Course of Aerospace Structures

Written test, September 9th, 2024

Name

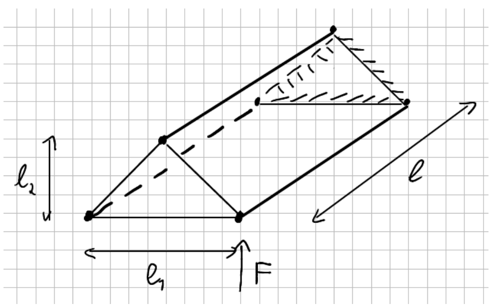
Surname

Person code: | | | | | | | | |

Exercise 1

The semi-monocoque structure in the figure has a three stringers with concentrated area equal to A each and three panels with thickness t. The structure is clamped at one extremity and loaded by the vertical force F, applied to the rightmost stringer, at the other end. Compute the value of the concentrated area A required so that the maximum absolute value of the axial stress is equal to the prescribed value σ_{zz}.

(Unit for result: mm^2)



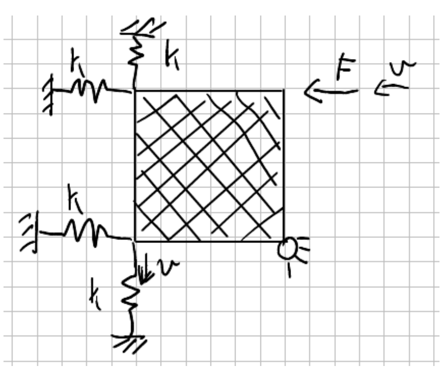
Data
$l_1 = 200$ mm
$l_2 = 150$ mm
$l = 2000$ mm
$t = 0.3$ mm
$E = 72000$ MPa
$\nu = 0.3$
$F = 10000$ N
$\sigma_{zz} = 400$ MPa

Answer

Exercise 2

The sketched square is infinitely rigid. It is hinged at the bottom-right corner, and is connected to two horizontal and two vertical springs, each with an unknown stiffness K, at the bottom-left and top left corners, as skethced. It is also loaded by the horizontal force F at the top-right corner. Compute the spring stiffness K so that the horizontal displacement $\overline{v}$ at the the point of application of the force F is equal to the prescribed value. Assume infinitesimal displacements and rotations.

(Unit for result: N/mm)

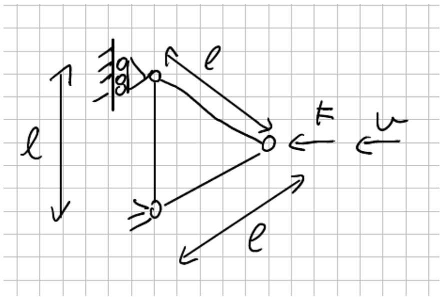


Data
$l = 100$ mm
$F = 1500$ N
$\overline{v} = 0.1$ mm

Answer

Exercise 3
Consider the three-hinged beam structure sketched in the figure, where each beam is hinged at its extremity, loaded by the concentrated force F. Compute the displacement v at the point of application of the force F. Neglect shear deformability.

(Unit for result: mm)

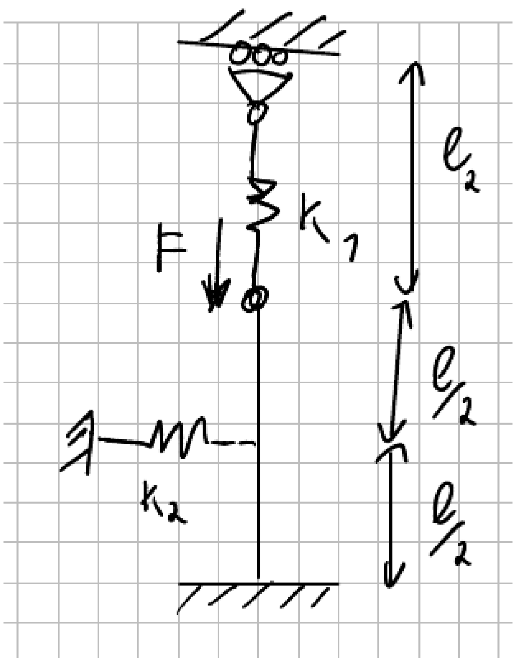


Data
$l = 1000$ mm
$EA = 1 \times 10^8$ N
$EI_{xx} = EI_{yy} = 1 \times 10^{13}$ N mm^2
$GJ = 7 \times 10^9$ N mm^2
$F = 1000$ N

Answer

Exercise 4
Consider the clamped beam sketched in the figure, with length l, and connected to the two springs with stiffness K_1 and K_2. Estimate, by using a polynomial approximation with one term for the beam transverse displacement, the critical buckling load F of the structure.

