

EXERCISE SESSION 9 - 22/11/22

Ribs Frames & Junctions

Ex 1

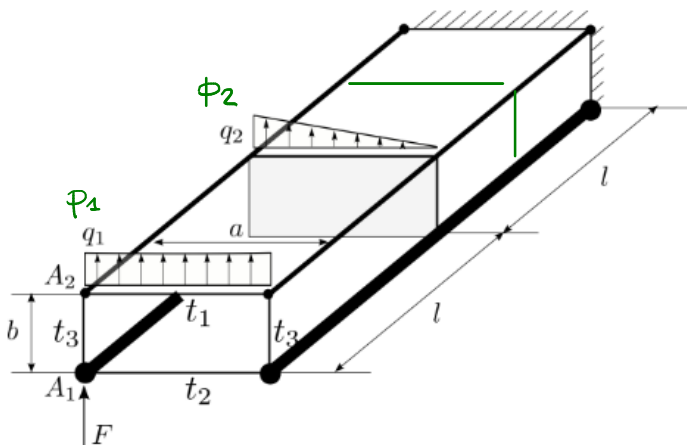
Course of Aerospace Structures

Written test, June 26th, 2019

Exercise 1

Consider the single-cell, thin-walled beam in the figure. The beam length is equal to $2l$; the section has dimensions $a \times b$, and is stiffened by four stringers, each characterized by lumped area equal to A_1 (at the bottom) and A_2 (at the top). The thickness of the panels is denoted with t_1 , t_2 and t_3 , as illustrated in the figure. Referring to the loading conditions reported in the sketch:

- determine the shear stresses in the panels and the axial stresses in the stringers for the section at a distance $l/2$ from the constraint;
- plot the internal actions on the rib at the mid-span, assuming that the rib can be modeled as a beam.



Data

$l = 1500 \text{ mm}$; $a = 400 \text{ mm}$; $b = 250 \text{ mm}$;
 $t_1 = 1 \text{ mm}$; $t_2 = 2 \text{ mm}$; $t_3 = 1.5 \text{ mm}$;
 $A_1 = 2000 \text{ mm}^2$; $A_2 = 1000 \text{ mm}^2$;
 $q_1 = 90 \text{ N/mm}$; $q_2 = 300 \text{ N/mm}$;
 $F = 10 \text{ kN}$;

q_1 & p_2

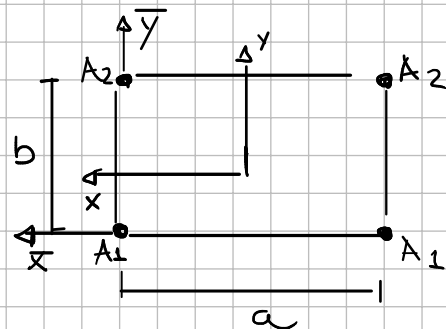
SECTION PROPERTIES

$$A_2 = A \quad A_1 = 2A$$

$$\bar{x}_{cg} = -\frac{a}{2}$$

$$\bar{y}_{cg} = \frac{1}{3}b$$

$$J_{xx} = \sum_i A_i y_i^2 = \frac{4}{3}Ab^2$$



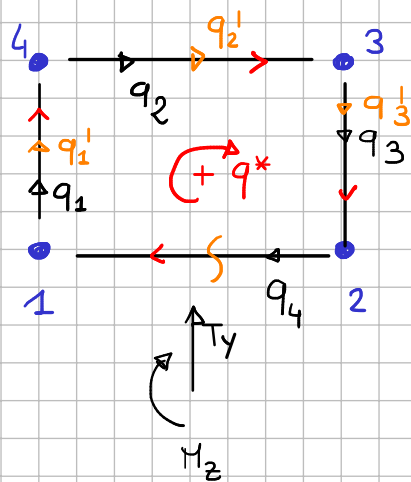
OPEN CELL FLUXES

$$S_{x_1} = S_{x_2} = -A_1 \frac{1}{3} b = -\frac{2}{3} Ab$$

$$S_{x_3} = S_{x_4} = A_2 \frac{2}{3} b = \frac{2}{3} Ab$$

$$q_1' = -q_3' = -T_y \cdot \frac{S_{x_1}}{J_{xx}} = \frac{1}{2} \frac{T_y}{b}$$

$$q_2' = 0$$



MOMENT EQUIVALENCE WRT ① ↺

$$\text{LHS Mom} = M_z - T_y \cdot \frac{a}{2}$$

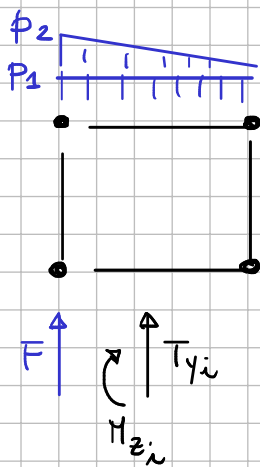
$$\text{RHS Mom} = 2q^* \cdot \Omega_{\text{CELL}} + 2q_3' \cdot \Omega_3$$

