Course of Aerospace Structures

Written test, August 29th, 2022

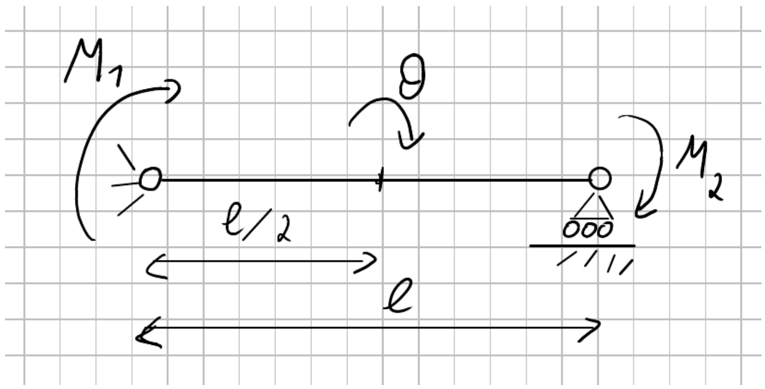
Name ______________________

Surname ______________________

Person code: | | | | | | | | |

Exercise 1

The beam sketched in the figure is loaded by the two concentrated moments M_1 and M_2. Compute the rotation θ in the middle of the beam.
(Unit for result: rad)

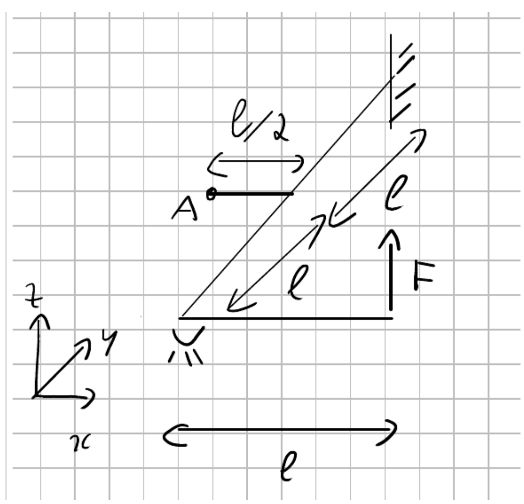


Data
$l = 400$ mm
$M_1 = 50000$ Nmm
$M_2 = 20000$ Nmm
$EJ = 6.0 \times 10^6$ Nmm^2
$EA = 7.2 \times 10^7$ N

Answer ______________________

Exercise 2

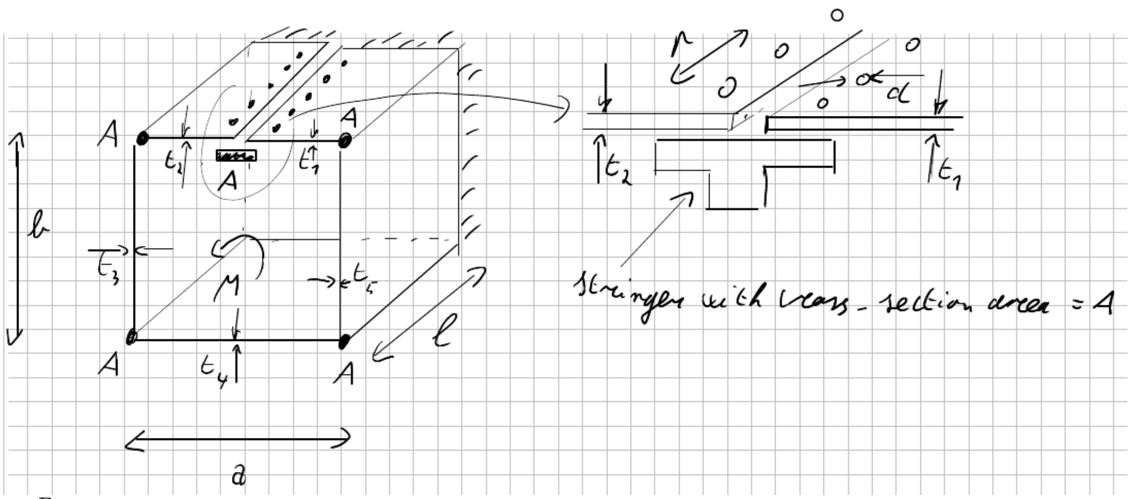
The beam sketched in the figure is loaded by the concentrated force F. Compute the beam displacement w in the z direction of point A.
(Unit for result: mm)



Data
$l = 1400$ mm
$F = 2500$ N
$EJ_{xx} = EJ_{yy} = 6.0 \times 10^{10}$ Nmm^2
$GJ = 8.0 \times 10^{10}$ Nmm^2
$EA = 1.0 \times 10^5$ N
Note: the beam cross-section stiffness properties are given in a local reference system.