(Unit for result: N)



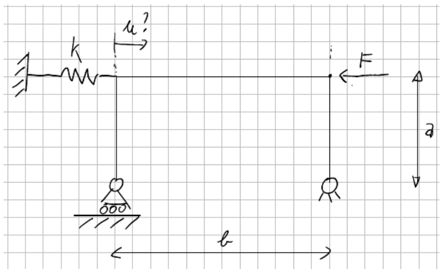
Data
$l = 1000$ mm
$GA^* = 1 \times 10^{10}$ N
$EI = 12 \times 10^9$ N mm^2
$EA = 1 \times 10^{10}$ N
$K_1 = 1.5 \times 10^7$ N/mm
$K_2 = 2500$ N/mm

Answer

Exercise 5

Consider the slender beam structure sketched in the figure, loaded by the concentrated force F. Compute the displacement u of the extremity of the spring. Neglect shear deformability.

(Unit for result: mm)



Data
$a = 1000$ mm
$b = 1500$ mm
$EA = 1 \times 10^{10}$ N
$EI_{xx} = EI_{yy} = 1 \times 10^{13}$ N mm^2
$GJ = 7 \times 10^{9}$ N mm^2
$K = 1 \times 10^{3}$ N/mm
$F = 1000$ N

Answer ______________________________

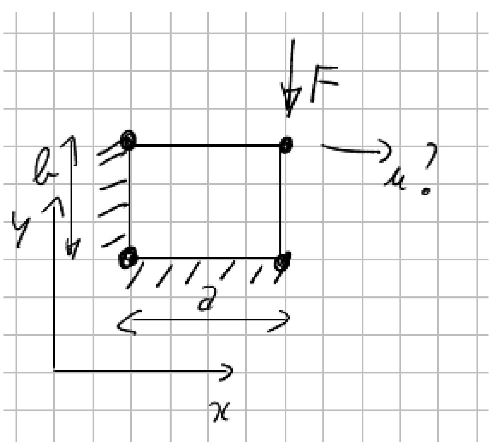
Exercise 6

The single bilinear finite element sketched in the figure has the displacement of the top left, bottom left and bottom right nodes completely constrained. The element has unit thickness, and the material works in a state of plane stress, so that

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{1}$$

The top right node is loaded by the vertical force F, as sketched. Compute the horizontal displacement u of the top right node.

(Unit for result: mm)



Data
$t = 1$ mm
$a = 4$ mm
$b = 3$ mm
$E = 72000$ MPa
$\nu = 0.3$
$F = 100$ N

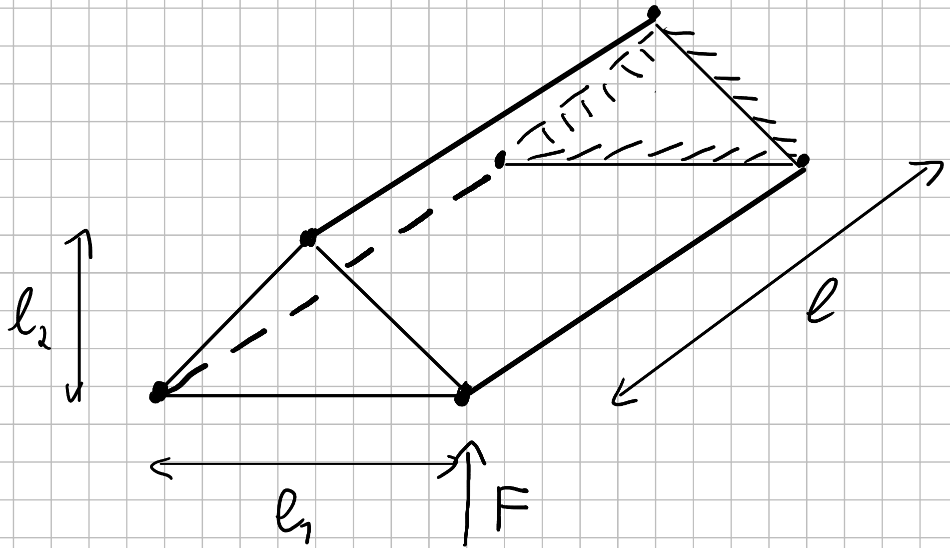
Answer ______________________________

True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. The torsional stiffness of a single-cell thin-walled cross section is proportional to the second power of the wall thickness t.
2. According to the de Saint-Venant solutions, it is possible to compute the torsional rotation in a beam only if the shear force F is applied through the shear center.
3. The axial stress σ_{zz} in a clamped beam loaded at the free extremity by a shear force F varies linearly with respect to the axial coordinate z.

