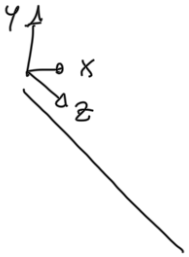
LAB 4

Hyperstatic Beams Systems I



PCVW → $\delta W_i = \delta W_e$

for beams $\delta W_e = (\delta F) \cdot (v) + (\delta M) \cdot (\varphi)$ (VIRTUAL / REAL)

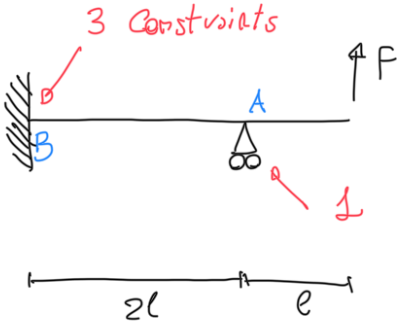
$\delta W_i = \int_V \delta \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}} \, dV =$ assume that we have (i) beams

number of beams

$= \sum_i \int_\ell \left(T_{xi}' \frac{T_{xi}}{GA^*} + T_{yi}' \frac{T_{yi}}{GA^*} + T_{zi}' \cdot \frac{T_{zi}}{EA} + M_{xi}' \frac{M_{xi}}{EJ_{xx}} + M_{yi}' \frac{M_{yi}}{EJ_{yy}} + M_{zi}' \frac{M_{zi}}{GJ_p^*} \right) \cdot dz_i$

for an Euler beam

1)



DATA
$\ell = 1000$ mm
$F = 4000$ N
$E = 200\,000$ MPa
$J_{xx} = 500\,000$ mm^4

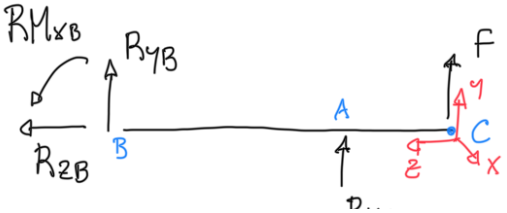
Let's find the RF in (A)

3 rigid DoF < 4 Constraints → Hyperstatic

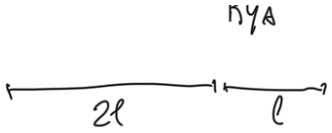
We cannot compute the RF by simply imposing the equilibrium.

- Reaction Forces

REAL



VIRTUAL



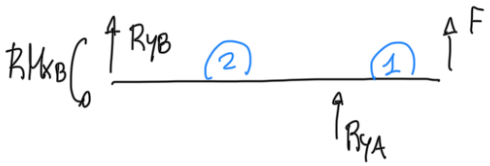
$$B\begin{cases} R_{zB} = \emptyset \\ R_{yB} + R_{yA} + F = \emptyset \\ RM_x + R_{yA}\cdot 2\ell + F\cdot 3\ell = \emptyset \end{cases}$$

2 Equations
3 Unknown → UNDETERMINED

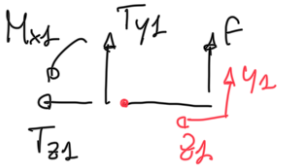
$$B\begin{cases} R'_{zB} = \emptyset \\ R'_{yB} = -1 \\ RM'_{xB} = -1\cdot 2\ell \end{cases}$$

It's not mandatory to have the same constraints in the REAL and VIRTUAL systems

- Internal Actions

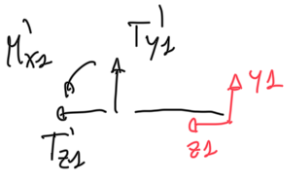


1) REAL



$$\begin{cases} T_{z1} = \emptyset \\ T_{y1} = -F \\ M_{x1} = -F\cdot z_1 \end{cases}$$

VIRTUAL

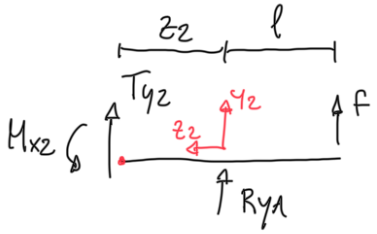


IT IS UNLOADED

$$\begin{cases} T'_{z1} = \emptyset \\ T'_{y1} = \emptyset \\ M'_{x1} = \emptyset \end{cases}$$

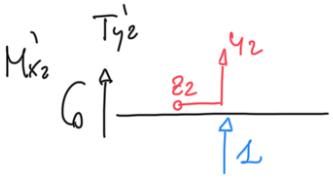
2)

REAL



$$\begin{cases} T_{y2} = -R_{yA} - F \\ M_{x2} = -R_{yA}\cdot z_2 - F\cdot(\ell + z_2) \end{cases}$$

