Course of Aerospace Structures

Written test, June ~~18~~ 17$^{\text{th}}$, 2024

Name ______________________

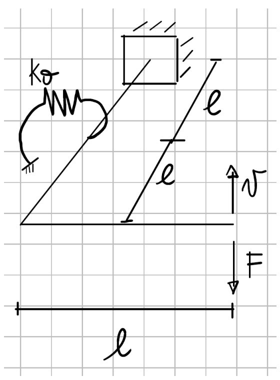
Surname ______________________

Person code: | | | | | | | | |

Exercise 1

The slender L-shaped beam structure in the figure is clamped at one extremity and has a torsional spring with torsional stiffness K_θ (such that $M = K_\theta \theta$) at a distance l from the clamp. Compute the sketched vertical displacement v due to the sketched force F . Neglect shear deformability.

(Unit for result: mm)



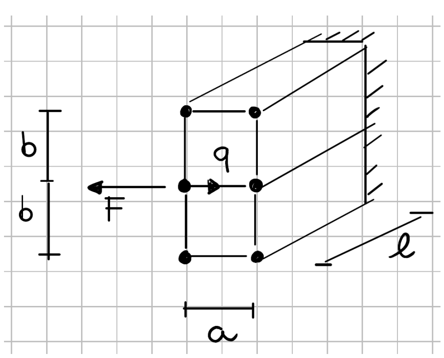
Data
$l = 1$ m
$K_\theta = 1 \times 10^{10}$ N mm
$EI = 1 \times 10^{15}$ N mm^2
$GJ = 7 \times 10^{9}$ N mm^2
$F = 150$ N

Answer ______________________

Exercise 2

Consider the two-cells semi-monocoque beam model sketched in the figure. All the six stringers have area equal to A, and all the seven panes have thickness equal to t. The beam is clamped at one extremity, and loaded by the horizontal force F at the other. Compute the shear flux q sketched in the figure.

(Unit for result: N mm^{-1})

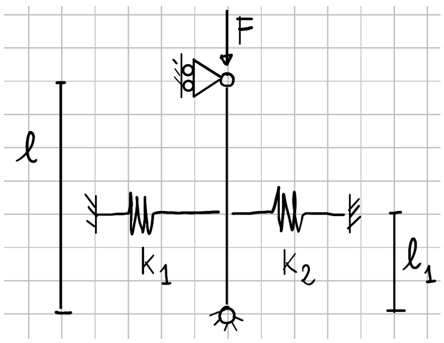


Data
$A = 100$ mm^2
$t = 1$ mm
$a = 250$ mm
$b = 100$ mm
$l = 1$ m
$E = 77000$ MPa
$\nu = 0.33$
$F = 50000$ N

Answer ______________________

Exercise 3
Consider the vertical slender beam model with length l sketched in the figure. It is constrained by an hinge at one extremity and a slider at the other. Two horizontal springs, with stiffness K_1 and K_2, are attached at a distance l_1 from the hinge. By approximating the transverse displacement with a polynomial function with only one unknown estimate the critical buckling compressive force F of the structure. Neglect shear deformability.

(Unit for result: N)

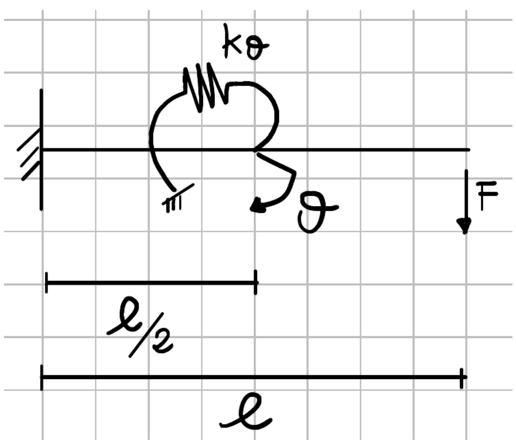


Data
$l = 2000$ mm
$l_1 = 750$ mm
$EA = 6 \times 10^{10}$ N
$EI_{xx} = EI_{yy} = 12 \times 10^{10}$ N mm^2
$GJ = 7 \times 10^9$ N mm^2
$K_1 = 75 \times 10^2$ N/mm
$K_1 = 25 \times 10^2$ N/mm

Answer

Exercise 4
Consider the slender cantilever beam sketched in the figure, with length l. It has a torsional spring, at a distance equal to $l/2$ from the clamp, that works for the bending rotation θ of the beam (i.e. the derivative of the transverse displacement). The beam is loaded by the concentrated vertical force F, as sketched. By assuming a quadratic transverse displacement approximation estimate the bending rotation θ at $l/2$. Neglect shear deformability.

