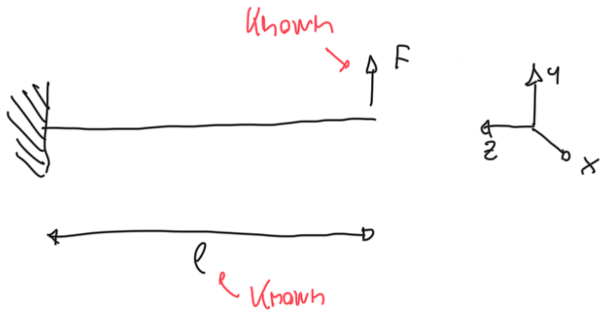
LAB 2

Isostatic Beams Systems I

Isostatic $\rightarrow$ n rigid DOF = n constrains

1) 

let's find:
- Reaction forces
- Internal Actions

2D Is the system isostatic? 3 rigid DOF, 3 constrains — YES!

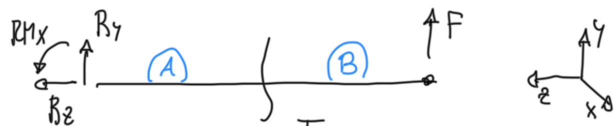
- we can compute the RF using the equilibrium eq.

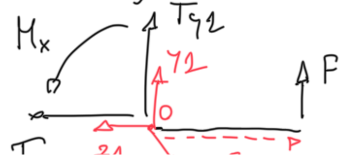
translation z:
" y:
rotation x wrt A:

$$\begin{cases} R_z = \emptyset \\ R_y + F = \emptyset \\ RM_x + F \cdot \ell = \emptyset \end{cases} \qquad \begin{cases} R_z = \emptyset \\ R_y = -F \\ RM_x = -F\ell \end{cases}$$

- Internal Actions

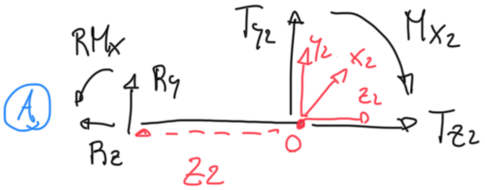
let's virtually cut our beam



B 

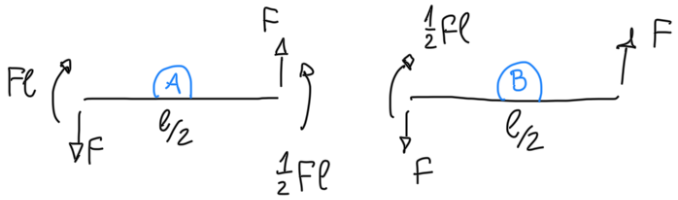
$$\begin{cases} T_{z1} = \emptyset \\ T_{y1} = -F \end{cases}$$

$M_{x1} = -F \cdot z_1$ wrt O



$$\text{wrt O} \begin{cases} T_{z2} = R_z = \emptyset \\ T_{y2} = -R_y = F \\ M_{x2} = RM_x - R_y \cdot z_2 = -F\ell + F \cdot z_2 \end{cases}$$

- Compute internal action in $\frac{\ell}{2}$ → $M_{x2} = -F\ell + F \cdot \frac{\ell}{2} = -F\frac{\ell}{2}$



EVERYTHING IS IN EQUILIBRIUM:
- FULL BEAM
- BEAM PARTS
- INTERFACE

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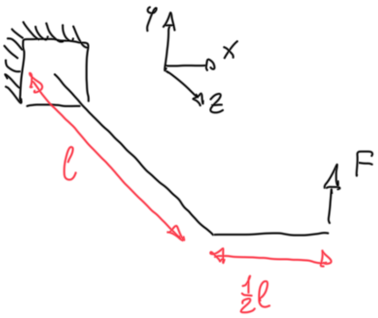
2)

