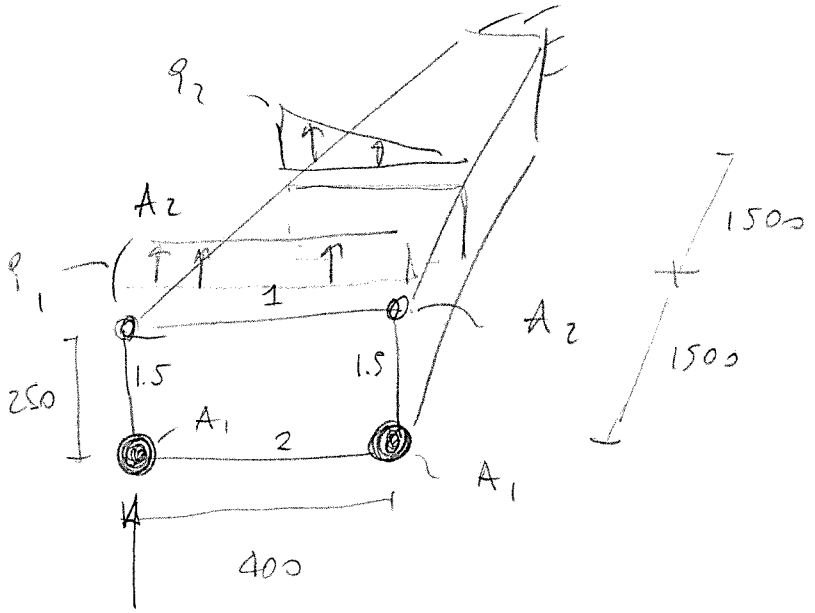
Ex 1



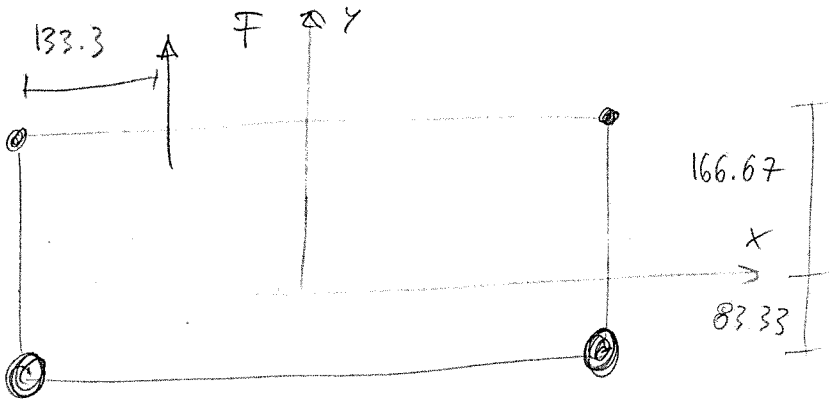
$A_1 = 2000\ mm^2$

$A_2 = 1000\ mm^2$

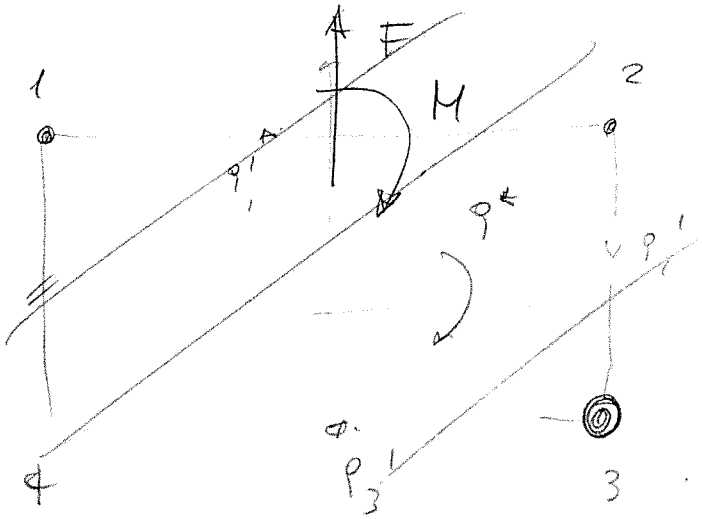
$q_1 = 90\ N/mm$

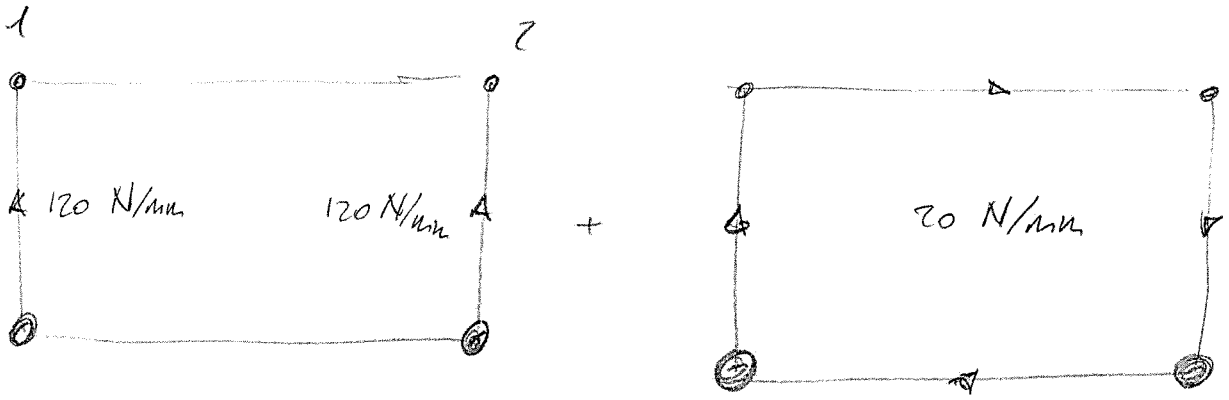
$q_2 = 300\ N/mm$