(Unit for result: rad)

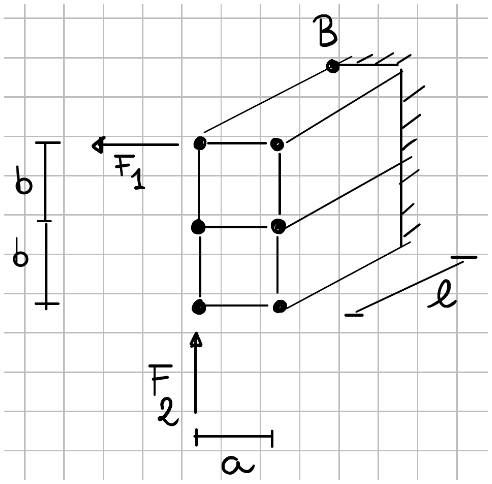


Data
$l = 1000$ mm
$K_\theta = 1 \times 10^{10}$ Nmm
$EI = 12 \times 10^9$ N mm^2
$F = 150$ N

Answer

Exercise 5
The same two-cells semi-monocoque beam model of Exercise 2 is loaded at its free end by the forces F_1 and F_2, as sketched. Compute the axial *strain* (ε_{zz}) of stringer B at the clamp (cfr. the figure).

(Unit for result: -)

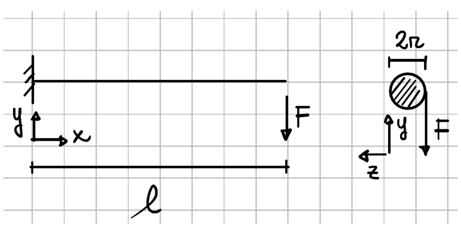


Data
$A = 100$ mm^2
$t = 1$ mm
$a = 250$ mm
$b = 100$ mm
$l = 1$ m
$E = 77000$ MPa
$\nu = 0.33$
$F_1 = 50000$ N
$F_2 = 2500$ N

Answer

Exercise 6
The clamped slender beam sketched in the figure has a circular cross-section. It is loaded by the vertical force F applied at the free end on the lateral side of the cross-section, cfr. the sketch. Compute the torsional rotation at the free end. Neglect shear deformability..

(Unit for result: rad)



Data
$l = 200$ mm
$r = 10$ mm
$EA = 2.5 \times 10^7$ N
$EI_{xx} = EI_{yy} = 6 \times 10^8$ N mm^2
$GJ = 4 \times 10^8$ N mm^2
$F = 2000$ N

Answer

True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

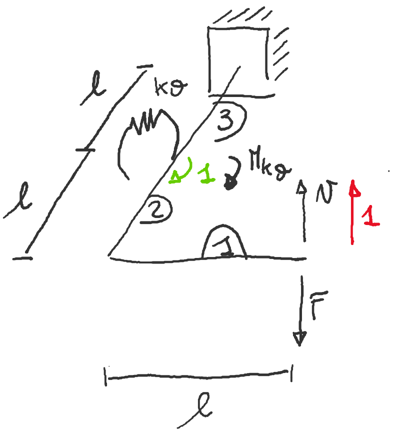
1. A beam model cannot be used for evaluating local effects due to the application of loads.
2. Essential boundary conditions are more important than natural ones.
3. Shear deformability effects are generally more relevant for thin-walled cross-section beams than for compact cross-section beams.

