Course of Aerospace Structures

Written test, February 07th, 2025

Name ______________________________

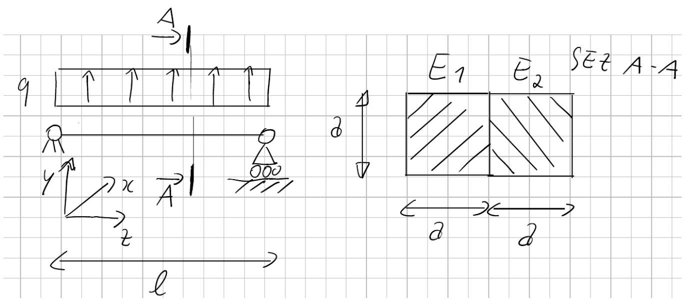
Surname ______________________________

Person code: | | | | | | | | |

Exercise 1

The simply supported beam in the figure cross section is made with two different materials, as sketched in Sez A-A. The beam is loaded with a distributed force per unit of length q. Compute the value of the vertical displacement v in the middle of the beam, i.e. at $z = l/2$.
Neglect shear deformability.

(Unit for result: mm)

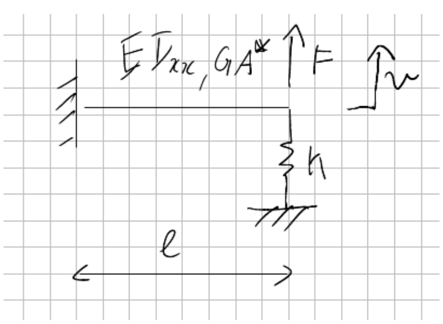


Data
$l = 1500$ mm
$a = 30$ mm
$E_1 = 70000$ MPa
$E_2 = 210000$ MPa
$\nu = 0.3$
$q = 1$ N/mm

Answer ______________________________

Exercise 2

The beam structure sketched in the figure is loaded by the force F, applied at the beam extremity. Compute the value of the displacement v in direction y at the point of application of the force. **Do not** neglect shear deformability; rather, **do account** for shear deformability. (Unit for result: mm)



Data
$l = 1000$ mm
$EI_{xx} = 1 \times 10^{12}$ N mm^2
$GA = 1 \times 10^7$ N
$k = 1000$ N/mm
$F = 100$ N

Answer ______________________________

Exercise 3

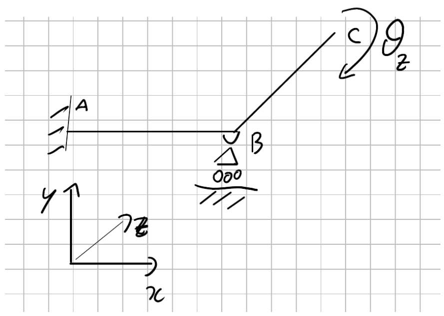
The continuous beam structure sketched in the figure has a prescribed rotation θ_z, around the z axis, at point C (a moment is applied there, leaving free the three displacement components and the other two rotation components).

Compute the reaction force in the y direction transmitted by the constraint of point B.

Neglect shear deformation. The coordinates of the points are given, with respect to the sketched reference system, in the data. The bending and torsional stiffness of the beams are given with respect to local reference systems, with the z axis aligned with the beam axis.

Be careful with the measurement units.

(Unit for result: N)



Data

$A : (0; 1000; 0)$ mm

$B : (1000; 1000; 0)$ mm

$C : (1000; 1000; 1000)$ mm

$\theta_z = 5^{\circ}$

$EI_{xx} = EI_{yy} = 1 \times 10^{10}$ N mm^2

$GJ = 2 \times 10^{11}$ N mm^2

Answer ______________________

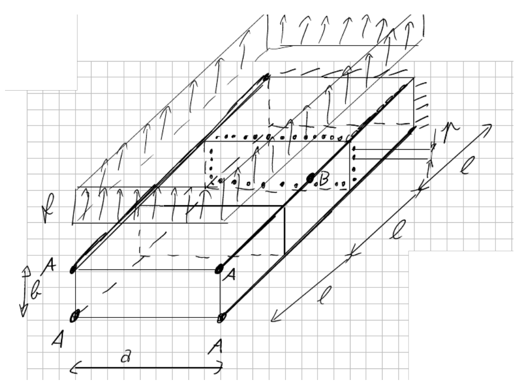
Exercise 4

The semi-monocoque beam sketched in the figure, with overall length $3l$, has four panels, each with thickness t, and four concentrated areas, each with area A. It is loaded on the upper panel, by force per unit of surface f. The load is introduced into the structure by four different ribs, with pitch equal to l (the first rib is at the clamp, the last one at the beam free extremity). The four panels are continuous all over the structure in the z direction.

The third rib from the free extremity (the one with all the dots near to it) is attached to the outer panels all around its boundary by means of rivets that are put in place with a pitch (a distance one from each other) equal to p. The rivets shank has a radius equal to r.

Compute the shear stress in the rivets that connect the rib to the lateral panel.

(Unit for result: MPa)



Data

$l = 1000$ mm

$a = 700$ mm

$b = 100$ mm

$t = 1$ mm

$A = 500$ mm^2

$E = 70000$ MPa

$\nu = 0.3$

$f = 0.01$ N/mm^2

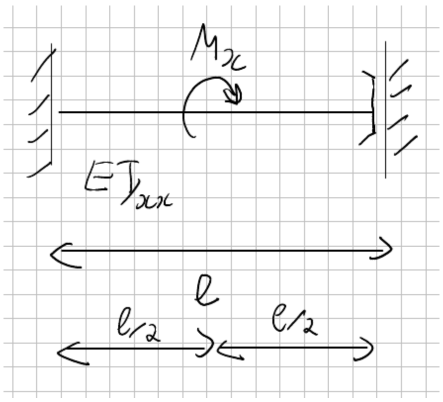
$r = 1.5$ mm

$p = 25$ mm

Answer ______________________

Exercise 5
Consider the slender beam structure sketched in the figure, loaded in the middle by the concentrated moment M_x. Estimate the transverse displacement v at the right extremity by assuming a suitable trigonometric approximation of the transverse displacement truncated to the first non null term.
Neglect shear deformability.

