Course of Aerospace Structures

Written test, July 5^{th}, 2023

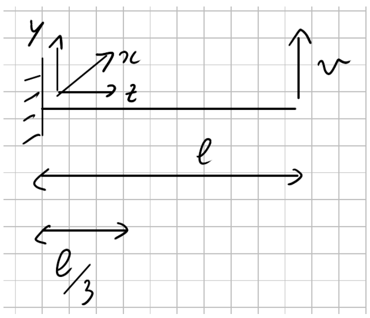
Name ______________________________

Surname ______________________________

Person code: | | | | | | | | |

Exercise 1

The beam sketched in the figure is clamped at one end. A displacement v is prescribed at the other end, still leaving the beam extremity free to rotate. This is (of course) accomplished by applying a force F at the extremity whose actual value of the force is not given (only the prescribed displacement v is known). Compute the beam bending moment M_{xx} at $z = l/3$ (from the clamp). Neglect shear deformation.
(Unit for result: N mm)

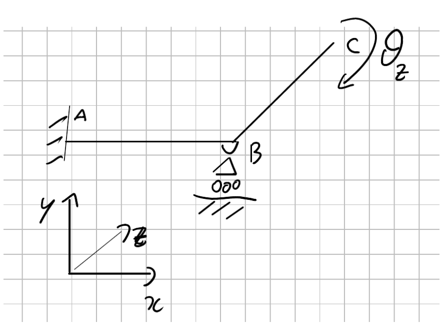


Data
$l = 3000$ mm
$v = 5$ mm
$EI_{xx} = EI_{yy} = 1 \times 10^{12}$ N mm^2
$EA = 1 \times 10^4$ N

Answer ______________________________

Exercise 2

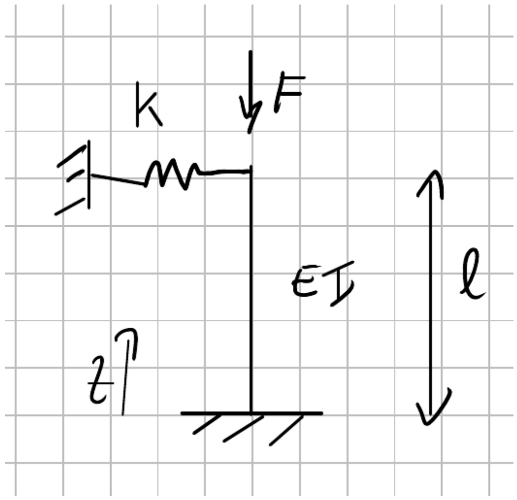
The beam structure sketched in the figure has a prescribed rotation θ_z, around the z axis, at point C (a moment is applied there, leaving free the three displacement components and the other two rotation components).
Compute the reaction force in the y direction transmitted by the constraint of point B.
Neglect shear deformation. The coordinates of the points are given, with respect to the sketched reference system, in the data. The bending and torsional stinness of the beams are given with respect to local reference systems, with the z axis aligned with the beam axis. Be careful with the measurement units.
(Unit for result: N)



Data
$A : (0; 1000; 0)$ mm
$B : (1000; 1000; 0)$ mm
$C : (1000; 1000; 1000)$ mm
$\theta_z = 5^{\text{o}}$
$EI_{xx} = EI_{yy} = 1 \times 10^{10}$ N mm^2
$GJ = 2 \times 10^{11}$ N mm^2

Answer ______________________________

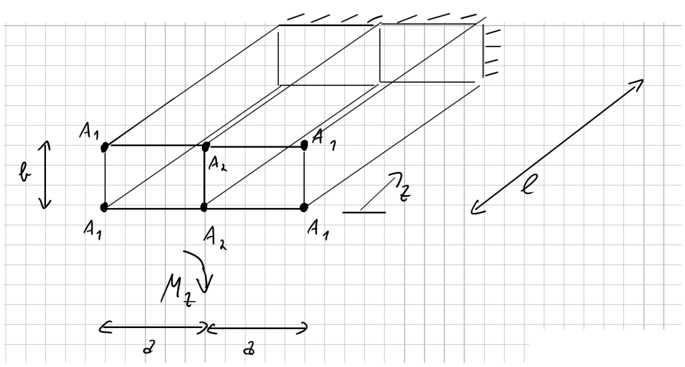
Exercise 3
Approximate the critical buckling load F of the structure sketched in the figure by resorting to a polynomial approximation of the beam transverse displacement truncated to one unknown coefficient.
The stiffness of the linear spring attached at the beam extremity is equal to k. Neglect shear deformation.
(Unit for result: N)