Multiple Choice questions
(Circle the correct answer)

1. Consider an Euler-Bernoulli cantilever beam model, loaded with a uniformly distributed load. The exact solution is:
   (a) trigonometric
   (b) polynomial (quadratic)
   (c) polynomial (cubic)
   (d) none of the above
2. A plane-strain constitutive law:
   (a) has null axial stress
   (b) has null axial strain
   (c) has null shear stress
   (d) has null shear strain
   (e) none of the above
3. A two-cell section modeled according to the semi-monocoque scheme can be solved by using:
   (a) shear flow equations only
   (b) shear flow equations together with the equivalence to internal moment
   (c) shear flow equations, equivalence to internal moment and the compatibility equation
   (d) none of the above

2024-06-18 Ex 1

Wednesday, June 12, 2024 7:31 PM



IN REAL SYST

SUBSTITUTE SPRING WITH REACTION

$$M_{k\vartheta} = -k_\vartheta \cdot \vartheta$$

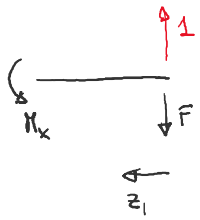
FROM WHICH

$$\vartheta = -\frac{M_{k\vartheta}}{k_\vartheta}$$

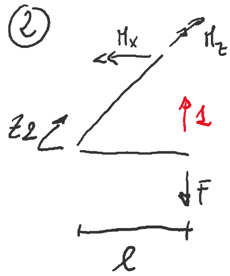
SOL

① $M_{x_1} = F \cdot z_1$

$M'_{x_1} = -z_1$



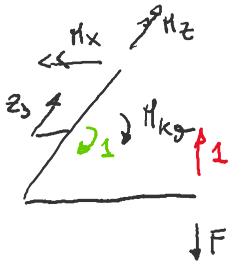
②



$M_{x_2} = F \cdot z_2$ $\quad M_{z_2} = -F \cdot \ell$

$M'_{x_2} = -z_2$ $\quad M'_{z_2} = 1 \cdot \ell$

③



$M_{x_3} = F \cdot (\ell + z_3)$ $\quad M_{z_3} = -F \cdot \ell - M_{k\vartheta}$

$M'_{x_3} = -(\ell + z_3)$ $\quad M'_{z_3} = (\ell + z_3)$

$M''_{x_3} = 0$ $\quad M''_{z_3} = -1$

PCVW (INDEPENDENT EQUATIONS SINCE RED IS FOR DISPC)

WHITE GREEN

$$\delta W_e = -1 \cdot \vartheta = -1 \cdot \frac{M_{k\vartheta}}{k_\vartheta}$$

$$\delta W_i = \int_0^\ell M''_{z_3} \cdot \frac{M_{z_3}}{GJ} \, dz_3 = \int_0^\ell \frac{F\ell + M_{k\vartheta}}{GJ} \, dz_3 = \frac{F\ell^2}{GJ} + \frac{M_{k\vartheta} \cdot \ell}{GJ}$$

$$M_{k\vartheta} = -\frac{F\ell^2}{GJ} \left(\frac{1}{k_\vartheta} + \frac{\ell}{GJ} \right)^{-1}$$

WHITE RED

$$\delta W_e = 1 \cdot v$$

$$\delta W_i = \int_0^l -\frac{F z_1^2}{EJ} dz_1 + \int_0^l -\frac{F z_2^2}{EJ} dz_2 + \int_0^l -\frac{F l^2}{GJ} dz_2 + \int_0^l -\frac{F(l+z_3)^2}{EJ} dz_3 - \int_0^l (l+z_3)\frac{Fl + M_{k\theta}}{GJ} dz_3$$

$$v = -\frac{1}{3}\frac{Fl^3}{EJ} - \frac{1}{3}\frac{Fl^3}{EJ} - \frac{Fl^3}{GJ} - \frac{Fl^3}{EJ} - \frac{Fl^3}{EJ} - \frac{1}{3}\frac{Fl^3}{EJ} - \frac{Fl^3}{GJ} - \frac{M_{k\theta} \cdot l^2}{GJ} - \frac{1}{2}\frac{Fl^3}{GJ} - \frac{1}{2}\frac{M_{k\theta} l^2}{GJ}$$

$$\boxed{v = -Fl^3\left(\frac{3}{EJ} + \frac{5}{2GJ}\right) - M_{k\theta} l^2 \frac{3}{2GJ}}$$

