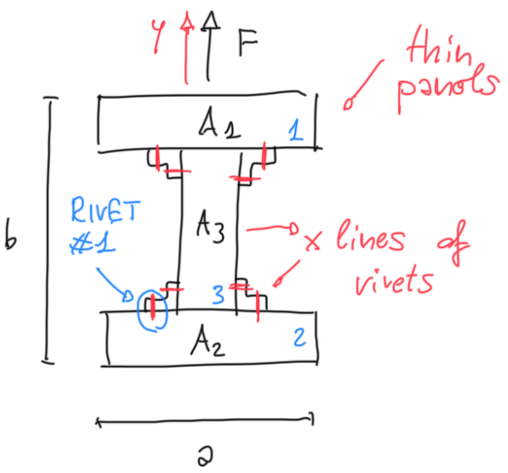
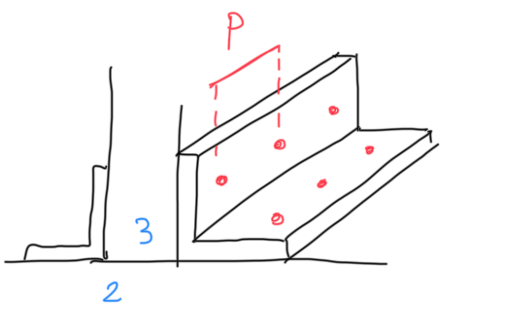
Ex 10 - Ribs and Junctions

1) EXAM 05/07/2023





Let's find the shear stress in rivet #1

DATA

$J_{xx} = 1.5 \times 10^8$ Nmm

$A_1 = A_2 = A = 5000 \text{ mm}^2$

$A_3 = 3500 \text{ mm}^2$

$p = 100$ mm

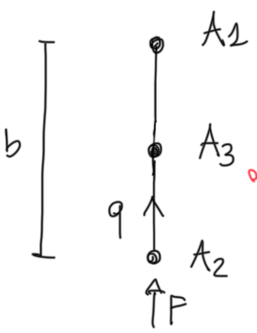
$\phi = 7$ mm rivet diameter

$a = 500$ mm

$b = 250$ mm

$F = 1000$ N

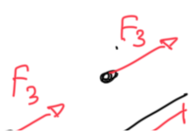
- Shear Fluxes in the panels

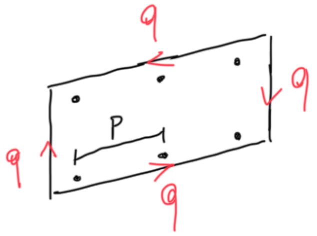


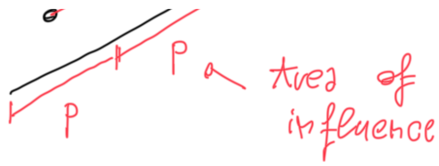
$$q = -F \frac{S_{x1}}{J_{xx}} = -F \cdot \left(- \frac{A \frac{b}{2}}{J_{xx}} \right)$$

here there is not a stringer (i.e. axial load), thus q is constant along all the panel side.

- Plate 3



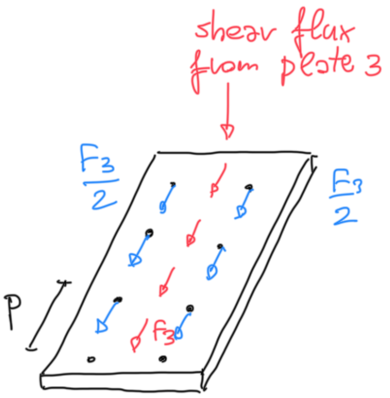




$$F_3 = q \cdot p \qquad \tau_3 = \frac{F_3}{A_{rivet}} = \frac{q \cdot p}{\pi \left(\frac{\phi}{2}\right)^2}$$

this is $\int q\, p$

- Plate 2



$$F_2 = \frac{F_3}{2} = \frac{q \cdot p}{2}$$

$$\tau_1 = \frac{F_2}{A_{rivet}} = \frac{\frac{1}{2} q \cdot p}{\pi \left(\frac{\phi}{2}\right)^2}$$

