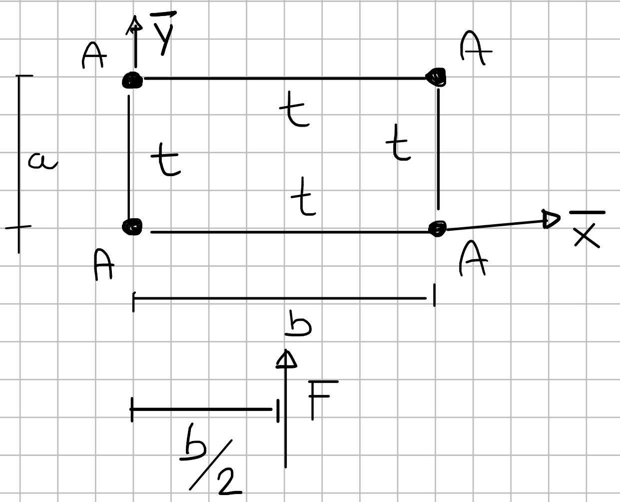
EXERCISE SESSION 6 — 19/10/22

Semi-monocoque approximation

Ex 1



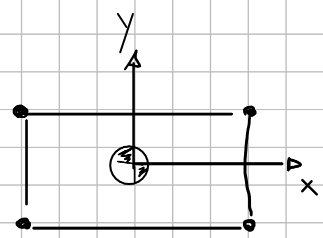
DATA

A, t, a, b are known.

1) CENTROID OF THE SECTION

$$x_{CG} = \frac{\sum_i A_i \bar{x}_i}{\sum_i A_i} \qquad x_{CG} = \frac{b}{2}$$

$$y_{CG} = \frac{\sum_i A_i \bar{y}_i}{\sum A_i} \qquad y_{CG} = \frac{a}{2}$$

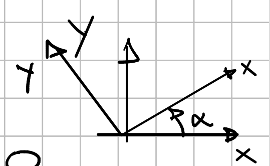


2) INERTIA MOMENT

$$J_{xx} = \sum_i A_i y_i^2 = Aa^2$$

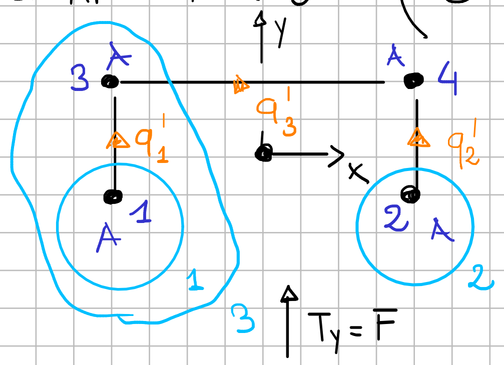
$$J_{yy} = \sum_i A_i x_i^2 = Ab^2$$

$$J_{xy} = \sum_i A_i x_i y_i = 0$$ ← SHOULD ALWAYS = 0



$$\alpha = \frac{1}{2} \operatorname{atan} \frac{2J_{xy}}{J_{yy} - J_{xx}}$$

3) SHEAR FLOWS (OPEN THE SECTION)



$$S_{x_i} = A_i y_i$$

$$S_{y_i} = A_i x_i$$

FOR EACH REGION $\quad q_i = -T_y \dfrac{\Sigma_j S_{x_j}}{J_{xx}} - T_x \dfrac{\Sigma_j S_{y_j}}{J_{yy}}$

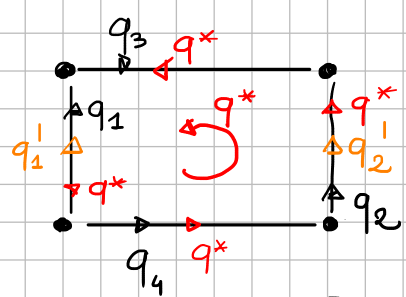
$$S_{x_1} = S_{x_2} = -A \cdot \frac{a}{2} \qquad S_{x_3} = S_{x_4} = A \cdot \frac{a}{2}$$

