• Ribs and frames

Ribs and frames are an essential component in a thin-walled structure for the reasons discussed next.
They are structural elements stiffening the thin-walled beam along its transverse direction. The nomenclature here adopted refers to

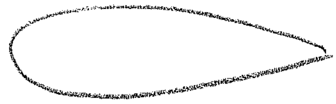
1. Ribs : transverse elements in wings/aerodynamic surfaces

2. Frames : transverse elements in fuselages or cylindrical shells of space launcher structures (sometimes denoted also as rings).

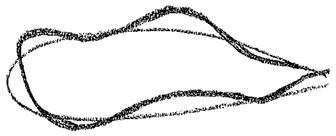
The roles played by ribs and frames in thin-walled aerostructures can be summarized as:

1. They preserve the shape of the section

The presence of transverse stiffening elements – infinitely stiff along the in-plane directions – ensures that the section shape is altered due to the applied loads, thus preserving the aerodynamic performance.



aerodynamic shape (preserved by ribs)

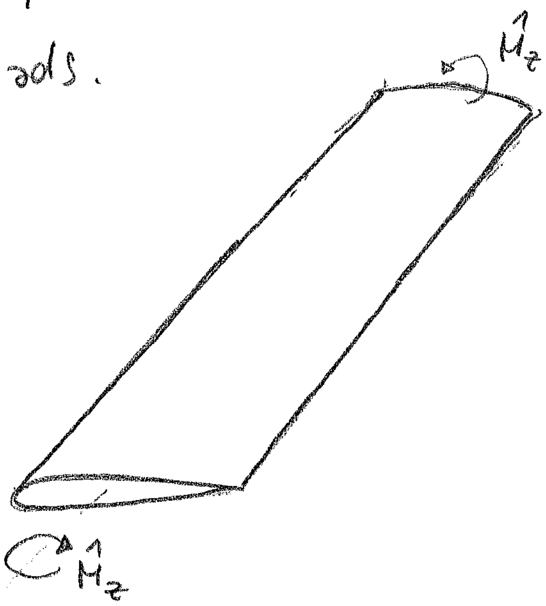


exaggerate deformed shape with altered aerodynamic properties

The situation depicted in the figure on the right is prevented thanks to the presence of ribs.

Note that this kind of behaviour is consistent with the results derived in the context of DSV theory for the torsion, where the section was found to be subyected to a rigid rotation preserving the shape of the section.

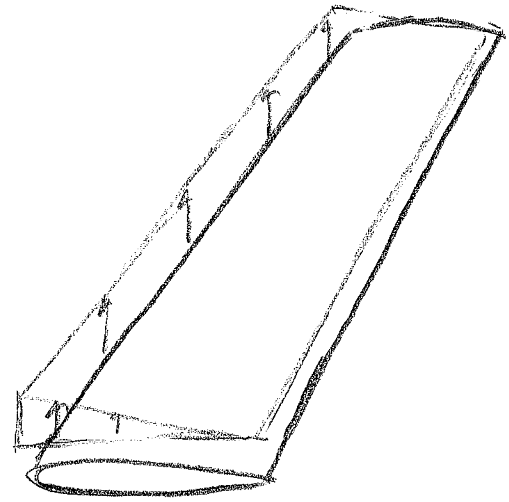
The presence of ribs guarantees that the section shape is preserved even when the DSV hypothesis are not fully respected, such as in the case of distributed loads.



The load is applied at the ends.

⇒ DSV hypothesis respected.

⇒ The section does not change shape

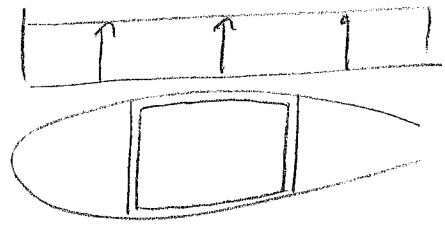


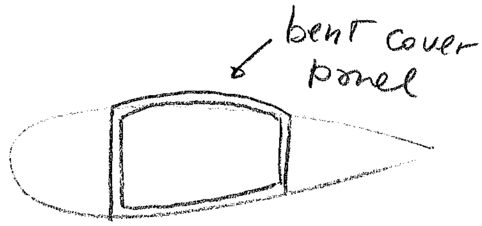
The load is distributed (consider the typical case of a distributed pressure)

⇒ DSV hypothesis not respected

This means that there is no need to consider the presence of ribs to guarantee that the section shape is not altered.

There is no guarantee that the shape will be preserved; indeed the distributed loads will alter the section as the cover panels are subjected to bending loads





Ribs are then necessary to overcome the in-plane weakness of the section, providing support to the panels.

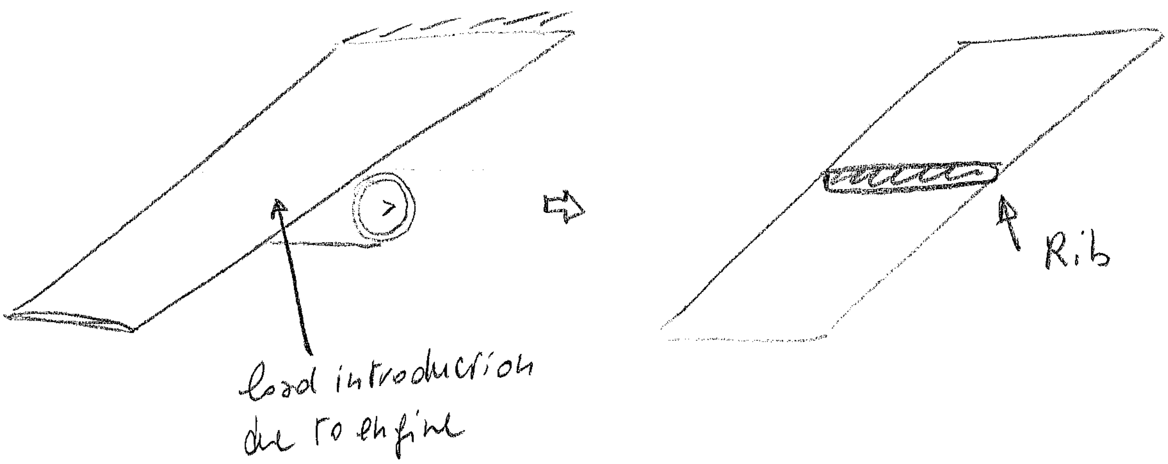
2. <u>They allow the introduction of loads</u>

As a consequence of the previous considerations, it is clear that ribs/frames are of paramount importance for allowing the introduction of loads into the structure. The loads will be, in general, in the form of:

a. Distributed loads

b. Concentrated loads

For a thin-walled sections, the load introduction has to be associated with a locally in-plane stiffer region (see also previous point)
During the design phase a number of ribs will be placed in corrispondence of those portions where concentrated loads need to be introduced. For instance:



In general the choice for the location of a number of ribs is dictated by the design itself. In particular, ribs will be introduced, at least, in correspondence of:

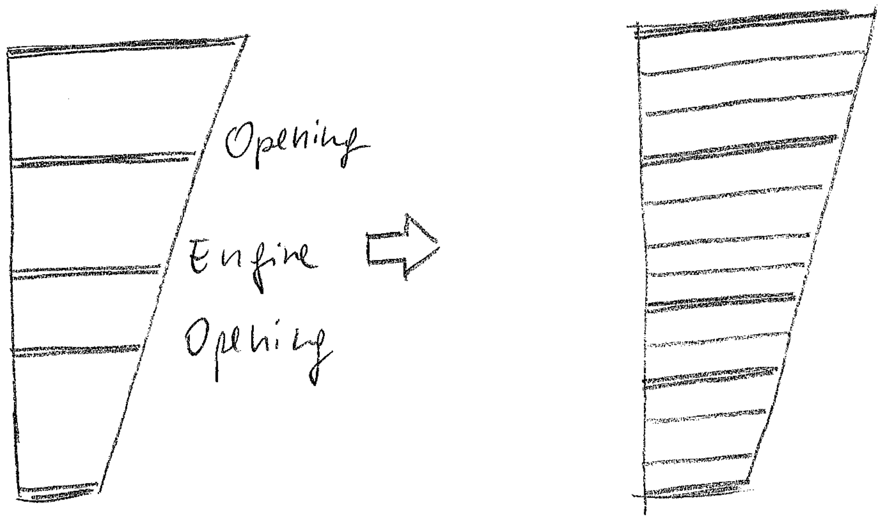
a. Engines

b. Junction between wing and fuselage

c. Openings

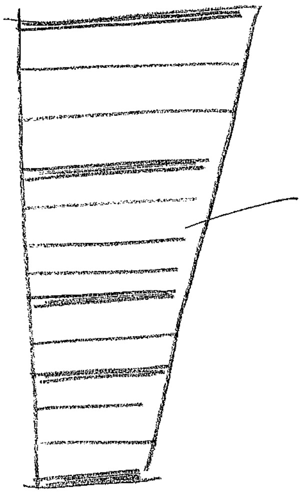
d. Actuators

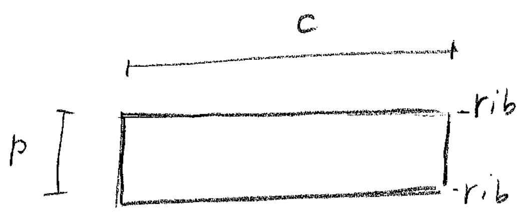
In order to guarantee proper load introduction of the distributed loads, a number of ribs will be then introduced to achieve a distribution which is approximately homogeneous



The regularity of the distribution guarantees that the loads are progressively introduced, in a smooth way, avoiding sudden changes of loads. Furthermore the ribs provide a support for the cover panels, which has a beneficial effect on the bending-induced displacement.

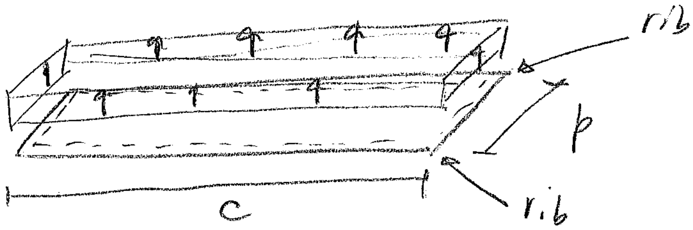
In this sense it is worth highlighting the relatively high ratio between the chord dimension and the pitch of the ribs:





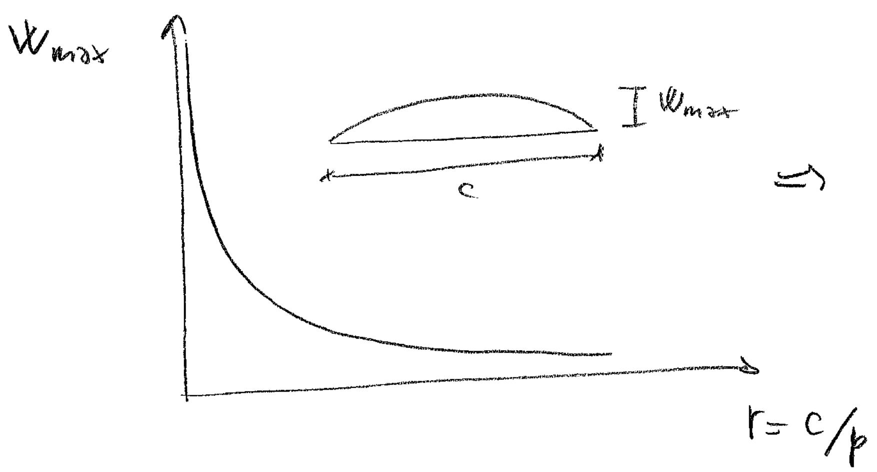
Define $r = c/p$ (aspect ratio)

