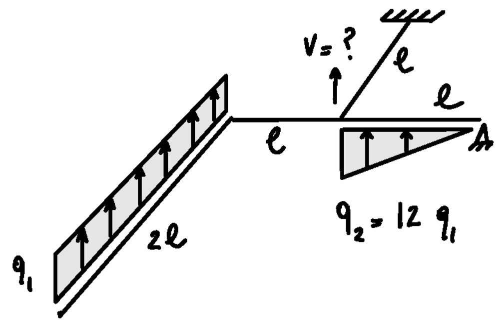
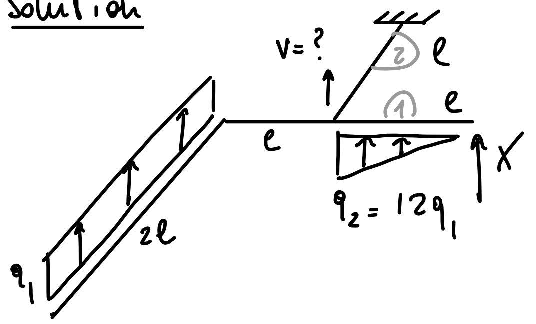
Exercise



The beams composing the structure in the figure have stiffnesses EJ and GJ. Shear deformability is negligible.
Determine the vertical displacement v as shown in the figure.
(Unit for result: mm)

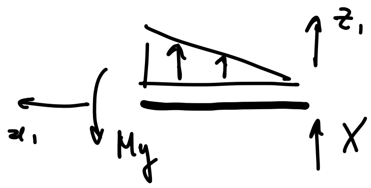
Data (solution for A = 0)
l = 1000. * (1 + A / 10); Units: mm
EJ = 1.0 * 1e12; Units: N mm^2
GJ = 1.0 * 1e12; Units: N mm^2
q1 = 1.0; Units: N / mm

Solution



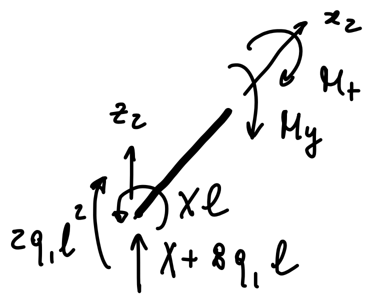
Rest system

- Beam 1



$$M_y = -X x_1 - 2 q_1 x_1^3/\ell$$

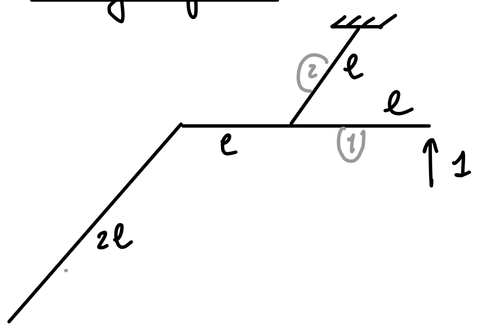
- Beam 2



$$M_y = -2 q_1 \ell^2 - (X + 8 q_1 \ell)\, x_2$$

$$M_t = X\ell$$

Dummy system # 1



- **Beam 1**

$'\delta M_y = -x_1$

- **Beam 2**

$'\delta M_y = -x_2$

$'\delta M_t = l$

By application of the PCVW:

$$\int_0^l {}'\delta M_y \frac{M_y}{EJ} dx_1 + \int_0^l {}'\delta M_y \frac{M_y}{EJ} dx_2 + \int_0^l {}'\delta M_t \frac{M_t}{GJ} dx_2 = 0$$

From which:

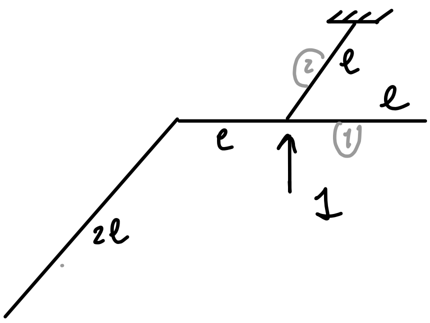
$$\frac{l^3}{15EJ}\left(5X + 6lq\right) + \frac{l^3}{3EJ}\left(X + 11\,l\,q\right) + \frac{l^3}{GJ} X = 0$$

and so:

$$X = -\frac{61\,GJ\,l\,q}{15EJ + 10\,GJ} = -2440\ N$$

A second dummy system is introduced for evaluating the displacement.

