Course of Aerospace Structures

Written test, June 13$^{\text{th}}$, 2023

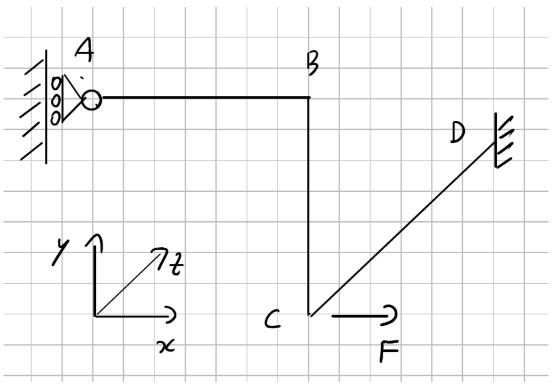
Name ______________________

Surname ______________________

Person code: | | | | | | | | |

Exercise 1

The beam structure sketched in the figure is loaded at point C by the force F, applied the x direction. Compute the displacement, in the x direction, of point C. Neglect shear deformation. The coordinates of the points are given, with respect to the sketched reference system, in the data.
(Unit for result: mm)

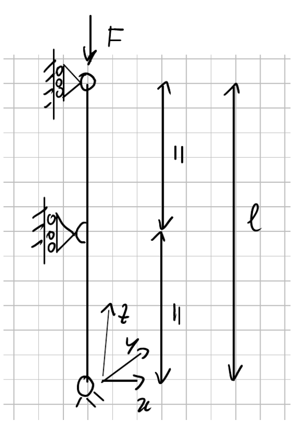


Data
$A : (0; 1000; 0)$ mm
$B : (1000; 1000; 0)$ mm
$C : (1000; 0; 0)$ mm
$D : (1000; 0; 1000)$ mm
$F = 1000.0$ N
$EI_{xx} = EI_{yy} = GJ = 1 \times 10^{10}$ N mm^2
$EA = 1 \times 10^4$ N

Answer ______________________

Exercise 2

Compute the critical buckling load F of the beam structure sketched in the figure. Note that the hinge and the two sliders prevent the out of plane displacement both in the x and y directions.
(Unit for result: N)

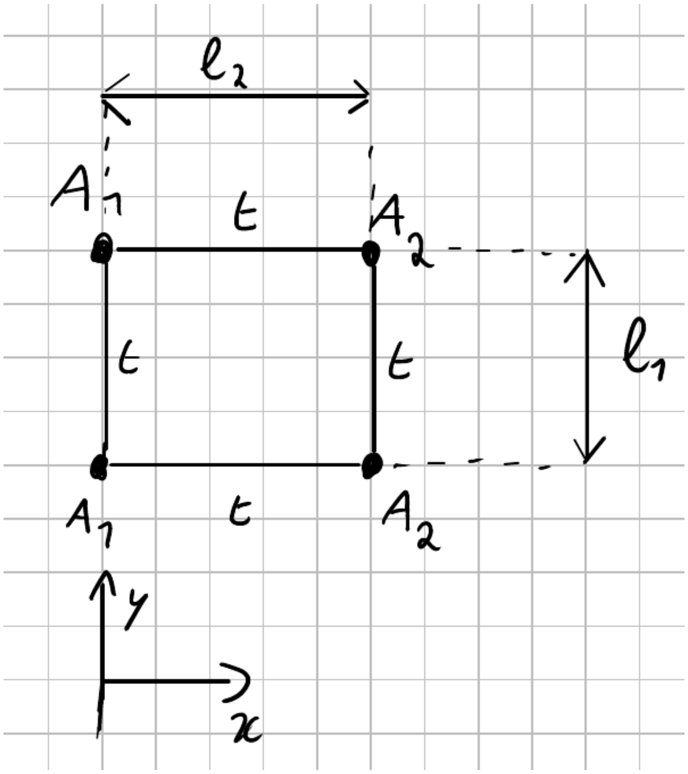


Data
$l = 4$ m
$EI_{xx} = 3 \times 10^{10}$ N mm^2
$EI_{yy} = 2 \times 10^{10}$ N mm^2

