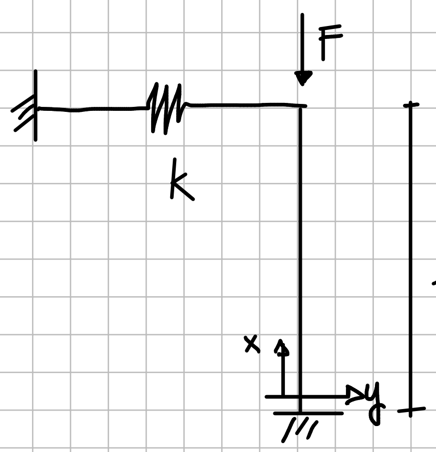
EXERCISE SESSION 13

Ex1 (EXAM 05/07/2023)



DATA

$EA = 1 \cdot 10^8$ N

$EJ = 1 \cdot 10^{12}$ Nmm^2

$l = 3000$ mm

$k = 500$ N/mm

FIND CRITICAL LOAD USING POLY APPROX

SOL INFINITESIMAL DISPL IN X
SMALL DISPL IN y

u_i $u_x = u$ $u_y = v$ $u_z = w$

GL STRAIN $\varepsilon_{ik} = \frac{1}{2}\left(u_{i/k} + u_{k/i} + \underline{u}_{/i} \cdot \underline{u}_{/k}\right)$

$$\varepsilon_{xx} = \frac{1}{2}\left(u_{/x} + u_{/x} + u_{/x}^2 + v_{/x}^2 + w_{/x}^2\right)$$

DISPLACEMENT IN X

$$\varepsilon_{xx} = u_{/x} + \frac{1}{2} v_{/x}^2$$

FOR EB BEAM $\begin{cases} u = u_0 - y \cdot v_{0/x} \\ v = v_0 \end{cases}$ WHERE $u_0 = u_0(x)$ $v_0 = v_0(x)$

$$\varepsilon_{xx} = \underbrace{u_{0/x} - y\, v_{0/xx}}_{\text{FOR INF DISPL}} + \frac{1}{2} v_{0/x}^2$$

FROM NOW ON I WILL LOSE SUBSCRIPT 0

$$\delta W_i = \int_V \delta\varepsilon_{xx} \cdot \sigma_{xx} \, dV$$

WHERE $\delta\varepsilon_{xx} = \delta u_{/x} + \underbrace{\frac{1}{2} \delta v_{/x} \cdot v_{/x} \cdot 2}_{} - y\,\delta v_{/xx}$

$$\delta(v_{/x} \cdot v_{/x}) = \delta v_{/x} \cdot v_{/x} + v_{/x}\,\delta v_{/x}$$

$$\delta W_i = \int_V \left[\left(\delta u_{/x} + \delta v_{/x} \cdot v_{/x}\right)\sigma_{xx} - \delta v_{/xx} \cdot y \cdot \sigma_{xx}\right] dV$$

KNOWN $\int_A \sigma_{xx}\, dA = N \qquad \int_A y\,\sigma_{xx}\, dA = M = -EJ\,v_{/xx}$

$$= \int_\ell \left(\delta u_{/x} + \delta v_{/x} \cdot v_{/x}\right) N + \delta v_{/xx} EJ v_{/xx}\, dx$$

$$\delta W_e = -\delta v(\ell)\, k\, v(\ell) - \delta u(\ell) \cdot F$$

KNOWN $\int_0^\ell \delta u_{/x} \cdot F\, dx = \delta u(\ell) F - \underbrace{\cancel{\delta u(0)}}_{=0} F$

PVW $\delta W_i = \delta W_e$

$$\underbrace{\int_0^\ell \delta u_{/x} (N+F)}_{} + \int_0^\ell \delta v_{/x} N v_{/x} + \delta v_{/xx} EJ v_{/xx}\, dx + \delta v(\ell)\, k\, v(\ell) = 0$$

STATING THE AXIAL EQ.

