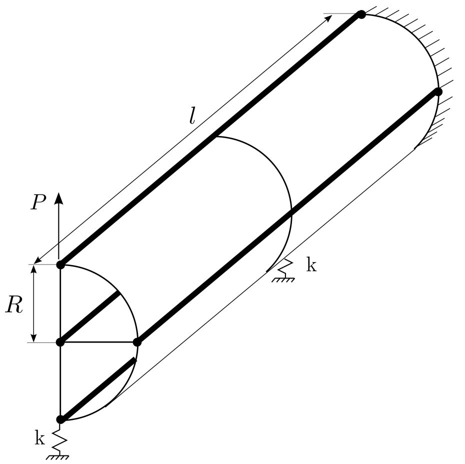
Course of Spacecraft Structures

Written test, June 26^{th}, 2018

Exercise 1

The thin-walled beam in the figure is characterized by a semi-circular section with five webs of thickness t and four stringers of area A (note: A is the equivalent stringer area, inclusive of the contribution due to the panels). The length of the beam is denoted with l. The material is homogeneous and isotropic with Young modulus E and Poisson's ratio 0.3. The beam is grounded with two linear elastic springs of stiffness k, as reported in the sketch, and is loaded with a concentrated force P applied at one of the two ends. Determine the reaction forces in correspondence of the springs by accounting for the beam shear deformability, and plot the internal actions in the beam.



Data

$E = 70\ GPa$; $\nu = 0.3$;

$A = 200\ mm^2$; $t = 1\ mm$;

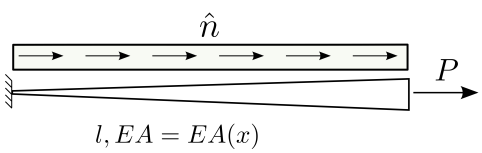
$R = 200\ mm$; $l = 2000\ mm$;

$k = 10^6\ N/mm$; $P = 1\ kN$;

Exercise 2

The bar in the figure is made of isotropic material of Young modulus E and Poisson's ratio ν. The section varies from A_0 to A_1 with a linear variation along the bar axis. The bar is loaded with a uniformly distributed load per unit length $\hat{n}$, and a concentrated force P is applied at the tip.

Determine stress and displacement at the midspan by using a Ritz solution with one degree of freedom. Compare the predicted stress with the exact solution and discuss how the Ritz solution is expected to behave if the number of trial functions is increased.



Data

$E = 70\ GPa$; $\nu = 0.3$;

$A_0 = 5\ mm^2$; $A_1 = 15\ mm^2$;

$l = 400\ mm$;

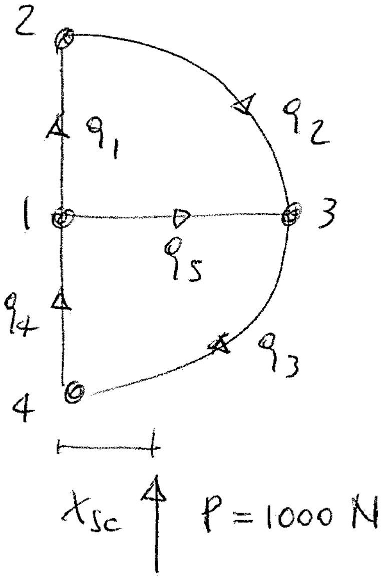
$P = 1\ kN$; $\hat{n} = 2\ N/mm$;

Question 1

Illustrate the procedure for evaluating the shear flows in a panel-to-panel junction with two rows of rivets. Assume that the two panels have thicknesses t_1 and t_2.

Exercise 1

Evaluation of section properties (GA^*)



By imposing :
- shear flow eqs
- equivalence to moment
- compatibility $(\theta_1' = \theta_2')$
- compatibility $(\theta_1' = 0)$

It is obtained : $x_{sc} = 122.20$ mm

the corresponding shear flows are:

$q_1 = q_4 = 1.5275$ N/mm

$q_2 = q_3 = -0.9725$ N/mm

From which

$$A^* = \frac{T_y^2}{\sum_i \frac{q_i^2 l_i}{t_i}} = 654.65 \text{ mm}^2$$

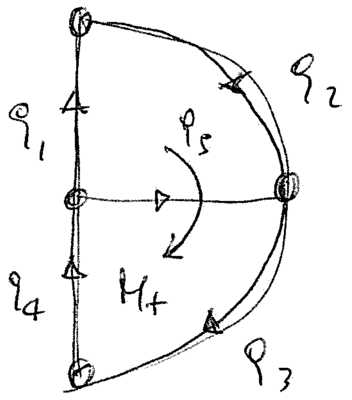
$GA^* = 1.76 \cdot 10^7$ N

EJ

$$EJ = E\left(2AR^2\right) = 1.12 \cdot 10^{12}\ Nmm^2$$

GJ

A torsional moment $M_t = 1 \cdot 10^6$ N·mm is considered



$$q_1 = q_2 = q_3 = q_4 = 7.96\ N/mm$$

$$q_5 = 0$$

The torsional constant is:

$$J_t = \frac{M_t}{\dfrac{1}{2\Omega_{cell_k}} \displaystyle\sum_{(k)} \frac{q_k l_k}{t_k}} = 1.54 \cdot 10^7\ mm^4$$

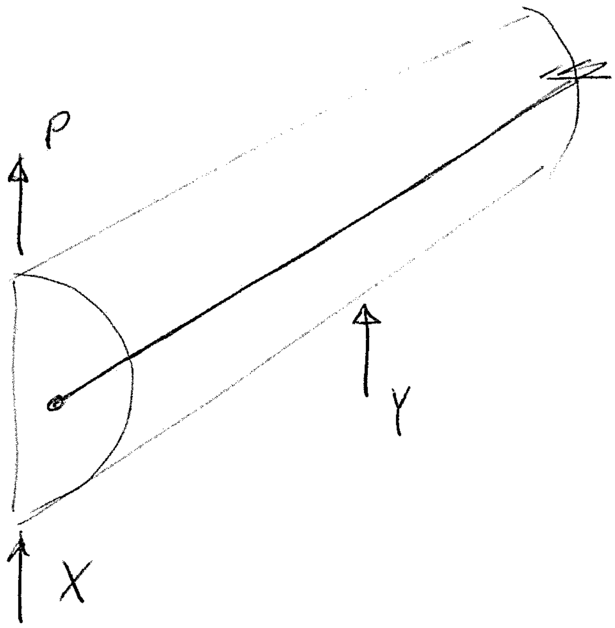
← summatory over panel belonging to cell #k

← can consider here cell #1 or #2

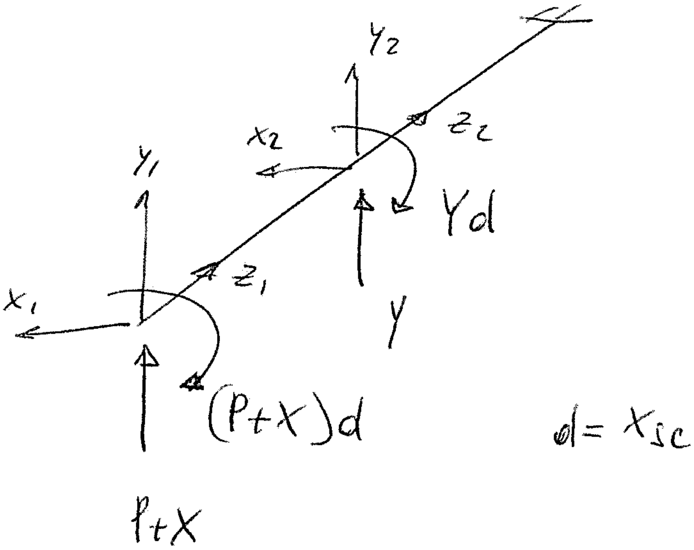
And so:

$$GJ = G \cdot J_t = 4.13 \cdot 10^{11}\ Nmm^2$$

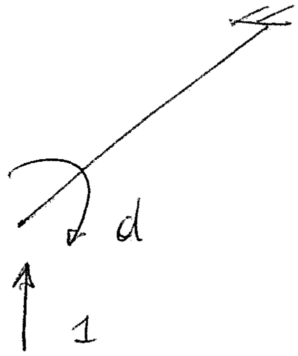
Reaction forces



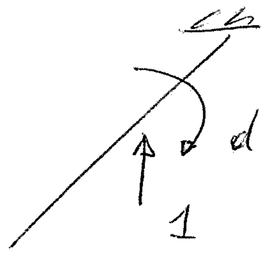
(beam axis running along the shear center positions)