Answer ______________________

Exercise 3

Consider the semi-monocoque cross-section sketched in the figure. All the paneles have thickness equal to t. Compute, with respect to the sketched reference system, the x coordinate of the shear center. (Unit for result: mm)

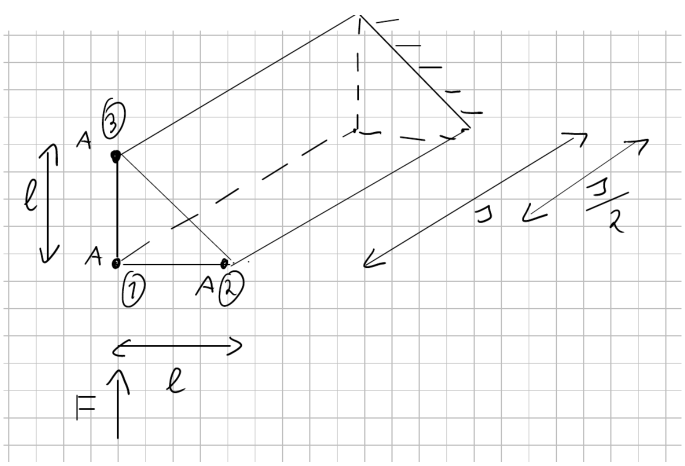


Data
$t = 1$ mm
$A_1 = 500$ mm^2
$A_2 = 250$ mm^2
$l_1 = 200$ mm
$l_2 = 300$ mm
$E = 70000$ MPa
$\nu = 0.3$

Answer ____________________

Exercise 4

Consider the 3-D beam semi-monocoque beam model sketched in the figure, loaded at its extremity by the vertical force F, that is aligned to the panel going from stringer #1 to stringer #3.
Compute the axial force in the concentrated area #1 (area equal to A) at a distance equal to $s/2$ from the clamp.
(Unit for result: N)

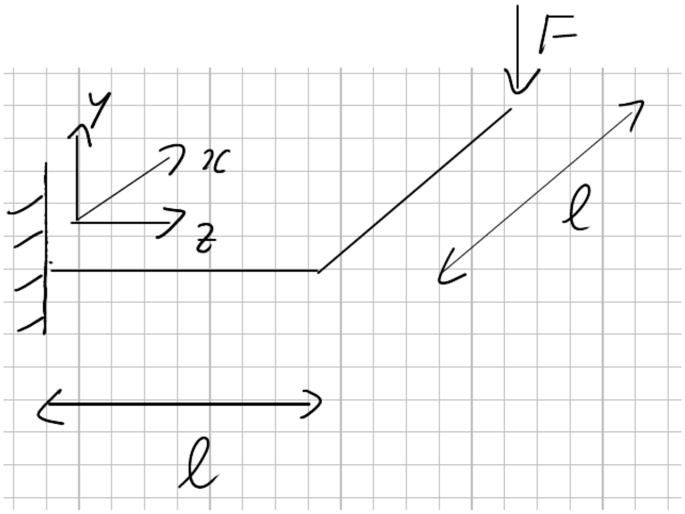


Data
$A = 100$ mm^2
$t = 1$ mm
$l = 0.3$ m
$s = 10$ m
$F = 1000$ N
$E = 70000$ MPa
$\nu = 0.3$;

Answer ____________________

Exercise 5

Consider the three-dimensional beam sketched in the figure, and loaded by the vertical force F. By resorting to the Ritz method, and adopting the simplest possible polynomial approximations for the vertical displacement and the torsional rotation, accounting for the torsional strain energy and for the bending strain energy of *both* beams, compute the approximated vertical displacement of the point of application of the force. Neglect shear deformability.
(Unit for result: mm)

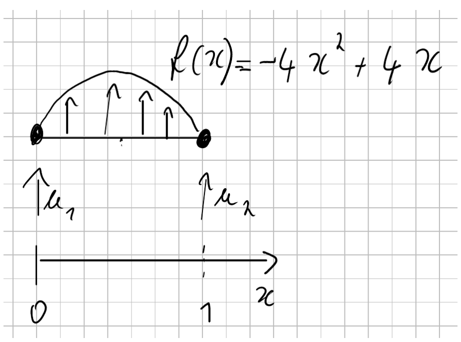


Data
$l = 1$ m
$F = 1000$ N
$GJ = 1 \times 10^{10}$ N mm^2
$EI_{xx} = EI_{yy} = 1 \times 10^{10}$ N mm^2

