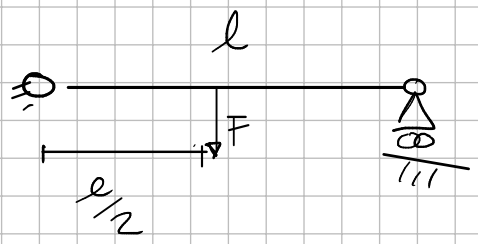
EXERCISE SESSION 2 — 23/09/2022

Isostatic Beam Systems

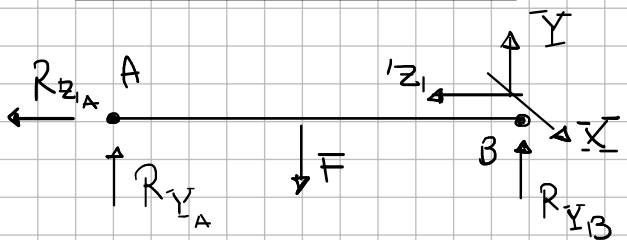
Ex 1



FIND VALUES:

- REACTION FORCES
- INTERNAL ACTIONS

REACTION FORCES



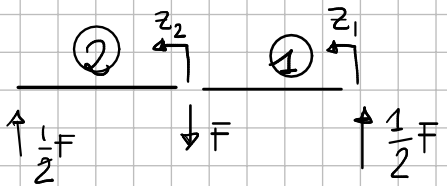
EQ EQS

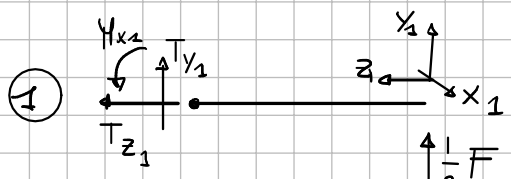
$$\begin{cases} Z: & R_{Z_A} = 0 \\ Y: & R_{Y_A} + R_{Y_B} - F = 0 \\ \text{ROT. } A: & -F\cdot\frac{l}{2} + R_{Y_B}\, l = 0 \end{cases}$$

$$\rightarrow R_{Y_B} = \frac{1}{2}F$$

$$\rightarrow R_{Y_A} = \frac{1}{2}F$$

INTERNAL ACTIONS

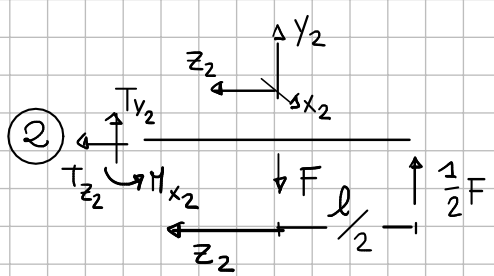




$$z_1: \quad T_{z_1}(z_1) = 0$$

$$y_1: \quad T_{y_1}(z_1) = -\frac{1}{2}F$$

$$\text{ROT. } x_1: \quad M_{x_1}(z_1) = -\frac{1}{2}F z_1$$

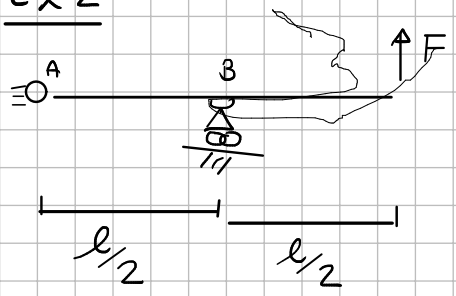


$$z_2: \quad T_{z_2}(z_2) = 0$$

$$y_2: \quad T_{y_2}(z_2) = -\frac{1}{2}F + F = +\frac{1}{2}F$$

$$\text{ROT } x_2: \quad M_{x_2}(z_2) = -\frac{1}{2}F\left(\frac{l}{2} + z_2\right) + F z_2 = -\frac{1}{4}Fl + \frac{1}{2}F z_2$$

