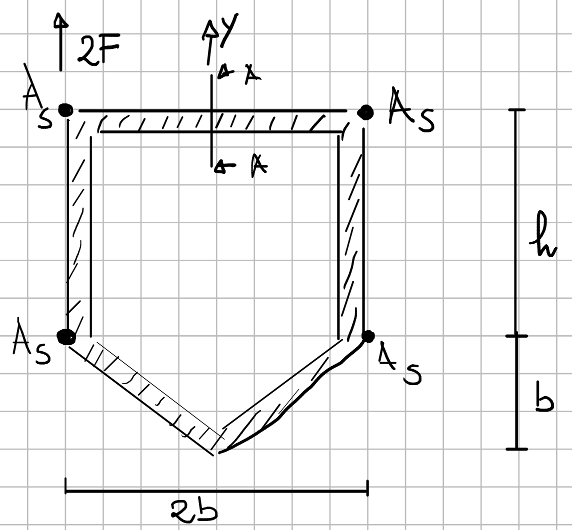
EXERCISE SESSION 10 — 02/12/22

Ribs Frames & Junctions



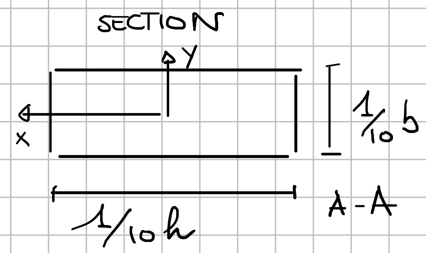
Evaluate internal action in rib (closed cell rib)

$A_s = 200 \ mm^2$

$h = 200 \ mm$

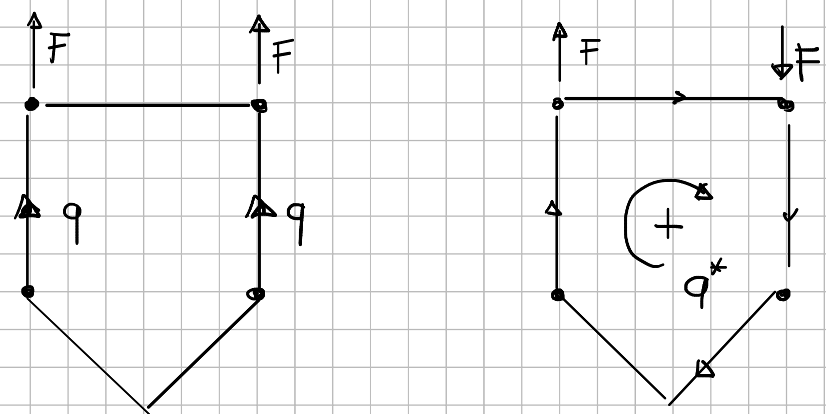
$b = 100 \ mm$

$F = 4000 \ N$



$E = 77000 \ MPa$

SEPARATE LOADS (SYMM. & ANTISYMM.)



SYMM.