Answer ______________________

Exercise 3
The semi-monocoque beam model in the figure is loaded at the free extremity by a concentrated moment M. It has five stringers, each with concentrated area A. The upper face is made by by two panels, that are connected through the stringer positioned in the middle at $a/2$. Carefully check the detailed sketch on the right, and in case of doubt ask for a clarification!
These two panels are connected to the stringer by means of rivets, spaced by p, and with a diameter d.
Compute the shear stress of one rivet.
(Unit for result: MPa)



Data

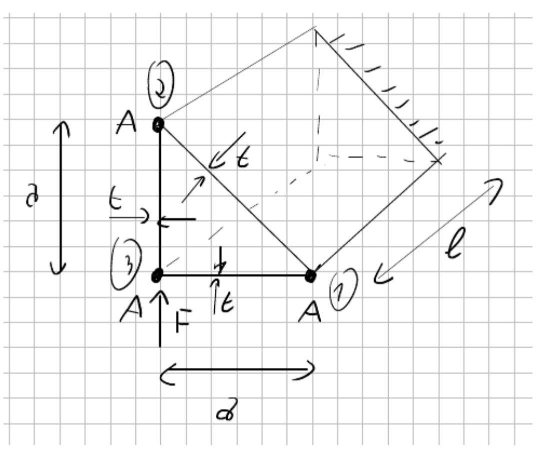
$a = 600$ mm
$b = 100$ mm
$t_1 = 1$ mm
$t_2 = 2$ mm

$t_3 = 2.5$ mm
$t_4 = 3$ mm
$t_5 = 1$ mm
$A = 100$ mm^2

$M = 1 \times 10^5$ Nmm
$p = 30$ mm
$d = 2$ mm

Answer

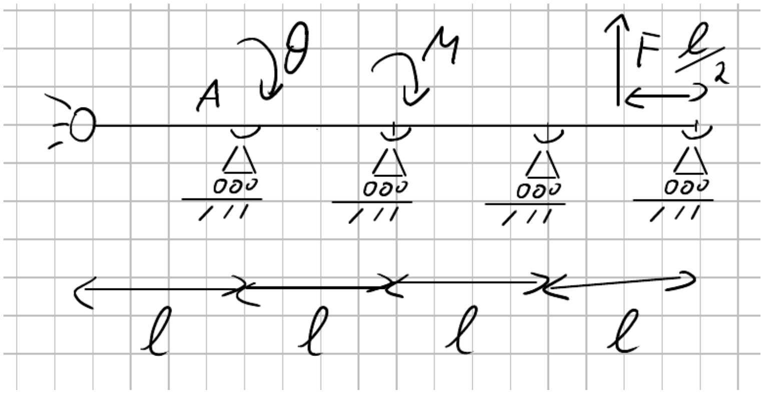
Exercise 4
Consider the semi-monocoque cross section model sketched in the figure.
Compute the axial stress of stringer number 2 at a distance of $l/2$ from the clamp.
(Unit for result: MPa)