As a simplified model, it can be assumed that the portion of the panel from rib to rib is a plate supported along the four edges. The effect of the distributed load can be then analyzed as:

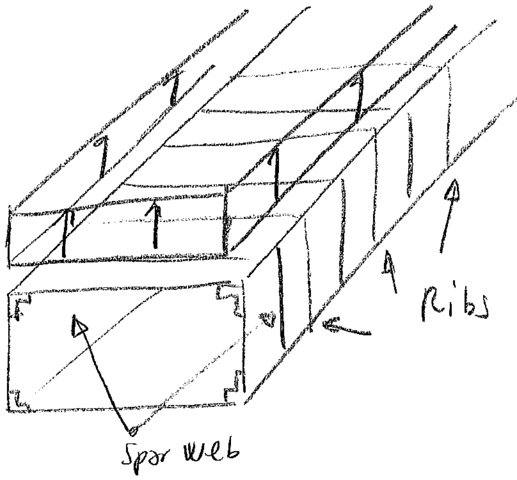


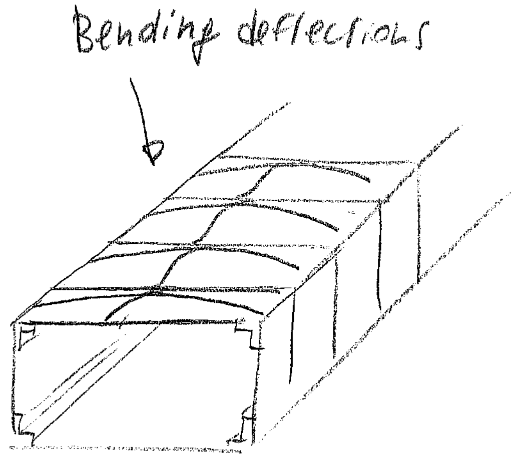
Idealization of the cover panel as a simply supported plate

An exact solution for this problem can be easily found (the procedure is not illustrated here), and it can be shown that the maximum bending displacement due to the pressure load is:

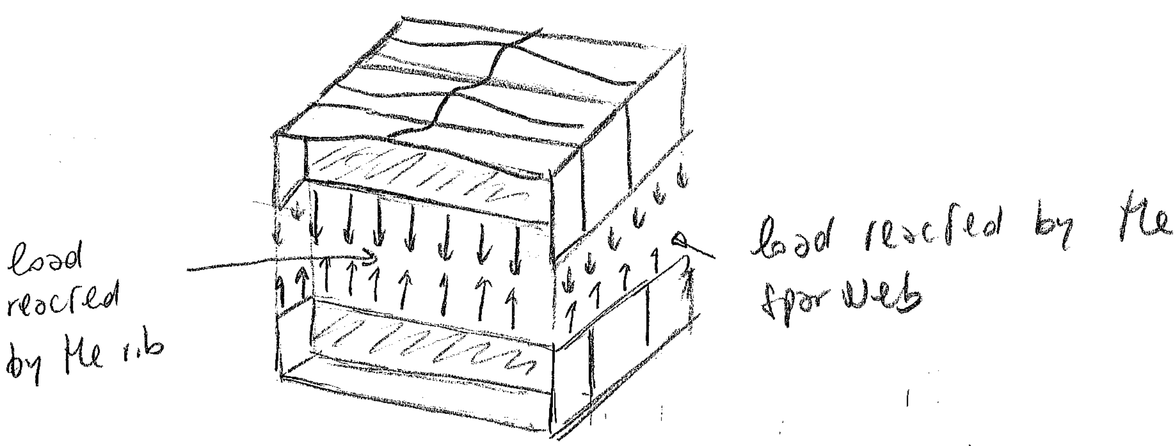


$\Rightarrow$ The increase of r has a drastic effect on the reduction of the bending displacements



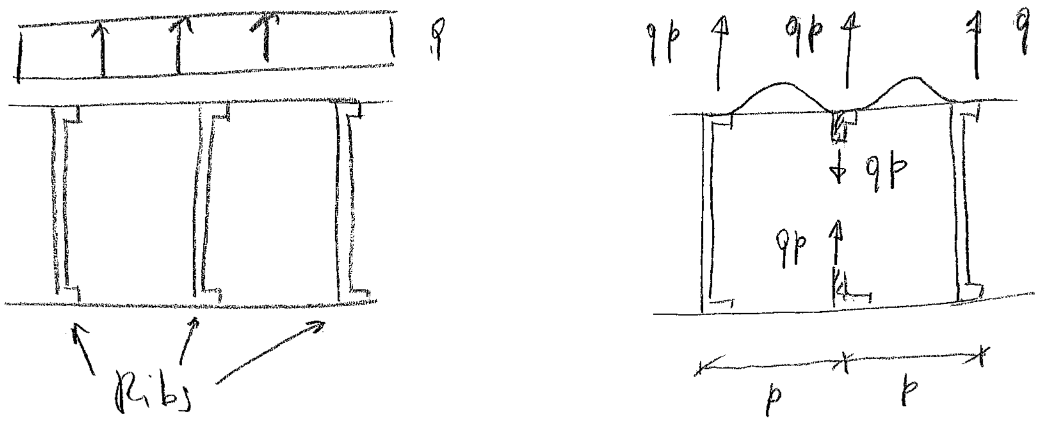


The distributed loads are then transferred from the panels to the ribs and the spar webs, which furnish support to the panels.



In general most of the introduction of load happens in correspondence of the ribs, although even the spars do provide a contribution to react the external loads transferred by the panels.
For this reason it can be assumed, at least as an initial scheme for analyzing the structure, that all of the distributed loads are introduced by the ribs. This assumption turns out to be especially true for typical configurations with high values of $r = c/p$. For $r = 1$ the load introduced by the rib would be equal to the load introduced by the spars

This assumption leads to the following scheme

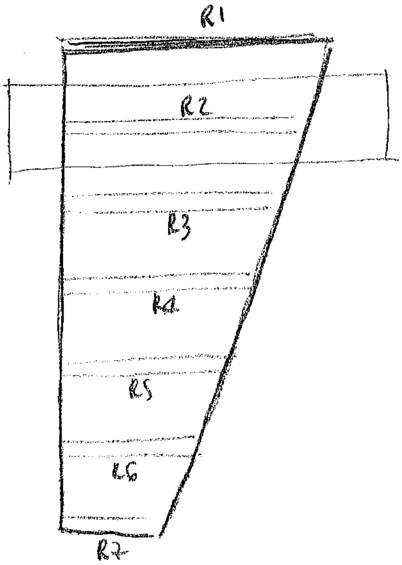


where the load is entirely transferred from the panels to the ribs (thus ignoring the amount of load introduced in the spars)

It is also noted that the bending of the panels can be seen as a local effect, which is not captured in the context of a DSV beam model.
The evaluation of the state of stress in a generic section will then be obtained by evaluating σ_{zz} according to the DSV solution and the shear stresses according to the semi-monocoque scheme.
These results do not include the local correction which is associated with the local bending of panels (whose evaluation can be performed with a detailed FEM model).

In the context of the semi-monocoque beam model it is then assumed that all the loads are introduced in correspondence of the ribs. Some of them (the concentrated ones) are effectively applied to ribs. The distributed ones will be condensed to a set of statically equivalent loads which are applied to ribs. As discussed, this operation has the effect of filtering out local effects (mainly due to the bending of panels), while preserving the overall behaviour of the structure.

For a generic structure, consider for instance a wing, the load introduction will be modeled as:



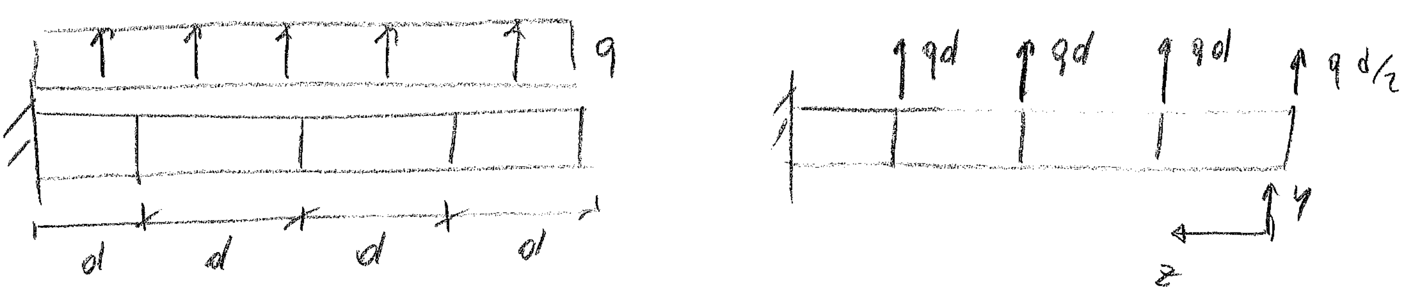
loads acting here should be reported to rib R2

(and similarly for the other ribs)

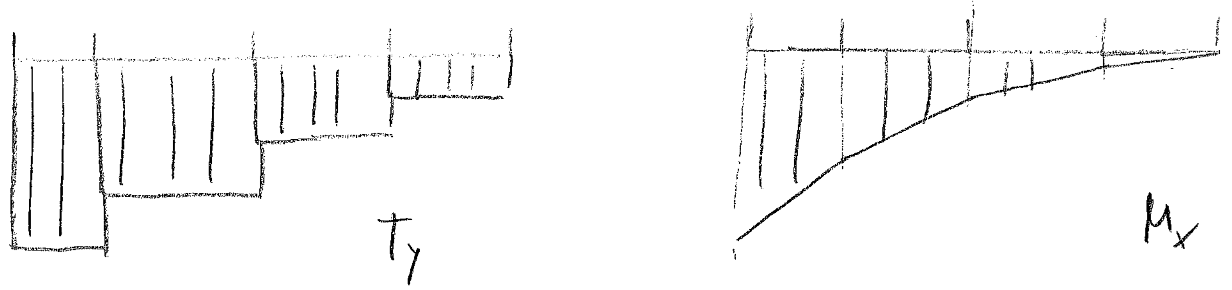
Accordingly the internal forces will be characterized by the following behaviour

- shear T_x, T_y piecewise constant
- moments M_x, M_y piecewise linear
- torsion M_z piecewise constant

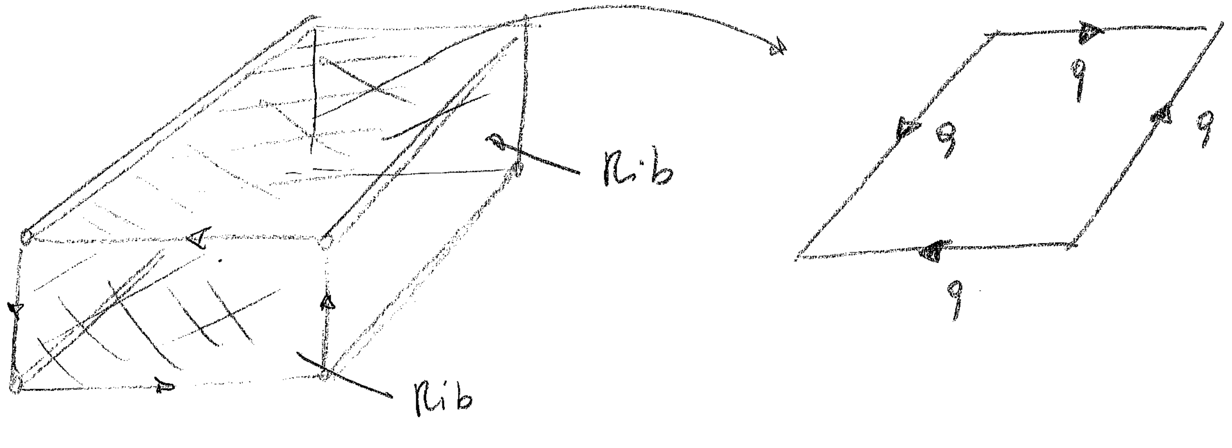
Consider, for instance, the following thin-walled beam.



