Course of Aerospace Structures

Written test, January 20th, 2025

Name ____________________________________

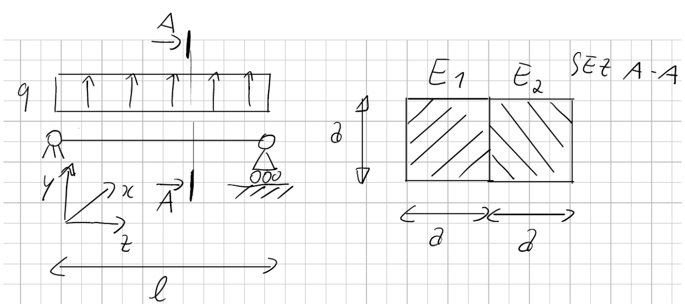
Surname ____________________________________

Person code: | | | | | | | | |

Exercise 1

The simply supported beam in the figure cross section is made with two different materials, as sketched in Sez A-A. The beam is loaded with a distributed force per unit of length q. Compute the maximum value, over the whole structure, of the axial stress σ_{zz}.
Neglect shear deformability.

(Unit for result: MPa)



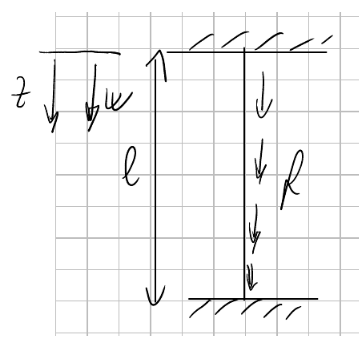
Data
$l = 1500$ mm
$a = 30$ mm
$E_1 = 70000$ MPa
$E_2 = 210000$ MPa
$\nu = 0.3$
$q = 1$ N/mm

Answer ____________________________________

Exercise 2

The beam in the figure is clamped at both ends, and is loaded with an distributed axial force per unit of length f. Compute the displacement of the point at $z = l/2$.

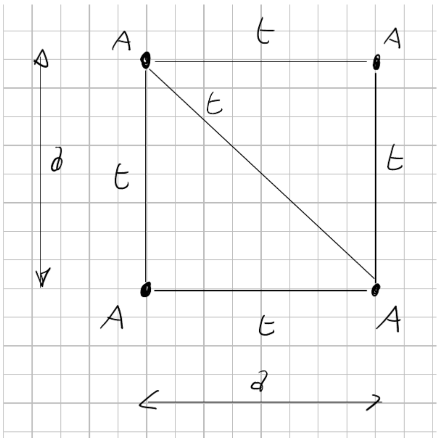
(Unit for result: mm)



Data
$l = 1000$ mm
$EA = 6 \times 10^9$ N
$f = 300$ N/mm

Answer ____________________________________

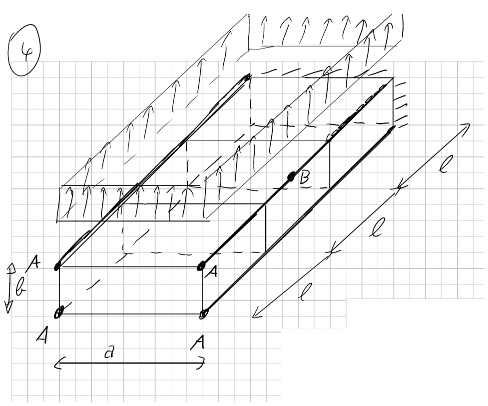
Exercise 3
Consider the semi-monocoque cross section sketched in the figure, with four concentrated areas, each with area A, five panels of thickness t and two cells.
Compute the torsional stiffness GJ of the cross section.
(Unit for result: N mm^2)



Data
$a = 500$ mm
$t = 2$ mm
$E = 70000$ MPa
$\nu = 0.3$
$A = 200$ mm^2

Answer

Exercise 4
The semi-monocoque beam sketched in the figure, with overall length $3l$, has four panels, each with thickness t, and four concentrated areas, each with area A. It is loaded on the upper panel, by force per unit of surface q. The load is introduced into the structure by four different ribs, with pitch equal to l (the first rib is at the clamp, the last one at the beam free extremity).
Compute the axial stress at the point B, as sketched in the figure, located at $z = 3/2l$ from the clamp.
(Unit for result: MPa)