ANTISYMM

$$q = \frac{T_y}{2h} = \frac{2F}{2h}$$

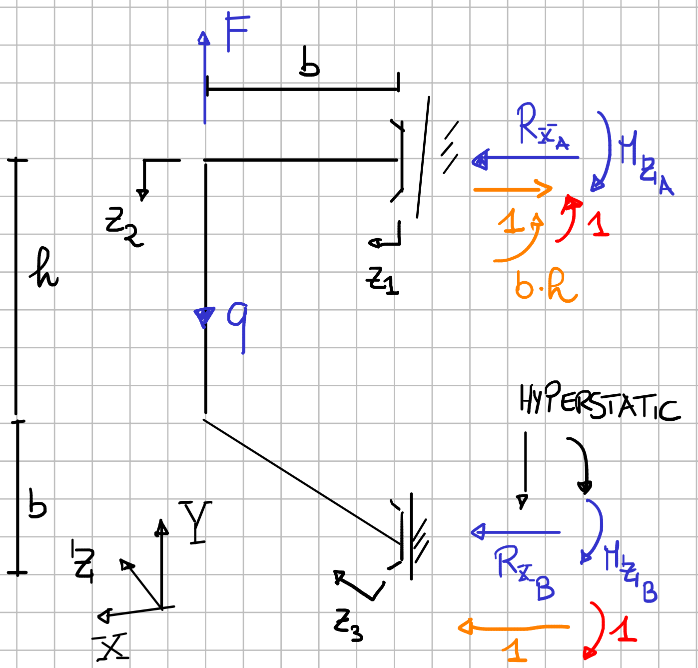
$$q^* = \frac{M_z}{2\Omega}$$

$$M_z = F \cdot 2b$$

$$\Omega = 2b \cdot h + \frac{1}{2} 2b \cdot b$$

INT. ACTIONS IN RIB.

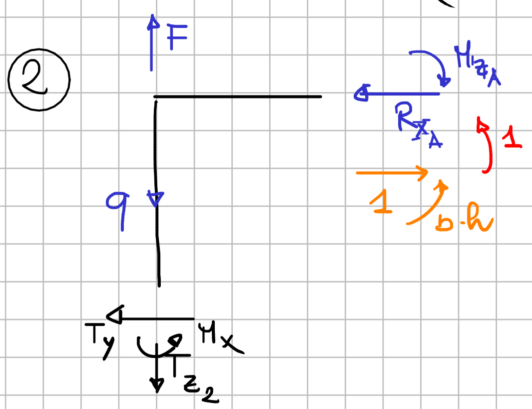
SYMM.



EQUILIBRIUM

$$\begin{cases} R_{\bar{X}_A} = - R_{\bar{X}_B} \\ M_{z_A} = - M_{z_B} - \cancel{F \cdot b} + \cancel{q \cdot h} \cdot b - R_{\bar{X}_B}(h+b) \end{cases}$$

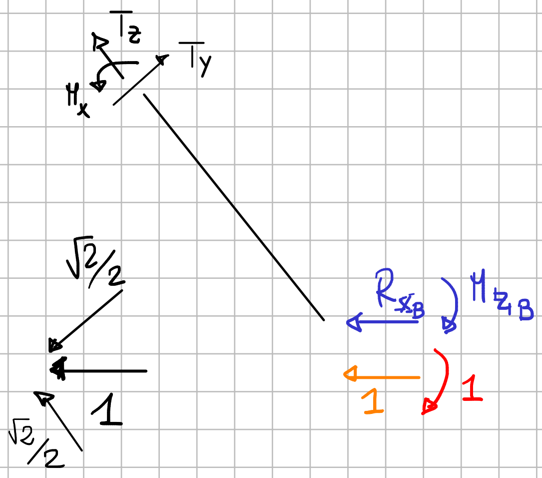
$$\begin{cases} T_{z_1} = - R_{\bar{X}_A} \\ T_{y_1} = 0 \\ M_{x_1} = M_{z_A} \end{cases} \qquad \begin{cases} T'_{z_1} = 1 \\ T'_{y_1} = 0 \\ M'_{x_1} = - b \cdot h \end{cases} \qquad \begin{cases} T''_{z_1} = 0 \\ T''_{y_1} = 0 \\ M''_{x_1} = -1 \end{cases}$$



$$\begin{cases} T_{z_2} = F - q \cdot z_2 \\ T_{y_2} = - R_{\bar{X}_A} \\ M_{x_2} = - R_{\bar{X}_A} \cdot z_2 + M_{z_A} \end{cases}$$

$$\begin{cases} T'_{z_2} = 0 \\ T'_{y_2} = 1 \\ M'_{x_2} = 1 \cdot z_2 - b \cdot h \end{cases} \qquad \begin{cases} T''_{z_2} = 0 \\ T''_{y_2} = 0 \\ M''_{x_2} = -1 \end{cases}$$

