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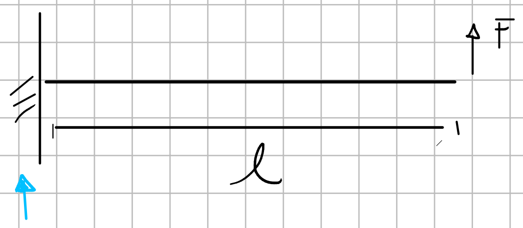
EXERCISE SESSION 1 – 16/09/2022

Isostatic beam systems

Ex 1

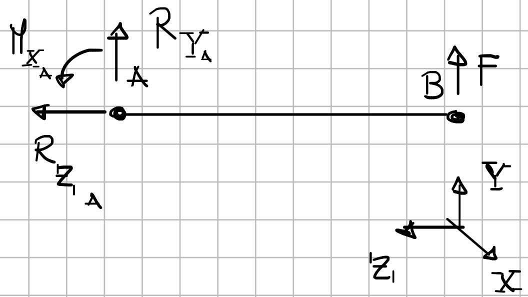


3 DoF CONSTRAINED

FIND VALUES OF
- CONSTRAINT REACTION FORCES
- INTERNAL ACTIONS

3 DoF — HORIZONTAL, VERTICAL (TRANSLATIONS), ROTATION
3 EQ EQS

1) REACTION FORCES



3 UNKNOWN : R_{Y_A} R_{Z_A} M_{X_A}

Z : $R_{Z_A} = 0$ $\qquad \Sigma F_Z = 0$

Y : $R_{Y_A} + F = 0$ $\qquad \Sigma F_Y = 0$

X_A : $M_{X_A} + F \cdot \ell = 0$ $\qquad \Sigma M_X = 0$ IN POINT A

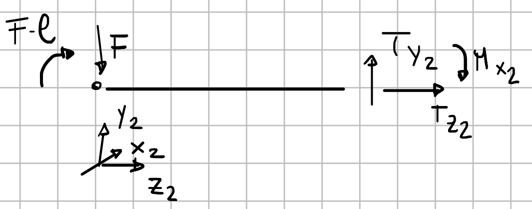
$$\boxed{\begin{cases} R_{Z_A} = 0 \\ R_{Y_A} = -F \\ M_{X_A} = -F \cdot \ell \end{cases}}$$

2) INTERNAL ACTIONS

$$\begin{cases} T_{y_1}(z_1) = -F \\ T_{z_1}(z_1) = 0 \\ M_{x_1}(z_1) = -F \cdot z_1 \end{cases}$$

THESE IN GENERAL TERMS ARE FUNCTION OF z_1

FROM CONSTRAINED SIDE:

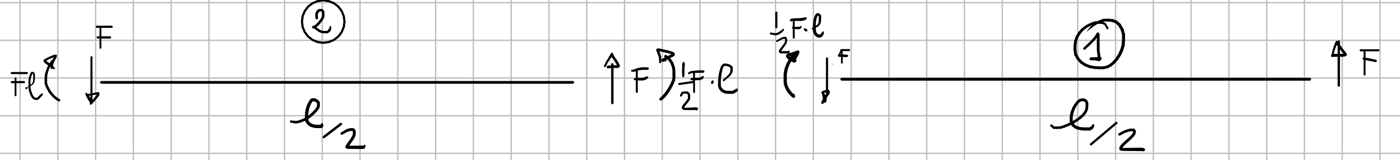


EQ EQS

$$\begin{cases} z_2: & T_{z_2} = 0 \\ y_2: & T_{y_2} = F \\ \text{ROT}: x_2 & M_{x_2} + F\ell - F \cdot z_2 = 0 \; \circledast \end{cases}$$

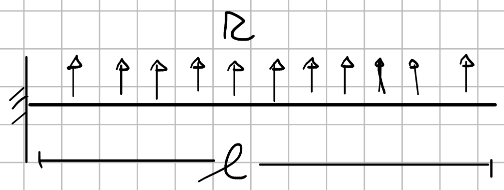
$$\circledast \quad M_{x_2} = F \cdot z_2 - F \cdot \ell$$