(Unit for result: mm)

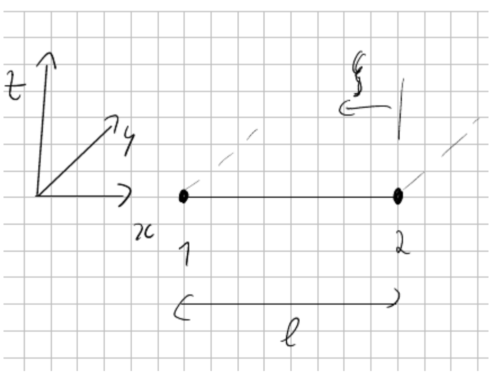


Data
$l = 3000$ mm
$EI_{xx} = 1 \times 10^{13}$ N mm^2
$M_x = 1 \times 10^5$ N mm

Answer ______________________

Exercise 6
The two node beam finite element in the figure interpolates linearly, and independently, the transverse displacements and the rotations. Compute the contribution stemming from the bending stiffness EI_{xx} to the diagonal term of the element stiffness matrix related to the rotation around axis y of node 1 ($K_{\theta_y \theta_y}$ of node 1).

(Unit for result: N mm)



Data
$l = 5$ mm
$EI_{xx} = 1 \times 10^{10}$ N mm^2

Answer ______________________

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. Shear locking affects beam finite elements based on Hermitian shape functions.
2. The natural vibration frequency of a tensioned beam is lower than that of an unloaded beam.
3. The Neumann boundary condition reads $\boldsymbol{\sigma} \cdot \mathbf{n} = \mathbf{f}$, where $\mathbf{f}$ is the applied load per unit of volume.

Multiple Choice questions

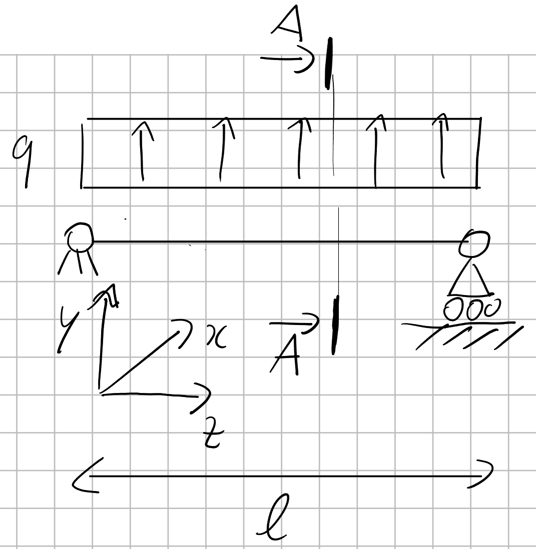
(Circle the correct answer)

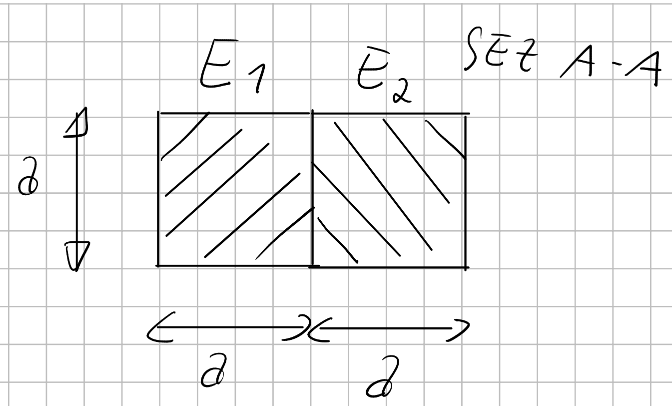
1. Knowing the axial stiffness EA, the two shear stiffness GA_x^* and GA_y^*, the torsional stiffness GJ and the two bending stiffness EI_{xx} and EI_{yy}
   (a) is enough to fully characterize the behavior of a beam cross section
   (b) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position of the shear center wrt to the cross section
   (c) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position of the beam principal axis wrt to the cross section
   (d) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position and orientation of the beam principal axis wrt to the cross section
   (e) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position of both the beam principal axis and shear center wrt to the cross section
   (f) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position of both the beam principal axis and shear center wrt to the cross section, together with the orientation of the principal axes
   (g) none of the above

2. The resultant over the cross section of a beam of the axial normal stress, $N = \int_A \sigma_{zz} \mathrm{d}A$ has the dimension of a force. It is:
   (a) work conjugated with the axial displacement
   (b) work conjugated with the derivative of the axial displacement
   (c) work conjugate with the derivative of the axial rotation
   (d) work conjugated with the shear deformation
   (e) none of the above

3. A riveted connection between two panels loaded in-plane cannot fail due to:
   (a) shear stress in the panels
   (b) shear stress in the rivet
   (c) axial stress in the rivet
   (d) bearing
   (e) axial stress in the panels
   (f) none of the above

