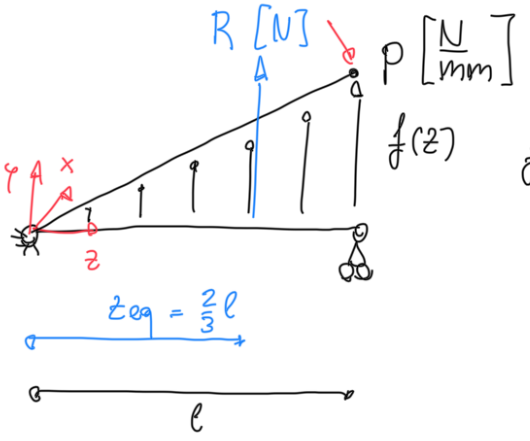
LAB 3

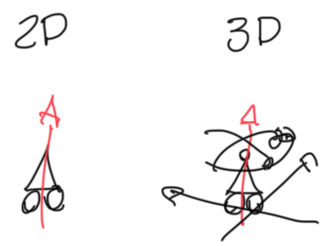
Isostatic Beams Systems II

1)

2D



$f(\ell) = p$



Let's find:
- Reaction forces
- Internal Actions

The system is Isostatic

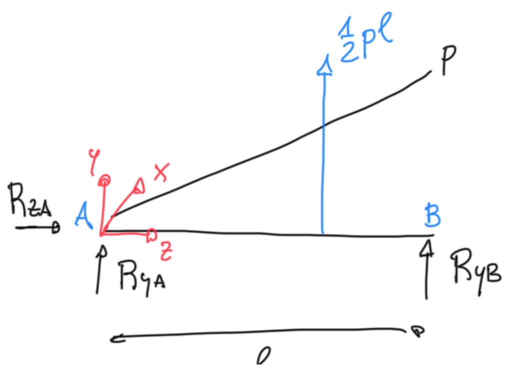
$$\left.\begin{aligned} f(z) &= m \cdot z + q \qquad m = p/\ell \\ f(\emptyset) &= \emptyset \quad \rightarrow \quad q = \emptyset \end{aligned}\right\} \rightarrow \boxed{f(z) = \frac{p}{\ell} \cdot z}$$

Eq force $R = \int_0^\ell f(z)\,dz = \int_0^\ell \frac{p}{\ell} z\,dz = \left[\frac{1}{2}\frac{p}{\ell} z^2\right]_0^\ell = \frac{1}{2} p \cdot \ell$

Eq application point

$$z_{eq} = \frac{1}{R}\int_0^\ell f(z) \cdot z \cdot dz = \frac{2}{3}\ell$$

- Reaction Forces



$z: \quad R_{zA} = \emptyset$

$y: \quad R_{yA} + R_{yB} + \frac{1}{2}p\ell = \emptyset$ *

rot x wrt A

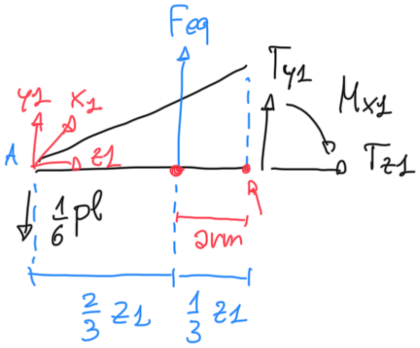
$$-R_{yB} \cdot \cancel{\ell} - \frac{1}{2}p\ell \cdot \frac{2}{3}\cancel{\ell} = \emptyset$$