Answer ______________________________

Exercise 6

The two-node finite element sketched in the figure is loaded by the distributed force $f(x)$. Compute the virtual work of the force for the virtual displacement of node #2. The overall lenght of the finite element is equal to 1 mm
(Unit for result: N mm)



Data
$f(x) = -4x^2 + 4x$ N/mm

Answer ______________________________

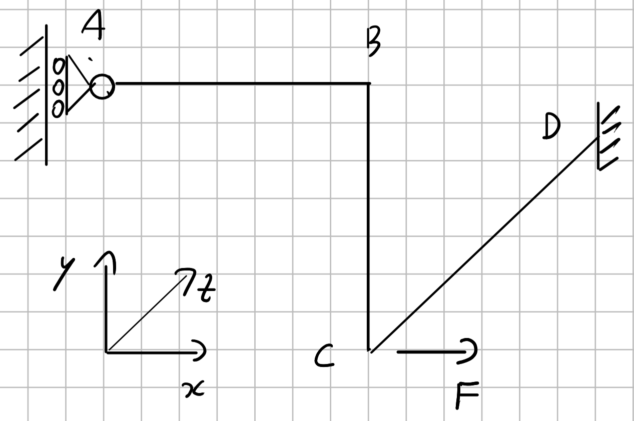
True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. whenever a slender structure works in plane strain the axial deformation is null:
2. the critical buckling force for a simply supported compressed beam is inversely proportional to its slenderness ($\propto 1/\lambda$):
3. the relation $\sigma_{zz} = E\varepsilon_{zz}$, where E is the elastic modulus, is correct only for a state of plane stress:

Multiple Choice questions
(Circle the correct answer)

1. A "simply supported" plate has:
   (a) transverse displacement and bending rotation prevented on one of its boundary sides
   (b) transverse displacement and bending rotation prevented on all of its boundary sides
   (c) transverse displacement prevented on two opposite boundary sides, bending rotation left free
   (d) transverse displacement prevented on all of its boundary sides, bending rotation left free
   (e) transverse displacement and bending rotation prevented on two opposite boundary sides, transverse displacement prevented and bending rotation left free on the other two
   (f) none of the above
2. When a clamped beam is loaded at its extemity by a shear force whose line of action goes through the shear center:
   (a) the shear stress is null
   (b) the axial stress is null
   (c) the bending deformation is null
   (d) the torsion is null
   (e) it is not possible to load a beam in that way unless it has a thin-walled opene cross-section
   (f) none of the above
3. When a torsional moment is applied to a thin-walled beam, without any other load:
   (a) the shear flows are null
   (b) the torsion is null
   (c) the torsion is different from zero, but only if the cross-section is free to warp
   (d) the torsion is different from zero
   (e) the torsion is different from zero only if the transverse shear deformability is not negligible
   (f) the beam is able to whitstand the torsional moment only if it is slender enough
   (g) none of the above

(1)