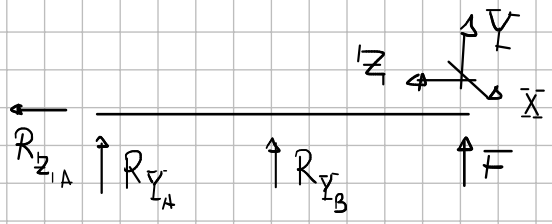
Ex 2



FIND VALUES:

- REACTION FORCES
- INTERNAL ACTIONS

REACTION FORCES

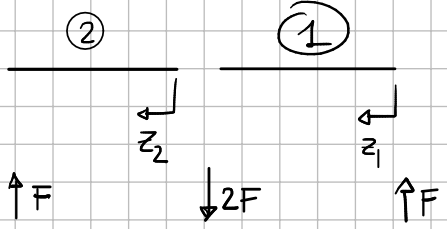


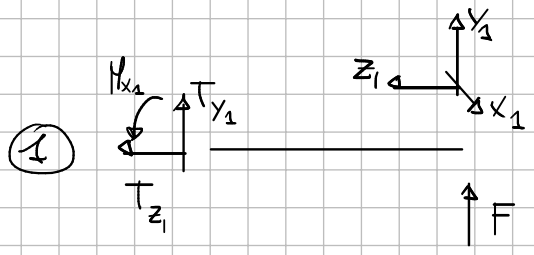
$$\begin{cases} Z: & R_{z_A} = 0 \\ Y: & R_{Y_A} + R_{Y_B} + F = 0 \\ \text{ROT. A}: & R_{Y_B} \cdot \frac{\ell}{2} + F \cdot \ell = 0 \end{cases}$$

$\longrightarrow R_{Y_B} = -2F$

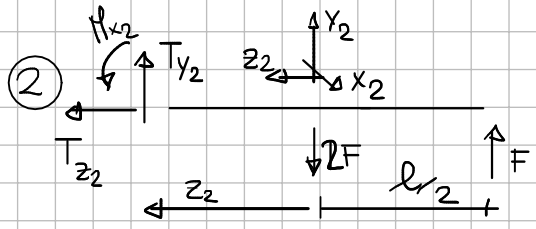
$\longrightarrow R_{Y_A} = F$

INTERNAL ACTIONS





$$\begin{cases} z_1: & T_{z_1}(z_1) = 0 \\ y_1: & T_{y_1}(z_1) = -F \\ \text{ROT } x_1: & M_{x_1}(z_1) = -F \cdot z_1 \end{cases}$$



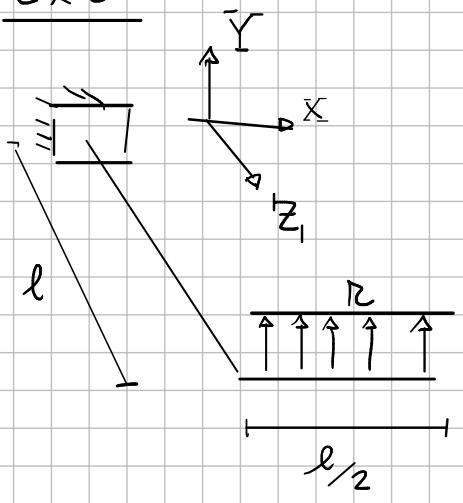
z_2: $T_{z_2}(z_2) = 0$

y_2: $T_{y_2}(z_2) = -F + 2F = F$

ROT x_2: $M_{x_2}(z_2) = 2F \cdot z_2 - F\left(\frac{\ell}{2} + z_2\right)$

$= 2Fz_2 - \frac{1}{2}F\ell - Fz_2 = Fz_2 - \frac{1}{2}F\ell$

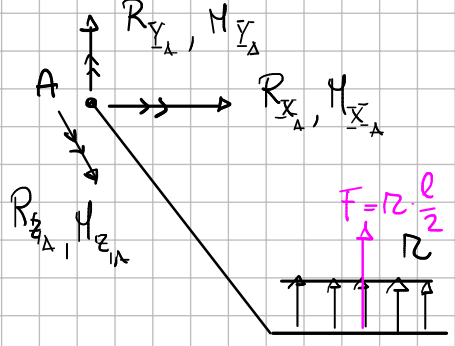
Ex 3



FIND VALUES:

- REACTION FORCES
- INTERNAL ACTIONS

REACTION FORCES



EQ EQS

$$\begin{cases} Z: & R_{Z_A} = 0 \\ Y: & R_{Y_A} + r \cdot \frac{\ell}{2} = 0 \rightarrow R_{Y_A} = -\frac{1}{2} r\ell \\ X: & R_{X_A} = 0 \end{cases}$$

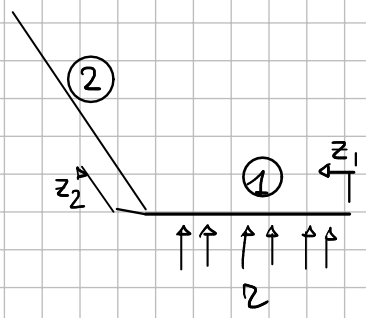
ROT IN A

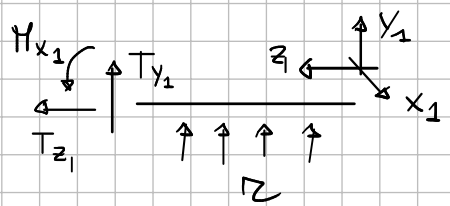
$$Z: \quad M_{Z_A} + r \cdot \frac{\ell}{2} \cdot \frac{\ell}{4} = 0 \longrightarrow M_{Z_A} = -\frac{1}{8} r\ell^2$$

$$Y: \quad M_{Y_A} = 0$$

$$X: \quad M_{X_A} - \frac{1}{2} r\ell \cdot \ell = 0 \longrightarrow M_{X_A} = +\frac{1}{2} r\ell^2$$

INT. ACTIONS



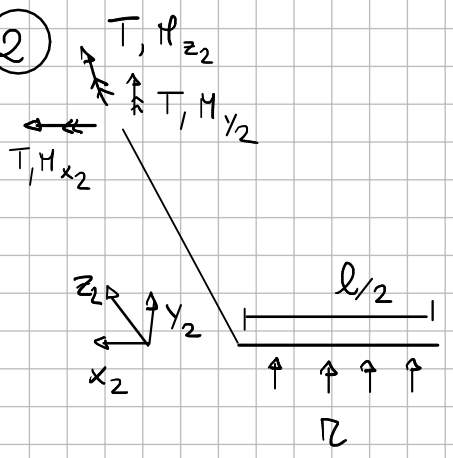


$$z_1: \quad T_{z_1}(z_1) = 0$$

$$y_1: \quad T_{y_1}(z_1) = -r \cdot z_1$$

$$\text{ROT. } x_1: \quad M_{x_1}(z_1) = -r \cdot z_1 \cdot \frac{z_1}{2} = -\frac{1}{2} r z_1^2$$

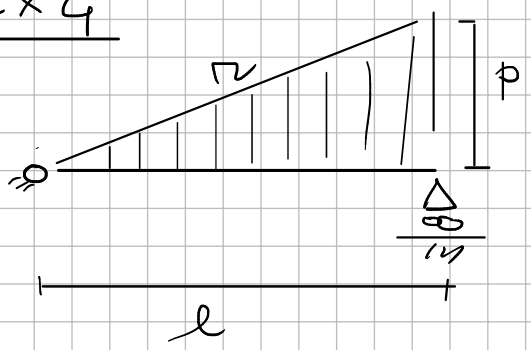
②



$$\begin{cases} z_2: & T_{z_2}(z_2) = 0 \\ y_2: & T_{y_2}(z_2) = -R \cdot \dfrac{l}{2} \\ x_2: & T_{x_2}(z_2) = 0 \end{cases}$$

$$\begin{cases} z_2: & M_{z_2}(z_2) = +R\dfrac{l}{2} \cdot \dfrac{l}{4} \\ y_2: & M_{y_2}(z_2) = 0 \\ x_2: & M_{x_2}(z_2) = -R\dfrac{l}{2} \cdot z_2 \end{cases}$$