1) $q_1' = -T_y \dfrac{S_{x_1}}{J_{xx}} = \dfrac{1}{2} \dfrac{F}{a}$

2) $q_2' = -T_y \dfrac{S_{x_2}}{J_{xx}} = \dfrac{1}{2} \dfrac{F}{a}$

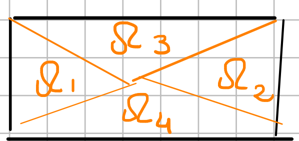
3) $q_3' = -T_y \dfrac{\cancel{S_{x_1} + S_{x_3}} = 0}{J_{xx}} = 0$

4) MOMENT EQUIVALENCE



$$M_z = \Sigma_i \, q_i \, 2\Omega_i$$

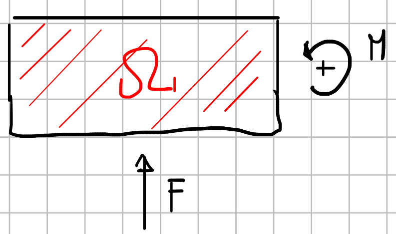
MOM. EQ FOR CG



$$F \cdot 0 = -\cancel{q_1' \cdot 2\Omega_1} + \cancel{q_2' \cdot 2\Omega_2} + q^* \cdot 2\Omega$$

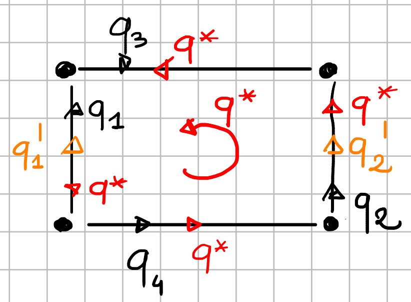
WITH $\quad \Omega_1 = \Omega_2 = \dfrac{1}{2} a \cdot \dfrac{1}{2} b$

$$\Omega = a \cdot b$$



$$\rightarrow q^* = \frac{F \cdot 0}{2\Omega} = 0$$

5) REAL FLUXES



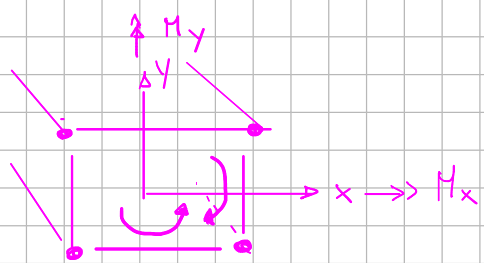
$$q_1 = q_1' - q^* = q_1'$$

$$q_2 = q_2' + q^* = q_2'$$

$$q_3 = -q^* = 0$$

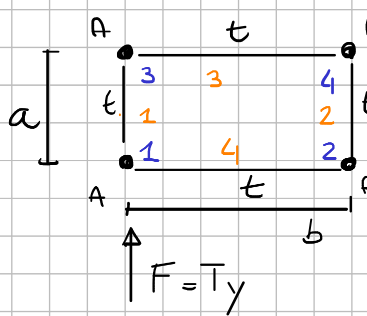
$$q_4 = +q^* = 0$$

6) AXIAL STRESS IN THE i-th STRINGER

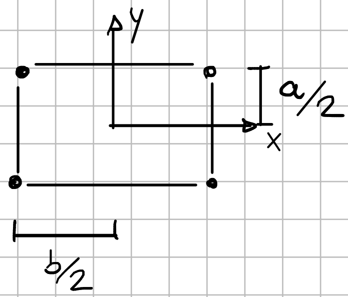


$$\sigma_i = \frac{T_z}{\Sigma_i A_i} + \frac{M_x}{J_{xx}} y_i - \frac{M_y}{J_{yy}} x_i$$