$N = -F$

$v(x) = C x^2 \qquad \delta v(x) = \delta c\, x^2$

$v_{/x}(x) = 2cx \qquad \delta v_{/x}(x) = \delta c \cdot 2x$

$v_{/xx}(x) = 2c \qquad \delta v_{/xx}(x) = \delta c \cdot 2$

$$\delta c \left[\int_0^\ell \left(2EJ\,2c - 2x F \cdot 2xc\right) dx + \ell^2 k c \ell^2 \right] = 0$$

$$\left[4EJ\ell - \frac{4}{3} F\ell^3 + k\ell^4 \right] c = 0 \longrightarrow C = 0 \quad (\text{NO DEF})$$

$$F = \frac{(4EJ + k\ell^3)\, 3}{4\ell^2} = 1.46 \cdot 10^6\ \text{N}$$

Contents


```
close all
clear variables
home
```

Data

```
EA = 10^8;       % N
EJ = 10^12;      % Nmm^2
l = 3000;        % mm
k = 500;         % N/mm
```

Sol

```
F = (3/l^3/4) * (4*EJ*l + k*l^4)
```

```
F =

   1.4583e+06
```

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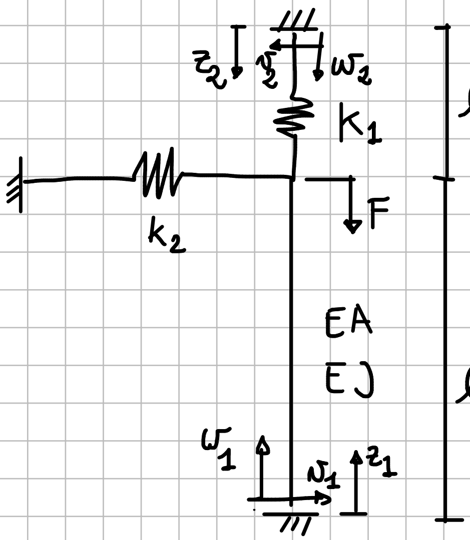
Ex 2 (13/02/2024)



$l = 2000$ mm

$EA = 6 \cdot 10^{10}$ N

$EJ = 12 \cdot 10^{10}$ N mm^2

$k_2 = 1$ N/mm

$k_1 = 1 \cdot 10^{7}$ N/mm

$l_1 = 1000$ mm

FIND CRITICAL LOAD WITH POLY APPROX

SOL TAKE AS KNOW $\delta W_i = \int_0^l \delta w_{/z} N dz + \int_0^l \delta v_{/zz} EJ v_{/zz} + \delta v_{/z} N v_{/z} \, dz$

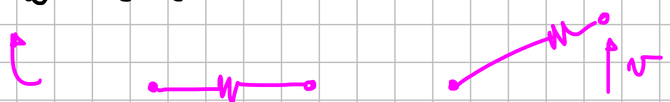
INTERNAL WORK FOR BEAM WITH NON LINEAR GL STRAIN INF z SMALL y

APPROX: TAKE LINEAR $w_1 = a_{w_1} \cdot z_1$ BC $a_{w_1} = -a_{w_2} \cdot \frac{l_1}{l} = a_w$ $a_{w_2} = -a_w \frac{l}{l_1}$

LINEAR $w_2 = a_{w_2} \cdot z_2$

QUADRATIC $v_1 = a_1 z_1^2$ BC $a_1 = -a_2 \frac{l_1}{l^2} = a$ $a_2 = -a \frac{l^2}{l_1}$

LINEAR $v_2 = a_2 z_2$



$$\delta W_i = \int_0^l \delta w_{1/z_1} N dz_1 + \int_0^l \delta v_{1/z_1} N v_{1/z_1} + \delta v_{1/z_1 z_1} EJ v_{1/z_1 z_1} dz$$ ~ BEAM

$$+ \int_0^{l_1} \delta w_{2/z_2} N_{k_1} dz_2 + \int_0^{l_1} \delta v_{2/z_2} N_{k_1} v_{2/z_2} dz_2$$ ~ SPRING k_1

