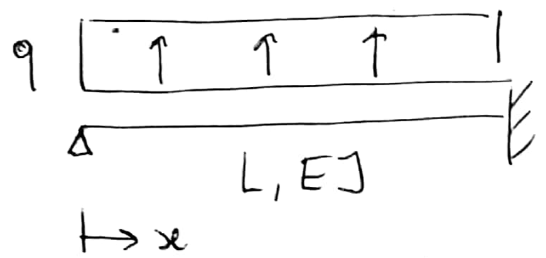


Find the minimum value EJ/EJ_0 such that the vertical displacement at $x = L/2$ is smaller than W_{max}

Data

$L = 1200\,(1 + B/10)$ mm

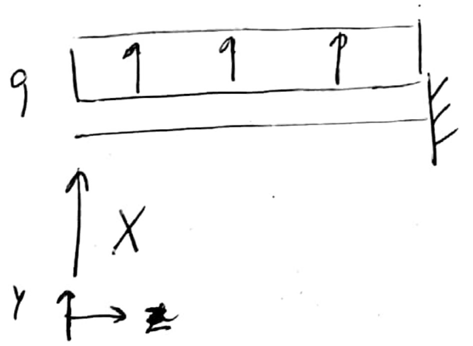
$EJ_0 = 6 \cdot 10^7$ N mm^2

$q = 1.0 \cdot (1 + C/10)$ N/mm

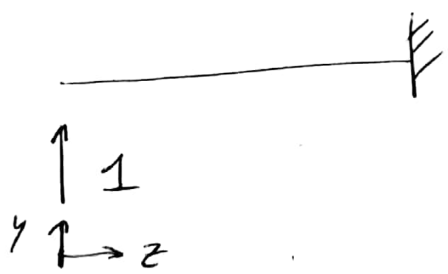
$W_{max} = 5.0$ mm

Solution ($B = C = 0$)

Find first the unknown reaction force X



Real

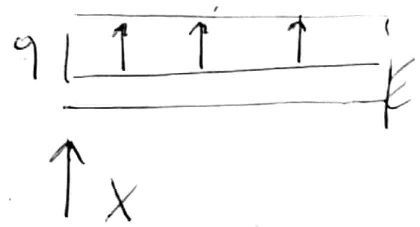


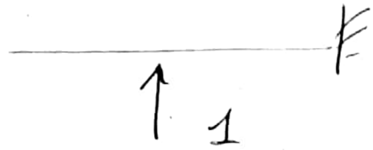
Dummy

$$M_x = -Xz - \frac{qz^2}{2} \qquad \delta M_x = -z$$

$$\int_0^L \frac{M_x \, \delta M_x}{EJ} \, dz = 0 \quad \Rightarrow \quad X = -\frac{3}{8} qL$$

Evaluate now the displacement:



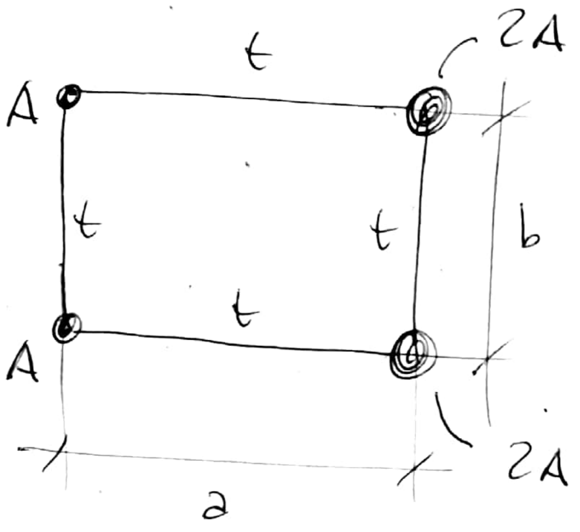


As before $\qquad \delta M_x = -z + L/2$

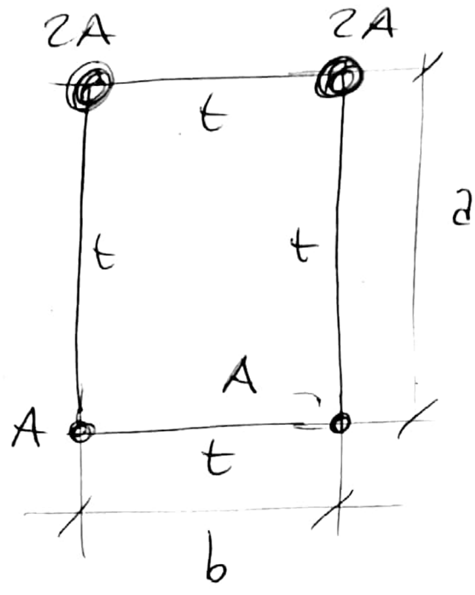
$$\int_{L/2}^{L} \frac{\delta M_x \, M_x}{EJ} \, dz = S = W_{max}$$ and so:

$$EJ = \frac{1}{W_{max}} \int_{L/2}^{L} \delta M_x \, M_x \, dz$$ from which

$$\frac{EJ}{EJ_0} = \frac{1}{EJ_0} \, \frac{L^3 (40X + 17qL)}{384 \, W_{max}}$$



Section #1



Section #2

Calculate the ratio between the torsional stiffness of the two sections GJ_2/GJ_1