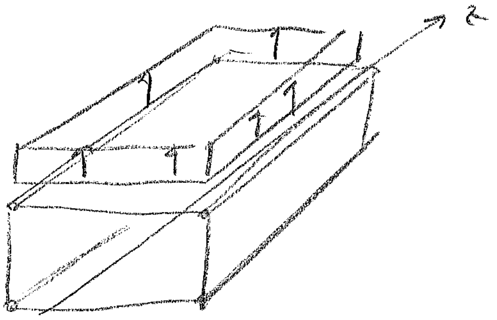
The internal actions T_y and M_x are then:



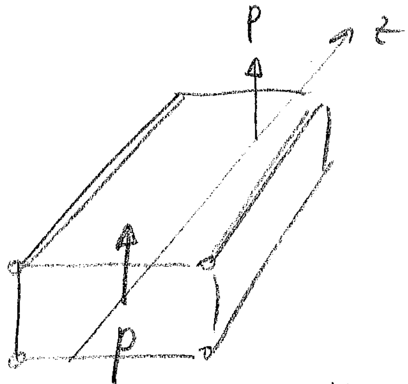
It is worth highlighting that the assumption of loads introduced by the ribs is consistent with the beam model and the semi-monocoque scheme adopted.



The generic panel is subjected to a constant shear flow q, which is constant even along the beam axis.
(recall that the shear stresses σ_{xz} and σ_{yz} are function of x and y, but not z). Clearly this is possible if and only if the loads are introduced at rib level

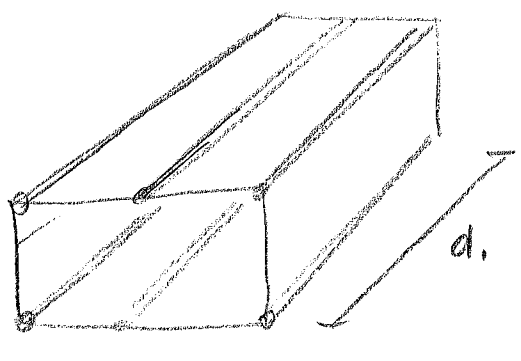


distributed load
=
non constant shear flows along z



loads reported to the ribs
=
constant shear flows along z
(consistent with the beam model)

3. <u>They stabilize panels and stringers with respect to their buckling response</u>



d_i: rib to rib distance
(pitch)

As a first approximation it can be assumed that the stringer would buckle according to the

Euler's buckling load:

$$P^{stringer}_{buck} = \frac{\pi^2 EJ}{\lambda^2}$$

λ: halfwave length of the buckled configuration (depends on the constraints)

Assuming the stringer as pinned to the ribs $\lambda^2 = d_1^2$.
(in any case λ is linear with d).
It follows that the reduction of the rib pitch has the effect of raising the buckling load.

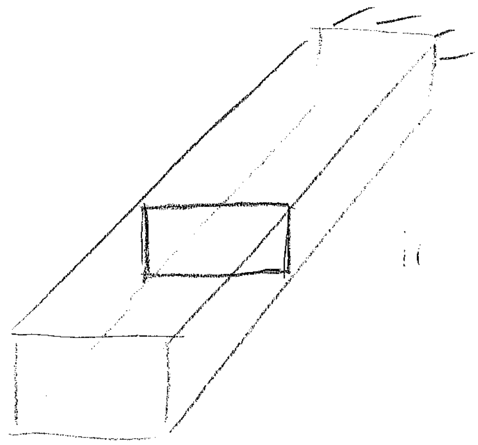
$$\frac{\pi^2 EJ}{d_1^2} < \frac{\pi^2 EJ}{d_2^2} \quad \text{if} \quad d_2 < d_1$$

A remark on the assumptions introduced

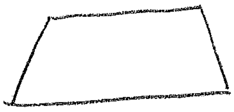
The beam model introduced so far is based on DSV results and has been specialized to the case of thin-walled beams for making possible the evaluation of the shear stresses.
Within the context of DSV solution, it was found that
$\sigma_{xx} = \sigma_{yy} = \sigma_{xy} = 0$ is part of the exact solution.
Ribs have now been introduced as in-plane infinitely stiff elements, which means that the section cannot change shape in correspondence of the ribs.

The impossibility of changing shape determines then the onset of not null σ_{xx}, σ_{yy} and σ_{xy}

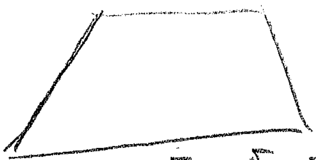
which is an <u>inconsistency</u> of the model.



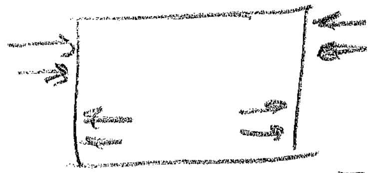
if no ribs were present the section highlighted would deform as:



However, if the presence of the rib is accounted for, the shape is preserved, meaning that an in-plane state of stress is induced for restoring the original shape

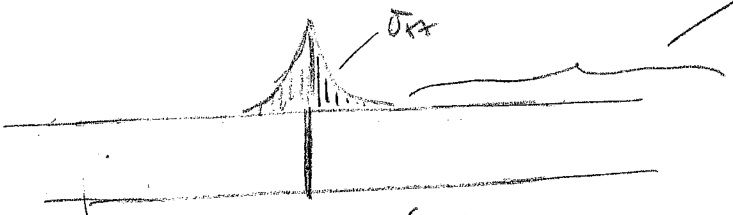


No rib $\sigma_{xx} = \sigma_{yy} = \sigma_{xy} = 0$



Rib $\Rightarrow$ $\sigma_{xy} \neq 0$, $\sigma_{yy} \neq 0$, $\sigma_{xx} \neq 0$

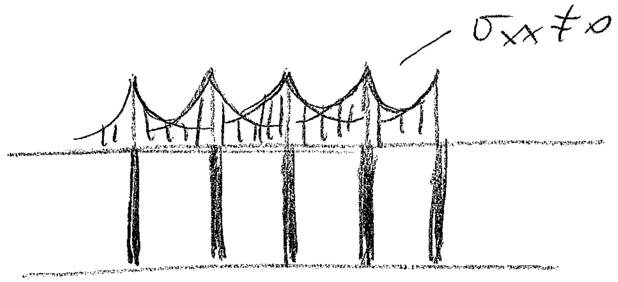
These effects fall outside the DSV beam model. They can be seen as local effects, which damp out moving away from the rib.



DSV hypotheses are here restored

Example: value of σ_{xx} moving away from the rib

Clearly, if the rib density is relatively high (as it often the case) the DSV assumptions may become questionable: the local effects do not have distance enough to damp out as those of the successive rib are immediately present.



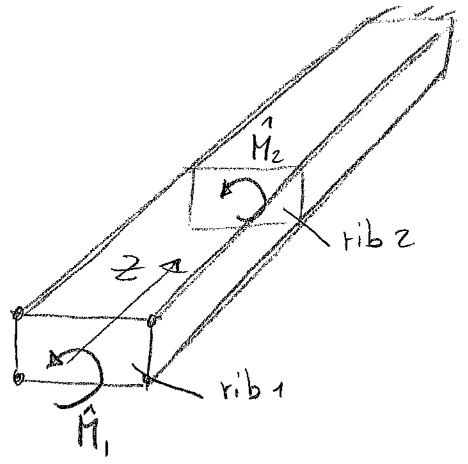
beam with closely spaced ribs

The high density of ribs does not allow σ_{xx}, σ_{yy} and σ_{xy} to reach zero value

From a structural behaviour standpoint, this means that the "real" structure, in comparison to the "ideal" DSV one, will be much stiffer: this due to the local stiffening of ribs which is not accounted for in the DSV model.

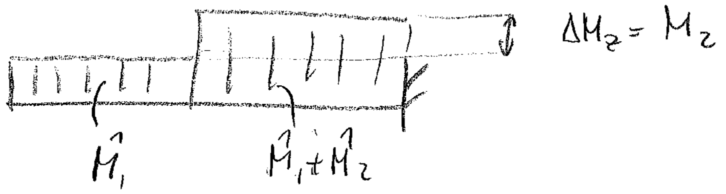
Loads on a rib

Consider a generic thin-walled beam, which is loaded as illustrated in the figure

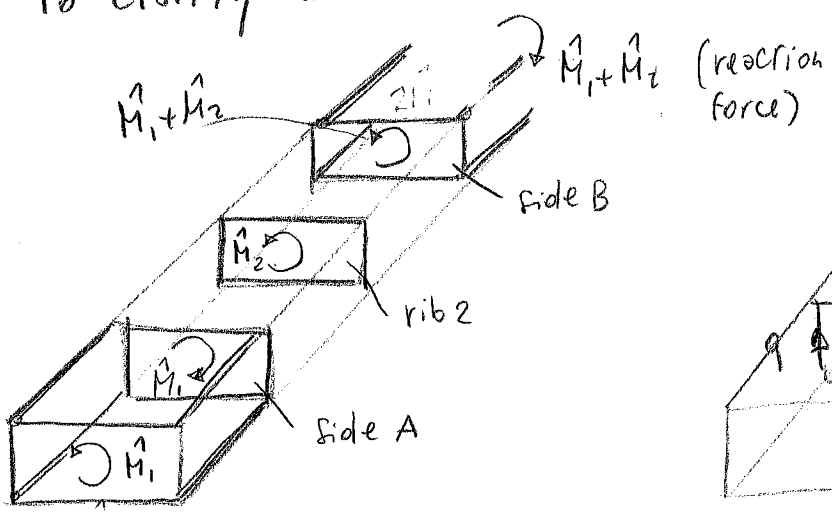


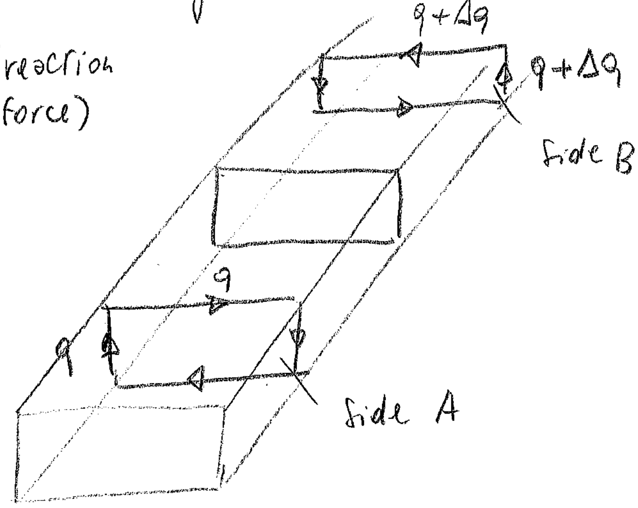
Both the torsional moments $\hat{M}$ are introduced in correspondence of the ribs

The plot of the internal torsional moment is then:



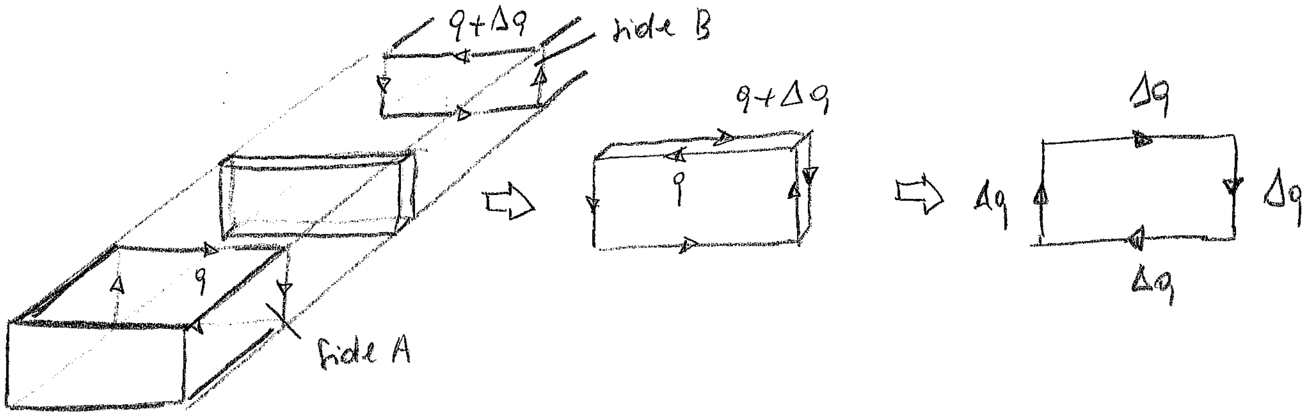
Extract now the middle rib from the structure, in order to clarify which are the loads acting on it.