VIRTUAL



$$\begin{cases} T'_{y2} = -1 \\ M'_{x2} = -1\cdot z_2 \end{cases}$$

- PCVW $\quad \delta W_e = \delta W_i$

$$\delta W_e = \delta F \cdot q = 1 \cdot \phi \qquad N \cdot mm$$

$$\delta W_i = \underbrace{\int_0^{\ell} M'_{x1} \cdot \frac{M_{x1}}{EJ_{xx}} \, dz_1}_{(1)} + \int_0^{2\ell} M'_{x2} \cdot \frac{M_{x2}}{EJ_{xx}} \, dz_2$$

$$Nmm \cdot \frac{Nmm}{\frac{N}{mm^2} \cdot mm^4} \cdot mm = N \cdot mm$$

$$= \frac{1}{EJ_{xx}} \int_0^{\ell} \phi \cdot [\dots] \, dz_1 + \frac{1}{EJ_{xx}} \int_0^{2\ell} (-z_2) \cdot (-R_{yA} \cdot z_2 - F\ell - F z_2) \cdot dz_2 = \phi$$

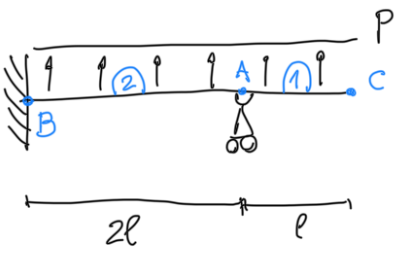
$$\int_0^{2\ell} (R_{yA} \cdot z_2^2 + F\ell z_2 + F z_2^2) \, dz_2 = \phi$$

$$\left[\frac{1}{3} R_{yA} \cdot z_2^3 + \frac{1}{2} F\ell z_2^2 + \frac{1}{3} F z_2^3 \right]_0^{2\ell} = \phi$$

$$\frac{8}{3} R_{yA} \cdot \ell^3 + 2F\ell^3 + \frac{8}{3} F\ell^3 = \phi$$

$$\frac{8}{3} R_{yA} + 2F + \frac{8}{3} F = \phi \qquad R_{yA} = -\frac{14}{8} F = -7000 \ N$$

2)



DATA

$\ell = 1000 \ mm$

$p = 1 \ N/mm$

$E = 200\,000 \ MPa$

$J_{xx} = 500\,000 \ mm^4$

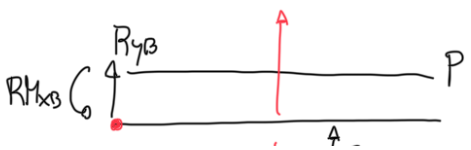
Let's find RF in (A)

4 Constraints ? Hyperstetic

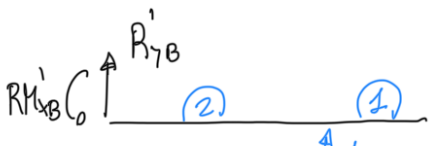
3 DoF

- RF

REAL



VIRTUAL



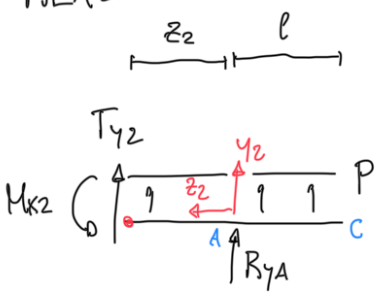
$$\begin{cases} R_{yB} + R_{yA} + 3p\ell = \emptyset \\ RM_{xB} = -3p\ell \cdot \frac{3\ell}{2} - R_{yA} \cdot 2\ell \end{cases} \qquad \begin{cases} R'_{yB} = -1 \\ RM'_{xB} = -1 \cdot 2\ell \end{cases}$$

- Internal Actions

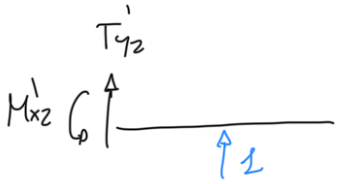
(1) Virtual system is unloaded → No virtual work

(2)

REAL



VIRTUAL



$$\begin{cases} T_{y2} = -R_{yA} - p(\ell + z_2) \\ M_{x2} = -R_{yA} \cdot z_2 - \frac{1}{2} p (\ell + z_2)^2 \end{cases} \qquad \begin{cases} T'_{y2} = -1 \\ M'_{x2} = -1 \cdot z_2 \end{cases}$$

- PCVW

$$\delta W_e = \emptyset$$

$$\delta W_i = \frac{1}{EJ_{xx}} \int_0^{2\ell} M'_{x2} \cdot M_{x2} \, dz_2 = \emptyset$$

$$\frac{1}{EJ_{xx}} \cdot \int_0^{2\ell} (-z_2) \cdot \left(-R_{yA} \cdot z_2 - \frac{1}{2} p (\ell + z_2)^2\right) dz_2 = \emptyset$$

$$\int_0^{2\ell} \left(R_{yA} \cdot z_2^2 + \frac{1}{2} p \ell^2 z_2 + \frac{1}{2} p z_2^3 + p \ell z_2^2\right) dz_2 = \emptyset$$