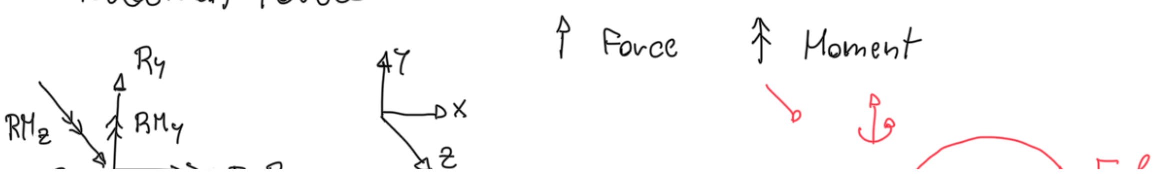


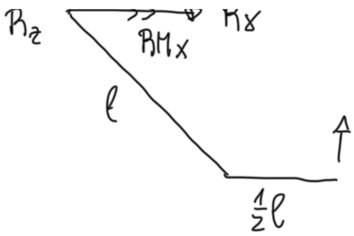
3D

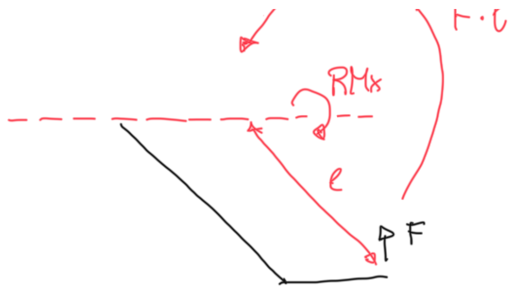
let's find: Reaction Forces
Internal Actions

- ISOSTATIC ? 3D: – 6 rigid DOF YES
  – 6 constraints

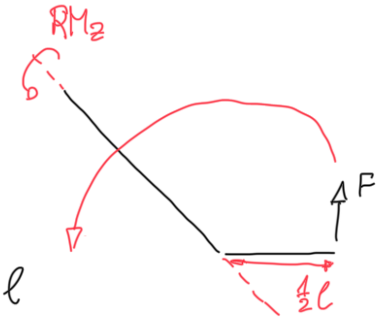
- Reaction Forces





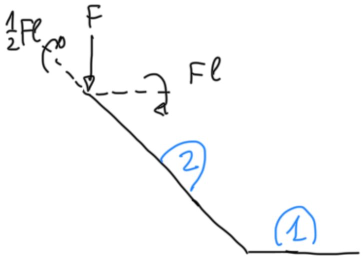


$$X: \quad R_x = 0$$
$$Y: \quad R_y + F = 0 \quad \rightarrow \quad R_y = -F$$
$$Z: \quad R_z = 0$$



$$\text{rot}\,X: \quad RM_x - F \cdot \ell = 0 \quad \rightarrow \quad RM_x = F \cdot \ell$$
$$\text{rot}\,y: \quad RM_y = 0$$
$$\text{rot}\,z: \quad RM_z + F \cdot \tfrac{1}{2}\ell = 0 \quad \rightarrow \quad RM_z = -\tfrac{1}{2} F \cdot \ell$$

- Internal Actions



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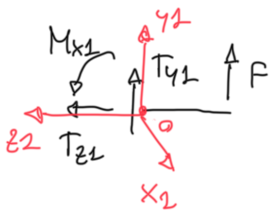
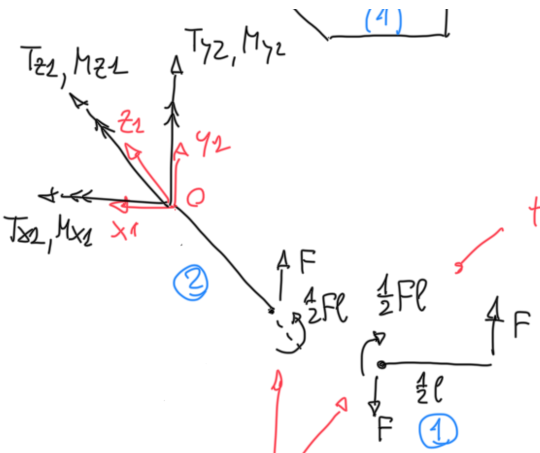


$$\begin{cases} T_{y1} = -F \\ T_{z1} = 0 \\ M_{x1} = -F \cdot z_1 \end{cases}$$ wrt O