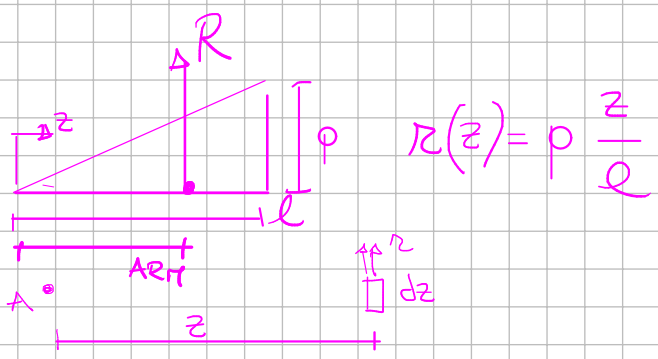
Ex 4



FIND VALUES:

- REACTION FORCES
- INTERNAL ACTIONS

REACTION FORCES

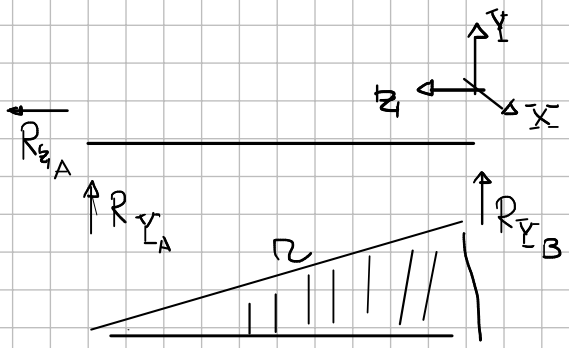


$$q(z) = p\frac{z}{l}$$

$$R = p \cdot \frac{l}{2}$$

$$M = \int_0^l z \cdot p \cdot \frac{z}{l}\, dz = \frac{1}{3}\frac{p}{l} z^3 \Big|_0^l = \frac{1}{3} p l^2$$

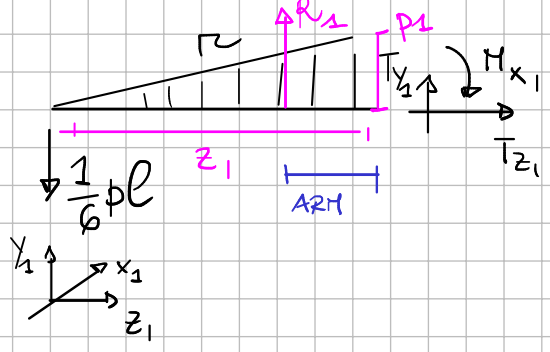
→ ARM OF THE MOMENT $\frac{2}{3} l$



EQ.	EQS
Z:	$R_{Z_A} = 0$
Y:	$R_{Y_A} + R_{Y_B} + \frac{1}{2} p l = 0$
ROTINA:	$R_{Y_B} \cdot l + \frac{1}{3} p l^2 = 0$

$$\rightarrow \begin{cases} R_{Y_B} = -\frac{1}{3} p l \\ R_{Y_A} = -\frac{1}{6} p l \end{cases}$$

INTERNAL ACTIONS



z_1: $T_{z_1}(z_1) = 0$

y_1: $T_{y_1}(z_1) = \frac{1}{6}pl - p\frac{z_1}{l} \cdot z_1 \cdot \frac{1}{2}$

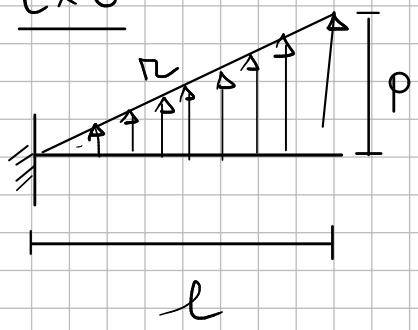
$= \frac{1}{6}pl - \frac{1}{2}p\frac{z_1^2}{l}$

(p_1, z_1)