Data
$EA = 1 \times 10^8$ N
$l = 3$ m
$EI_{xx} = EI_{yy} = 1{\times}10^{12}$ N mm^2
$k = 500$ N/mm

Answer

Exercise 4
Consider the 3-D semi-monocoque beam model sketched in the figure, loaded at its extremity by the torsional moment M_z. All the panels have thickness equal to t.
Compute torsional rotation angle θ_z of the cross-section at $z = \frac{2}{3}l$ from the free extremity.
(Unit for result: rad)



Data
$t = 1$ mm
$E = 70000$ MPa
$\nu = 0.3$
$a = 1000$ mm
$b = 500$ mm
$l = 6000$ mm
$A_1 = 500$ mm^2
$A_2 = 1000$ mm^2
$M_z = 1 \times 10^9$ N mm

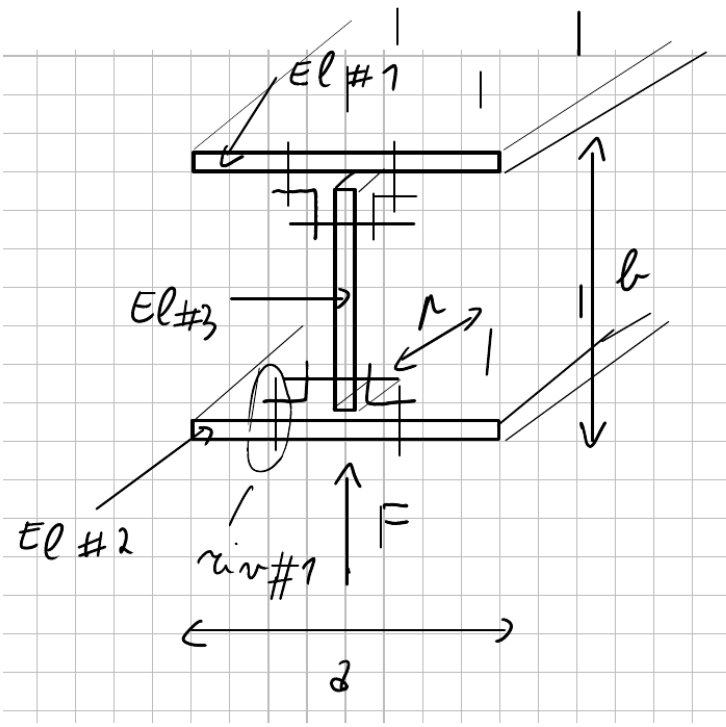
Answer

Exercise 5

Consider the three-dimensional beam sketched in the figure, and loaded by the vertical force F, applied along one of the two axis of symmetry of the cross-section. The two horizontal plates have a cross-section area equal to $A(El\#1)$ and $A(El\#2)$, respectively and are placed at a distance equal to b. The vertical plate has a cross-section area equal to $A(El\#3)$. The overall barycenter moment of inertia of the beam cross section with respect to the horizontal axis is equal to I_{xx}. The horizontal plates are connected to the vertical one by means of four L-shaped stringers (with negligible cross-section area), placed at the two sides of the vertical plate, as sketched in the figure. One solid rivet for each stringer connects the stringer to the horizontal plates. The vertical plate is connected to the two stringers in the upper position by a single rivet; a single rivet is used as well in order to connect the vertical plate to the two stringers in the lower position. All the rivet connections are repeated along the beam axis with a pitch equal to p. The rivets shank diameter is equal to ϕ.

Compute the average shear stress τ in the shank of rivet #1 ($riv\#1$ in the figure).