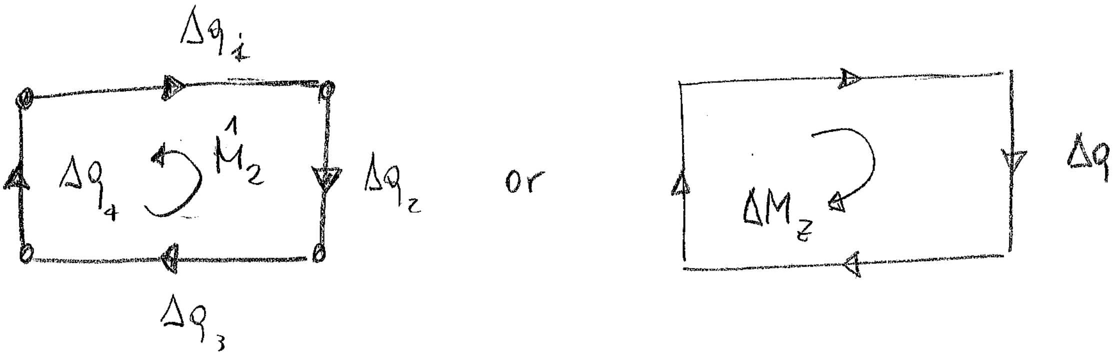


The shear flows on side A and B will be directed as reported in the figure

Accordingly the rib is subjected to



As observed the shear flows acting on the rib are:



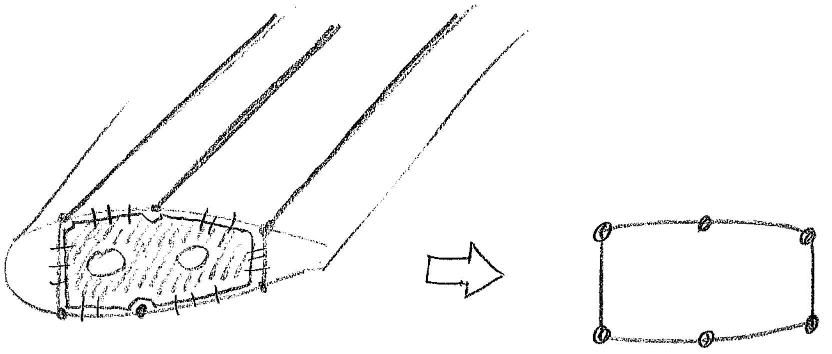
This means that the shear flows acting on the rib are

1. in equibrium with the load introduced
2. equivalent to the jump of internal action in correspondence of the rib

When a plot of the internal actions is available, it is then possible to consider the jumps of internal actions to determine the loads introduced in the rib.

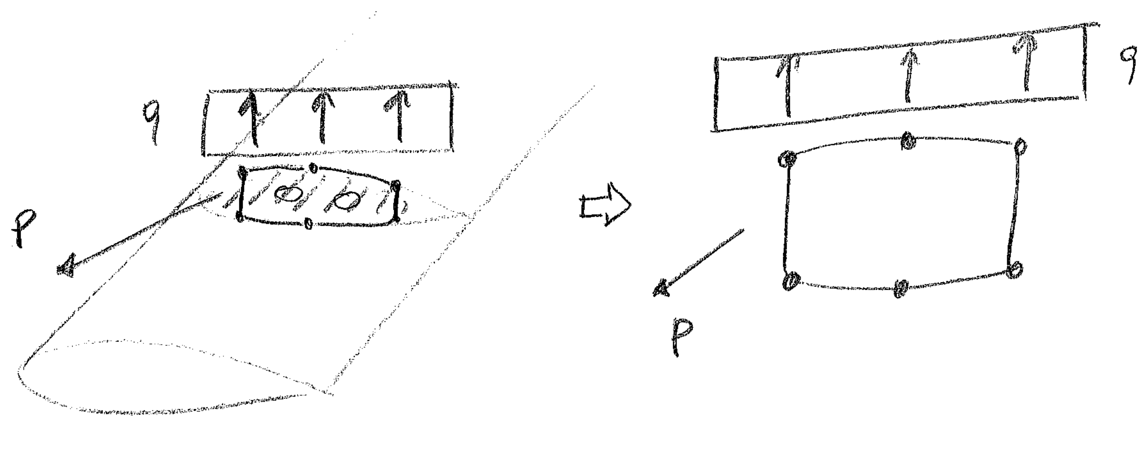
The procedure is then the following:

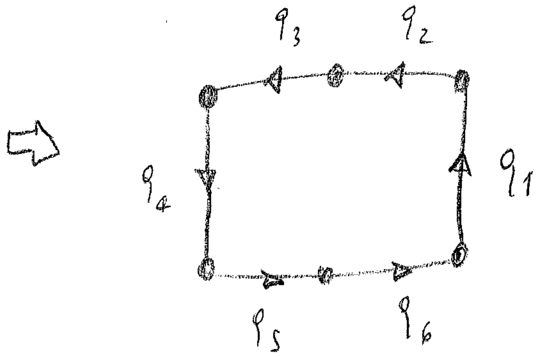
1. Consider the boom section where the rib is attached



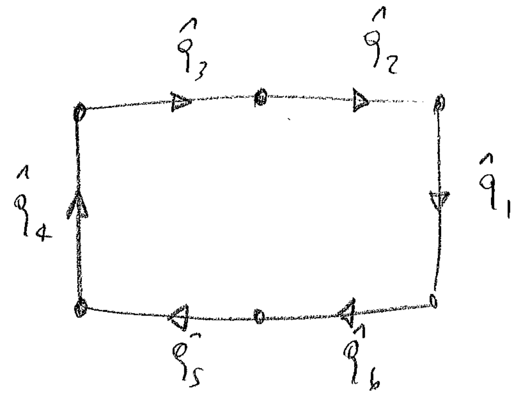
Section where the rib is connected

2. Apply the loads acting on the rib (e.g. concentrated loads, distributed forces ...) and determine the <u>equivalent</u> shear flows



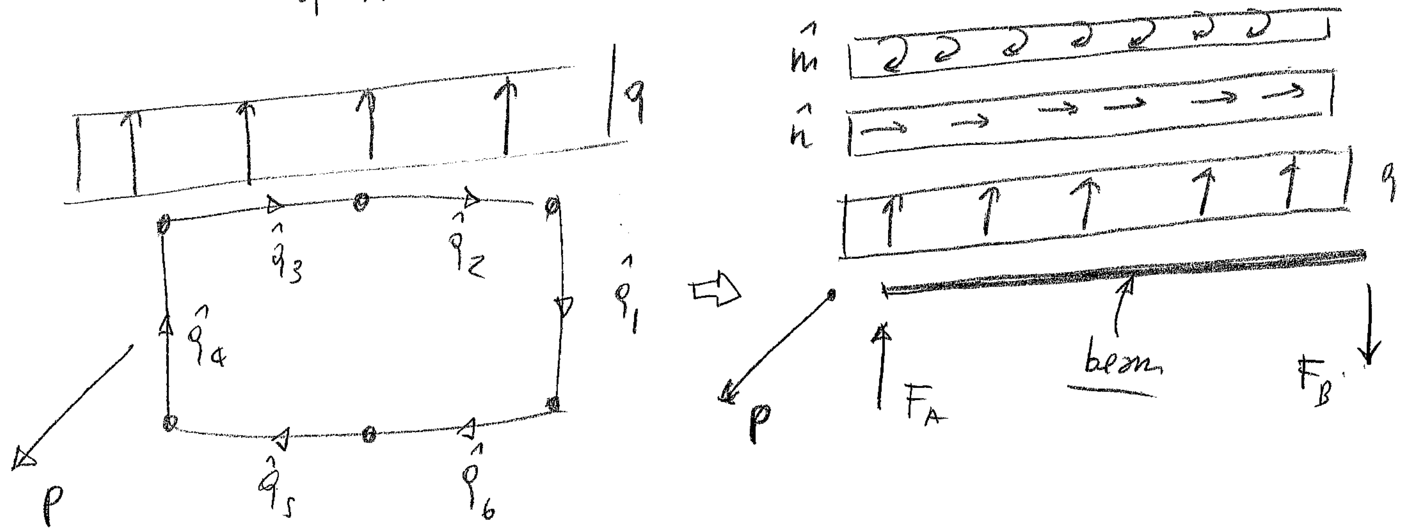


3. As seen, the shear flows on the rib side are in <u>equilibrium</u> with the external loads (and <u>equivalent</u>). It is then necessary to reverse the shear flows obtained at step 2.



