Course of Aerospace Structures

Written test, Jan 25$^{\text{th}}$, 2023

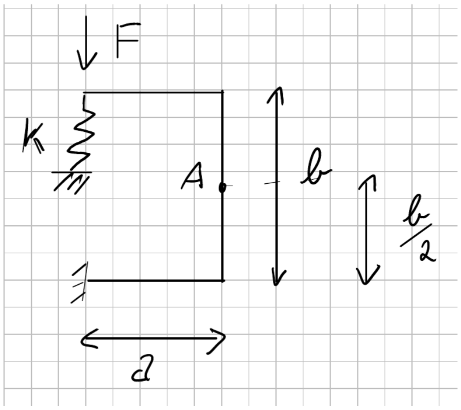
Name ______________________________

Surname ______________________________

Person code: | | | | | | | | |

Exercise 1

The beam sketched in the figure, with bending stiffness EI, is loaded by the concentrated force F.
Compute the bending moment at point A.
(Unit for result: N mm)

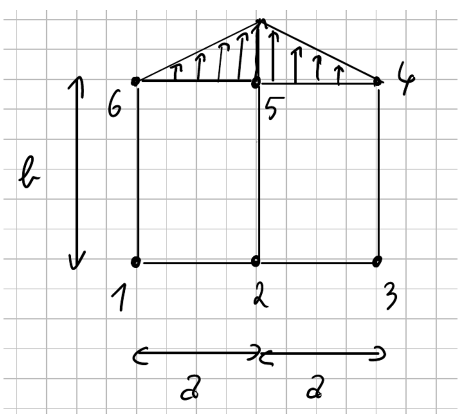


Data
$a = 400$ mm
$b = 800$ mm
$EA = 1.0 \times 10^5$ N
$EI = 6.0 \times 10^{10}$ Nmm^2
$K = 100$ N/mm
$F = 5000$ N

Answer ______________________________

Exercise 2

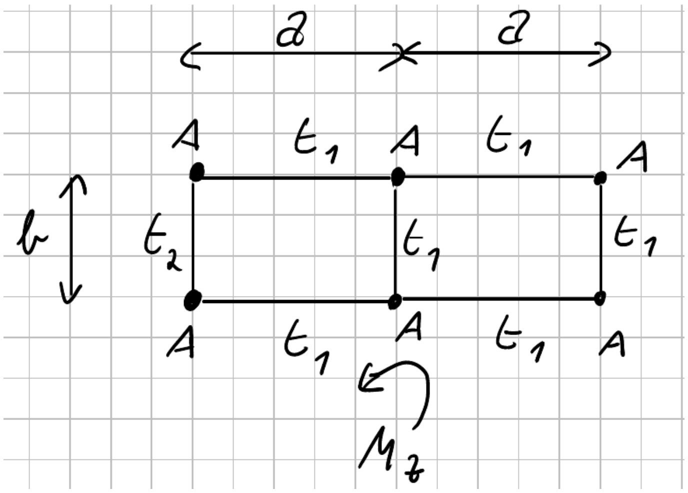
The FE mesh sketched in the figure has two bilinear planar element and ~~eight~~ nodes. It is loaded by the distributed load sketched in the figure, with a maximum load per unit of length equal to $\overline{f}$.
Compute the virtual external work for the unitary virtual vertical displacement of node 5.
(Unit for result: N mm)



Data
$a = 4$ mm
$b = 6$ mm
$\overline{f} = 20$ N/mm

Answer ______________________________

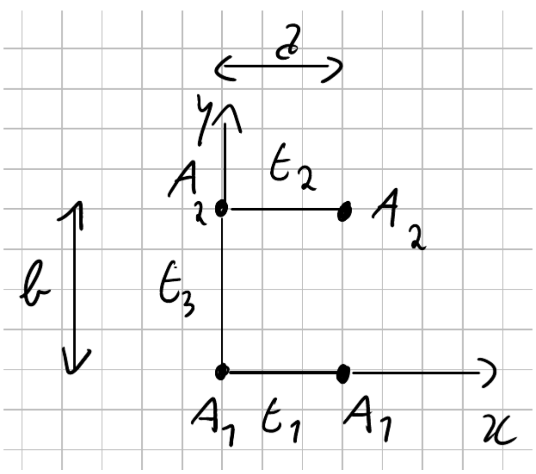
Exercise 3
The semi-monocoque beam cross section model in the figure is loaded by a concentrated moment M_z. It has six stringers, each with concentrated area A. Compute the derivative of the cross section rotations angle with respect to the beam axis.
(Unit for result: rad/mm)