Dummy system #2



• Beam #2

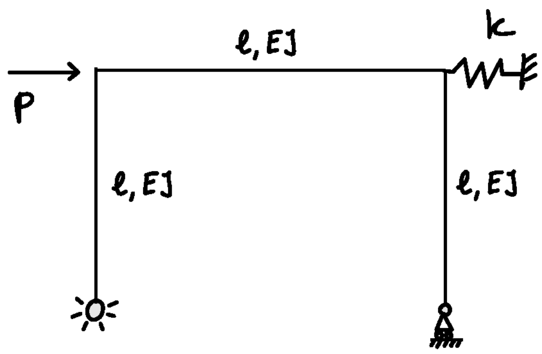
$$^{2}\delta M_y = -x_2$$

And by application of the PCVW:

$$\int_0^l M_y \frac{^{2}\delta M_y}{EJ} dx_2 = s \quad , \quad \text{from which:}$$

$$s = \frac{l^3}{3EJ}\left(X + 11\, l\, q\right) = 2.85 \text{ mm}$$

Exercise

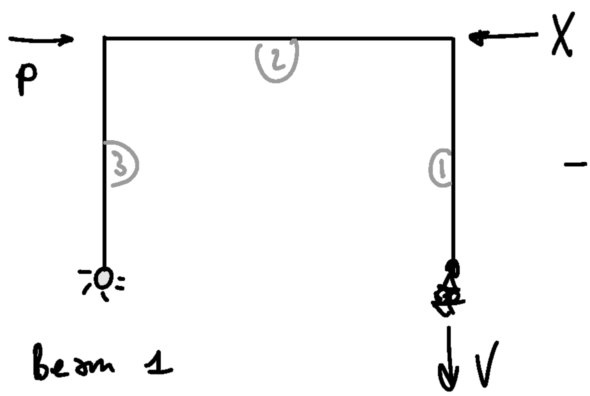


The structure in the figure is composed of three beams with same length l and bending stiffness EJ.
Shear deformability is negligible and so is the contribution due to the axial stiffness.
Determine the strain energy stored in the spring.
(Unit for result: N mm)

Data (solution for C = 0)
l = 1000. * (1 + C / 10); Units: mm
EJ = 1.0 * 1e12; Units: N mm^2
k = 1000.; Units: N / mm
P = 1000.; Units: N

