

Determine the axial force at $x = L/2$

Data

$$L = 1200 (1 + D/10) \text{ mm}$$

$$EA_1 = 2.0 \cdot 10^4 \text{ N}$$

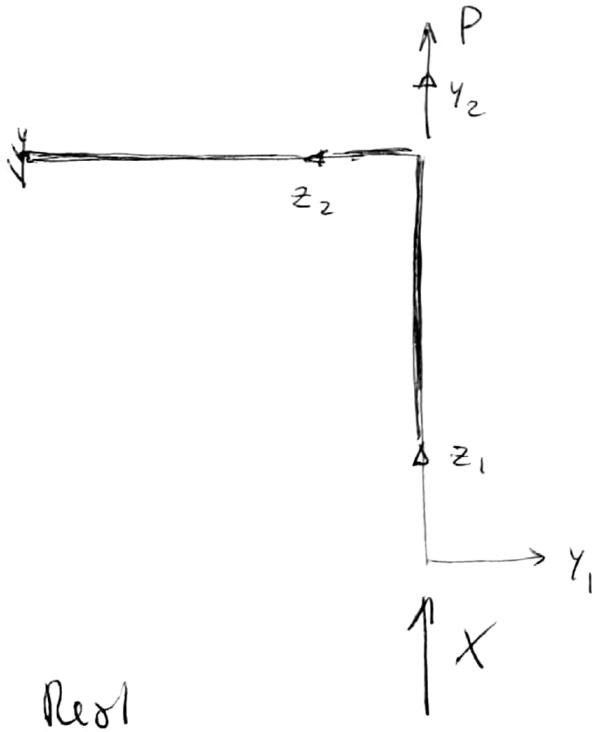
$$EA_2 = 1.0 \cdot 10^4 \text{ N}$$

$$EA_3 = 2.9 \cdot 10^4 \text{ N}$$

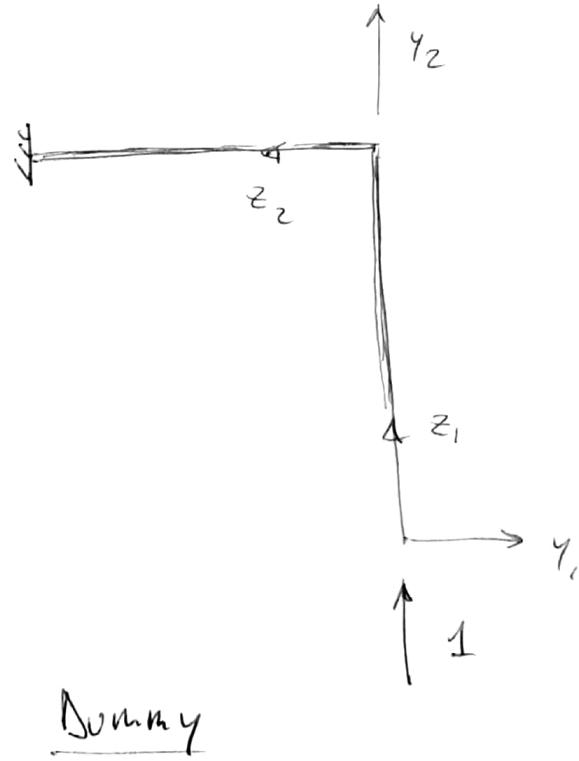
$$EJ = 1.5 \cdot 10^{10} \text{ N mm}^2$$

$$P = 1200 (1 + F/10) \text{ N}$$

Solution ($D = F = 0$)