Data
$a = 600$ mm
$b = 300$ mm
$A = 500$ mm^2
$t_1 = 1$ mm
$t_2 = 2$ mm
$E = 70000$ MPa
$\nu = 0.3$
$M_z = 5 \times 10^6$ Nmm

Answer ____________________

Exercise 4
Consider the semi-monocoque cross section model sketched in the figure.
Compute the x positions of the shear center.
(Unit for result: mm)



Data
$a = 400$ mm
$b = 800$ mm
$t_1 = 1$ mm
$t_2 = 2$ mm
$t_3 = 3$ mm
$A_1 = 500$ mm^2
$A_2 = 1000$ mm^2
$E = 70000$ MPa
$\nu = 0.3$

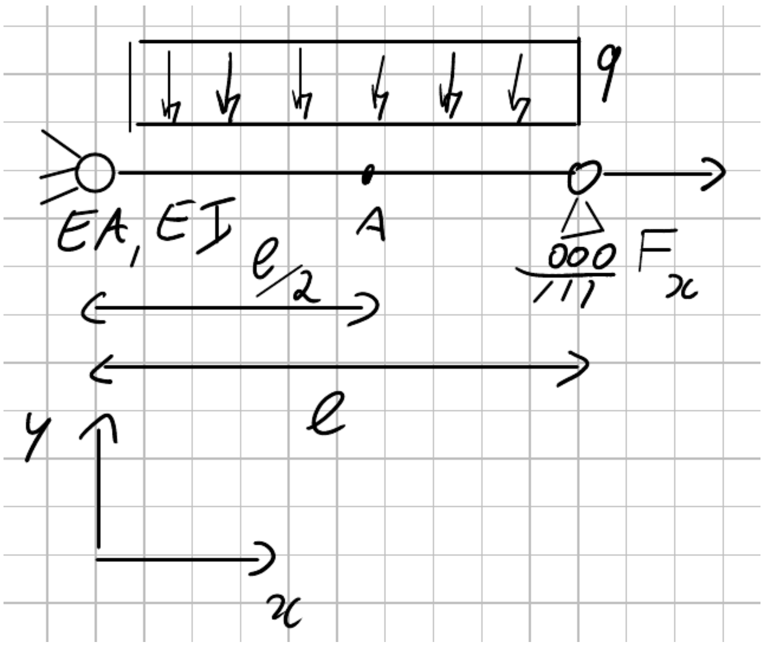
Answer ____________________

Exercise 5

The beam sketched in the figure is loaded by the concentrated force F at $x = l$ and by the distributed force for unit of length q.

By resorting to the displacement method, and using a trigonometric approximation with only one term, estimate the vertical displacement in the y direction of point A (the point at $x = l/2$ from the hinge). Account for the axial pre-stress.

(Unit for result: mm)



Data

$q = 1.2$ N/mm

$F_x = 10000$ N

$EA = 1 \times 10^8$ N

$EI = 1 \times 10^{11}$ Nmm^2

$l = 4000$ mm

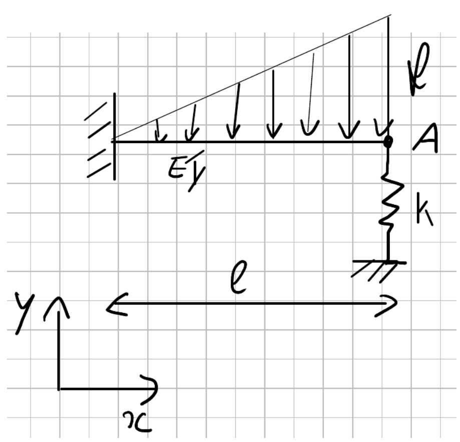
Answer ________________________

Exercise 6

The beam sketched in the figure is loaded by a triangular force per unit of length, going from 0 N/mm to f N/mm.

By resorting to the displacement method, and using a polynomial approximation with only one term, compute the vertical displacement in the y direction of point A.