If we compute internal actions at $z_1 = \ell/2$ $z_2 = \ell/2$



THE FACES ARE IN EQUILIBRIUM WHEN WE OPEN A BEAM, EACH PART IS IN EQUILIBRIUM

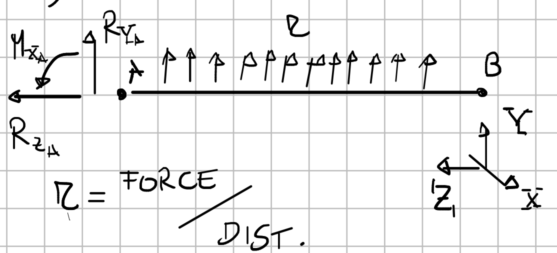
Ex 2



FIND:

- REACTION FORCES
- INTERNAL ACTIONS

1) FIND REACTION FORCES



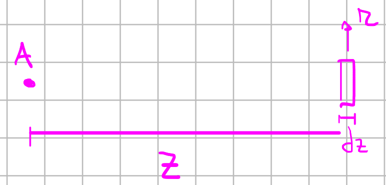
$r = \text{FORCE}/\text{DIST.}$

EQ. EQS

$$\begin{cases} R_{z_A} = 0 \\ R_{Y_A} + r \cdot l = 0 \\ M_{x_A} + \underbrace{r \cdot l}_{} \cdot \left[\frac{l}{2}\right]_{\to \text{ARM}} = 0 \end{cases}$$

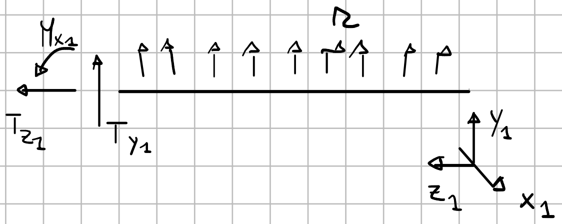
RESULTANT OF DISTR. FORCE $\quad R = r \cdot l$

$$M_R = \int_0^l r \cdot z \cdot dz$$



$$\boxed{\begin{cases} R_{z_A} = 0 \\ R_{Y_A} = -r \cdot l \\ M_{x_A} = -\frac{1}{2} r l^2 \end{cases}}$$

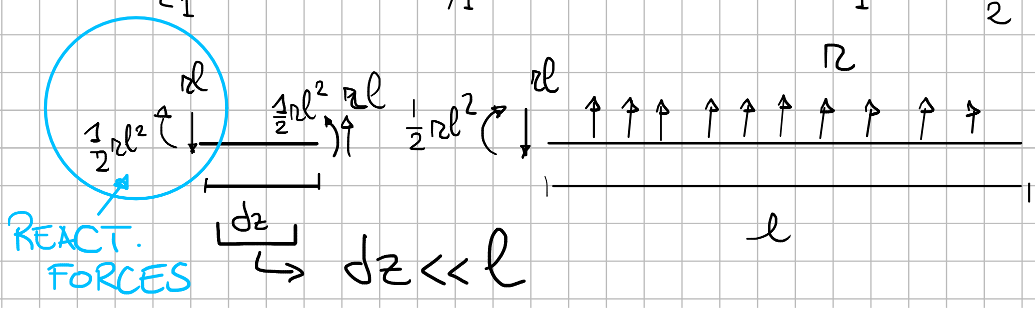
2) COMPUTE INTERNAL ACTIONS



$$\begin{array}{ll} z_1: & \\ y_1: & \\ \text{ROT } x_1: & \end{array} \begin{cases} T_{z_1}(z_1) = 0 \\ T_{y_1}(z_1) = -r z_1 \\ M_{x_1}(z_1) = -\frac{1}{2} r z_1^2 \end{cases}$$

If we substitute inside here $z_1 = l$

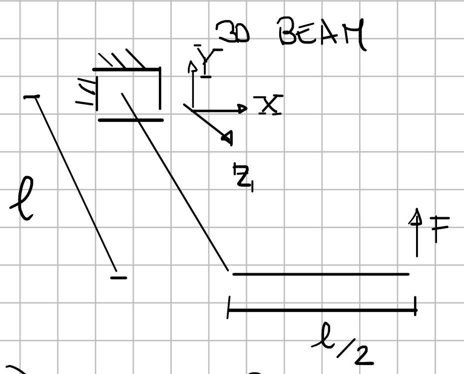
$T_{z_1} = 0 \qquad T_{y_1} = -rl \qquad M_{x_1} = -\frac{1}{2} r l^2$



$dz \ll l$

We obtain the reaction forces (and not something in eq. with them)