(Unit for result: MPa)



Data

I_{xx}(barycentric) $= 1.5 \times 10^8$ mm^4

$A(El\#1) = 5000$ mm^2

$A(El\#2) = 5000$ mm^2

$A(El\#3) = 3500$ mm^2

$a = 500$ mm

$b = 240$ mm

$p = 100$ mm

$\phi = 7$ mm

$F = 10000$ N

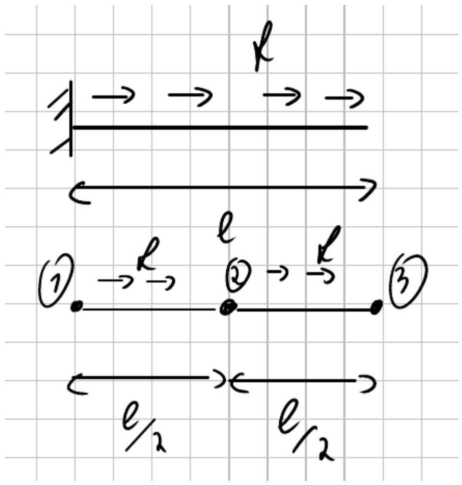
Answer ______________________________

Exercise 6

The clamped beam in the figure, with lenght l, is loaded with a constant distributed axial force per unit of lenght f. The beam response is approximated by means of a finite element model with two linear elements and three nodes, as sketched at the bottom of the figure.

Compute the horizontal displacement of node #3 at the free extremity of the beam. Be careful with the measurement units.

(Unit for result: mm)



Data
$l = 3$ m
$EA = 7 \times 10^7$ N
$f(x) = 10$ N/mm

Answer

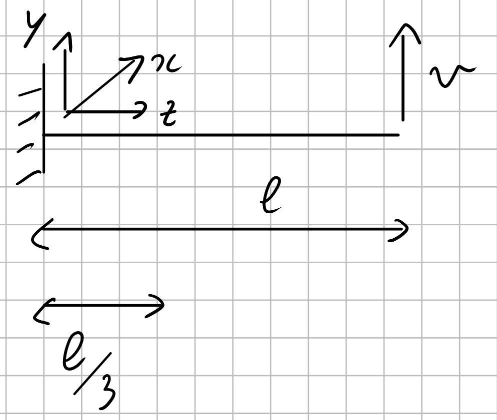
True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. Any structure with one dimension much larger than the other two can likely be modeled as a beam:
2. According to the Timoshenko beam model, the transverse shear stresses are linear on the cross section:
3. The position of the shear center of a closed-cell section can be evaluated using the shear flow equations and the equivalence to internal moment:

Multiple Choice questions
(Circle the correct answer)

1. A system of slender beams can be modeled by beams finite elements:
   (a) never
   (b) if the structure can sustain the loads through an internal axial load path
   (c) only if shear deformability is not negligible
   (d) always
   (e) none of the above
2. An Euler-Bernoulli cantilever beam with uniform stiffness is clamped at one extremity and loaded with a concentrated force at the tip. The solution obtained using a displacement-based method based on polynomial functions with two unknown coefficients is:
   (a) exact
   (b) an approximation of the exact solution with errors below 10 %
   (c) an approximation of the exact solution with errors depending on the problem data
   (d) none of the above
3. The PCVW allows to:
   (a) find the compatible solution among the equilibrated ones
   (b) find the equilibrated solution among the compatible ones
   (c) find the compatible and equilibrated solutions among all the possible independent stress and displacement fields
   (d) none of the above

(1)



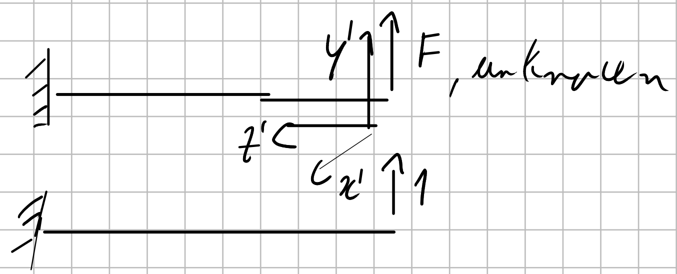
Prescribed displacement u at the free end