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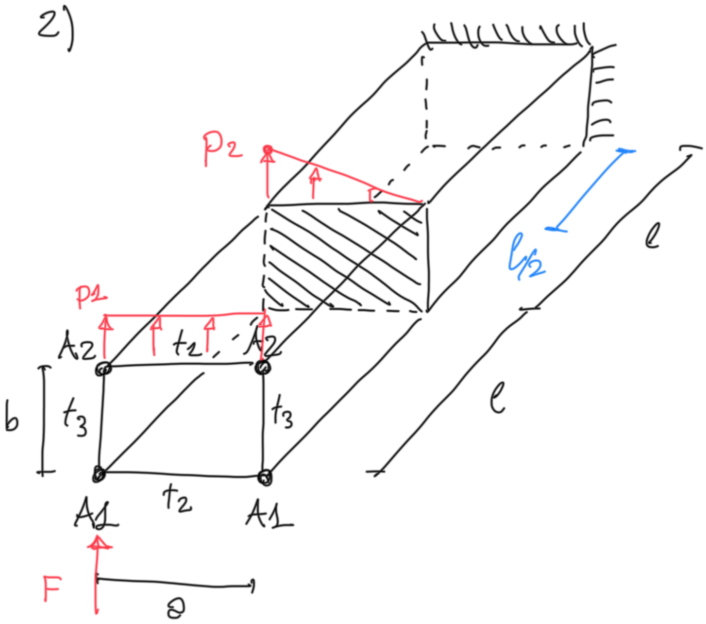
2)



DATA

$l = 1500$ mm

$a = 400$ mm

$b = 250$ mm

$t_1 = 1$ mm

$t_2 = 2$ mm

$t_3 = 1.5$ mm

$A_1 = 2000\ mm^2$

$A_2 = 1000\ mm^2$

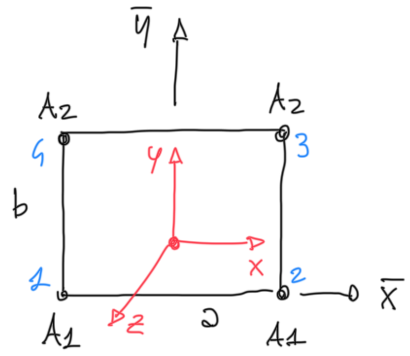
$F = 10$ kN

$p_1 = 90$ N/mm

$p_2 = 300$ N/mm

- q_i and σ_{zz} at $l/2$ from the clamp?
- Plot the internal actions in the rib

- Section Properties



$$\bar{y}_c = \frac{2A_2 b}{2A_1 + 2A_2} = \frac{1}{3} b$$

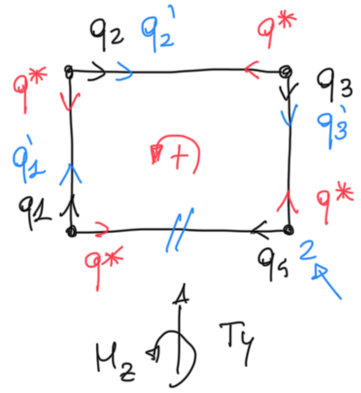
$$J_{xx} = 2A_2\left(\frac{2}{3}b\right)^2 + 2A_1\left(-\frac{1}{3}b\right)^2 = \frac{4}{3} A_2 b^2$$

$$S_{x1} = S_{x2} = -A_1 \cdot \frac{b}{3} = -\frac{2}{3} \underline{A_2} b$$

$$S_{x3} = S_{x4} = \frac{2}{3} A_2$$

- Open Cell Fluxes

Let's compute them for a generic T_y and M_z



$$q_1' = -T_y \frac{S_{x1}}{J_{xx}} = \frac{T_y}{2b} = -q_3'$$

$$q_2' = \emptyset$$

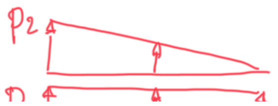
- Moment Equilibrium wrt 2

$$LHS = M_z - T_y \cdot \frac{a}{2}$$

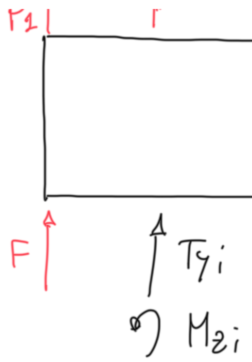
$$RHS = 2q^* \Omega_{cell} - 2 q_1' \Omega_1 \qquad \Omega_{cell} = a \cdot b \qquad \Omega_1 = \frac{a \cdot b}{2}$$

