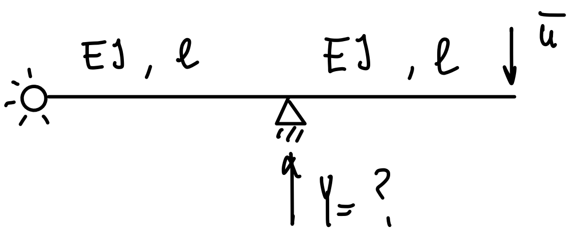
Exercise 6



Determine the reaction force in the figure.
The structure is loaded with a prescribed displacement $\bar{u}$.

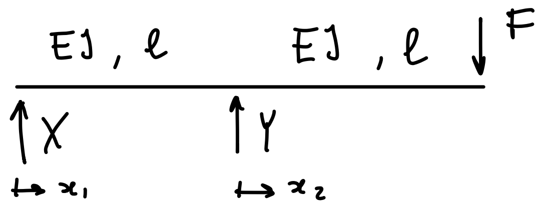
Data

$\ell = 1500\left(1 + E/_{10}\right)$ mm

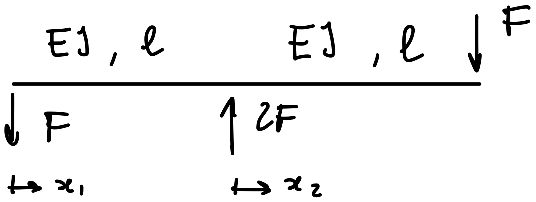
$\bar{u} = 10$ mm

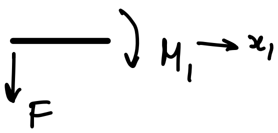
$EJ = 10^{11}$

Real system

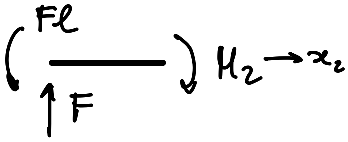


$$\begin{cases} X + Y = F \\ Yl - 2Fl = 0 \end{cases} \Rightarrow \begin{cases} X = F - Y = -F \\ Y = 2F \end{cases}$$



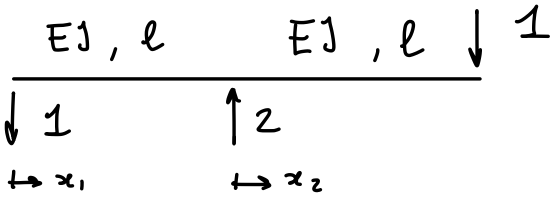


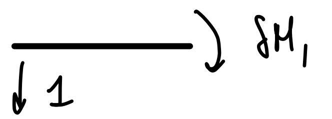
$M_1 = F x_1$



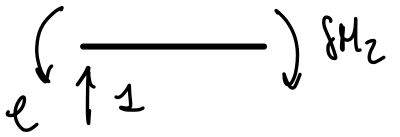
$M_2 = Fl - F x_2$

Dummy system





$\delta M_1 = x_1$



$\delta M_2 = l - x_2$

By application of the PCVW

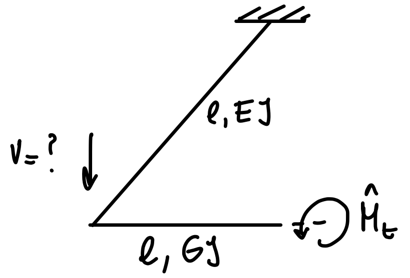
$$\int_0^l \delta M_1 M_1 / EJ \, dx_1 + \int_0^l \delta M_2 M_2 / EJ \, dx_2 = \bar{u}$$

From which:

$$F = \frac{3 EJ \bar{u}}{2 l^3} = 444.44 \text{ N}$$

And so: $Y = 2F = 888.88$ N

Exercise 7



Determine the vertical displacement (positive in the downward direction) due to a torsional moment $\hat{M}_t$

