Course of Aerospace Structures

Written test, January 16$^{\text{th}}$, 2025

Name ______________________________

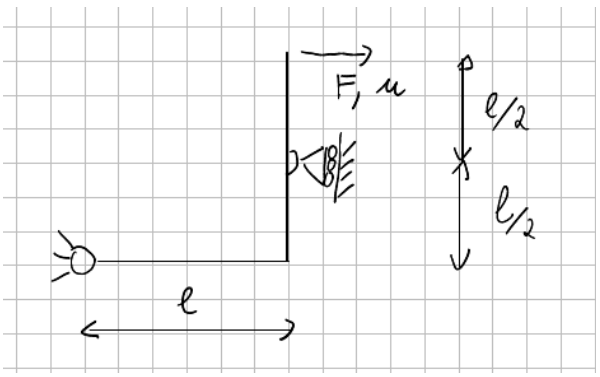
Surname ______________________________

Person code: ☐☐☐☐☐☐☐☐

Exercise 1

The L-shaped beam in the figure is loaded by the force F at the free extremity. Compute the displacement u at the point of application of the external load.
Neglect shear deformability.

(Unit for result: mm)



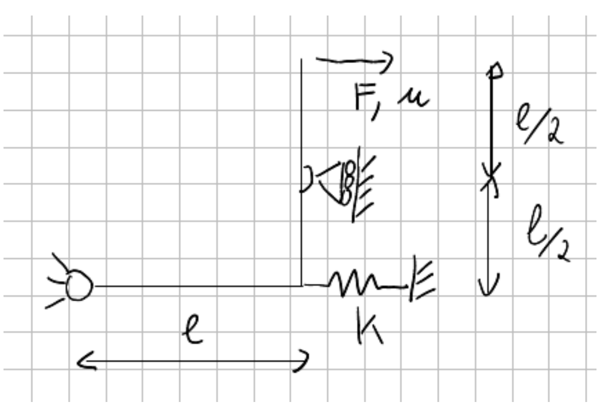
Data
$l = 1000$ mm
$F = 1000$ N
$EI = 1 \times 10^{11}$ N mm^2
$EA = 1 \times 10^{6}$ N

Answer ______________________________

Exercise 2

The L-shaped beam in the figure is loaded by the force F at the free extremity. Compute the displacement u at the point of application of the external load.
Neglect shear deformability.

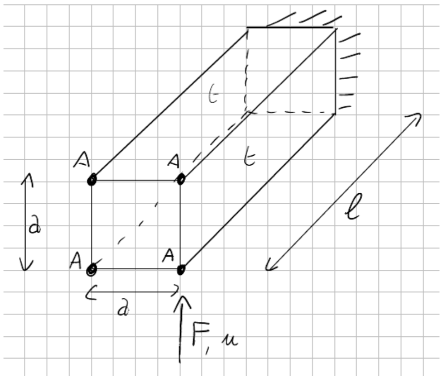
(Unit for result: mm)



Data
$l = 1000$ mm
$F = 1000$ N
$EI = 1 \times 10^{11}$ N mm^2
$EA = 1 \times 10^{6}$ N
$K = 1 \times 10^{5}$ N/mm

Answer ______________________________

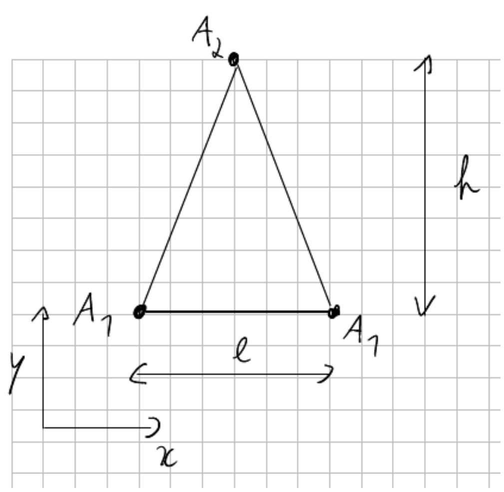
Exercise 3
The semi-monocoque beam sketched in the figure, with overall length l, has four panels, each with thickness t, and four concentrated areas, each with area A. It is loaded by the concentrated force F, applied along the right panel, as shown in the figure.
Compute the vertical displacement of the right bottom concentrated area.
Be careful with the measurement units.
(Unit for result: mm)



Data
$l = 1000$ mm
$E = 70000$ MPa
$\nu = 0.3$
$a = 100$ mm
$t = 0.4$ mm
$A = 1000$ mm^2
$F = 1000$ N

Answer ______________________

Exercise 4
The semi-monocoque beam cross section sketched in the figure, has three panels, each with thickness t, and three concentrated areas; each of the two areas at the bottom is equal to A_1. The cross section is loaded by the bending moment M_x.
Compute the value of the top area A_2 such that the axial stress there is equal to the design value σ_y.
(Unit for result: mm^2)

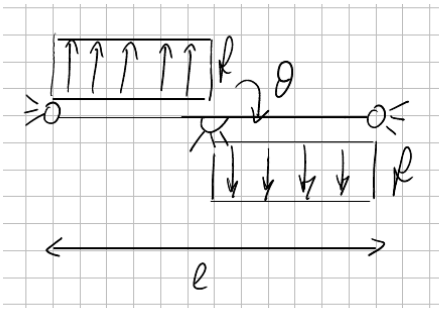


Data
$\sigma_y = 120$ MPa
$t = 1$ mm
$l = 100$ mm
$h = 200$ mm
$A_1 = 100$ mm^2
$E = 70000$ MPa
$\nu = 0.3$
$M_x = 1 \times 10^6$ N mm

Answer ______________________

Exercise 5
Consider the slender beam structure of the figure, loaded the distributed force per unit of length f, as sketched.
Estimate the rotation θ at the middle hinge by assuming a suitable trigonometric approximation of the transverse displacement truncated to the first non null term.
Neglect shear deformability.
Be careful with the measurement units.

(Unit for result: deg)

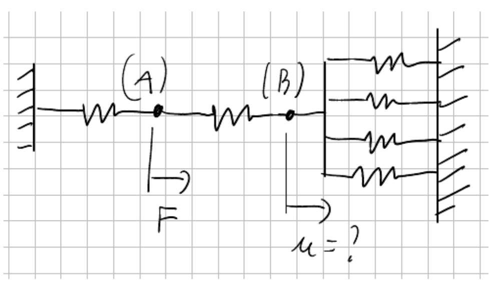


Data
$l = 3000$ mm
$f = 100$ N/mm
$EI = 1 \times 10^{11}$ N mm

Answer ______________________

Exercise 6
The spring system drawn in the figure, each with stiffness K, is loaded at point A by the horizontal force F. Compute the displacement in the horizontal direction of point B.