$$M_z - \cancel{T_y \frac{a}{2}} = 2q^* ab - \cancel{2 \frac{T_y}{2b} \cdot \frac{ab}{2}} \qquad q^* = \frac{M_z}{2ab}$$

- Lumped Forces



F $\qquad$ P_1 $\qquad$ d

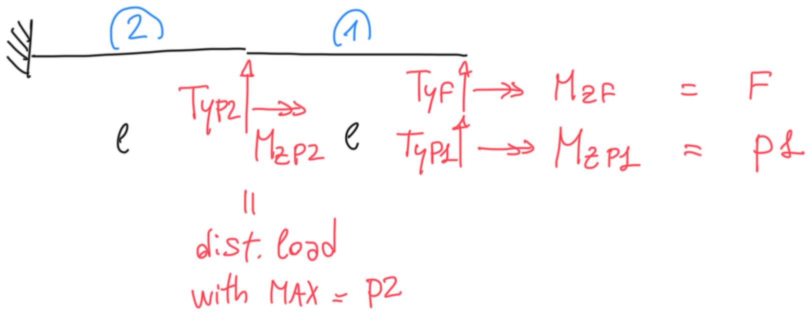


$T_{yF} = F$ $\quad$ $T_{yP1} = p_1 \cdot a$

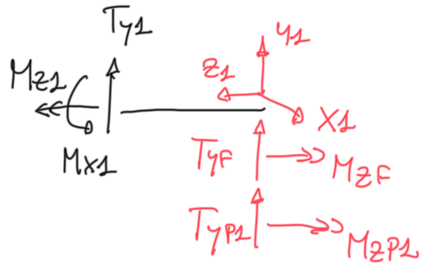
$M_{zF} = -F \cdot \frac{a}{2}$ $\quad$ $M_{zP1} = \emptyset$

$$T_{yP2} = \tfrac{1}{2} p_2 \cdot a \qquad M_{zP2} = -\left(\tfrac{1}{2} p_2 \cdot a\right) \cdot \tfrac{1}{6} a$$

- Internal Actions in the beam

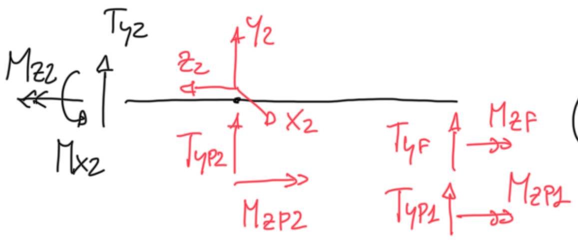


1



$$\begin{cases} T_{y1} = -T_{yF} - T_{yP1} \\ M_{x1} = -(T_{yF} + T_{yP1}) \cdot z_2 \\ M_{z1} = M_{zF} + M_{zP1} \end{cases}$$

2



$$\begin{cases} T_{y2} = -T_{yP2} - T_{yP1} - T_{yF} \\ M_{x2} = -T_{yP2} \cdot z_2 - (T_{yP1} + T_{yF}) \cdot (\ell + z_2) \\ M_{z2} = M_{zP2} + M_{zP1} + M_{zF} \end{cases}$$