Data

$a = 200\,(1 + F/10)$ mm

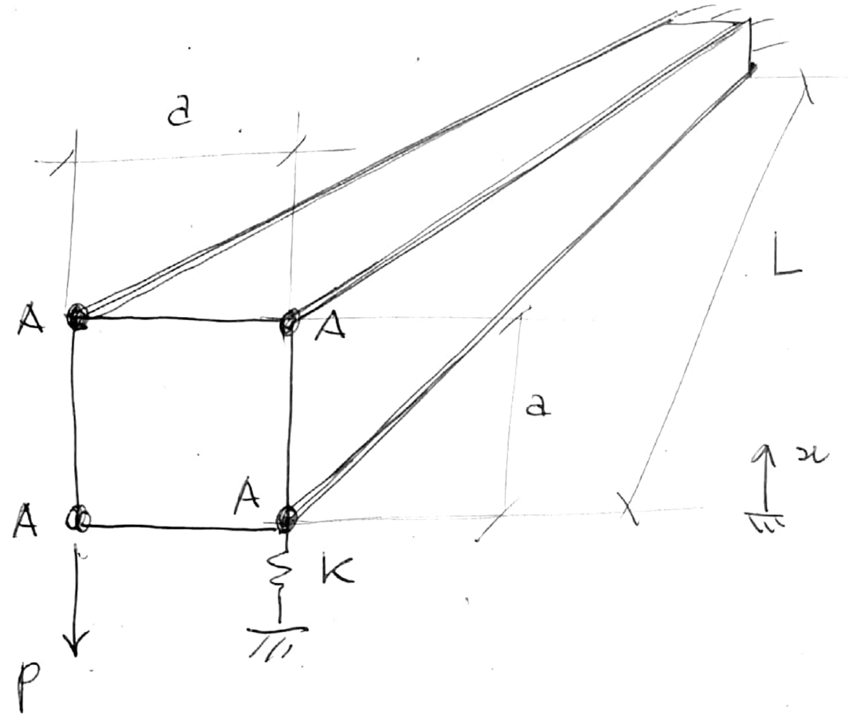
$b = 100$ mm

$A = 500\,(1 + A/10)$ mm^2

$t = 1$ mm

Solution

$$GJ_2/GJ_1 = 1.0$$



Determine the vertical displacement in correspondence of the spring. (refer to the x axis in the figure)

<u>Data</u>

$A = 250\ mm^2$

$a = 150\ mm$

$t = 0.6\ (1 + B/10)\ mm$

$L = 1200\ (1 + F/10)\ mm$

$k = 100\ N/mm$

$E = 72000\ MPa$

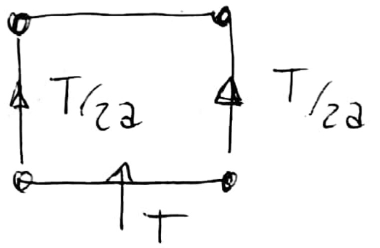
$\nu = 0.3$

$P = 1700\ N$

Solution ($B = F = 0$)

Section properties can be evaluated straightforwardly by exploiting the section symmetries