Ex 2



1) CENTROID

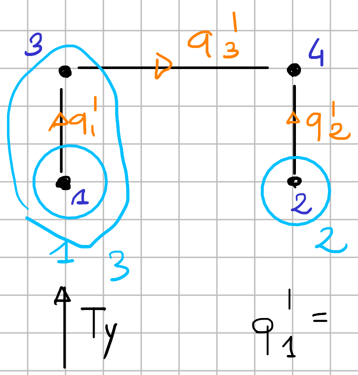


2) INERTIA

$$J_{xx} = Aa^2$$

$$J_{yy} = Ab^2$$

3) OPEN SECTION

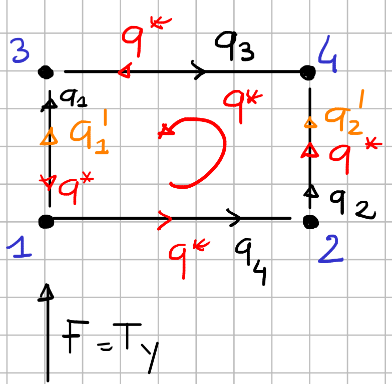


$$S_{x_1} = S_{x_2} = -A\frac{a}{2}$$

$$S_{x_3} = S_{x_4} = A\frac{a}{2}$$

$$q_1' = q_2' = -T_y\frac{S_{x_1}}{J_{xx}} = \frac{1}{2}\frac{T_y}{a}$$

4) MOMENTS EQUIVALENCE



WRT 1

$$F \cdot 0 = q_2' \cdot \overbrace{2ab \cdot \frac{1}{2}}^{\Omega_2} + q^* \cdot 2ab$$

$$q^* = -q_2' \cdot \frac{ab}{2ab} = -\frac{1}{2}q_2' = -\frac{1}{4}\frac{F}{a}$$

$$q_1 = q_1' - q^* = \frac{F}{2a} + \frac{1}{4}\frac{F}{a} = \frac{3}{4}\frac{F}{a}$$

$$q_2 = q_2' + q^* = \frac{F}{2a} - \frac{1}{4}\frac{F}{a} = \frac{1}{4}\frac{F}{a}$$

$$q_3 = -q^* = \frac{1}{4}\frac{F}{a}$$

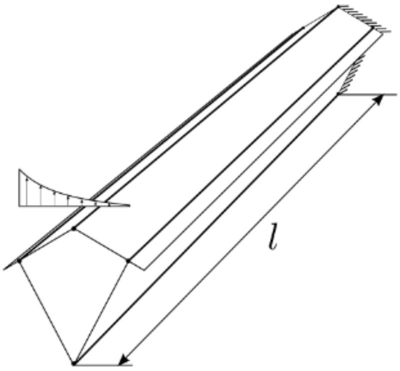
$$q_4 = +q^* = -\frac{1}{4}\frac{F}{a}$$

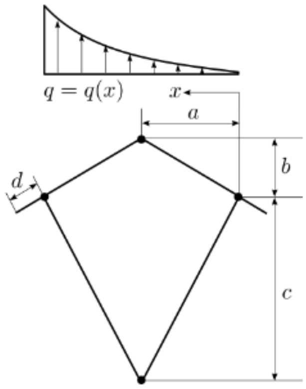
Ex 3 (OLD EXAMS 15 - 7 - 2019)

Exercise 1

A thin-walled beam is fixed at one end and loaded at the free end with a distributed force per unit length q. The total length is l, while the geometry of the section is depicted in the figure. The lumped stringers have area A, while the thickness of the panels is denoted with t.

By considering a semi-monocoque approximation, determine the shear and the axial stresses acting at the mid-span section. $(\ell/2)$





Data

$l = 6000$ mm;

$a = 200$ mm; $b = 200$ mm;

$c = 400$ mm; $d = 50$ mm;

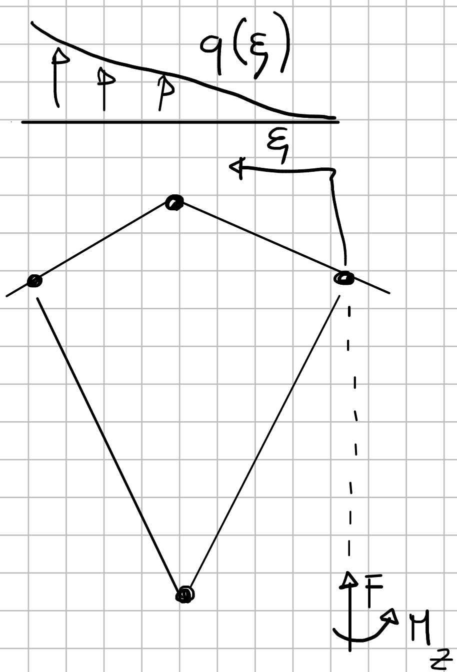
$t = 1.5$ mm;

$A = 1000$ mm^2;

$q = 150\left(\frac{x}{400}\right)^2$ N/mm,

$x \in [0, 400]$ mm;