(Unit for result: mm)



Data
$K = 10000$ N/mm
$F = 10000$ N

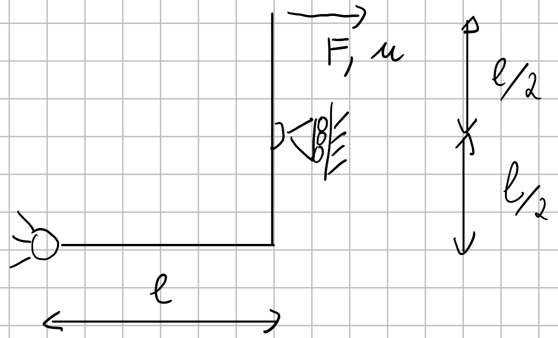
Answer ______________________

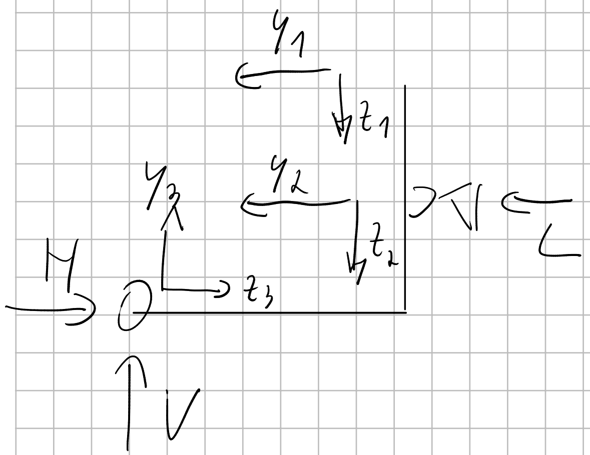
True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. The compressive buckling stress of an Euler-Bernoulli beam is proportional to the minimum bending stiffness $EI_{\min}$ of the cross section.
2. Consider a clamped Timoshenko beam loaded by a concentrated force applied at the free extremity. The shear strain at the clamp is null.
3. The torsional stiffness of a square cross section is equal to GJ, where G is the shear modulus and J the polar area moment of the cross section.

Multiple Choice questions
(Circle the correct answer)

1. The elastic problem can be formulated in terms of displacements:
   (a) always;
   (b) only for statically determinate problems;
   (c) only within the framework of the finite element method;
   (d) never;
   (e) none of the above.
2. The linear static response of simply-supported beam with stiffness EJ and loaded with a uniform load:
   (a) can be analyzed by imposing symmetry conditions;
   (b) cannot be analyzed by imposing symmetry conditions because even vibration modes are anti-symmetric;
   (c) can be analyzed by imposing symmetry conditions because odd vibration modes are symmetric;
   (d) none of the above.
3. In a finite element procedure, the stress tensor:
   (a) cannot be computed;
   (b) is part of the solution;
   (c) can be recovered from the solution;
   (d) none of the above.





real

$V = 0$

$L = 2F$

$H = F$

$M_{x_1} = F z$

$M_{x_2} = F \frac{l}{2} - F z$

$T_{z_3} = -F$

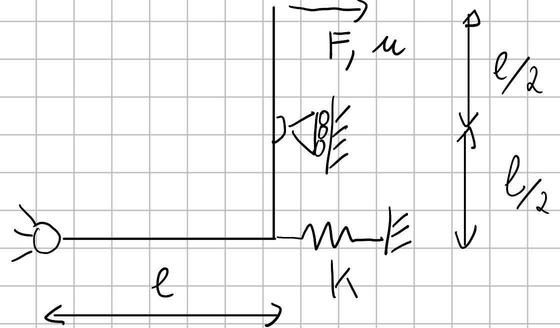
$M'_{x_1} = z$

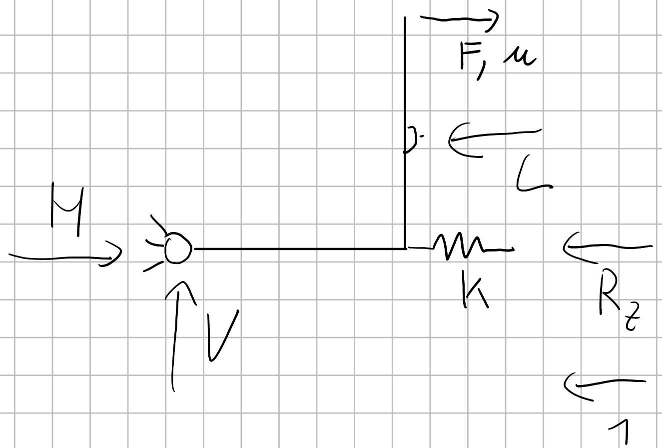
$M'_{x_2} = \frac{l}{2} - z$

$T'_{z_3} = -1$

$$\int_0^{l/2} \frac{F z^2}{E\cancel{I}} \, dz_1 + \int_0^{l/2} \frac{F\left(\frac{l}{2} - z\right)^2}{E\cancel{I}} \, dz_2 + \int_0^{l} \frac{F}{EA} \, dz_3 = u$$

$$\frac{1}{3} \frac{l^3}{8} \frac{F}{E\cancel{I}} + \frac{F}{E\cancel{I}} \left(\cancel{\frac{l^3}{8}} + \frac{1}{24} l^3 - \cancel{\frac{l^3}{8}} \right) + \frac{F}{EA} = u$$





real

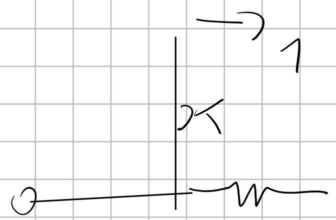
$$H = F + R_z$$

dummy

$$H' = 1$$

$$\int_0^l \frac{F + R_z}{EA} + \frac{R_z}{K} = 0$$

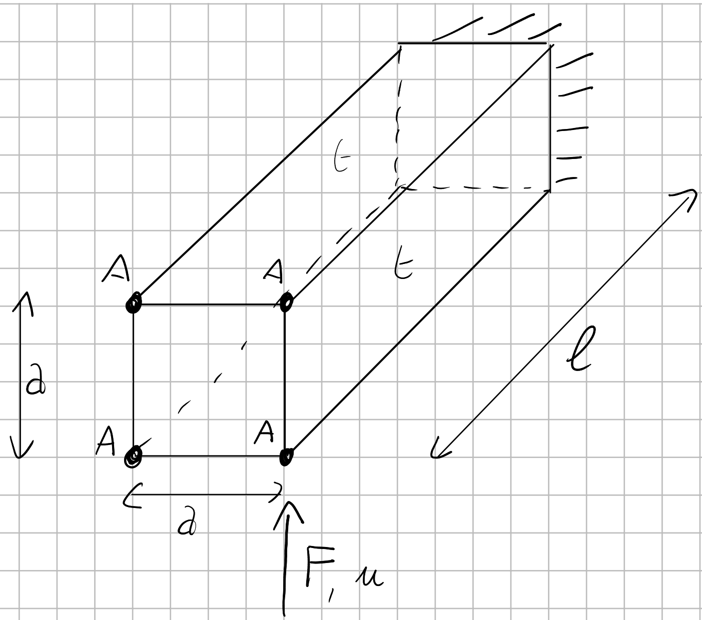
$$\frac{Fl}{EA} + R_z\left(\frac{l}{EA} + \frac{1}{k}\right) = \emptyset \qquad R_z = \frac{-Fl}{EA}\left(\frac{l}{EA} + \frac{1}{k}\right)^{-1}$$



