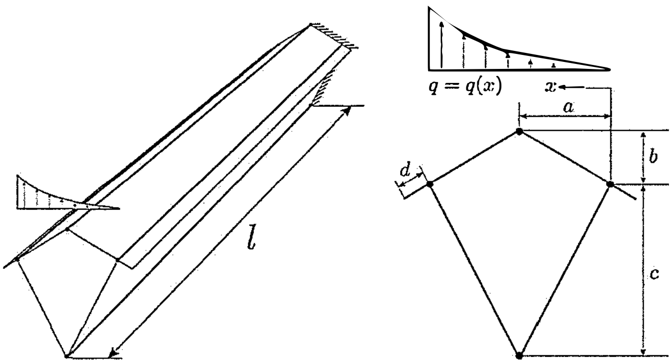
Course of Spacecraft Structures

Written test, July 15^{th}, 2019

Exercise 1

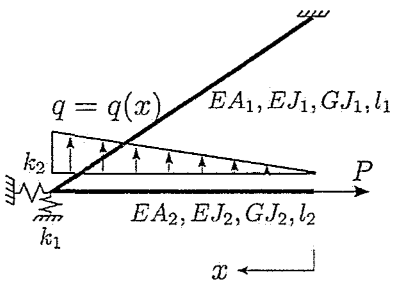
A thin-walled beam is fixed at one end and loaded at the free end with a distributed force per unit length q. The total length is l, while the geometry of the section is depicted in the figure. The lumped stringers have area A, while the thickness of the panels is denoted with t.
By considering a semi-monocoque approximation, determine the shear and the axial stresses acting at the mid-span section.



Data
$l = 6000$ mm;
$a = 200$ mm; $b = 200$ mm;
$c = 400$ mm; $d = 50$ mm;
$t = 1.5$ mm;
$A = 1000$ mm^2;
$q = 150 \left(\frac{x}{400}\right)^2$ N/mm,
$x \in [0, 400]$ mm;

Exercise 2

A structure is composed of two beams. The geometric and elastic properties, calculated with respect to the principal axes, are denoted as EA_i, EJ_i and GJ_i, with $i = 1, 2$, as illustrated in the figure. Note, $EJ_i = EJ_{xx} = EJ_{yy}$.The structure is fixed at one end, and elastically constrained by means of two linear springs of stiffness k_1 and k_2. Considering a loading condition with an axial force P applied at the free end and a linearly varying distributed load $q = q(x)$, determine the horizontal and vertical displacement in correspondence of the two springs.