A: (0; 1000; 0) mm

B: (1000; 1000; 0) mm

C: (1000; 0; 0) mm

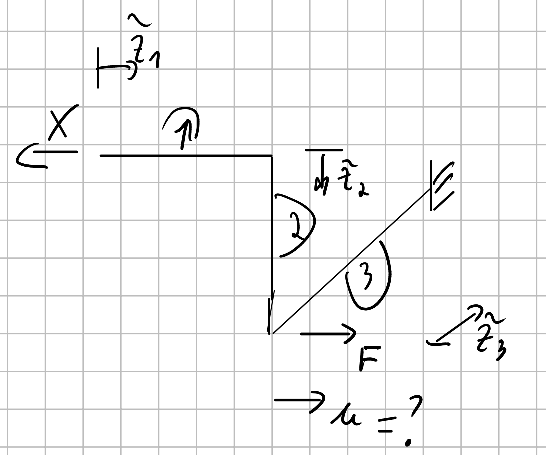
D: (1000; 0; 1000) mm

$F = 1000$ N

$EI_{xx} = EI_{yy} = GJ = 1E6$

$EA = 1E9$

$l = 1000$ mm



$N_1 = X \qquad M_{yy1} = M_{zz1} = M_{xx1} = \emptyset$

$N_2 = \emptyset \qquad M_{xx2} = X\,\tilde{z}_2 \qquad M_{yy2} = \emptyset$

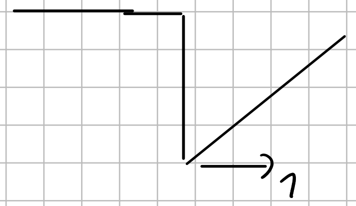
$N_3 = \emptyset \qquad M_{xx3} = (X - F)\,\tilde{z}_3$

$M_{zz3} = X \cdot l$

$$\int_0^l \frac{X}{EA}\, d\tilde{z}_1 + \int_0^l \frac{X\tilde{z}_2^2}{EI_{xx}}\, d\tilde{z}_2 + \int_0^l \frac{(X-F)\tilde{z}_3^2}{EI_{xx}} + \frac{X\,l^2}{GJ}\, d\tilde{z}_3$$

$$u = X\left(\frac{l}{EA} + \frac{1}{3}\frac{l^3}{EI_{xx}} + \frac{1}{3}\frac{l^3}{EI_{xx}} + \frac{l^3}{GJ}\right) - \frac{1}{3}\frac{F\,l^3}{EI_{xx}} = \emptyset$$

$$X = \frac{\frac{1}{3}\frac{F l^3}{E J_{xx}}}{\left(\frac{l}{EA} + \frac{2}{3}\frac{l^3}{E J_{xx}} + \frac{l^3}{GJ}\right)}$$

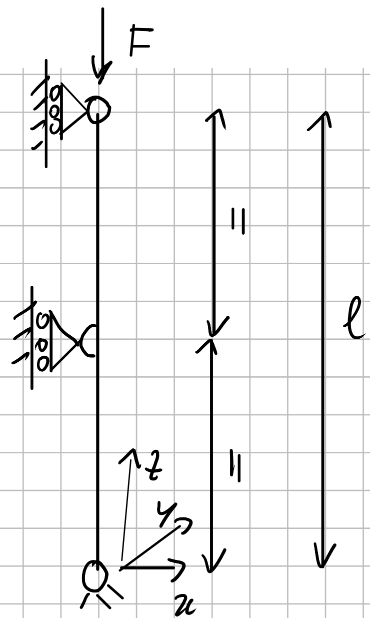


$$\tilde{M}_{xx_3} = -\tilde{z}_3$$

$$\int_0^l \frac{(F-X)\,\tilde{z}_3^{\,2}}{E J_{xx}} = u$$

$$u = \frac{1}{3}\frac{(F-X)\,\tilde{z}_3^{\,3}}{E J_{xx}} = 29{,}167 \text{ mm}$$

(2)



