Course of Aerospace Structures

Written test, Feb 15th, 2023

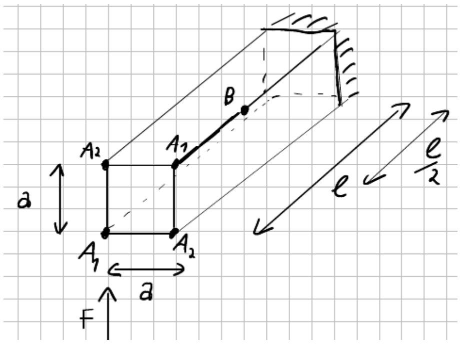
Name ____

Surname ____

Person code: | | | | | | | | |

Exercise 1

The semi-monocoque beam sketched in the figure is loaded at the free end by the vertical force F, applied along the lefmost panel.
Compute the upper right stringer axial stress σ_{zz} at point B.
(Unit for result: MPa)

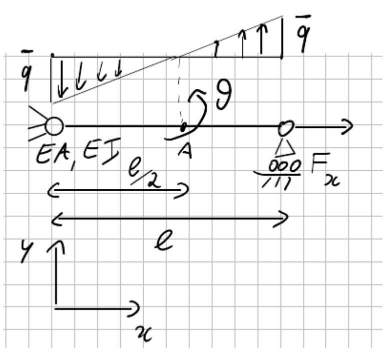


Data
$a = 400$ mm
$l = 12000$ mm
$E = 70000.0$ MPa
$\nu = 0.3$
$A_1 = 200$ mm^2
$A_2 = 400$ mm^2
$F = 5000$ N

Answer ____

Exercise 2

The beam sketched in the figure is loaded by a distributed force for unit of length with butterfly distribution and null resultant $q(x) = -\overline{q} + 2\overline{q}x/l$.
By resorting to the displacement method, and using a trigonometric approximation for the vertical displacement v, with the first and second term (i.e. $v \approx a\sin(\pi x/l) + b\sin(2\pi x/l)$), estimate the rotation θ at point A (the point at $x = l/2$ from the hinge). **Account for the axial pre-stress.**
Note: $\int \sin(\frac{c\pi x}{l})x\mathrm{d}x = -\frac{lx}{c\pi}\cos(\frac{c\pi x}{l}) + \frac{l^2}{\pi^2 c^2}\sin(\frac{c\pi x}{l})$.
(Unit for result: rad)

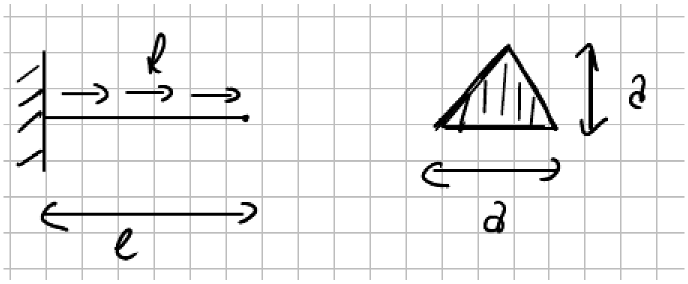


Data
$\overline{q} = 100$ N/mm
$F_x = 10000$ N
$EA = 1 \times 10^8$ N
$EI = 1 \times 10^{11}$ Nmm^2
$l = 4000$ mm

Answer ____

Exercise 3

The beam in the figure, with the sketched triangular cross section, is loaded by a distributed traction force per unit of length f. By resorting to a FE model of the beam with *one* linear element estimate the axial displacement of the free end.
(Unit for result: mm)

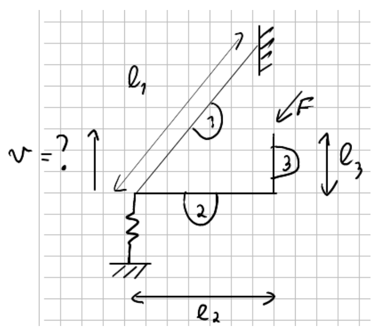


Data
$a = 1$ cm
$l = 1$ m
$f = 100 \times 10^2$ N/m
$E = 70000$ MPa
$\nu = 0.3$

Answer ______________________

Exercise 4

Consider the 3-D beam sketched in the figure, loaded by the force F.
The axial stiffness EA, torsional stiffness GJ and bending stiffnesses EI_{xx} and EI_{yy} are the same for the three beams. The spring has stiffness K.
Compute the vertical displacement v at the extremity of beam 1.
(Unit for result: mm)