③



$$\begin{cases} T_{z_3} = -\dfrac{\sqrt{2}}{2} R_{\bar{x}_B} \\ T_{y_3} = \dfrac{\sqrt{2}}{2} R_{\bar{x}_B} \\ M_{x_3} = M_{z_B} + \dfrac{\sqrt{2}}{2} R_{\bar{x}_B} \cdot z_3 \end{cases} \qquad \begin{cases} T_{z_3}' = -\dfrac{\sqrt{2}}{2} \\ T_{y_3}' = \sqrt{2}/2 \\ M_{x_3}' = \sqrt{2}/2 \cdot z_3 \end{cases} \qquad \begin{cases} T_{z_3}'' = 0 \\ T_{y_3}'' = 0 \\ M_{x_3}'' = 1 \end{cases}$$

PCVW REAL DUMMY 1

$$\delta W_{e_1} = 0$$

$$\delta W_{i_1} = \sum_i \int_{l_i} T_{z_i}' \frac{T_{z_i}}{EA} dz_i + \sum_i \int_{l_i} M_{x_i}' \frac{M_{x_i}}{EJ} dz_i$$

$$l_1 = b \qquad l_2 = h \qquad l_3 = \sqrt{2} \cdot b$$

$$A = \frac{1}{100} b \cdot h \qquad J = \frac{1}{12} \frac{1}{10} \cdot h \left(\frac{1}{10} b\right)^3$$

PCVW REAL - DUMMY

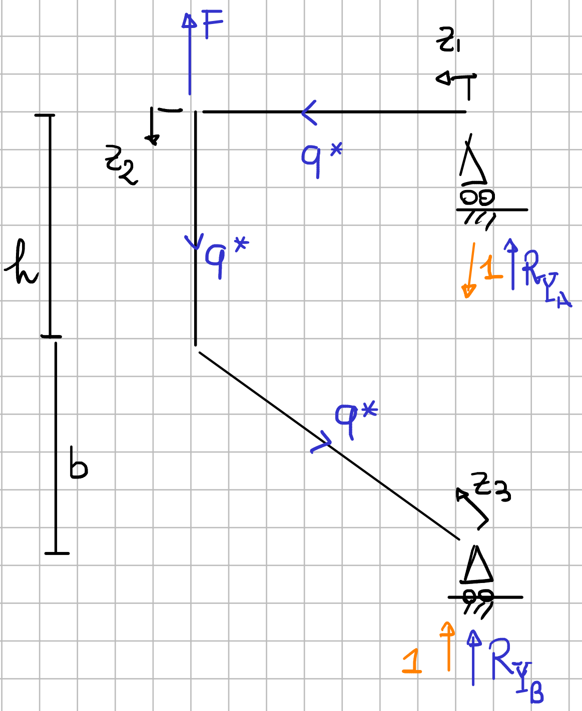
$$\delta W_{e_2} = 0$$

$$\delta W_{i_2} = \sum_i \int_{l_i} T_{z_i}'' \frac{T_{z_i}}{EA} dz_i + \sum_i \int_{l_i} M_{x_i}'' \frac{M_{x_i}}{EJ} dz_i$$

SOLVE

$$\begin{cases} \delta W_{e_1} = \delta W_{i_1} \\ \delta W_{e_2} = \delta W_{i_2} \end{cases} \longrightarrow \begin{matrix} R_{\bar{x}_B} = 0 \\ M_{z_B} = 0 \end{matrix}$$

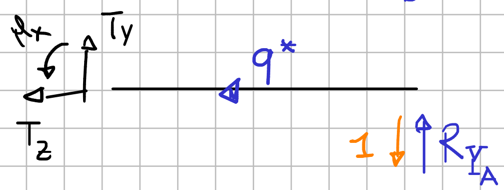
ANTISYMM.



EQUILIBRIUM

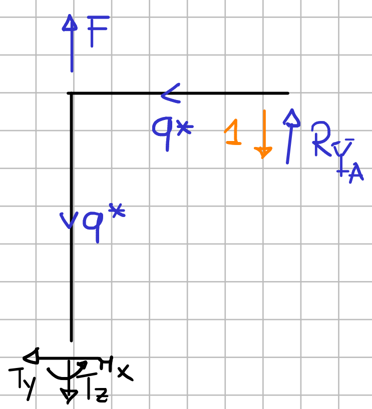
$$R_{\bar{Y}_A} = - R_{\bar{Y}_B} - F + q^* \cdot h + q^* \cdot b$$