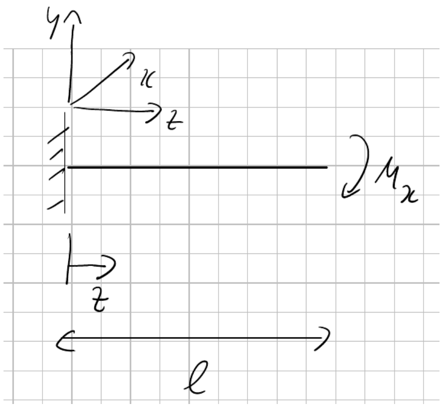


Data
$l = 1000$ mm
$a = 700$ mm
$b = 100$ mm
$t = 1$ mm
$A = 500$ mm^2
$E = 70000$ MPa
$\nu = 0.3$
$q = 0.01$ N/mm^2

Answer

Exercise 5
Consider the slender beam structure sketched in the figure, loaded at the free extremity by the concentrated moment M_x. The bending stiffness varies linearly along the beam, $EI_{xx} = a + bz$ Estimate the transverse displacement v at the extremity by assuming a polynomial approximation of the transverse displacement truncated to the first non null term. Neglect shear deformability.

(Unit for result: mm)

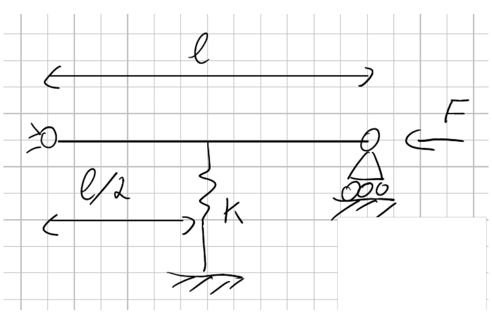


Data
$l = 3000$ mm
$a = 1 \times 10^{13}$ N mm^2
$b = 3 \times 10^9$ N mm
$M_x = 1 \times 10^5$ N mm

Answer ______________________

Exercise 6
The single simply supported beam sketched in the figure, with bending stiffness EI_{xx}, has a supporting spring in the middle with stiffness K. The beam is loaded by the compressive force F. Estimate the critical buckling value of F by assuming a trigonometric expansion of the transverse displacement, truncated to one term.
Neglect shear deformability.

(Unit for result: N)



Data
$l = 3000$ mm
$EI_{xx} = 1 \times 10^{10}$ N mm^2
$K = 100$ N/mm

Answer ______________________

True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. The assumption of plane strain implies that a component of strain is null.
2. The Neumann boundary conditions are satisfied in a weak sense by the Principle of Virtual Work.
3. According to the semi-monocoque model, the shear stress in the panels can be computed from the axial derivative of the axial stress σ_{zz}.

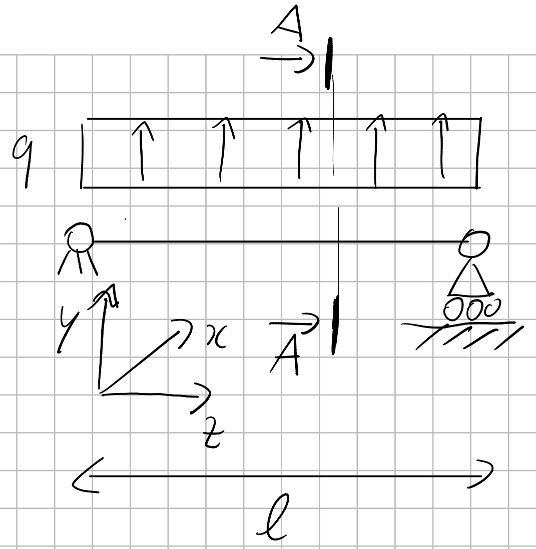
Multiple Choice questions
(Circle the correct answer)

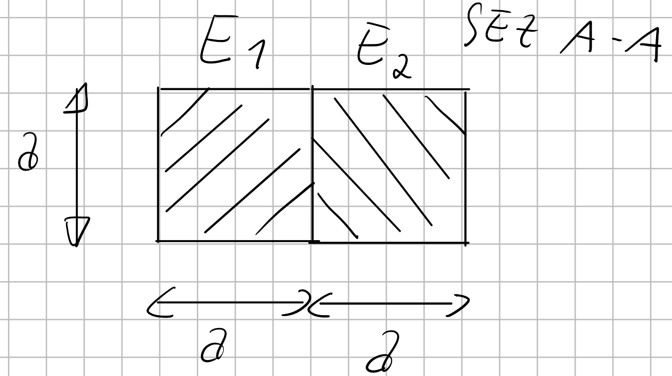
1. The shear flux in a thin panel is equal to:
   (a) the shear stress in the panel divided by the panel thickness
   (b) the shear stress in the panel multiplied by the panel thickness
   (c) the shear stress in the panel
   (d) the derivative of the axial stress in the panel
   (e) none of the above