$$R_{yB} = -\frac{1}{3} p\ell$$

$$* \; R_{yA} = -\frac{1}{6} p\ell$$

- Internal Actions

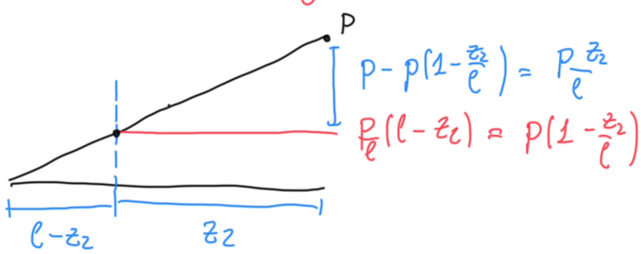
from the left



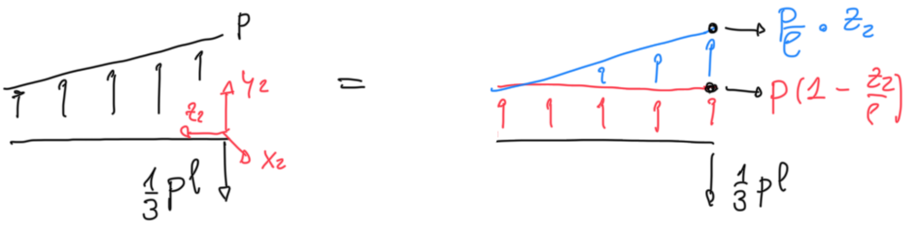
$$T_{z1} = \emptyset$$

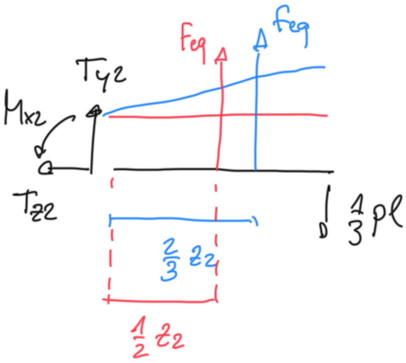
$$T_{y1} = \frac{1}{6} p\ell - \overbrace{\frac{1}{2} \overbrace{\frac{p}{\ell} \cdot z_1}^{\text{height}} \cdot \overbrace{z_1}^{\text{base}}}^{\text{area of the triangle}}$$

$$M_{x1} = \frac{1}{6} p\ell \cdot z_1 - \underbrace{\frac{1}{2} \frac{p}{\ell} z_1^2}_{\text{magnitude}} \cdot \underbrace{\frac{1}{3} z_1}_{\text{arm}}$$



from the right



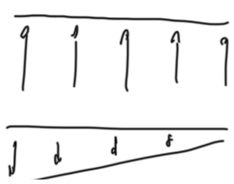


$$T_{z2} = \emptyset$$

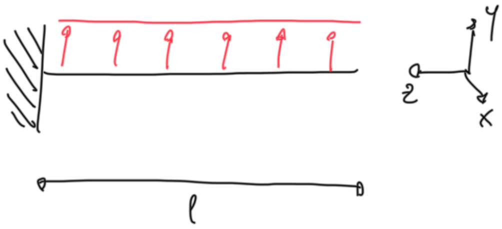
$$T_{y2} = -\frac{1}{2} \frac{p}{\ell} z_2 \cdot z_2 - p\left(1 - \frac{z_2}{\ell}\right) \cdot z_2 + \frac{1}{3} p\ell$$

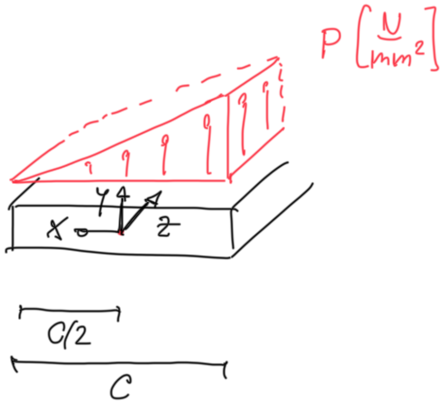
$$M_{x2} = \frac{1}{3} p\ell \cdot z_2 - \frac{1}{2} \frac{p}{\ell} z_2^2 \cdot \frac{2}{3} z_2 - p\left(1 - \frac{z_2}{\ell}\right) \cdot z_2 \cdot \frac{1}{2} z_2$$

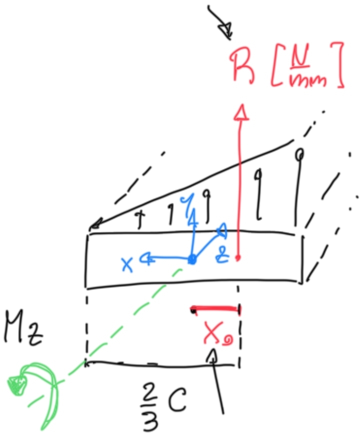
another option



2)