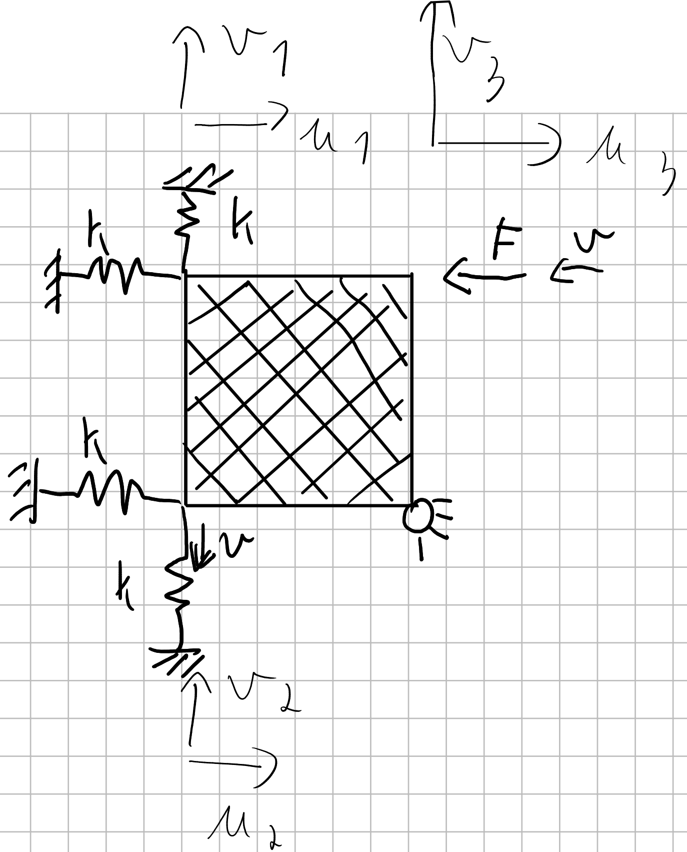
Multiple Choice questions
(Circle the correct answer)

1. According to the analytical solution, the shear stress transmitted by a glued connection is:
   (a) higher at the extremities
   (b) lower at the extremities
   (c) constant
   (d) described by a sin function
   (e) described by a cos function
   (f) described by a quadratic polynomial function
   (g) none of the above
2. An Euler-Bernoulli cantilever beam with uniform stiffness is clamped at one extremity and loaded with a transverse (shear) concentrated force at the tip. The bending solution obtained using a displacement-based method based on polynomial functions with two unknown coefficients is:
   (a) approximated
   (b) exact
   (c) wrong, because with two unknown coefficients we cannot impose the natural boundary conditions
   (d) none of the above
3. A "simply supported" plate has:
   (a) vertical (normal to the plate) displacement and bending rotation prevented on one of its boundary sides
   (b) vertical displacement and bending rotation prevented on all of its boundary sides
   (c) vertical displacement prevented on two opposite boundary sides, bending rotation left free
   (d) vertical displacement prevented on all of its boundary sides, bending rotation left free
   (e) vertical displacement and bending rotation prevented on two opposite boundary sides, vertical displacement prevented and bending rotation left free on the other two
   (f) none of the above