$\ell = 4\ m$

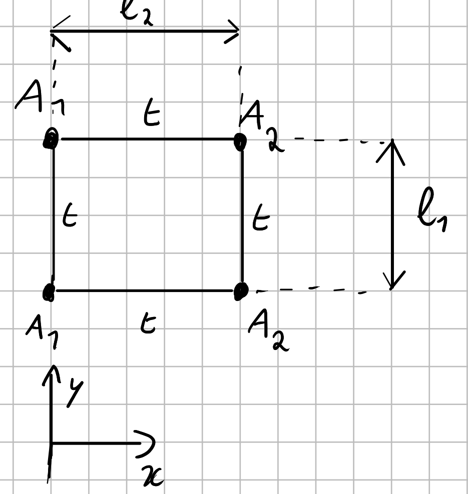
$EI_{xz} = 3\ E10\ N\ mm^2$

$EI_{yy} = 2\ E10\ N\ mm^2$

$\tilde{\ell} = \frac{\ell}{2} = 2\ m = 2000\ mm$

$$F = \frac{\pi^2 EI_{yy}}{\tilde{\ell}^2} = 49348\ N$$

3



$t = 1$ mm

$A_1 = 500$ mm^2

$A_2 = 250$ mm^2

$l_1 = 200$ mm

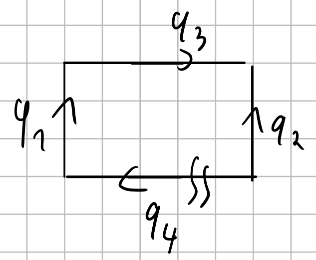
$l_2 = 300$ mm

$E = 70000$ MPa

$\nu = 0{,}3$

$$x_{CG} = \frac{\cancel{2} A_2 l_2}{\cancel{2}(A_1 + A_2)}$$

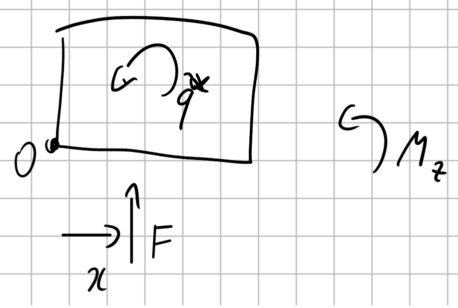
$$I_{xx_{CG}} = 2(A_1 + A_2)\left(\frac{l_1}{2}\right)^2$$



$$\tilde{q}_1 = \frac{A_1 \cdot \frac{l_1}{2}}{I_{xx_{CG}}} F = \quad N/mm$$

$$\tilde{q}_2 = \frac{A_2 \cdot \frac{l_1}{2}}{I_{xx_{CG}}} F = \quad N/mm$$

$$\tilde{q}_3 = 0 \quad N/mm$$



$$F \cdot x = \tilde{q}_2 \cdot l_1 \cdot l_2 + 2 q^* l_1 l_2$$

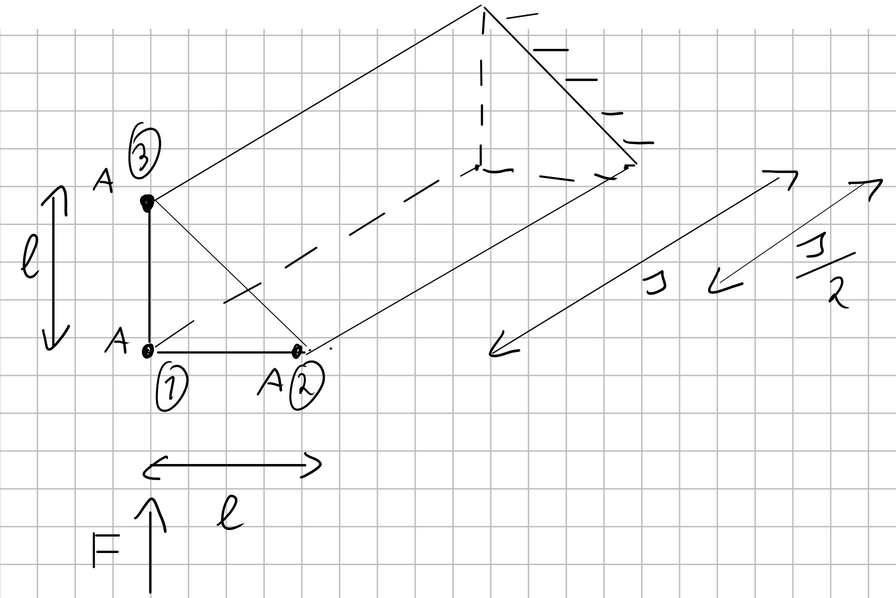
$$q^* = \frac{Fx}{2\,l_1 l_2} - \frac{\tilde{q}_2}{2}$$

$$\theta' = \cancel{\frac{1}{2\Omega_c G}}\left[\frac{-\tilde{q}_1 l_1}{\cancel{t}} + \tilde{q}_2 \frac{l_1}{\cancel{t}} + q^* \frac{(2l_1 + 2l_2)}{\cancel{t}}\right] = 0$$

$$-\tilde{q}_1 l_1 + \tilde{q}_2 l_1 - \frac{\tilde{q}_2}{\cancel{2}}(\cancel{2}l_1 + \cancel{2}l_2) + \frac{Fx}{\cancel{2}l_1 l_2}\cancel{2}(l_1 + l_2) = 0$$

$$x = \left[\tilde{q}_1 l_1 - \tilde{q}_2 l_1 + \tilde{q}_2(l_1 + l_2)\right]\frac{l_1 l_2}{(l_1 + l_2)\,F} = 140 \quad mm$$

