$$\int_0^L \delta u_{1/x}\, EA_1\, u_{1/x}\, dx_1 + \int_0^L \delta w_{2/xx}\, EJ\, w_{2/xx}\, dx_2 = \delta u_1(L)\, P_x$$

which leads to:

$$\left(EA_1 L^3 + 4 EJL\right) q_2 = P_x L^2$$

From which

$$S = w_2(L) = q_2 L^2 = 1.12 \text{ mm}$$



Model rib #2 as a beam, and evaluate the bending moment at $x = a/2$, as M/M_{ref}.

Data

$a = 700$ mm

$b = 200$ mm

$c = 100$ mm

$L = 2000$ mm

$t = 0.6$ mm

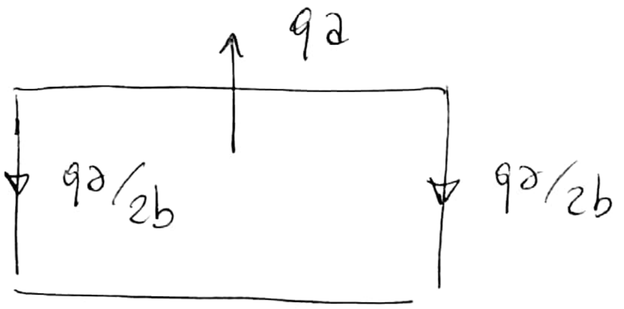
$A = 700$ mm^2

$P = 2000$ N

$q = 2(1 + F/10)$ N/mm

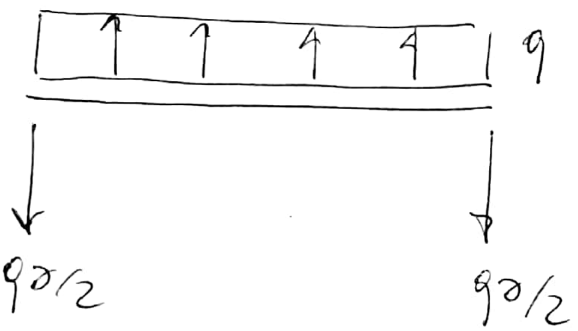
$M_{ref} = 9.8 \cdot 10^5$ N·mm

The equilibrating shear flows on Rib #2 are obtained as:

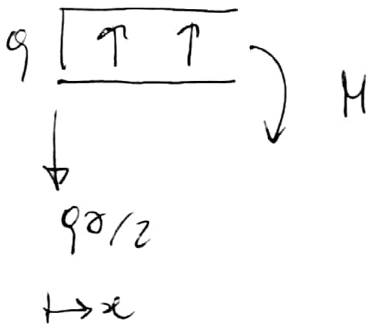


equilibrating flows

So:



beam model



$$M(x) = \frac{qa}{2}x - q\frac{x^2}{2}$$

So: $$\frac{M(a/2)}{M_{ref}} = 0.125$$

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- The Principle of Virtual Works can be applied only for hyperelastic constitutive laws.

  <u>False</u>

- An hyperelastic constitutive law is not necessarily linear.

  <u>True</u>

- The assumption of infinitesimal displacements implies that the equilibrium conditions are referred to the undeformed configuration.

  <u>True</u>

- The equivalence between the Principle of Virtual Works and the Principle of Minimum Potential Energy

  <u>holds for hyperelastic material law</u>

- The shear flows acting on the rib are:

  <u>the flows equilibrating the applied load</u>

- In finite elements, the hourglass phenomenon

  <u>can be due to a excessively low number of integration point</u>