1





$$EJ_{xx} = \frac{1}{12} a^4 (E_1 + E_2)$$

$$T_y = -\frac{q\ell}{2} + qz$$

$$\int_0^\ell \frac{M_x M_x'}{EJ_{xx}} = 1 \cdot v$$

$$M_x = +\frac{q\ell}{2} z - \frac{1}{2} q z^2$$

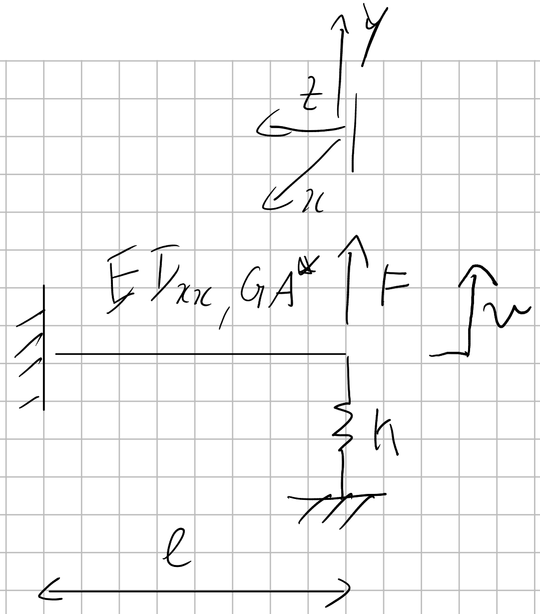
$$T'_y = +\frac{1}{2} \qquad 0 \le z \le \frac{\ell}{2}$$

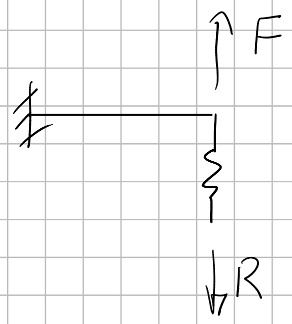
$$-\frac{1}{2} \qquad \frac{\ell}{2} < z < \ell$$

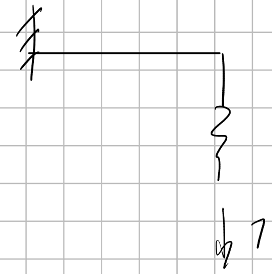
$$M'_x = \frac{1}{2} z \qquad 0 < z < \frac{\ell}{2}$$

$$\frac{1}{2} z - \left(z - \frac{\ell}{2}\right) \qquad \frac{\ell}{2} < z < \ell$$

(2)







$$T_y = -F + R$$

$$M_x = -Fz + Rz$$

$$T'_y = 1$$

$$M'_x = z$$

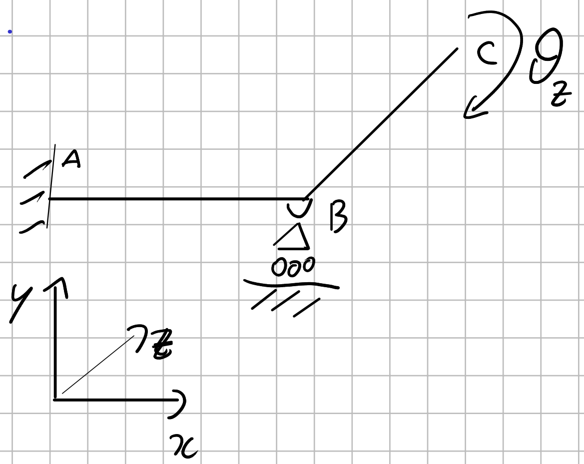
$$\int_0^l \left(\frac{M_x \, M_x'}{EI_{xx}} + \frac{T_y \, T'_y}{GA^*} \right) dz + \frac{R \cdot 1}{K} = \emptyset$$

$$1{,}4333E-3\,R - 0{,}04333 = \emptyset$$

$R = \ldots$

$$v = \frac{R}{K}$$

3



A : (0, 1000, 0) mm

B : (1000, 1000, 0) mm

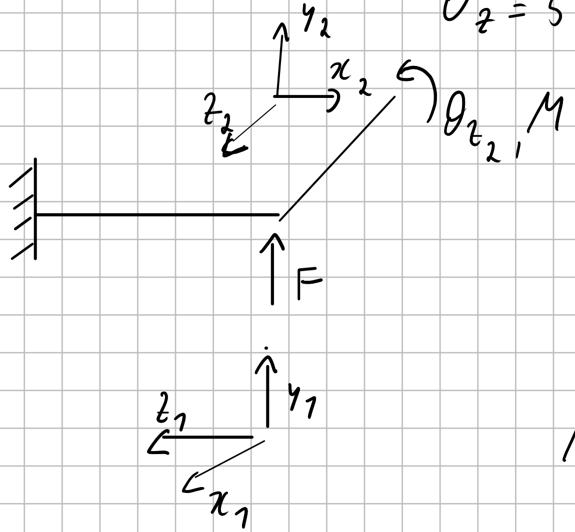
C : (1000, 1000, 1000) mm

prescribed $\theta_z = 5^\circ$

$EI_{xx} = EI_{yy} = 1E10 \ N\,mm^2$

$GJ = 2E11 \ N\,mm^2$

compute reaction force at point B



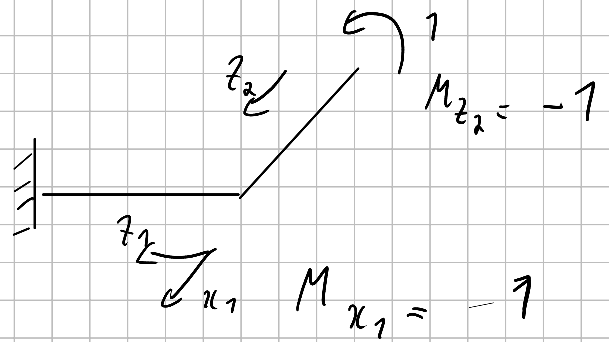
$$\theta_z = 5^\circ = \frac{5}{180} \cdot \pi \text{ rad} \simeq 0{,}08727 \text{ rad}$$