Resl



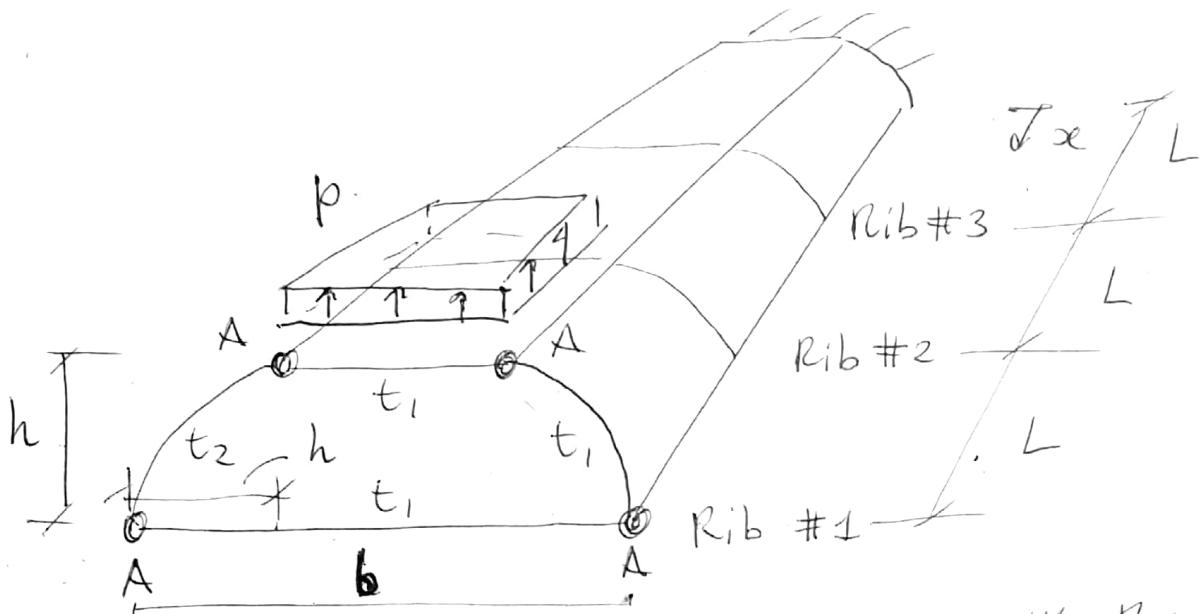
Dummy

$$N = -X \quad \delta N = -1 \quad (\text{beam } 1)$$

$$M_x = -(P + X) z_2 \quad \delta M_x = -z_2 \quad (\text{beam } 2)$$

$$\int_0^L \frac{N \delta N}{EA_3} dz_1 + \int_0^L \frac{M_x \delta M_x}{EI} dz_2 = 0$$

$$X = - \frac{PL^2/3}{\frac{EI}{EA_3} + L^2/3} = -577.59 \text{ N}$$



Evaluate the shear flow in the panel with thickness t_2 at $x=0$.

Data

$$A = 700 \text{ mm}^2 \quad t_2 = 1.5 \text{ mm}$$

$$b = 600 \text{ mm} \quad L = 1800 (1 + A/10) \text{ mm}$$

$$h = 150 \text{ mm} \quad p = 2.5 \cdot 10^{-2} (1 + B/10) \text{ N/mm}^2$$

$$t_1 = 1 \text{ mm}$$

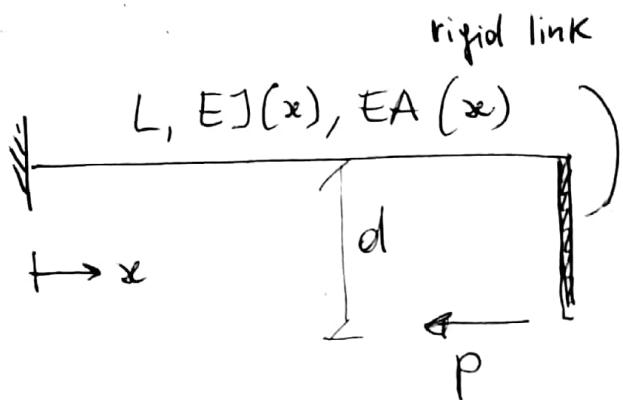
Solution ($A = B = 0$)

The shear at $x=0$ is

$$T = p \cdot L (b - 2h)$$

And from the symmetry of the section,

$$q = \frac{T}{2h} = 45 \text{ N/mm}$$



Using the Ritz method and a 1-dof polynomial approximation for any displacement

Component, determine the strain energy.

Data

$$L = 1200 \text{ mm}$$

$$d = 45 \text{ mm}$$

$$EA_0 = 1.15 \cdot 10^6 \text{ N}$$

$$EA_1 = 6.5 \cdot 10^5 (1 + B/10) \text{ N}$$

$$EI_0 = 1.5 \cdot 10^9 \text{ Nmm}^2$$

$$EI_1 = 7.5 \cdot 10^9 \text{ Nmm}^2$$

$$P = 800 \text{ N}$$

Note, EA and EI vary linearly with x from EA_0/EI_0 to EA_1/EI_1 .