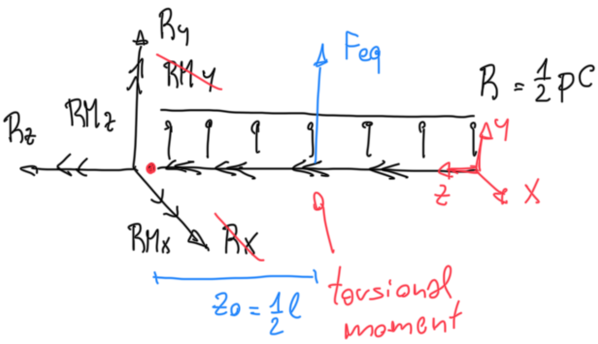
$R = \frac{1}{2}p \cdot c \qquad x_0 = \frac{1}{2}c - \frac{2}{3}c = -\frac{1}{6}c$

$\underbrace{M_z = R \cdot x_0} = -\frac{1}{2}pc \cdot \frac{1}{6}c = -\frac{1}{12}pc^2$

Moment x unit length

$\left[\frac{N}{mm^2}\, mm^2 = \frac{N.mm}{mm}\right]$

- Reaction Forces



$z: \quad R_z = \emptyset$

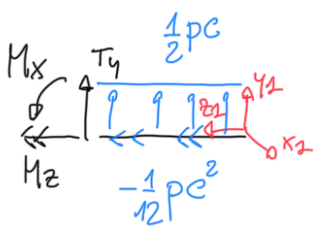
$y: \quad R_y = -\frac{1}{2}pcl$

$rot\,x: \quad RM_x = -\frac{1}{2}pcl \cdot \frac{1}{2}l$

$rot\,z: \quad RM_z = \frac{1}{12}pc^2 \cdot l$

$\left[\frac{Nmm}{mm}\right]$

- Internal Actions

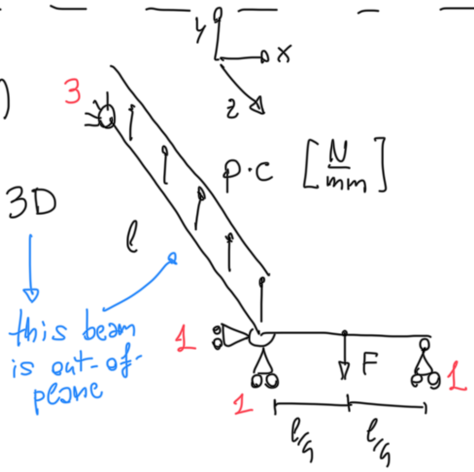


$T_y = -\frac{1}{2}pc \cdot z_1$

$M_x = -\frac{1}{2}pc z_1 \cdot \frac{1}{2}z_1$

$M_z = \frac{1}{12}pc^2 \cdot z_1$

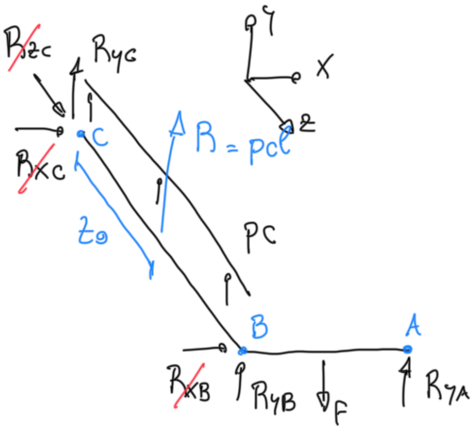
3)



find: • RF
• AI

Isostatic

• Reaction Forces



$R = pc\cdot\ell \qquad z_0 = \frac{\ell}{2}$

$z: \quad R_{zc} = \emptyset$

$x: \quad R_{xB} + R_{xc} = \emptyset \quad \rightarrow \quad R_{xB} = R_{xc} = \emptyset$

$y: \quad R_{yA} + R_{yB} + R_{yc} - F + pc\ell = \emptyset$ **

Rot Equilibrium wrt C

rot x: $\quad pc\ell\cdot\frac{\ell}{2} + R_{yB}\cdot\ell - F\cdot\ell + R_{yA}\cdot\ell = \emptyset$ *

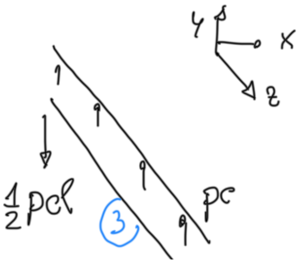
rot y: $\quad R_{xB}\cdot\ell = \emptyset$