$$\text{WHERE } \Omega_{\text{CELL}} = a \cdot b$$

$$\Omega_3 = \frac{1}{2} a \cdot b$$

$$\text{SOLVE } \text{LHS Mom} = \text{RHS Mom} \rightarrow q^*(T_y, M_z)$$

LUMP FORCES @ X=0



$$T_{y_F} = F$$

$$M_{z_F} = F \cdot \frac{a}{2}$$

$$T_{y_{p_1}} = p_1 \cdot a$$

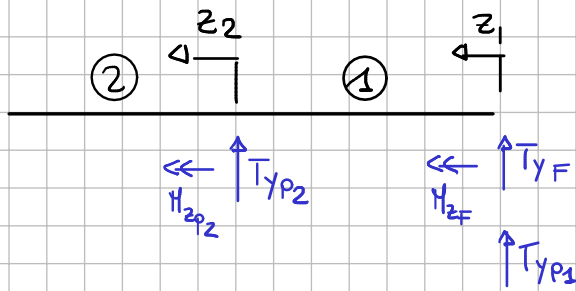
$$M_{z_{p_1}} = 0$$

$$T_{y_{p_2}} = \frac{1}{2} p_2 \cdot a$$

$$M_{z_{p_2}} = \frac{1}{12} p_2 a^2$$

$$= \frac{1}{2} p_2 a \cdot \frac{1}{6} a$$

BEAM INTERNAL ACTIONS



$$\textcircled{1} \quad T_{y_1} = -T_{yF} - T_{yP_1}$$

$$M_{x_1} = -(T_{yF} + T_{yP_1}) z_1$$

$$M_{z_1} = -M_{zF}$$

$$\textcircled{2} \quad T_{y_2} = -T_{yP_1} - T_{yF} - T_{yP_2}$$

$$M_{x_2} = -(T_{yP_1} + T_{yF})(z_2 + \ell) - T_{yP_2} \cdot z_2$$

$$M_{z_2} = -M_{zF} - M_{zP_2}$$

SHEAR FLUXES @ $z_2 = \frac{\ell}{2} \rightarrow$ EVALUATE FLUXES FOR

$$\begin{aligned} T_y &= T_{y_2} \left(\frac{\ell}{2} \right) \\ M_z &= M_{z_2} \left(\frac{\ell}{2} \right) \end{aligned}$$

$$q_1 = q_1' + q^* = -242 \frac{N}{mm}$$

$$q_2 = q_2' + q^* = -30 \frac{N}{mm}$$

$$q_3 = q_3' + q^* = -182 \frac{N}{mm}$$

$$q_4 = q^* = -30 \frac{N}{mm}$$

Axial stress in stringers @ $z_2 = \frac{l}{2}$ $M_{x_2}(\frac{l}{2})$

$$\sigma_{zz_i} = \frac{T_z}{\sum_i A_i} + \frac{M_x}{J_{xx}} y_i - \frac{M_y}{J_{yy}} x_i$$

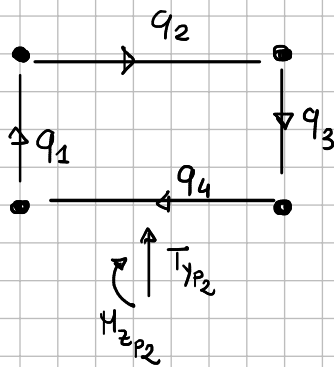
$$\sigma_{zz_1} = \sigma_{zz_2} = - \frac{M_{x_2}(\frac{l}{2})}{J_{xx}} \cdot \frac{1}{3} b = 148,5 \text{ MPa}$$

$$\sigma_{zz_3} = \sigma_{zz_4} = + \frac{M_{x_2}(\frac{l}{2})}{J_{xx}} \cdot \frac{2}{3} b = -297 \text{ MPa}$$

Internal action in RIB

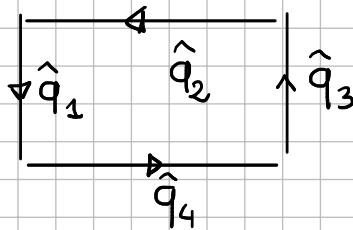
EVALUATE EQUIVALENT FLOWS FOR THE LOAD APPLIED

BY THE RIB



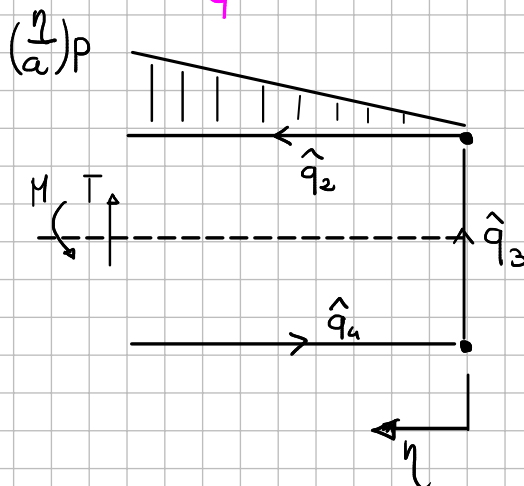
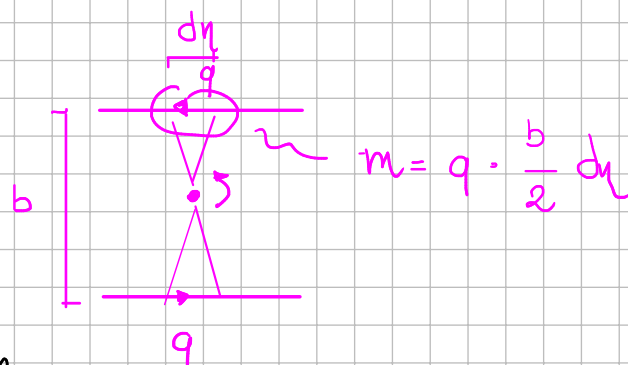
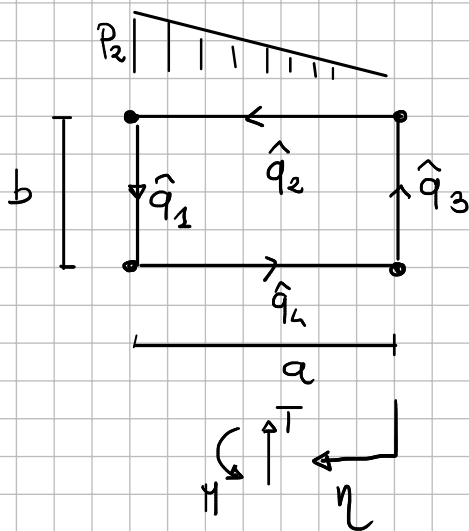
$$q_i = q_i(T_{yP_2}, M_{z_2})$$