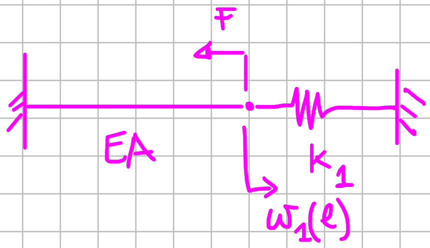
$$\delta W_e = \underbrace{- \delta w_1(l) F}_{\text{EXTERNAL FORCE}} - \underbrace{\delta v_1(l) k v_1(l)}_{\text{SPRING } k_2}$$

KNOWN N $N_{k_1} = k_1 \cdot w_2(l_1) = -k_1 \cdot a_w \cdot l$

$N = EA \, w_{1/z_1} = EA a_w$

- $$\delta a_w \left(\int_0^{\ell} EA\, a_w \, dz_1 + \int_0^{\ell_1} \left(-\frac{\ell}{\ell_1}\right)\left(k_1 \cdot \left(-a_w \cdot \frac{\ell}{\ell_1}\right)\cdot \ell_1\right) dz_2 \right) = -\delta a_w \cdot F \cdot \ell$$ ⇝ FIND a_w

a_w CAN BE FOUND AS



$$F + \frac{EA}{\ell} \cdot w_1(\ell) + k_1 w_1(\ell) = 0$$

$$w_1(\ell) = \underbrace{-\frac{F}{EA+k_1\ell}}_{a_w} \cdot \ell = a_w \cdot \ell$$

- $$\delta a \left(\int_0^{\ell} 2z_1 \cdot 2z_1 a \underbrace{EA \cdot \left(-\frac{F}{EA+k_1\ell}\right)}_{N = EA\, a_w = EA\, w_{/z_1}} + 4EJ a \, dz_1 + \right)$$

$$\hookrightarrow + \int_0^{\ell}\left(-\frac{\ell^2}{\ell_1}\right)\cdot a\left(-\frac{\ell^2}{\ell_1}\right)\cdot \overbrace{\left(-k_1 \cdot a_w \cdot \ell\right)}^{N_{k_1}} dz_2 = -\delta a \cdot \ell^2 k_2 \cdot a \cdot \ell^2$$

SOLVE → $a = 0$

↳ $F_{CRIT} = 244000\ N$ $\quad a \neq 0$

Contents


```
close all
clear variables
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Data

```
l = 2000;
EA = 6*10^10;
EJ = 12*10^10;
k_1 = 10^7;
k_2 = 1;
l_1 = 1000;
```

Sol

```
syms a_w a z_1 z_2 F N

w_1 = a_w*z_1;
w_2 = -a_w*l/l_1*z_2;

v_1 = a*z_1^2;
v_2 = -a*l^2/l_1^2*z_2;
```

PVW 1

```
a_w_r = solve(int(EA*a_w, z_1, 0, l) + int((-l/l_1)*k_1*subs(w_2, z_2, l_1), z_2, 0, l_1) == -F*l, a_w);

simplify(int(2*z_1*2*a*z_1*EA*a_w_r + 2*EJ*2*a, z_1, 0, l)  == -l^2*k_2*a*l^2 - int((-l^2/l_1)*a*(-l^2/l_1)*(-k_1*a_w_r*l), z_2, 0, l_1))
```