$l = 3000$ mm

$EJ = 1E12$ N mm^2

$v = 5$ mm

Compute the bending moment M_{xx} at $z = \frac{l}{3}$ from the clamp



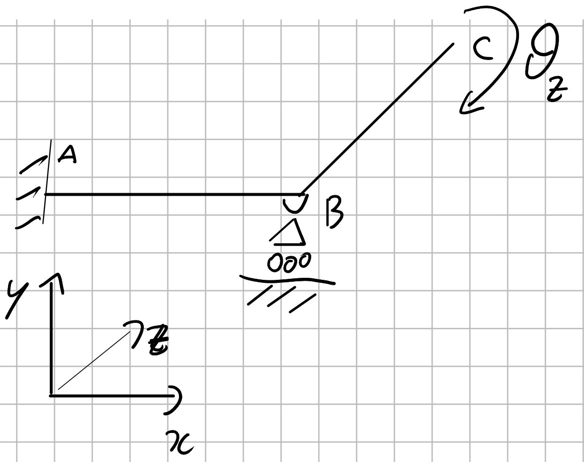
$$M_{x'x'} = -F z$$

$$M'_{x'x'} = -z$$

$$\int_0^l \frac{F z^2}{EJ} dz' = u \qquad \frac{F l^3}{3EJ} = v \quad \Rightarrow \quad F = \frac{3 v EJ}{l^3} = 555,6 \text{ N}$$

$$M_{xx} = -F \cdot \frac{2}{3} l = -1,11 E6 \text{ N mm}$$

(2)



A : (0, 1000, 0) mm

B : (1000, 1000, 0) mm

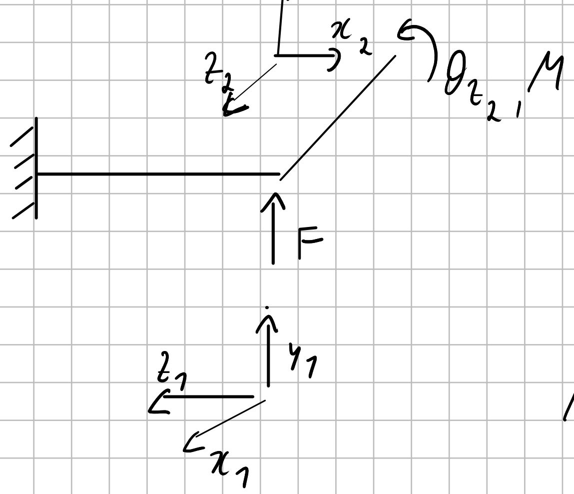
C : (1000, 1000, 1000) mm

prescribed $\theta_z = 5^0$

$EI_{xx} = EI_{yy} = 1E10$ N mm^2

$GJ = 2E11$ N mm^2

compute reaction force at point B



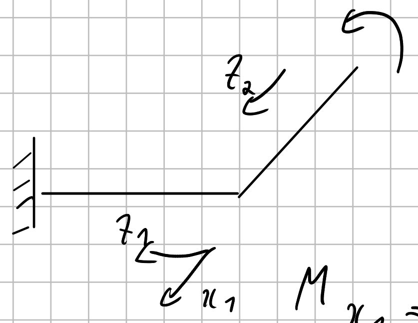
$$\theta_z = 5^0 = \frac{5}{180} \cdot \pi \text{ rad} \simeq 0{,}08727 \text{ rad}$$

$$\theta_{z_2}^{(0)} = -\theta_z = -0{,}08727 \text{ rad}$$

$M_{z_2} = -M$, where M is unknown

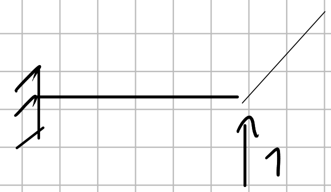
F unknown

$M_{x_1} = -M - F z_1$



$M_{z_2} = -1$

$M_{x_1} = -1$



$M_{x_1} = -z_1$

$\ell = 1000$ mm

$$\begin{cases} \displaystyle\int_0^{\ell} \frac{M}{GJ}\, dz_2 + \int_0^{\ell} \frac{M + F z_1}{E J_{xx}}\, dz_1 = -0{,}08727 \\ \displaystyle\int_0^{\ell} \frac{M z_1 + F \cdot z_1^2}{E J_{xx}}\, dz_1 = 0 \end{cases}$$