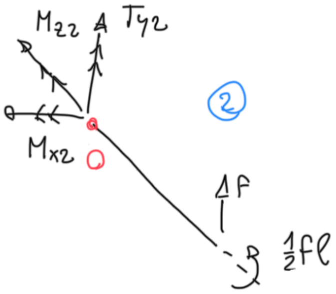
2 I) PIECEWISE APPROACH

to compute the IA of 2 we will use IA of 1 computed in its extremity ($z_1 = \frac{1}{2}\ell$)



these are the IA computed for $z_1 = \frac{1}{2}l$

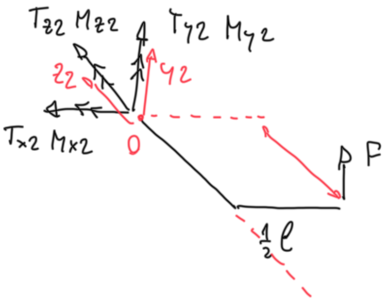
we are imposing the equilibrium at the interface



$$\begin{cases} T_{x2} = \emptyset \\ T_{y2} = -F \\ T_{z2} = \emptyset \\ M_{x2} = -F \cdot z_2 \\ M_{y2} = \emptyset \\ M_{z2} = \frac{1}{2} Fl \end{cases}$$

wrt O

II) GLOBAL APPROACH



$$\begin{aligned} T_{x2} &= \emptyset \\ T_{y2} &= -F \\ T_{z2} &= \emptyset \\ M_{x2} &= -F \cdot z_2 \\ M_{y2} &= \emptyset \\ M_{z2} &= F \cdot \tfrac{1}{2}l \end{aligned}$$

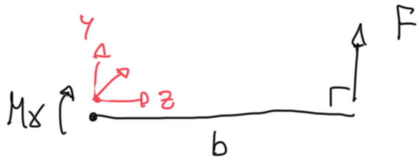
wrt O

When computing a moment is the sign of the arm relevant?

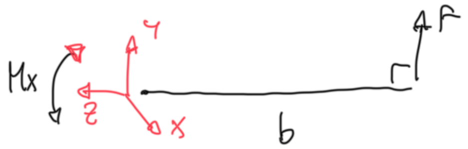
VECTOR PRODUCT

$M = F \times b \qquad |M| = |F| \cdot |b| \cdot \sin\theta$

$a \times (-b) = -(a \times b) \qquad (-a) \times b = -(a \times b)$

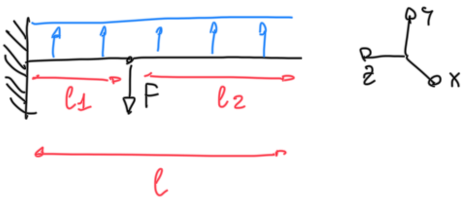


$M_x = F \cdot b = Fb$



$M_x = F \cdot (-b) = -Fb$

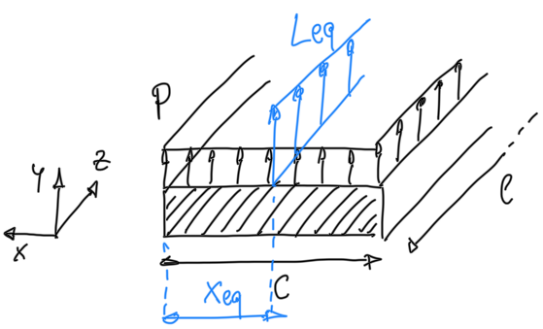
3) a pressure P is applied on the beam's top surface



2D

let's find: RF, IA

This is the beam section



Let's find a line load [N/m] L_{eq} which is equivalent to p [N/m²].