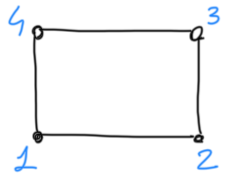
- Shear fluxes in $z_2 = \frac{\ell}{2}$

$$q_1 = q_1' - q^* = \frac{T_{y2}}{2b} - \frac{M_{z2}}{2ab} = -242 \ \text{N/mm}$$

$$q_2 = q_2' - q^* = -30 \quad N/mm$$

$$q_3 = q_3' - q^* = 182 \quad N/mm$$

$$q_4 = -q^* = -30 \quad N/mm$$



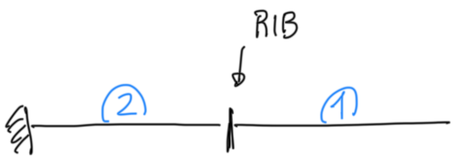
- Axial Stress in stringers at $z_2 = \ell/2$

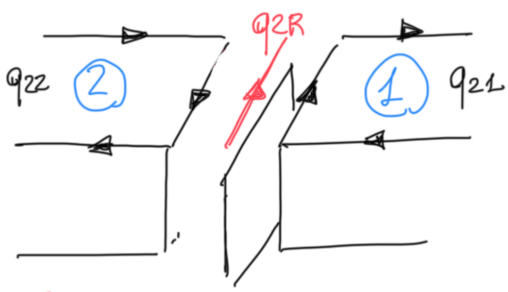
✓ $z_2 = \ell/2$

$$\sigma_{zz1} = -\frac{M_{x2}}{J_{xx}} \cdot \frac{1}{3} b = 148.5 \ MPa = \sigma_{zz1}$$

$$\sigma_{zz3} = \sigma_{zz4} = \frac{M_{x2}}{J_{xx}} \cdot \frac{2b}{3} = -297 \ MPa$$

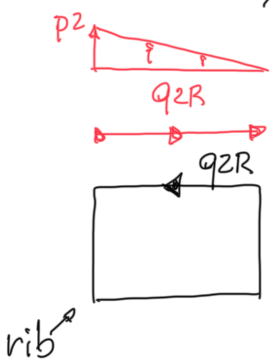
- Internal Actions in the rib





$$q_{22} = q_{21}$$

$$q_{2R} = q_{22} - q_{21}$$



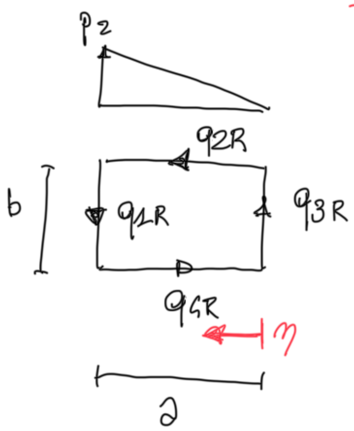
the rib equilibrates:

- the external force P2
- the shear fluxes jump in the panel

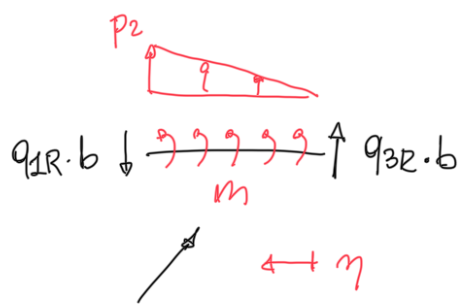
the rib must be in equilibrium

- Beam model of the rib

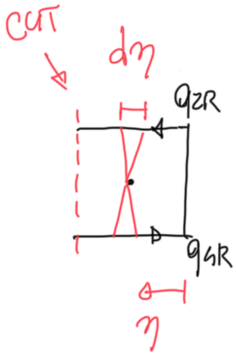
This model allows an estimation of the internal actions in the rib, along one of its sides



TBN: $q_{2R} = q_{3R} = q^*$



distributed bending moment given by q_{2R} and q_{4R}



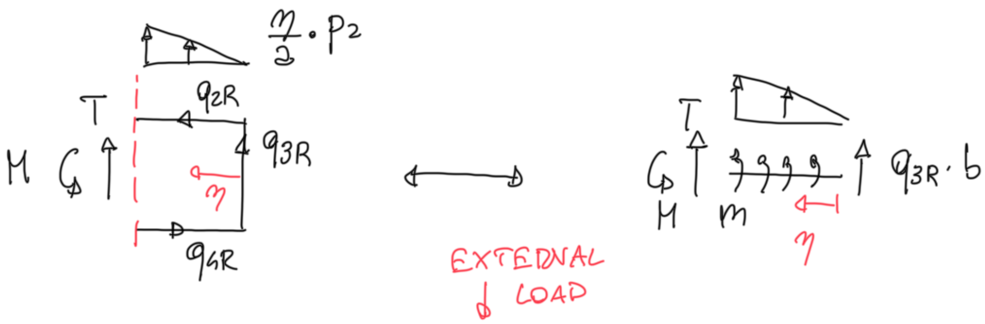
$$m = 2\, q_{2R} \cdot \Omega + 2\, q_{4R} \cdot \Omega =$$