4. A beam model can be adopted as a preliminary way for analyzing the rib. To this aim consider

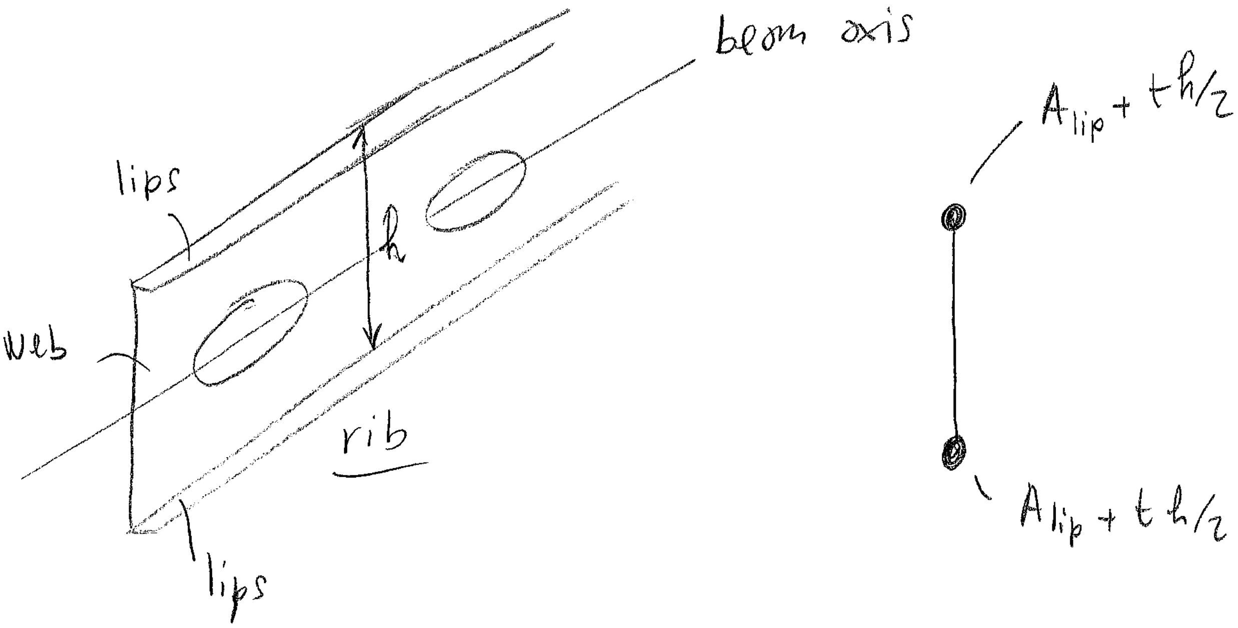
1. the external loads applied to the rib
2. the equilibrating shear flows obtained at step 3.



where: $\hat{m}, \hat{n}$ are the distributed loads obtained from the shear flows

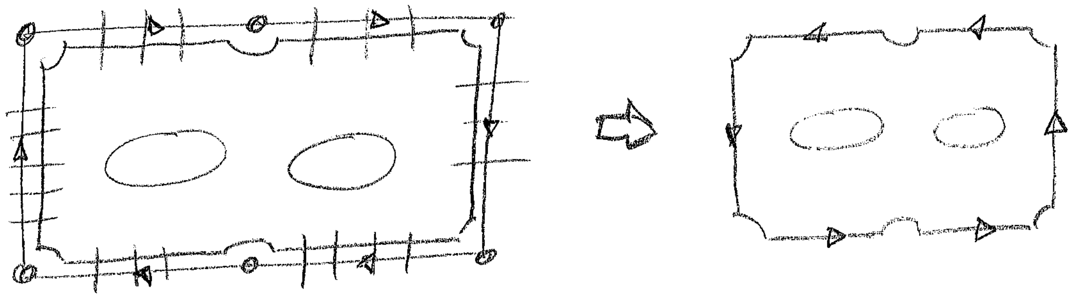
F_A, F_B are the force resultants along the webs

5. Once the internal actions on the beam model of the rib are available, it is possible to evaluate the stresses.
To this aim, dependently on the kind of rib under investigation, it is possible to adopt an approximate scheme for evaluating the shear stresses based on the semi-monocoque scheme.



Remarks

It is worth highlighting that the analysis of the rib is conducted by evaluating the section where rib is attached. This is because the two parts are connected with junctions and so there is an exchange of shear flows along the junction lines

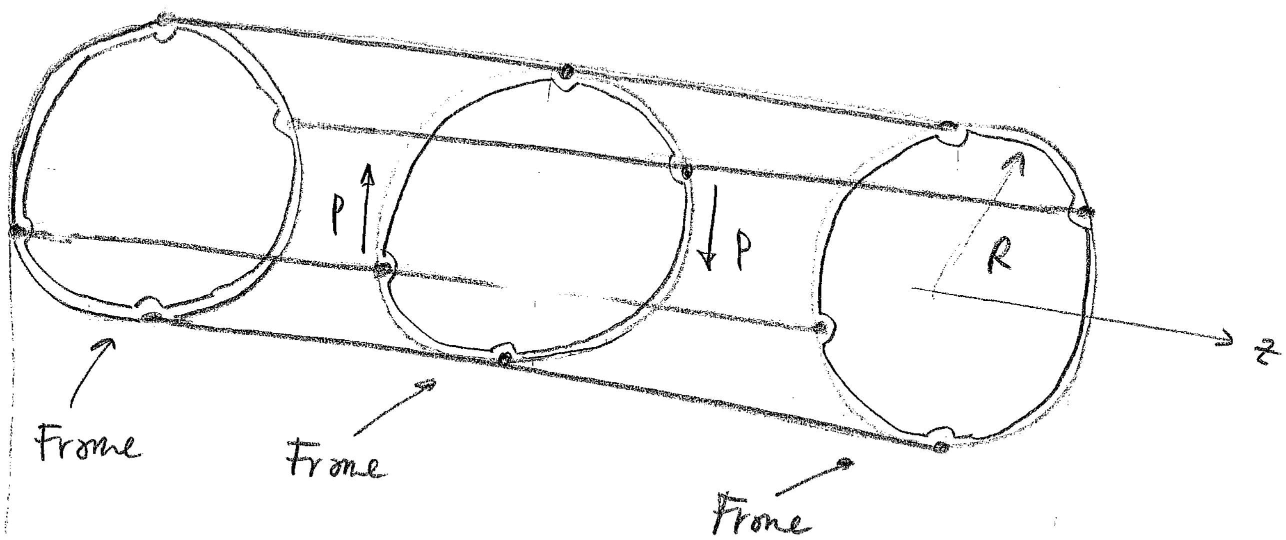


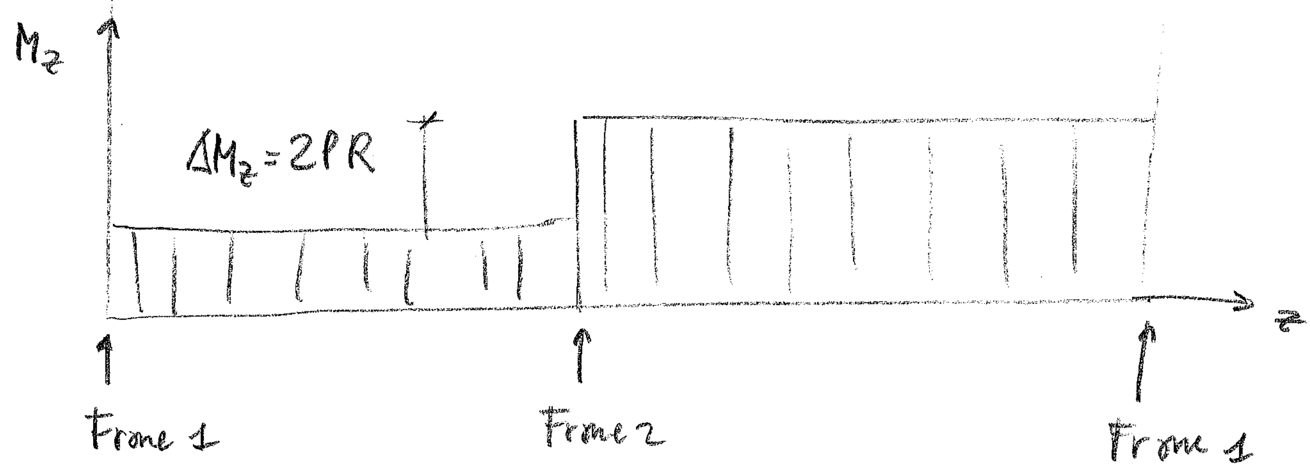
- Analysis of frames

The structural role of frames is corresponding to the role played by ribs in the context of wing-like structures. Frames are commonly referred to cylindrical structures such as fuselages or cylindrical parts of space launchers.

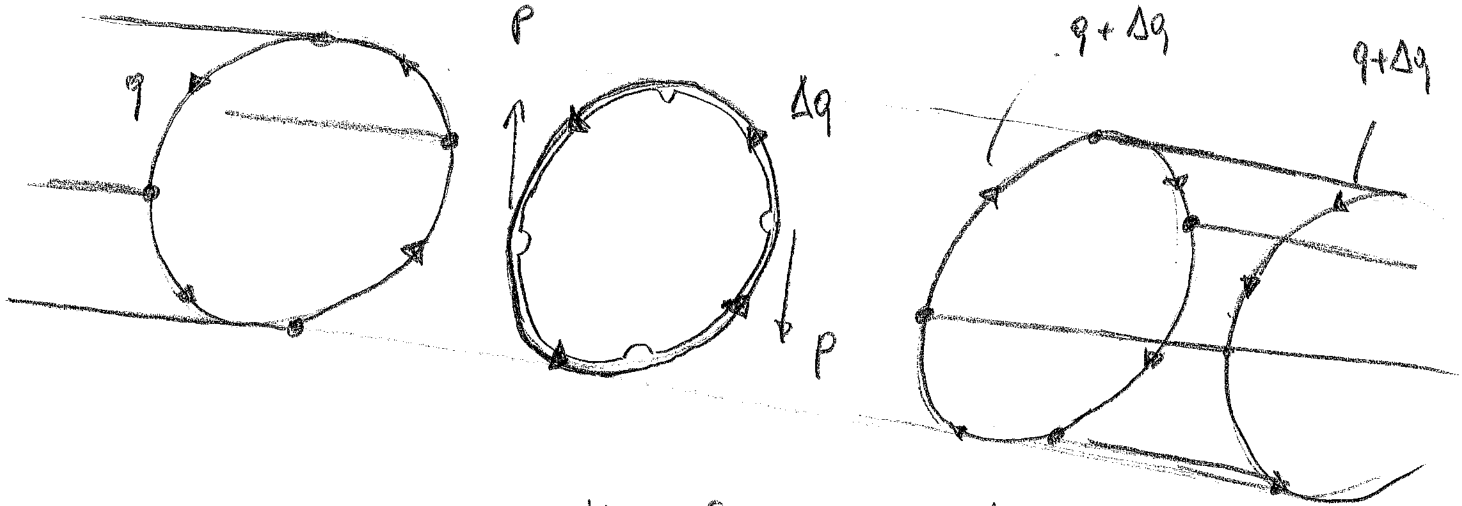
The analysis procedure is then analogous to the procedure outlined for ribs, as the load introduction and load transfer mechanisms happen in a similar manner.

Consider, as an example, a portion of a cylindrical structure longitudinally stiffened by the stringers and transversally by the frames





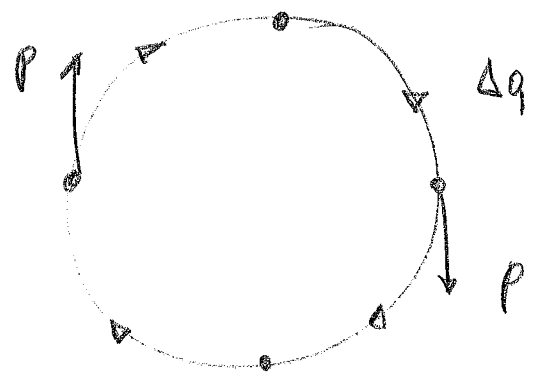
By "extracting" the second frame from the structure, it is highlighted how the internal shearing stresses are exchanged



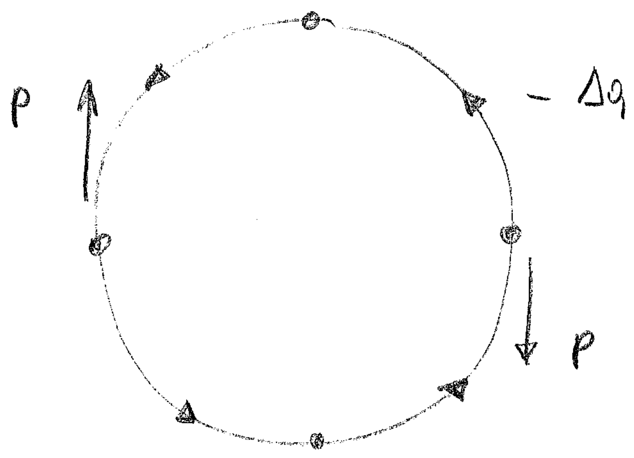
As in the case of ribs, the frame is subjected to the applied loads which are in equilibrium with the circulating shear flows. The shear flows in the frame can thus be seen as equivalent to the jump of internal action.