L_{eq} must have:

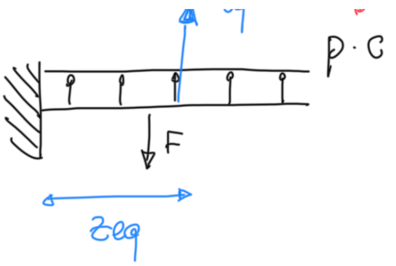
- the right MAGNITUDE to give an equivalen FORCE distribution
- the right POINT OF APPLICATION to give an equivalent MOMENT distribution.

b)

$$L_{eq} = \int_0^c p(x,z)\,dx = \int_0^c p\,dx = [p \cdot x]_0^c = p \cdot c$$

$$X_{eq} = \frac{1}{L_{eq}} \int_0^c p(x,y) \cdot x \cdot dx = \frac{1}{pc} \int_0^c p\,x \cdot dx = \frac{1}{pc} \cdot \frac{1}{2} \cdot pc^2 = \frac{1}{2} \cdot C$$

• Feq   d

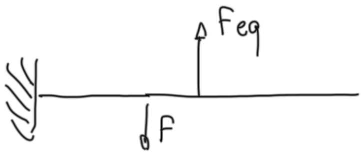


Let's find a concentrated load Feq [N] which is equivalent to Leq [N/m].

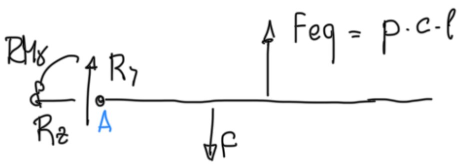
$$F_{eq} = \int_0^\ell L_{eq} \cdot dz = \int_0^\ell pc \cdot dz = \left[pcz \right]_0^\ell = pc\ell$$

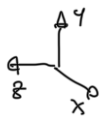
$$\underline{z_{eq}} = \frac{1}{F_{eq}} \cdot \int_0^\ell L_{eq} \cdot z \cdot dz = \frac{1}{pc\ell} \cdot \int_0^\ell pc \cdot z\, dz =$$

$$= \frac{1}{pc\ell} \cdot \left[\frac{1}{2} pcz^2 \right]_0^\ell = \frac{1}{2} \frac{1}{pc\ell} \cdot pc\ell^2 = \frac{1}{2}\ell$$



- Reaction Forces





$$R_y - F + pc\ell = \emptyset$$

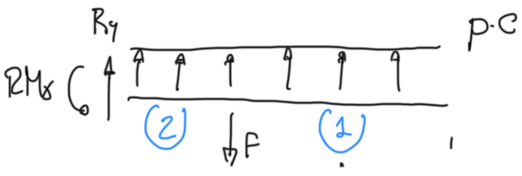
$$R_y = F - pc\ell$$

$$R_z = \emptyset$$

wrt A $\quad RM_x - F \cdot \ell_1 + pc\ell \cdot \frac{\ell}{2} = \emptyset$

$$RM_x = F \cdot \ell_1 - pc\frac{\ell^2}{2}$$

- Internal Actions



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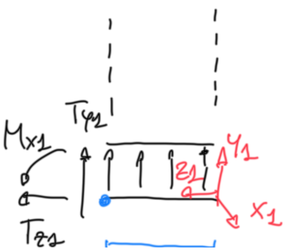
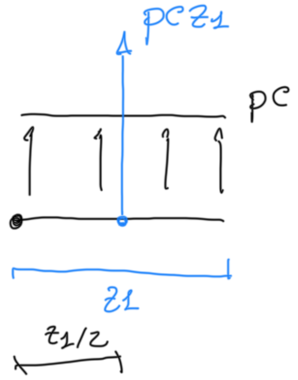