$F = 10\ kN$



$F = 60\ kN$

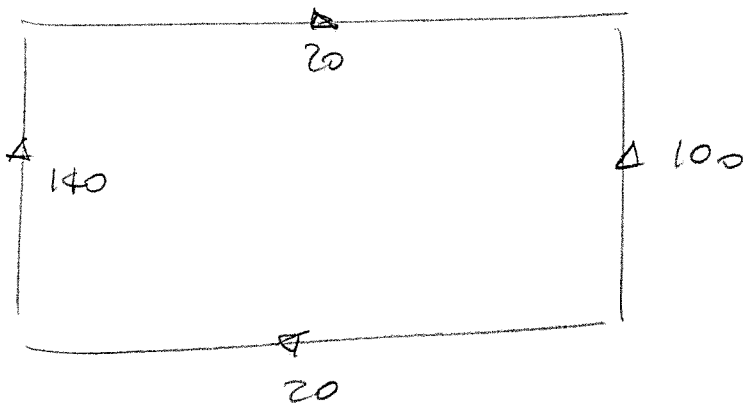




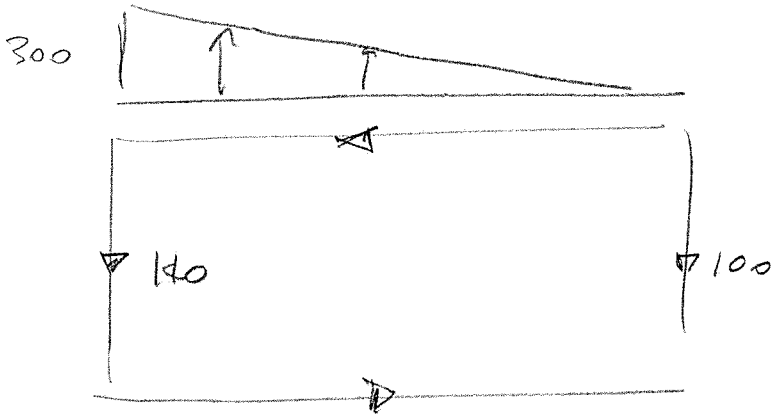
$M = 2q^* \Omega$ $\quad q^* = \frac{M}{2\Omega} = 20$ N/mm

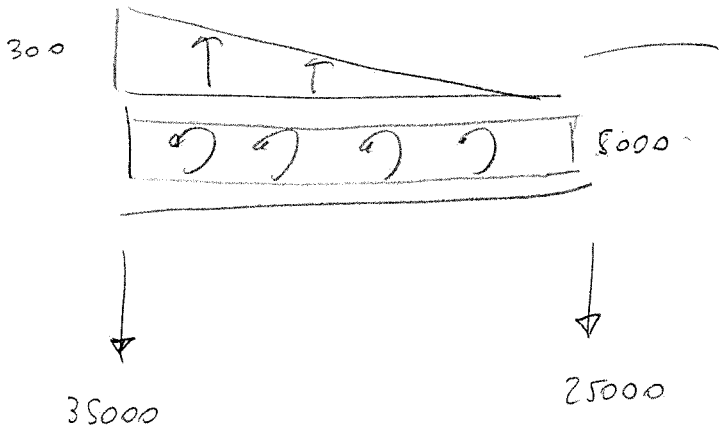
$\Omega = 1 \cdot 10^5$ mm²

Flush:



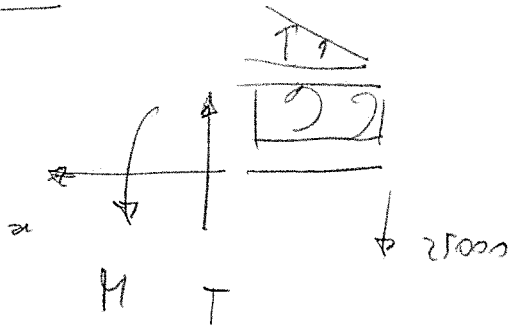
Centre





$p = 300 \frac{x}{400}$

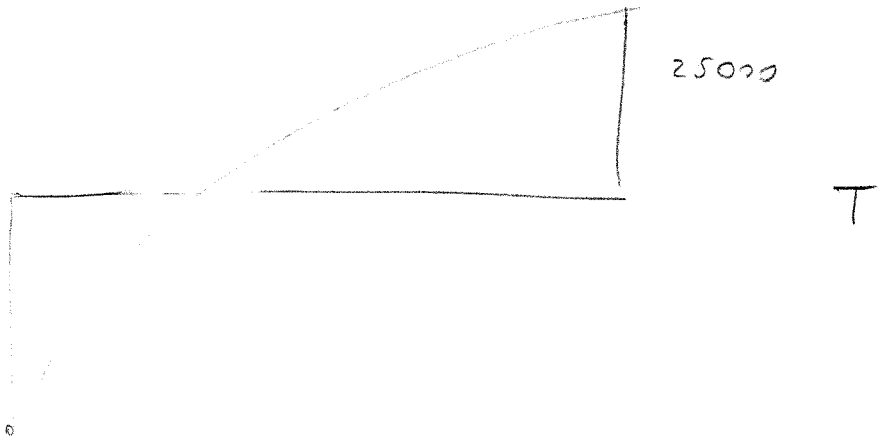
Axial influence



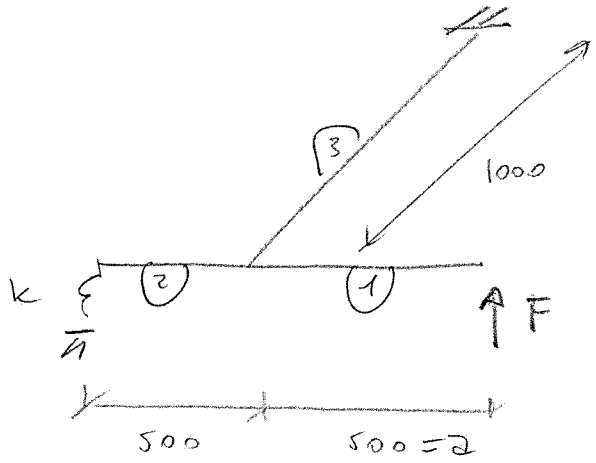
$$T = 25000 - \frac{300}{800} x^2$$

$$M = 25000 x - 5000 x - \frac{3}{24} x^3$$

$$= 20000 x - \frac{3}{24} x^3$$



Ex 2



$k = 400\ N/mm$

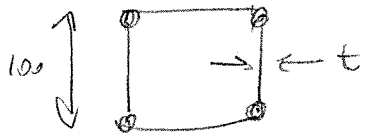
$F = 2000\ N$

$A = 200\ mm^2$

$t = 1\ mm$

$E = 70\ GPa$

$a = 500\ mm$



$EJ = 1.4 \cdot 10^{11}\ N mm^2$

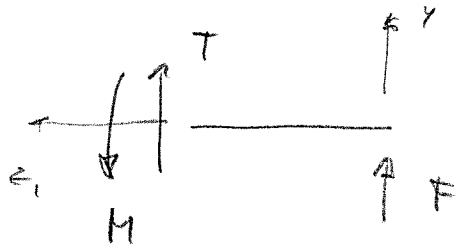
$$J_t = \frac{4\Omega^2}{\oint \frac{1}{t}\,dl} = \frac{4\Omega^2 t}{4l} = \Omega^2 \frac{t}{l} = 1 \cdot 10^6\ mm^4$$