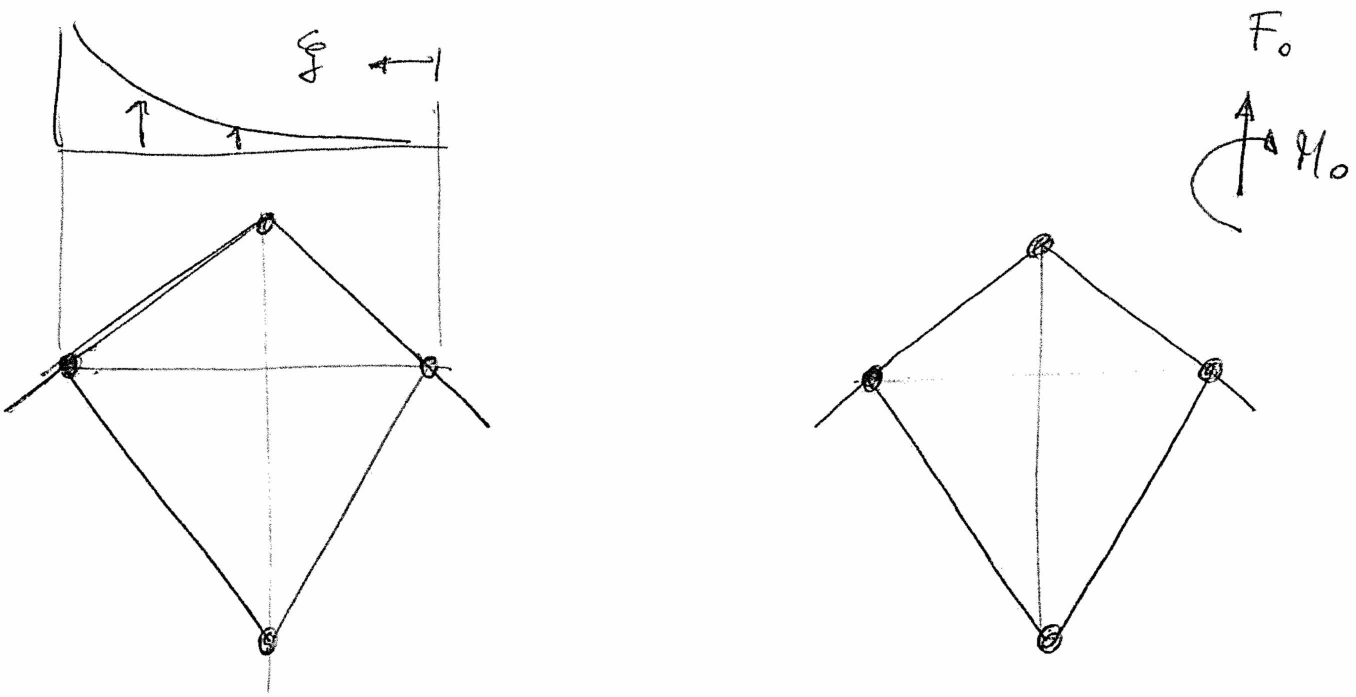


Data
$l_1 = 1000$ mm; $l_2 = 800$ mm;
$EA_1 = 5.7 \times 10^7$ N;
$EJ_1 = 9.0 \times 10^8$ N mm^2;
$GJ_1 = 4.0 \times 10^8$ N mm^2;
$EA_2 = 2.8 \times 10^7$ N;
$EJ_2 = 7.0 \times 10^8$ N mm^2;
$GJ_2 = 2.0 \times 10^8$ N mm^2;
$k_1 = 2.8$ N/mm; $k_2 = 3.5 \times 10^4$ N/mm;
$q = 6 \times 10^{-3} \frac{x}{800}$ N/mm; $P = 40$ kN;

Question 1

Discuss and illustrate the equivalence between the indefinite equilibrium equations for a 3D solid along with the relevant boundary conditions and the Principle of Virtual Work.