0) FIND EQUIVALENT FORCE & MOMENTS FROM q



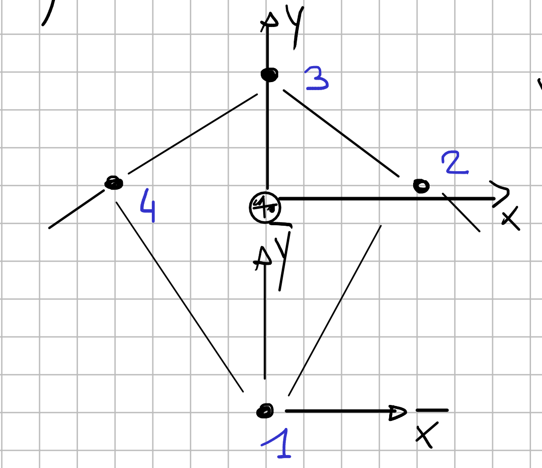
$$q(\xi) = 150\left(\frac{\xi}{400}\right)^2 \ \frac{N}{mm}$$

$$F = \int_0^{2a} q \, d\xi$$

$$= 150 \, \frac{1}{160000} \, \frac{1}{3} \, \xi^3 \Big|_0^{2a} = 20 \cdot 10^3 \ N$$

$$M_z = -\int_0^{2a} q \, \xi \, d\xi = -6 \cdot 10^6 \ N\,mm$$

1) CENTROID



$$y_{CG} = \frac{\Sigma_i A_i y_i}{\Sigma_i A_i} = \frac{2A \cdot c + A(b+c)}{4A} = 350 \text{ mm}$$

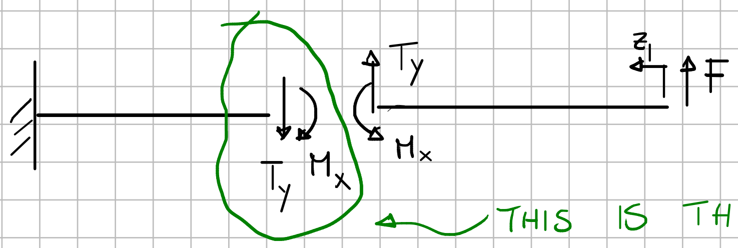
$$x_{CG} = 0$$

2) INERTIA MOMENT OF THE SECTIONS

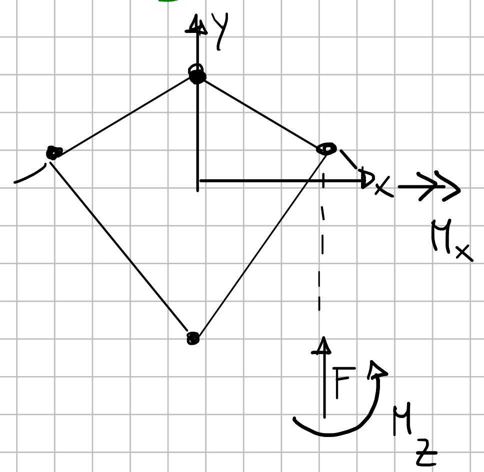
$$J_{xx} = \Sigma_i A_i y_i^2 = 190 \cdot 10^6 \text{ mm}^4$$