(Unit for result: mm)



Data

$f = 14$ N/mm

$l = 2000$ mm

$EI = 1 \times 10^{10}$ Nmm^2

$K = 100$ N/mm

Answer ________________________

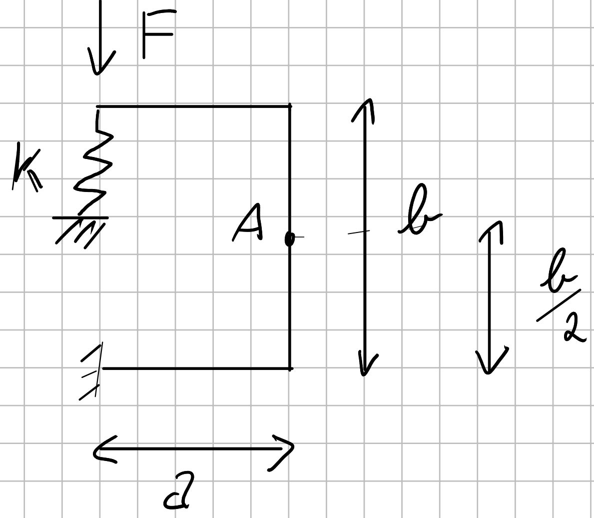
True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. a beam is loaded with a shear force; the torsional moment is equal to the value of the force multiplied by the distance between the force line of action and the barycenter:
2. the bending moment of over-constrained beam structures cannot be computed by resorting to the PVW:
3. the axial stress of a beam transmitting a given bending moment M_x is function of the beam material elasic modulus E:

Multiple Choice questions
(Circle the correct answer)

1. "Differential bending" is related to:
   (a) the different bending behavior of beams around the principal axis x and y
   (b) torsional stiffness
   (c) interaction between axial and bending stiffness of a beam
   (d) the derivative of the axial stress in the panel of a thin-walled cross-section
   (e) none of the above
2. The PCVW:
   (a) cannot be applied to staticaly determined systems
   (b) can be applied to statically determined systems, but only in order to compute the reaction forces and moments
   (c) can be applied to statically determined systems only in order to compute the displacemnt and/or the rotation of a given point
   (d) is equivalent to the PVW for statically determined systems
   (e) none of the above
3. Hermitian shape functions:
   (a) are used to approximate the torsional moment using the Ritz method
   (b) are required in order to build Euler-Bernoulli beam FEs
   (c) are special C^2 shape functions required to build high-performance beam FEs
   (d) need to be avoided because they reduce the order of convergence
   (e) are useless
   (f) none of the above

1



$a = 400\ mm$

$b = 800\ mm$

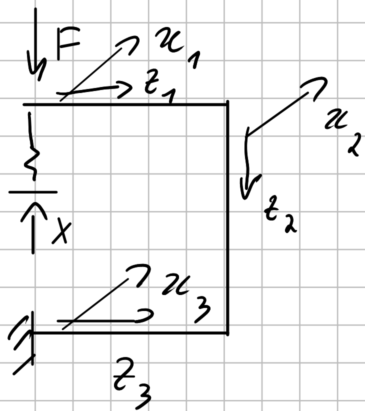
$EA = 1E5\ N$

$EI = 6E10\ N\,mm^4$

$k = 100\ N/mm$

$F = 5000\ N$

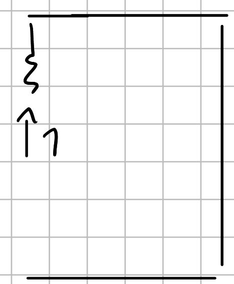
$M_f(A) = ?$



$M_{x_1} = (F - \cancel{X})\, z_1$

$M_{x_2} = (F - X)\, a$

$M_{x_3} = (F - X)\, z_3$



$M'_{x_1} = -z_1$

$M'_{x_2} = -a$

$M'_{x_3} = -z_3$

$$\int_0^b \frac{(-F+X)}{EA}\,dx + \int_0^a \frac{(-F+X)\, z_1^2}{EI} + \frac{(X-F)\, a^2 b}{EI} + \int_0^a \frac{(-F+X)\, z_3^2}{EI} + \frac{X}{k} = \cancel{0}$$