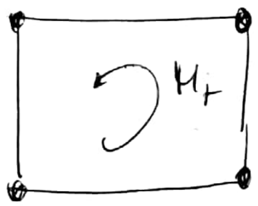
Shear



$$A^* = 2at$$

$$GA^* = 2atG$$

Torsional

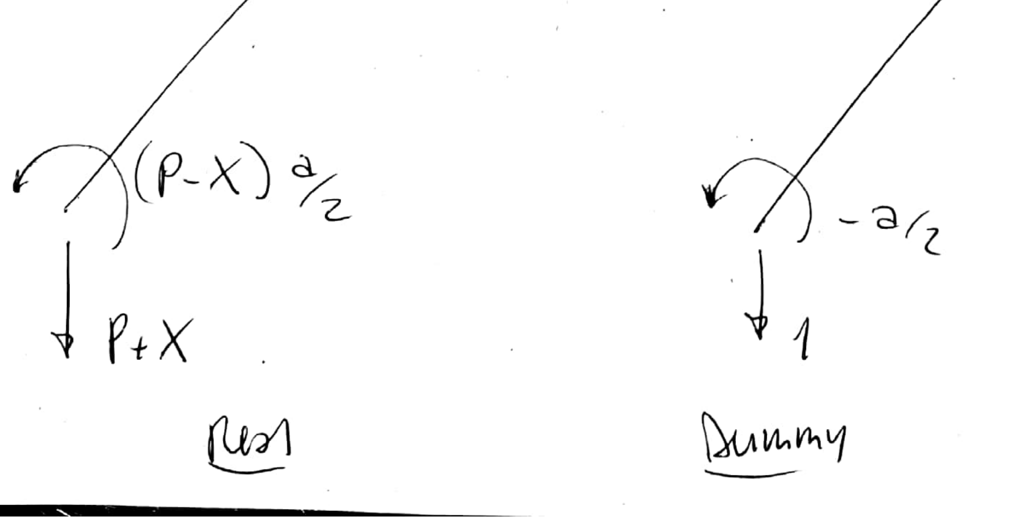


$$GJ = Ga^3t$$

Bending

$$EJ = EAa^2$$

Evaluate now the unknown reaction force X in correspondence of the spring



$T = P + X \qquad \delta T = 1$

$M = (P+X)z \qquad \delta M = z$

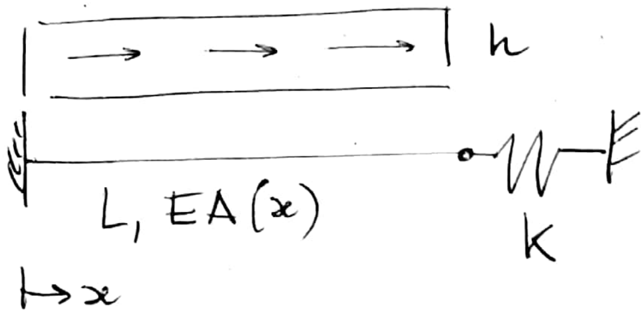
$M_t = (P-X)\,a/2 \qquad \delta M_t = -a/2$

$$\int_0^L \left(\frac{\delta T\, T}{GA^*} + \frac{\delta M\, M}{EJ} + \frac{\delta M_t\, M_t}{GJ} \right) dz + \frac{X}{k} = 0$$

From which:

$$X = P \, \frac{a^2/4GJ - L^2/3EJ - 1/GA^*}{1/GA^* + L^2/3EJ + a^2/4GJ + 1/kL} = -222.55\ N$$

$S = X/k = -2.23\ mm$



Using the Ritz method and a 2-dof polynomial approximation determine the strain energy

Data

$L = 2700$ mm

$n = 2(1 + F/10)$ N/mm

$EA_0 = 2.8 \cdot 10^5$ N

$EA_1 = 6.5 \cdot 10^5$ N

$k = 300(1 + B/10)$ N/mm

Solution (B = F = 0)

$u(x) = q_1 x + q_2 x^2$, so:

$$\int_0^L \delta u_{/x} \, EA(x) \, u_{/x} \, dx + \delta u(L) \, k \, u(L) =$$

$$= \int_0^L \delta u \, n \, dx$$

with $EA(x) = \alpha + \beta x$

and $\alpha = EA_0$; $\beta = \dfrac{EA_1 - EA_0}{L}$

Upon substitution:

$$K_{11} = \alpha L + \beta L^2/2 + kL^2$$

$$K_{12} = K_{21} = \alpha L^2 + 2/3\, \beta L^3 + kL^3$$

$$K_{22} = 4/3\, \alpha L^3 + \beta L^4 + kL^4$$

$$f_1 = L^2/2\, h$$

$$f_2 = L^3/3\, h$$

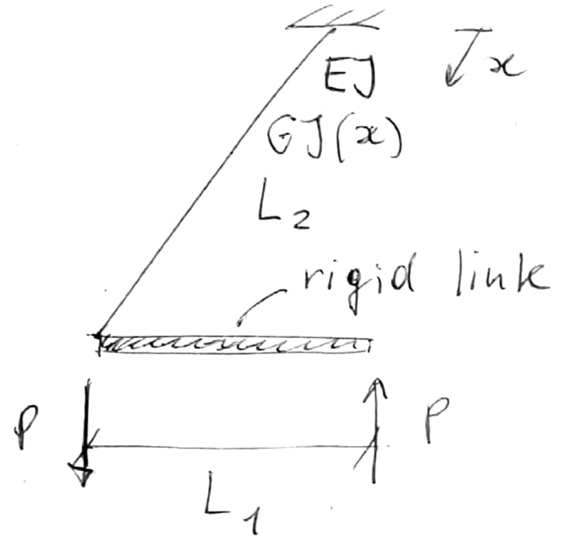
The problem is then: $\underline{\underline{K}}\, \underline{q} = \underline{f}$, from which:

$$q_1 = 9.23 \cdot 10^{-3}$$

$$q_2 = -2.51 \cdot 10^{-6}$$

The strain energy is:

$$U = \frac{1}{2}\, \underline{q}^T\, \underline{\underline{K}}\, \underline{q} = 17151 \text{ Nmm}$$