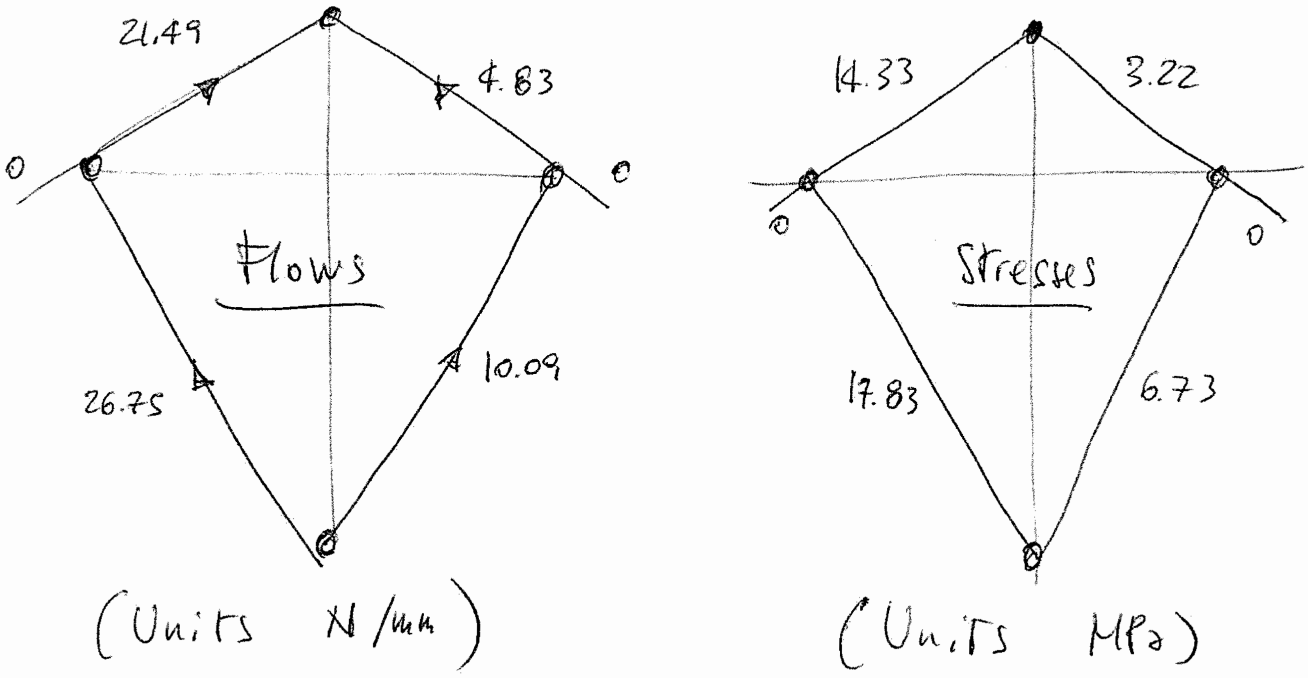
Exercise 1



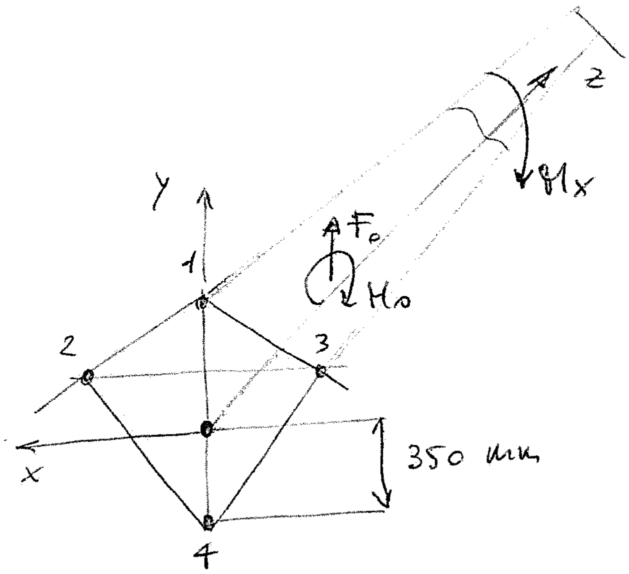
$$F_o = \int_0^{400} 150 \left(\frac{\xi}{400}\right)^2 d\xi = 20 \text{ kN}$$

$$M_o = \int_0^{400} 150 \left(\frac{\xi}{400}\right)^2 \xi \, d\xi = 6 \cdot 10^6 \text{ Nmm}$$

The shear flows and stresses are obtained as:



Axial stress in the stringers



the vertical position of the principal axis origin is at 350 mm from the bottom

the horizontal position is available from the symmetry of the section

$$M_x = -F_0 z$$

At the mid-span section $z = 3000$ mm, so:

$$M_x = -6.00 \cdot 10^7 \text{ Nmm}$$

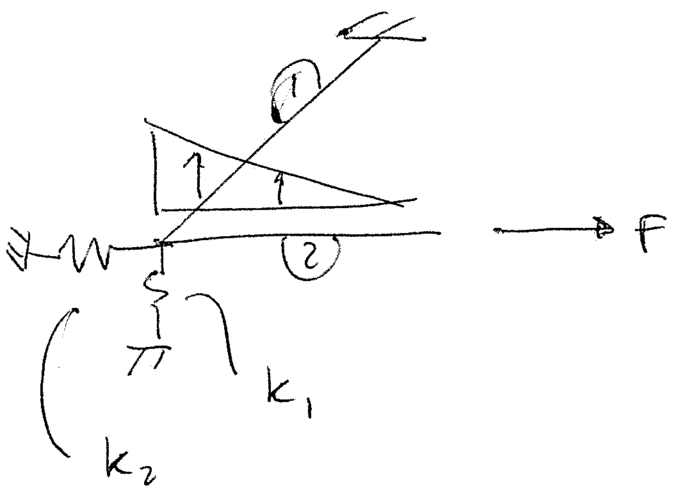
the stresses on the stringers are:

$$\sigma_{xx_1} = \frac{M_x}{J_{xx}}(600-350) = -78.95 \text{ MPa}$$

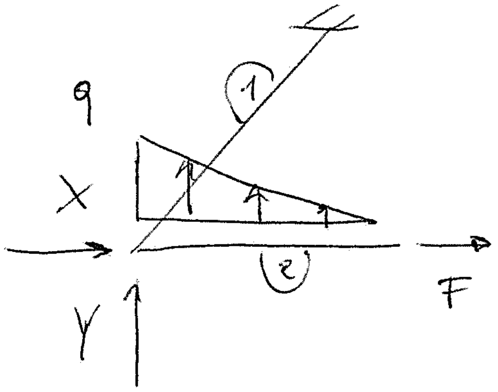
$$\sigma_{xx_2} = \sigma_{xx_3} = \frac{M_x}{J_{xx}}(400-350) = -15.79 \text{ MPa}$$

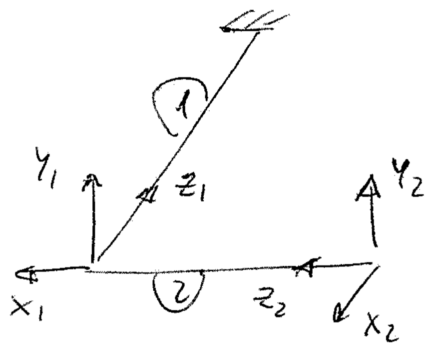
$$\sigma_{xx_4} = \frac{M_x}{J_{xx}}(-350) = 110.53 \text{ MPa}$$

Exercise 2



Real





Ref. systems

$$q(z_2) = a z_2 \quad \text{with} \quad a = 7.5 \cdot 10^{-6}\ N/mm^2$$

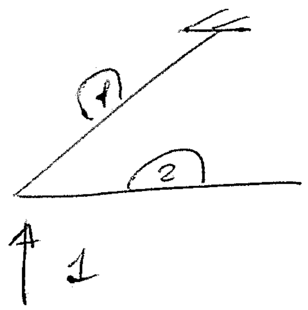
$$M_x^{(2)} = -a \frac{z_2^3}{6}$$

$$M_x^{(1)} = -\left(\frac{a l_2^2}{2} + Y \right) z_1$$

$$M_y^{(1)} = -\left(F + X \right) z_1$$

$$M_z^{(1)} = \frac{a l_2^3}{6}$$

Dummy #1



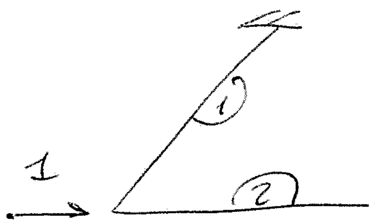
$${}^{1}\delta M_x^{②} = 0$$

$${}^{1}\delta M_x^{①} = -z_1$$

$${}^{1}\delta M_y^{①} = 0$$

$${}^{1}\delta M_z^{①} = 0$$

Dummy #2



$${}^{2}\delta M_x^{②} = 0$$

$${}^{2}\delta M_x^{①} = 0$$

$${}^{2}\delta M_y^{①} = -z$$

$${}^{2}\delta M_z^{①} = 0$$

PCVW

$$\left(\frac{l_1^3}{3} + \frac{EJ_1}{k_1}\right) Y = -\frac{R l_1^3}{3}$$

$$\left(R = q l_2^2/2\right)$$

$$\left(\frac{l_1^3}{3} + \frac{EJ_2}{k_2}\right) X = -\frac{F l_1^3}{3}$$

From which:

$X = -40$ kN $\Rightarrow$ $\delta = -Y/k_1 = 0.44$ mm

$Y = -1.22$ N $\qquad$ $\delta = -X/k_2 = 1.14$ mm