$GJ = 2.7 \cdot 10^{10}\ N mm^2$

$GA^* = 200\ mm^2$

$GA^* = 5.38 \cdot 10^6\ N$

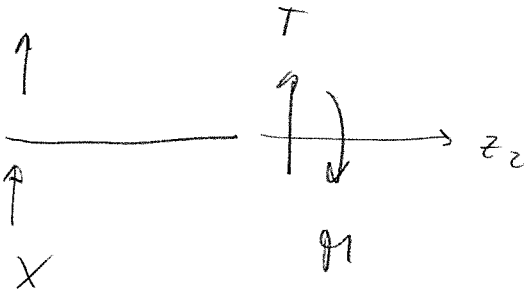
Trave 1



$$T^1 = -F$$

$$M^1 = -Fz_1$$

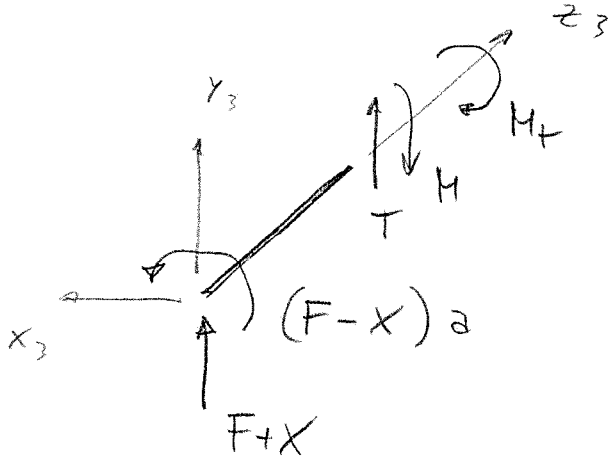
Trave 2



$$T^2 = -X$$

$$M^2 = -Xz_2$$

Trave 3



$$T^3 = -(F+X)$$

$$M^3 = -(F+X)z_3$$

$$M_t^3 = (F-X)a$$

Dummy

$$\delta T^1 = 0$$

$$\delta M^1 = 0$$

$$\delta T^2 = -1$$

$$\delta M^2 = -z_2$$

$$\delta T^3 = -1$$

$$\delta M^3 = -z_3$$

$$\delta M_t^3 = -a$$

PLVC

$$\int_0^a \left(\frac{X}{GA} + X \frac{z_2^2}{EJ} \right) dz_2 + \int_0^l \frac{F+X}{GA} + \frac{F+X}{EJ} z_3^2 + \frac{(X-F)a^2}{GJ}$$

$$+ \frac{X}{k} = 0$$

$$X \left[\left(\frac{a}{GA} + \frac{a^3}{3EJ} \right) + \frac{l}{GA} + \frac{l^3}{3EJ} + \frac{a^2 l}{GJ} + \frac{1}{k} \right] =$$

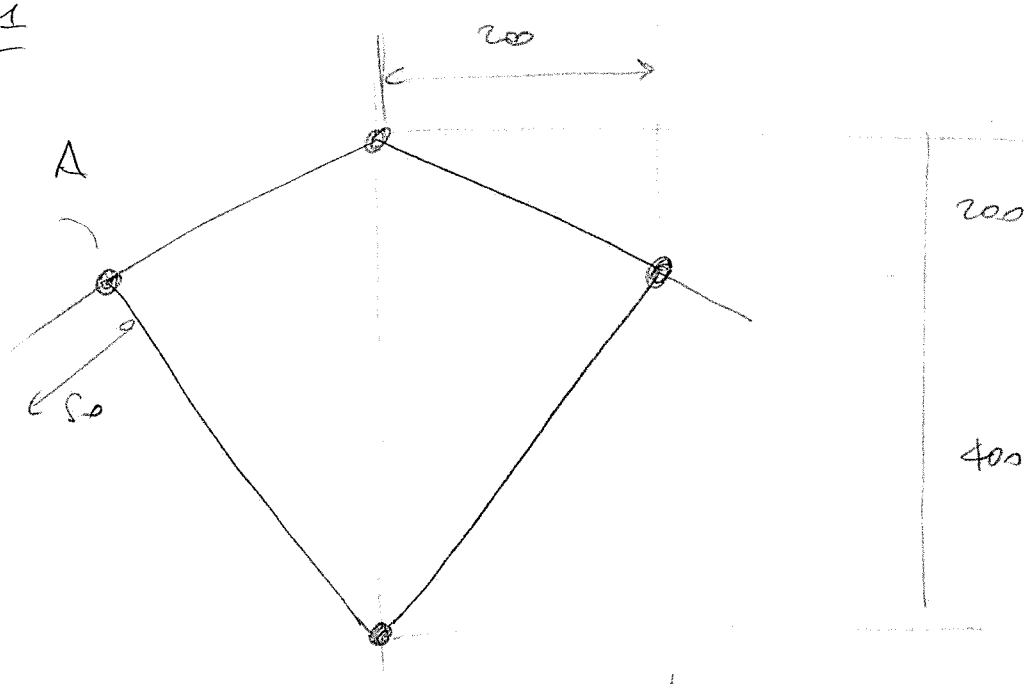
$$= -F \left(\frac{l}{GA} + \frac{l^3}{3EJ} - \frac{a^2 l}{GJ} \right)$$

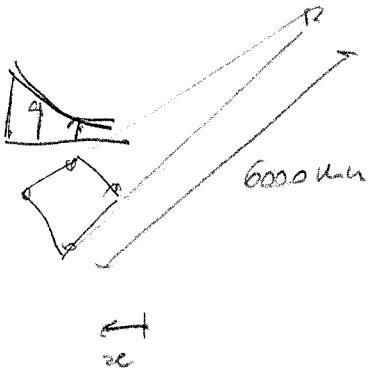
$X = 909.51 \ N$

Spostamento:

$$\delta = \frac{X}{k} = 2.27 \ mm$$

Ex 1





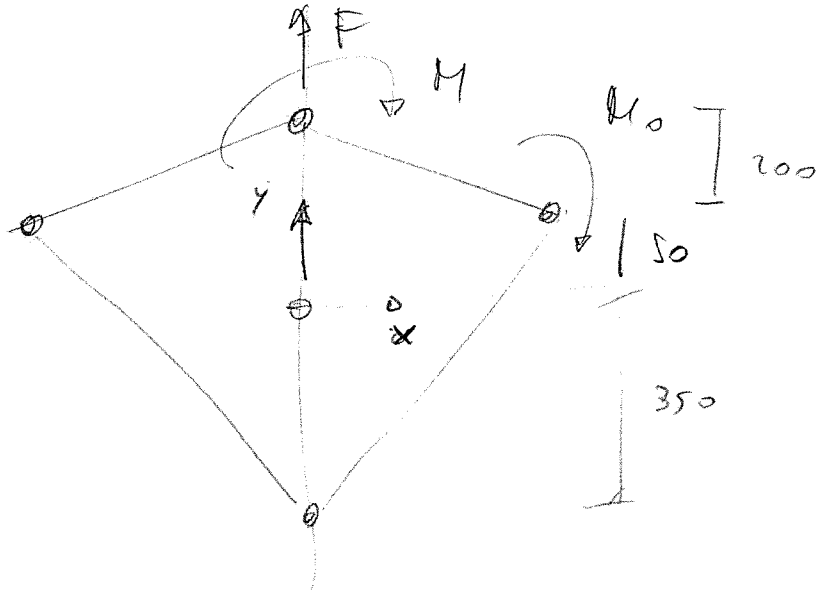
x

$t = 1.5$

$A = 1000\ mm^2$

$q = 150 \left(\frac{x}{400}\right)^2$

Trova sforzi di taglio e σ @ $z = 3000$ mm

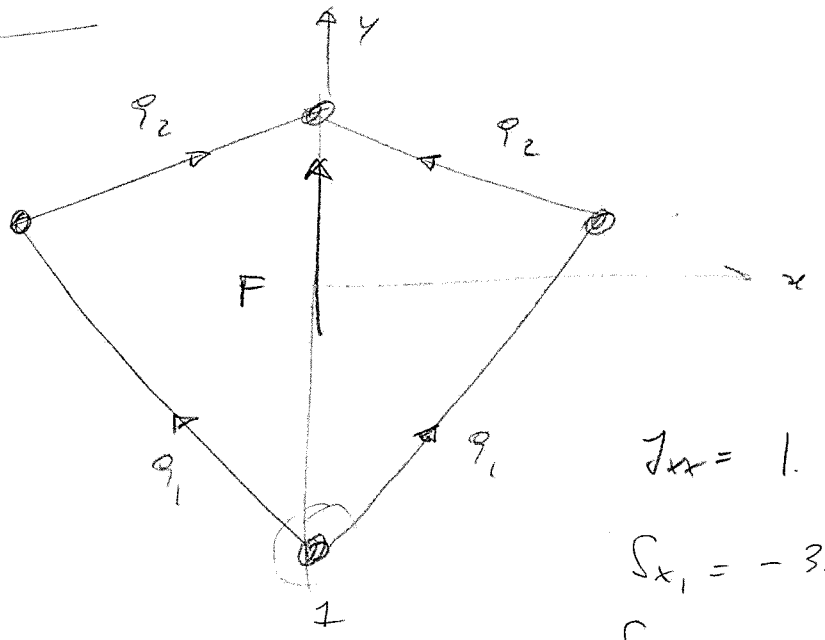


Risultanti:

$$F = \frac{150}{400^2} \frac{x^3}{3}\Big|_0^{400} = 20\ kN$$

$$M_0 = \int_0^{400} \frac{150}{400^2} x^3 dx = \frac{150}{400^2} \frac{400^4}{4} = 6 \cdot 10^6\ Nmm \quad (\text{wrt } x=0)$$