(4)



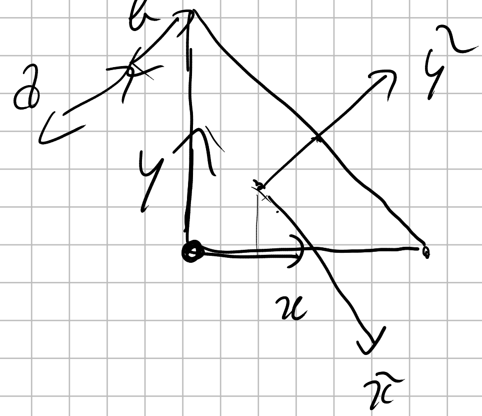
$A = 100 \ mm^2$

$t = 1 \ mm$

$l = 1 \ m = 1000 \ mm$

$F = 1000 \ N$

$s = 10 \ m = 10\,000 \ mm$



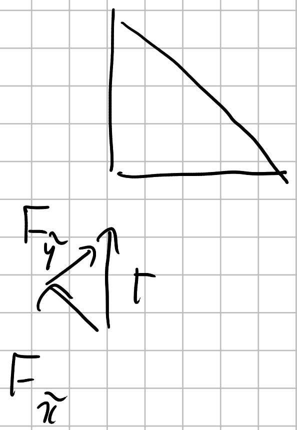
$$x_{CG} = \frac{1}{3} l$$

$$y_{CG} = \frac{1}{3} l$$

$$a = \frac{\sqrt{2}}{3} l$$

$$a + b = \frac{l}{\sqrt{2}} \qquad b = \left(\frac{1}{\sqrt{2}} - \frac{2}{\sqrt{2}\,3} \right) l = \frac{1}{3\sqrt{2}} l$$

$$I_{\tilde{x}\tilde{x}} = \cancel{2}A \cdot \frac{1}{9 \cdot \cancel{2}} l^2 + A \cdot \frac{2}{9} l^2 = A \frac{l^2}{3}$$

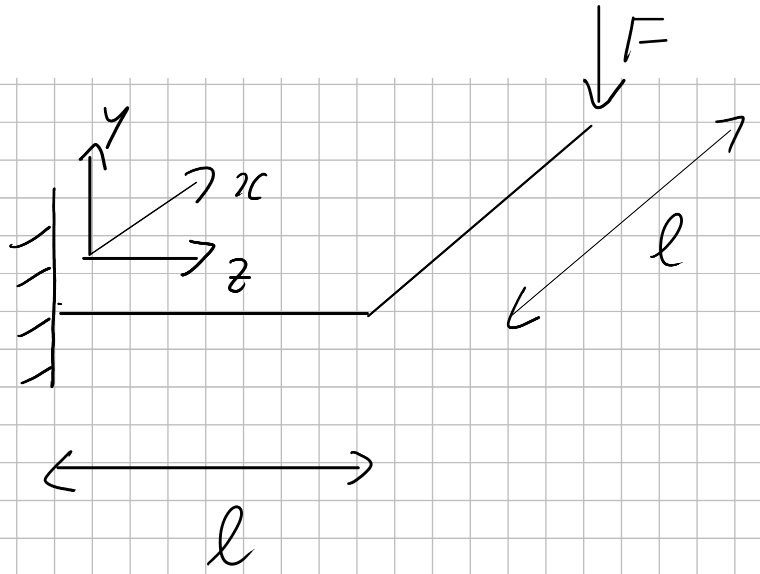


$$F_{\tilde{y}} = \frac{F}{\sqrt{2}}$$

$$M_{\tilde{x}} = F_{\tilde{y}} \cdot \frac{l}{2}$$

$$M_1 = \frac{M_{\tilde{x}}}{I_{\tilde{x}\tilde{x}}} \cdot A \cdot a = 16667 \quad N$$