$$\frac{F\ell}{\ell_2 A} = \bar{\sigma}$$

$$A = \frac{F\ell}{\ell_2} \frac{1}{\bar{\sigma}}$$



$$u_3 = -\vartheta \ell$$

$$u_1 = -\vartheta \ell$$

$$v_1 = -\vartheta \ell$$

$$u_2 = 0$$

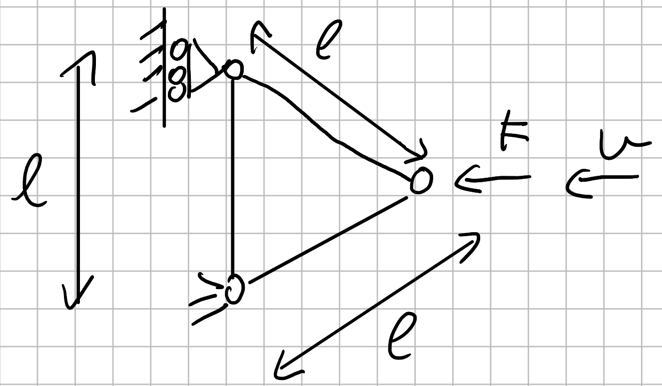
$$v_2 = -\vartheta \ell = -\bar{v}$$

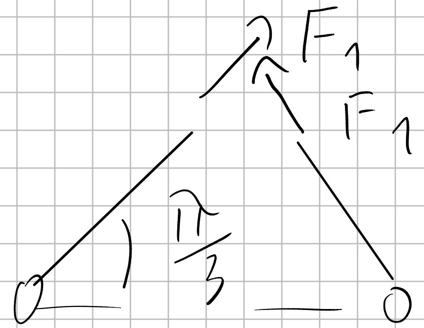
$$\vartheta = \frac{\bar{v}}{\ell}$$

$$\delta v_1 k v_1 + \delta u_1 k u_1 + \delta u_2 k u_2 + \delta v_2 k v_2 = -\delta u_3 F$$

$$\delta\vartheta \ell k (\bar{v} + \bar{v} + \bar{v}) = \delta\vartheta \ell F$$

$$3\bar{v}\cancel{\ell}K = \cancel{\ell}F \qquad K = \frac{F}{3\bar{v}}$$

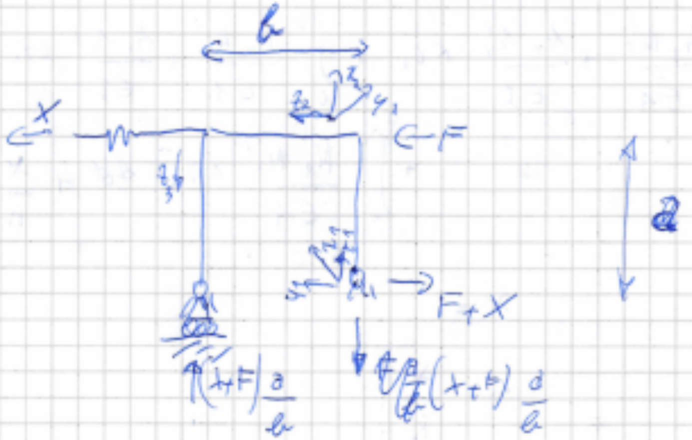


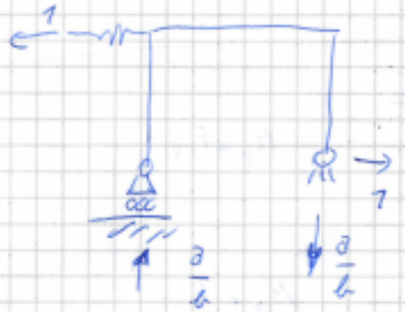


$$2 F_1 \sin \frac{\pi}{3} = F$$

$$F_1 = \frac{F}{2 \sin \frac{\pi}{3}}$$

$$\frac{F}{4\sin^2\frac{\pi}{3}} \cdot \ell \cdot 2\,\frac{1}{EA} + \frac{F\cos^2\frac{\pi}{3}}{4\sin^2\frac{\pi}{3}} \cdot \frac{\ell}{EA} = v$$





$N_1 = (X+F)\,\frac{a}{b}$ $\quad N'_1 = \frac{a}{b}$

$T_{y_1} = F+X$ $\quad T'_{y_1} = 1$

$M_{z_1} = -(F+X)\cdot z$ $\quad M'_{z_1} = -z$

$T_{z_2} = (X+F)\,\frac{a}{b}$ $\quad T'_{z_2} = \frac{a}{b}$

$N_{z_2} = X$ $\quad N'_{z_2} = 1$

$M_{y_2} = -(X+F)\,\frac{a\,z}{b} + (X+F)\,a$ $\quad M'_{y_2} = -\frac{a}{b}\cdot z + a$