2. The axial stiffness for a thin-walled beam:
   (a) is null
   (b) is generally larger than that of a corresponding (same material and cross-section area of the material) compact section
   (c) is generally smaller than that of a corresponding (same material and cross-section area of the material) compact section
   (d) is equal to that of a corresponding (same material and cross-section area of the material) compact section
   (e) can be neglected
   (f) is infinite
   (g) none of the above

3. A "simply supported" plate has:
   (a) vertical (normal to the plate) displacement and bending rotation prevented on one of its boundary sides
   (b) vertical displacement and bending rotation prevented on all of its boundary sides
   (c) vertical displacement prevented on two opposite boundary sides, bending rotation left free
   (d) vertical displacement prevented on all of its boundary sides, bending rotation left free
   (e) vertical displacement and bending rotation prevented on two opposite boundary sides, vertical displacement prevented and bending rotation left free on the other two
   (f) none of the above

1)





$$EJ_{xx} = \frac{1}{12} a^4 (E_1 + E_2)$$

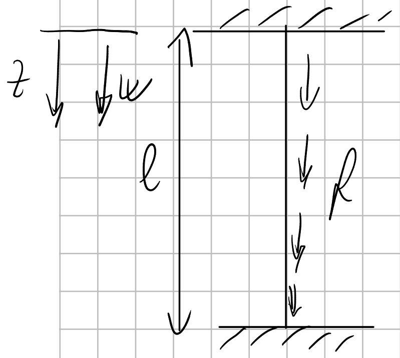
$$T_y = -\frac{ql}{2} + qz$$

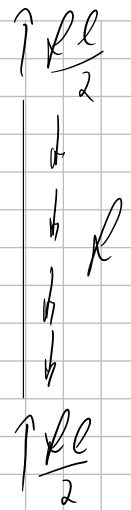
$$\sigma_{x_1} = \frac{M_x\left(\frac{l}{2}\right) \cdot \frac{a}{2}}{EJ_{xx}} \cdot E_1$$

$$M_x = +\frac{ql}{2} z - \frac{1}{2} q z^2$$

$$\sigma_{x_2} = \frac{M_x\left(\frac{l}{2}\right) \cdot \frac{a}{2}}{EJ_{xx}} \cdot E_2$$

(2)





$$N = \frac{pl}{2} - pz$$

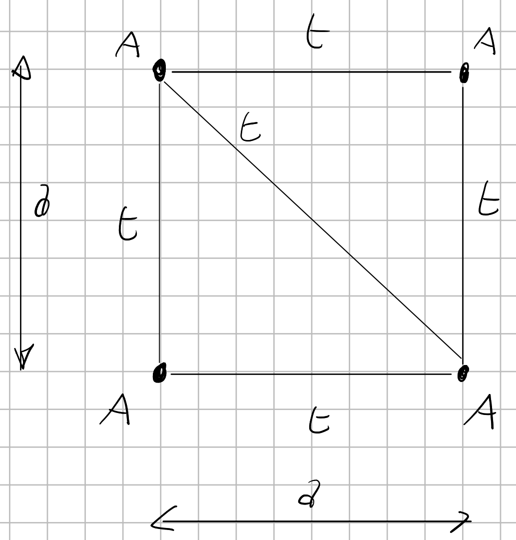
$$\varepsilon_{zz} = \frac{N}{EA}$$

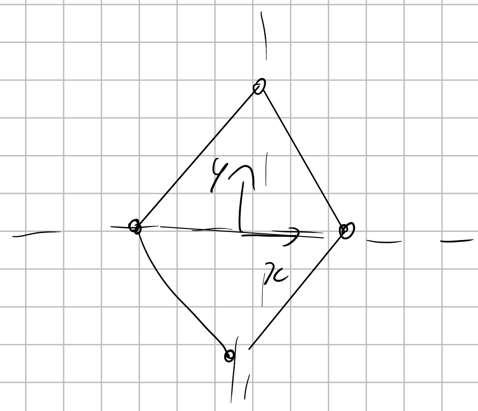
$$w\left(\frac{l}{2}\right) = \int_0^{l/2} \varepsilon_{zz}\, dz = \frac{1}{EA}\int_0^{l/2}\left(\frac{pl}{2} - pz\right) dz$$

$$= \frac{1}{EA}\left(\frac{pl}{2} z - \frac{1}{2} p z^2\right)\Bigg|_0^{l/2}$$

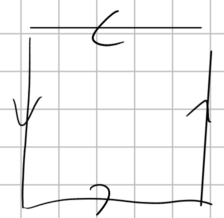
$$= \frac{1}{EA}\left(\frac{pl^2}{4} - \frac{1}{8} p l^2\right) = \frac{1}{EA}\frac{pl^2}{8}$$