Solution ($B=0$)

$$u = q_1 x; \quad w = q_2 x^2$$

$$EI(x) = \alpha + \beta x$$

$$EA(x) = \delta + \gamma x$$

$$\alpha = EI_0 \quad \beta = \frac{EI_1 - EI_0}{L}$$

$$\delta = EA_0 \quad \gamma = \frac{EA_1 - EA_0}{L}$$

$$\int_0^L (\delta u_{xx} EA(x) U_{xx} + \delta w_{xx} EI(x) W_{xx}) dx = \\ = - \delta u(L) P - \delta w_{xx}(L) Pd$$

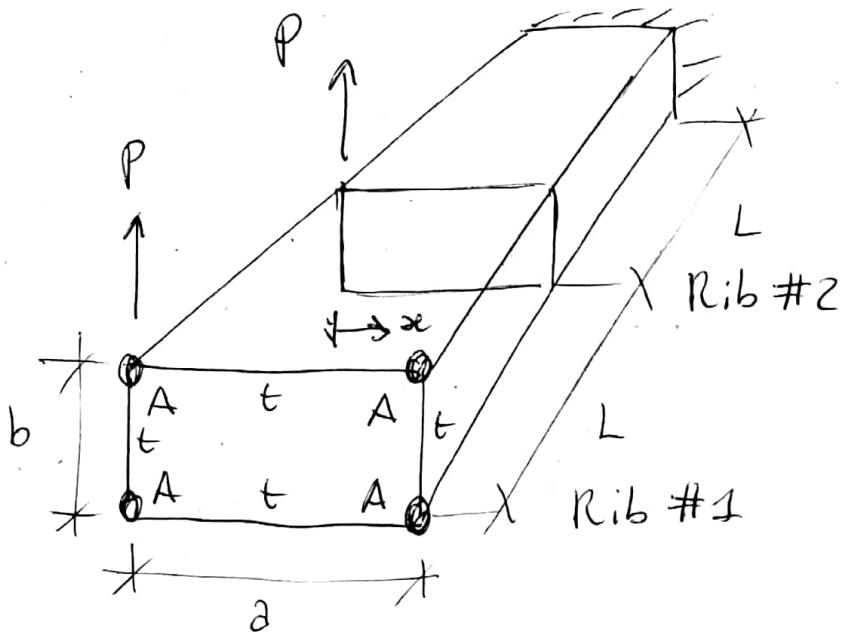
After substitution of the approximations,

$$\begin{bmatrix} \delta L + \gamma L^2/2 & 0 \\ 0 & 4(\alpha L + \beta L^2/2) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} -PL \\ -2LPd \end{bmatrix}$$

$$\underline{\underline{K}} \quad \underline{\underline{q}} = \underline{\underline{f}}$$

Solving for $\underline{\underline{q}}$ and evaluating U provides:

$$U = \frac{1}{2} \underline{\underline{q}}^T \underline{\underline{K}} \underline{\underline{q}} = 599.47 \text{ Nmm}$$



Model Rib#2 as a beam and evaluate the bending moment at $x = \frac{a}{3}$

Data

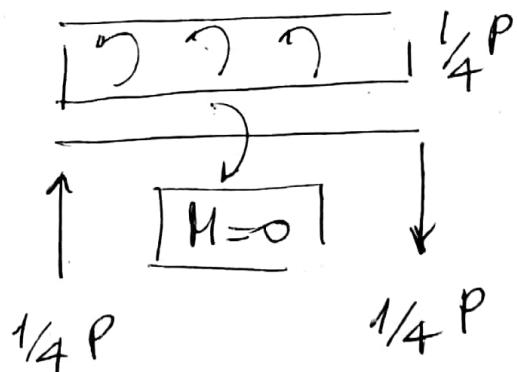
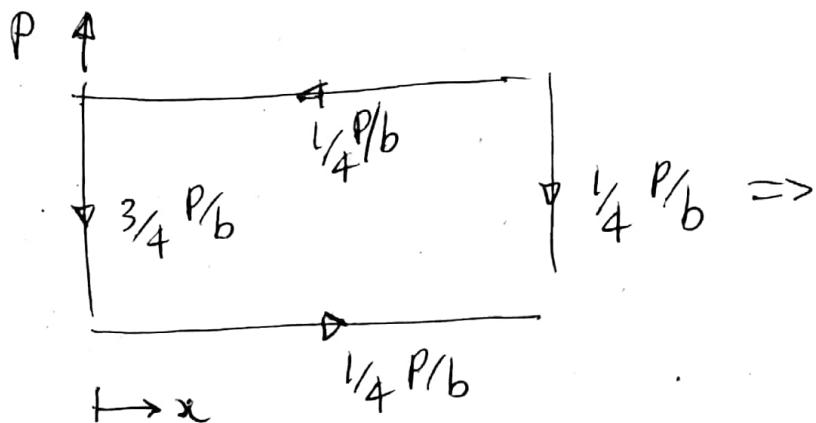
$$a = 500 (1 + A/I_0) \text{ mm} \quad A = 500 \text{ mm}^2$$