Data
$A = 400$ mm^2
$a = 200$ mm
$t = 2$ mm
$l = 6000$ mm
$F = 2000$ N

Answer

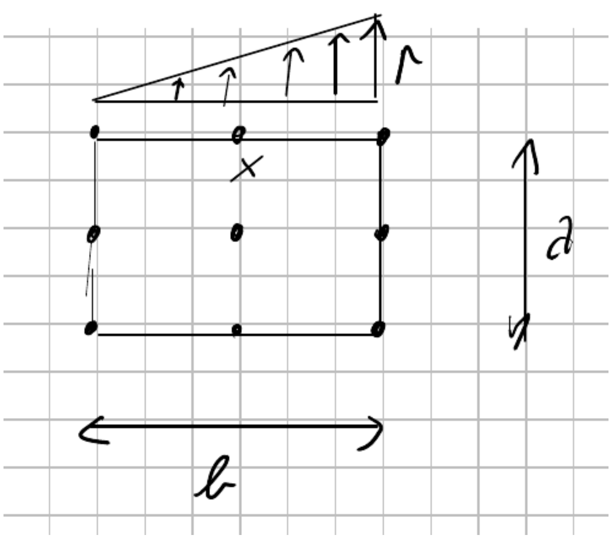
Exercise 5
The beam sketched in the figure is loaded by the concentrated moment M at $z = 2l$ from the leftmost hinge and by the concentrated force F at $z = 7/2l$.
By resorting to the displacement method, and using a trigonometric approximation with only one term, estimate the rotation θ of point A (the point at $z = l$ from the leftmost hinge).
(Unit for result: rad)



Data
$l = 1000$ mm
$EJ = 6 \times 10^8$ Nmm^2
$M = 4 \times 10^5$ Nmm
$F = 2000$ N

Answer

Exercise 6
The nine-nodes parabolic element sketched in the figure is loaded by a traingular force per unit of length, going from 0 N/mm to p N/mm.
Compute the virtual work of the load for a unit virtual displacement in the direction of the load for the upper middle node (i.e. the node identified by an "X" in the sketch).
(Unit for result: Nmm)



Data
$a = 2$ mm
$b = 3$ mm
$p = 10$ N/mm

Answer

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. the axial stress σ_{zz} is always null for a beam in plane strain:

2. the reaction forces of over-constrained structures cannot be computed by resorting to the displacement method:

3. the stress σ_{zz} of an isotropic material is only a function of the corresponding deformation ε_{zz}:

Multiple Choice questions

(Circle the correct answer)

1. The shear stress in a thin panel is equal to:

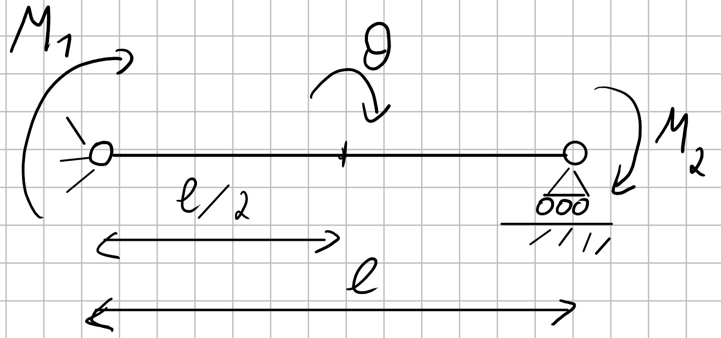
   (a) the shear flux divided by the panel thickness
   (b) the shear flux multiplied by the panel thickness
   (c) the shear flux
   (d) the derivative of the axial stress in the panel
   (e) none of the above

2. The shear deformability of a thin-walled beam is:

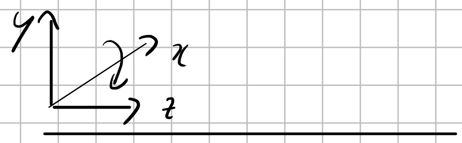
   (a) always negligible
   (b) always more significant than the bending stiffness
   (c) equal to the axial stiffness
   (d) equal to the bending stiffness
   (e) equal to the torsional stiffness
   (f) often not negligible
   (g) none of the above

3. The axial stiffness for a thin-walled beam:

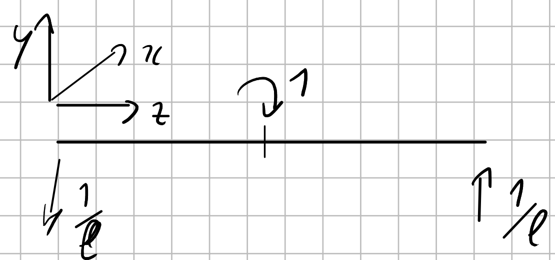
   (a) is null
   (b) is generally larger with respect to a corresponding (same material and cross-section area of the material) compact section
   (c) is generally smaller with respect to a corresponding (same material and cross-section area of the material) compact section
   (d) is equal to that of a corresponding (same material and cross-section area of the material) compact section
   (e) can be neglected
   (f) none of the above