Ex 3

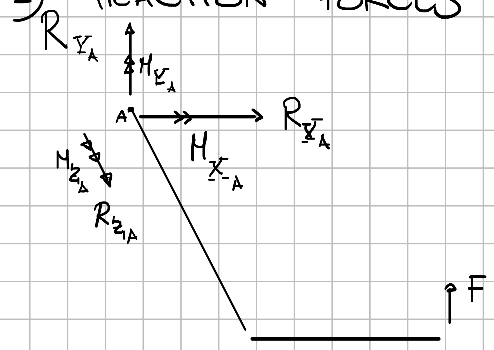


FIND :

- REACTION FORCES
- INTERNAL ACTIONS

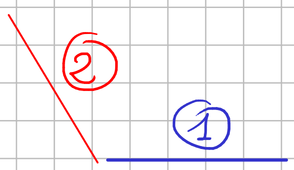
6 DOF   6 EQ. EQS'

1) REACTION FORCES

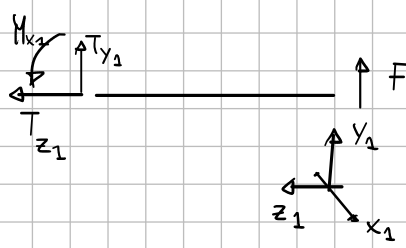


$$
\begin{cases}
\text{TRANSL. } X : & R_{X_A} = 0 \\
Y : & R_{Y_A} = -F \\
Z_1 : & R_{Z_{1_A}} = 0 \\
\text{ROT IN A } X : & M_{X_A} - F \cdot \ell = 0 \rightarrow M_{X_A} = F \cdot \ell \\
Y : & M_{Y_A} = 0 \\
Z_1 : & M_{z_A} + F \cdot \frac{\ell}{2} = 0 \rightarrow M_{z_A} = -\frac{1}{2} F \ell
\end{cases}
$$

2) INTERNAL ACTIONS



PART 1 :



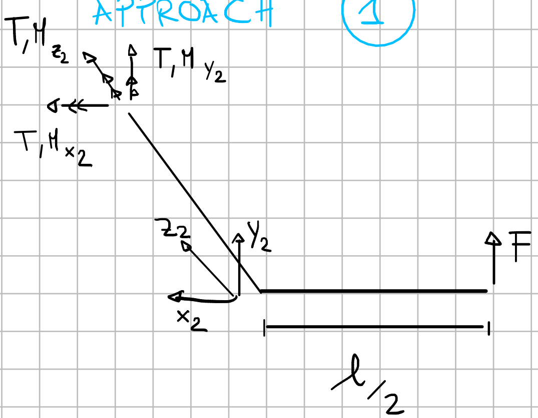
$$T_{z_1}(z_1) = 0$$

$$T_{y_1}(z_1) = -F$$

$$M_{x_1}(z_1) = -F \cdot z_1$$

PART 2

APPROACH 1

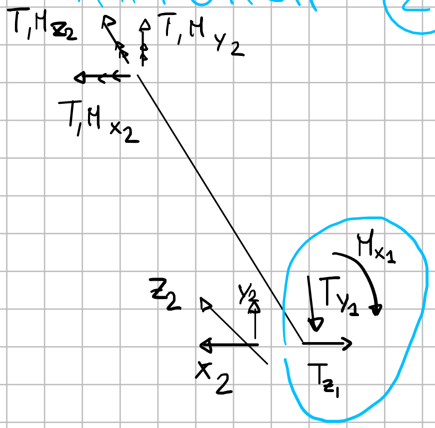


6 EQ. EQS'

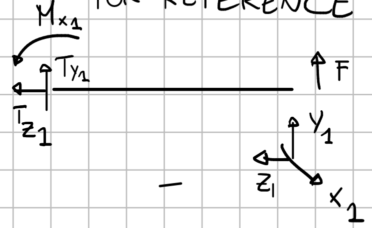
$$
\begin{cases}
T_{z_2} = 0 \\
T_{y_2} = -F \\
T_{x_2} = 0
\end{cases}
\qquad
\begin{cases}
M_{z_2} = F \cdot \ell/2 \\
M_{y_2} = 0 \\
M_{x_2} = -F \cdot z_2
\end{cases}
$$