3





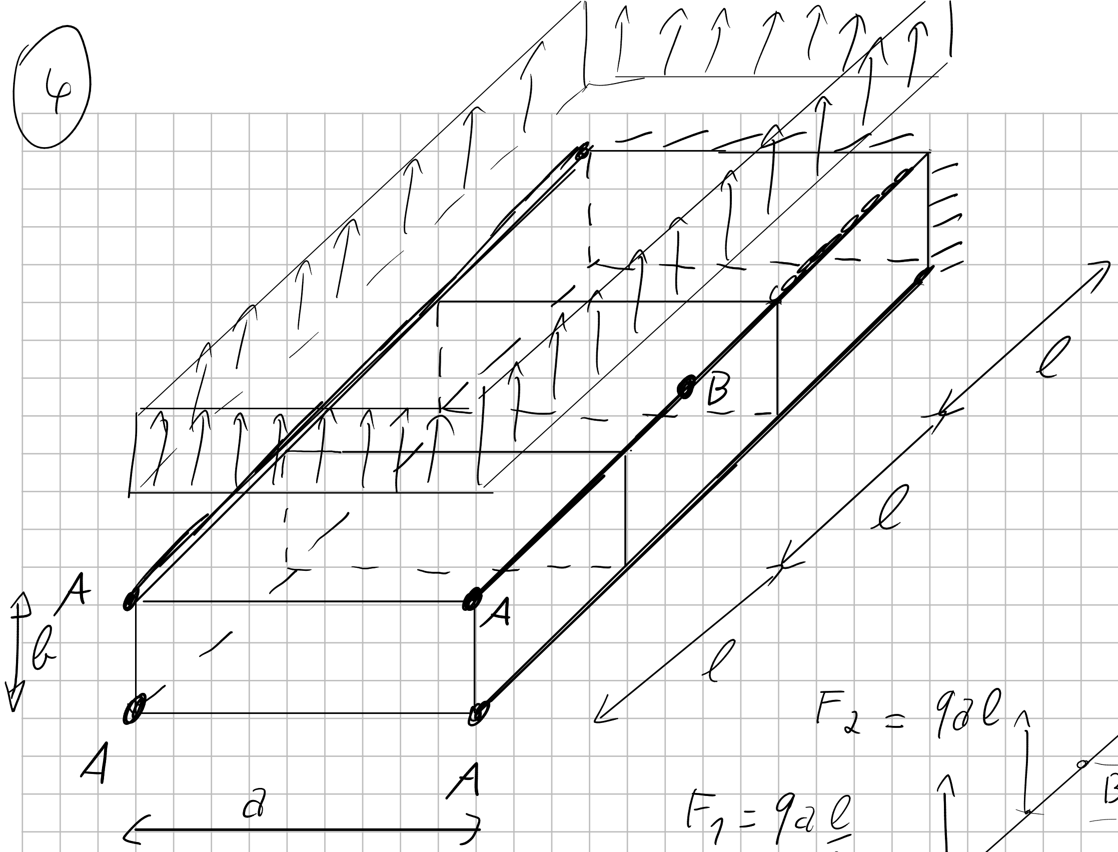
$\Rightarrow$



$$q = \frac{M_t}{2a^2}$$

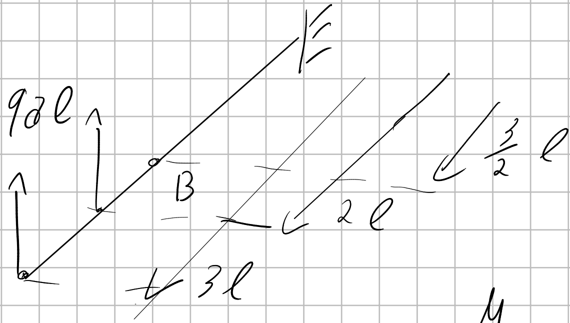
$$\theta' = \frac{1}{\cancel{2}a^{\cancel{2}}G}\,\frac{q\,\cancel{4}^{2}\,\cancel{a}}{t} = \frac{M_t\,\cancel{2}}{\cancel{2}\,a^2\,a G t} = \frac{M_t}{a^3 G t}$$