$M = M_0 - 200\,F = 2 \cdot 10^6$ N·mm



$J_{xx} = 1.9 \cdot 10^8$ N mm²

$S_{x_1} = -3.5 \cdot 10^5$ mm²

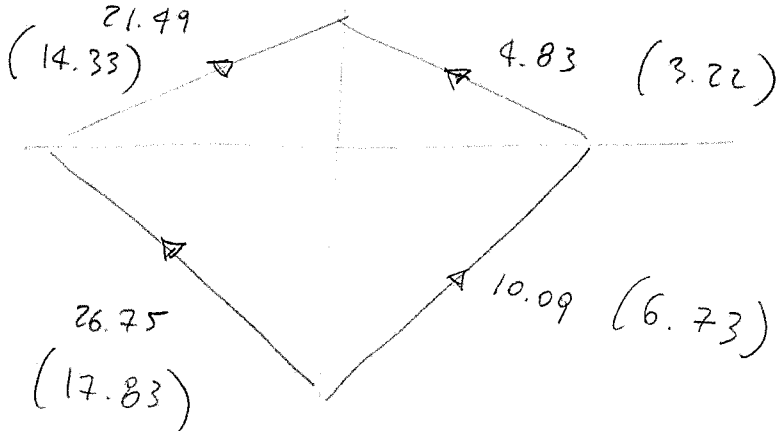
$S_{x_2} =$

$$2q_1 = -\frac{S_{x_1}}{J_{xx}} F \Rightarrow q_1 = 18.42 \text{ N/mm}$$

$$+2q_2 = +\frac{S_{x_2}}{J_{xx}} F \Rightarrow q_2 = 13.16$$

$\Omega = 600 \cdot 200 = 1.2 \cdot 10^5$ mm²

$M = 2q^* \Omega \qquad q^* = \dfrac{M}{2\Omega} = 8.33$ N/mm



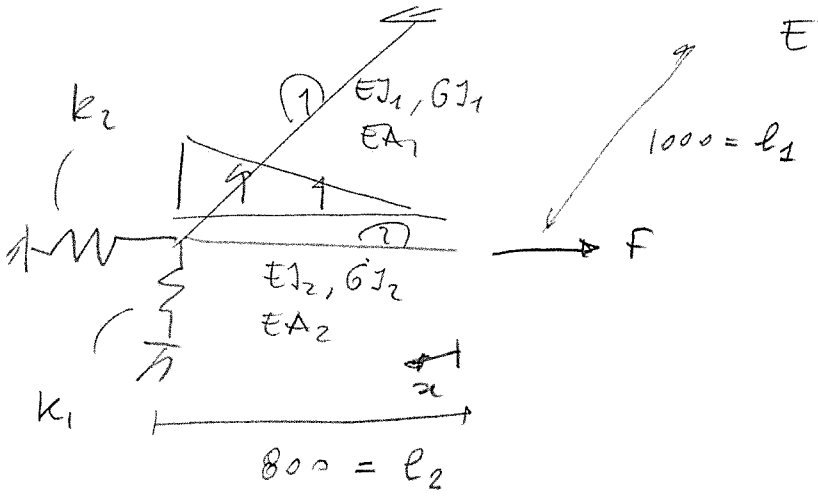
Tra parentesi sforzi di taglio

Azioni di concio:

$\sigma_x^1 = -78.95$ MPa	$N_1 = -78.95$ kN
$\sigma_x^2 = -15.79$ MPa	$N_2 = -15.79$ kN
$\sigma_x^3 = -15.79$ MPa	$N_3 = -15.79$ kN
$\sigma_x^4 = 110.53$ MPa	$N_4 = 110.53$ kN