APPLY THE EQUILIBRATING FLOWS TO THE RIB

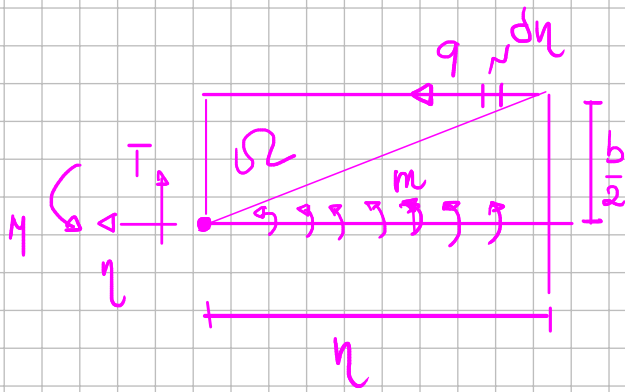


$$q_i \hat{=} q_i$$

BEAM MODEL FOR RIB



TWO WAY TO COMPUTE INTERNAL MOMENT



WAY ①:
COLLAPSE IN ONE
LINE AS
DISTR. MOMENT

$$m = q \cdot \frac{b}{2} \quad \text{CONTRIB. FOR } d\eta$$

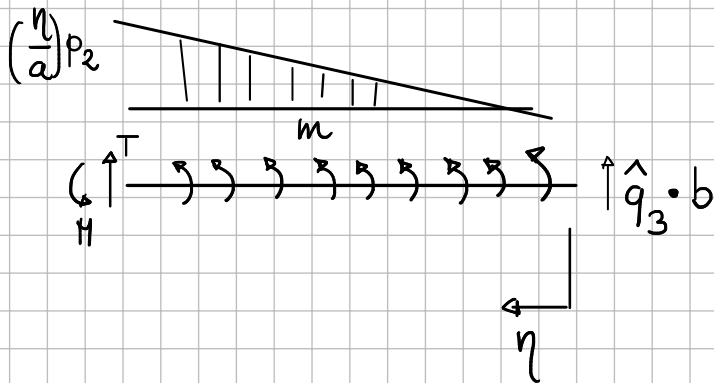
$$M = - \int_0^\eta m d\eta = - \frac{1}{2} q b \eta$$

WAY ② USE MOH EQUIVALENCE FORMULATION $2q\Omega$

$$M = -2q \cdot \Omega$$

$$= -2q \cdot \frac{1}{2} \frac{b}{2} \cdot \eta = - \frac{1}{2} q b \eta$$

Pay attention to the sign: beam internal action impose equilibrium not equivalence.



$$m = \hat{q}_2 \cdot \frac{b}{2} + \hat{q}_4 \cdot \frac{b}{2}$$

$$T(\eta) = - \hat{q}_3 \cdot b - \frac{1}{2} \left(\frac{\eta}{a} \right) p_2 \cdot \eta$$

$$M(\eta) = - m \cdot \eta - \hat{q}_3 \cdot b \cdot \eta - \frac{1}{2} \left(\frac{\eta}{a} \right) p_2 \cdot \eta \cdot \frac{1}{3} \eta$$

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```
clear variables
close all
home
```

Data

```
l = 1500;    % mm
a = 400;     % mm
b = 250;     % mm
t_1 = 1;     % mm
t_2 = 2;     % mm
t_3 = 1.5;   % mm
A_1 = 2000;  % mm^2
A_2 = 1000;  % mm^2
% On the given data this was defined as q_1 and q_2 renamed to avoid
% confusion with fluxes
p_1 = 90;    % N/mm
p_2 = 300;   % N/mm
F = 10000;   % N
```

Section properties

```
x_cg = -a/2;
y_cg = (2*A_2*b)/(2*A_2+2*A_1);

J_xx = 2*A_1*(1/3*b)^2 + 2*A_2*(2/3*b)^2;

S_x1 = -A_1*(1/3)*b;
S_x2 = S_x1;
S_x3 = A_2*(2/3)*b;
S_x4 = S_x3;
```

Open cell fluxes