$$= 2\left(q_{2R} + q_{4R}\right) \cdot \left(\frac{1}{2}\frac{b}{2}\, d\eta\right)$$

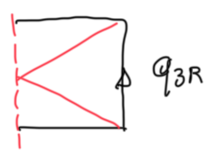
area of the triangle

$d\eta = 1 \rightarrow m =$ bending momen × unit length given by q_{2R} and q_{4R}

- Internal Actions in the rib



EXTERNAL LOAD

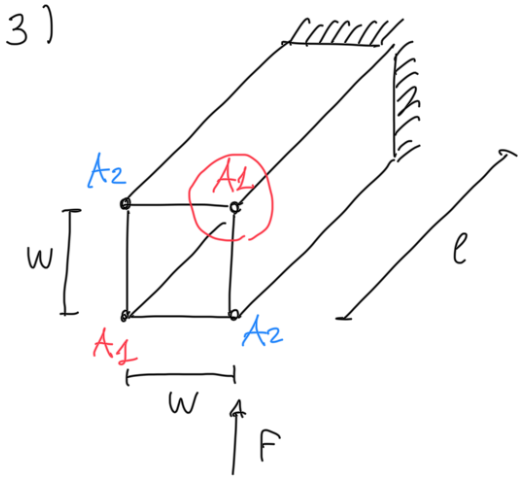


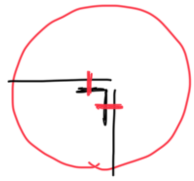
$$T = -q_{3R} \cdot b - \frac{1}{2}\left(\frac{\eta}{a} p_2\right) \cdot \eta$$

$$M = -2\, q_{3R}\left(\frac{1}{2} b \eta\right) - \left(\frac{1}{2}\left(\frac{\eta}{a} \cdot p_2\right)\eta\right) \cdot \left(\frac{1}{3}\eta\right) - \underbrace{\int_0^{\eta} m \, d\eta}_{m \cdot \eta}$$

EXTERNAL LOAD

3)





DATA

$A_1 = 300\ mm^2 = 2A$

$A_2 = 150\ mm^2 = A$

$t = 1.5\ mm$ for all the panels

$w = 100\ mm$

$E = 72\,000\ MPa$

$\nu = 0.3$

$\ell = 1500\ mm$

$d_1 = 2\ mm\ ,\ p_1 = 10\ mm$

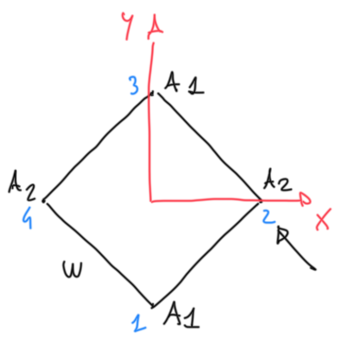
$d_2 = 3\ mm\ ,\ p_2 = 12\ mm$

$F = 15\ kN$

σ_{zz} at $\frac{\ell}{2}$?

shear stress in the rivets ?

