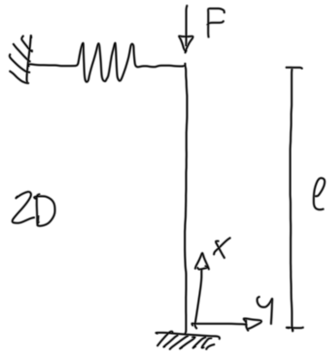
EX 13 – Displacement Methods and Instability

1) EXAM 05/07/2023



DATA

$EA = 1 \cdot 10^8$ N

$EJ = 1 \cdot 10^{12}$ Nmm^2

$l = 3000$ mm

$K = 500$ N/mm

Let's find the critical load using a polynomial approximation. → PVW

for an Euler-Bernoulli beam

$u(x)$ x - disp

$v(x)$ y - disp

$$\begin{cases} u(x) = u_0(x) - y \cdot v_0(x)_{/x} \\ v(x) = v_0(x) \end{cases}$$ *

In the assumption of FINITE displacement:

→ Green-Lagrange Strain Tensor

$$\varepsilon_{ik} = \underbrace{\frac{1}{2}\left(u_{i/k} + u_{k/i}\right.}_{\text{linear strain}} + \left.\underline{u}_{/k} \cdot \underline{u}_{/i} \right)$$

displacement vector

disp in x

$$\varepsilon_{xx} = \frac{1}{2}\left(u_{/x} + u_{/x} + \cancel{(u_{/x})^2} + (v_{/x})^2 + (w_{/x})^2 \right)$$

in a 3D case

Let's assume $\begin{cases} \text{infinitesimal disp in } x \\ \text{finite disp in } y \end{cases}$

$$\varepsilon_{xx} = u_{/x} + \frac{1}{2}(v_{/x})^2$$

*
$$\varepsilon_{xx} = \underline{u_{0/x} - y \cdot v_{0/xx}} + \frac{1}{2}(v_{0/x})^2$$

- Virtual Internal Work

$$\delta(v_{/x} \cdot v_{/x}) = \delta v_{/x} \cdot v_{/x} + v_{/x} \cdot \delta v_{/x}$$

$$\delta W_i = \int_V \delta\varepsilon_{xx} \cdot \sigma_{xx}\, dV$$

$$\delta\varepsilon_{xx} = \delta u_{/x} - y \cdot \delta v_{/xx} + \cancel{\frac{1}{2}}(\delta v_{/x} \cdot v_{/x} \cdot \cancel{2})$$

$$\delta W_i = \int_V \left[(\delta u_{/x} + \delta v_{/x} v_{/x}) \cdot \sigma_{xx} - \delta v_{/xx} \cdot y \cdot \sigma_{xx} \right] dV$$

Knowing that $\int_A \sigma_{xx}\, dA = N$ and $\int_A y \cdot \sigma_{xx}\, dA = \overbrace{M}^{\text{BEAM}} = -EJ \cdot v_{/xx}$

$$\delta W_i = \overbrace{\int_\ell \left[(\delta u_{/x} + \delta v_{/x} \cdot v_{/x}) \cdot N + \delta v_{/xx} EJ\, v_{/xx} \right]}^{\text{BEAM}} dx + \overbrace{\delta v(\ell) \cdot K \cdot v(\ell)}^{\text{SPRING}}$$

- Virtual External Work

$$\delta W_e = -\int_0^\ell \delta u_x \cdot F\, dx = -\delta u(\ell) \cdot F$$

- PVW $\quad \delta W_i = \delta W_e$

$$\underbrace{\int_0^\ell \delta u_{/x} (N + F)\, dx}_{\text{AXIAL}} + \underbrace{\int_0^\ell (\delta v_{/x} N v_{/x} + \delta v_{/x} EJ v_{/x})\, dx + \delta v(\ell) \cdot K \cdot v(\ell)}_{\text{BENDING}} = \emptyset$$

- $N = -F$ *

- poly approx + BC

$$v(x) = c \cdot x^2 \qquad \delta v(x) = \delta c \cdot x^2$$

$V(x)_{/x} = 2 \cdot c x \qquad \delta V(x)_{/x} = \delta c \cdot 2x$

$V(x)_{/xx} = 2c \qquad \delta V(x)_{/xx} = \delta c \cdot 2$

$$\delta c \cdot \left[\int_0^{\ell} (2 EJ \cdot 2c - 2x \cdot F^{*} \cdot 2x\, c)\, dx + \ell^2 \cdot k \cdot c\, \ell^2 \right] = \emptyset$$

$$\left[4 EJ\ell - \frac{4}{3} F \ell^3 + K \ell^4 \right] \cdot c = \emptyset$$

unknown unknown

$$\begin{cases} c = \emptyset & \text{no transversal displacement} \\ F = \dfrac{(4EJ + K\ell^3) \cdot 3}{4 \ell^2} = 1.46 \times 10^6 \text{ N} \end{cases}$$