Data

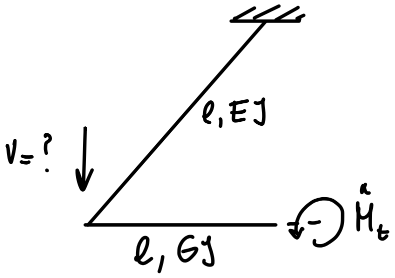
$l = 1200$ mm

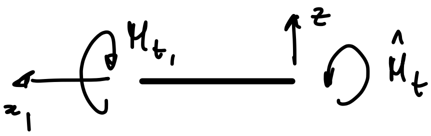
$EJ = 10^{10}$ Nmm^2

$GJ = 10^8$ Nmm^2

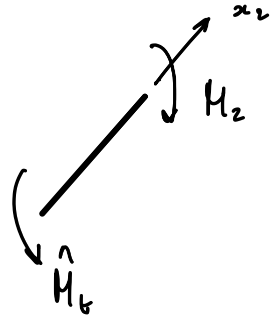
$\hat{M}_t = 10^5 (1 + F/10)$ Nmm

Real system



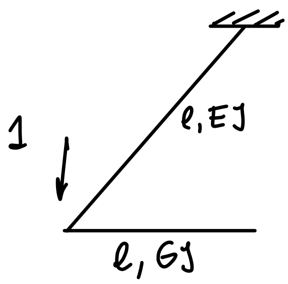


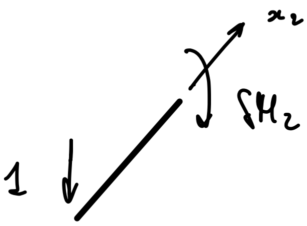
$$M_{t_1} = \hat{M}_t$$



$$M_2 = \hat{M}_t$$

Dummy system





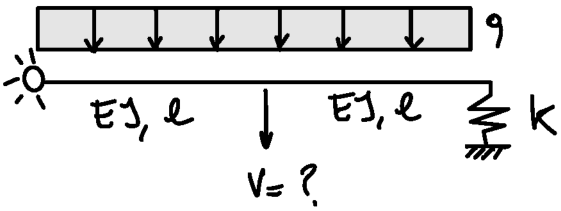
$$\delta M_2 = x_2$$

By application of the PCVW

$$\int_0^{\ell} \delta M_z \, M_z / EJ \; dx_z = V \quad , \quad \text{from which:}$$

$$V = \frac{M_t}{EJ}\frac{\ell^2}{2} = 7.20 \text{ mm}$$

Exercise 8



Determine the vertical displacement v.

Data

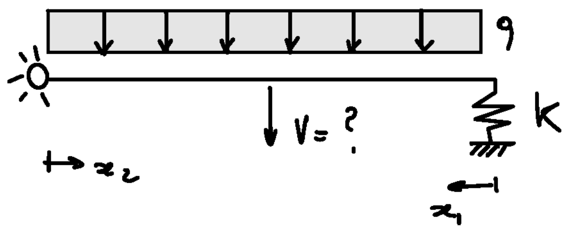
$l = 1200$ mm

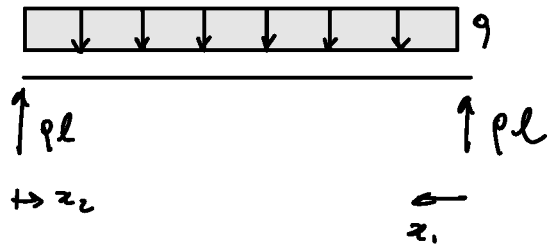
$EJ = 10^{12}$ Nmm^2

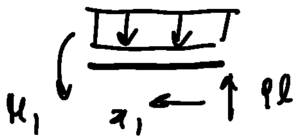
$k = 750\,(1+G/10)$ N/mm

$q = 12$ N/mm

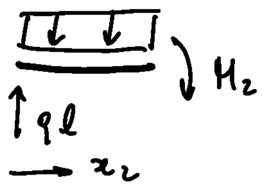
<u>Real system</u>





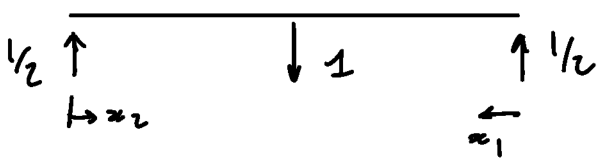


$$M_1 = -ql\,x_1 + q\,x_1^2/2$$



$$M_2 = -ql\,x_2 + q\,x_2^2/2$$

<u>Dummy system</u>



$$\delta M_1 = -\tfrac{1}{2}x_1$$

$$\delta M_2 = -\tfrac{1}{2}x_2$$

By application of the PCVW:

$$\int_0^l \delta M_1 M_1 / EJ \, dx_1 + \int_0^l \delta M_2 M_2 / EJ \, dx_2 + \delta F_s \, F_s / k = V$$