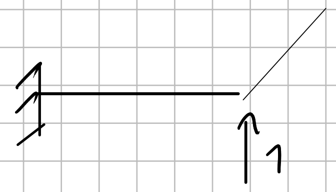
$$\theta_{z_2}^{(0)} = -\theta_z = -0{,}08727 \text{ rad}$$

$M_{z_2} = -M$, where M is unknown

F unknown

$$M_{x_1} = -M - F z_1$$





$$M_{x_1} = -z_1$$

$$l = 1000 \text{ mm}$$

$$\begin{cases} \displaystyle\int_0^l \frac{M}{GJ}\, dz_2 + \int_0^l \frac{M + F z_1}{E J_{xx}}\, dz_1 = -0,08727 \\[2ex] \displaystyle\int_0^l \frac{M z_1 + F z_1^2}{E J_{xx}}\, dz_1 = 0 \end{cases}$$

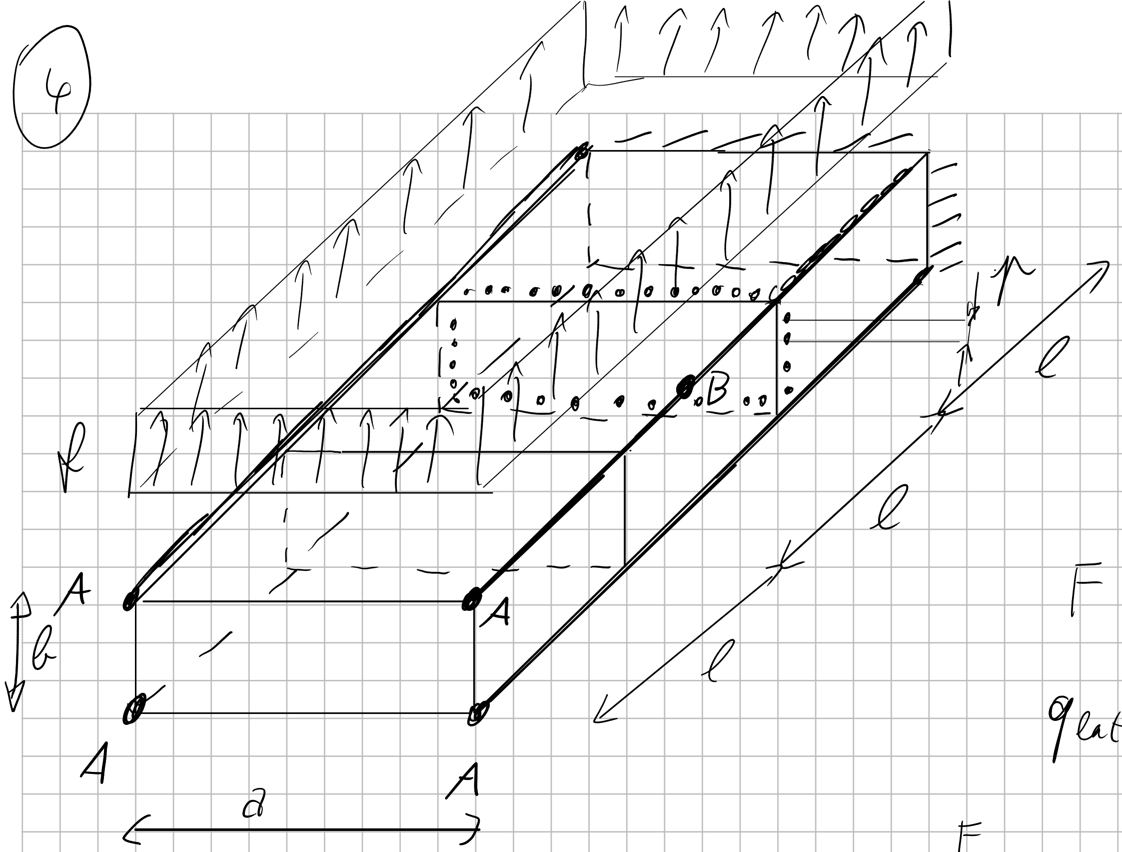
$$\begin{cases} \displaystyle\frac{M}{GJ} \cdot l + \frac{M}{E J_{xx}} l + F \frac{l^2}{2 E J_{xx}} = -0,08727 \\[2ex] \displaystyle M \frac{l^2}{2 E J_{xx}} + F \frac{l^3}{3 E J_{xx}} = 0 \end{cases}$$

$$
\begin{bmatrix} \dfrac{l}{GJ} + \dfrac{l}{EJ_{xx}} & \dfrac{l^2}{2EJ_{xx}} \\[2ex] -\dfrac{l^2}{2EJ_{xx}} & \dfrac{l^3}{3EJ_{xx}} \end{bmatrix} \begin{Bmatrix} M \\[2ex] F \end{Bmatrix} = \begin{Bmatrix} -0{,}08727 \\[2ex] 0 \end{Bmatrix}
$$