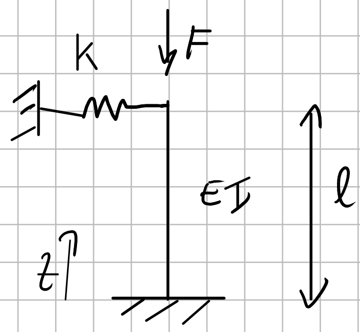
$$\begin{cases} \displaystyle\frac{M}{GJ} \cdot \ell + \frac{M}{E J_{xx}} \ell + F \frac{\ell^2}{2 E J_{xx}} = -0{,}08727 \\ \displaystyle M \frac{\ell^2}{2 E J_{xx}} + F \frac{\ell^3}{3 E J_{xx}} = 0 \end{cases}$$

$$\begin{bmatrix} \frac{l}{GJ} + \frac{l}{EJ_{xx}} & \frac{l^2}{2EJ_{xx}} \\ \frac{l^2}{2EJ_{xx}} & \frac{l^3}{3EJ_{xx}} \end{bmatrix} \begin{Bmatrix} M \\ F \end{Bmatrix} = \begin{Bmatrix} -0{,}087 27 \\ 0 \end{Bmatrix}$$

$M = -2{,}909\,E6 \ N\,mm$

$F = 4{,}364\,E3 \ N$

③



Approximate the critical buckling load F by resorting to a polynomial approximation of the transverse displacement with only one unknown coefficient

$EA = 1E8$ N
$l = 3$ m
$EJ = 1E12$ N mm^2
$k = 500$ N/mm

$$v = a z^2$$

$$\int_0^l \left(\delta v'' EI v'' - F \delta v' v' \right) dz + \delta v(l)\, k\, v(l) = 0$$

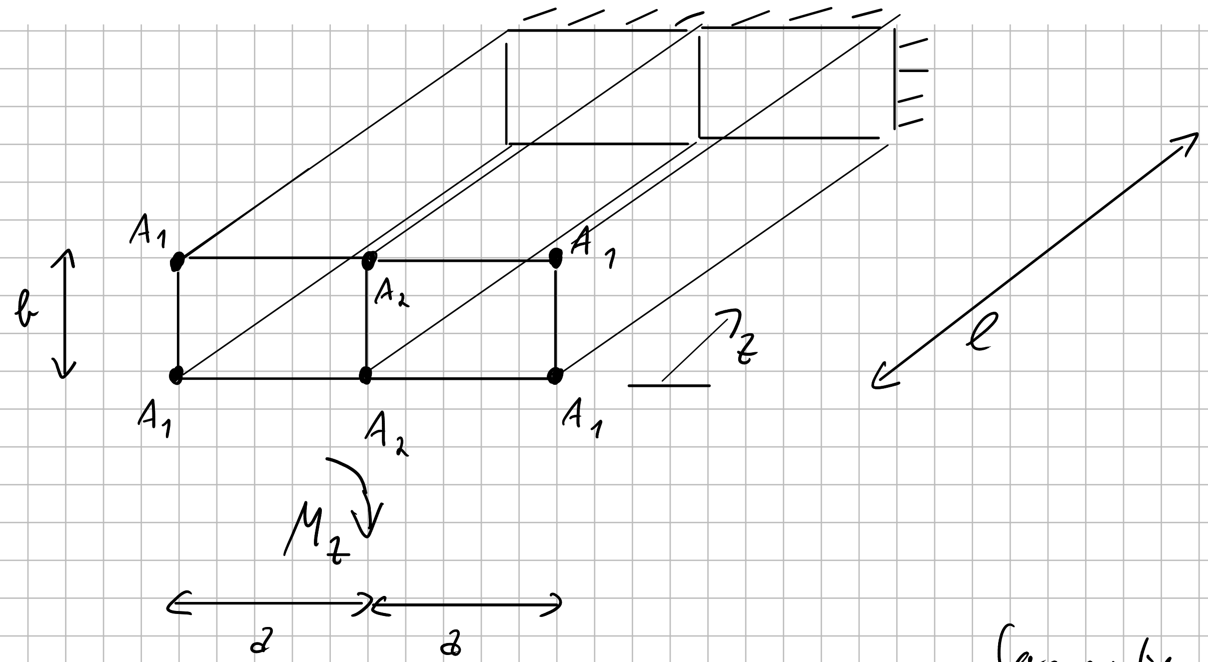
$$v'' = 2a$$
$$v' = 2az$$
$$v(l) = a l^2$$

$$\int_0^l \left(4 \delta a\, EJ\, a - 4F\, \delta a\, z^2 a \right) dz + \delta a\, k\, l^4 a = 0$$

$$\delta a \left(4 EJ l + k l^4 - \frac{4}{3} l^3 F \right) a = 0$$

$$\frac{4}{3} l^3 F = 4 EJ l + k l^4 \Rightarrow F = \frac{4EJl + kl^4}{\frac{4}{3} l^3} = 1{,}458E6 \text{ N}$$