(Real)



(dummy #1)



(dummy #2)

Real system

$$T_y^{(1)} = -(P+X)$$

$$M_x^{(1)} = -(P+X)\,z_1$$

$$M_z^{(1)} = -(P+X)\,d$$

$$T_y^{(2)} = -(P+X+Y)$$

$$M_x^{(2)} = -(P+X)\,l/2 - (P+X+Y)\,z_2$$

$$M_z^{(2)} = -(P+X+Y)\,d$$

Dummy #1

$^1\delta T_y^{①} = -1$ $\quad$ $^1\delta T_y^{②} = -1$

$^1\delta M_x^{①} = -z_1$ $\quad$ $^1\delta M_x^{②} = -\ell/2 - z_2$

$^1\delta M_z^{①} = -d$ $\quad$ $^1\delta M_z^{②} = -d$

Dummy #2

$^2\delta T_y^{①} = 0$ $\quad$ $^2\delta T_y^{②} = -1$

$^2\delta M_x^{①} = 0$ $\quad$ $^2\delta M_x^{②} = -z$

$^2\delta M_z^{①} = 0$ $\quad$ $^2\delta M_z^{②} = -d$

PCVW

The equations are:

$$\left(\frac{\ell^3}{3EJ} + \frac{\ell}{GA^*} + \frac{d^2\ell}{GJ} + \frac{1}{k}\right)X + \left(\frac{\ell}{2GA^*} + \frac{5}{48}\frac{\ell^3}{EJ} + \frac{d^2\ell}{2GJ}\right)Y$$

$$+ \left(\frac{\ell^3}{3EJ} + \frac{\ell}{GA} + \frac{d^2\ell}{GJ}\right)P = 0$$

$$\left(\frac{\ell}{2GA^*} + \frac{5\ell^3}{48EJ} + \frac{d^2\ell}{2GJ}\right)X + \left(\frac{\ell}{2GA^*} + \frac{\ell^3}{24EJ} + \frac{d^2\ell}{2GJ} + \frac{1}{k}\right)Y$$

$$+ \left(\frac{5\ell^3}{48EJ} + \frac{d^2\ell}{2GJ} + \frac{\ell}{2GA^*}\right)P = 0$$

From which: $X = -998.71\ N$

$Y = -2.74\ N$

Exercise 2

the trial function is taken as:

$$u = c\left(\frac{x}{\ell}\right) = c\phi \qquad \text{with} \quad \phi = \frac{x}{\ell}$$

Stiffness matrix (1×1)

$$\delta W_i = \int_0^\ell \delta u_{,x}\, EA\, u_{,x}\, dx$$

$$= \delta c\, k\, c$$

$$\text{with} \quad k = \int_0^\ell \phi_{,x}\, EA\, \phi_{,x}\, dx$$

$$= \int_0^\ell \frac{1}{\ell^2}\left(EA_0 + E\frac{A_1 - A_0}{\ell}x\right) dx$$

$$= \frac{EA_0}{\ell} + E\frac{A_1 - A_0}{2\ell} =$$

External forces

$$\delta W_e = \int_0^\ell \delta u\, \hat{n}\, dx + \delta u(\ell)\, P$$

$$= \delta c\, f$$

$$\text{with} \quad f = \int_0^\ell \phi\, \hat{n}\, dx + \phi(\ell)\, P$$

$$= \frac{\hat{n}\ell}{2} + P$$

Solution

$$c = F/k = 0.80 \quad mm$$

Displacement & stress at $x = \ell/2$

$$u = \phi(\ell/2)\cdot c = \frac{1}{2}\, c = 0.40 \quad mm$$

$$\sigma = E u_{/x} = E\, \phi_{/x}(\ell/2)\, c$$

$$= E\, 1/\ell \cdot c = 140 \quad MPa$$

Exact stress value: $\sigma = \dfrac{P + \hat{n}\,\ell/2}{A(\ell/2)} = 140 \quad MPa$