• Section properties



if we rotate the section, it becomes symmetric wrt x and y $\rightarrow J_{xy} = \emptyset$

$$J_{xx} = 2A_1\left(\frac{\sqrt{2}}{2}w\right)^2 = A_1 w^2 = 2Aw^2$$

$$J_{yy} = 2A_2\left(\frac{\sqrt{2}}{2}w\right)^2 = A_2 w^2 = Aw^2$$

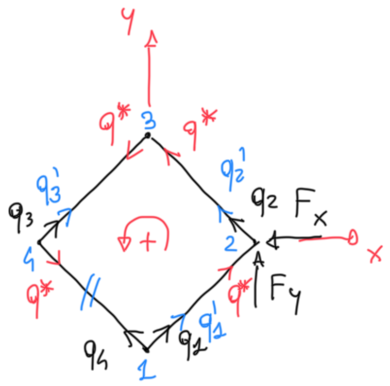
$$S_{x1} = -S_{x3} = -A_1\frac{\sqrt{2}}{2}w = -\sqrt{2}Aw$$

$$S_{x2} = S_{x4} = \emptyset$$

$$S_{y1} = S_{y3} = \emptyset$$

$$S_{y2} = -S_{y4} = A_2\frac{\sqrt{2}}{2}w = \frac{\sqrt{2}}{2}Aw$$

- Open Cell Fluxes



$$q_1' = -F_y \frac{S_{x1}}{J_{xx}} - F_x \cancel{\frac{S_{y1}}{S_{yy}}} \rightarrow \emptyset$$

$$q_2' = -F_y \frac{S_{x1} + \cancel{S_{x2}}}{J_{xx}} - F_y \frac{\cancel{S_{y1}} + S_{y2}}{J_{yy}}$$

$$q_3' = -F_y \cancel{\frac{S_{x4}}{J_{xx}}} - F_x \frac{S_{y4}}{J_{yy}}$$

- Moment Equivalence wrt 2

$$LHS = \emptyset$$

$$RHS = 2q^* \cancel{w}^2 - \cancel{2} q_3' \frac{\cancel{w}^2}{\cancel{2}}$$

$$q^* = \frac{q_3'}{2} = \frac{1}{2}\left(F_x \cdot \frac{\sqrt{2}}{2} w\right) = \frac{\sqrt{2}}{4} F_x$$

- Closed Cell

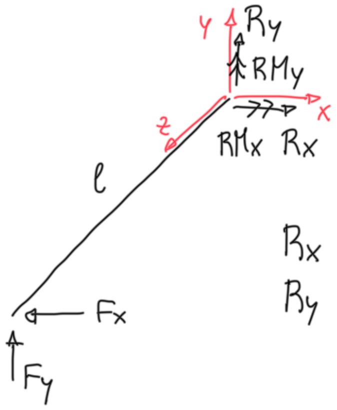
$$q_1 = q_1' + q^*$$

$$q_2 = q_2' + q^*$$

$$q_3 = q_3' - q^*$$

$$q_4 = -q^*$$

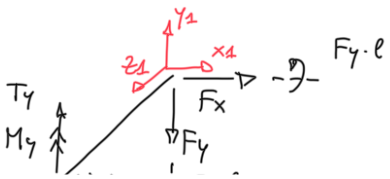
- Reaction Forces and Internal Actions in the beam



we don't care about RM_z because it doesn't act on stringers

$$R_x = F_x \qquad RM_x = F_y \cdot \ell$$

$$R_y = -F_y \qquad RM_y = F_x \cdot \ell$$



$$T_x = -F_x$$

$$T_y = F_y$$

$$M_x = F_y(z - \ell)$$

M_x T_x $F_x \cdot \ell$

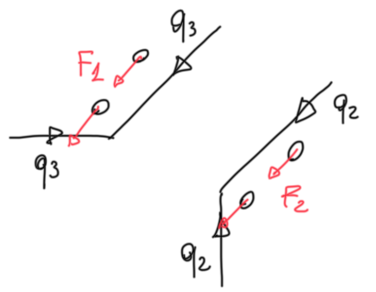
$$M_y = F_x (z - \ell)$$

- Stress in stringers at $z = \frac{\ell}{2}$

$$\sigma_{zz1} = + \frac{M_x(\frac{\ell}{2})}{J_{xx}} \cdot y = + \frac{F_y \cdot \frac{\ell}{2}}{J_{xx}} \left(+ \frac{\sqrt{2}}{2} w \right) = 187.5 \ MPa = - \sigma_{zz3}$$

$$\sigma_{zz2} = - \frac{M_y}{J_{yy}} \cdot x = + \frac{(+F_x \cdot z_1)}{J_{yy}} \cdot \left(\frac{\sqrt{2}}{2} w \right) = 375.0 \ MPa = - \sigma_{zz4}$$