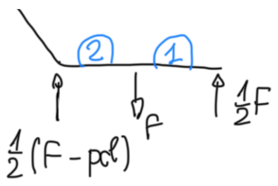
rot z: $\quad F\cdot\frac{\ell}{4} - R_{yA}\cdot\frac{\ell}{2} = \emptyset \qquad R_{yA} = \frac{1}{2}F$

* $R_{yB} = \frac{1}{2}(F - pc\ell)$

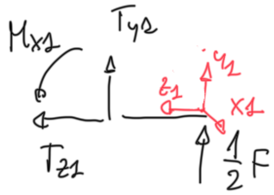
** $R_{yc} = -\frac{1}{2}pc\ell$

• Internal Actions



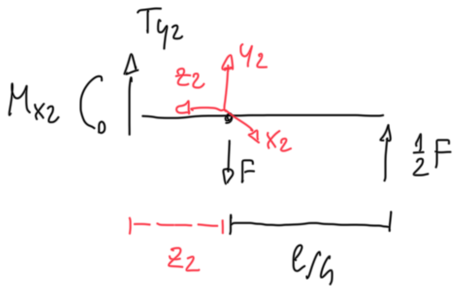


1



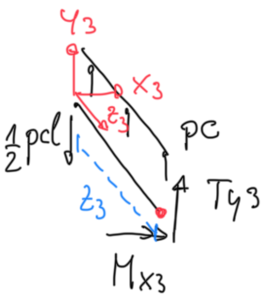
$$\begin{cases} T_{z1} = \emptyset \\ T_{y1} = -\frac{1}{2}F \\ M_{x1} = -\frac{1}{2}F \cdot z_1 \end{cases}$$

2



$$\begin{cases} T_{y2} = F - \frac{1}{2}F = \frac{1}{2}F \\ M_{x2} = F \cdot z_2 - \frac{1}{2}F \cdot \left(\frac{\ell}{4} + z_2\right) \end{cases}$$

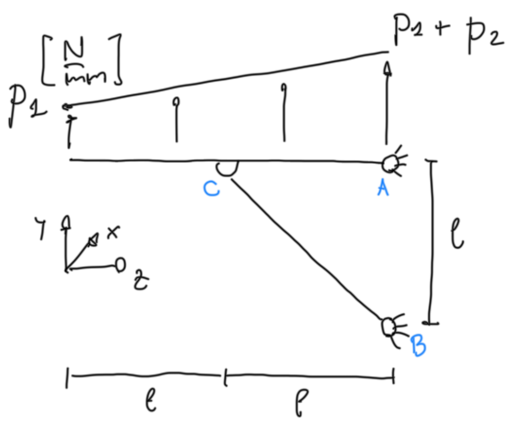
3



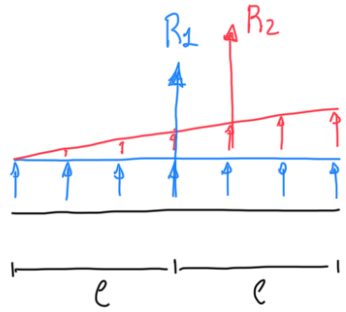
$$T_{y3} = \frac{1}{2}pc\ell - pc \cdot z_3$$

$$M_{x3} = \frac{1}{2}pc\ell \cdot z_3 - pc \cdot z_3 \cdot \frac{z_3}{2}$$

2D

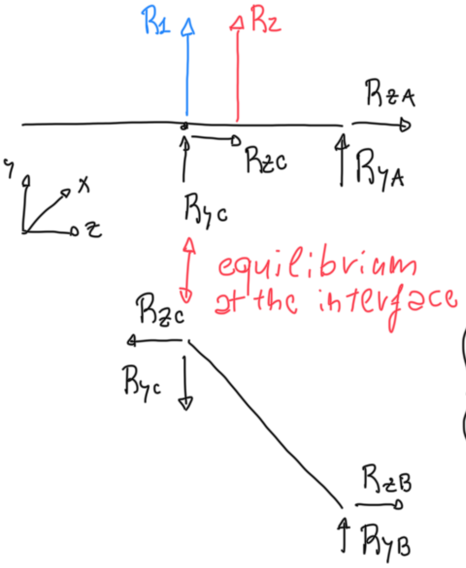


Let's find:
- Reaction Forces
- Internal Actions



$R_1 = 2p_1\ell$
$R_2 = p_2\ell$

- Reaction Forces



$$\begin{cases} z: \; R_{zC} + R_{zA} = 0 & (1) \\ y: \; R_{yC} + R_{yA} + R_1 + R_2 = 0 & (2) \\ \text{rot } X \text{ wrt } A: \; R_{yC} \cdot \cancel{\ell} + R_1 \cdot \ell + R_2 \cdot \frac{2}{3} \cancel{\ell} = 0 & (3) \end{cases}$$

$$\begin{cases} z: \; R_{zB} - R_{zC} = 0 & (4) \\ y: \; R_{yB} - R_{yC} = 0 & (5) \\ X_B: \; -R_{yC} \cancel{\ell} - R_{zC} \cancel{\ell} = 0 & (6) \end{cases}$$