Ex 2

$F = 4\ 10^4\ N$



$EJ_{xx}^{1} = EJ_{yy}^{1} = 9.\ 10^8\ N/mm^2$

$GJ_1 = 4 \cdot 10^8\ N/mm^2$

$EJ_2 = 7 \cdot 10^8\ N/mm^2$

$GJ_2 = 2 \cdot 10^8\ N/mm^2$

$EA_2 = 2.8 \cdot 10^7\ N$

$EA_1 = 5.7 \cdot 10^7\ N$

$k_1 = 2.8 \cdot\ N/mm$

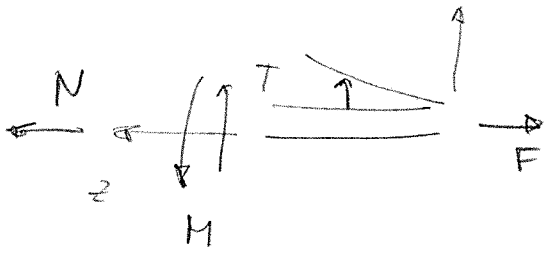
$k_2 = 3.5 \cdot 10^4\ N/mm$

$q(x) = 6 \cdot 10^{-5} \left(\frac{x}{800}\right)\ N/mm$

$q(x) = ax$

$a = 7.5 \cdot 10^{-6}$

Trave 2

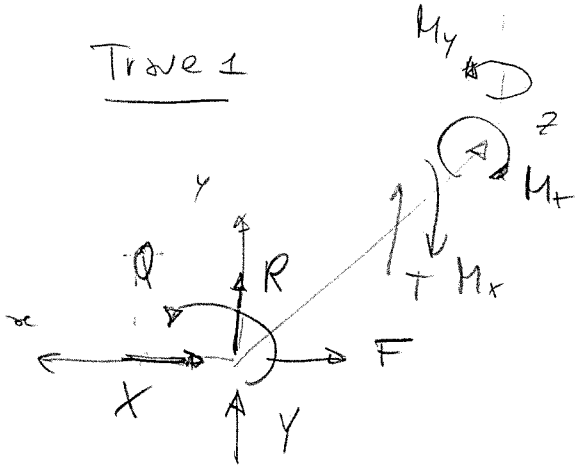


$N = F$

$T = -\frac{ax^2}{2}$

$M = -\frac{ax^3}{6}$

Trave 1



$R = \frac{a l_2^2}{2} = 2.4\ \ N$

$Q = \frac{a l_2^3}{6} = 640\ \ N/mm$

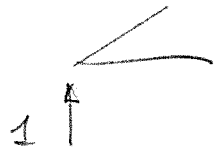
$$T = -(R+Y)$$

$$M_x = -(R+Y)z$$

$$M_y = -(X+F)z$$

$$M_t = 0$$

Dummy # 1



Trave 2

Trave 1

${}^1\delta M_x^1 = -z$

Dummy # 2

Trave 2

Trave 1

${}^2\delta M_y^1 = -z$

PLVC

1) $$\int_0^{\ell_1} \frac{(R+Y)z^2}{EJ_1}\,dz + \frac{Y}{k_1} = 0$$

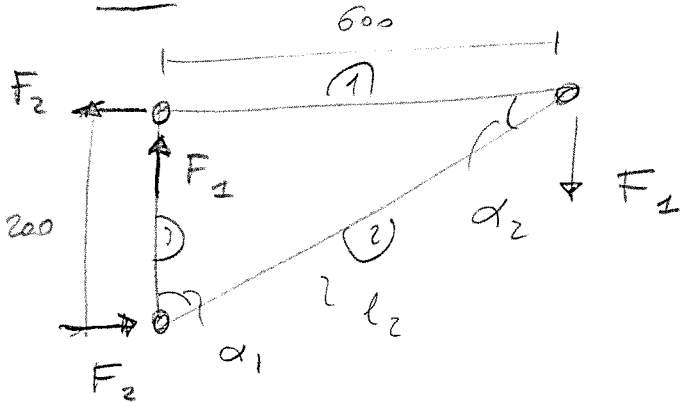
$$\left(\frac{\ell_1^3}{3} + \frac{EJ_1}{k_1}\right) Y = -\frac{R\ell_1^3}{3} \qquad \left(6.55\,Y = -8\right)$$

$$Y = -1.22 \Rightarrow s = -Y/k_1 = +0.44\ \text{mm}$$

2) $$\left(\frac{\ell_1^3}{3} + \frac{EJ_1}{k_2}\right) X = -\frac{F\ell_1^3}{3} \qquad \left(3.33\,X = -1.33\cdot 10^5\right)$$

$$X = -40\ \text{kN} \Rightarrow s = -X/k_2 = 1.14\ \text{mm}$$

Ex3



$F_1 = 1 \cdot 10^4 \, N$

$F_2 = 3 \cdot 10^4 \, N$

$A = 400 \, mm^2$

Sol.

$$l_2 = \sqrt{l_1^2 + l_2^2} = 632.5 \, mm$$

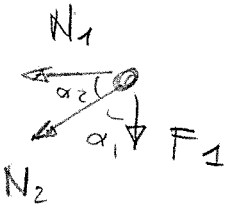
$$\alpha_1 = \arccos \frac{200}{l_2} = 71.57°$$

$$\alpha_2 = 18.43$$

Trave 1

$$N = F_2 = 3 \cdot 10^4 \Rightarrow \sigma = \frac{N}{A} = 75 \, MPa$$

Trave 2



$$N + N_1 \cos\alpha_2 + F_1 \cos\alpha_1 = 0$$

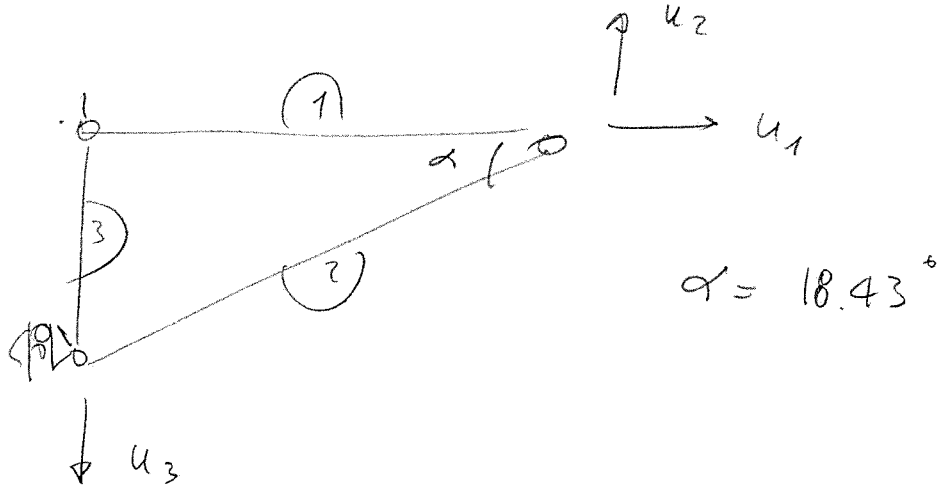
$$N_2 = -3.16 \cdot 10^4 \, N$$

$$\sigma = -79 \, MPa$$

Trave 3

$$N_3 = F_1 \Rightarrow \sigma = \frac{F_1}{A} = \frac{25}{\cancel{133.33}} \, MPa$$