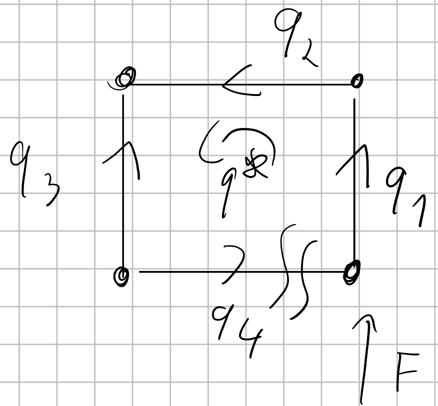
$$\Rightarrow N = F + R_z \qquad N' = 1 + R_z' \quad \text{with}$$

$$R_z' = \frac{-1 \cdot l}{EA}\left(\frac{l}{EA} + \frac{1}{k}\right)^{-1}$$

$$\int_0^{l/2} \frac{F z^2}{EI}\, dz_1 + \int_0^{l/2} \frac{F\left(\frac{l}{2} - z\right)^2}{EI}\, dz_2 + \int_0^{l} \frac{N N'}{EA}\, dz_3 + \frac{R_z R_z'}{k} = u$$



$$EI = 4A\,\frac{a^2}{4}\cdot E = A a^2 E$$

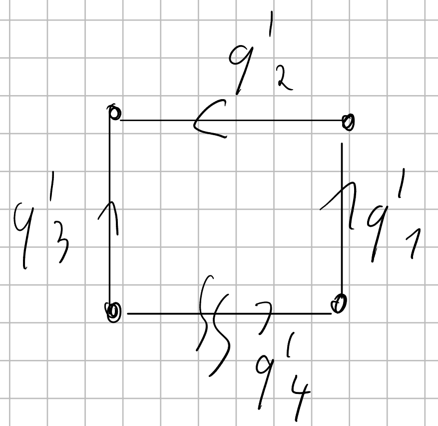


real

$$q^* = F\cdot\frac{\cancel{a}}{2}\cdot\frac{1}{2a^{\cancel{2}}} = \frac{F}{4a}$$

$$q_1 = \frac{F}{2a} + q^* \qquad q_2 = q^*$$

$$q_3 = \frac{F}{2a} - q^* \qquad q_4 = q^*$$



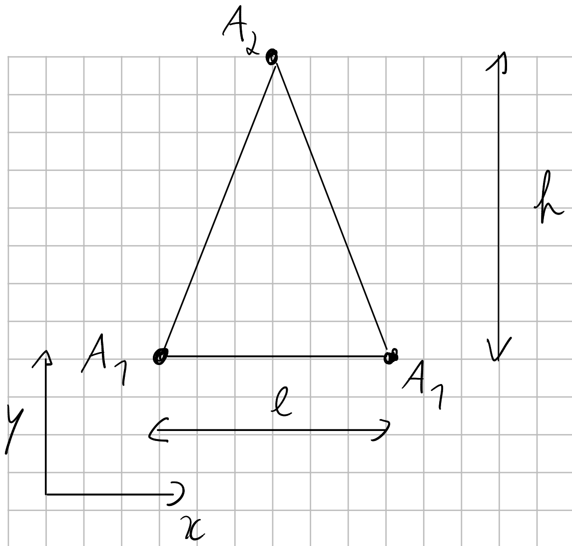
$$q^{*\prime} = \frac{1}{4a}$$

$$q'_1 = \frac{1}{2a} + q^{*\prime} \qquad q'_2 = q^{*\prime}$$

$$q'_3 = \frac{1}{2a} - q^{*\prime} \qquad q'_4 = q^{*\prime}$$

$$\int_0^l \frac{Fz^2}{EI}\, dz +$$

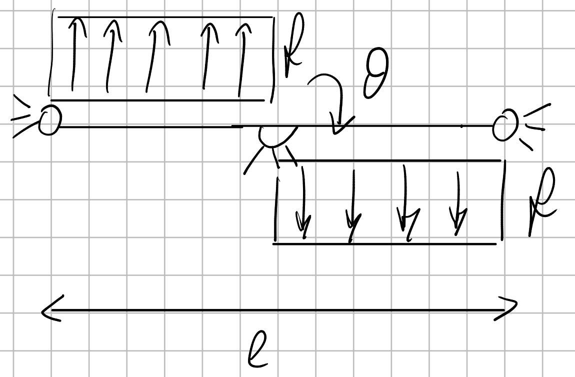
$$\int_0^l \int_0^a \frac{q_1 q_1'}{tG} + \frac{q_2 q_2'}{tG} + \frac{q_3 q_3'}{tG} + \frac{q_4 q_4'}{tG} = u = \frac{1}{3}\frac{Fl^3}{EI} \quad \frac{12}{16} \quad \frac{Fl}{atG}$$

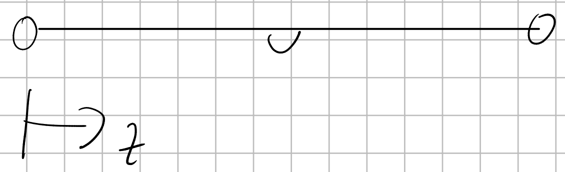


$$F_{A_2} = \frac{M_x}{h}$$

$$\sigma_{A_2} = \sigma_y = \frac{M_x}{A\,h}$$

$$A_2 = \frac{M_x}{h\,\sigma_y}$$





$$v = a \sin\left(\frac{2\pi z}{l}\right)$$

$$v'' = -\frac{4\pi^2}{l^2}\, a \sin\left(\frac{2\pi z}{l}\right)$$

$$\int_0^l \delta a \; EI \; \frac{16\pi^4}{l^4} \sin^2\left(\frac{2\pi z}{l}\right) dz = 2\int_0^{l/2} \delta a \cdot \sin\left(\frac{2\pi z}{l}\right) p \, dz$$