```
ans =

F == 244000 | a == 0
```

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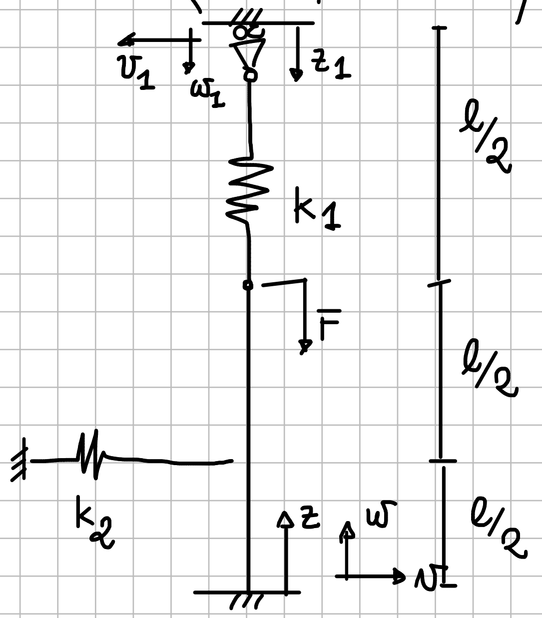
Ex 3 (EXAM 09/09/2024)



DATA

$l = 1000$ mm

$GA^* = 1 \cdot 10^{10}$ N

$EJ = 12 \cdot 10^9$ Nmm^2

$EA = 1 \cdot 10^{10}$ N

$k_1 = 1.5 \cdot 10^7$ N/mm

$k_2 = 2500$ N/mm

FIND CRIT LOAD

POLY APPROX

SOL APPROX

$$\begin{cases} \text{LINEAR} & w_1(z_1) = a_{w_1} \cdot z_1 \\ \text{LINEAR} & w(z) = a_w \cdot z \\ \text{QUADR} & v(z) = a z^2 \\ \text{CONST} & v_1(z) = a_1 \end{cases}$$

$$BC \rightsquigarrow \begin{cases} w_1(l/2) = -w(l) & \rightsquigarrow \quad a_{w_1} = -2a_w \\ v_1(l/2) = -v(l) & \rightsquigarrow \quad a_1 = -a \cdot l^2 \end{cases}$$

$$\delta W_i = \int_0^l \delta w_{/z} N \, dz + \int_0^{l/2} \delta w_{1/z_1} N_{k_1} \, dz_1 + \int_0^l \left(\delta v_{/zz} EJ v_{/zz} + \delta v_{/z} N v_{/z} \right) dz$$

$$\delta W_e = - \delta w(l) \cdot F - \delta v\left(\frac{l}{2}\right) k_2 v\left(\frac{l}{2}\right)$$

KNOWN $N = EA \cdot a_w$ $\quad N_{k_1} = k_1 \cdot w_1\left(\frac{l}{2}\right) = -k_1 a_w \cdot l$

$$a_w = -\frac{F}{EA + k_1 l} \quad \rightsquigarrow \text{SAME AS Ex 2}$$

$$\delta a \cdot \int_0^l \left(4EJa + 4az^2 \cdot N \right) dz = -\delta a \frac{l^2}{4} k_2 a \frac{l^2}{4}$$

$$N = EA \cdot \left(-\frac{F}{EA + k_1 l} \right)$$

SOLVE THIS FOR $a \neq 0$ $\rightsquigarrow F = F_{CRIT} = 3.830 \cdot 10^5$ N

```
close all
clear variables
home

l = 1000;
EJ = 12E+09;
EA = 1E+10;
k_1 = 1.5E+7;
k_2 = 2500;

syms F a_w z z_1
LHS = [
    int(EA*a_w, z, 0, l) + int(2*k_1*a_w*l, z_1, 0, l/2) + l*F;
    int(4*EJ + 4*EA*a_w*z^2, z, 0, l) + l^2/4*l^2/4*k_2
    ];
RHS = [0; 0];
[a_w, F] = solve(LHS == RHS, [a_w, F]);
F = double(F)
```