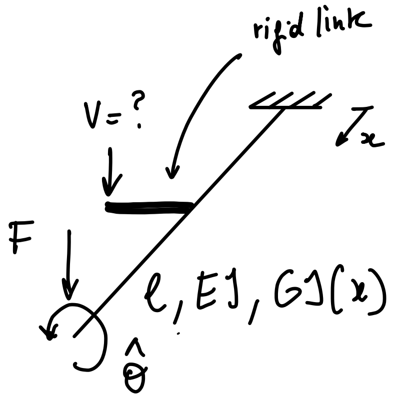
where $\delta F_s = 1/2$

$F_s = ql$

which leads to:

$$V = \frac{5}{24} \frac{ql^4}{EJ} + \frac{pl}{2k} = 14.78 \text{ mm}$$

Exercise 11



Determine the vertical displacement V in correspondence of the tip of the rigid link in the figure.

The rigid body is located at $x = l/2$, i.e. in the middle of the beam.

Use Ritz with polynomial expansion and the lowest possible number of dofs.

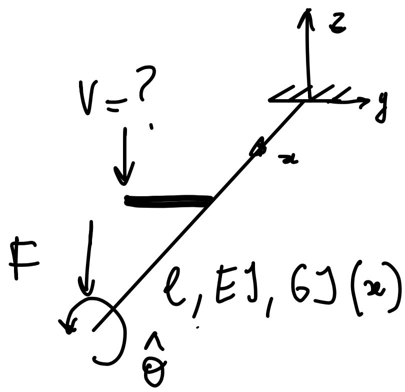
Data

$l = 1200$ mm $GJ_0 = 10^{10}$ Nmm^2 $\hat{\theta} = 10$ deg

$L = 200$ mm $GJ_1 = 2 \cdot 10^{10}$ Nmm^2

$EJ = 10^{11}$ Nmm^2 $F = 5000\,(1 + B/10)$

Solution



Essential conditions

$\theta(0)=0 \qquad w(0)=0$

$\theta(\ell)=\hat{\theta} \qquad w'(0)=0$

The trial functions are then constructed as:

1) $w = a_0 + a_1 \left(x/\ell\right) + a_2 \left(x/\ell\right)^2$

and imposing $w(0)=0$ and $w'(0)=0$ one obtains

$w = a_2 \left(x/\ell\right)^2 \qquad$ so: $w'' = a_2 \, 2/\ell^2$

2) The expansion can be represented as:

$\theta = \theta_H + \theta_p$

θ_p: term restoring the non homog. condition

θ_H: expansion where non homog conditions are replaced by homog. ones.

So: $\theta_H(0) = 0$

$\theta_H(\ell) = 0$

$\theta_H = b_0 + b_1 (x/\ell) + b_2 (x/\ell)^2$, so:

$\theta_H(0) = b_0 = 0$

$\theta_H(\ell) = b_1 + b_2 = 0 \Rightarrow b_1 = -b_2$

$\theta_H(\ell) = \left[(x/\ell)^2 - (x/\ell) \right] b_2$

$\theta_p(x) = \frac{x}{\ell} \hat{\theta}$, so:

$\theta(x) = \left[(x/\ell)^2 - (x/\ell) \right] b_2 + (x/\ell) \hat{\theta}$

$$\theta' = \left(\frac{2x}{\ell^2} - \frac{1}{\ell} \right) b_2 + \frac{1}{\ell} \hat{\theta}$$

$$= \frac{1}{\ell} \left(\frac{2x}{\ell} - 1 \right) b_2 + \frac{1}{\ell} \hat{\theta}$$

By application of the PVW

$$\int_0^\ell \delta w'' EJ w'' \, dx + \int_0^\ell \delta\theta' GJ \theta' \, dx = -\delta w(\ell) F$$

From which: $\underline{\underline{K}}\,\underline{u} = \underline{F}$, where:

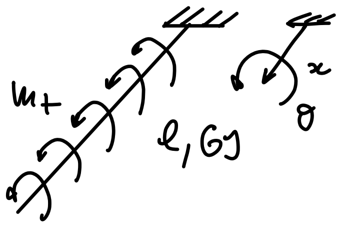
$$\underline{\underline{K}} = \begin{bmatrix} 4EJ/\ell^3 & 0 \\ 0 & (2GJ_0 + GJ_1)/6\ell \end{bmatrix}$$

$$\underline{F} = \begin{Bmatrix} -F \\ -\dfrac{GJ\hat{\theta}}{6\ell} \end{Bmatrix} \qquad \underline{u} = \begin{Bmatrix} a_2 \\ b_2 \end{Bmatrix}$$

And so the displacement reads

$$V = -w(\ell/2) + \theta(\ell/2)\,L = 27.22 \text{ mm}$$

Exercise 12



Evaluate the rotation θ at $x = l/2$ solving the problem exactly (circular bar)

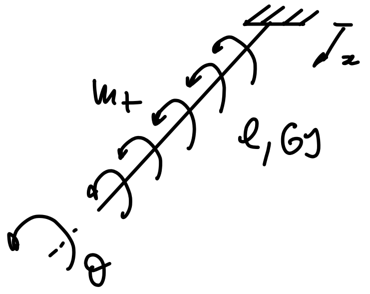
Report the result in deg

Data

$l = 1500$ mm $GJ = 10^9$ Nmm^2

$m_t = 200\,(1 + B/10)$ N

Exercise



The governing eqs. are:

$$\begin{cases} GJ\theta'' + m_t = 0 \\ \theta(0) = 0 \\ GJ\theta'(l) = 0 \end{cases}$$

The solution of the system is found as:

$$\theta = \theta_H + \theta_p$$

a) $GJ\theta_H'' = 0 \quad , \quad \theta_H' = a_0 \quad , \quad \theta_H = a_0 x + a_1$

b) $GJ\theta_p'' = -m_t \quad , \quad \theta_p' = -\dfrac{m_t}{GJ} x \quad , \quad \theta_p = -\dfrac{1}{2}\dfrac{m_t}{GJ} x^2$

So:

$$\theta = a_1 + a_0 x - \frac{1}{2}\frac{m_t}{GJ} x^2$$

Imposing:

$\theta(0) = 0 \implies a_1 = 0$

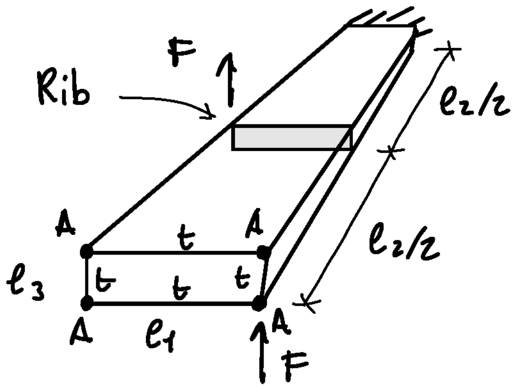
$\theta'(l) = 0 \implies a_0 = \dfrac{m_t l}{GJ}$

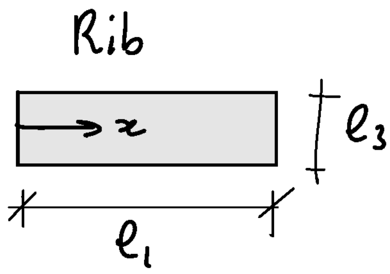
therefore: $\theta = \frac{m_t}{GJ}\left(\ell x - \frac{1}{2}x^2\right)$

and so:

$$\theta(\ell/2) = \frac{m_t}{GJ}\,\frac{3\ell^2}{8} = 9.67 \text{ deg}$$

Exercise 24





Model the rib as a beam and determine the bending moment at $x = l_1/2$

Data

$l_1 = 500$ mm

$l_2 = 1500$ mm

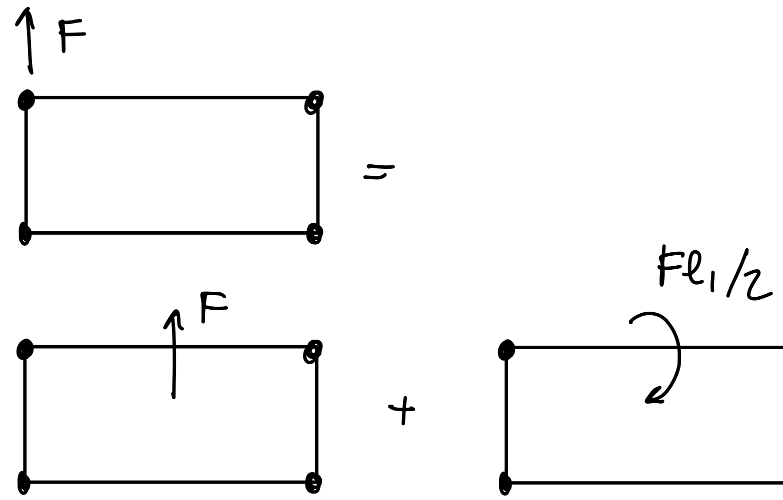
$l_3 = 150$ mm

$t = 1.0$ mm

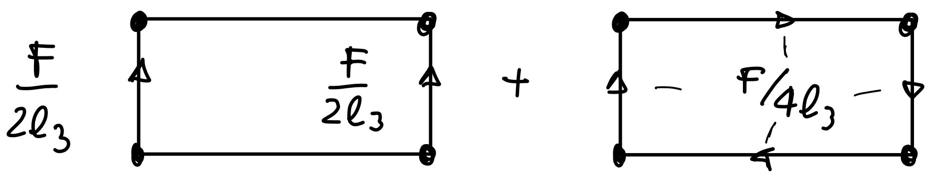
$A = 500$ mm^2

$F = 6000$ N

Solution



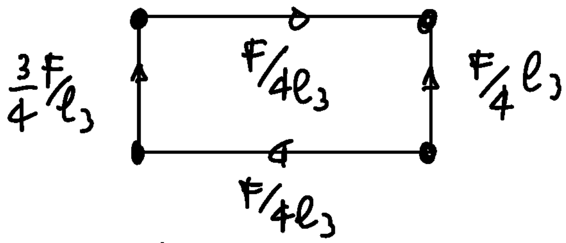
The shear flows are readily determined as:



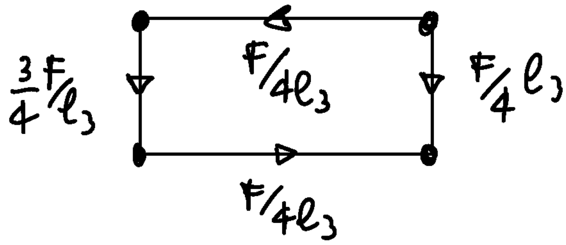
where the shear flows for torsion were derived as:

$$\frac{Fl_1}{2} = 2q\Omega_c \Rightarrow q = \frac{Fl_1}{4l_1 l_3} = \frac{F}{4l_3}$$

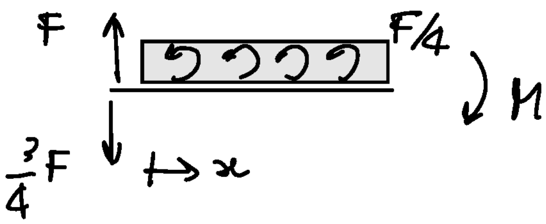
By superposition of effects



the equilibrating flows are:



So the beam model of the rib is



From which:

$$M(x) = F/4\,x - \frac{F}{4}x = 0$$

- A truss is fixed at both the ends and is loaded with a uniformly distributed axial load. The axial displacement:
  - is quadratic
- The natural boundary conditions associated with the Timoshenko beam model:
  - involve shear and bending equilibrium
- The transverse shear deformability for a thin-walled beam:
  - is generally larger with respect to a corresponding (same dimensions and bending stiffness) compact section
- The assumption of plane stress imply that the deformation along the thickness is zero:
  - False
- The equilibrium equations can be obtained by integrating by parts the Principle of Virtual Work:
  - True
- According to the semi-monocoque scheme, the shear stresses are constant along the thickness of the panel:
  - True