4



$F_2 = q a l$

$F_1 = q a \frac{l}{2}$

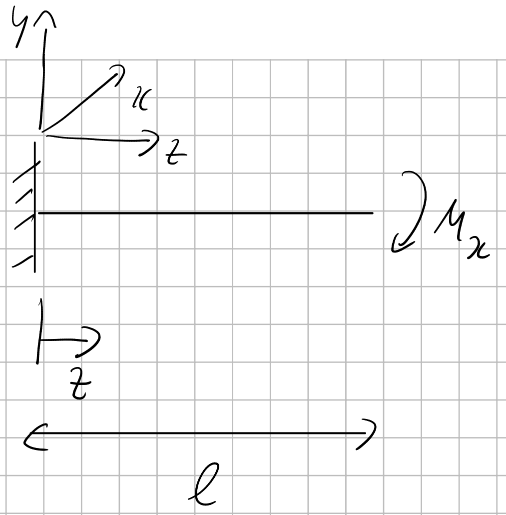


$I_{xx} = 4 A \cdot \left(\frac{b}{2}\right)^2$

$M_{xx} = F_1 \frac{3}{2} l + F_2 \frac{l}{2}$

$\sigma_{z7} = - \frac{M_{xx}}{I_{xx}} \cdot \frac{b}{2}$

5



$$EI_{xx} = a + bz$$

$$v = c z^2 \qquad \delta v'' = 2\delta c \qquad \delta v' = 2\delta c\, z$$

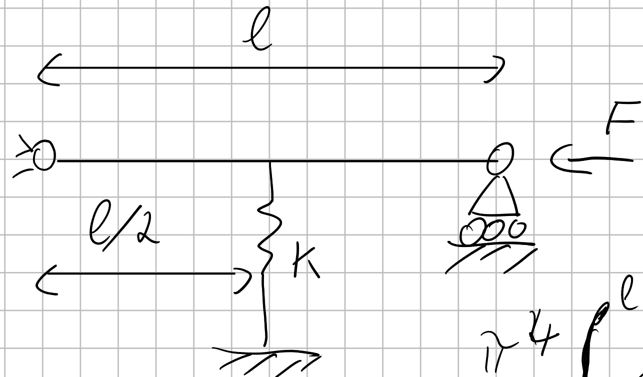
$$\int_0^l \delta c \; 4 EI_{xx} \, c \, dz = -\delta c \, 2 l \cdot M_x$$

$$\delta c \int_0^l (4a + 4bz)\, dz \, c = -\delta c \, 2 l M_x$$

$$= \delta c \left(4al + 2bl^2 \right) c = -\delta c \, 2 l M_x$$

$$c = \frac{-2 l M_x}{4al + 2bl^2} \qquad v(l) = c\, l^2$$

⑥



$$v = a \sin\left(\frac{\pi z}{\ell}\right) \qquad v'' = -\frac{\pi^2}{\ell^2} \sin\left(\frac{\pi z}{\ell}\right)$$

$$\frac{\pi^4}{\ell^4}\int_0^\ell \delta a \sin^2\left(\frac{\pi z}{\ell}\right) E I_{xx}\, a\, dz - \frac{\pi^2}{\ell^2}\int \delta a \cos^2\left(\frac{\pi z}{\ell}\right) a F\, dz$$

$$+ \delta a \sin^2\left(\frac{\pi}{2}\right) K a = \emptyset$$

$$\delta a \left(\frac{\pi^4}{\cancel{\ell^4}_3} E I_{xx} \frac{\cancel{\ell}}{2} - \frac{\pi^2}{\ell^2}\frac{\ell}{2} F + K\right) a = \emptyset$$

$$\delta a \left(\frac{\pi^4 E I_{xx}}{2\ell^3} + K - \frac{\pi^2}{2\ell} F\right) a = 0$$

$$F = \frac{\frac{\pi^4 E I_{xx}}{2\ell^3} + K}{\frac{\pi^2}{2\ell}}$$

- The assumption of plane strain implies that a component of strain is null
  - True
- The Newmann boundary conditions are satisfied in a weak sense by the Principle of Virtual Work
  - True
- According to the semi-monocoque model, the shear stress in the panels can be computed from the axial derivative of the axial stress σ_{zz}:
  - False