3 EQ FOR TRANSL.   3 EQ FOR ROT.

APPROACH ②



FOR REFERENCE



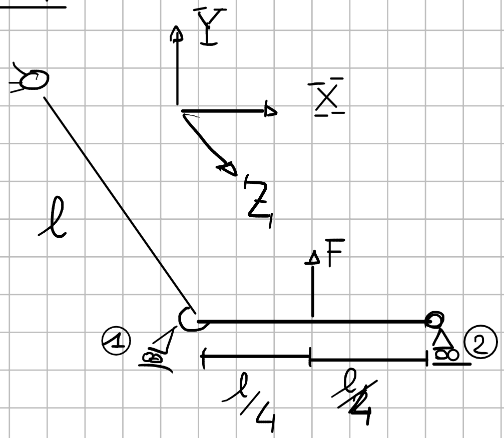
MUST BE EVALUATED IN $z_1 = \frac{\ell}{2}$

$$\begin{cases} T_{y_2}(z_2) = T_{y_1}\left(\ell/2\right) \\ T_{z_2}(z_2) = 0 \\ T_{x_2}(z_2) = 0 \end{cases} \qquad \begin{cases} M_{y_2}(z_2) = 0 \\ M_{z_2}(z_2) = -M_{x_1}\left(\frac{\ell}{2}\right) \\ M_{x_2}(z_2) = T_{y_1}\left(\frac{\ell}{2}\right) \cdot z_2 \end{cases}$$

PART ①

$$\begin{cases} T_{z_1}(z_1) = 0 \\ T_{y_1}(z_1) = -F \\ M_{x_1}(z_1) = -F \cdot z_1 \end{cases} \longrightarrow \begin{cases} T_{y_2}(z_2) = -F \\ M_{z_2}(z_2) = F \cdot \frac{\ell}{2} \\ M_{x_2}(z_2) = -F \cdot z_2 \end{cases}$$

Ex 4



THE BEAM IS CONTINUOS

SLIDE 1 DOES NOT BREAK THE BEAM!

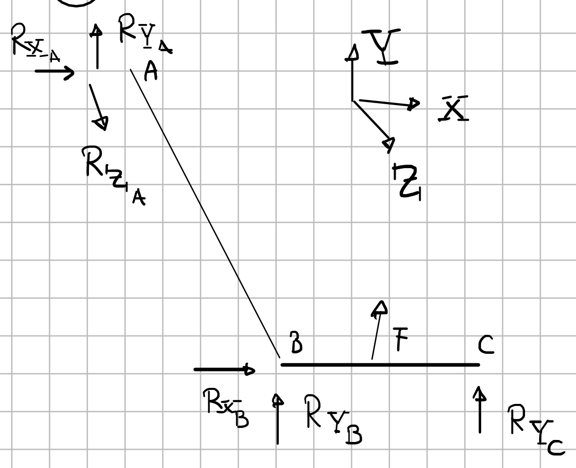
FIND :

- REACTION FORCES
- INTERNAL ACTIONS

1 LOCKS X, Y

2 LOCKS Y

1 REACTION FORCES



EQ EQS

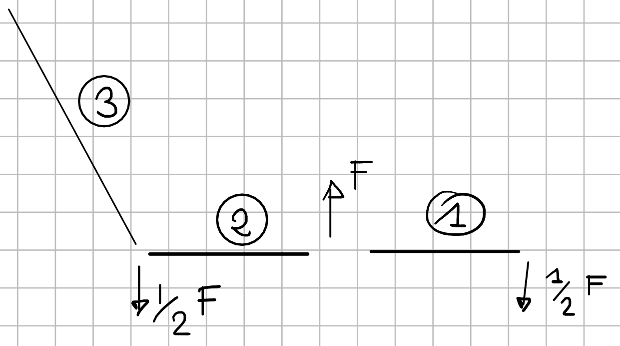
$$\begin{cases} 1\ X: & R_{X_A} + R_{X_B} = 0 \\ 2\ Y: & R_{Y_A} + R_{Y_B} + R_{Y_C} + F = 0 \\ 3\ Z: & R_{Z_A} = 0 \end{cases} \quad \text{TRANS.}$$