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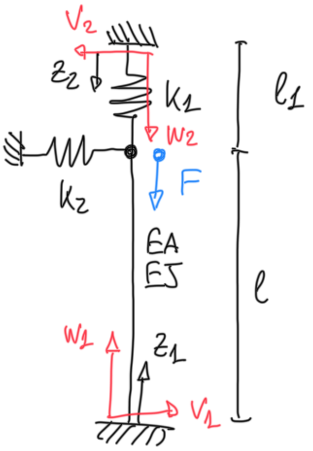
2) EXAM 13/02/2024



DATA

$\ell = 2000$ mm

$EA = 6 \cdot 10^{10}$ N

$EJ = 12 \cdot 10^{10}$ N mm^2

$K_1 = 1 \cdot 10^7$ N/mm

$K_2 = 1$ N/mm

$\ell_1 = 1000$ mm

Let's find the critical load F using a polynomial approximation

- Poly Approx + external BC

$w_1 = a_{w1} \cdot z_1$

$w_2 = a_{w2} \cdot z_2$

$V_1 = a_1 \cdot z_1^2$

$V_2 = a_2 \cdot z_2$

Internal BC - vertical disp

$a_{w1} \cdot \ell = - a_{w2} \cdot \ell_1$

$a_{w1} = - a_{w2} \cdot \dfrac{\ell_1}{\ell} = a_w \qquad a_{w2} = - a_w \dfrac{\ell}{\ell_1}$

trasversal disp

$a_1 \cdot \ell^2 = - a_2 \cdot \ell_1 \qquad a_2 = - a \dfrac{\ell^2}{\ell_1}$

$$a_1 = -a_2 \cdot \frac{l_1}{l^2} = a$$

- Virtual Internal Work

axial internal action in the beam

$$\delta W_i = \underline{\int_0^l \delta w_{1/z_1} \cdot N \, dz_1} + \underline{\int_0^l \left(\delta v_{1/z_1 z_1} EJ \cdot v_{1/z_1 z_1} + \delta v_{1/z_1} N \, v_{1/z_1} \right.}$$ BEAM

axial internal action in spring 1

$$+ \underline{\int_0^{l_1} \delta w_{2/z_2} N_{K1} \, dz_2} + \underline{\int_0^{l_1} \delta v_{2/z_2} N_{K1} v_{2/z_2}}$$ SPRING 1

$$+ \underline{\delta v_1(l) \cdot K \cdot v_1(l)}$$ SPRING 2

Knowing that $N = EA \, w_{1/z_1} = EA \cdot a_w$

$$N_{K1} = K_1 \cdot w_2(l_1) = -K_1 \cdot a_w \cdot l$$

- Virtual External Work

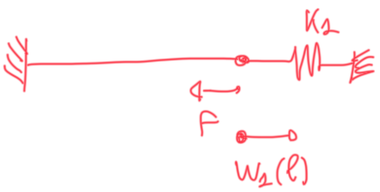
$$\delta W_e = \underline{- \delta w_1(l) \cdot F}$$

they are dependant *

unknown   unknown

- $$\delta a_w \left(\int_0^l EA \cdot a_w \, dz_2 + \int_0^{l_1} \left(-\frac{l}{l_1}\right) \cdot \left(K_1 \cdot \left(-a_w \cdot \frac{l}{l_1}\right) \cdot l_1 \right) dz_2 = -\delta a_w \cdot F \right.$$

AXIAL SYSTEM



AXIAL EQUILIBRIUM

$$F + \frac{EA}{l} \cdot w_1(l) + K_1 \cdot w_1(l) = \emptyset$$

$$* \quad w_1(l) = \frac{-F}{\underline{EA + K_1 l}} \cdot l = a_w \cdot l$$

- $$\delta a \cdot \left(\int_0^l \left(2 z_1 \cdot 2 z_1 \cdot a \cdot \underbrace{EA \cdot \left(-\frac{F}{EA + K_1 l} \right)}_{N = EA \cdot a_w = EA \, w_{1/z_1}} + 4 EJ \cdot a \right) dz_1 \right.$$ BEAM

$$+\int_0 \left(\left(-\frac{\ell^2}{\ell_1}\right)\cdot \underbrace{a\left(-\frac{\ell^2}{\ell_1}\right)}_{a_z}\cdot \underbrace{\left(-K_1\cdot a w\cdot \ell\right)}_{N_{K1}}\right)\cdot dz_1 = -\delta a\cdot \ell^2 K_2\, a\, \ell^2$$

solve $\begin{cases} a = \emptyset & \text{no transversal displacement} \\ a \neq \emptyset & F = 244\,000 \text{ N} \end{cases}$