(3) $R_{yC} = -R_1 - R_2 = -(2p_1 + \frac{2}{3}p_2)\ell$

(6) $R_{zC} = -R_{yC} = (2p_1 + \frac{2}{3}p_2)\ell$

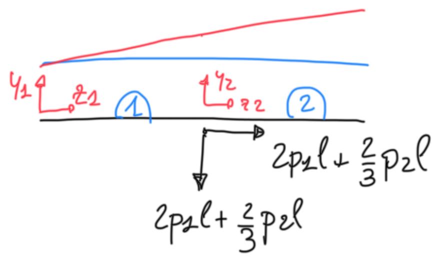
(1) $R_{zA} = -R_{zC} = -(2p_1 + \frac{2}{3}p_2)\ell$

(2) $R_{yA} = -R_{yC} - R_1 - R_2 = (2p_1 + \frac{2}{3}p_2)\ell - 2p_1\ell - p_2\ell = -\frac{1}{3}p_2\ell$

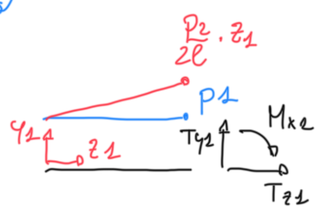
(4) $R_{zB} = R_{zC} = (2p_1 + \frac{2}{3}p_2)\ell$

(5) $R_{yB} = R_{yC} = -(2p_1 + \frac{2}{3}p_2)\ell$

- Internal Actions

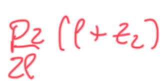


(1)

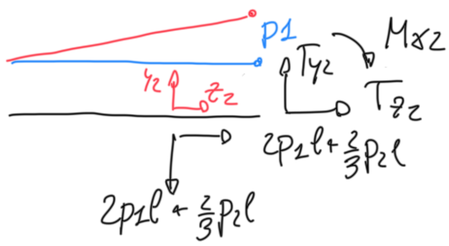


$$\begin{cases} T_{z1} = 0 \\ T_{y1} = -p_1 \cdot z_1 - \frac{1}{2} \cdot \frac{p_2}{2\ell} \cdot z_1 \\ M_{x1} = -\frac{1}{2} p_1 z_1^2 - \frac{1}{6} \frac{p_2}{2\ell} z_1^2 \end{cases}$$

(2)



$T_{z2} = 0$



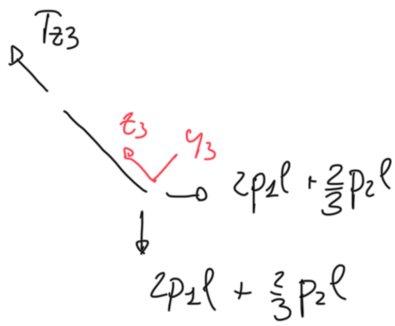
$$T_{y2} = -p_1(l+z_2) - \frac{1}{2}\frac{p_2}{2l}(l+z_2) + 2p_1 l + \frac{2}{3}p_2 l$$

$$T_{z2} = -2p_1 l - \frac{2}{3}p_2 l$$

$$T_{y2} = -p_1(l+z_2) - \frac{1}{2}\frac{p_2}{2l}(l+z_2) + 2p_1 l + \frac{2}{3}p_2 l$$

$$M_{x2} = -\frac{1}{2}p_1(l+z_2)^2 - \frac{1}{6}\frac{p_2}{2l}(l+z_2)^2 + \left(2p_1 l + \frac{2}{3}p_2 l\right)\cdot z_2$$

3



$$T_{y3} = \varnothing$$

$$M_{x3} = \varnothing$$

$$T_{z3} = \sqrt{2}\cdot\left(2p_1 l + \frac{2}{3}p_2 l\right)$$