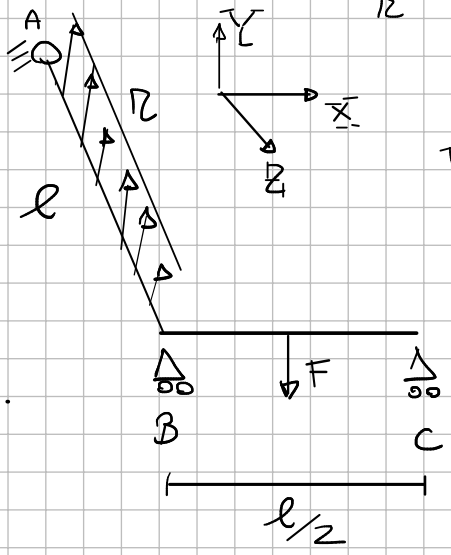
$$M_{x_1}(z_1) = \frac{1}{6}plz_1 - \frac{1}{2}p\frac{z_1^2}{l} \cdot \frac{1}{3}z_1$$

(ARM)

Ex 5



Ex 6

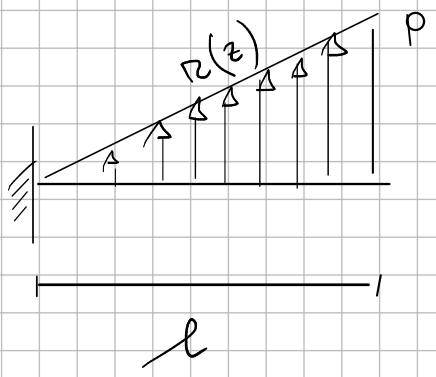


p CONST. DISTR. Y

CONST.

TRANSL $\begin{cases} XYZ \leftarrow A \\ X,Y \leftarrow B \\ Y \leftarrow C \end{cases}$

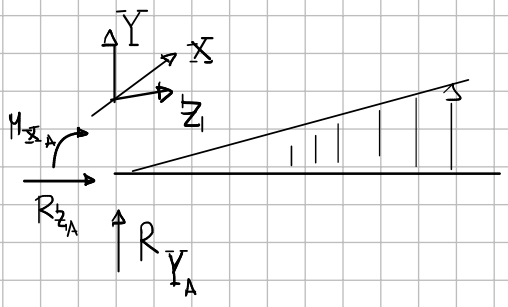
Ex 5



FIND:

- REACT. FORCES
- INT. ACTIONS

① REACTION FORCES



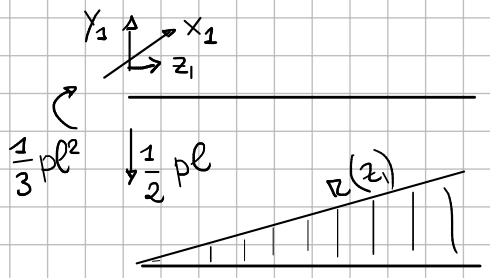
$$p(z_1) = p\left(\frac{z_1}{l}\right)$$