```
syms T_y
q_1_p = -T_y*(S_x1/J_xx);
q_2_p = -T_y*((S_x1+S_x4)/J_xx);
q_3_p = -q_1_p;
```

Moment equivalence wrt 1

```
syms M_z q_s
Omega_cell = a*b;
Omega_3 = .5*a*b;
LHS_Mom = M_z - T_y*(a/2);
RHS_Mom = 2*q_s*Omega_cell + 2*q_3_p*Omega_3;

q_s = solve(LHS_Mom == RHS_Mom, q_s);

q_1 = q_1_p + q_s;
q_2 = q_2_p + q_s;
q_3 = q_3_p + q_s;
q_4 = q_s;
```

Lump forces to centroid

```
T_y_F = F;
M_z_F = F*.5*a;

T_y_p1 = p_1*a;
M_z_p1 = 0;

T_y_p2 = .5*p_2*a;
M_z_p2 = (1/12)*p_2*a^2;
```

Beam internal actions

```
syms z_1 z_2
T_y_1 = -T_y_F-T_y_p1;
M_x_1 = -(T_y_F + T_y_p1)*z_1;
M_z_1 = -M_z_F-M_z_p1;

T_y_2 = -T_y_F-T_y_p1-T_y_p2;
M_x_2 = -(T_y_F + T_y_p1)*(1+z_2) - T_y_p2*z_2;
M_z_2 = -M_z_F-M_z_p1-M_z_p2;
```

Shear fluxes for $z_2 = l/2$

```
q_1_d = double(subs(q_1, [T_y, M_z], [T_y_2, M_z_2]))
q_2_d = double(subs(q_2, [T_y, M_z], [T_y_2, M_z_2]))
q_3_d = double(subs(q_3, [T_y, M_z], [T_y_2, M_z_2]))
q_4_d = double(subs(q_4, [T_y, M_z], [T_y_2, M_z_2]))
```

```
q_1_d =  
-242
```

```
q_2_d =  
-30
```

```
q_3_d =
```

q_4_d =

-30

Axial stress in stringers for $z_2 = l/2$

```
s_zz1 = double(-(subs(M_x_2, z_2, .5*1)/J_xx)*b/3)
s_zz2 = double(s_zz1)
s_zz3 = double((subs(M_x_2, z_2, .5*1)/J_xx)*2*b/3)
s_zz4 = double(s_zz3)
```

s_zz1 =

148.5000

s_zz2 =

148.5000

s_zz3 =

-297.0000

s_zz4 =

-297.0000

Internal action in rib

```
syms eta
q_1_r = double(subs(q_1, [T_y, M_z], [T_y_p2, M_z_p2]));
q_2_r = double(subs(q_2, [T_y, M_z], [T_y_p2, M_z_p2]));
q_3_r = double(subs(q_3, [T_y, M_z], [T_y_p2, M_z_p2]));
q_4_r = double(subs(q_4, [T_y, M_z], [T_y_p2, M_z_p2]));

m = q_2_r*b/2 + q_4_r*b/2;
T = -q_3_r*b - .5*(eta/a)*p_2*eta
M = -m*eta - q_3_r*b*eta - .5*(eta/a)*p_2*eta*(1/3)*eta

syms x
figure(1)
hold on
title('Shear in rib', 'Interpreter','latex')
fplot(subs(T, eta, (x+a/2)), [-a/2, a/2], 'Color','#0072BD')
xlabel('$x$ [mm]', 'Interpreter','latex')
ylabel('T [N]', 'Interpreter','latex')
grid on
ax = gca();
ax.XDir='reverse';
```

```

ax.TickLabelInterpreter='latex';
ax.Box = 1;

figure(2)
hold on
title('Bending moment in rib', 'Interpreter','latex')
fplot(subs(M, eta, (x+a/2)), [-a/2, a/2], 'Color','#0072BD')
xlabel('$x$ [mm]', 'Interpreter','latex')
ylabel('M [Nmm]', 'Interpreter','latex')
grid on
ax = gca();
ax.XDir='reverse';
ax.TickLabelInterpreter='latex';
ax.Box = 1;