$N_{z_3} = -(X+F)\,\frac{a}{b}$ $\quad N'_{z_3} = -\frac{a}{b}$

$$\int_0^a \left(\frac{N_1 N_1'}{EA} + \frac{M_1 M_1'}{EI}\right) dz_1 + \int_0^b \left(\frac{N_2 N_2'}{EA} + \frac{M_2 M_2'}{EI}\right) dz_2$$

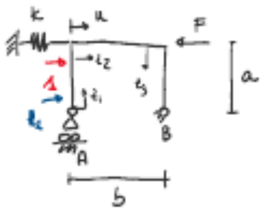
$$+ \int_0^a \frac{N_3 N_3'}{EA}\, dz_3 + \frac{X}{k} = \emptyset$$

$X = \ldots$

$u = \frac{X}{k}$

2024-07-08 Ex 3

Wednesday, July 3, 2024 11:25 AM



Sol

EQUIL.

$X: \quad H_x - F + R_{x_B} = 0$

$Y: \quad R_{Y_A} + R_{Y_B} = 0$

$M_B^{\circlearrowleft}: \quad +F \cdot a - H_x \cdot a - R_{Y_A} \cdot b = 0$

$$\boxed{R_{Y_A} = (F - H_x)\frac{a}{b}}$$

$1 + R'_{X_B} = 0$

$R'_{Y_A} + R'_{Y_B} = 0$

$-1 \cdot a - R'_{Y_A} \cdot b = 0$

$$\boxed{R'_{Y_A} = -\frac{a}{b}}$$

$z_1: \; T_{z_1} = -(F - H_x)\frac{a}{b}$

$T'_{z_1} = +\frac{a}{b}$

$z_2: \; T_{z_2} = -H_x$

$T'_{z_2} = -1$

$M_{x_2} = -(F - H_x)\frac{a}{b} \cdot z_2$

$M'_{x_2} = +\frac{a}{b} z_2$

$z_3: \; T_{z_3} = (F - H_x)\frac{a}{b}$

$T'_{z_3} = -\frac{a}{b}$

$M_{x_3} = -H_x \cdot z_3 + F z_3 - (F - H_x)\frac{a}{\cancel{b}} \cdot \cancel{b}$

$M'_{x_3} = -1 \cdot z_3 + \frac{a}{\cancel{b}} \cdot \cancel{b}$

M_x

T_z

PCVW

$$1 \cdot -\frac{H_x}{k} = \int_0^a -\frac{F - H_x}{EA} \cdot \frac{a^2}{b^2} dz_1 +$$

$$+ \int_0^b \frac{H_x}{EA} - \frac{F - H_x}{EJ} \cdot \frac{a^2}{b^2} \cdot z_2^2 \, dz_2 +$$

$$+ \int_0^a -\frac{F - H_x}{EA} \frac{a^2}{b^2} dz_3 +$$

$$+ \int_0^a \frac{(F - H_x) z_3 - (F - H_x) a}{EJ} \cdot (-z_3 + a) \, dz_3$$

$$-\frac{H_x}{k} = \frac{H_x a^3}{EAb^2} - \frac{F a^3}{EAb^2} + \frac{H_x}{EA} \cdot b + \frac{1}{3}\frac{H_x a^2 b^{\cancel{3}}}{EJ \cancel{b^2}} +$$

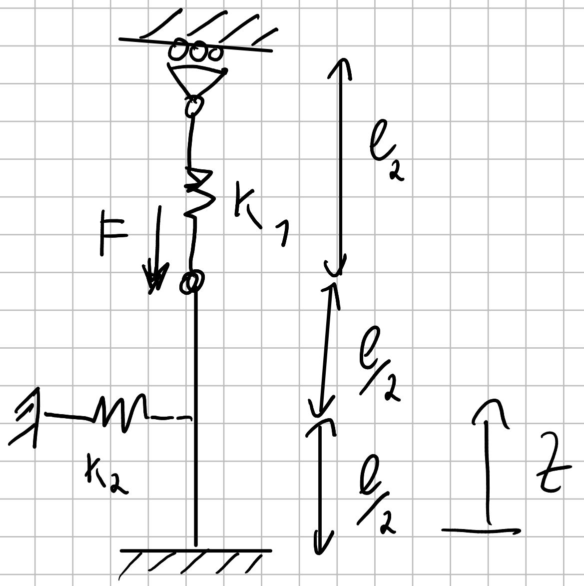
$$- \frac{F a^2 b^{\cancel{3}}}{3EJ b^{\cancel{2}}} + \frac{H_x a^3}{EAb^2} - \frac{F a^3}{EAb^2} +$$