1. The shear flux in a thin panel is equal to
   (a) the shear stress in the panel divided by the panel thickness
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   (c) the shear stress
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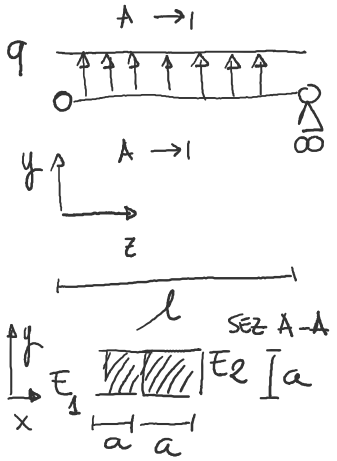
(e) vertical displacement and bending rotation prevented on two opposite boundary sides, vertical displacement prevented and bending rotation left free on the other two

(f) none of the above

2025-01-20 Ex 1

Wednesday, January 15, 2025 2:02 PM



$a = 30\ mm$

$l = 1500\ mm$

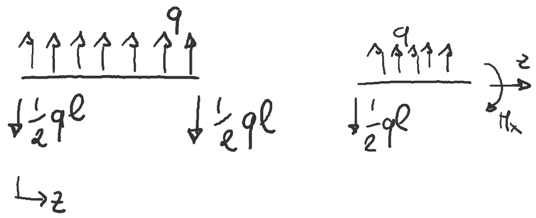
$E_1 = 70000\ MPa$

$E_2 = 210000\ MPa$

$\nu = 0.3$

$q = 1N/mm$

SOL



$$M_x(z) = \frac{1}{2}qlz - qz \cdot \frac{z}{2}$$

$$M_x'(z) = \frac{1}{2}ql - qz \rightsquigarrow M_x'(z) = 0$$

$$z = \frac{1}{2}l$$

$$M_{x_{MAX}} = \frac{1}{2}ql \cdot \frac{1}{2}l - q\frac{1}{2}l \cdot \frac{\frac{1}{2}l}{2}$$

$$= \frac{1}{4}ql^2 - \frac{1}{8}ql^2 = \frac{1}{8}ql^2$$

$$\varepsilon_{zz_{MAX}} = \frac{M_{x_{MAX}}}{E\,J_{xx}} \cdot \frac{a}{2} \qquad E = \frac{E_1 + E_2}{2}$$

$$J_{xx_1} = \int_{-a/2}^{a/2}\int_{-a/2}^{a/2} y^2\,dx\,dy$$

$$= \int_{-a/2}^{a/2} y^2 dy \cdot \int_{-a/2}^{a/2} dx$$

$$= \frac{1}{3}\left(\frac{a^3}{8} + \frac{a^3}{8}\right) \cdot \left(\frac{a}{2} + \frac{a}{2}\right)$$

$$= \frac{1}{12}a^4$$

$$= \frac{}{12} a^4$$

$$J_{xx_2} = \frac{1}{12} a^4 = J_{xx_1}$$

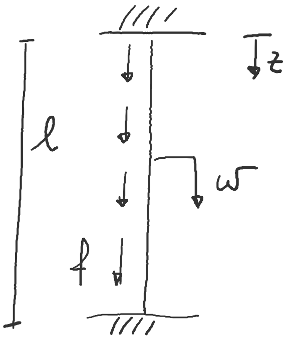
$$J_{xx} = \frac{1}{6} a^4 = J_{xx_1} + J_{xx_2} = 2 J_{xx_1}$$

$$\varepsilon_{zz_{MAX}} = \frac{3\cancel{6} M_{x_{MAX}}}{E a^{\cancel{4}3}} \cdot \frac{\cancel{a}}{2} = \frac{3}{8} \frac{q\ell^2}{E a^3}$$

$$\sigma_{xx_{MAX}} = E_2 \cdot \varepsilon_{zz_{MAX}} = \frac{3}{8} \frac{q\ell^2}{E a^3} \cdot E_2$$