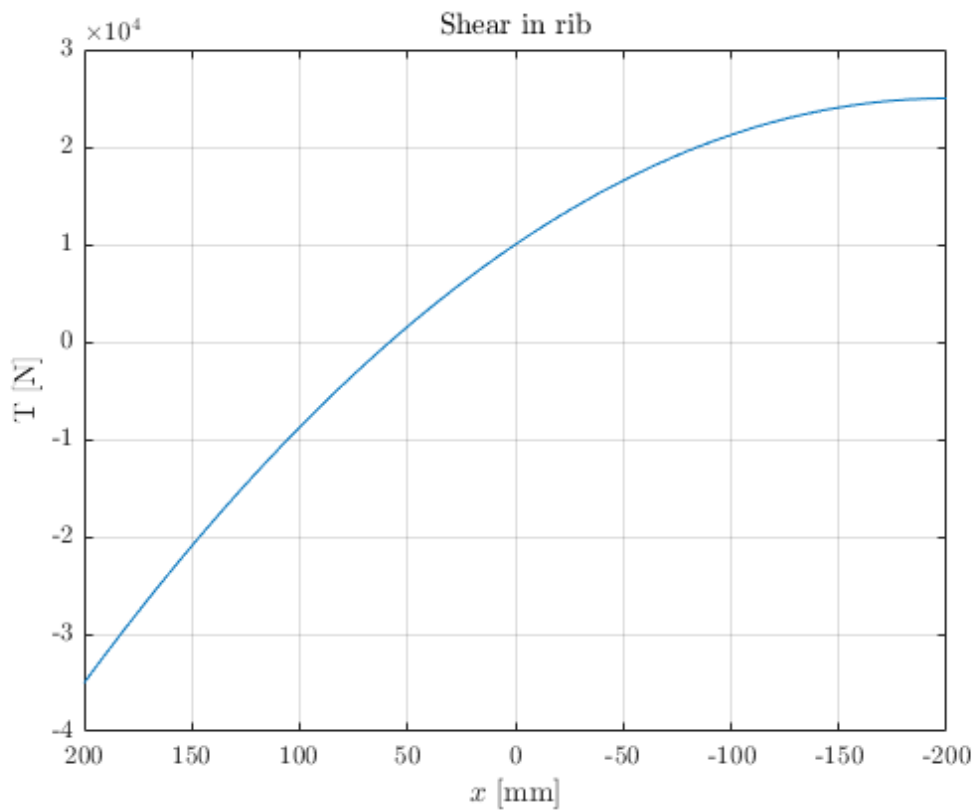
```

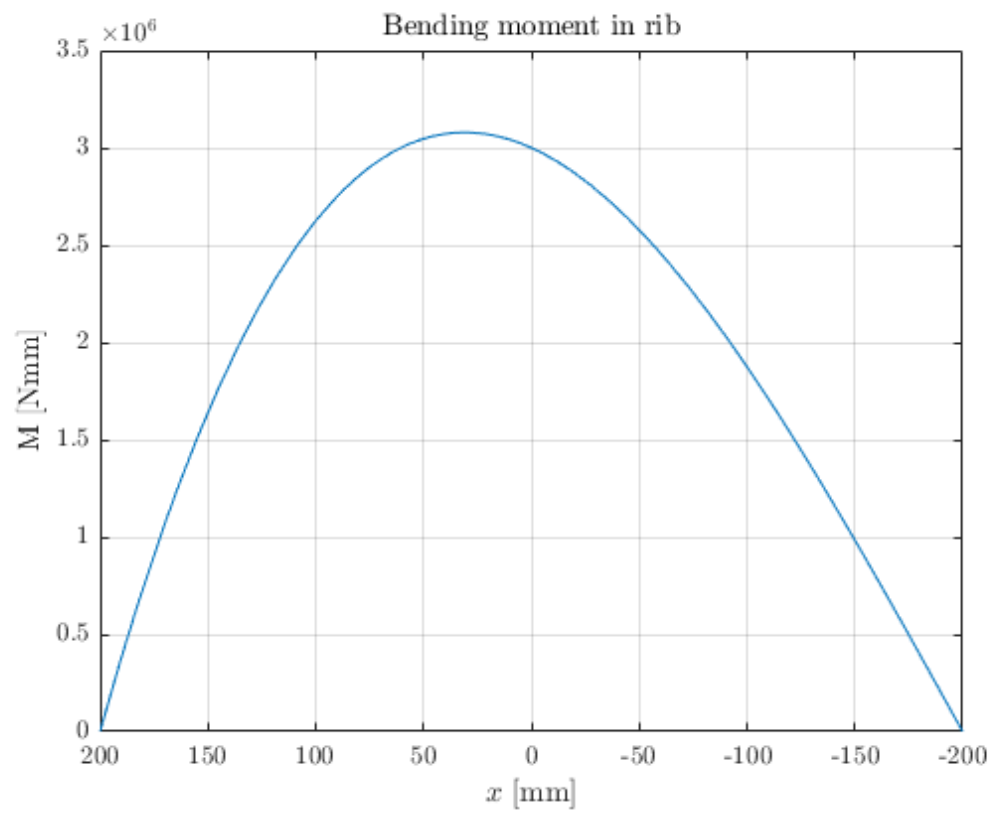
T =

$$25000 - (3 \cdot \eta^2)/8$$

M =

$$- \eta^3/8 + 20000 \cdot \eta$$

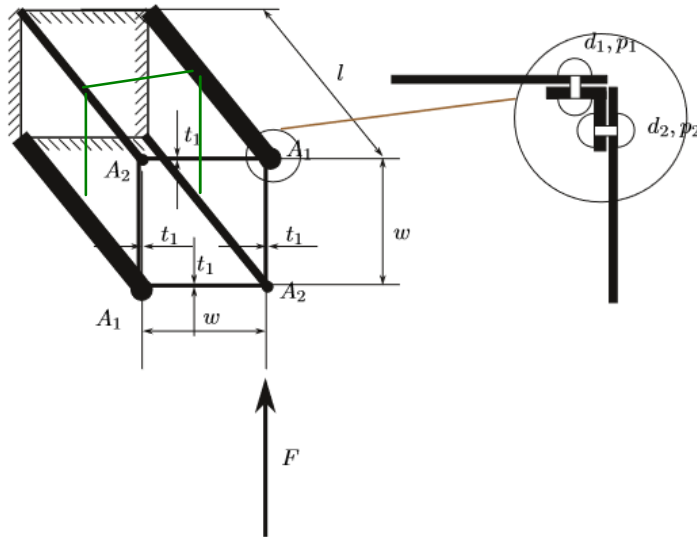




AEROSPACE STRUCTURES

Written test February 11 2019

- The semi-monomocoque wing of Figure 1 is loaded by a concentrated force F . Refer to Table 1 for the problem data, and assume reasonable values for any missing constant. Compute
 - the stress of the stringers at a distance $l/2$ from the loaded end;
 - the average shear stress of the two rivets with diameter d_1 and d_2 connecting the panels to the stringer. The rivets are spaced with pitch p_1 and p_2 , respectively. Assume the area A_1 to belong to the L-shaped stringer only.

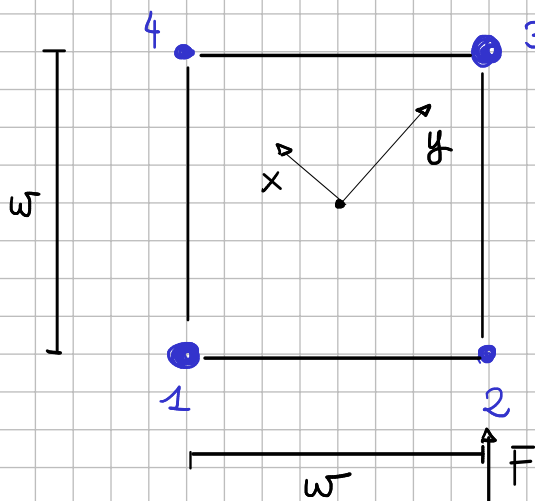


A_1	300 mm ²	l	1500 mm
A_2	150 mm ²	d_1	2 mm
t_1	1.5 mm	d_2	3 mm
w	100 mm	p_1	10 mm
E	72000 MPa	p_2	12 mm
ν	0.3	F	15 kN

Table 1: Semi-monomocoque wing data

Figure 1: Loaded wing

SECTION PROPERTIES



$$A_1 = 2A \quad A_2 = A$$

$$J_{xx} = 2A w^2$$

$$J_{yy} = A w$$

$$S_{x_1} = -2A \frac{\sqrt{2}}{2} w$$

$$S_{x_2} = 0$$

$$S_{x_3} = 2A \frac{\sqrt{2}}{2} w$$

$$S_{x_4} = 0$$

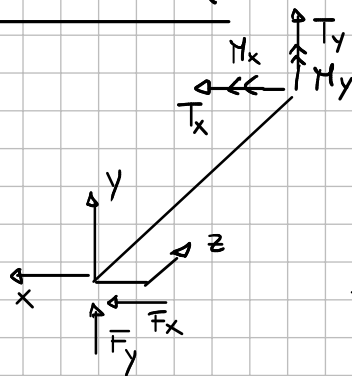
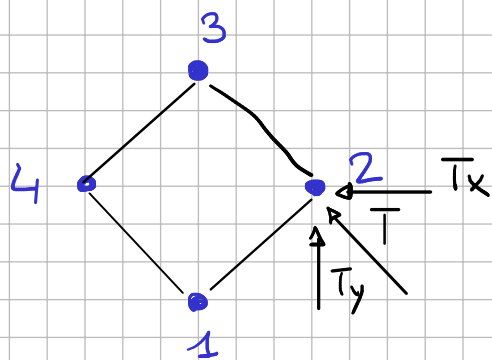
$$S_{y_1} = 0$$

$$S_{y_2} = -A \frac{\sqrt{2}}{2} w$$

$$S_{y_3} = 0$$

$$S_{y_4} = A \frac{\sqrt{2}}{2} w$$

INTERNAL ACTION FOR THE BEAM