Solution

Real system



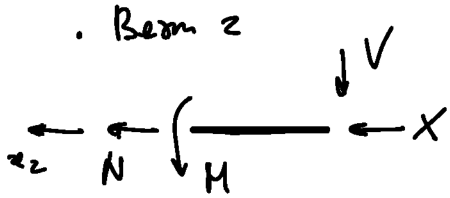
$$-Vl + Xl - Pl = 0$$

$$V = X - P$$

- beam 1

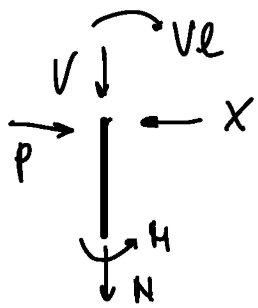
$$N = +V$$

- Beam 2



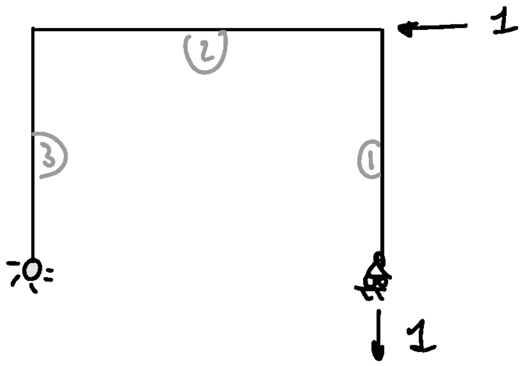
$$N = -X$$
$$M = +Vx_2$$

- Beam 3



$$N = -V$$
$$M = +Vl + (P - X)x_3$$

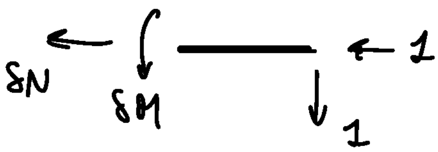
Dummy system



- Beam 1

  $\delta N = 1$

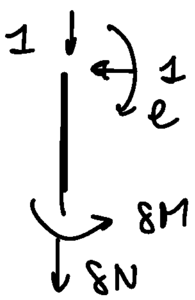
- Beam 2



$\delta N = -1$

$\delta M = x_2$

- Beam 3



$\delta N = -1$

$\delta M = l - x_3$

The PCVW reads

$$\int_0^\ell \delta N \frac{N}{EA} dx_1 + \int_0^\ell \delta N \, N/_{EA} \, dx_2 + \int_0^\ell \delta N \, N/_{EA} \, dx_3 +$$

$$+ \int_0^\ell \delta M \, M/_{EJ} \, dx_2 + \int_0^\ell \delta M \, M/_{EJ} \, dx_3 + X/_k = 0$$

$$\frac{\ell}{EA}\left(2V + X\right) + \frac{\ell^3}{3EJ} V + \frac{\ell^3}{6EJ}\left(P + 3V - X\right) + X/_k = 0$$

And substituting $V = X - P$:

$$\frac{\ell}{EA}\left(3X - 2P\right) + \frac{\ell^3}{3EJ} 2\left(X - P\right) + X/_k = 0$$

From which:

$$X = \frac{2EA\,k\ell^3 + 6EJ\,k\ell}{2EA\,k\ell^3 + 9EJ\,k\ell + 3EA\,EJ} P$$

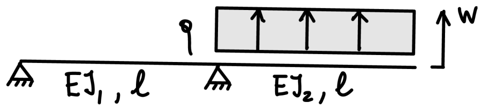
If the energy contribution of the axial stiffness is neglected:

$$X = \frac{2k\ell^3}{2k\ell^3 + 3EJ} P = 400 \text{ N}$$

And the strain energy reads:

$$U_{spring} = \frac{1}{2} X^2/k = 80 \text{ Nmm}$$

Exercise

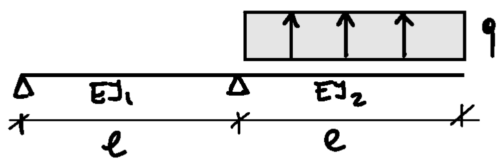


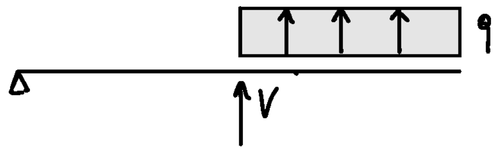
The structure in the figure is composed of two beams with bending stiffnesses equal to EJ1 and EJ2.
Determine the bending stiffness EJ2 such that the vertical displacement at the free end is equal to w when the structure is loaded with a uniformly distributed force per unit length.
Report the result as EJ2 / EJ1.
(Unit for result: adim)

Data

l = 1000.; Units: mm
EJ1 = 1.0 * 1e12; Units: N mm^2
q = 1.0; Units: N / mm
w = 0.5 * (1 + A / 10); Units: mm