Data
$E = 70000$ MPa
$\nu = 0.3$
$l_1 = 1000$ mm
$l_2 = 500$ mm
$l_3 = 250$ mm
$EA = 800 * E$ N
$GJ = 1000 * G$ Nmm^2
$EI_{xx} = EI_{yy} = 10000 * E$ Nmm^2
$K = 10$ N/mm
$F = 1000$ N

Answer ______________________

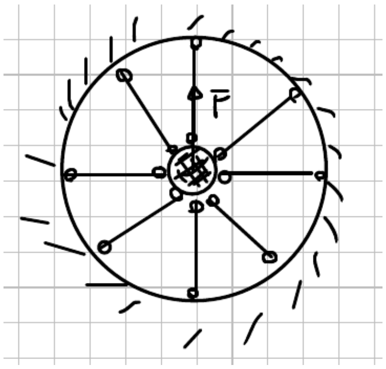
Exercise 5

Enterprise's antimatter containment chamber (the inner circle in the figure, to be considered as rigid) is held in place by a web of eight equal beams of lenght l, each **hinged at both extremities**. The outer ring onto which the beams are hinged is rigid and fully constrained. The beams material have eleastic modulus E and Poisson coefficient ν. Their cross section area is A, their bending stiffness are E_{xx} and E_{yy}, and their torsional stiffness is GJ

The beams are arranged in such a way that the angle between them is equal to 45 deg. The first vertical beam is aligned with a force F that is applied to the antimatter containment chamber.

Compute the overall vertical displacement of the antimatter containment chamber in the direction of the force.

(Unit for result: mm)



Data

$l = 1$ m

$A = 150$ mm^2

$E = 210000$ MPa

$\nu = 0.3$

$EI_{xx} = EI_{yy} = 1000 * E$ Nmm^2

$GJ = 1000 * G$ Nmm^2

$F = 1 \times 10^5$ N

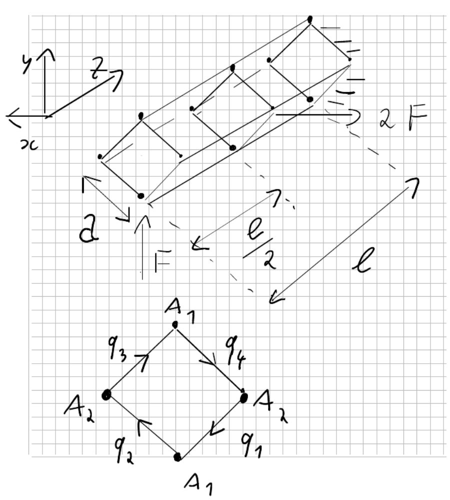
Answer

Exercise 6

The semi-monocoque beam sketched in the figure, with the square cross section drawn below, is loaded by a vertical force F aligned with the vertical cross-section diagonal and applied at the free end, and an horizontal force $2F$ aligned with the horizontal cross-section diagonal and applied in the middle of the beam. All panels have the same thickness t.

Compute the shear stress τ in the panel 4 (the one with flux q_4) at $z = 3/4l$ (i.e. at a distance of $1/4l$ from the clamp).

(Unit for result: MPa)