ROT. IN A

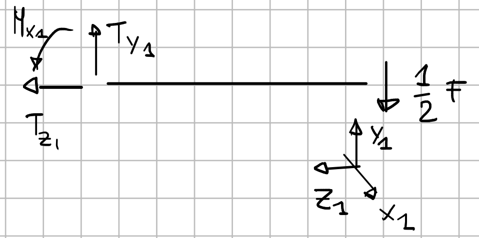
$$\begin{cases} 4\ X: & R_{Y_B} \cdot \ell + R_{Y_C} \cdot \ell + F \cdot \ell = 0 \\ 5\ Y: & R_{X_B} \cdot \ell = 0 \\ 6\ Z: & F \cdot \frac{\ell}{4} + R_{Y_C} \cdot \frac{\ell}{2} = 0 \end{cases}$$

$$\begin{cases} 6: & R_{Y_C} = -\frac{1}{2} F \\ 5 & R_{X_B} = 0 \\ 3 & R_{Z_A} = 0 \\ 4 & R_{Y_B} = -\frac{1}{2} F \\ 1 & R_{X_A} = 0 \\ 2 & R_{Y_A} = 0 \end{cases}$$

2) COMPUTE INT. ACTIONS

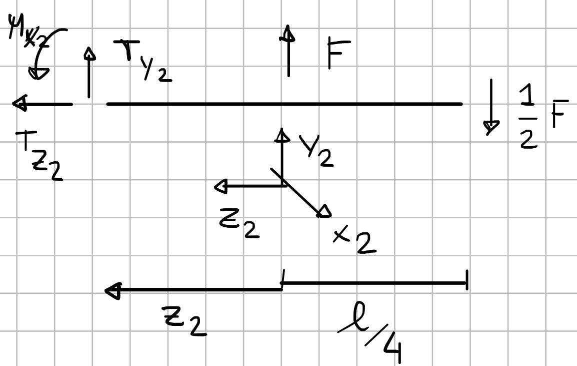


PART 1



$$\begin{cases} T_{z_1} = 0 \\ T_{y_1}(z_1) = \frac{1}{2} F \\ M_{x_1}(z_1) = \frac{1}{2} F \cdot z_1 \end{cases}$$

PART 2

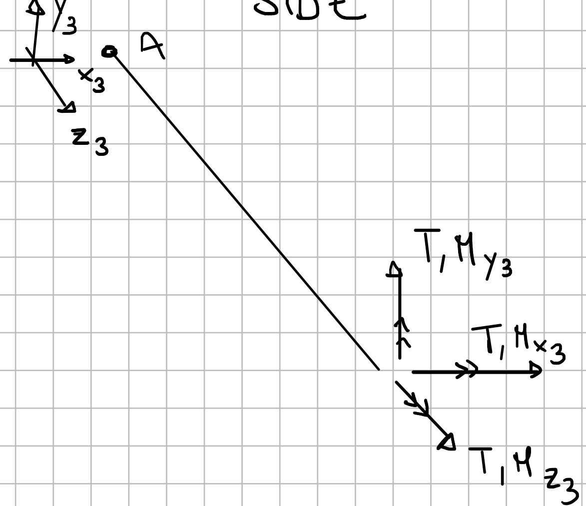


$$\begin{cases} T_{z_2}(z_2) = 0 \\ T_{y_2}(z_2) = -\frac{1}{2} F \\ M_{x_2}(z_2) = \frac{1}{2} F \left(\frac{l}{4} + z_2\right) - F z_2 \end{cases}$$

PART 3

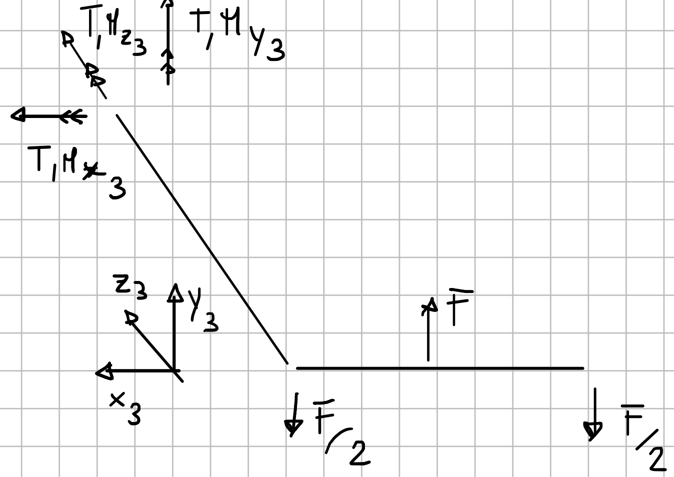
UNLOADED:

COMING FROM CONSTR. SIDE



NO LOADS TO BE EQUILIBRATED

COMING FROM PART 2



THE SYSTEM OF FORCE IS ALREADY EQUILIBRATED