$$M = -2{,}909\,E6 \text{ N mm}$$

$$F = 4{,}364\,E3 \text{ N}$$

4

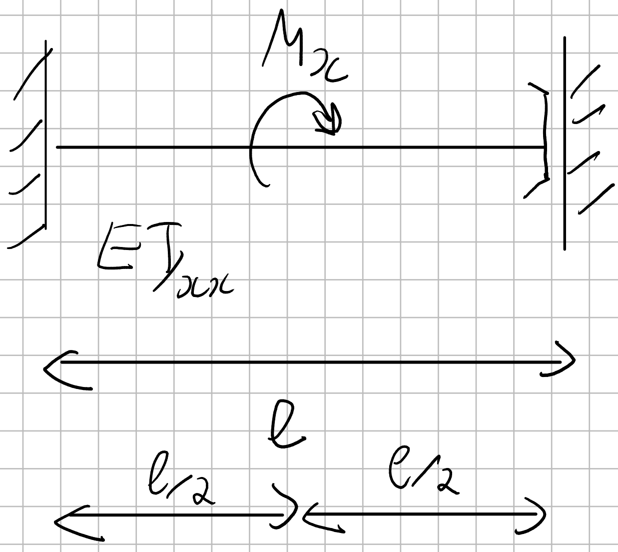


$$F = p \cdot a \cdot l$$

$$q_{lat} = \frac{F}{2b}$$

$$F_{smuc} = q_{lat} \cdot r$$

$$\tilde{\tau} = \frac{F_{smuc}}{\pi r^2}$$



$$v = \left(1 - \cos\left(\frac{\pi z}{l}\right)\right) c$$

$$v' = \frac{\pi}{l} \sin\left(\frac{\pi z}{l}\right) c$$

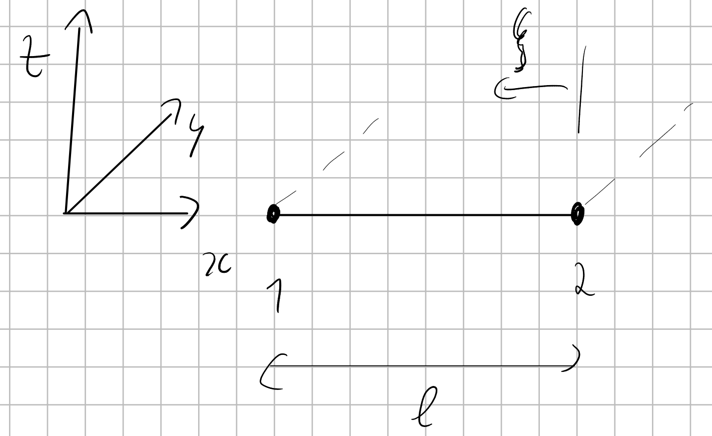
$$v'' = \frac{\pi^2}{l^2} \cos\left(\frac{\pi z}{l}\right) c$$

$$\int_0^l \delta c \, \frac{\pi^4}{l^4} \cos^2\left(\frac{\pi z}{l}\right) EJ_{xx} \, c \, dz = \delta c \, \frac{\pi}{l} \cdot 1 \, (-M_x)$$

$$\frac{\pi^{\cancel{4}\,3}}{l^{\cancel{4}}\, 2} \, \frac{\cancel{l}}{2} \, EJ_{xx} \, c = \frac{-\cancel{\pi}}{\cancel{l}} \, M_x \qquad c = \frac{-M_x \cdot 2 l^2}{\pi^3 EJ_{xx}}$$

$$v(l) = 2c$$

⑥



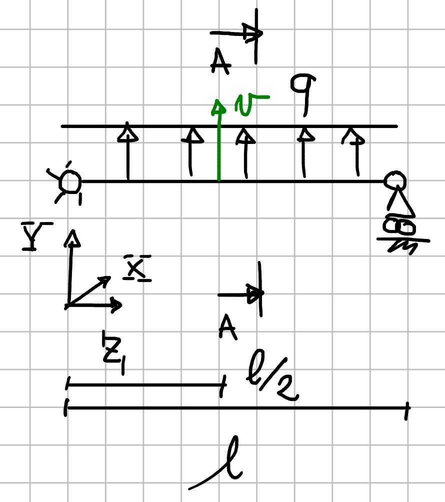
$$K_{\theta_{y1}\cdot\theta_{y1}}$$

$$\varphi_y = \theta_{y1} \cdot \frac{\xi}{l} \qquad \frac{\varphi_y}{\partial x} = -\frac{\varphi_y}{\partial \xi} = -\frac{\theta_{y1}}{l}$$

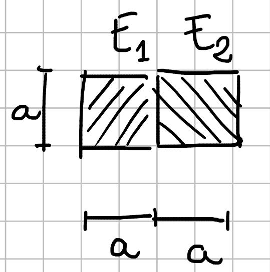
$$K_{\theta_{11}\,\theta_{y1}} = \int_0^l \frac{EI_{yy}}{l^2}\,dx = \frac{EI_{yy}}{l}.$$

EXAM 07/02/2025

Ex 1



SEZ A-A



DATA

$\ell = 1500$ mm $v = ?$

$a = 30$ mm

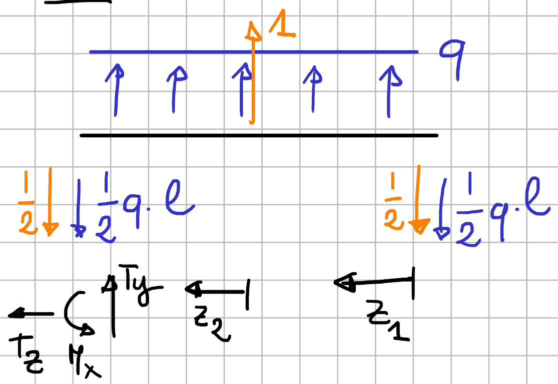
$E_1 = 70\,000$ MPa

$E_2 = 210\,000$ MPa

$\nu = .3$

$q = 1$ N/mm

SOL



BEAM 1

$$M_{x_1} = +\frac{1}{2} q \cdot \ell \cdot z_1 - q \cdot z_1 \cdot \frac{1}{2} z_1 = \frac{1}{2} q \left(\ell z_1 - z_1^2\right)$$