The analysis procedure is then summarized as:

1. Determine the equivalent shear flows

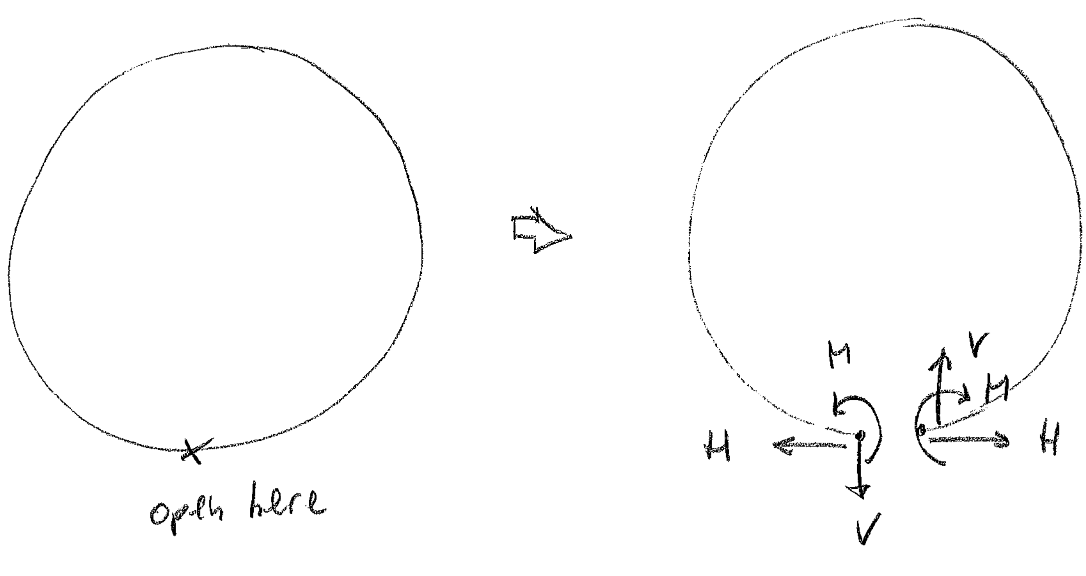


2. Obtain the equilibrium shear flows by reversing the equivalent ones



3. Determine the frame internal actions by considering the frame subject to the self-equilibrated load system obtained at step 2.

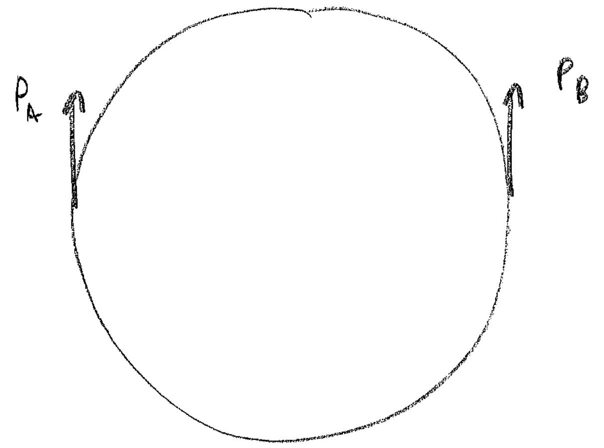
It is worth noting that a frame is a closed structure which is, in the context of a plane analysis, a 3 times statically indetermined structure.
The evaluation of the internal action requires then the initial evaluation of the three unknown internal actions associated with a cut in a generic point of the frame



H, V, M to be determined

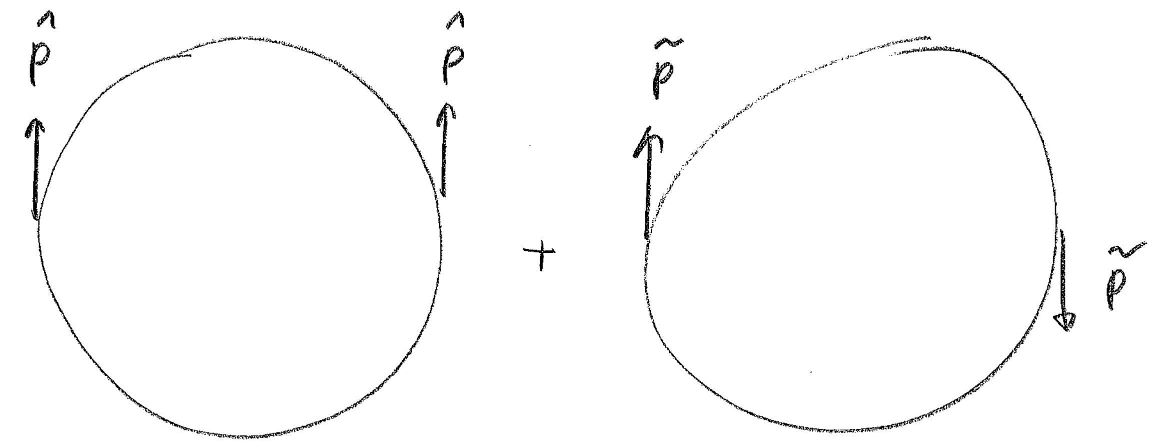
A simple strategy for simplifying the calculations consists in exploiting the symmetry of the structure and separating the load set into the <u>sum of</u> a <u>symmetric</u> contribution and an <u>anti-symmetric</u> one

Consider the generic case here below



with $P_A \neq P_B$

the system of load can be seen as

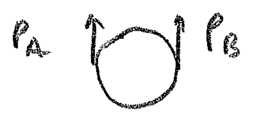


$$\hat{P} = \frac{P_A + P_B}{2} \qquad \tilde{P} = \frac{P_A - P_B}{2}$$

<u>symmetric loading condition</u>   <u>anti-symmetric loading condition</u>

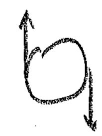
(clearly, this separation can be done for <u>any</u> load set)

The advantage of this separation consists in the possibility of solving two problems which are simpler in comparison to the original one.



Initial problem
(3 times statically indetermined)

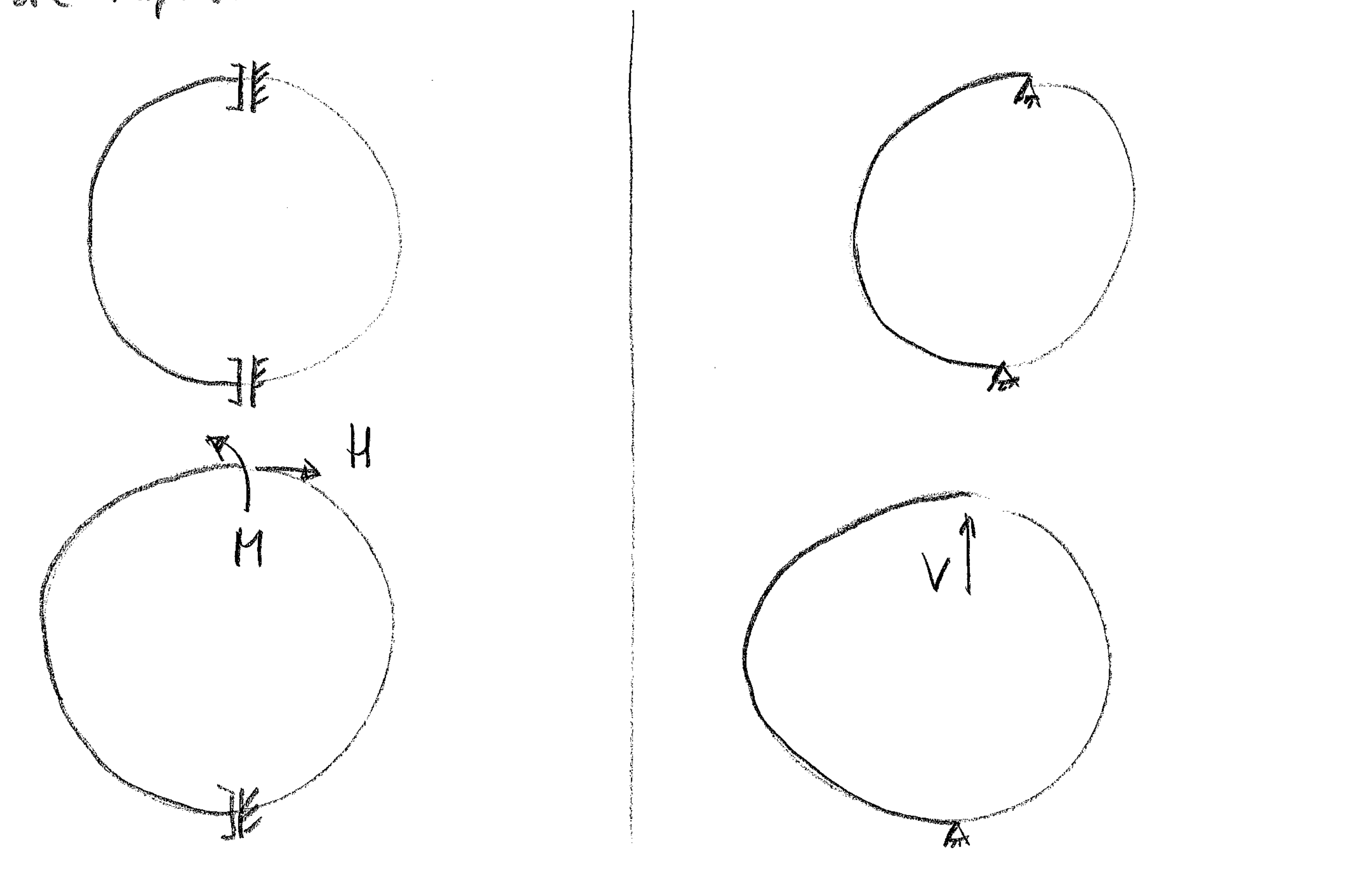
Symmetric load
(2 times statically inder)
+



Anti-symmetric load
(1 time statically inder)

Indeed the frames are generally symmetric structures that
a. loaded symmetrically respond symmetrically
b. loaded anti-symmetrically " anti-symmetrically

More specifically the symmetry / anti-symmetry conditions are imposed as:

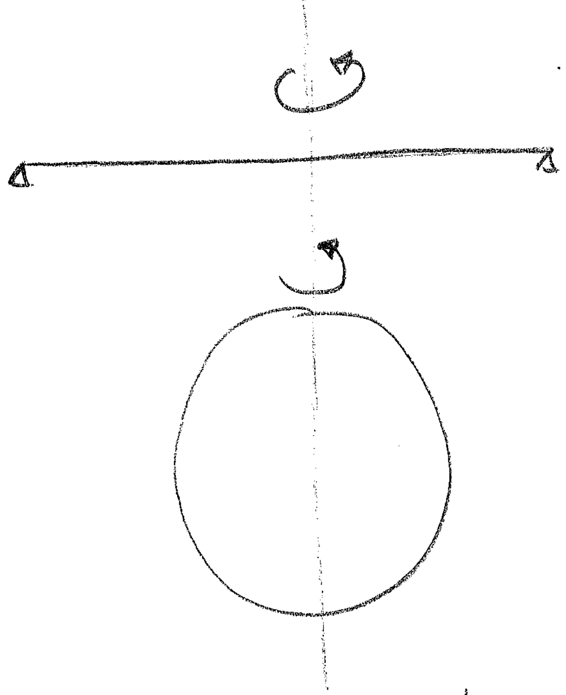


Symmetries and anti-symmetries

As in the case of frames, the possibility of exploiting symmetries in structures is generally a good mean for simplifying the calculations.