$$J_{yy} = \dots$$

3) INTERNAL ACTIONS AT MIDSPAN



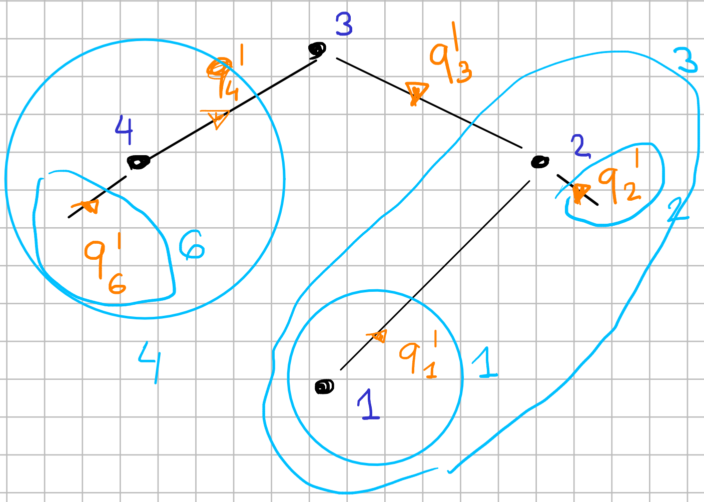
THIS IS THE FACE ASKED



$$T_y = -F = -20 \cdot 10^3 \text{ N}$$

$$M_x = -F \cdot \frac{\ell}{2} = -60 \cdot 10^6 \text{ Nmm}$$

OPEN CELL



2-6) q_2' & q_6' $= 0$

$$q_2' = -F \frac{0}{J_{xx}} = 0$$

1) $S_{x_1} = -A \cdot 350mm$

2) $S_{x_2} = S_{x_4} = A \cdot 50mm$

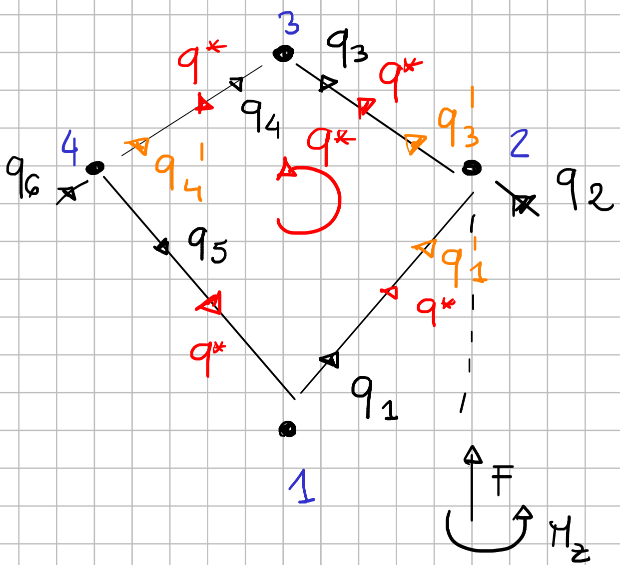
3) $S_{x_3} = A \cdot 250$

1) $q_1' = -F \dfrac{S_{x_1}}{J_{xx}} = \dfrac{700}{19} \; N/mm$

3) $q_3' = -F \cdot \dfrac{S_{x_1} + S_{x_2}}{J_{xx}} = \dfrac{600}{19} \; \dfrac{N}{mm}$

4) $q_4' = -F \cdot \dfrac{S_{x_4}}{J_{xx}} = -\dfrac{100}{19} \; \dfrac{N}{mm}$

CLOSED CELL



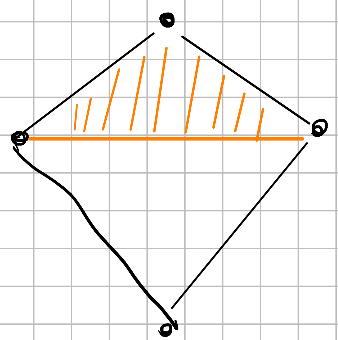
WRT 2

$$M_z = q^* \cdot 2\Omega_{CELL} - q_4' \cdot 2\Omega_4$$

$$\Omega_{CELL} = a \cdot c + a \cdot b = 120 \cdot 10^3 \ mm^2$$

$$\Omega_4 = a \cdot b = 40 \cdot 10^3 \ mm^2$$

$$q^* = \frac{M_z + q_4' \cdot 2\Omega_4}{2\Omega_{CELL}} = -26.75 \ \frac{N}{mm}$$



$$q_1 = q_1' + q^* = 10.09 \ \frac{N}{mm}$$

$$q_2 = 0$$

$$q_3 = 4.83 \ \frac{N}{mm}$$

$$q_4 = q_4' - q^* = 21.49 \ \frac{N}{mm}$$

$$q_5 = q^* = -26.75 \ \frac{N}{mm}$$

$$q_6 = 0$$

STRESS IN AXIAL STRINGER

$$\sigma_{z1} = 110.53 \ MPa \longleftarrow \sigma'_{zz_i} = \frac{M_x}{J_{xx}} y_i$$

$$\sigma_{zz_2} = \sigma_{zz_4} = -15.79 \ MPa$$

$$\sigma_{zz_3} = -78.95 \ MPa$$