Soluzione agli spostamenti



$\alpha = 18.43°$

$$\delta l_2 = \begin{bmatrix} c & s & s \end{bmatrix} \begin{Bmatrix} \delta u_1 \\ \delta u_2 \\ \delta u_3 \end{Bmatrix}$$

$$\delta \underline{u}^T \begin{bmatrix} c \\ s \\ s \end{bmatrix} \begin{bmatrix} c & s & s \end{bmatrix} \frac{EA}{\ell_2} \underline{u} +$$

$$+ \begin{bmatrix} EA/\ell_1 & & \\ & 0 & \\ & & EA/\ell_3 \end{bmatrix} \underline{u} = \begin{Bmatrix} 0 \\ -F \\ 0 \end{Bmatrix}$$

$$\Rightarrow \underline{u} = \begin{Bmatrix} 0.64 \\ -4.26 \\ 0.07 \end{Bmatrix} \quad mm$$

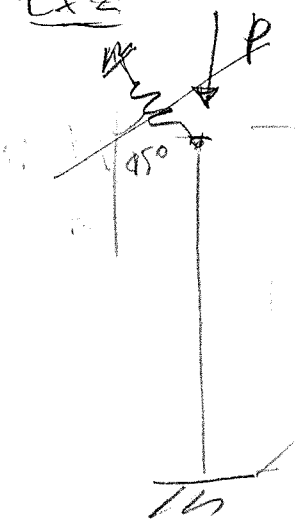
$$\underline{\underline{k}} = 1 \cdot 10^5 \begin{bmatrix} 0.87 & 0.13 & 0.13 \\ & 0.44 & 0.44 \\ & & 1.44 \end{bmatrix} \quad N/mm$$

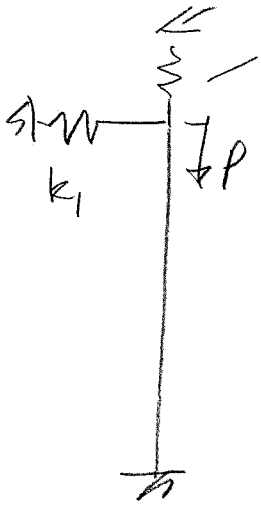
$\sigma_1 = 75 \quad MPa \qquad \left(\frac{\delta l}{\ell_i} : \varepsilon \right)$

$\sigma_2 = -79 \quad MPa$

$\sigma_3 = 75 \quad MPa$

Ex 2

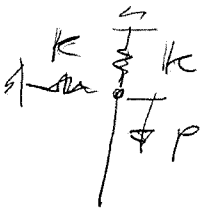


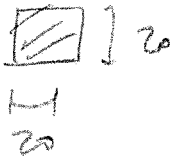


$k = 1\ N/mm$

$l = 1200\ mm$

$E = 70\ GPa$





$EJ = 9.33 \cdot 10^8\ N mm^2$

$EA = 2.8 \cdot 10^7\ N$

$u = \left(1 - \cos \frac{\pi x}{2l}\right) q$ | $u = x^2 q$

$u'' = \left(\frac{\pi}{2l}\right)^2 \cos \frac{\pi x}{2l}\, q$

$u' = \frac{\pi}{2l} \sin \frac{\pi x}{2l}\, q$

$\int_0^l \delta u'' EJ u'' = \int_0^l \left(\frac{\pi}{2l}\right)^4 \cos^2 \frac{\pi x}{2l}\, q\, \delta q\, EJ$

$= \left(\frac{\pi}{2l}\right)^4 \frac{l}{2} EJ\, q\, \delta q$

$\int_0^l \delta u' N u' = \int_0^l \left(\frac{\pi}{2l}\right)^2 \sin^2 \, q\, \delta q\, N =$

$= \left(\frac{\pi}{2l}\right)^2 \frac{l}{2} N\, q\, \delta q$

$$N = -\frac{EA/\ell \cdot P}{\frac{EA}{\ell} + k_2} = -\gamma P \qquad \gamma = 0.7$$

$$\left[\left(\frac{\pi}{2\ell}\right)^2 EJ - \gamma P + k_1 \frac{2}{\ell}\left(\frac{2\ell}{\pi}\right)^2\right] q = 0$$

$$P = \frac{\left(\frac{\pi}{2\ell}\right)^2 EJ + k_1 2/\ell \left(\frac{2\ell}{\pi}\right)^2}{\gamma} = 3.67 \text{ kN}$$

Usando x^2

$u' = 2x$

$u'' = 2$

$u(\ell) = \ell^2$

$$\int_0^\ell \delta u'' EJ u'' = 4\ell EJ\, q$$

$$\int_0^\ell \delta u' N u' = N \int 4x^2 = N \frac{4}{3}\ell^3 = \frac{4}{3}\ell^3 N$$

$$= -\frac{4}{3}\ell^3 \gamma P$$

$$\left(4\ell EJ - \frac{4}{3}\ell^3 \gamma P + k_1 \ell^4\right) q = 0$$

de aqui

$P = 4.06$ kN