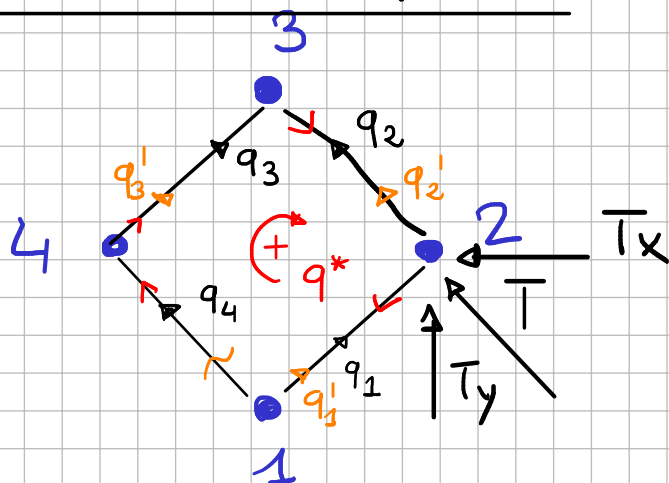


$$F_x = \frac{\sqrt{2}}{2} \bar{F} = F_y$$

$$T_y = -F_y \quad M_x = -F_y \cdot z$$

$$T_x = -F_x \quad M_y = F_x \cdot z$$

OPEN CELL FLUXES



$$q_1' = -T_y \frac{S_{x1}}{J_{xx}} - T_x \frac{S_{y1}}{J_{yy}}$$

$$q_2' = -T_y \frac{S_{x1} + S_{x2}}{J_{xx}} - T_x \frac{S_{y1} + S_{y2}}{J_{yy}}$$

$$q_3' = -T_y \frac{S_{x4}}{J_{xx}} - T_x \frac{S_{y4}}{J_{yy}}$$

MOM. EQUIVALENCE WRT (2) (+)

$$\text{LHS-Mom} = 0$$

$$\text{RHS-Mom} = 2q^* \cdot \Omega_{\text{CELL}} + 2q_3' \cdot \Omega_3$$

$$\text{WHERE } \Omega_{\text{CELL}} = \omega^2$$

$$\Omega_3 = \frac{1}{2} \omega^2$$

$$\text{SOLVE } \Gamma \quad q^*$$

STRESS IN STRINGERS @ $z = \frac{l}{2}$

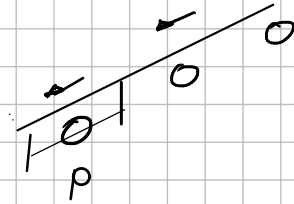
$$\sigma_{zz_1} = -\sigma_{zz_3} = -\frac{M_x}{J_{xx}} \cdot \frac{\sqrt{2}}{2} w = 187.5 \text{ MPa}$$

$$\sigma_{zz_2} = -\sigma_{zz_4} = +\frac{M_y}{J_{yy}} \cdot \frac{\sqrt{2}}{2} w = 375.0 \text{ MPa}$$

SHEAR STRESS IN RIVETS

EACH RIVET CARRIES:

$$F_i = q_i \cdot p_i$$



$$\tau_i = \frac{F_i}{\pi \cdot (d_i/2)^2}$$

$$\tau_1 = \frac{q_3 \cdot p_1}{\pi (d_1/2)^2} = 119,37 \text{ MPa}$$

WHERE $q_3 = q_3' + q^*$

$$\tau_2 = \frac{q_2 p_2}{\pi (d_2/2)^2} = 190,99 \text{ MPa}$$

$$q_2 = q_2' - q^*$$

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clear variables
close all
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```

Data