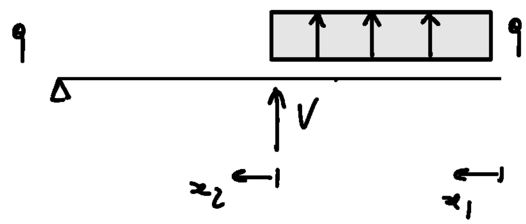
Solution

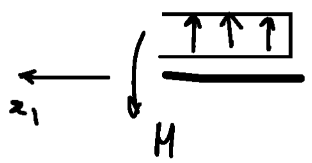




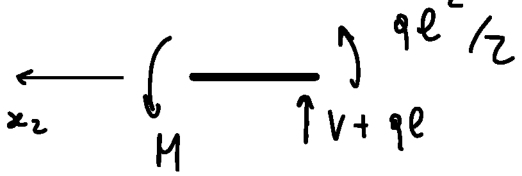
$$q l \frac{3}{2} l + V l = 0 \quad \Rightarrow \quad V = -\frac{3}{2} q l$$

Rest system



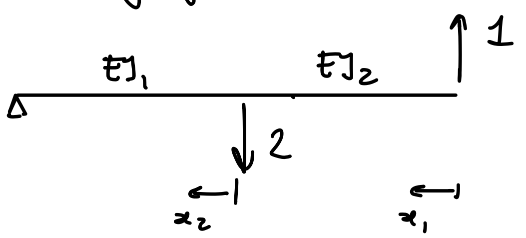


$$M = -q x_1^2/2$$



$$M = -q l^2/2 - (V + q l)\, x_2$$

Dummy system



$$\delta M = -x_1 \qquad (0 \le x_1 \le l)$$

$$\delta M = +x_2 - l \qquad (0 \le x_2 \le l)$$

By application of the PCVW:

$$\int_0^l \delta M \, M/EJ_2 \, dx_1 + \int_0^l \delta M \, M/EJ_1 \, dx_2 = w$$

From which:

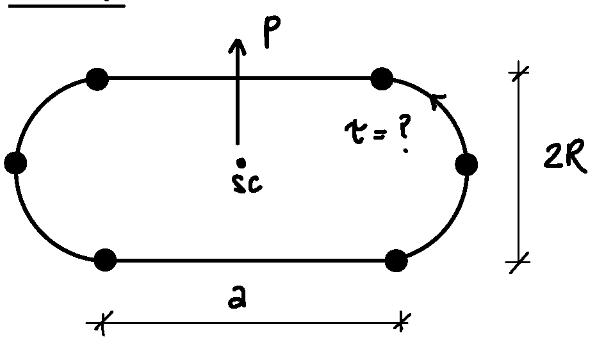
$$\frac{l^3}{12EJ_1}\left(2V + 5ql\right) + \frac{ql^4}{8EJ_2} = w$$

And upon substitution of V:

$$\frac{ql^4}{8EJ_2} + \frac{ql^4}{6EJ_1} = w \quad , \text{ from which:}$$

$$EJ_2 = \frac{ql^4}{8w - \frac{4ql^4}{3EJ_1}} \quad , \text{ so } \frac{EJ_2}{EJ_1} = 0.375$$

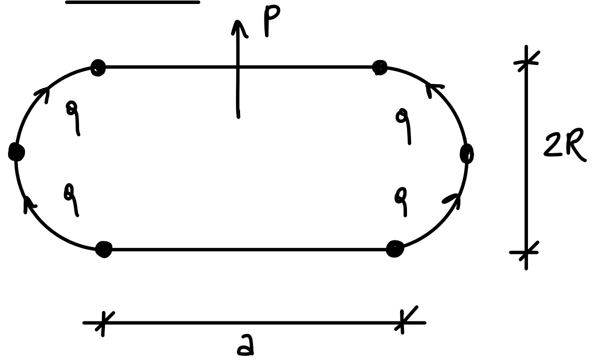
Exercise



The thin-walled beam in the figure is subjected to an internal shear force equal to P. The force is referred to the shear center of the section. The area of the lumped stringers, including the contribution of the panels, is equal to A; the panels have thickness equal to t.
By using a semi-monocoque approximation of the section, determine the shear stress in the panel indicated in the figure.
(Unit for result: MPa)

Data (solution for A = 0)
a = 200.; Units: mm
R = 45.; Units: mm
t = 1.2 * (1 + A / 10); Units: mm
P = 9000.; Units: N