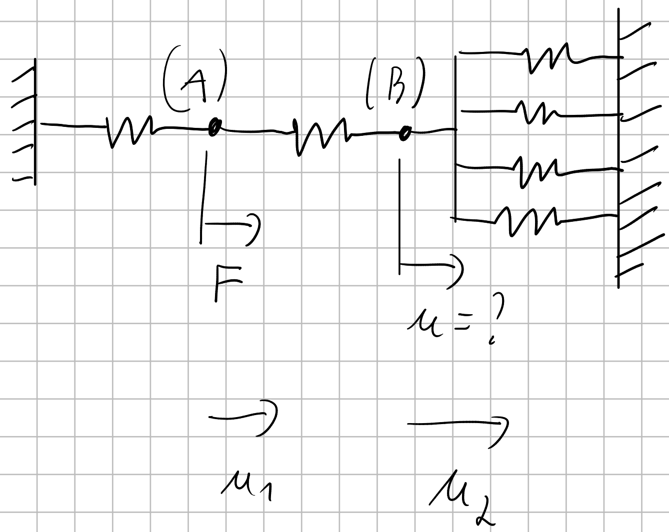
$$\delta a \, EI \, \frac{8\pi^4}{l^4} \, a = \delta a \, 2p \, \frac{l}{2\pi} \, -\cos\left(\frac{2\pi z}{l}\right)\Bigg|_0^{l/2}$$

$$= \delta a \, \frac{\cancel{4}^{2} p l}{\cancel{2}\pi}$$

$$a = \frac{\cancel{2} p l^4}{\cancel{8}^{4} \pi^5 EI} = \frac{p l^4}{4\pi^5 EI}$$

$$v'\left(z = \frac{l}{2}\right) = -\vartheta = \frac{2\pi}{l} \cdot a = \frac{-p l^3}{2\pi^4 EI} = \frac{-p l^3}{2\pi^4 EI}$$

$$\vartheta = \frac{p l^3}{2\pi^4 EI}$$



$$K \begin{bmatrix} 2 & -1 \\ -1 & 5 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

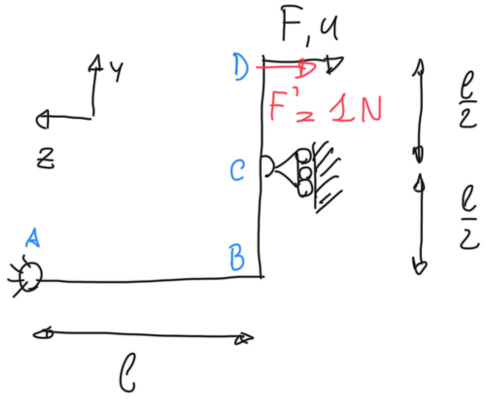
$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \frac{1}{9K} \begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix} \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

$$u_2 = \frac{F}{9K}$$

AEROSPACE STRUCTURE

WRITTEN TEST 16/06/2025

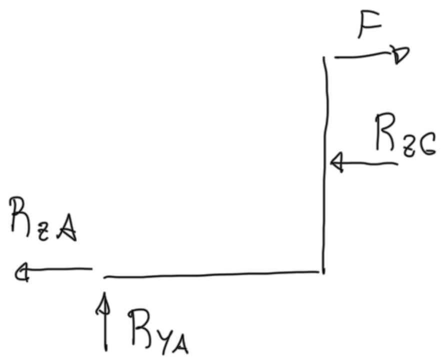
Exercise 1



DATA

$l = 1000$ mm
$F = 1000$ N
$EI = 1 \times 10^{11}$ Nmm^2
$EA = 1 \times 10^{6}$ N

1) Reaction Forces



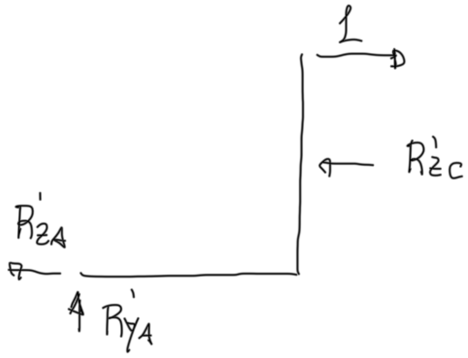
REAL

Y $\quad R_{YA} = 0$

Z $\quad R_{ZA} + R_{ZC} - F = 0$
$\quad R_{ZA} + 2F - F = 0$
$\quad R_{ZA} = -F$

rot X in A $\quad R_{ZC} \cdot \frac{l}{2} - F \cdot l = 0$
$\quad R_{ZC} = 2F$

DUMMY



Y $\quad R'_{YA} = 0$

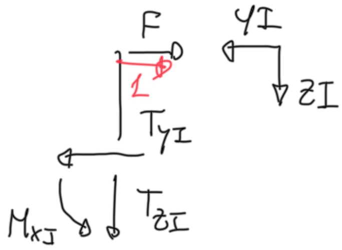
Z $\quad R'_{ZA} + R'_{ZC} - 1 = 0$

rot X in A $\quad R'_{ZC} \cdot \frac{l}{2} - 1 \cdot l = 0 \qquad R'_{ZC} = 2$ N

$\rightarrow R'_{ZA} = -1$ N

2) INTERNAL ACTION

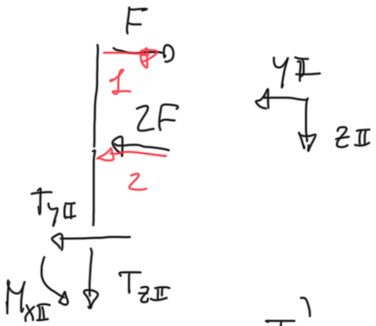
I) REAL



$$T_{yI} = F \qquad M_{xI} = F \cdot z_I$$
$$T_{zI} = 0$$

DUMMY $\quad T'_{yI} = 1 \quad T'_{zI} = 0 \quad M'_{xI} = z_I$

II) REAL

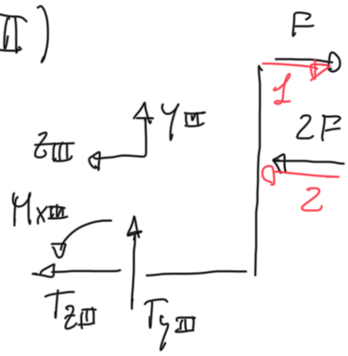


$$T_{yII} = F - 2F = -F$$
$$T_{zII} = 0$$
$$M_{xII} = F \cdot \left(\frac{\ell}{2} + z_{II} \right) - 2F \cdot z_{II}$$
$$= F \left(\frac{\ell}{2} - z_{II} \right)$$

$$T'_{yII} = 1 - 2 = -1 \qquad T_{zII} = 0$$

$$M'_{xII} = 1 \cdot \left(\frac{\ell}{2} + z_{II} \right) - 2 \cdot z_{II} = \frac{\ell}{2} - z_{II}$$

III)



REAL

$$T_{yIII} = 0 \qquad T_{zIII} = F - 2F = -F$$

$$M_{xIII} = F \cdot \ell - 2F \cdot \frac{\ell}{2} = 0$$

DUMMY $\quad T'_{yIII} = 0 \quad T'_{zIII} = 1 - 2 = -1$

$$M'_{xIII} = \ell - 2 \frac{\ell}{2} = 0$$

3) PCVW

REAL $\quad \delta W_e = 1 \cdot u$

$$\delta W_i = \int_0^{\frac{\ell}{2}} M'_{xI} \cdot M_{xI} \, dz_I + \int_0^{\frac{\ell}{2}} M'_{xII} \cdot M_{xII} \, dz_{II} + \int_0^{\ell} T'_{zIII} \cdot T_{zIII} \, dz_{III} =$$

$$= \frac{F}{EI}\int_0^{\frac{\ell}{2}} z_I^2 \cdot dz_I + \frac{F}{EI}\int_0^{\frac{\ell}{2}} \left(\frac{\ell}{2} - z_{II}\right)^2 dz_{II} + \frac{F}{EA}\int_0^{\ell} dz_{III} =$$

$$= \frac{F}{EI} \cdot \left[\frac{z_I^3}{3}\right]_0^{\frac{\ell}{2}} + \frac{F}{EI}\int_0^{\frac{\ell}{2}} \left(\frac{\ell^2}{4} + z_{II}^2 - \ell z_{II}\right) dz_{II} + \frac{F}{EA}\left[z_{III}\right]_0^{\ell} =$$

$$= \frac{F \cdot \ell^3}{24 EI} + \frac{F}{EI}\left[\frac{\ell^2 z_{II}}{4} + \frac{z_{II}^3}{3} - \frac{\ell z_{II}^2}{2}\right]_0^{\frac{\ell}{2}} + \frac{F \cdot \ell}{EA} =$$