```
A_1 = 300; %mm^2
A_2 = 150; %mm^2
t_1 = 1.5; %mm
w = 100; %mm
E = 72000; %MPa
nu = 0.3;
l = 1500; %mm
d_1 = 2; %mm
d_2 = 3; %mm
p_1 = 10; %mm
p_2 = 12; %mm
F = 15000; %N
```

Section properties

```
k = sqrt(2)/2;
J_xx = 2*A_1*(k*w)^2;
J_yy = 2*A_2*(k*w)^2;

S_x1 = -A_1*k*w;
S_x2 = 0;
S_x3 = -S_x1;
S_x4 = 0;

S_y1 = 0;
S_y2 = -A_2*k*w;
S_y3 = 0;
S_y4 = -S_y2;
```

Internal actions

```
syms z

F_x = k*F;
F_y = F_x;
```

```

T_x = -F_x;
T_y = -F_y;

M_x = -F_y*z;
M_y = F_x*z;

```

Open cell fluxes

```

q_1_p = -T_x*(S_y1/J_yy) - T_y*(S_x1/J_xx);
q_2_p = -T_x*((S_y1+S_y2)/J_yy) - T_y*((S_x1+S_x2)/J_xx);
q_3_p = -T_x*(S_y4/J_yy) - T_y*(S_x4/J_xx);

```

Moment equivalence wrt 2

```

syms q_s

Omega_cell = w^2;
Omega_3 = .5*w^2;

LHS_mom = 0;
RHS_mom = 2*q_s*Omega_cell + 2*q_3_p*Omega_3;

[q_s] = solve(LHS_mom == RHS_mom, q_s);

```

Closed cell fluxes

```

q_1 = q_1_p - q_s;
q_2 = q_2_p - q_s;
q_3 = q_3_p + q_s;
q_4 = + q_s;

```

Axial stresses in stringers

```

s_zz1 = double(-(subs(M_x, z, 1/2)/J_xx)*k*w)
s_zz2 = double(+ (subs(M_y, z, 1/2)/J_yy)*k*w)
s_zz3 = double(-s_zz1)
s_zz4 = double(-s_zz2)

```