2024-06-18 Ex 2

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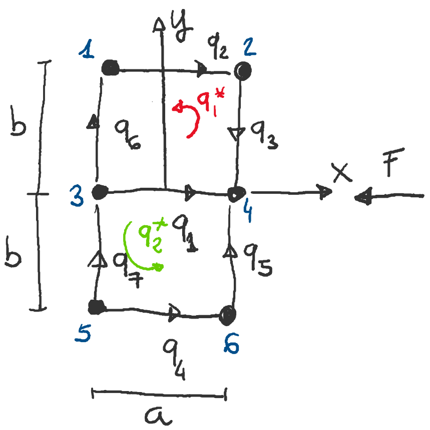
The fact that it is a beam do not enter in the exercise solution



WITH THIS REF. SYST:

$$\boxed{T_x = -F}$$

OPEN 6/7

$$S_{y_1} = S_{y_3} = S_{y_5} = -S_{y_2} = -S_{y_4} = -S_{y_6} = -\frac{1}{2}Aa$$

$$J_{yy} = 6 \cdot A\left(\frac{a}{2}\right)^2 = \frac{3}{2}Aa^2$$

$$q_2' = q_4' = q_1' = -T_x \frac{S_{y_1}}{J_{yy}} = +\frac{1}{3}\frac{T_y}{a} = q'$$

$$q_3' = q_5' = 0 \quad \left(\overset{\text{SINCE}}{S_{y_1} + S_{y_2} = 0}\right)$$

MOM. EQUIV. WRT 3

$$0 = 2ab \cdot q_1^* + 2ab\, q_2^* \rightarrow q_1^* = -q_2^* = q^*$$

COMPATIBILITY

$$\theta_1' = \frac{1}{2Gtab}\left(q_1^* \, 2(a+b) + \cancel{(1-1)q'}\cdot a - q_2^* \cdot a\right)$$

$$\theta_2' = \frac{1}{2Gtab}\left(q_2^* \cdot 2(a+b) + \cancel{(1-1)q'}\cdot a - q_1^* \cdot a\right)$$

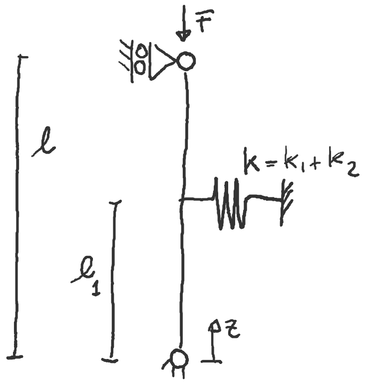
$$\theta_1' = \theta_2' \Rightarrow q_1^* \cancel{(3a+2b)} = q_2^* \cdot \cancel{(3a+2b)}$$

$$\begin{cases} q_1^* = q_2^* \\ q_1^* = -q_2^* \end{cases} \quad q^* = q_1^* = q_2^* = 0$$

$$\boxed{q_1 = q_1' = +\frac{1}{3}\frac{T_y}{a} = -\frac{F}{3a}}$$

2024-06-18 Ex 3

Thursday, June 13, 2024 6:50 PM



<u>SOL</u>

PVW WITH EB BEAM & NL GL STRAIN TENSOR FOR INFINITESIMAL DISPL. IN z & SMALL DISPL IN y

PVW $\delta w \longrightarrow N = -F \leftarrow$ AXIAL EQ.

$$\delta v \rightarrow \delta w_i = \int_0^\ell \delta v_{/zz}\, EJ\, v_{/zz} + \delta v_{/z}\, N\, v_{/z}\, dz$$

$$\delta w_e = -\delta v(\ell_1) \cdot k \cdot v(\ell_1)$$

TAKE $v = Az^2 + Bz + C$

WITH BC: $v(0) = 0$

$v(\ell) = 0$

$$\longrightarrow v = A(z^2 - z\ell) \qquad \delta v = \delta A(z^2 - z\ell)$$

$$v_{/z} = A(2z - \ell) \qquad \delta v_{/z} = \delta A(2z - \ell)$$

$$v_{/zz} = 2A \qquad \delta v_{/zz} = 2\delta A$$

$\rightarrow$ PVW

$$\cancel{\delta A}\int_0^\ell 2EJ\,2A - (2z-\ell)F \cdot A(2z-\ell) =$$

$$-\cancel{\delta A}(\ell_1^2 - \ell_1 \cdot \ell)k \cdot A(\ell_1^2 - \ell_1\ell)$$