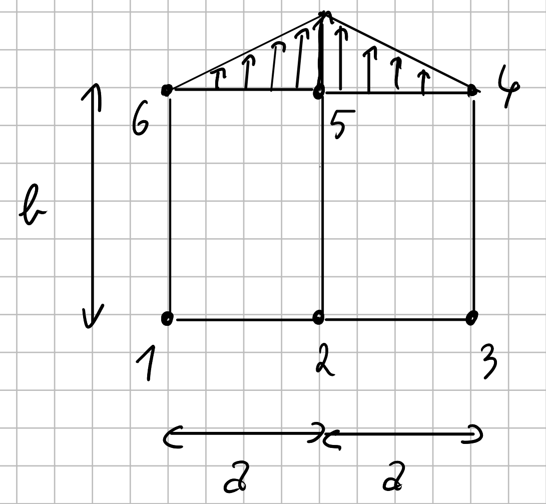
$$-\frac{EI\, Fb}{EA} + \frac{X}{EA} - \frac{1}{3} a^3 F + \frac{1}{3} a^3 X + a^2 b\, X - a^2 b\, F - \frac{1}{3} a^3 F + \frac{1}{3} a^3 X + \frac{EI}{K} X = 0$$

$$\left(\frac{EI\,h}{EA}+\frac{2}{3}a^3+a^2h+\frac{EI}{k}\right)X=\left(\frac{2}{3}a^3+a^2h+\frac{hEI}{EA}\right)F$$

$$X=\left(\frac{2}{3}a^3+a^2h+\frac{hEI}{EA}\right)\Big/\left(\frac{hEI}{EA}+\frac{2}{3}a^3+\frac{EI}{k}\right)F$$

$$M=(F-X)\,a=9{,}59\,E5\ \text{Nmm}$$

2

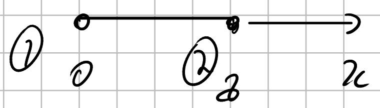


$a = 4 \text{ mm}$

$b = 6 \text{ mm}$

$\bar{p} = 20 \text{ N/mm}$

$\delta \mathcal{L}_e (\text{node } 5) = ?$



$$N_1 = 1 - \frac{x}{a}$$

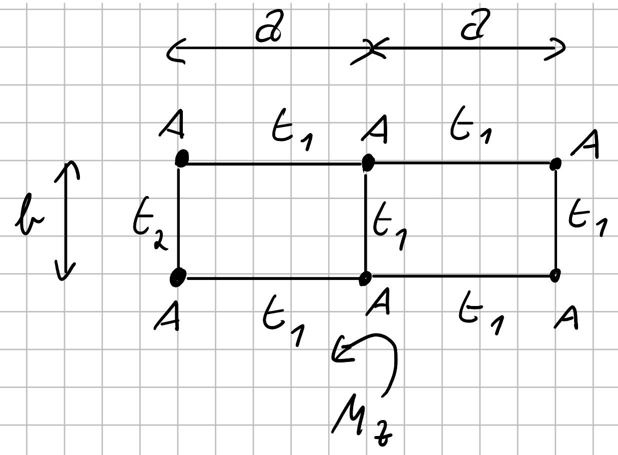
$$N_2 = \frac{x}{a}$$

$$\delta u = N_1 \delta u_1 + N_2 \delta u_2$$

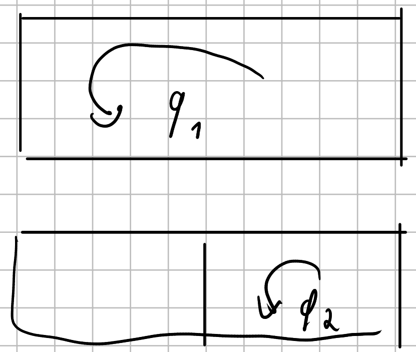
$$\int_0^a \delta u_2 \, N_2(x) p(x) \, dx = \delta u \int_0^a \frac{x}{a} \cdot \frac{\bar{p}}{a} \cdot x \, dx = \frac{1}{3} a \bar{p}$$

$$\delta \mathcal{L}_i (\text{node } 5) = \frac{2}{3} a \bar{p} \quad \text{N mm}$$