Data

$F = 1 \times 10^5$ N

$l = 5$ m

$a = 50$ cm

$A_1 = 1500$ mm^2

$A_2 = 1000$ mm^2

$t = 2$ mm

$E = 70000$ MPa

$\nu = 0.3$

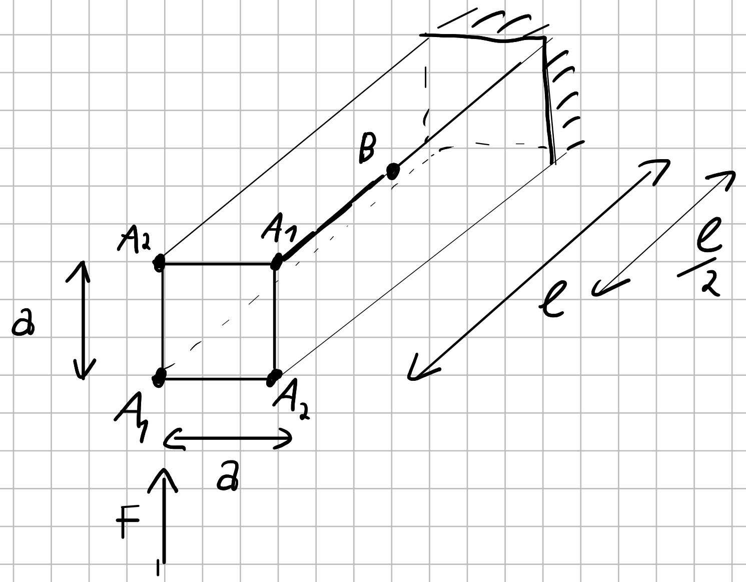
Answer

True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. thin plates typically work in plane strain conditions:
2. the stress field predicted by a FE solution is continuous:
3. the cross section of a beam subject to shear has an out-of-plane warping:

Multiple Choice questions
(Circle the correct answer)

1. A riveted connection between two panels loaded in-plane cannot fail because of:
   (a) shear stress in the panels
   (b) shear stress in the rivet
   (c) axial stress in the rivet
   (d) bearing stress in the rivet
   (e) axial stress in the panels
   (f) none of the above
2. Shear deformability is important for:
   (a) slender compact cross-section beams
   (b) thin-walled beams
   (c) thin panels
   (d) any kind of beam
   (e) any kind of panel
   (f) none of the above
3. The critical buckling compression force for the Euler instability of a beam is function of:
   (a) only the beam length and the constraints
   (b) only the beam bending stiffness and the constraints
   (c) only the beam torsional stiffness and the constraints
   (d) only the beam axial stiffness and the constraints
   (e) the beam length, the axial stiffness and the constraints
   (f) the beam length, the bending stiffness and the constraints
   (g) the beam length, the bending stiffness, the cross-section area and the constraints
   (h) the beam length, the torsional stiffness and the constraints
   (i) none of the above



$a = 400 \ mm$

$l = 12000 \ mm$

$E = 70000 \ MPa$

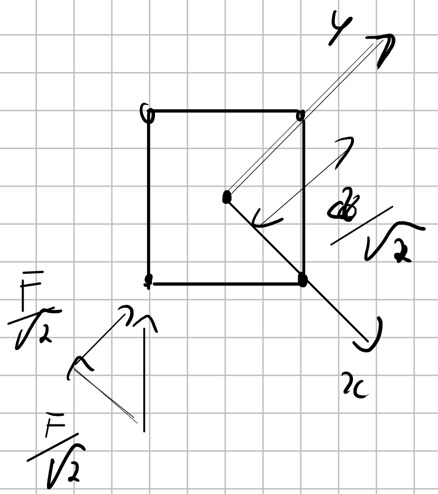
$\nu = 0{,}3$

$A_1 = 200 \ mm^2$

$A_2 = 400 \ mm^2$

$F = 5000 \ N$

Axial stress at point B, located at $\frac{l}{2}$ from the clamp

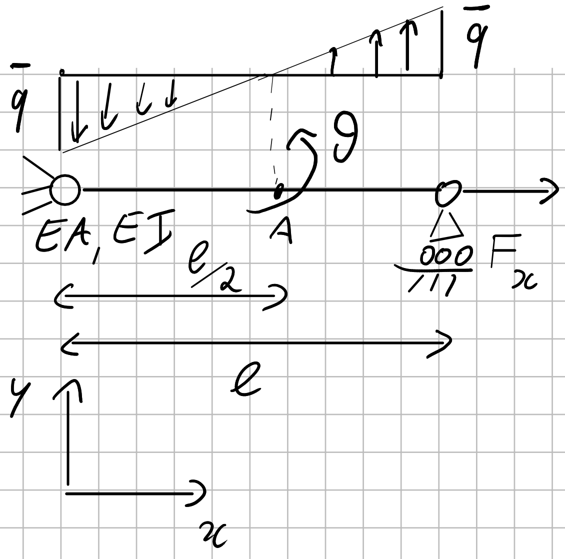


$$M_y(l/2) = -\frac{F}{\sqrt{2}} \cdot \frac{l}{2}$$

$$I_{xx} = A_1 \cdot 2\,\frac{a^2}{2} = A_1 \cdot a^2$$

$$\sigma_{zz}(B) = \frac{M_y(l/2)}{I_{xx}} \cdot \frac{a}{\sqrt{2}} = -\frac{F}{\sqrt{2}} \cdot \frac{l}{2}\,\frac{1}{A_1 a^2} \cdot \frac{a}{\sqrt{2}} = \frac{-F\,l}{4 A_1 a}$$