Solution



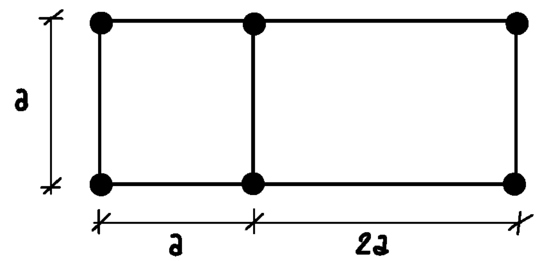
Due to the symmetry of the structure, the shear flows are readily found as shown in the figure, where:

$$q \cdot 2R \cdot 2 = P \quad , \text{ so: } q = \frac{P}{4R} = 50 \text{ N/mm}$$

The shear stress in the panel is then:

$$\tau = q / t_2 = 41.67 \text{ MPa}$$

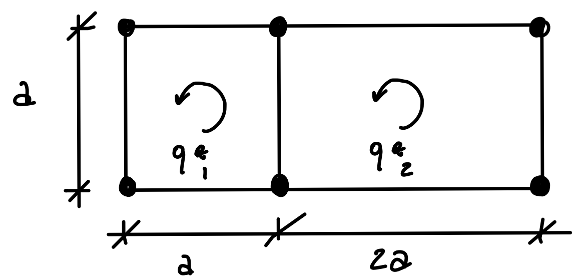
Exercise



Consider the two-cell section in the figure. The area of the lumped stringers, including the contribution of the panels, is equal to A; the panels have thickness equal to t. The shear modulus of the material is G.
Determine the torsional stiffness GJ. For this purpose, adopt a semi-monocoque approximation of the section.
Report the result as GJ / GJref, where GJref is reference value available in the data.
(Unit for result: adim)

Data (solution of C = 0)
a = 100. * (1 + C / 10); Units: mm
t = 1.5; Units: mm
A = 400.; Units: mm^2
G = 27000.; Units: MPa
GJref = 1.0 * 1e10; Units: N mm^2

Solution



- Equiv. moment:

$$2\Omega_1 q_1^* + 2\Omega_2 q_2^* = M_t \qquad \text{with } \Omega_1 = a^2;\ \Omega_2 = 2\Omega_1$$

So:

$$q_1^* + 2q_2^* = M_t / 2\Omega_1$$

- Compatibility

$$2G\Omega_1 t\,\theta_1' = q_1^*\, 4a - q_2^*\, a$$

$$2G\Omega_2 t\,\theta_2' = q_2^*\, 6a - q_1^*\, a$$

Imposing $\theta_1' = \theta_2'$:

$$8q_1^* - 2q_2^* = 6q_2^* - q_1^*, \text{ from which}$$

$$9q_1^* - 8q_2^* = 0 \Rightarrow q_1^* = \frac{8}{9} q_2^*$$

$$8/9\, q_2^* + 2q_2^* = M_t / 2\Omega_1$$

$$q_2^* = \frac{9}{26}\frac{M_t}{2\Omega_1} \quad ; \quad q_1^* = \frac{4}{13}\frac{M_t}{2\Omega_1}$$

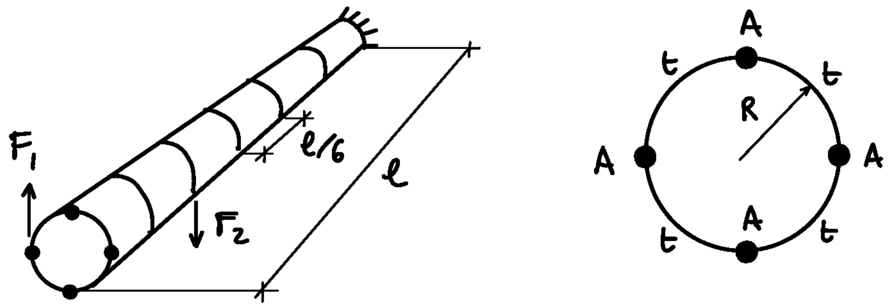
$$\theta_1' = \frac{1}{26\Omega_1 t}\left(4q_1^* - q_2^*\right) a$$