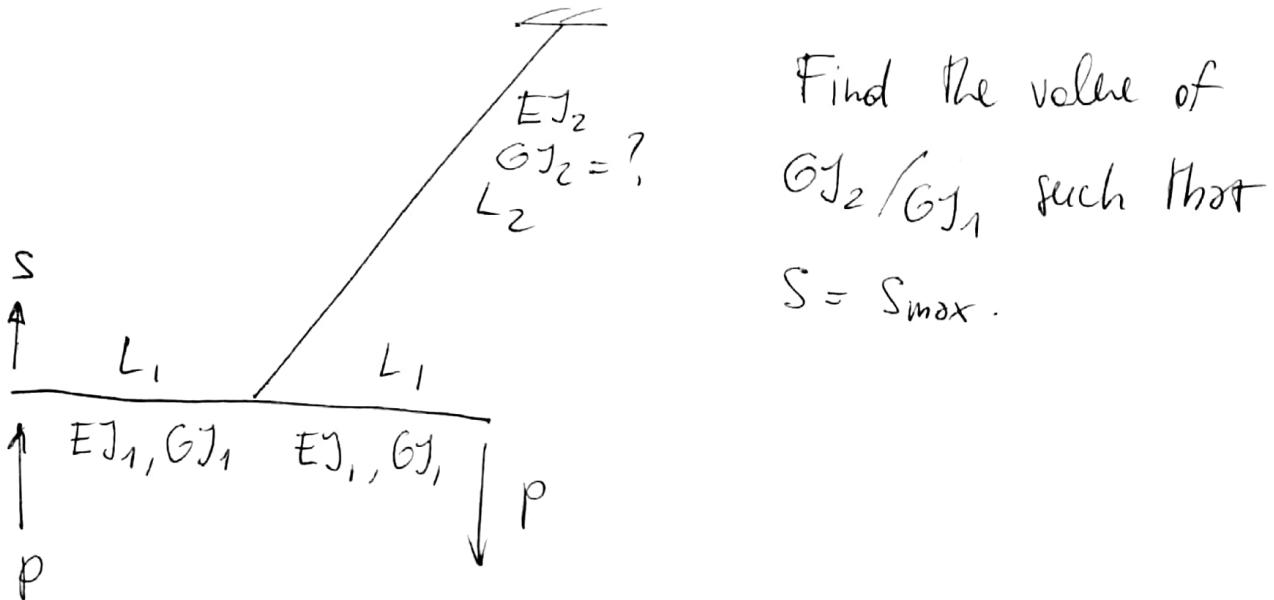
$$b = 250 \text{ mm} \quad L = 2000 \text{ mm}$$

$$t = 0.6 \text{ mm} \quad P = 1000 \text{ N}$$

Solution ($A=0$)

The equilibrating shear flows are found as:





Δeff

$$L_1 = 300 \text{ mm}$$

$$GJ_1 = 9 \cdot 10^7 \text{ Nmm}^2$$

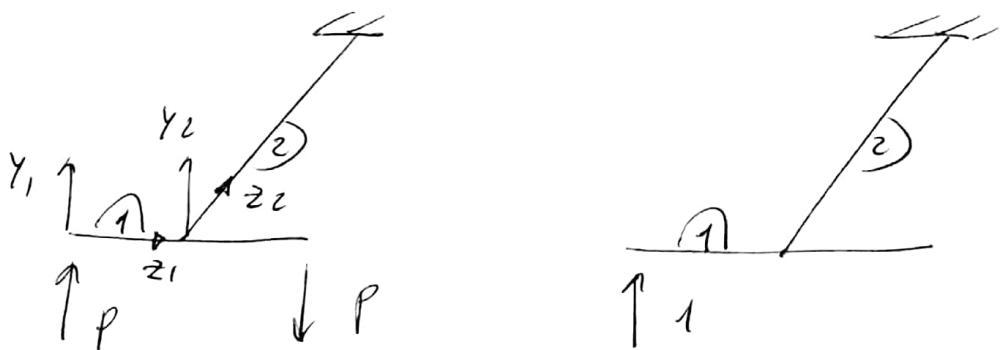
$$L_2 = 750 \text{ mm}$$

$$P = 1200 \text{ N}$$

$$EJ_1 = 2 \cdot 10^9 \text{ Nmm}^2$$

$$S_{\max} = 15 \left(1 + F_{10} \right)$$

Solution ($F=0$)