Using the Ritz method and a 1-dof polynomial approximation, determine the strain energy.
(GJ varies linearly from GJ_0 at $x=0$ to GJ_1 at $x=L_2$)

Data

$L_1 = 180$ mm

$L_2 = 1800$ mm

$GJ_0 = 1.1 \cdot 10^8$ N mm^2

$GJ_1 = 3.3 \cdot 10^8$ N mm^2

$P = 50(1 + D/10)$ N

Solution ($D=0$)

$$\theta = q_1 x$$

$$GJ(x) = \alpha + \beta x; \quad \alpha = GJ_0; \quad \beta = \frac{GJ_1 - GJ_0}{L_2}$$

$$\int_0^L \delta\theta' \, GJ(x)\, \theta' \, dx = \delta\theta(L_2) P L_1, \quad \text{so:}$$

$$\left(\alpha L_2 + \beta \frac{L_2^2}{2}\right) q_1 = P L_1 L_2$$

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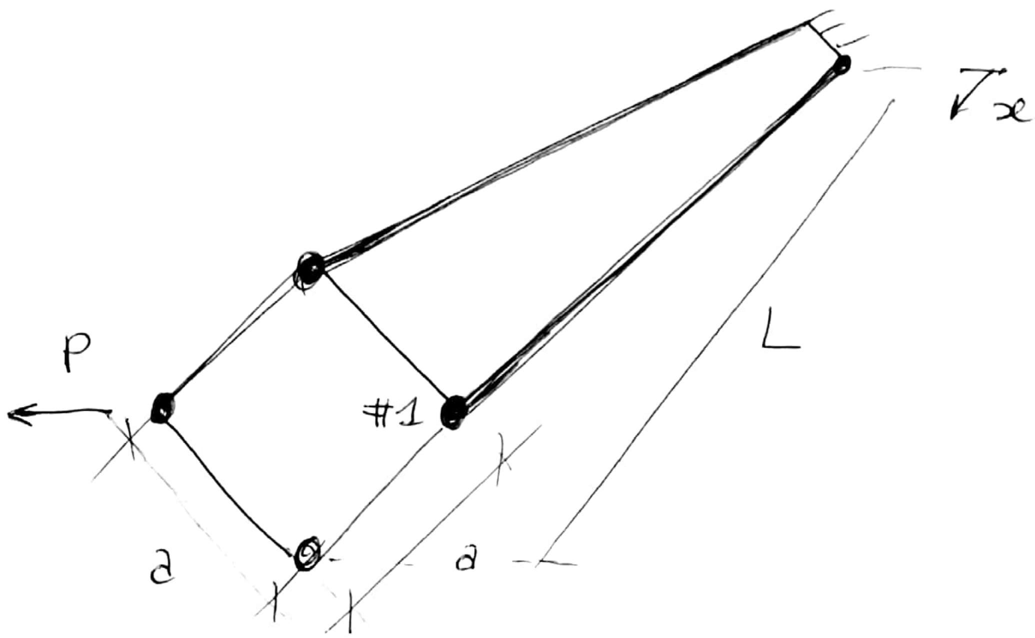
$q_1 = 4.091 \cdot 10^{-5}$, and

$$U = \frac{1}{2} \left(\alpha L_2 + \beta L_2^2 / 2 \right) q_1^2 = 331.36 \ N\,mm$$



Evaluate the axial stress in stringer #1 at $x = 0$.

Data

$a = 180$ mm $L = 1600\,(1 + A/10)$ mm

$A = 350$ mm^2 $P = 1300\,(1 + F/10)$ N

Solution $(A = F = 0)$

$J_{xx} = A a^2$

the bending moment at $x = 0$ is $M = PL$, so:

$$\sigma = \frac{M}{J_{xx}} \frac{a\sqrt{2}}{2} = 23.35 \text{ MPa}$$

- The solution due to De Saint Venant does not account for local effects because local effects are always negligible.

  False

- The Hooke's law is a constitutive law for linear elastic materials.

  True

- Shear deformability effects are generally more relevant for thin-walled beams than for compact beams.

  True

- The elastic problem can be formulated in terms of displacements:

  Always

- The shear center of open section thin-walled beam section modeled according to the semi-monocoque scheme

  can be determined by imposing the equivalence of torsional moment

- The Timoshenko beam model

  accounts for shear deformability