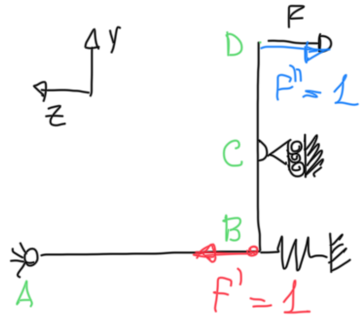
$$= \frac{F \cdot \ell^3}{24 EI} + \frac{F}{EI} \cdot \left(\cancel{\frac{\ell^3}{8}} + \frac{\ell^3}{24} - \cancel{\frac{\ell^3}{8}}\right) + \frac{F \cdot \ell}{EA} =$$

$$= \frac{F \cdot \ell^3}{24 EI} + \frac{F\ell^3}{24 EI} + \frac{F\ell}{EA} = \frac{F\ell^3}{12 EI} + \frac{F\ell}{EA} = \delta W_e' = u$$

$$u = \frac{10^3 \cdot 10^9}{12 \cdot 10^{11}} + \frac{10^3 \cdot 10^3}{10^6} = \frac{10^{12}}{12 \cdot 10^{11}} + 1 = \frac{10}{12} + 1 = \frac{11}{6} =$$

$$= 1.833 \text{ mm}$$

EXERCISE 2

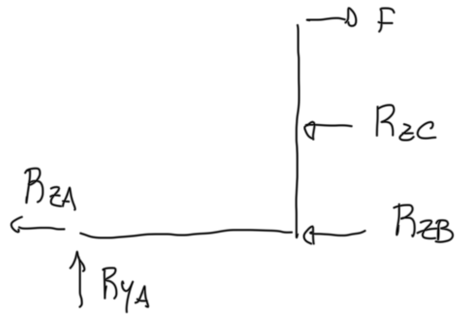


DATA

$\ell = 1000$ mm
$F = 1000$ N
$EI = 1 \times 10^{11}$ Nmm^2
$EA = 1 \times 10^6$ N
$K = 1 \times 10^5$ N/mm

1) REACTION FORCES

REAL



$Y \quad R_{yA} = 0$

$Z \quad R_{zA} + R_{zB} + R_{zC} - F = 0$

rot X in $A \quad R_{zC} \cdot \frac{\cancel{\ell}}{2} - F \cdot \cancel{\ell} = 0 \quad R_{zC} = 2F$

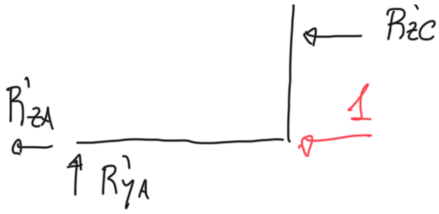
DUMMY 1

Y $R'_{yA}=0$

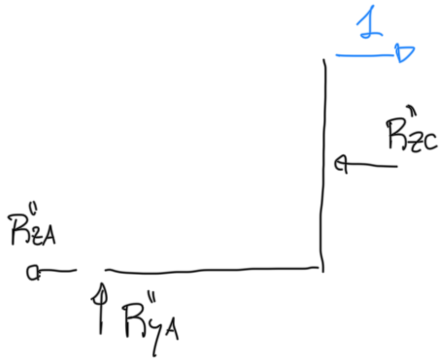
Z $R'_{zA}+R'_{zC}+1=0$

rot X in A $R'_{zC}\cdot\frac{l}{2}=0$ $\rightarrow$ $R'_{zC}=0$

$R'_{zA}=-1$



DUMMY 2

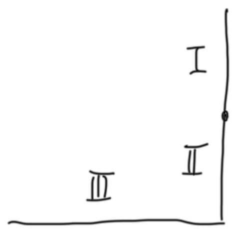


Y $R''_{yA}=0$

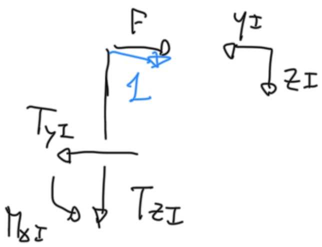
rot X in A $R''_{zC}=2$

Z $R''_{zA}+2-1=0$ $R''_{zA}=-1$

2) INTERNAL ACTIONS



I) REAL



$T_{zI}=0$

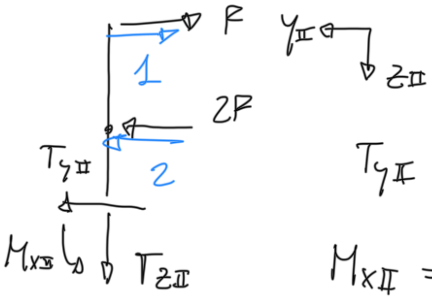
$T_{yI}=F$

$M_{xI}=F\cdot z_I$

DUMMY 1 UNLOADED

DUMMY 2 $T''_{zI}=0$ $T''_{yII}=1$ $M''_{xI}=z_I$

II) REAL



$T_{yII}=-F$ $T_{zII}=0$

$M_{xII}=F\left(\frac{l}{2}+z_{II}\right)-2F\cdot z_{II}=$

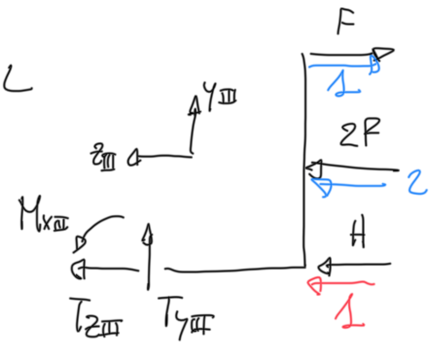
$=F\left(\frac{l}{2}-z_{II}\right)$

DUMMY 1 UNLOADED

DUMMY 2 $T''_{y_{II}} = -1 \qquad T''_{z_{II}} = 0$

$$M''_{x_{II}} = \frac{\ell}{2} + z_{II} - 2z_{II} = \frac{\ell}{2} - z_{II}$$

III) REAL



rename $R_{zB} = H$

$$T_{y_{III}} = 0 \qquad T_{z_{III}} = F - 2F - H = -F - H \qquad M_{x_{III}} = 0$$