A common case is given by plane symmetries, which are characterized by one (or more) axis which can be considered for rotating the structure of π rad without modifying the structure itself.

As examples consider



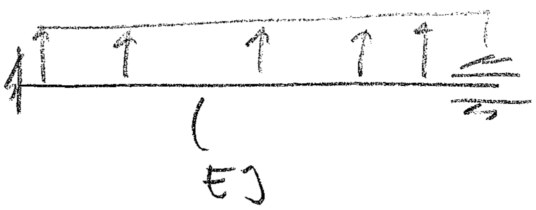
After the rotation takes place, the structure is still equal to its original configuration

It is worth highlighting that symmetry consideration involve

1. the geometry
2. the material and, to a more general extent, the stiffnesses
3. the boundary conditions

Whenever one or more of these conditions does not respect symmetry requirements, the analysis cannot be simplified by exploiting any kind of symmetry consideration.

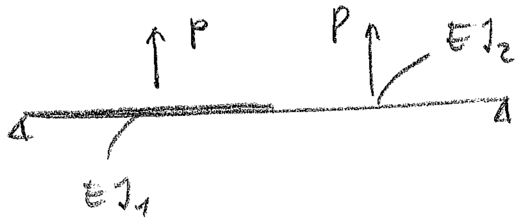
Example 1.



- The structure is geometrically and materially symmetric, but boundary conditions not.

The problem cannot be simplified.

Example 2.



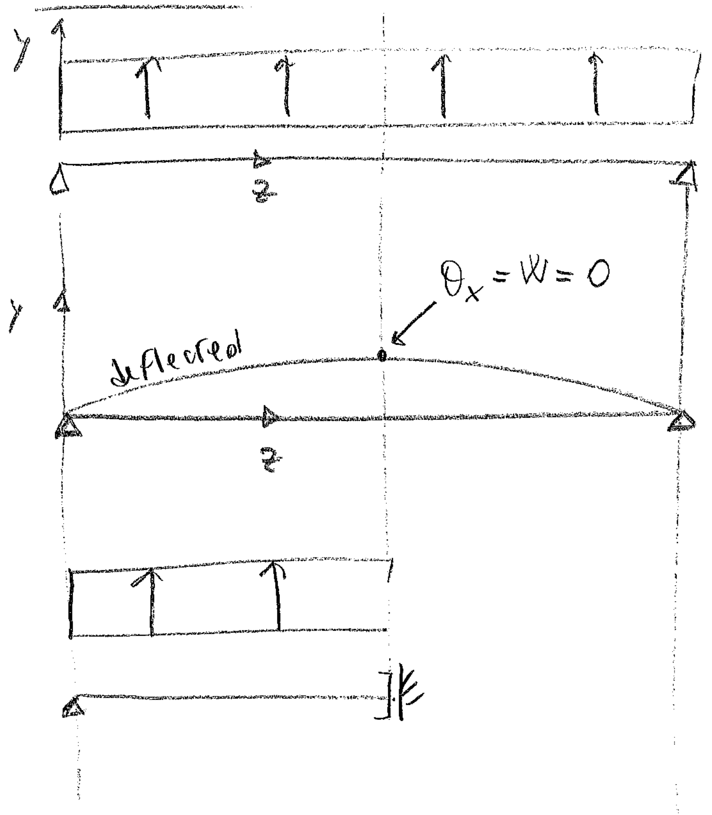
- the structure is symmetric with respect to geometry and boundary conditions, but not in terms of stiffnesses

The problem cannot be simplified

On the contrary the satisfaction of the three previous requirements allow to simplify the problem. In particular

1. Symmetric structure loaded symmetrically.
   $\longrightarrow$ symmetric response
2. Symmetric structure loaded anti-symmetrically
   $\longrightarrow$ anti-symmetric response

Example 1

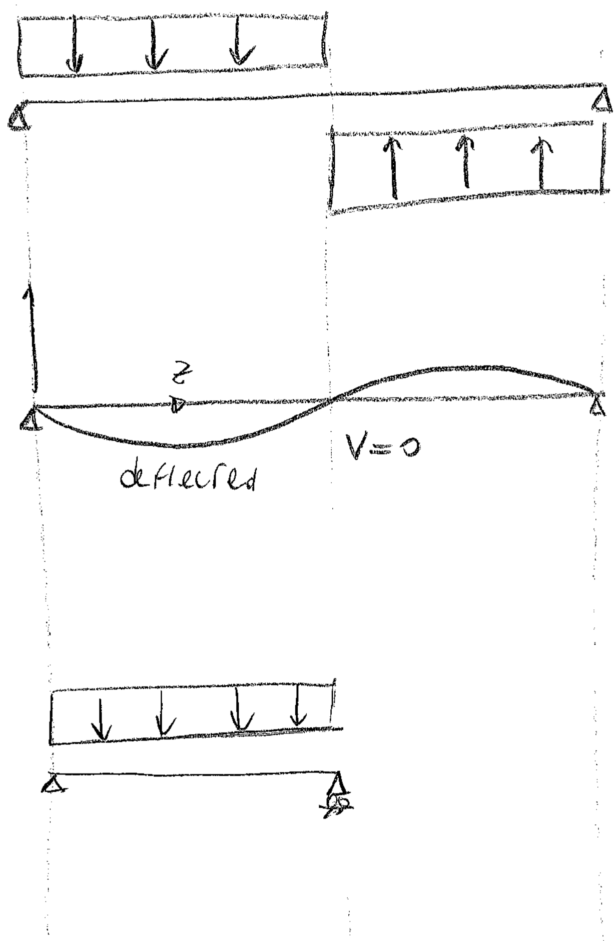


- Symmetric structure
- Symmetric load

It is intuitive to understand that the symmetry of the response implies that $\theta_x = W = 0$

The problem can be simplified by introducing a constraint imposing the symmetry conditions

Example 2



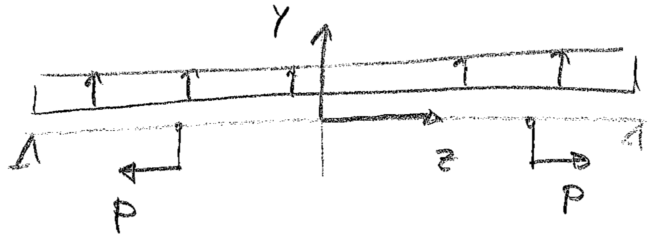
- Symmetric structure
- Anti-symmetric load

In this case the anti-symmetry of the response determines a null bending deflection in the middle portion

The problem can then be simplified with a simply-support condition in the middle.

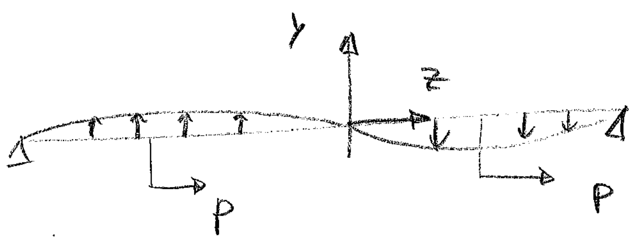
The 2D planar case can then be formalized as

1. <u>Symmetric structure with symmetric load</u>



$$v(z) = v(-z) \Rightarrow v(z) \neq 0$$
$$w(z) = -w(-z) \Rightarrow w(0) = 0$$
$$\theta_x(z) = -\theta_x(-z) \Rightarrow \theta_x(0) = 0$$

2. <u>Symmetric structure with anti-symmetric load</u>

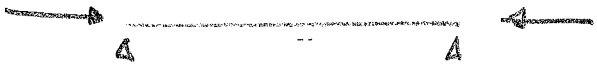


$$v(z) = -v(-z) \Rightarrow v(0) = 0$$
$$w(z) = w(-z) \Rightarrow w(0) \neq 0$$
$$\theta_x(z) = \theta_x(-z) \Rightarrow \theta_x(0) \neq 0$$

An important remark

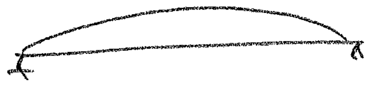
The application of symmetry/anti-symmetry conditions should be carefully conducted out of linearity assumptions.
A typical example is given by the buckling analysis where it is intuitive to notice that a symmetric structure loaded symmetrically may respond anti-symmetrically

With this regard consider a simply-supported beam loaded in compression

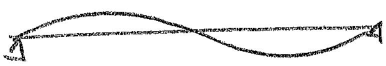


Symmetric structure
Symmetric load

The buckling modes are:

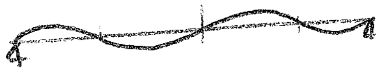


symmetric 1st mode



anti-symm. 2nd mode

symm. 3rd mode

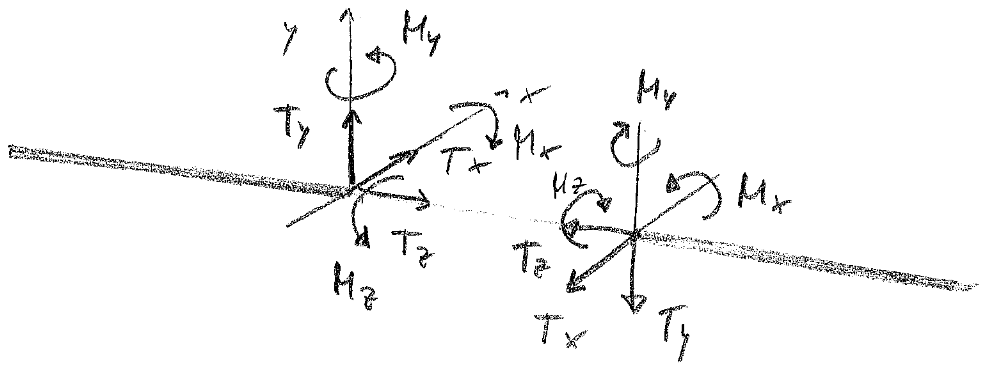


anti-symm 4th mode

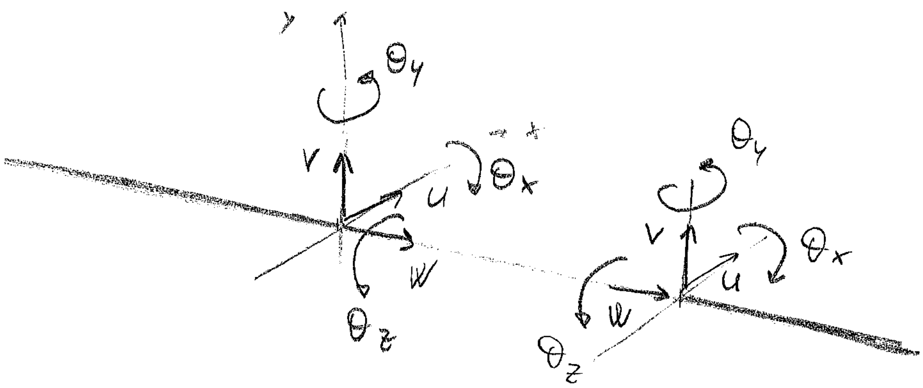
It is then clear that the application of symmetry conditions would determine the <u>loss of all the even buckling modes</u>.