5

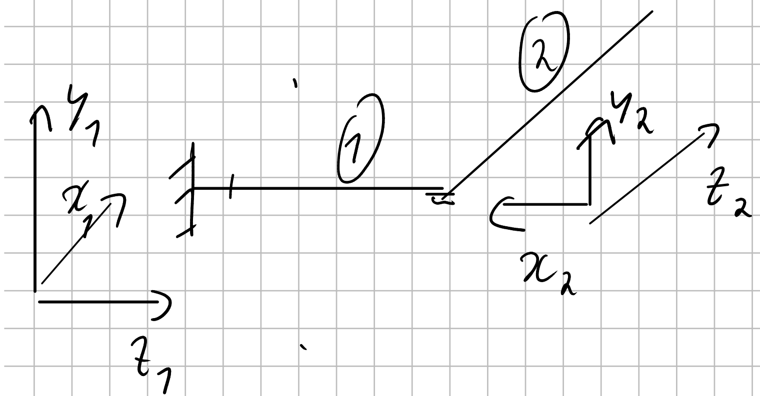


$GJ = 1E10$

$EI_{xx} = EI_{yy} = 1E10$

$l = 1\ m$

$F = 1000\ N$



$$\theta_1 = a\, z_1$$

$$v_1 = b\, z_1^2$$

$$v_2 = c + d\, z_2 + e\, z_2^2$$

$$v_2(0) = v_1(l) \Rightarrow c = b\, l^2$$

$$v_2'(0) = \theta_1(l)$$

$$d = a\,l$$

$$v_2 = b\,l^2 + a\,l\,z_2 + e\,z_2^2 \qquad \theta' = a$$

$$v_1'' = 2b \qquad v_2'' = 2e$$

$$v_2(l) = b\,l^2 + a\,l^2 + e\,l^2$$

$$\int_0^l \delta\theta_1'\,GJ\,\theta_1' + \delta v_1''\,E\bar{I}\,v_1''\,dz_1 + \int_0^l \delta v_2''\,E\bar{I}\,v_2'' = -\delta v_2(l)\,F$$

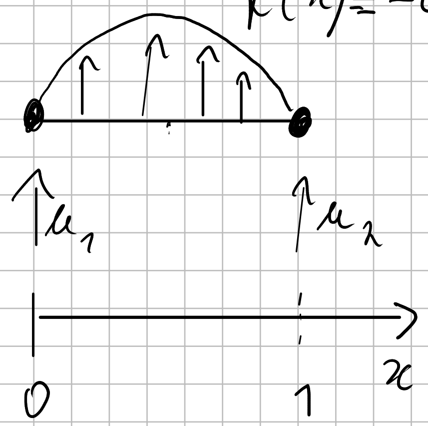
$$\delta a\,GJ\,l\,a + \delta b\,4E\bar{I}\,l\,b + \delta e\,4E\bar{I}\,l\,e = -(\delta a + \delta b + \delta e)\,l^2 F$$

$$a = \frac{-\ell F}{GJ} \qquad b = \frac{-\ell F}{4EI} \qquad c = \frac{-\ell F}{4EI}$$

$$v_2 = -450 \quad mm$$

6

$$p(x) = -4x^2 + 4x$$



$$N_1(x) = 1 - x$$

$$N_2(x) = x$$

$$\int_0^1 \delta u_2 \, N_2 \, p(x) \, dx$$

$$= \delta u_2 \int_0^1 x(-4x^2 + 4x) \, dx$$

$$= \delta u_2 \left(-1 + \frac{4}{3}\right) = \frac{1}{3}$$

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. whenever a slender structure works in plane strain the axial deformation is null:
   - True
2. the critical buckling force for a simply supported compressed beam is inversely proportional to its slenderness ($\propto 1/\lambda$):
   - False
3. the relation $\sigma_{zz} = E\varepsilon_{zz}$, where E is the elastic modulus, is correct only for a state of plane stress:
   - False

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(Circle the correct answer)

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3. When a torsional moment is applied to a thin-walled beam, without any other load:

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