3



$a = 600$ mm
$b = 300$ mm
$A = 500$ mm^2
$t_1 = 1$ mm
$t_2 = 2$ mm
$E = 70000$ MPa
$\nu = 0{,}3$
$M_z = 5E6$ N mm
$\dot{\vartheta} = ?$



$$\dot{\vartheta}_1 = \frac{1}{4abG}\left(\frac{4a+b}{t_1} + \frac{b}{t_2}\right) q_1 + \frac{1}{4abG}\left(\frac{2a+2b}{t_1}\right) q_2$$

$$\dot{\vartheta}_2 = \frac{1}{2abG}\left(\frac{2a+b}{t_1}\right) q_1 + \frac{1}{2abG}\left(\frac{2a+2b}{t_1}\right) q_2$$

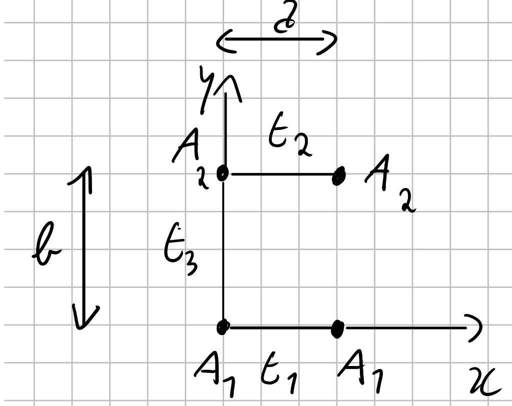
$$\begin{cases} \dot{\vartheta}_1 - \dot{\vartheta}_2 = 0 \\ 4 \partial b\, q_1 + 2 \partial b\, q_2 = M_z \end{cases}$$

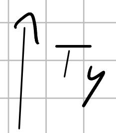
$q_1 = 7{,}25 \; N/mm$

$q_2 = -0{,}6 \; N/mm$

$\dot{\Theta} = 1E-6 \; rad/mm$

④





$a = 400$ mm
$b = 800$ mm
$t_1 = 1$ mm
$t_2 = 2$ mm
$A_1 = 500$ mm^2
$A_2 = 1000$ mm^2
$E = 70000$ MPa
$\nu = 0{,}3$

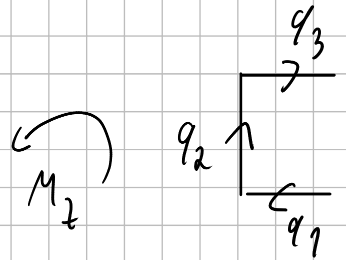
x coord of shear center

$$x_{CG} = \frac{a}{2}$$

$$y_{CG} : \quad \frac{A_2 \cdot (b - y_{CG}) - A_1 (y_{CG})}{A_1 + A_2} = 0$$

$$y_{CG} = \frac{b\, A_2}{A_1 + A_2}$$

$$I_{xx} = 2 A_2 (b - y_{CG})^2 + 2 A_1 y_{CG}^2$$



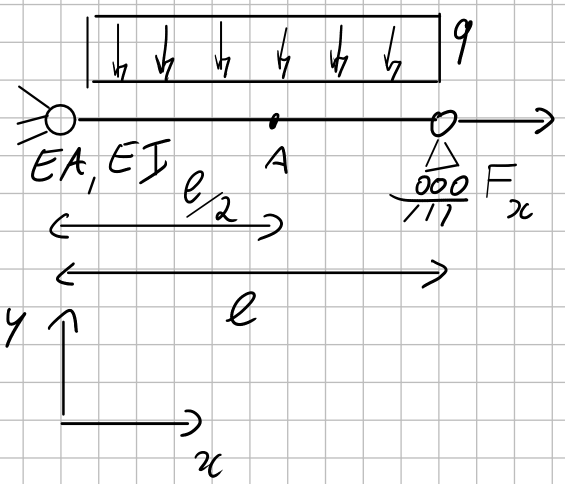
$$q_2 = \frac{T_y}{b}$$

$$q_1 = \frac{-T_y\, S'_x}{I_{xx}} \qquad S'_x = -A_1\, y_{CG}$$

$$q_3 = q_1$$

$$-q_3 \cdot b \cdot d = T_y \cdot x_{sc} \qquad x_{sc} = -\frac{q_3\, b}{T_y} = -200 \text{ mm}$$

(5)