$$F_2 = \frac{M_1 + M_2}{\ell}$$



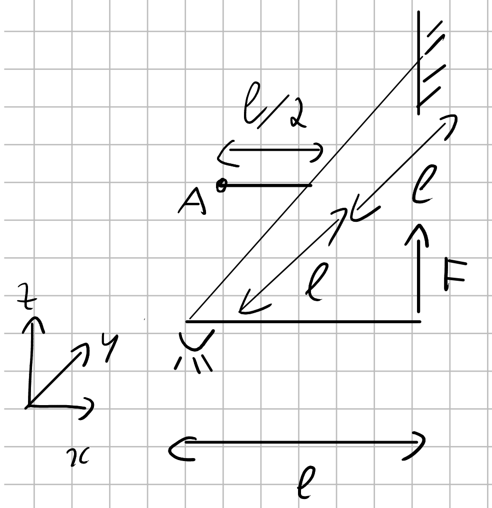
$$M_x = -M_1 + F_2\, z$$

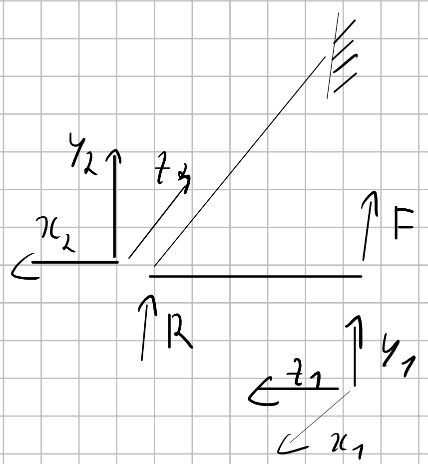


$$0 < z < \frac{\ell}{2} \qquad M'_{x_1} = \frac{1}{\ell}\, z$$

$$\frac{\ell}{2} < z < \ell \qquad M'_{x_2} = -1 + \frac{1}{\ell}\, z$$

$$\vartheta = \int_0^{\ell/2} \frac{M'_{x_1}\, M_x}{EJ}\, dz + \int_{\ell/2}^{\ell} \frac{M'_{x_2}\, M_x}{EJ}\, dz$$

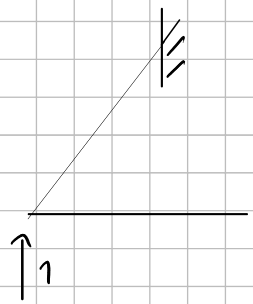




$$M_{x_1} = -F z_1$$

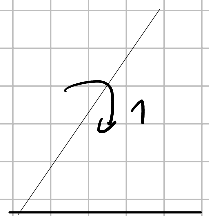
$$M_{z_2} = F l$$

$$M_{x_2} = -(F+R) z_2$$



$$M'_{x_2} = -z_2$$

$$\int_0^{2l} \frac{(F+R)\, z_2^2}{EJ}\, dz_2 = 0 \quad \Rightarrow \quad R = -F$$