Real

Dummy

$$M = -Pz, \quad \delta M = -z, \quad (\text{beam 1})$$

$$M_z = -2PL_1, \quad \delta M_z = -L_1, \quad (\text{beam 2})$$

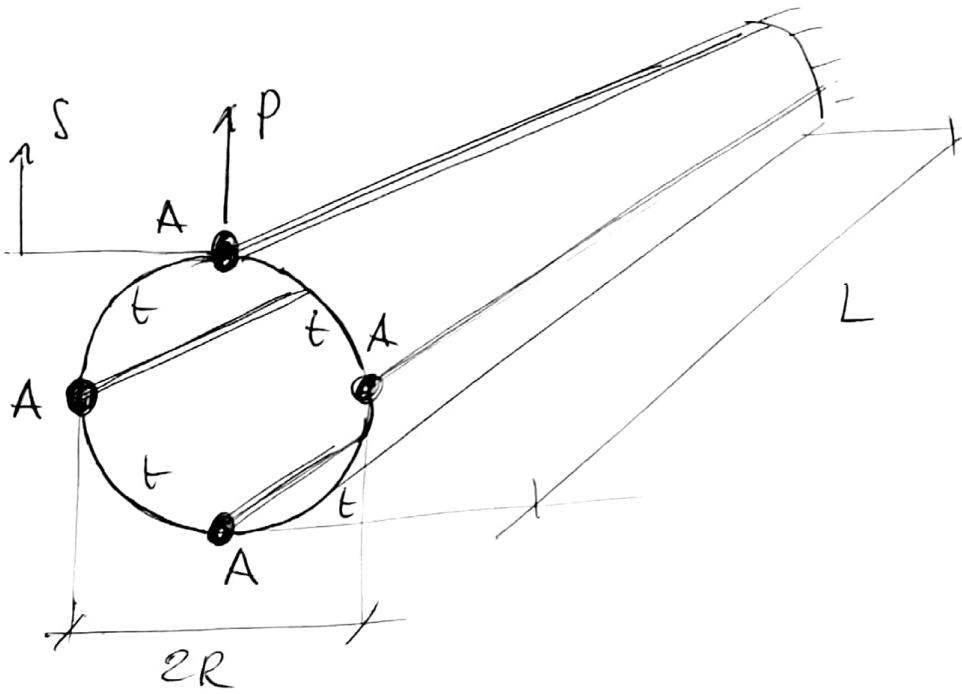
$$\int_0^{L_1} \frac{\delta M M}{EJ_1} dz_1 + \int_0^{L_2} \frac{\delta M_z M_z}{GJ_2} dz_2 = S = S_{\max}$$

From which:

$$GJ_2 = \frac{2PL_1^2 L_2}{S_{\max} - PL_1^3 / 3EJ_1}$$

and

$$\frac{GJ_2}{GJ_1} = 187.5$$



Evaluate the contribution of shear deformability to the displacement s .

Data

$$L = 2300 \text{ mm}$$

$$R = 150 \text{ mm}$$

$$t = 0.8(1 + A/10) \text{ mm}$$

$$P = 1700(1 + B/10) \text{ N}$$

$$G = 27000 \text{ MPa}$$

Solution ($A=B=0$)

$$A^* = \frac{\theta t R}{\pi}$$

$$s_{\text{shear}} = \frac{PL}{6A^*} = 0.4739 \text{ mm}$$

- A beam model cannot be used for evaluating local effects due to load introduction.

True

- The semi-monocoque approximation provides the exact shear stress distribution along the panels' thickness.

False

- Essential boundary conditions are more important than natural ones.

False

- The semi-inverse approach for the De Saint Venant solution for isotropic, homogeneous beams
leads to the exact solution of the problem.

- The shear center of beam section with one closed cell

requires application of the compatibility equation $\theta'=0$

- The trial functions in the Ritz method

must be part of a complete set