$$= -187 \ MPa$$



$\bar{q} = 100 \ N/mm$

$F_x = 10000 \ N$

$EA = 1E8 \ N$

$EI = 1E11 \ N \ mm^2$

$l = 4000 \ mm$

$q(x) = -\bar{q} + \frac{2\bar{q}}{l} x$

Rotation ϑ of point A, located at $l/2$

use a 2-term trigonometric approximation of the transverse displacement and the PVW

$$\int \sin\left(\frac{k\pi x}{l}\right) x \, dx = -\frac{l}{k\pi} x \cos\left(\frac{\pi k x}{l}\right) + \frac{l^2}{\pi^2 k^2} \sin\left(\frac{\pi k x}{l}\right)$$

$$\int_0^l \left(\delta w'' EI\, w'' + \delta w'\, w'\, F_x\right) dx = \int_0^l \delta w\, q\, dx$$

$$w = a \sin\left(\frac{\pi x}{l}\right) + b \sin\left(\frac{2\pi x}{l}\right)$$

$$w' = \frac{\pi}{l} a \cos\left(\frac{\pi x}{l}\right) + \frac{2\pi}{l} b \cos\left(\frac{2\pi x}{l}\right)$$

$$w'' = -\frac{\pi^2}{l^2} a \sin\left(\frac{\pi x}{l}\right) - \frac{4\pi^2}{l^2} b \sin\left(\frac{2\pi x}{l}\right)$$

$$\int_0^l -\delta a \sin\left(\frac{\pi x}{l}\right) \bar{q}\, dx = -\delta a \frac{l}{\pi} \cos\left(\frac{\pi x}{l}\right)\Big|_0^l \bar{q} = -\delta a \frac{l}{\pi} 2\bar{q}$$

$$\int_0^l \delta a \sin\left(\frac{\pi x}{l}\right) \frac{2\bar{q}}{l} x\, dx = \frac{\delta a 2\bar{q}}{l}\left[\left(\frac{l}{\pi}\right)^2 \sin\left(\frac{\pi x}{l}\right) - \frac{l}{\pi} x \cos\left(\frac{\pi x}{l}\right)\right]\Big|_0^l = 2\delta a \frac{l}{\pi} q$$

$$\int_0^l -\delta b \sin\left(\frac{2\pi x}{l}\right) \bar{q}\, dx = -\delta b \frac{l}{2\pi} \cos\left(\frac{2\pi x}{l}\right)\Big|_0^l = \delta b \cdot \emptyset$$

$$\int_0^l \delta b \sin\left(\frac{2\pi x}{l}\right) \frac{2\bar{q}}{l} x\, dx = \frac{\delta b 2\bar{q}}{l}\left[\left(\frac{l}{2\pi}\right)^2 \sin\left(\frac{2\pi x}{l}\right) - \frac{l}{2\pi} x \cos\left(\frac{2\pi x}{l}\right)\right]\Big|_0^l = \frac{\delta b 2\bar{q}}{\cancel{l}}\left(-\frac{l^{\cancel{2}}}{2\pi}\right)$$

$$= \delta b\left(\frac{-l\bar{q}}{\pi}\right)$$

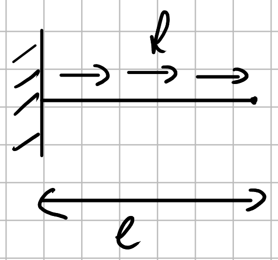
$$\delta a \left(\frac{\pi}{\ell}\right)^4 EI \, \frac{1}{2} \ell \, a + \delta a \left(\frac{\pi}{\ell}\right)^2 \frac{1}{2} \ell F_x \, a + \delta b \left(\frac{2\pi}{\ell}\right)^4 EI \, \frac{1}{2} \ell \, b + \delta b \left(\frac{2\pi}{\ell}\right)^2 \frac{1}{2} F_x \ell \, b = \delta b \left(\frac{-\ell \bar{q}}{\pi}\right)$$