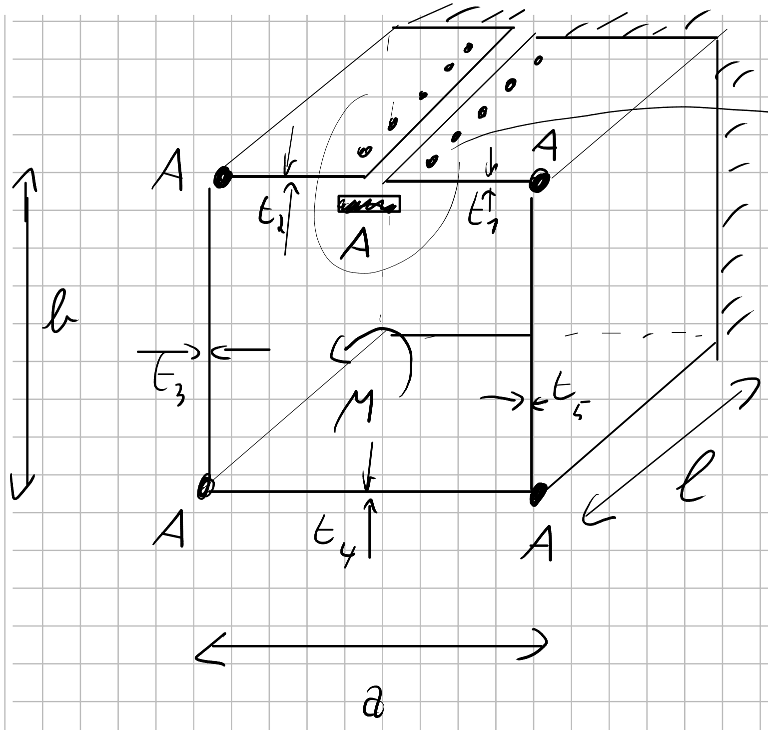


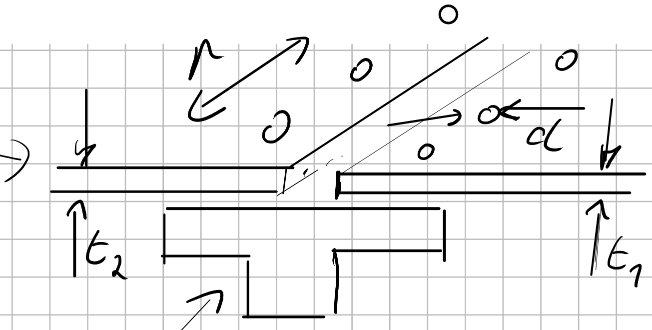
$$M_{x_2} = 0 \qquad M'_{z_2} = -1$$

$$\theta(z_2 = l) = \int_l^{2l} \frac{-Fl}{GJ}\, dz_2 = \frac{-Fl^2}{GJ}$$

$w_2(z_2 = l) = 0$ because $M_{z_2} = 0$

$$w = \theta(z_2 = l) \cdot \frac{l}{2} = -\frac{Fl^3}{GJ2}$$



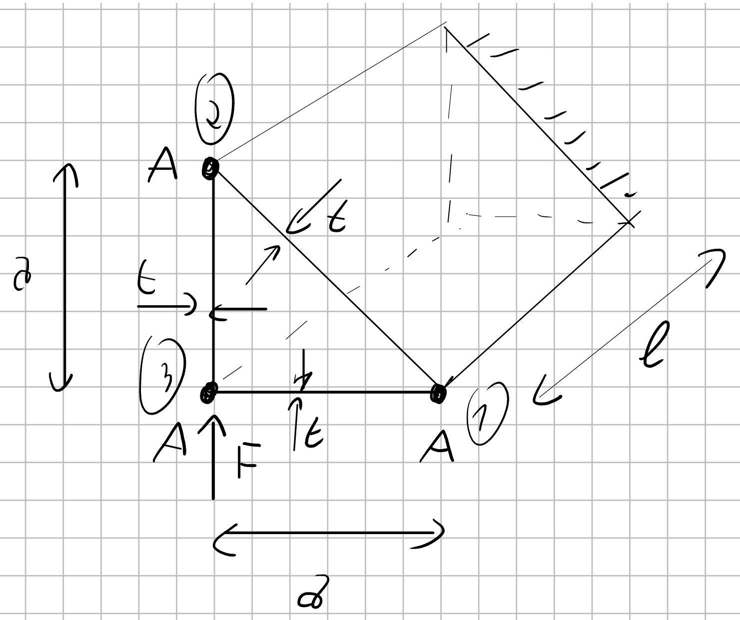


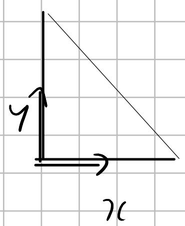
Stringer with cross-section area = A

$$q = \frac{M}{2ab}$$

$$F = q \cdot r$$

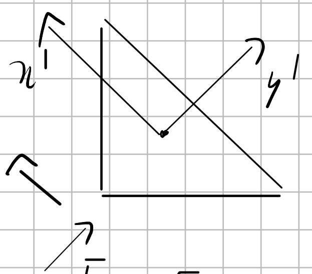
$$\tau = \frac{F}{\pi \frac{d^2}{4}}$$





$$x_{CG} = \frac{a}{3}$$

$$y_{CG} = \frac{a}{3}$$



$$F_{x'} = \frac{F}{\sqrt{2}}$$

$$F_{y'} = \frac{F}{\sqrt{2}}$$

45° for symmetry

$$J_{x'x'} = A\left(\frac{a}{3}\sqrt{2}\right)^2 + 2A\left(\frac{a}{\sqrt{2}} - \frac{a}{3}\sqrt{2}\right)^2 = \frac{1}{3} A a^2$$