```
F =

   3.8297e+05
```

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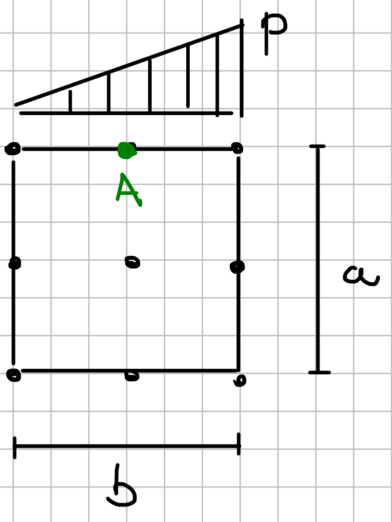
Ex 4 (EXAM 29/08/2022)



DATA

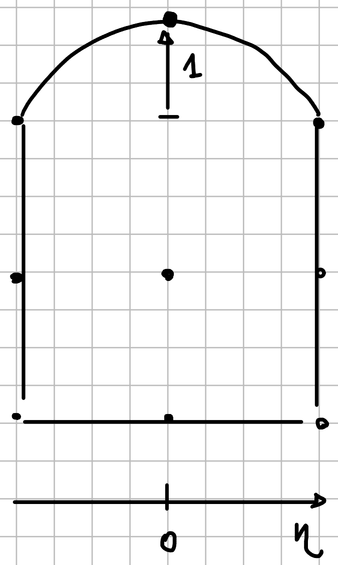
$a = 2\,mm$

$b = 3\,mm$

$p = 10\,N/mm$

FIND THE VIRTUAL WORK OF THE FORCE FOR A UNITARY VIRTUAL DISPL. OF A IN THE VERTICAL DIRECTION

SOL PARABOLIC ELEMENT



DEFORMED SHAPE FOR UNITARY DISPL v_A

$$v(\eta) = \left(-\frac{4\eta^2}{b^2} + 1\right) v_A$$

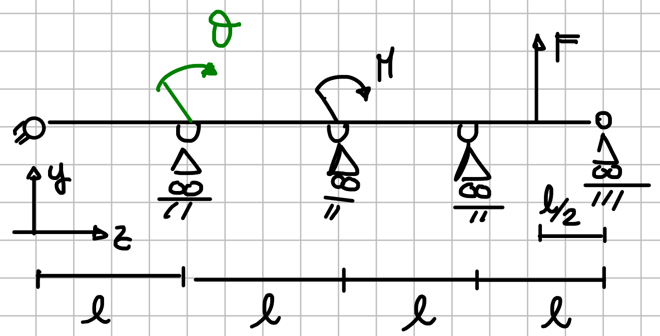
$$f(\eta) = \frac{p}{b}\cdot\eta + \frac{p}{2}$$

$$\delta W_e = \int_{-\frac{b}{2}}^{b/2} \delta v \cdot f \, d\eta$$

$$= \int_{-b/2}^{b/2} \delta v_A \cdot \left(-\frac{4\eta^2}{b^2} + 1\right)\cdot\left(\frac{p}{b}\eta + \frac{p}{2}\right) d\eta$$

$$= \delta v_A \cdot \boxed{\frac{1}{3} b p}$$

Ex 5 (EXAM 23/08/2022)



DATA

$l = 1000$ mm

$EJ = 6 \cdot 10^{8}$ Nmm^2

$M = 4 \cdot 10^{5}$ Nmm

$F = 2000$ N

FIND ϑ WITH TRIG. APPROX WITH 1 TERM

SOL $\quad v(z) = C \cdot \sin\left(\frac{\pi z}{l}\right)$

$$\delta W_i = \int_0^{4l} \delta v_{/zz} \, EJ \, v_{/zz} \, dz$$

$$= \delta c \; EJ \frac{\pi^4}{l^4} \cdot c \int_0^{4l} \sin^2\left(\frac{\pi z}{l}\right) dz = \delta c \cdot EJ \cdot c \cdot \frac{2\pi^4}{l^3}$$

$$\delta W_e = -\delta v_{/z}(2l) \cdot M + \delta v\left(\frac{7}{2}l\right) \cdot F$$

$$= \delta c \left(-\frac{\pi}{l} \cdot M - F\right)$$

PVW $\quad \delta W_i = \delta W_e \longrightarrow C = -\frac{F + \pi/l \cdot M}{2\pi^4/l^3 \, EJ}$

$$\vartheta = -v_{/z}(l) = +C \cdot \frac{\pi}{l} = -\frac{F + \pi/l \, M}{2\pi^4/l^3 \, EJ} \cdot \frac{\pi}{l}$$