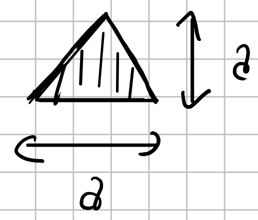
$$\delta a: \quad \left[\left(\frac{\pi}{\ell}\right)^4 EI \, \frac{1}{2} \ell + \left(\frac{\pi}{\ell}\right)^2 F_x \, \frac{1}{2} \ell\right] a = 0 \quad \Rightarrow \quad a = 0$$

$$\delta b: \quad \left[\left(\frac{2\pi}{\ell}\right)^4 EI \, \frac{1}{2} \cancel{\ell} + \left(\frac{2\pi}{\ell}\right)^2 \frac{1}{2} F_x \cancel{\ell}\right] b = -\frac{\cancel{\ell} \bar{q}}{\pi}$$

$$b = -\frac{\bar{q}}{\pi} \cdot \frac{2}{\left(\frac{2\pi}{\ell}\right)^4 EI + \left(\frac{2\pi}{\ell}\right)^2 F_x}$$

$$\vartheta = w'\left(\frac{\ell}{2}\right) = b \, \frac{2\pi}{\ell} \cos(\pi) = -\frac{2\pi}{\ell} b = 0{,}15 \text{ rad}$$





$a = 1\ cm$

$l = 1\ m$

$p = 100E2\ N/m$

$E = 70000\ MPa$

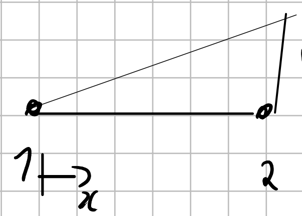
Make a FE element model of the beam with a single linear bam element and estimate di displacement of the extremity

$a = 1\ cm = 10\ mm$

$p = 100\ E2\ N/m = 10\ N/mm$

$l = 1\ m = 1000\ mm$

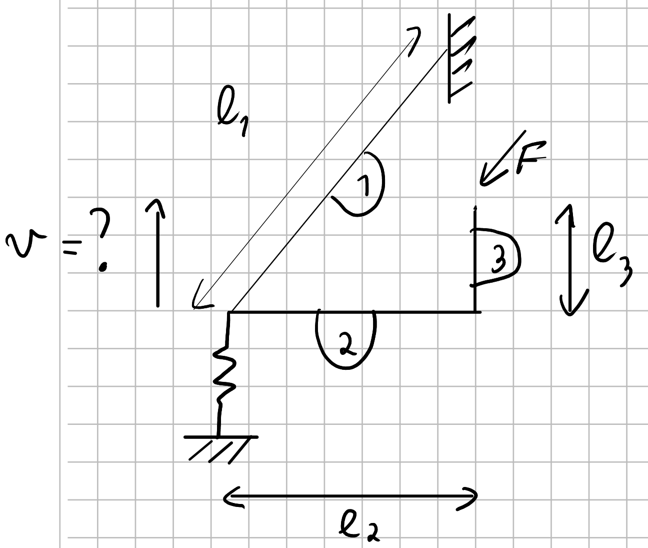
$A = \frac{a^2}{2}$ $EA = E \cdot A$



$N_2(x) = \frac{x}{1000}$

$$\delta \mathcal{L}_e = \int_0^{1000} \delta u_2 \frac{x}{1000} \cdot p\, dx = \delta u_2\ 500\, p$$

$$\frac{EA}{l} u_2 = 500\, p \qquad u_2 = \frac{500 \cdot l\, p}{EA} = 1{,}43\ mm$$



$\nu = 0{,}3$

$E = 70\,000 \ MPa$

$l_1 = 1000 \ mm$

$l_2 = 500 \ mm$

$l_3 = 250 \ mm$

$EA_1 = EA_2 = EA_3 = 500 \cdot E \ N$

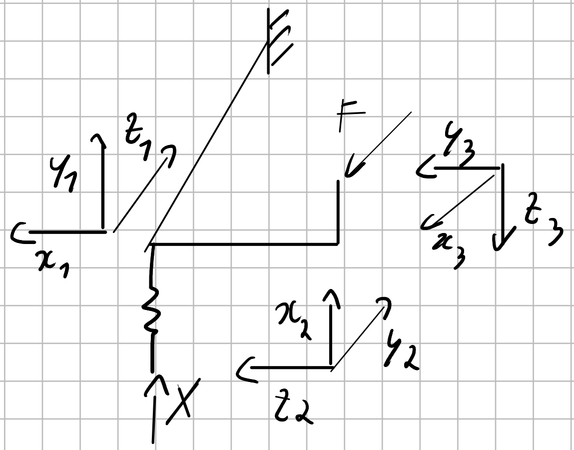
$GJ_1 = GJ_2 \ GJ_3 = 1000 \cdot G \ N\,mm^2$