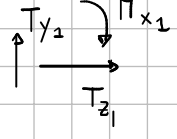
$$R_{z_A} = 0$$

$$R_{y_A} = -p \cdot \frac{1}{2} l = -\frac{1}{2} pl$$

$$M_{x_A} = p \cdot \frac{1}{2} l \cdot \frac{2}{3} l = \frac{1}{3} pl^2$$

② INT. ACTIONS





$$T_{z_1} = 0$$

$$T_{y_1} = \frac{1}{2} pl - p\left(\frac{z_1}{l}\right) \cdot \frac{1}{2} z_1$$

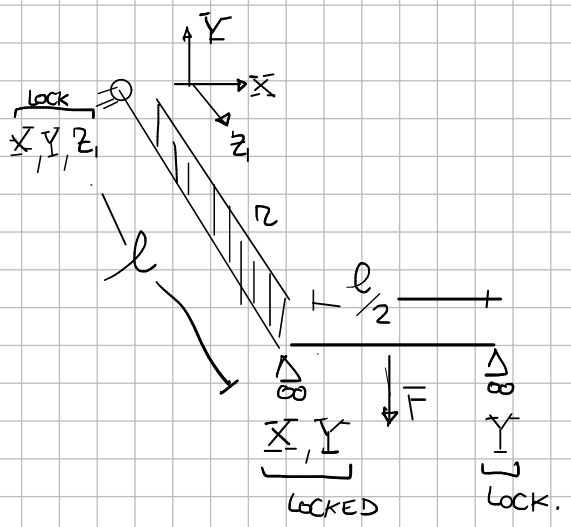
$$M_{x_1} = -\frac{1}{3} pl^2 + \frac{1}{2} plz_1 + - p\left(\frac{z_1}{l}\right) \cdot \frac{1}{l} z_1 \cdot \frac{1}{3} z_1$$

↳ ξ

$$\rightarrow T_{y_1} = \frac{1}{2} pl - \frac{1}{2} p \frac{z_1^2}{l}$$

$$M_{x_1} = -\frac{1}{3} pl^2 + \frac{1}{2} plz_1 - \frac{1}{6} p \frac{z_1^3}{l}$$

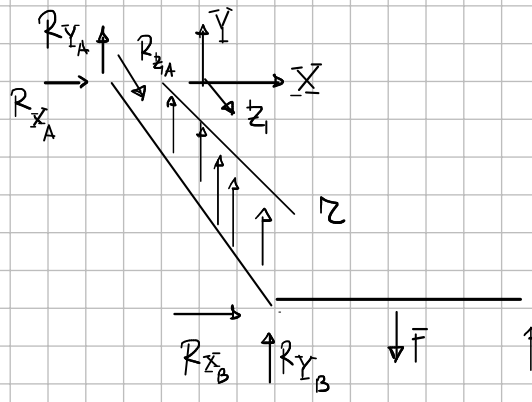
Es 6



FIND:

- REACT. FORCES
- INT. ACTIONS

(1) REACTION FORCES



EQ. EQS

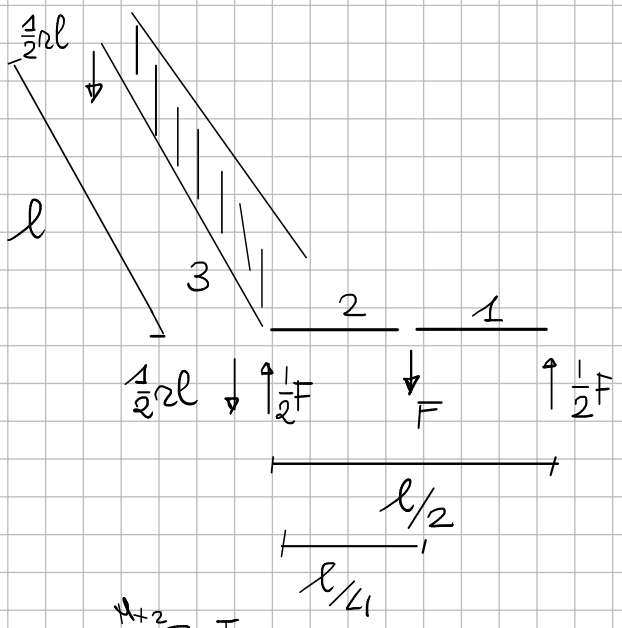
$$\begin{cases} R_{Z_A} = 0 \\ R_{Y_A} + R_{Y_B} + R_{Y_C} + r l - F = 0 \quad (*) \\ R_{X_A} + R_{X_B} = 0 \rightarrow R_{X_A} = 0 \end{cases}$$