$$J_{y'y'} = 2A\left(\frac{a}{\sqrt{2}}\right)^2 = A a^2$$

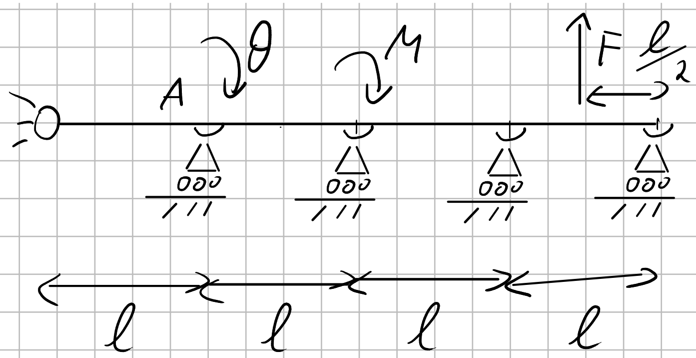
$$M_{x'}\left(\frac{l}{2}\right) = -F_{y'} \cdot \frac{l}{2}$$

$$M_{y'}\left(\frac{l}{2}\right) = F_{x'} \cdot \frac{l}{2}$$

$$x'(2) = \frac{a}{\sqrt{2}}$$

$$y'(2) = \frac{a}{3\sqrt{2}}$$

$$\sigma_{z'z'}\left(2, z = \frac{l}{2}\right) = \frac{M_{x'} \cdot y'}{J_{x'x'}} - \frac{M_{y'} \cdot x'}{J_{y'y'}}$$



$$v = c \sin\left(\frac{\pi z}{l}\right)$$

$$v' = c \frac{\pi}{l} \cos\left(\frac{\pi z}{l}\right)$$

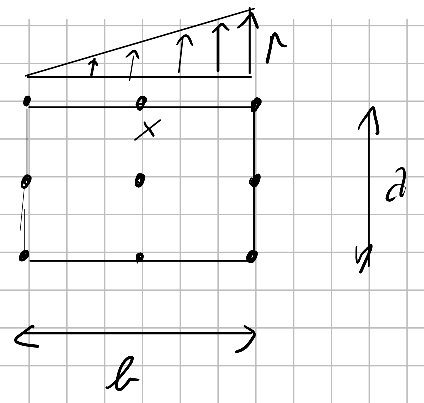
$$v'' = -c \frac{\pi^2}{l^2} \sin\left(\frac{\pi z}{l}\right)$$

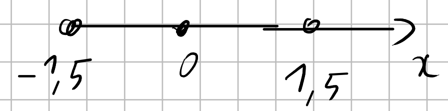
$$\int_0^{4l} \delta c \sin^2\left(\frac{\pi z}{l}\right) c \, EJ \frac{\pi^4}{l^4} \, dz = -\delta c \frac{\pi}{l} \cos(2\pi) M + \delta c \sin\left(7\frac{\pi}{2}\right)$$

$$= -\delta c \frac{\pi}{l} M - \delta c F$$

$$\frac{2 l \pi^4 EJ}{l^4} c = -M \frac{\pi}{l} - F \qquad c = \left(-M\frac{\pi}{l} - F\right) \Big/ \left(\frac{2 l \pi^4 EJ}{l^4}\right)$$

$$\theta = -v' = c \frac{\pi}{l}$$





$$M_x = 1 - \frac{x^2}{2,25}$$

$$p = \frac{r}{b}\left(x + \frac{b}{2}\right)$$

$$\delta f_e = \int_{-1,5}^{1,5} \left(1 - \frac{x^2}{2,25}\right)\left(x + 1,5\right) \cdot \frac{r}{3}\, dx = \dots = r$$

- The axial stress σ_{zz} is always null for a beam in plane strain:
  - False
- The reaction forces of over-constrained structures cannot be computed by resorting to the displacement method:
  - False
- The stress σ_{zz} of an isotropic material is only a function of the corresponding deformation ε_{zz}:
  - False

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