2025-01-20 Ex 2

Wednesday, January 15, 2025 2:34 PM

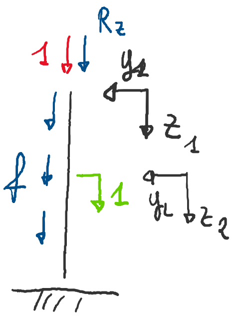


$l = 1000\ mm$

$EA = 6 \cdot 10^9 N$

$f = 300\ N/mm$

Sol



$T_{z_1} = -f \cdot z_1 - R_z$

$T_{z_1}' = -1$

$T_{z_2} = -f \cdot z_2 - f \cdot l/2 - R_z$

$T_{z_2}' = -1$

$T_{z_2}'' = -1$

PCVW

• • $\delta w_e' = 1 \cdot 0$

$$\delta w_i' = \int_0^{l/2} \frac{f z_1 + R_z}{EA} dz_1 + \int_0^{l/2} \frac{f z_2 + f l/2 + R_z}{EA} dz_2$$

$$\rightarrow \frac{1}{2} f \cdot \frac{l^2}{4} + R_z \frac{l}{2} + \frac{1}{2} f \frac{l^2}{4} + \frac{1}{4} f l^2 + R_z \frac{l}{2} = 0$$

$$R_z = -\left(\frac{1}{8} + \frac{1}{8} + \frac{1}{4}\right) l f = -\frac{1}{2} f l$$

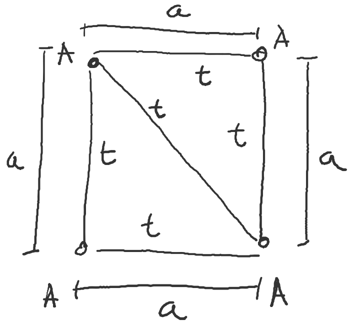
• • $\delta w_e' = w \cdot 1$

$$\delta w_i = \int_0^{l/2} \frac{f \cdot z_2 + \cancel{\frac{1}{2} f l} - \cancel{\frac{1}{2} f l}}{EA} dz_2 = \frac{1}{2} \frac{f l^2}{4EA} = \frac{f l^2}{8EA}$$

$$w = \frac{f l^2}{8EA}$$

2025-01-20 Ex 3

Wednesday, January 15, 2025 2:51 PM



$a = 500$ mm

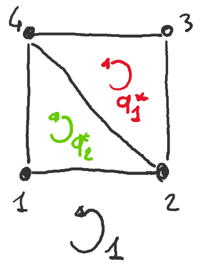
$t = 2$ mm

$E = 70000$ MPa

$\nu = .3$

$A = 200$ mm^2

SOL OPEN CELL FLXS = 0



MOM EQ. WRT. 2 +

$$1 = 2q_1^* \cdot \frac{1}{2} a^2 + 2q_2^* \cdot \frac{1}{2} a^2$$

ROT.

$$\vartheta_1' = \frac{1}{2 \cdot \frac{1}{2} a^2 \cdot G \cdot t} \left(2q_1^* \cdot a + q_1^* \cdot \sqrt{2} a - q_2^* \cdot \sqrt{2} a \right)$$

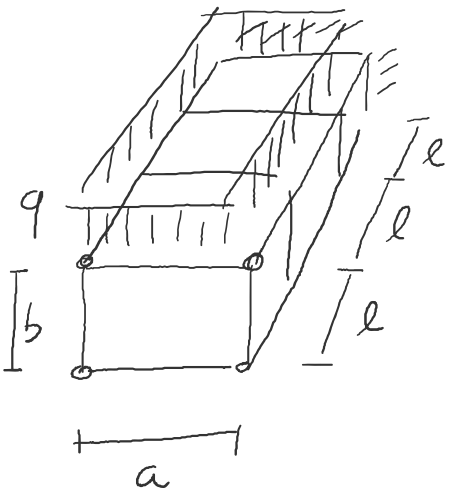
$$\vartheta_2' = \frac{1}{2 \frac{1}{2} a^2 G t} \cdot \left(+2q_2^* \cdot a + q_2^* \sqrt{2} a - q_1^* \sqrt{2} a \right)$$

$$\begin{cases} \vartheta_1' = \vartheta_2' \\ \text{LHS\_MOM} = \text{RHS\_MOM} \end{cases} \rightarrow q_1^*,\ q_2^*$$

$$GJ = \frac{M_t}{\vartheta_1'} = \frac{1}{\vartheta_1'}$$

2025-01-20 Ex 4

Wednesday, January 15, 2025 3:06 PM



$l = 1000 mm$

$a = 700 mm$

$b = 100 mm$

$t = 1 mm$

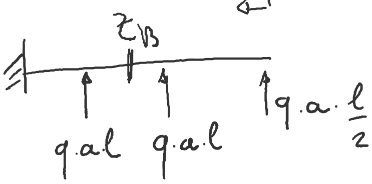
$A = 500 mm^2$

$E = 70000 MPa$

$\nu = 0.3$

$q = 0.01 N/mm^2$

Sol



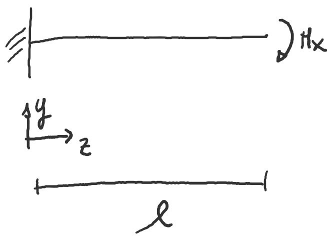
$$M_x(z_B) = -q \cdot a \cdot l \cdot \frac{l}{2} - q \cdot a \cdot \frac{l}{2} \cdot \frac{3l}{2}$$