$$+ \frac{1}{3}\frac{H_x a^3}{EJ} - \frac{1}{3}\frac{F a^3}{EJ} - \frac{1}{2}\frac{H_x a^3}{EJ} + \frac{1}{2}\frac{F a^3}{EJ} +$$

$$+ \frac{1}{2}\frac{F a^3}{EJ} - \frac{1}{2}\frac{H_x a^3}{EJ} - \frac{F a^3}{EJ} + \frac{H_x a^3}{EJ}$$

SOLVE FOR H_x

$$\rightarrow \boxed{u = -\frac{H_x}{k} = -7.699 \cdot 10^{-2} \text{ mm}}$$



$$k_3 = \frac{EA}{\ell}$$

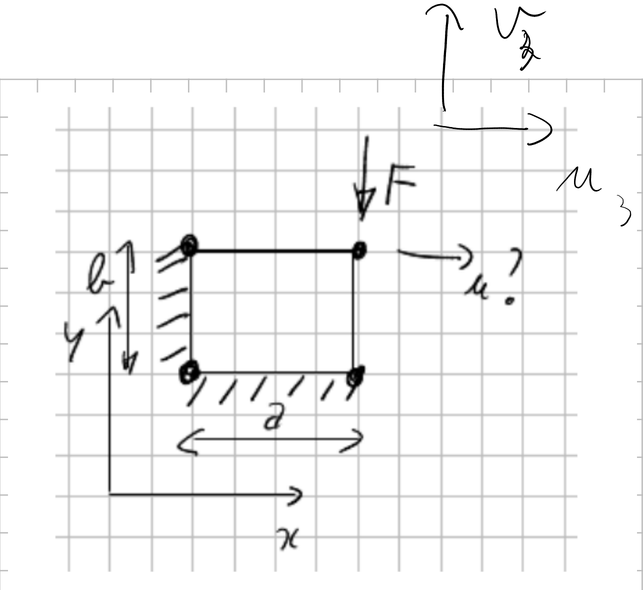
$$N_{11} = \frac{F}{k_1 + k_3} \cdot k_1$$

$$N_3 = \frac{F}{k_1 + k_3} \cdot k_3$$

$$v_3 = a z^2$$

$$\int_0^{\ell} \left(\delta a \, 4 EI \, a - F_3 \, \delta a \, 4 z^2 a \right) dz + \delta a \frac{\ell^4}{16} a k_2 = 0$$

$$F = \ldots$$



$$u(x,y) = \frac{xy}{ab}\, u_3$$

$$v(x,y) = \frac{xy}{ab}\, v_3$$

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = \frac{y}{ab}\, u_3$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = \frac{x}{ab}\, v_3$$

$$\gamma_{xy} = \frac{x}{ab}\, u_3 + \frac{y}{ab}\, v_3$$

$$G = \frac{E}{2(1+\nu)}$$

$$C = \frac{1}{E}\begin{bmatrix} 1 & -\nu & 0 \\ -\nu & 1 & 0 \\ 0 & 0 & \frac{E}{G} \end{bmatrix}$$

$$E_\varepsilon = C^{-1}$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \underbrace{\begin{bmatrix} \frac{y}{ab} & 0 \\ 0 & \frac{x}{ab} \\ \frac{x}{ab} & \frac{y}{ab} \end{bmatrix}}_{B} \begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix}$$

$$\delta \begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix}^T \underbrace{\int_0^a \int_0^b B^T E_E \, B \, dy \, dx}_{K} \begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -F \end{Bmatrix}$$

$$\begin{Bmatrix} u_3 \\ v_3 \end{Bmatrix} = K^{-1} \begin{Bmatrix} 0 \\ -F \end{Bmatrix}$$

True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. The torsional stiffness of a single-cell thin-walled cross section is proportional to the second power of the wall thickness t.
   - False
2. According to the de Saint-Venant solutions, it is possible to compute the torsional rotation in a beam only if the shear force F is applied through the shear center.
   - False
3. The axial stress σ_{zz} in a clamped beam loaded at the free extremity by a shear force F varies linearly with respect to the axial coordinate z.
   - True

Multiple Choice questions
(Circle the correct answer)

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   (a) approximated
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