$q = 1{,}2$ N/mm

$F_x = 10000$ N

$EA = 1E8$ N

$EI = 1E11$ N mm^2

$l = 4000$ mm

vertical displacement in y direction of point A, located at $l/2$

use a 1-term trigonometric approximation of the transverse displacement and the PVW

$$\int_0^l \left(\delta w'' EI\, w'' + \delta w' w' F_x\right) dx = \int_0^l -\delta w\, q\, dx$$

$$w = a \sin\left(\frac{\pi x}{l}\right)$$

$$w' = \frac{\pi}{l} a \cos\left(\frac{\pi x}{l}\right)$$

$$w'' = -\frac{\pi^2}{l^2} w$$

$$\delta a \left(\frac{\pi}{l}\right)^4 EI \frac{1}{2} l\, a + \delta a \left(\frac{\pi}{l}\right)^2 \frac{1}{2} l\, F_x\, a = -\delta a \frac{l}{\pi} \cos\left(\frac{\pi x}{l}\right)\Big|_0^l \, q$$

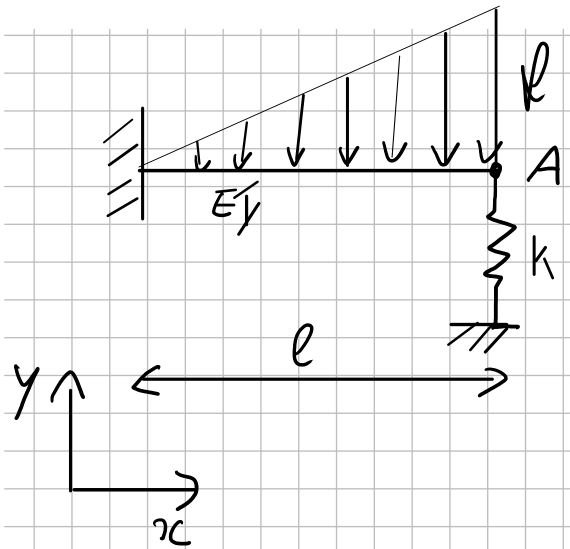
$$= -\delta a \frac{l}{\pi} 2 q$$

$$\left[\left(\frac{\pi}{l}\right)^4 EI \frac{1}{2} l + \left(\frac{\pi}{l}\right)^2 F_x \frac{1}{2} l\right] a = -2 q \frac{l}{\pi}$$

$$a = \frac{2 q \frac{l}{\pi}}{[\quad]} = -34{,}55 \text{ mm}$$

$$w\left(\frac{l}{2}\right) = a = -34{,}55 \text{ mm}$$

6



$p = 14 \ N/mm$

$l = 2000 \ mm$

$EJ = 1E10 \ N \ mm^2$

$k = 100 \ N/mm$

$v(z=l) = ?$

$$v = a z^2 \qquad v'' = 2a$$

$$\delta a \int_0^l 4 EJ \, dz \, a + \delta a \, l^4 k a = \delta a \int_0^l -\frac{p}{l} z z^2 \, dz \, a$$

$$\delta a \left(4 EJ \, l + k l^4 \right) a = \delta a \left(-\frac{1}{4} \frac{p}{\cancel{l}} l^{\cancel{4}\,3} \right) a$$

$$a = \frac{-\frac{1}{4} p l^3}{4 EJ \, l + k l^4}$$

$$v = a l^2 = -66,67 \ mm$$

- beam is loaded with a shear force; the torsional moment is equal to the value of the force multiplied by the distance between the force line of action and the barycenter
  - False
- the bending moment of over-constrained beam structures cannot be computed by resorting to the PVW
  - False
- the axial stress of a beam transmitting a given bending moment M_x is function of the beam material elastic modulus E
  - False
- "Differential bending" is related to:
  - the different bending behavior of beams around the principal axis x and y
  - torsional stiffness
  - interaction between axial and bending stiffness of a beam
  - the derivative of the axial stress in the panel of a thin-walled cross-section
  - none of the above
- The PCVW:
  - cannot be applied to staticaly determined systems
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  - is equivalent to the PVW for statically determined systems
  - none of the above
- Hermitian shape functions:
  - are used to approximate the torsional moment using the Ritz method
  - are required in order to build Euler-Bernoulli beam FEs
  - are special C^2 shape functions required to build high-performance beam FEs
  - need to be avoided because they reduce the order of convergence
  - are useless
  - none of the above