$$J_{xx} = 4 \cdot A \left(\frac{b}{2}\right)^2 = Ab^2$$

$$\sigma_{z_B} = \frac{M_x(z_B)}{J_{xx}} \cdot \frac{b}{2}$$

2025-01-20 Ex 5

Wednesday, January 15, 2025 3:14 PM



$\ell = 3000$ mm

$a = 1 \cdot 10^{13}\ N\,mm^2$

$b = 3 \cdot 10^{9}\ N\,mm$

$M_x = 1 \cdot 10^{5}\ N\,mm$

<u>SOL</u>

$$v(z) = A z^2$$

PVW $\quad \delta W_e = -\delta v'(\ell) \cdot M_x$

$$\delta W_i = \int_0^\ell \delta v'' \, EJ \, v'' \, dz$$

$$\rightarrow \cancel{\delta A} \int_0^\ell 2 EJ(z) 2A \, dz = -\cancel{\delta A} \cdot 2\ell \cdot M_x$$

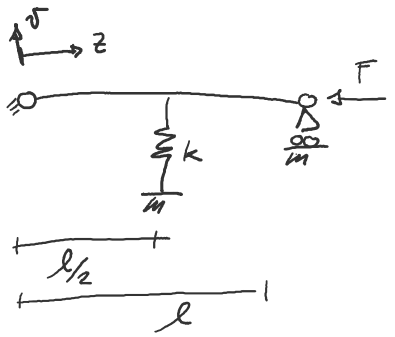
$$A = -\frac{\cancel{2}\ell \cdot M_x}{\cancel{2}\underbrace{\ell \int_0^\ell EJ(z)\,dz}_{1}} = -\frac{M_x}{2a + b\ell}$$

$$\int_0^\ell a + bz \, dz = a\ell + \frac{1}{2} b\ell^2$$

$$v(\ell) = -\frac{M_x}{2a + b\ell} \cdot \ell^2$$

2025-01-20 Ex 6

Wednesday, January 15, 2025 3:21 PM



$\ell = 3000$ mm

$EJ = 1 \cdot 10^{10}$ Nmm^2

$k = 100$ N/mm

SOL

FROM AXIAL EQUILIBRIUM $N = -F$

$$v(z) = A\sin\left(\frac{\pi z}{\ell}\right)$$

$$v'(z) = A\frac{\pi}{\ell}\cos\left(\frac{\pi z}{\ell}\right)$$

$$v''(z) = -A\frac{\pi^2}{\ell^2}\sin\left(\frac{\pi z}{\ell}\right)$$

$$\delta W_i = \int_0^\ell \delta v'' EJ v'' + \delta v' N v' \, dz$$

$$= \delta A \cdot \int_0^\ell \frac{\pi^4}{\ell^4} EJ \sin^2\left(\frac{\pi z}{\ell}\right) \cdot A + \dots$$

$$\dots - \frac{\pi^2}{\ell^2} F \cos^2\left(\frac{\pi z}{\ell}\right) \cdot A \, dz$$

$$= \delta A \left[\frac{\pi^4}{\ell^4} EJ \cdot \frac{\ell}{2} \cdot A - \frac{\pi^2}{\ell^2} \cdot F \cdot \frac{\ell}{2} \cdot A\right]$$

$$\delta W_e = -\delta v\left(\frac{\ell}{2}\right) \cdot k \cdot v\left(\frac{\ell}{2}\right)$$

$$= -\delta A \cdot k \cdot A \cdot \underbrace{\sin\left(\frac{\pi}{2}\right)}_{1}$$

$$= -\delta A \cdot k \cdot A$$

FOR $A \neq 0$

$$\longrightarrow \quad F = \left(\frac{\pi^4}{\cancel{\ell^4}\,\ell^3} \cdot EJ \cdot \frac{\cancel{\ell}}{2} + k\right) \cdot \left(\frac{\pi^2}{\ell^2} \cdot \frac{\ell}{2}\right)^{-1}$$

$$F = \frac{\pi^2}{\ell^2} \cdot EJ + \frac{2k \cdot \ell}{\pi^2}$$