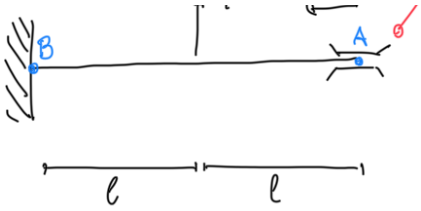
$$\left[\frac{1}{3} R_{yA} z_2^3 + \frac{1}{4} p \ell^2 z_2^2 + \frac{1}{8} p z_2^4 + \frac{1}{3} p \ell z_2^3\right]_0^{2\ell} = \emptyset$$

$$\frac{8}{3} R_{yA} \ell^3 + p\ell^4 + 2p\ell^4 + \frac{8}{3} p \ell^4 = \emptyset \qquad R_{yA} = -\frac{17}{8} p \ell = -2125\,N$$

3) A F  SLIDER

DATA

$\ell = 1000\ mm$



✓ it constrains:
- y translation
- x rotation

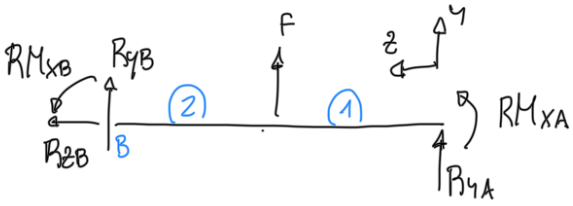
F = 6000 N
E = 200 GPa
J_{xx} = 500 000 mm^4

Let's find ALL the RFs in (A)

- 3 DoF < 5 Constraints → Hyperstatic
  We will need 2 virtual systems

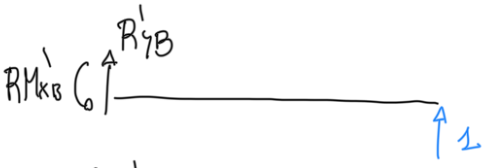
- RF

REAL



$$\begin{cases} R_{zB} = \phi \\ R_{yB} = -F - R_{yA} \\ RM_{xB} = -Fl - R_{yA} \cdot 2l - RM_{xA} \end{cases}$$

VIRTUAL 1



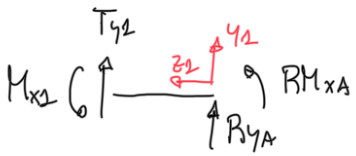
$$\begin{cases} R'_{yB} = -1 \\ RM'_{xB} = -2l \end{cases}$$

VIRTUAL 2

$$RM''_{xB} = -1$$

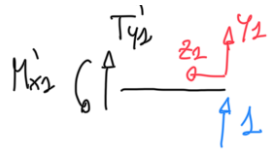
- Internal Actions

(1) REAL



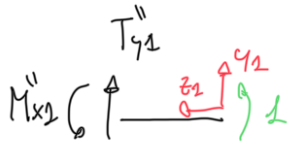
$$\begin{cases} T_{y1} = -R_{yA} \\ M_{x1} = -R_{yA} \cdot z_1 - RM_{xA} \end{cases}$$

VIRTUAL 1



$$\begin{cases} T_{y1} = -1 \\ M'_{x1} = -1 \cdot z_1 = -z_1 \end{cases}$$

VIRTUAL 2

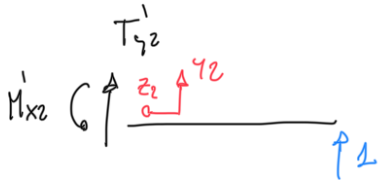


$$\begin{cases} T''_{y1} = \phi \\ M''_{x1} = -1 \end{cases}$$

(2) REAL

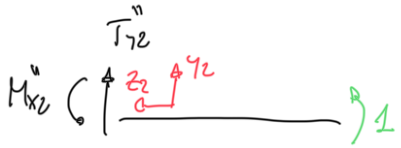
$$\begin{cases} T_{y2} = -F - R_{yA} \\ M_{xz} = -F \cdot z_2 - R_{yA}(l + z_2) - RM_{xA} \end{cases}$$

VIRTUAL 1



$$\begin{cases} T'_{y2} = -1 \\ M'_{x2} = -(l + z_2) \end{cases}$$

VIRTUAL 2



$$\begin{cases} T''_{y2} = \phi \\ M''_{x2} = -1 \end{cases}$$

- PCVW

VIRTUAL 1

$$\delta W_e = \phi$$

$$\delta W_i = \int_0^l \left(M'_{x1} \frac{M_{x1}}{EJ_{xx}}\right) dz_1 + \int_0^l \left(M'_{x2} \cdot \frac{M_{x2}}{EJ_{xx}}\right) dz_2 = \phi$$

VIRTUAL 2

$$\delta W_e = \emptyset$$

$$\delta W_i = \int_0^{\ell} \left(M_{x1}'' \cdot \frac{M_{x1}}{EJ_{xx}} \right) dz_1 + \int_0^{\ell} \left(M_{x2}'' \cdot \frac{M_{x2}}{EJ_{xx}} \right) dz_1 = \emptyset$$

$$R_{yA} = -2000\ N$$

$$RM_{xx} = 1000\,000\ Nmm$$