$EI_{xx1} = EI_{xx2} = EI_{xx3} = EI_{yy1}$

$= EI_{yy2} = EI_{yy3} = 10000 \cdot E \ N\,mm^2$

$K = 10 \ N/mm$

$F = 1000 \ N$



$M_{y3} = F z_3$

$M_{z2} = -F l_3$

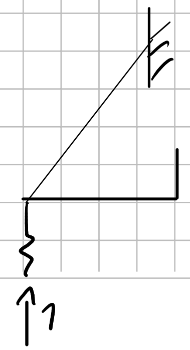
$M_{x2} = -F z_2$

$M_{y1} = F l_2$

$M_{x1} = -X z_1 + F l_3$

$M'_{x1} = -z_1$

you need not to compute these □ because □ is the only one $\neq \emptyset$ with X and it works for □

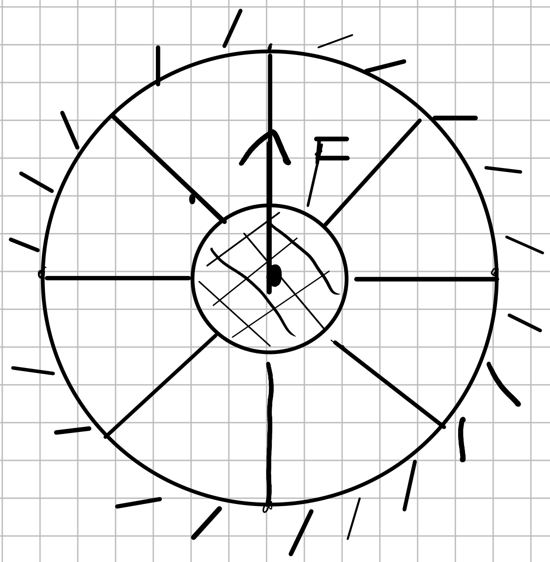


$$\int_0^{l_1} \frac{X z_1^2}{EI_{xx1}} - \frac{F l_3 z_1}{EI_{xx1}} \, dz_1 + \frac{X}{K} = 0$$

$$\left(\frac{1}{3}\,l_1^{\,3} + \frac{EI_{x_1x_1}}{K}\right) X = \frac{1}{2} F l_3\, l_1^{\,2}$$

$$X = \frac{\frac{1}{2} F l_3\, l_1^{\,2}}{\left(\frac{1}{3}\,l_1^{\,3} + \frac{EI_{x_1x_1}}{K}\right)}$$

$$v = -\frac{X}{K} = -\ 37\ mm$$



$l = 1\ m$

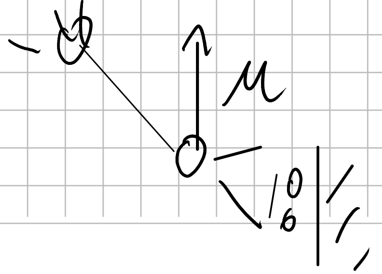
$A = 150\ mm^2$

$E = 210000\ MPa$

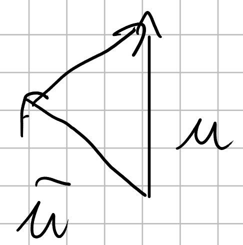
$F = 1E5\ N$

horizontal displacement = 0

vertical displacement:



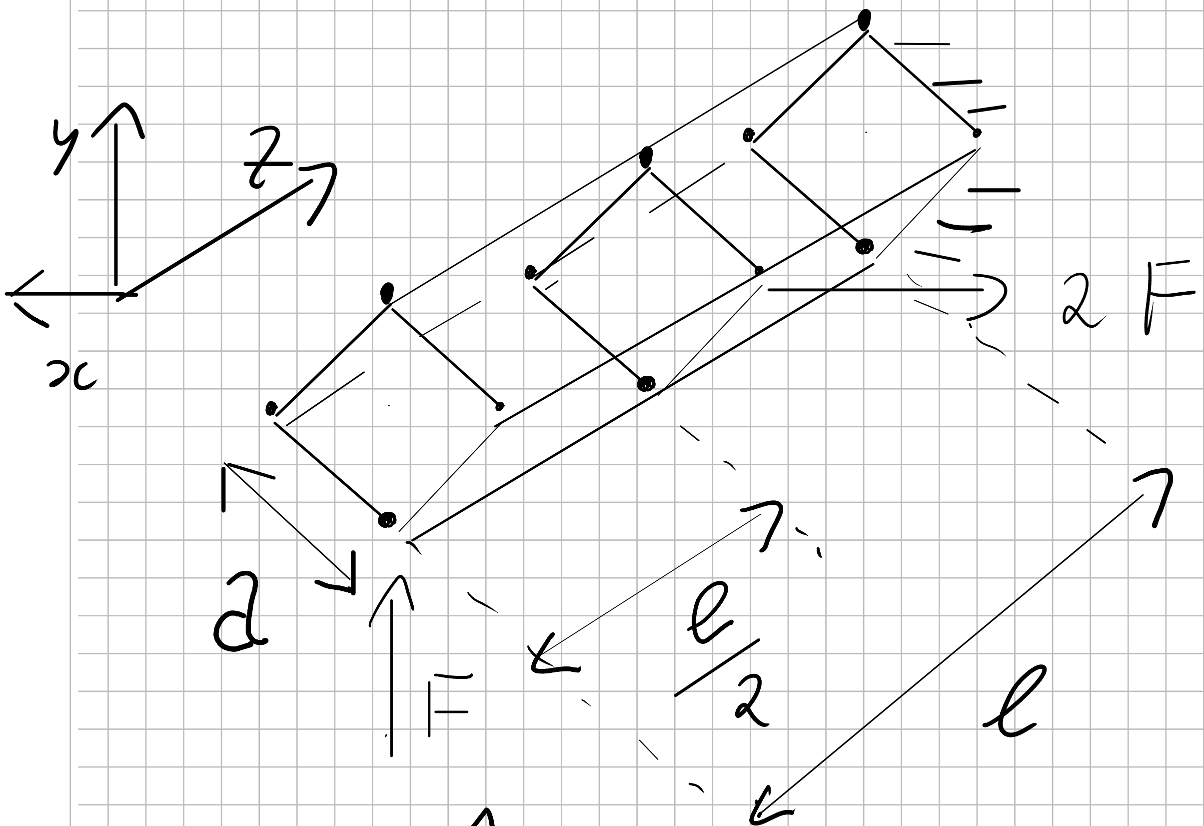
$$k = \frac{EA}{l}$$

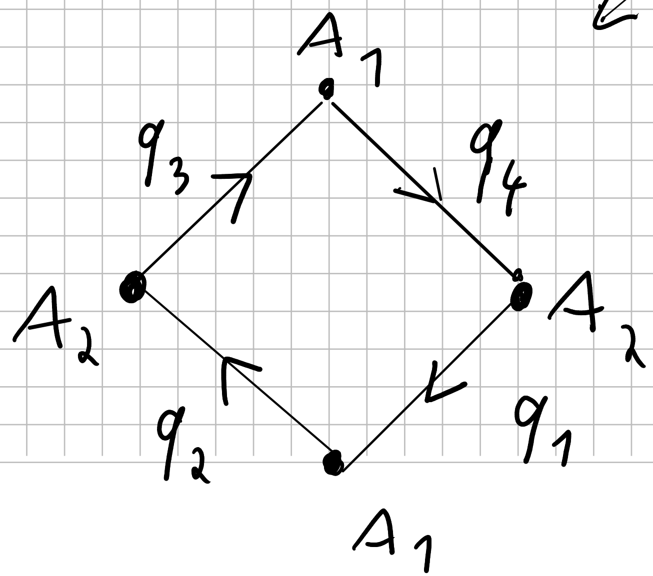


$$\tilde{u} = \frac{u}{\sqrt{2}}$$

$$\left(2 + \frac{4}{2}\right) K u = F$$

$$u = \frac{F}{K\left(2 + \frac{4}{2}\right)} = 0{,}79 \text{ mm}$$





$F = 1E5\ N$

$l = 5\ m$

$a = 50\ cm$

$A_1 = 1500\ mm^2$

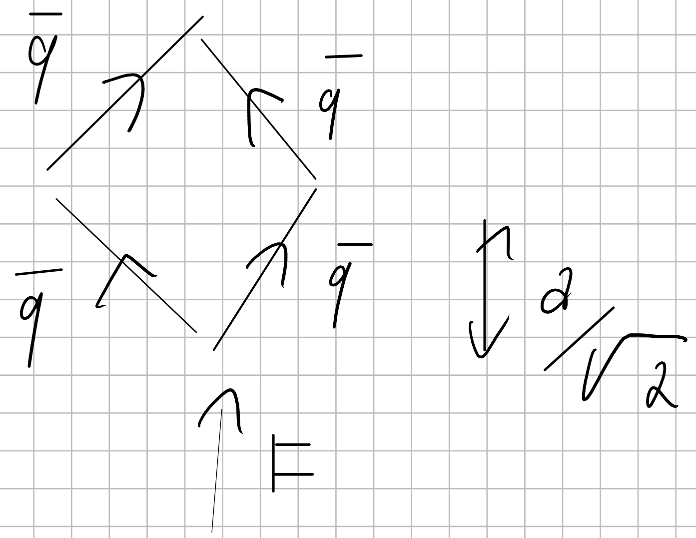
$A_2 = 1000\ mm^2$

$t_1 = t_2 = t_3 = t_4 = 2\ mm$

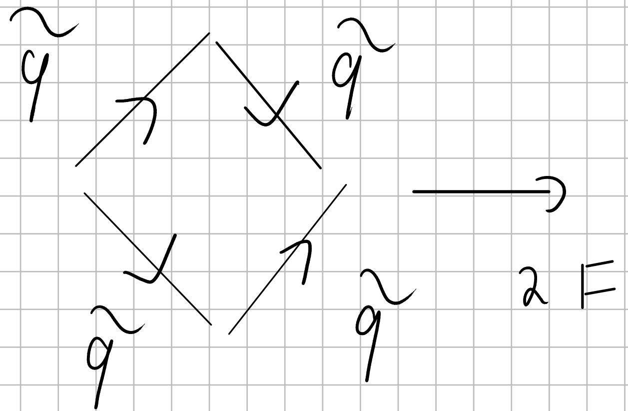
$E = 70000\ MPa$

$\nu = 0{,}3$

$T_4 = ?$



$$\bar{q} = \frac{F}{2} \cdot \frac{\sqrt{2}}{2a} = \frac{F}{2\sqrt{2}\,a}$$



$$\tilde{q} = \frac{2F}{2} \cdot \frac{\sqrt{2}}{2a} = \frac{F}{\sqrt{2}\,a}$$

$$q_4 = \tilde{q} - \bar{q} = \frac{F}{2\sqrt{2}\,a}$$

$$\tau_4 = \frac{q_4}{t_4} = 35{,}35 \text{ MPa}$$

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. thin plates typically work in plane strain conditions:
   - False
2. the stress field predicted by a FE solution is continuous:
   - False
3. the cross section of a beam subject to shear has an out-of-plane warping:
   - True

Multiple Choice questions

(Circle the correct answer)

1. A riveted connection between two panels loaded in-plane cannot fail because of:
   (a) shear stress in the panels
   (b) shear stress in the rivet
   (c) axial stress in the rivet
   (d) bearing stress in the rivet
   (e) axial stress in the panels
   (f) none of the above
2. Shear deformability is important for:
   (a) slender compact cross-section beams
   (b) thin-walled beams
   (c) thin panels
   (d) any kind of beam
   (e) any kind of panel
   (f) none of the above
3. The critical buckling compression force for the Euler instability of a beam is function of:
   (a) only the beam length and the constraints

(b) only the beam bending stiffness and the constraints

(c) only the beam torsional stiffness and the constraints

(d) only the beam axial stiffness and the constraints

(e) the beam length, the axial stiffness and the constraints

(f) the beam length, the bending stiffness and the constraints

(g) the beam length, the bending stiffness, the cross-section area and the constraints

(h) the beam length, the torsional stiffness and the constraints

(i) none of the above