4



$t = 1$ mm

$E = 70000$ MPa

$\nu = 0{,}3$

$a = 1000$ mm

$b = 500$ mm

$\ell = 6000$ mm

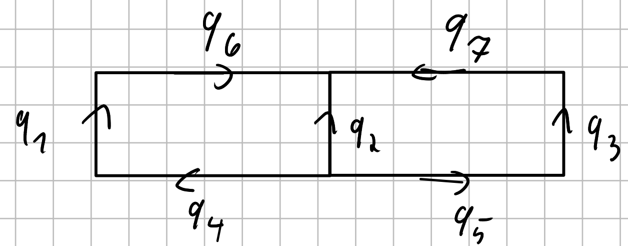
$A_1 = 500$ mm^2

$A_2 = 1000$ mm^2

$M_z = 1E9$ N mm

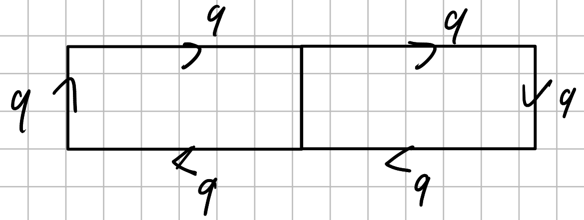
Compute θ_z at $z = \frac{2}{3}\ell$ from the free extremity

$$G = \frac{E}{2(1+\nu)} = 2{,}6923\,E4 \text{ MPa}$$



$q^*_1 = q^*_2$ for symmetry considerations

$\Rightarrow q_2 = \emptyset$



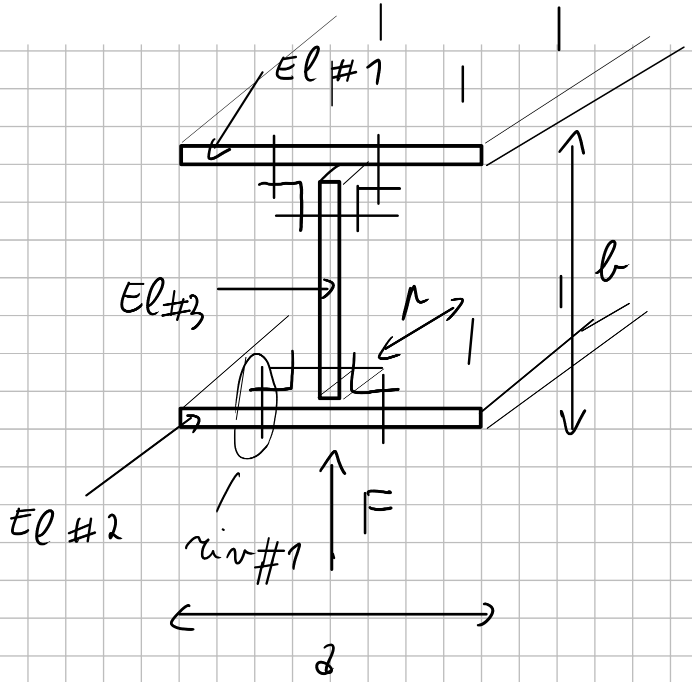
$$4abq = M_z$$

$$q = \frac{M_z}{4ab} = 500 \quad N/mm$$

$$\dot{\vartheta}_1 = \frac{1}{2abG}\left(\frac{2qa}{t} + \frac{qb}{t}\right) = 4{,}6429E-5 \quad rad/mm$$

$$\vartheta(\ell/3 \text{ from the clamp}) = \dot{\vartheta} \cdot \frac{\ell}{3} = 0{,}09286 \, rad$$