$$M'_{x_1} = +\frac{1}{2} z_1$$

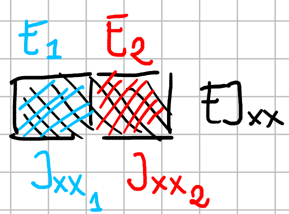
BEAM 2

$$M_{x_2} = \frac{1}{2} q \cdot \ell \left(\frac{\ell}{2} + z_2\right) - q \cdot \left(z_2 + \frac{\ell}{2}\right) \cdot \frac{1}{2}\left(z_2 + \frac{\ell}{2}\right) = \frac{1}{2} q \left(\frac{\ell^2}{2} + \cancel{z_2 \ell} - z_2^2 - \cancel{z_2 \ell} - \frac{\ell^2}{4}\right)$$

$$M'_{x_2} = \frac{1}{2}\left(z_2 + \frac{\ell}{2}\right) - 1 \cdot z_2 = \frac{1}{2}\left(\frac{\ell}{2} - z_2\right) \qquad = \frac{1}{2} q \left(\frac{\ell^2}{4} - z_2^2\right)$$

FROM COMPATIBILITY THE STRAINS ARE IDENTICAL

THE TWO PARTS ARE CONNECTED AND BEHAVE AS A SINGLE BEAM



$$EJ_{xx} = E_1 J_{xx_1} + E_2 J_{xx_2}$$

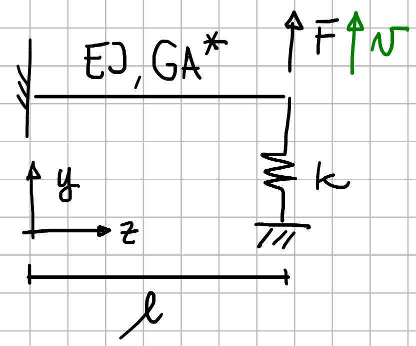
PCVW

$$\delta W_e' = 1 \cdot v$$

$$\delta W_i' = \int_0^{l/2} M_{x_1} \cdot \frac{M_{x_1}'}{EJ_{xx}} \, dz_1 + \int_0^{l/2} M_{x_2} \cdot \frac{M_{x_2}'}{EJ_{xx}} \, dz_2$$

$$\delta W_e' = \delta W_i' \quad \rightsquigarrow \quad v$$

Ex. 2



DATA

$l = 1000\ mm$

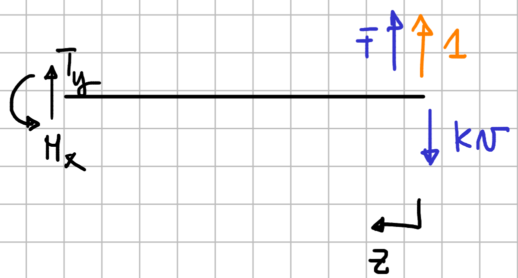
$EJ_{xx} = 1 \cdot E+12\ N mm^2$

$GA^* = 1E+07\ N$

$k = 1000\ N/mm$

$F = 100\ N$

SOL



$$T_y = -F + kN \qquad T_y' = -1$$

$$M_x = -F \cdot z + kN \cdot z \qquad M_x' = -1 \cdot z$$

PCVW

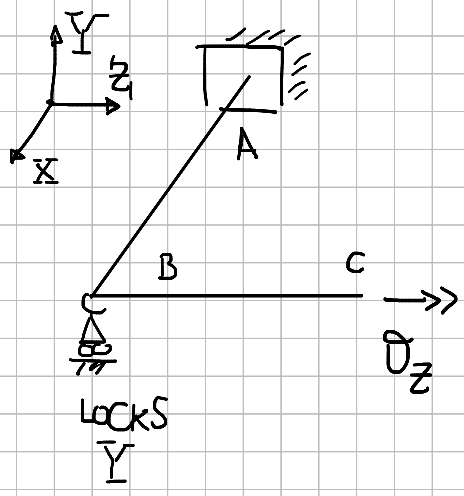
$$\delta W_e' = 1 \cdot N$$

$$\delta W_i' = \int_0^l T_y \cdot \frac{T_y'}{GA^*} + M_x \cdot \frac{M_x'}{EJ_{xx}}\, dz$$

$$= \frac{Fl}{GA^*} - \frac{kN \cdot l}{GA^*} + \frac{1}{3}\frac{Fl^3}{EJ_{xx}} - \frac{1}{3}\frac{kNl^3}{EJ_{xx}}$$

$$N = \left(\frac{Fl}{GA^*} + \frac{1}{3}\frac{Fl^3}{EJ_{xx}}\right)\left(1 + \frac{kl}{GA^*} + \frac{1}{3}\frac{kl^3}{EJ_{xx}}\right)^{-1}$$