```

s_zz1 =

187.5000

```

```

s_zz2 =

375.0000

```

```

s_zz3 =

-187.5000

```

```

s_zz4 =

```

-375.0000

Rivet shear stress

```
tau_1 = double((q_3*p_1)/(pi*(.5*d_1)^2))  
tau_2 = double((q_2*p_2)/(pi*(.5*d_2)^2))
```

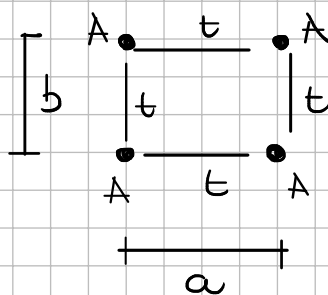
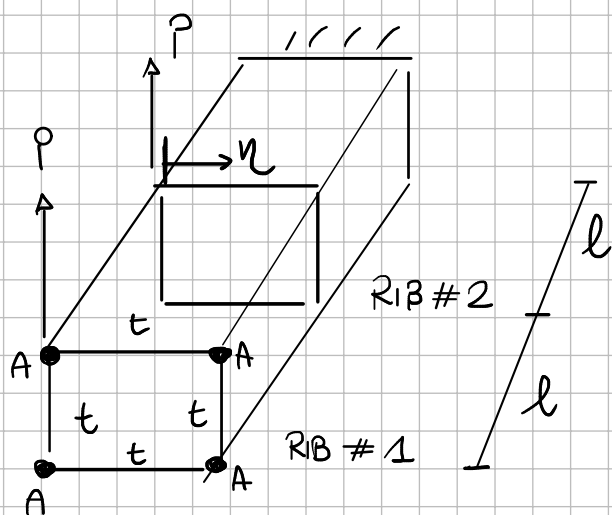
tau_1 =

119.3662

tau_2 =

-190.9859

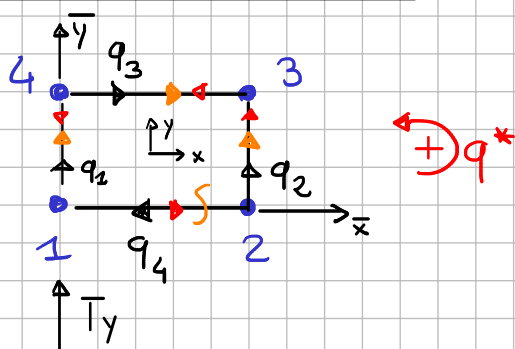
Ex 3



$$\begin{aligned} a &= 500 \text{ mm} & A &= 500 \text{ mm}^2 \\ b &= 250 \text{ mm} & l &= 2000 \text{ mm} \\ t &= 0.6 \text{ mm} & P &= 1000 \text{ N} \end{aligned}$$

FIND BENDING MOM OF THE RIB FOR $\eta = \frac{a}{3}$ IN RIB #2

OPEN CELL FLUX



$$q_1' = q_2' = \frac{T_y}{2b}$$

$$q_3' = 0$$

MOM EQ. WRT (1) \curvearrowright

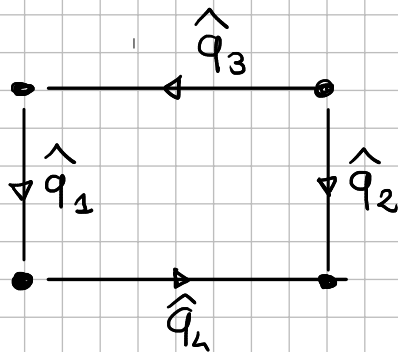
$$0 = 2q^* \cdot \Omega_{\text{CELL}} + 2q_2' \cdot \Omega_2$$

$$0 = 2q^* \cdot ab + 2q_2' \cdot \frac{1}{2}ab \rightarrow q^* = -\frac{1}{4b} T_y$$

INTERNAL ACTIONS IN RIB

WE COMPUTE EQUIVALENT FLUXES FOR $T_y = P$

IMPOSE EQUILIBRATING FLUXES TO RIB

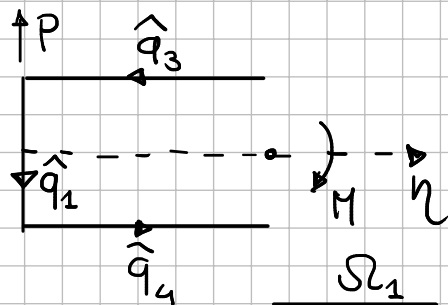


$$\hat{q}_1 = q_1(P) = +q_1' - q^* = +\frac{3}{4b} P$$

$$\hat{q}_2 = q_2(P) = q_2' + q^* = \frac{1}{4b} P$$

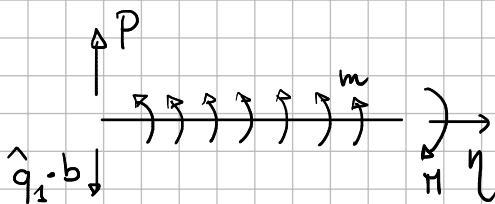
$$\hat{q}_3 = \hat{q}_4 = q_3 = q_4 = -q^*(P) = \frac{1}{4b} P$$

MODEL AS BEAM THE RIB



$$\begin{aligned} \mathcal{H} &= -P \cdot \eta + 2\hat{q}_1 \cdot b \cdot \frac{1}{2}\eta + 2\hat{q}_4 \cdot \frac{b}{2} \cdot \frac{1}{2}\eta + 2\hat{q}_3 \cdot \frac{b}{2} \cdot \frac{1}{2}\eta \\ &= -P \cdot \eta + \frac{3}{4} \frac{P}{b} \cdot b \cdot \eta + \frac{1}{8} \frac{P}{b} b \cdot \eta + \frac{1}{8} \frac{P}{b} \cdot b \cdot \eta \\ &= 0 \end{aligned}$$

If we collapse to one line



WHERE

$$m = \frac{1}{4} P = \hat{q}_3 \cdot \frac{b}{2} + \hat{q}_4 \cdot \frac{b}{2}$$

$$\mathcal{H} = -P \cdot \eta + \hat{q}_1 \cdot b \cdot \eta + m \cdot \eta = -P \cdot \eta + \frac{3}{4} P \eta + \frac{1}{4} P \eta = 0$$