DUMMY 1

$$T'_{y_{III}} = 0 \qquad T'_{z_{III}} = -1 \qquad M'_{x_{III}} = 0$$

DUMMY 2

$$T''_{y_{III}} = 0 \qquad T''_{z_{III}} = -1 \qquad M''_{x_{III}} = 0$$

3) PCVW

DUMMY 1

$$\delta W'_e = 1 \cdot u_B = -H/K$$

$$\delta W'_i = \int_0^\ell T'_{z_{III}} \cdot \frac{T_{z_{III}}}{EA} \cdot dz_{III} = \frac{F+H}{EA} \int_0^\ell dz_{III} =$$

$$= \frac{F+H}{EA} \cdot \ell = -\frac{H}{K}$$

$$\rightarrow \frac{F\ell}{EA} + \frac{H\ell}{EA} + \frac{H}{K} = 0 \qquad H\left(\frac{\ell}{EA} + \frac{1}{K}\right) = -\frac{F\ell}{EA} \qquad H \cdot \frac{\ell K + EA}{K\,EA} = -\frac{F\ell}{EA}$$

$EA \quad EA \quad k \quad = \quad k \quad EA \quad K \cdot EA \quad EA$

ALWYS $\neq 0$

$$\rightarrow H = -\frac{F\ell}{\cancel{EA}} \cdot \frac{\cancel{EA} \cdot k}{\ell k + EA} = -\frac{F\ell k}{\ell k + EA} = -\frac{10^3 \cdot 10^3 \cdot 10^5}{10^3 \cdot 10^5 + 10^6} = -\frac{10^{11}}{10^8 + 10^6} =$$

$$= -\frac{10^{11}}{10^6 (10^2 + 1)} = -\frac{10^5}{101} = -990{,}1 \text{ N}$$

the lower beam is in tension

DUMMY 2

$$\delta W_e'' = u$$

$$\delta W_i'' = \int_0^{\frac{\ell}{2}} M''_{xI} \cdot \frac{M_{xI}}{EI} \, dz_I + \int_0^{\frac{\ell}{2}} M''_{xII} \cdot \frac{M_{xII}}{EI} \cdot dz_{II} + \int_0^{\ell} T''_{zIII} \cdot \frac{T_{zIII}}{EA} \, dz_{III} =$$

$$= \frac{F}{EI} \int_0^{\frac{\ell}{2}} z_I^2 \, dz_I + \frac{F}{EI} \int_0^{\frac{\ell}{2}} \left(\frac{\ell}{2} - z_{II}\right)^2 dz_{II} + \frac{F+H}{EA} \int_0^{\ell} dz_{III} =$$

$$= \frac{F}{EI} \left[\frac{z_I^3}{3} \right]_0^{\frac{\ell}{2}} + \frac{F}{EI} \int_0^{\frac{\ell}{2}} \left(\frac{\ell^2}{4} + z_{II}^2 - \ell z_{II} \right) dz_{II} + \frac{F+H}{EA} \cdot \ell =$$

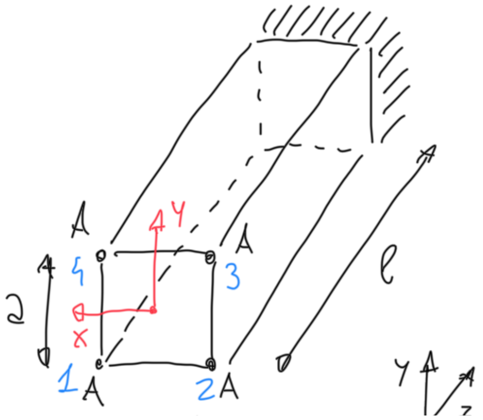
$$= \frac{F\ell^3}{24EI} + \frac{F}{EI} \left[\frac{\ell^2}{4} z_{II} + \frac{z_{II}^3}{3} - \ell \frac{z_{II}^2}{2} \right]_0^{\frac{\ell}{2}} + \frac{F+H}{EA} \cdot \ell =$$

$$= \frac{F\ell^3}{24 \cdot EI} + \cancel{\frac{F\ell^3}{8EI}} + \frac{F\ell^3}{24 \cdot EI} - \cancel{\frac{F\ell^3}{8EI}} + \frac{F+H}{EA} \cdot \ell =$$

$$= \frac{F\ell^3}{12 EI} + \frac{F+H}{EA} \cdot \ell = u$$

$$u = \frac{10^3 \cdot 10^9}{12 \cdot 10^{11}} + \frac{10^3 - 990.1}{10^6} \cdot 10^3 = \frac{5}{6} + 0.0099 \simeq 0.843 \text{ mm}$$

EXERCISE 3



DATA

$\ell = 1000$ mm

$E = 70000$ MPa

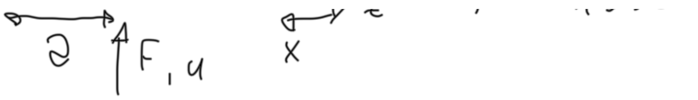
$\nu = 0.3$

$a = 100$ mm

$t = 0.4$ mm

$A = 1000$ mm^2

$F = 1000$ N



1) CENTROID

$$C(x_c, y_c) \qquad x_c = \frac{a}{2} \quad y_c = \frac{a}{2}$$

2) INERTIAS

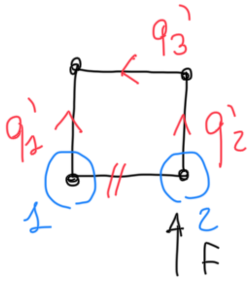
$$J_{xx} = \sum_i A_i \cdot y_i^2 = 2A \cdot \left(\frac{a}{2}\right)^2 + 2A \cdot \left(-\frac{a}{2}\right)^2 =$$

$$= \frac{4Aa^2}{4} = Aa^2$$

$$S_{xi} = A_i \cdot y_i \qquad S_{x1} = S_{x2} = -\frac{1}{2}Aa$$

$$S_{x3} = S_{x4} = \frac{1}{2}Aa$$

3) OPEN CELL FLUXES

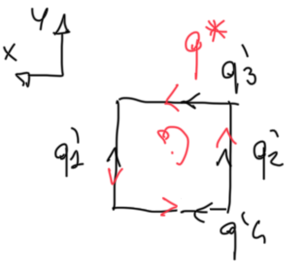


1) $$q_1' = -F\frac{S_{x1}}{J_{xx}} = +\frac{1}{2}\frac{F}{a}$$

2) $$q_2' = -F\frac{S_{x2}}{J_{xx}} = +\frac{1}{2}\frac{F}{a}$$

$$q_3' = q_4' = 0$$

4) MOMENTUM EQUIVALENCE at 2



$$F \cdot \frac{a}{2} = q^* \cdot 2\Omega_{cell} + q_2' \cdot \cancel{2}\Omega_2 - q_1' \cancel{/} 2 \cdot \Omega_1$$

$$F \cdot \frac{a}{2} - 2q^* a^2 = 0 \qquad q^* = +\frac{F}{4a} = \frac{5}{2}$$

$$q_1 = -q^* + q_1' = -\frac{F}{4a} + \frac{F}{2a} = +\frac{F}{4a} = +\frac{5}{2} \text{ N/mm}$$

$$q_2 = q^{*} + q_2 = +\frac{1}{4a} + \frac{1}{2a} = +\frac{3}{4}\frac{5}{a} = \frac{15}{2} \ \text{N/mm}$$

$$q^3 = q^* = +\frac{5}{2} \ \text{N/mm} \qquad q^4 = -q^* = -\frac{5}{2} \ \text{N/mm}$$

$$\frac{d\theta}{dz} = \frac{1}{2\Omega_{cell}} \cdot \sum_i \frac{q_i \cdot l_i}{G \cdot t_i} = \frac{\cancel{a}}{2 \cdot a^2 G \cdot t} \cdot \left| -\cancel{\frac{5}{2}} + \cancel{\frac{5}{2}} + \frac{15}{2} + \frac{5}{2} \right|$$