- Symmetries and anti-symmetries in 3D beams

The discussion has been restricted to the 2D case so far. It is now possible to extend the same ideas to the case of beams in a three-dimensional space.
In this case the displacement of a generic point is fully characterized by three displacement components and three rotations. Similarly the exchange of internal forces is described in terms of three forces and three moments



forces and moments



displacements and rotations

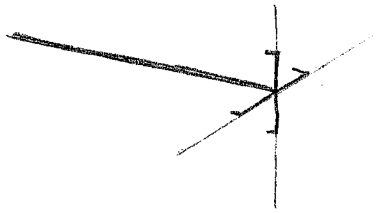
Understanding symmetry and anti-symmetry condition can be simpler if the analysis is referred to the displacements and rotations. In particular:

$$\left.\begin{array}{l} u \neq 0 \\ v \neq 0 \\ w = 0 \\ \theta_x = 0 \\ \theta_y = 0 \\ \theta_z \neq 0 \end{array}\right\} \text{symmetric response} \qquad \left.\begin{array}{l} u = 0 \\ v = 0 \\ w \neq 0 \\ \theta_x \neq 0 \\ \theta_y \neq 0 \\ \theta_z = 0 \end{array}\right\} \text{anti-symmetric response}$$

The null displacement/rotation components characterizing the symmetric/anti-symmetric behaviour can be imposed with proper boundary conditions. It follows that a reaction force/moment exists for any component set to zero

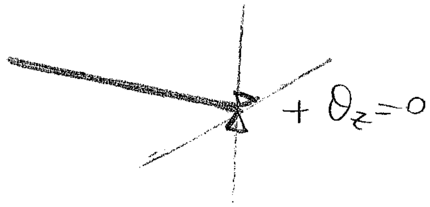
$w = 0 \Rightarrow R_z \neq 0$
$\theta_x = 0 \Rightarrow M_x \neq 0$
$\theta_y = 0 \Rightarrow M_y \neq 0$

symmetric

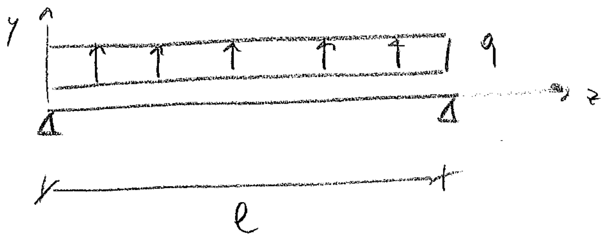


$u = 0 \Rightarrow R_x \neq 0$
$v = 0 \Rightarrow R_y \neq 0$
$\theta_z = 0 \Rightarrow M_z \neq 0$

anti-symmetric

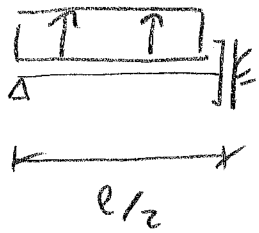


Example



Evaluate the internal actions

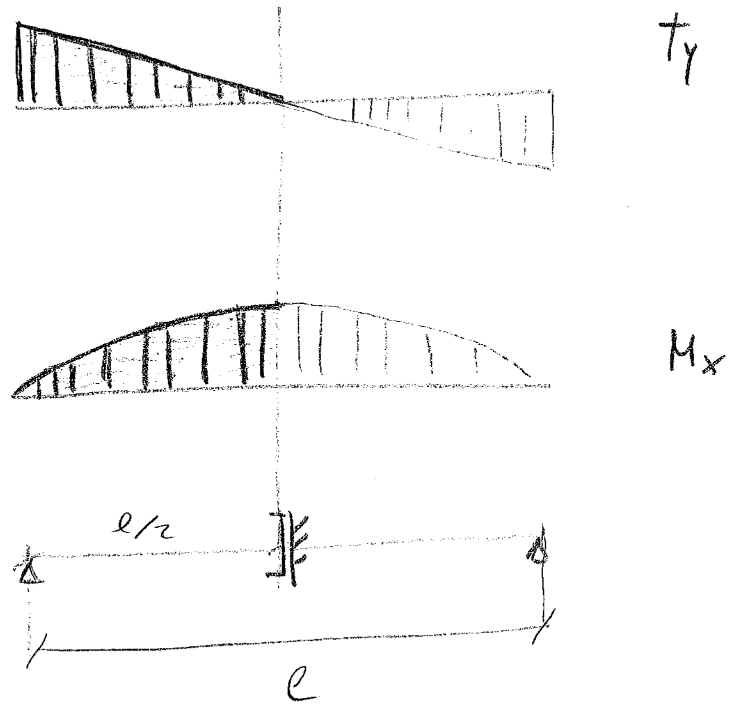
The problem can be analyzed considering half of the structure subjected to symmetry constraints



$$T_y(z) = -qz + ql/2$$

$$M_x(z) = -q\frac{z^2}{2} + q\frac{l}{2}z$$

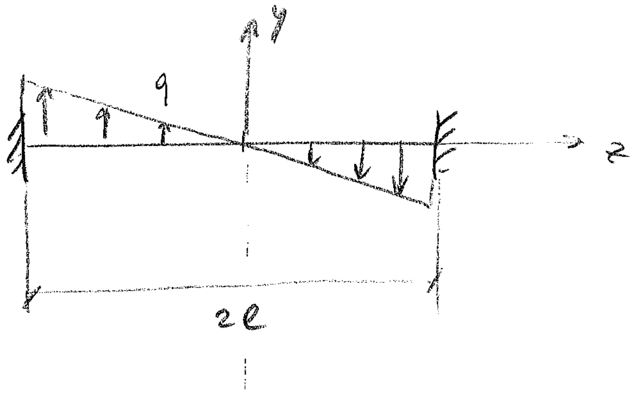
The internal actions are then:



Note: v is symmetric $\Rightarrow$ T_y is anti-symmetric

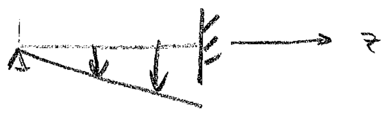
θ_x is anti-symmetric $\Rightarrow$ M_x is symmetric

Example



Evaluate the internal actions by exploiting the symmetries / anti symmetries

$$q(z) = -\frac{\bar{q}}{l} z$$

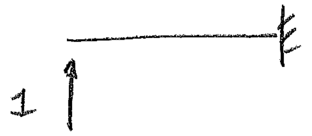


Real

$$T_y(z) = \frac{\bar{q} z^2}{2l} - Y$$

$$M_x(z) = \frac{\bar{q} z^3}{6l} - Yz$$

Dummy



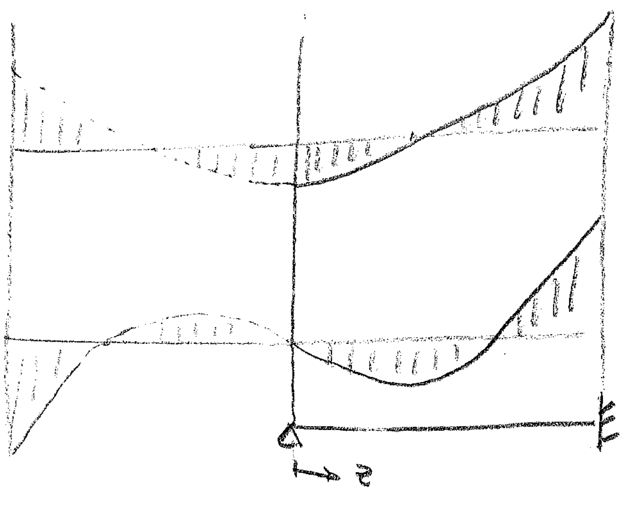
$$\delta T_y(z) = -1$$

$$\delta M_x(z) = -z$$

PCVW:

$$\int_0^l \delta M_x \frac{M_x}{EJ} dz = 0 \quad \Rightarrow \quad Y = \frac{\bar{q} l}{10}$$

Internal actions



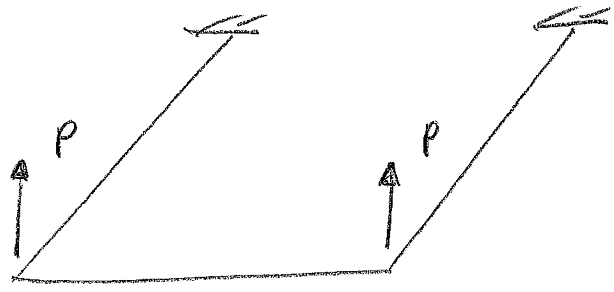
$$T_y = \frac{\bar{q} z^2}{2l} - \frac{\bar{q} l}{10}$$

$$M_x = \frac{\bar{q} z^3}{6l} - \frac{\bar{q} l}{10} z$$

Note: V is anti-symmetric $\Rightarrow$ T_y is symmetric

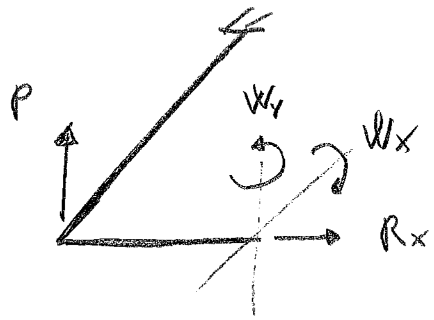
θ_x is symmetric $\Rightarrow$ M_x is anti-symmetric

Example



The structure is symmetric with a symmetric load set

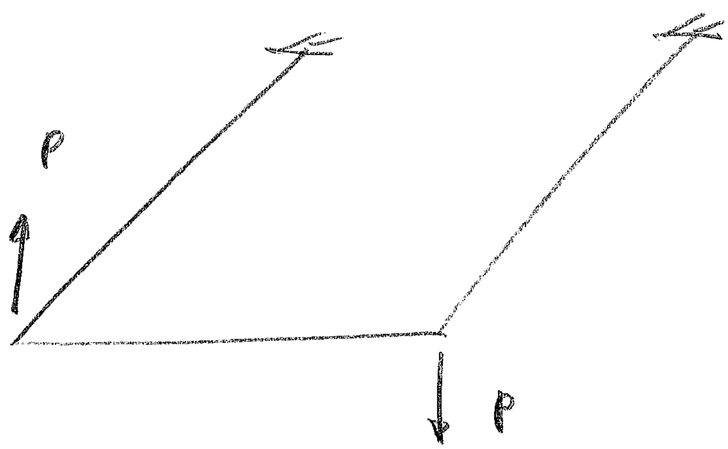
The problem can be simplified as



It is straightforward to notice that $W_y = W_x = R_x = 0$ for this specific problem (apply the PCVW as an exercise for verifying this result)

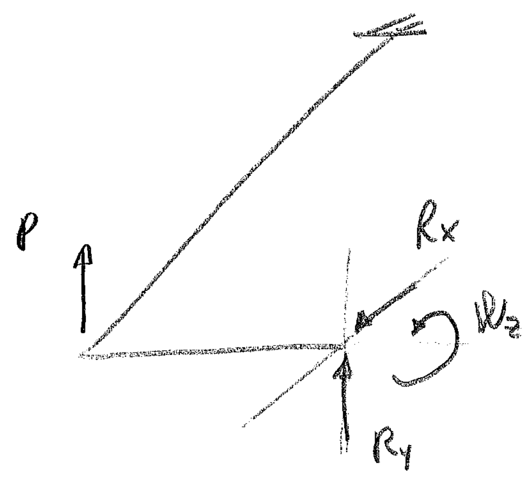
W_y, W_x, R_x reaction forces due to symmetry constraints

Example



The structure is symmetric with an anti-symmetric load set

The anti-symmetry condition can be imposed as:



The three unknown reactions R_x, R_y and W_y can be obtained by applying the PCVW three times