Ex 3



DATA

A: $(0, 1000, 0)$ mm

B: $(1000, 1000, 0)$ mm

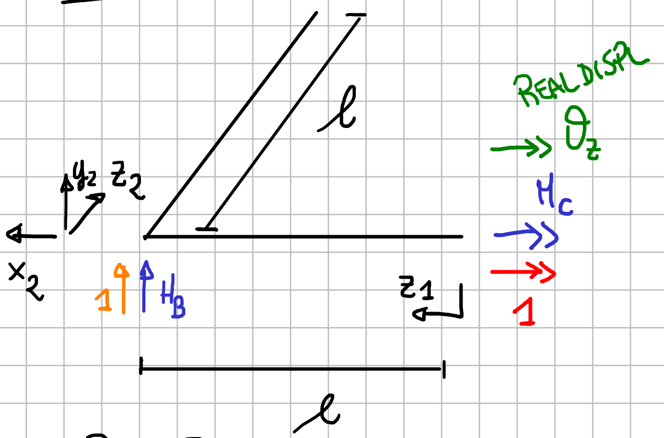
C: $(1000, 1000, 1000)$ mm

$\vartheta_z = 5^\circ \rightsquigarrow$ USE rad

$EJ_{xx} = EJ_{yy} = 1E+10 \ N\,mm^2$

$GJ = 2E+11 \ N\,mm^2$

SOL



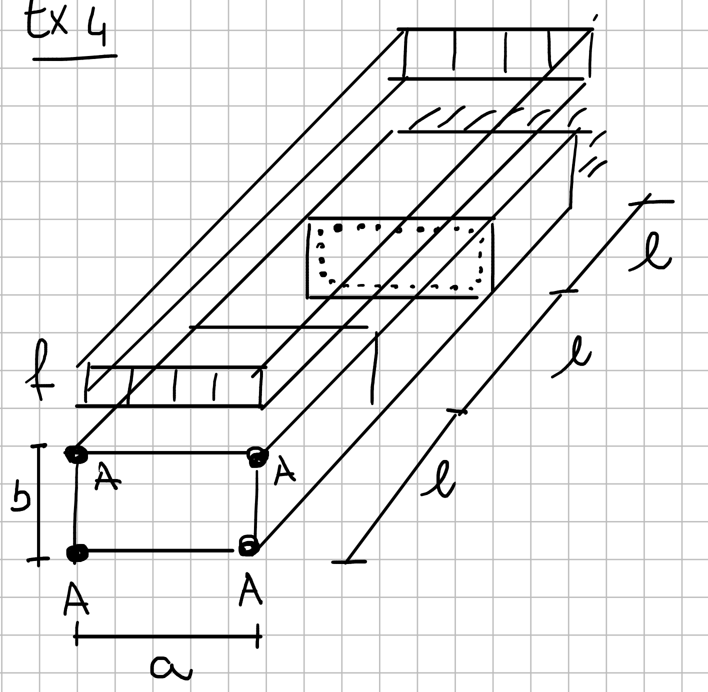
- $M_{z_1} = M_C \qquad M_{x_2} = M_C - M_B \cdot z_2$
- $M'_{z_1} = 0 \qquad M'_{x_2} = -1 \cdot z_2$
- $M''_{z_1} = 1 \qquad M''_{x_2} = 1$

$\ell = 1000$ mm

PCVW

$$\begin{Bmatrix} 0 \\ \vartheta_z \end{Bmatrix} = \begin{Bmatrix} \dfrac{1}{EJ_{xx}} \displaystyle\int_0^\ell M_B \cdot z_2^2 - M_C \cdot z_2 \, dz_2 \\ \dfrac{1}{GJ} \cdot \displaystyle\int_0^\ell M_C \, dz_1 + \dfrac{1}{EJ_{xx}} \displaystyle\int_0^\ell M_C - M_B \cdot z_2 \, dz_2 \end{Bmatrix} \rightsquigarrow \text{SOLVE FOR } M_B \ \& \ M_C$$

Ex 4



DATA

$l = 1000$ mm

$a = 700$ mm

$b = 100$ mm

$t = 1$ mm

$p = 25$ mm

$A = 500$ mm^2

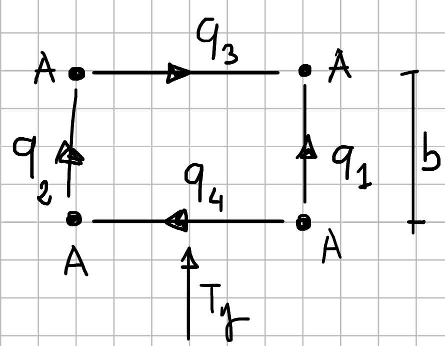
$E = 70000$ MPa

$\nu = 0.3$

$f = 0.01$ N/mm^2

$r = 1.5$ mm

SOL



FROM SYMMETRY: $q_3 = q_4 = 0$ $\quad q_1 = q_2$

$$q_1 = q_2 = \frac{T_y}{2b}$$ → FORCE EQUIVALENCE

THE RIVETS TRANSFER ONLY THE LOAD INTRODUCED BY THE RIB AS THE EXTERNAL PANEL ARE CONTINUOS IN Z

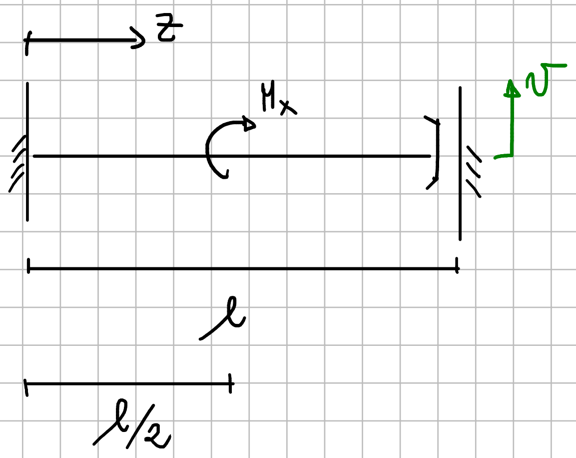
→ THE RIB TRANSFERS TO THE VERTICAL PANEL A FLUX:

$$\hat{q}_1 = \underbrace{f \cdot a \cdot l}_{\text{FORCE INTRODUCED BY RIB. } (T_y)} \cdot \frac{1}{2b} = \frac{1}{2} f \cdot l \cdot \frac{a}{b}$$