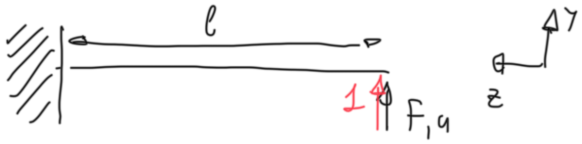
$$= \frac{5}{aGt} = \frac{5 \cdot 2 \cdot (1+\nu)}{100\, E \cdot 0.4} = \frac{1.3}{10 \cdot 70 \cdot 10^3 \cdot 0.4}$$

$$\theta = \frac{d\theta}{dz} \cdot l = 0{,}00464 \ \text{rad}$$

6) VERTICAL DISPLACEMENT DUE TO TORSION

$$\Delta z = \theta \cdot \frac{a}{2} = 6.96 \times 10^{-3} \cdot 50 = 0{,}232$$

7) FLEXURAL DISPLACEMENT DUE TO BENDING



- INTERNAL ACTIONS

REAL $\quad T_y = -F \quad T_z = 0 \quad M_x = -F \cdot z_I$

DUMMY $\quad T_y' = -1 \quad T_z' = 0 \quad M_x' = -z_I$

- PCVW

$$\delta W_e = u_{fl}$$

$$\delta W_i = \int_0^l T_y' \cdot \frac{T_y}{GA^*} \, dz_I + \int_0^l M_x' \cdot \frac{M_x}{EJ} \cdot dz_I =$$

$$= \frac{F}{GA} [z_I]_0^l + \frac{F}{EJ} \left[\frac{z_I^3}{3} \right]_0^l = \frac{Fl}{GA^*} + \frac{Fl^3}{3EJ} = u_{fl}$$

$J = J_{xx} = J_{yy}$

$A^* = 2 \cdot 2 \cdot t =$

$= 80 \, mm^2$

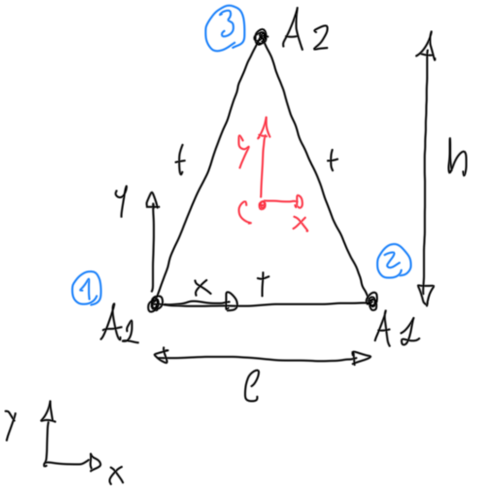
$$u_{fl} = \frac{2Fl(1+\nu)}{EA^*} + \frac{Fl^3}{3EJ} =$$

$$= \frac{2 \cdot \cancel{10^3} \cdot 10^3 \cdot 1{,}3}{70 \cdot \cancel{10^3} \cdot 80} + \frac{10^3 \cdot 10^9}{3 \cdot 70 \cdot 10^3 \cdot 10^7} =$$

$$= 0{,}464 + \frac{100}{210} = 0.94 \quad mm$$

$$u_{TOT} = u_{fl} + u_{tor} = 0.94 + 0.232 = 1.172 \, mm$$

EXERCISE 4



DATA

$\sigma_y = 120$ MPa

$t = 1$ mm

$l = 100$ mm

$h = 200$ mm

$A_1 = 100 \, mm^2$

$E = 70000$ MPa

$\nu = 0.3$

$M_x = 1 \times 10^6 \, N mm^{\cancel{2}}$

AXIAL STRESS

$$\sigma_{zz_i} = \frac{T_z}{\Sigma_i A} + \frac{M_x}{J_{xx}} \cdot y_i + \frac{M_y}{J_{yy}} \cdot x_i$$

1) CENTROID

$C(x_C, y_C) \qquad x_C = \frac{l}{2}$

$$y_C = \frac{\Sigma_i A_i \cdot y_i}{\Sigma_i A_i} = \frac{A_2 h}{2A_1 + A_2}$$

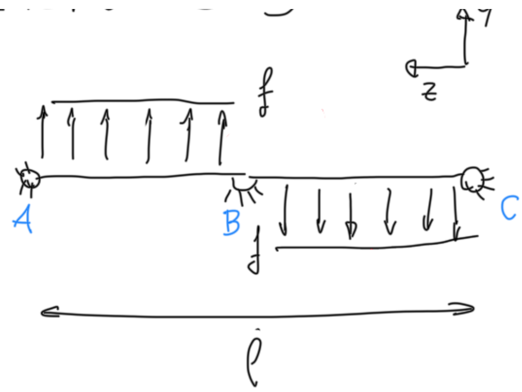
wrt ①

$$J_{xx} = \sum_i A_i \cdot y_i = A_2 \cdot h$$

$$\sigma_{zz} = \frac{M_x}{J_{xx}} \cdot y = \sigma_y$$

$$\sigma_y = \frac{M_x}{A_2 \cdot h^2} \cdot \cancel{h} \rightarrow A_2 = \frac{M_x}{\sigma_y \cdot h} = \frac{10^6}{120 \cdot 200} = 41.67 \text{ mm}$$

EXERCISE 5 ... DATA



$\ell = 3000$ mm
$f = 100$ N/mm
$EI = 1 \times 10^{11}$ Nmm^2

approx $\quad u(z) = c \cdot \phi(z)$

$\phi(z) = \sin(c_1 z + c_2)$

ESSENTIAL BC

$\phi(0) = 0 \quad \to \quad c_2 = 0$
$\phi(\frac{\ell}{2}) = 0 \quad \to \quad c_1 \cdot z = \pi \qquad c_1 = \frac{2\pi}{\ell}$
$\phi(\ell) = 0$

$\to \phi(z) = \sin\left(\frac{2\pi z}{\ell}\right) \quad \to \quad u(z) = c \cdot \sin\left(\frac{2\pi z}{\ell}\right)$

$u'(z) = \frac{du(z)}{dz} = \frac{2\pi c}{\ell} \cdot \cos\left(\frac{2\pi z}{\ell}\right)$