$$= M_t \frac{23}{104}\frac{1}{Ga^3 t}$$

$$GJ = M_t / \theta_1' = G \; 104/_{23}\, a^3 t = 1.83 \cdot 10^{11} \ \text{Nmm}^2$$

$$GJ / GJ_{ref} = 18.31$$

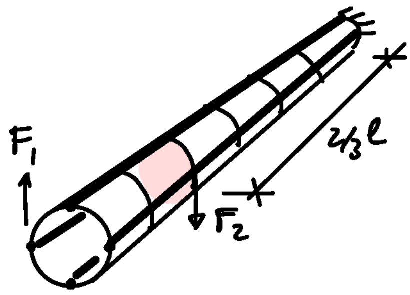
Exercise



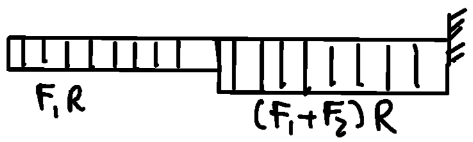
The thin-walled beam in the figure is loaded with two concentrated forces F1 and F2. The lumped area of the stringers, including the contribution of the panels, is equal to A. The thickness of panels is t. The shear modulus of the material is G.
By using a semi-monocoque approximation, determine the rotation of the section at a distance equal to 1 / 3 l from the free end.
Report the absolute value of the rotation angle expressed in degrees.
(Unit for result: deg - absolute value)

Data (solution for E = 0)
l= 2500.; Units: mm
R = 25.; Units: mm
t = 0.6; Units: mm
A = 500; Units: mm^2
G = 27000.; Units: MPa
F1 = 2000.; Units: N
F2 = 5000. * (1 + E / 10); Units: N

Solution



The position of the shear center is readily available due to the section symmetry. The torsional moment evaluated with respect to the shear center is:



The torsional constant of the section is evaluated referring to the Bredt's formula:

$$J = \frac{4\Omega^2}{\oint_p 1/t \, ds} \quad \text{where } \Omega = \pi R^2, \text{ so:}$$

$$= \frac{4\pi^2 R^4 t}{2\pi R} = 2\pi R^3 t$$

Recalling that the torsion is $\theta' = M_t / GJ$, it follows that:

$$\theta' = (F_1 + F_2) R / GJ$$

and the rotation of the section at $x = 2/3$ is

$\theta = \theta' \, 2/3 \, \ell = 10.51$ deg

- A structure is modeled using finite elements. It is unconstrained and subjected to a set of loads in self equilibrium. The solution of the linear static problem:
  - can be obtained after constraining the structure isostatically
  - ~~is defined up to a rigid body motion; thus, not being unique, can never be obtained~~
  - ~~is stress-free~~
  - ~~can be obtained only if the loads are concentrated~~
- Consider a truss fixed at one end and free the other, and loaded with a uniformly distributed traction. The finite element solution obtained with quadratic elements:
  - ~~is an approximation of the exact solution~~
  - is exact for both displacement and axial force
  - ~~is exact for the displacement, but approximated for the axial force~~
  - ~~is exact for the the axial force, but approximated for the displacement~~
  - ~~is exact for the the displacement and strain, but approximated for the axial force~~
- The torsional stiffness of a single-cell thin-walled beam:
  - ~~is zero according to the semi-monocoque approximation~~
  - ~~requires first the shear center position to be evaluated~~
  - can be evaluated using the Bredt's formula
  - ~~can be evaluated using Eulero's formula~~
- The polynomial order of the finite element shape functions does not affect the rate of convergence of the solution
  - ~~True~~
  - False
- The torsional stiffness of an open section profile modeled using the semi-monocoque scheme is null
  - True
  - ~~False~~
- Consider a Euler-Bernoulli beam, whose static solution is obtained using the FE method. The approximating functions need to be C^2
  - ~~True~~
  - False