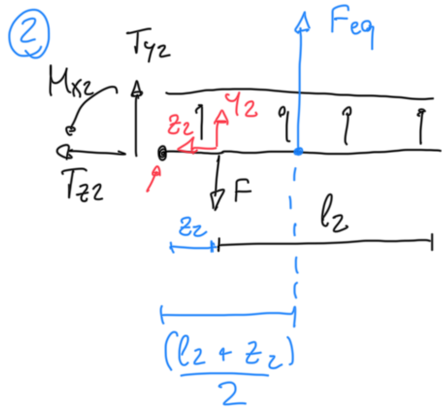
$$T_{y1} = -p \cdot c \cdot z_1$$

$$T_{z1} = \emptyset$$

z_1

$$M_{x1} = -\widehat{pcz_1}\cdot\widehat{\frac{z_1}{2}} = -pc\frac{z_1^2}{2}$$





$$T_{y2} = F - pc(l_2 + z_2)$$

$$T_{z2} = \emptyset$$

$$M_{x2} = +Fz_2 - pc(l_2+z_2)\cdot\frac{(l_2+z_2)}{2}$$

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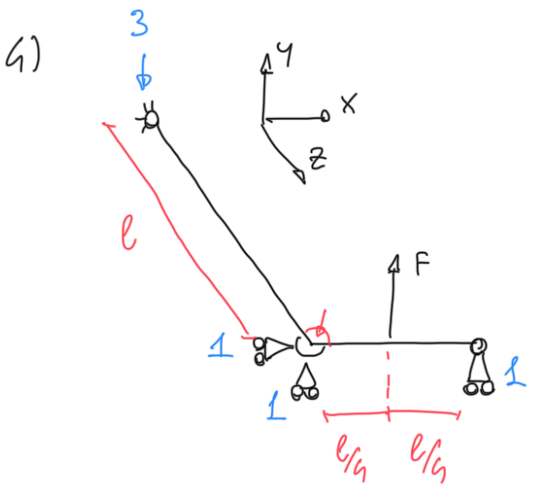
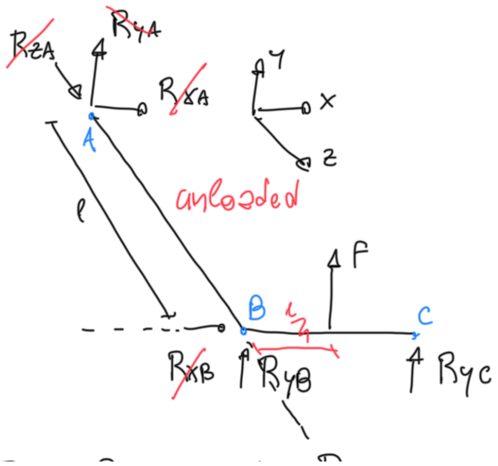
4)



Let's find : RF, IA

The system is isostatic

- Reaction Forces



External loads don't generate translations in x or rotations in y

$$\begin{cases} z: & R_{zA} = \emptyset \\ x: & R_{xA} = R_{xB} = \emptyset \\ y: & R_{yA} + R_{yB} + R_{yC} + F = \emptyset \; * \end{cases}$$

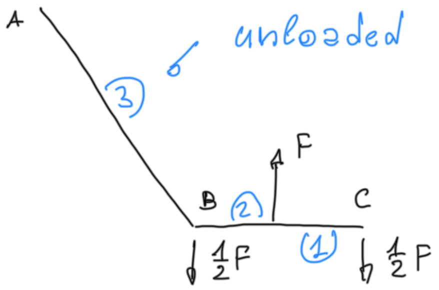
Rot Eq wrt B

rot x $\quad R_{yA} \cdot \ell = \emptyset \qquad R_{yA} = \emptyset$

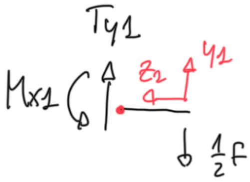
rot z $\quad F \cdot \frac{\ell}{4} + R_{yC} \cdot \frac{\ell}{2} = \emptyset \qquad R_{yC} = -\frac{1}{2}F$

* $R_{yB} + R_{yC} + F = \emptyset \qquad R_{yB} - \frac{1}{2}F + F = \emptyset \qquad R_{yB} = -\frac{1}{2}F$

- Internal Actions

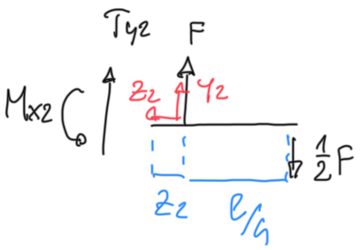


1



$$\begin{cases} T_{y1} = \frac{1}{2}F \\ M_{x1} = \frac{1}{2}F \cdot z_1 \end{cases}$$

2



$$\begin{cases} T_{y2} = \frac{1}{2}F + F = -\frac{1}{2}F \\ M_{x2} = \frac{1}{2}F \cdot \left(\frac{\ell}{4} + z_2\right) - F \cdot z_2 \end{cases}$$