$u''(z) = \frac{d^2u(z)}{dz^2} = -\frac{4\pi^2}{\ell^2} c \cdot \sin\left(\frac{2\pi z}{\ell}\right)$

$dW_e = \int_0^{\frac{\ell}{2}} f \cdot \delta u(z)\, dz + \int_{\frac{\ell}{2}}^{\ell} (-f) \cdot (-\delta u(z))\, dz = 2 \cdot \int_0^{\frac{\ell}{2}} f \delta u(z) =$

$= 2 f \delta c \cdot \int_0^{\frac{\ell}{2}} \sin\left(\frac{2\pi z}{\ell}\right) = -\cancel{2} f \delta c \cdot \left[\frac{\ell}{\cancel{2}\pi} \cos\left(\frac{2\pi z}{\ell}\right)\right]_0^{\frac{\ell}{2}} =$

$= -f \frac{\delta c \cdot \ell}{\pi} (1+1) = -\frac{2}{\pi} \cdot f \ell \cdot \delta c$

ANTI SYMM
$\emptyset$

$dW_i = \int_0^{\frac{\ell}{2}} \delta u''(z) \cdot M(z) \cdot dz \cdot 2 = \int_0^{\frac{\ell}{2}} \delta u''(z) \cdot EI \cdot u''(z)\, dz =$

$= \int_0^{\frac{\ell}{2}} + \frac{4\pi^2}{\ell^2} \cdot \delta c \sin\left(\frac{2\pi z}{\ell}\right) \cdot EI \cdot \frac{4\pi^2}{\ell^2} \cdot c \cdot \sin\left(\frac{2\pi z}{\ell}\right) dz =$

$= \frac{16\pi^4}{\ell^4} \cdot c \cdot \delta c \cdot EI \cdot \int \sin^2\left(\frac{2\pi z}{\ell}\right) dz = \qquad \sin^2(x) = \frac{1 - \cos(2x)}{2}$

$$= \sim \cdot \left[\frac{1}{2}z - \frac{\ell}{8\pi}\sin\left(\frac{4\pi z}{\ell}\right)\right]_0^{\frac{\ell}{2}} =$$

$$= \frac{\cancel{16}^{4}\pi^4}{\ell^{\cancel{4}3}} EI \cdot C \cdot \delta C \cdot \left(\frac{\ell}{\cancel{4}} - \cancel{0}\right) = \frac{8\pi^4 EI}{\ell^3} \cdot C \cdot \delta C$$

$$dW_e = dW_i \qquad -\frac{2fL}{\pi} \cdot \cancel{\delta c} = \frac{8\pi^4 EI}{L^3} \cdot c \cdot \cancel{\delta c}$$

$$c = -\frac{\cancel{2}fL}{\pi} \cdot \frac{L^3}{\cancel{8}^{4}\pi^4 EI} = -\frac{fL^4}{4\pi^5 EI} = \frac{-10^2 \cdot 81 \cdot 10^{\cancel{12}}}{4\pi^5 \, \cancel{10^{11}}} = -\frac{81 \cdot 10^3}{4\pi^5}$$

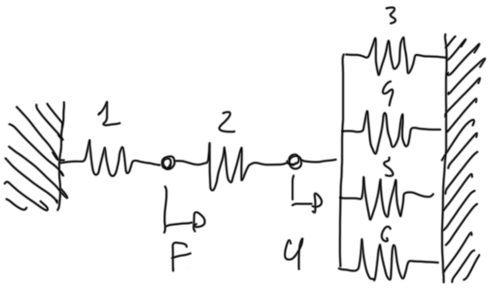
$$\simeq 66{,}17\,mm$$

$$u'(z) = \frac{2\pi c}{\ell} \cdot \cos\frac{2\pi z}{\ell}$$

$$u'\left(\frac{\ell}{2}\right) = \frac{2\pi}{\ell} \cdot (-1) \cdot c = -\frac{2\pi}{3000} \cdot 66.17 = 0.138 \text{ rad} =$$

$$= 7.906^\circ$$

EXERCISE 6



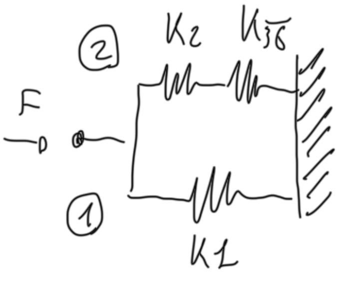
DATA

$K = 10\,000 \ N/mm$

$F = 10\,000 \ N$

$K_{\overline{36}} = 4K \qquad \frac{1}{K_{\overline{26}}} = \frac{1}{K_2} + \frac{1}{K_{\overline{36}}} = \frac{1}{K} + \frac{1}{4K} = \frac{5}{4K} \qquad K_{\overline{26}} = \frac{4}{5}K$

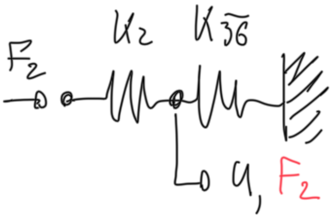
$K_{TOT} = K_1 + K_{26} = \frac{9}{5}K$



$F = K_{TOT}\, U_{TOT} = (K_1 + K_{\overline{26}})\, u_{TOT}$

$= (K_1 + K_{26})\, u_{TOT}$

(2) $F_2 = K_{26} \cdot U_{TOT} = \frac{K_{26}}{K_{TOT}} \cdot F = \frac{4}{5} \cdot \frac{5}{9} \cdot F = \frac{4}{9} F$



SERIES

$K_2 \cdot u_2 = K_{\overline{36}} \cdot u_{\overline{36}} = F_2$

$u_2 = \frac{F_2}{K_{\overline{36}}} = \frac{\cancel{4}}{9} F \cdot \frac{1}{\cancel{4}K} = \frac{F}{9K} = \frac{1}{9} \ mm$

- The compressive buckling stress of an Euler-Bernoulli beam is proportional to the minimum bending stiffness $EI_{\min}$ of the cross section.
  - False
- Consider a clamped Timoshenko beam loaded by a concentrated force applied at the free extremity. The shear strain at the clamp is null.
  - False
- The torsional stiffness of a square cross section is equal to GJ, where G is the shear modulus and J the polar area moment of the cross section.
  - False

1. The elastic problem can be formulated in terms of displacements:
   (a) always;
   (b) only for statically determinate problems;
   (c) only within the framework of the finite element method;
   (d) never;
   (e) none of the above.
2. The linear static response of simply-supported beam with stiffness EJ and loaded with a uniform load:
   (a) can be analyzed by imposing symmetry conditions;
   (b) cannot be analyzed by imposing symmetry conditions because even vibration modes are anti-symmetric;
   (c) can be analyzed by imposing symmetry conditions because odd vibration modes are symmetric;
   (d) none of the above.
3. In a finite element procedure, the stress tensor:
   (a) cannot be computed;
   (b) is part of the solution;
   (c) can be recovered from the solution;
   (d) none of the above.