EACH RIVET CARRIES $\quad F = \hat{q}_1 \cdot p = \frac{1}{2} f \cdot l \frac{a}{b} \cdot p$

SHEAR STRESS $\quad \tau = \frac{F}{A} = \frac{1}{2} f \cdot l \frac{a}{b} p \frac{1}{\pi r^2}$

Ex 5



DATA

$l = 3000\ mm$

$M_x = 1E+05\ Nmm$

$EJ_{xx} = 1E+13\ Nmm^2$

FIND $v(l)$

SOL

DISPL. APPROX: $v = A\overbrace{\left(1 - \cos\left(\frac{\pi z}{l}\right)\right)}^{\phi(z)}$ $\rightsquigarrow$ BC $v(0) = 0$, $v'(0) = 0$, $v'(l) = 0$

PVW: $\delta W_e = -\delta v'\left(\frac{l}{2}\right) \cdot M_x$

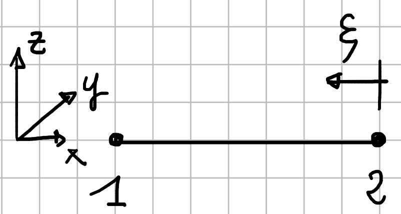
$\delta W_i = \int_0^l \delta v'' EJ v'' dz$

$$-\delta A \cdot \phi'\left(\frac{l}{2}\right) \cdot M_x = \delta A \int_0^l \phi'' EJ \phi'' dz\, A$$

$$\rightsquigarrow A = -\frac{M_x \phi'(l/2)}{\int_0^l \phi'' EJ \phi'' dz}$$

$$v(l) = A \cdot (1 - \cos(\pi)) = 2A$$

Ex 6



DATA

$l = 5$ mm

$EJ = 1E{+}10 \; N\,mm^2$

INTERPOLATION $\underline{\text{FCN} \quad N_1 = \xi / l \quad \text{DOF} \quad \vartheta_{y_1}}$ THIS IS THE NODAL ROTATION

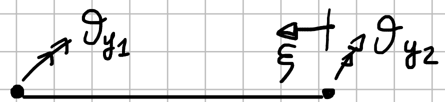
$\rightarrow \delta W_i = \int_0^l \delta\vartheta_{y/\xi} \; EJ \; \vartheta_{y/\xi} \, d\xi$ $\qquad \vartheta_y(\xi) = N_1 \cdot \vartheta_{y_1}$

EQUIVALENT TO $w_{/zz}$ FOR EB BEAM AS THE ROTATION IS $w_{/z}$

$\rightsquigarrow K_{\vartheta_{y_1}\vartheta_{y_1}} = \int_0^l \frac{1}{l} EJ \frac{1}{l} d\xi = \frac{EJ}{l}$

FULL DERIVATION

$\vartheta_y(\xi) = \underline{N} \cdot \underline{q}$



$\underline{N} = \frac{1}{l}\begin{bmatrix} \xi & l-\xi \end{bmatrix} \qquad \underline{q} = \begin{Bmatrix} \vartheta_{y_1} \\ \vartheta_{y_2} \end{Bmatrix}$

$$\delta \underline{q}^T \cdot \underbrace{\int_0^l \underline{N}^T_{/\xi} \, EJ \, \underline{N}_{/\xi} \, d\xi}_{} \; \underline{q} = \delta W_i$$

$$\underline{\underline{K}} = \begin{bmatrix} 1/l^2 & -1/l^2 \\ -1/l^2 & 1/l^2 \end{bmatrix} \int_0^l EJ \, d\xi$$

THIS ELEMENT IS THE REQUEST

- Shear locking affects beam finite elements based on Hermitian shape functions.
  - False
- The natural vibration frequency of a tensioned beam is lower than that of an unloaded beam.
  - False
- The Neumann boundary condition reads $\boldsymbol{\sigma} \cdot \boldsymbol{n} = \boldsymbol{f}$, where $\boldsymbol{f}$ is the applied load per unit of volume
  - False

1. Knowing the axial stiffness EA, the two shear stiffness GA_x^* and GA_y^* , the torsional stiffness GJ and the two bending stiffness EI_{xx} and EI_{yy}
   (a) is enough to fully characterize the behavior of a beam cross section
   (b) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position of the shear center wrt to the cross section
   (c) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position of the beam principal axis wrt to the cross section
   (d) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position and orientation of the beam principal axis wrt to the cross section
   (e) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position of both the beam principal axis and shear center wrt to the cross section
   (f) is not enough to fully characterize the behavior of a beam cross section, because one needs to know the position of both the beam principal axis and shear center wrt to the cross section, together with the orientation of the principal axeshe shear stress in the panel multiplied by the panel thickness
   (g) none of the above
2. The resultant over the cross section of a beam of the axial normal stress, $N = \int_A \sigma_{zz} \mathrm{d}A$ has the dimension of a force. It is:
   (a) work conjugated with the axial displacement
   (b) work conjugated with the derivative of the axial displacement
   (c) work conjugate with the derivative of the axial rotation
   (d) work conjugated with the shear deformation
   (e) none of the above

3. A riveted connection between two panels loaded in-plane cannot fail due to:

   (a) shear stress in the panels
   (b) shear stress in the rivet
   (c) axial stress in the rivet
   (d) bearing
   (e) axial stress in the panels
   (f) none of the above