TAKE $A \neq 0$

$$4EJ\ell - \int_0^\ell (2z-\ell)^2 F dz = (\ell_1^2 - \ell_1\ell)^2 k$$

$$4EJ\ell - \frac{4}{3}F\ell^3 - F\ell^3 + \frac{4}{2}F\ell^3 = k(\ell_1^2 - \ell_1\ell)^2$$

$$F = \frac{-k(\ell_1^2 - \ell_1\ell)^2 - 4EJ\ell}{2\ell^3 - \ell^3 - \frac{4}{3}\ell^3}$$

2024-06-18 Ex 4

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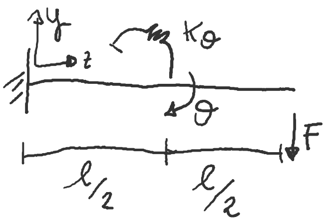

SOL

ASSUME $v(z) = Az^2$ DUE TO BC

$v(0) = v'(0) = 0$

PVW

$$\delta W_i = \int_0^\ell \delta v_{/zz} \cdot EJ \, v_{/zz} \, dz$$

$$\delta W_e = -\delta v(\ell) \cdot F - \delta v_{/z}\left(\frac{\ell}{2}\right) k_\theta \cdot v_{/z}\left(\frac{\ell}{2}\right)$$

$$\rightarrow \cancel{\delta A} \cdot 4EJ\ell A = -\cancel{\delta A}\left(F\ell^2 + \cancel{4} \cdot \frac{\ell}{\cancel{2}} k_\theta \frac{\ell}{\cancel{2}} A\right)$$

$$A = -\frac{F\ell^2}{k\ell^2 + 4EJ\ell} = -\frac{F\ell}{k\ell + EJ4}$$

$$\vartheta = -v_{/z}\left(z = \frac{\ell}{2}\right) = \frac{F\ell^2}{k\ell + EJ4}$$

2024-06-18 Ex 5

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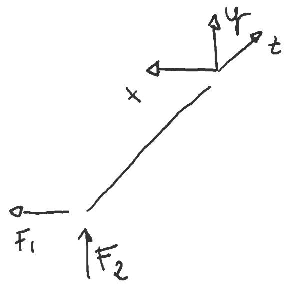

$$M_x = -F_2 \cdot \ell$$

$$M_y = +F_1 \cdot \ell$$

$$J_{xx} = 4Ab^2$$

$$J_{yy} = \frac{3}{2}Aa^2$$

$$\varepsilon_{zz} = \frac{1}{E}\sigma_{zz} =$$

$$= \frac{1}{E}\left(+\frac{M_x}{J_{xx}} \cdot b - \frac{M_y}{J_{yy}} \cdot \frac{a}{2}\right)$$

2024-06-18 Ex 6

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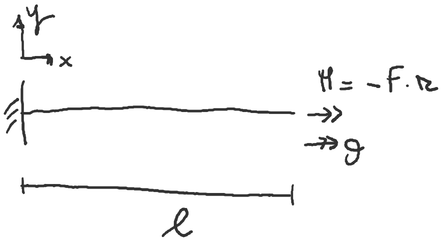

$\vartheta(x) = Ax$

$\vartheta'(x) = A$

PVW

$\rightarrow \delta w_i = \int_0^l \delta\vartheta' GJ \vartheta' dx$

$\delta w_e = \delta\vartheta(l) \cdot M$

$\int_0^l GJA dx = l \cdot (-F \cdot r)$

$GJAl = l \cdot (-F \cdot r)$

$A = -\frac{F \cdot r}{GJ}$ (ALSO CAN BE OBTAINED FROM $M = GJ\vartheta'$)

$\rightarrow \vartheta(l) = -\frac{Fr}{GJ} l$

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. A beam model cannot be used for evaluating local effects due to the application of loads.
   - True
2. YEssential boundary conditions are more important than natural ones.
   - False
3. Shear deformability effects are generally more relevant for thin-walled cross-section beams than for compact cross-section beams.
   - True

Multiple Choice questions

(Circle the correct answer)

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   (a) trigonometric
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