5



I_{xx} (barycentric) = 1,5 E 8 mm^4

A(El #1) = 5000 mm^2

A(El #2) = 5000 mm^2

A(El #3) = 3500 mm^2

p = 100 mm

a = 500 mm

b = 240 mm

neglect the areas of ⅂ ⌈ ⌋ L elements

ϕ rivets = 7 mm

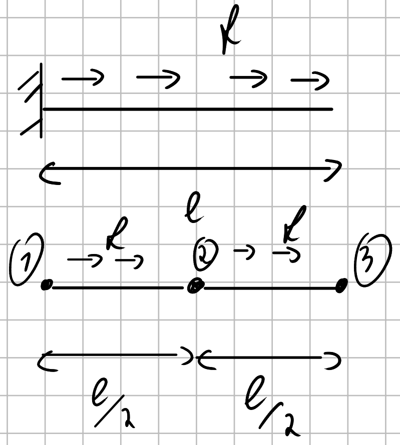
compute the shear stress in rivet #1

$$|q| = \frac{A(El\#2)\cdot\frac{b}{2}\,F}{I_{xx}} = 40\ N/mm$$

$$F_{rivet\#} = \frac{q}{2}\cdot p = 2000\ N$$ for symmetry

$$\tau = \frac{F_{rivet}}{\frac{\phi^2\pi}{4}} = 51,97\ MPa$$

(6)

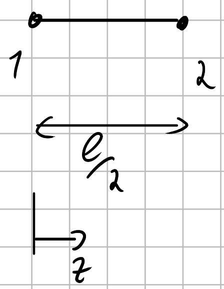


$l = 3 \; m$

$EA = 7E5 \quad N$

$p = 10 \quad N/mm$

$u(\text{node } \#3) = ?$



$$N_1(z) = \frac{2}{l}\left(-z + \frac{l}{2}\right)$$

$$N_2(z) = \frac{2z}{l}$$

$$u = u_1 N_1(z) + u_2 N_2(z)$$

$$u' = u_1\left(-\frac{2}{l}\right) + u_2 \frac{2}{l}$$

Stiffness: 1 element

$$\int_0^{l/2} \delta u' \, EA \, u' \, dz = \int_0^{l/2} \begin{Bmatrix} \delta u_1 \\ \delta u_2 \end{Bmatrix}^T \begin{bmatrix} -2/l \\ 2/l \end{bmatrix} EA \begin{bmatrix} -\frac{2}{l} & \frac{2}{l} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} dz$$

$$= \begin{Bmatrix} \delta u_1 \\ \delta u_2 \end{Bmatrix}^T EA \begin{bmatrix} 2/l & -2/l \\ -2/l & 2/l \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}$$

$$\delta \mathcal{L}_e \text{ 1 element} = \int_0^{l/2} \begin{Bmatrix} \delta u_1 \\ \delta u_2 \end{Bmatrix}^T \left[\frac{-2z}{l} + 1 \, , \; \frac{2z}{l} \right]^T p \, dz$$

$$= \begin{Bmatrix} \delta u_1 \\ \delta u_2 \end{Bmatrix}^T \left[-\frac{l}{4} + \frac{l}{2} \quad \frac{l}{4} \right]^T p = \begin{Bmatrix} \delta u_1 \\ \delta u_2 \end{Bmatrix}^T \begin{Bmatrix} l/4 \\ l/4 \end{Bmatrix} p$$

after assembling the two elements and constraining the clamped node

$$\begin{Bmatrix} \delta u_2 \\ \delta u_3 \end{Bmatrix}^T \begin{bmatrix} 4/l & -2/l \\ -2/l & 2/l \end{bmatrix} EA \begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} \delta u_2 \\ \delta u_3 \end{Bmatrix}^T \begin{Bmatrix} l/2 \\ l/4 \end{Bmatrix} p$$

$$\begin{Bmatrix} u_2 \\ u_3 \end{Bmatrix} = \begin{Bmatrix} 0{,}4821 \\ 0{,}6429 \end{Bmatrix} \text{ mm}$$

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. Any structure with one dimension much larger than the other two can likely be modeled as a beam:
   - True
2. According to the Timoshenko beam model, the transverse shear stresses are linear on the cross section:
   - False
3. The position of the shear center of a closed-cell section can be evaluated using the shear flow equations and the equivalence to internal moment:
   - False

Multiple Choice questions

(Circle the correct answer)

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(b) find the equilibrated solution among the compatible ones

(c) find the compatible and equilibrated solutions among all the possible independent stress and displacement fields

(d) none of the above