ROT. IN A

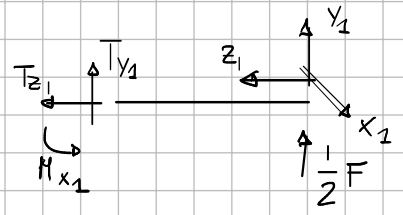
$$\begin{cases} \bar{X}: -R_{Y_B} \cdot l - R_{Y_C} \cdot l + F \cdot l - r \cdot l \cdot \frac{l}{2} = 0 \rightarrow R_{Y_B} = -\frac{1}{2}F + F - \frac{1}{2} r l \\ \qquad\qquad = \frac{1}{2}(F - r l) \\ \bar{Y}: R_{X_B} \cdot l = 0 \rightarrow R_{X_B} = 0 \\ \bar{Z}: -F \cdot \frac{l}{4} + R_{Y_C} \cdot \frac{l}{2} = 0 \rightarrow R_{Y_C} = \frac{1}{2}F \end{cases}$$

$$(*) \quad R_{Y_A} = \cancel{F} - r l - \frac{1}{\cancel{2}}\cancel{F} + \frac{1}{2} r l - \frac{1}{\cancel{2}}\cancel{F} = -\frac{1}{2} r l$$

② INTERNAL ACTIONS

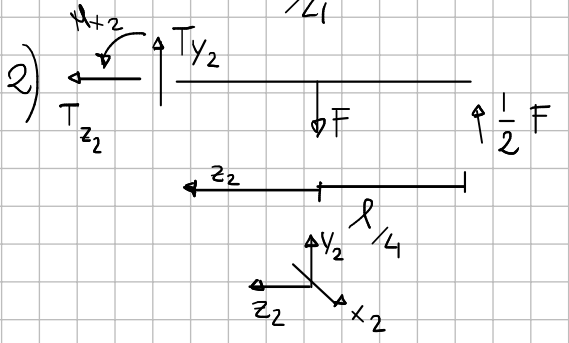


1)



$$\begin{cases} T_{z_1} = 0 \\ T_{y_1} = -\frac{1}{2}F \\ M_{x_1} = -\frac{1}{2}F z_1 \end{cases}$$

2)



$$T_{z_2} = 0$$

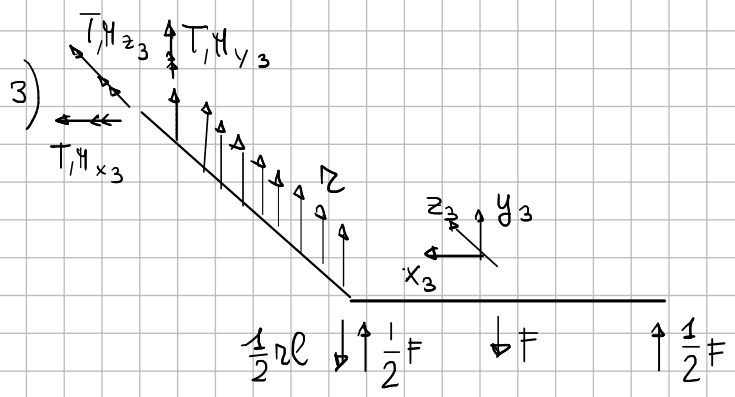
$$T_{y_2} = \frac{1}{2}F = F - \frac{1}{2}F$$

$$M_{x_2} = -\frac{1}{2}F\left(\frac{l}{4} + z_2\right) + F \cdot z_2$$

$$= -\frac{1}{8}Fl - \frac{1}{2}Fz_2 + Fz_2$$

$$= \frac{1}{2}Fz_2 - \frac{1}{8}Fl$$

3)



$$\begin{cases} T_{z_3} = 0 \\ T_{y_3} = -r \cdot z_3 + \frac{1}{2}rl \underbrace{- \frac{1}{2}F + F - \frac{1}{2}F}_{=0} \\ T_{x_3} = 0 \end{cases}$$

$$\begin{cases} M_{z_3} = -F \cdot \frac{l}{4} + \frac{1}{2}F \cdot \frac{l}{2} = 0 \\ M_{y_3} = 0 \\ M_{x_3} = \frac{1}{2}rlz_3 - r \cdot z_3 \cdot \frac{1}{2}z_3 + \underbrace{\left(-\frac{1}{2}F + F - \frac{1}{2}F\right)}_{=0} z_3 \end{cases}$$