- Shear Stress in Rivets



$$F_1 = q_3 \cdot p_1 \qquad \tau_1 = \frac{F_1}{\pi \left(\frac{d_1}{2} \right)^2} = \frac{q_3 \, p_1}{\pi \left(\frac{d_1}{2} \right)^2} = 119.37 \ MPa$$

$$F_2 = q_2 \cdot p_2 \qquad \tau_2 = \frac{F_2}{\pi \left(\frac{d_2}{2} \right)^2} = \frac{q_2 \cdot p_2}{\pi \left(\frac{d_2}{2} \right)^2} = 190.99 \ MPa$$

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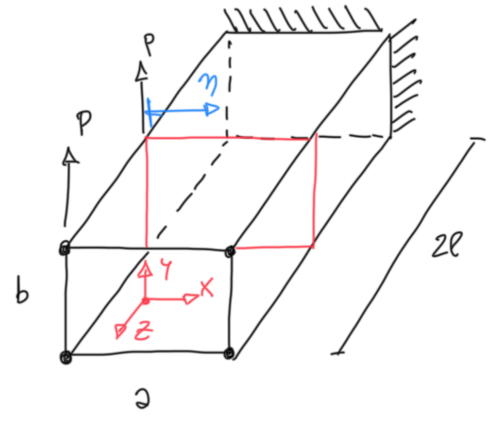
4)



DATA

$a = 500$ mm

$b = 250$ mm

$t = 0.6$ mm

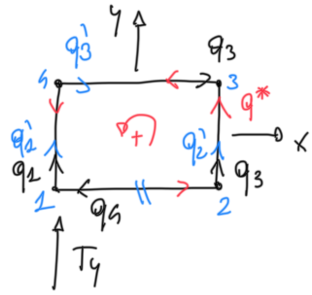
$A = 500$ mm^2

$\ell = 2000$ mm

$P = 1000$ N

Let's find the moment in the rib at $\eta = \frac{a}{3}$

- Open Cell Fluxes



$$q_1' = q_2' = \frac{T_y}{2b} \qquad q_3' = \emptyset$$

- Moment Equivalence wrt (1)

$$LHS = \emptyset$$

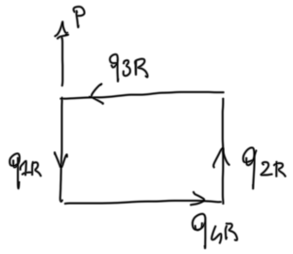
$$RHS = 2q^* ab + \cancel{2}\,\frac{T_y}{\cancel{2b}} \cdot \frac{a\cancel{b}}{2} \qquad q^* = -\frac{T_y}{4b}$$

- Total Fluxes

$$q_1 = q_1' - q^* = \frac{T_y}{2b} + \frac{T_y}{4b} = \frac{3}{4}\frac{T_y}{b} \, * \qquad q_3^* = -q^* = \frac{T_y}{4b}$$

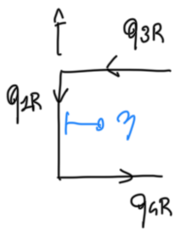
$$q_2 = q_2' + q^* = \frac{T_y}{4b} \qquad q_4 = -q^* = \frac{T_y}{4b}$$

- Internal Action in the rib



$$* \; q_{1R} = \frac{3}{4}\frac{P}{b} \qquad q_{3R} = \frac{P}{4b}$$

$$q_{2R} = \frac{P}{4b} \qquad q_{4R} = \frac{P}{4b}$$



$$M(\eta) = 2 \cdot q_{1R}\left(\frac{b\eta}{2}\right) + 2\, q_{3R}\left(\frac{b\eta}{4}\right) + \ldots$$

$$\ldots + 2 \cdot q_{4R}\left(\frac{b\eta}{4}\right) - P \cdot \eta$$

$$M\left(\frac{a}{3}\right) = \emptyset$$