1



$$\begin{cases} T_{z_1} = - q^* \cdot z_1 \\ T_{y_1} = - R_{\bar{Y}_A} \\ M_{x_1} = - R_{\bar{Y}_A} \cdot z_1 \end{cases} \qquad \begin{cases} T'_{z_1} = 0 \\ T'_{y_1} = 1 \\ M'_{x_1} = 1 \cdot z_1 \end{cases}$$

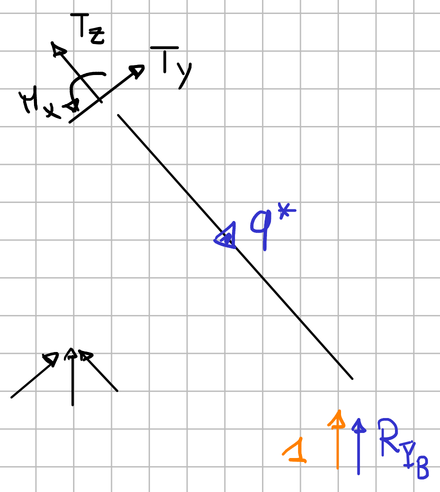
2



$$\begin{cases} T_{z_2} = F - q^* \cdot z_2 + R_{\bar{Y}_A} \\ T_{y_2} = - q^* \cdot b \\ M_{x_2} = - q^* \cdot b \cdot z_2 - R_{\bar{Y}_A} \cdot b \end{cases}$$

$$\begin{cases} T'_{z_2} = -1 \\ T'_{y_2} = 0 \\ M'_{x_2} = 1 \cdot b \end{cases}$$

③



$$\begin{cases} T_{z_3} = -\dfrac{\sqrt{2}}{2} R_{Y_B} + q^* \cdot z_3 \\ T_{y_3} = -\dfrac{\sqrt{2}}{2} R_{Y_B} \\ M_{x_3} = -\dfrac{\sqrt{2}}{2} R_{Y_B} \cdot z_3 \end{cases}$$

$$\begin{cases} T'_{z_3} = -\sqrt{2}/2 \\ T'_{y_3} = -\sqrt{2}/2 \\ M'_{x_3} = -\dfrac{\sqrt{2}}{2} \cdot z_3 \end{cases}$$

PCVW

$$\delta W_e = 0$$

$$\delta W_i = \sum_i \int_{l_i} T'_{z_i} \cdot \frac{T_{z_i}}{EA} \, dz_i + \sum_i \int_{l_i} M'_{x_i} \cdot \frac{M_{x_i}}{EJ} \, dz_i$$

SOLVING $\delta W_e = \delta W_i \longrightarrow R_{Y_B}$

FIND THE INT. ACTIONS BY SUBSTITUTING R_{x_B}, R_{Y_B}, $M_{z_{1B}}$

THE INT. ACTIONS ARE SUPERPOSITION OF THE TWO SYMM + ANTISYMM

Contents


```
clear variables
close all
home
```

Data

```
A_s = 200;  % mm^2
h = 200;    % mm
b = 100;    % mm
F = 4000;   % N
E = 77000;  % MPa
A = h*b/100;    % mm^2
J = (h/10)*(b/10)^3/12; % mm^4
```

Fluxes

```
q = 2*F/2/h;
M_z = F*2*b;
Omega = 2*b*h+.5*2*b*b;
q_s = M_z/2/Omega;
```

Internal action symmetric

```
syms R_XB M_ZB z_1 z_2 z_3

R_XA = -R_XB;
M_ZA = -M_ZB - R_XB*(h+b);

symm.T_z1 = -R_XA;
symm.T_y1 = 0;
symm.M_x1 = M_ZA;

symm.T_z1_p = 1;
symm.T_y1_p = 0;
symm.M_x1_p = -b*h;

symm.T_z1_pp = 0;
symm.T_y1_pp = 0;
symm.M_x1_pp = -1;



symm.T_z2 = F-q*z_2;
symm.T_y2 = -R_XA;
symm.M_x2 = M_ZA - R_XA*z_2;
```

```
symm.T_z2_p = 0;
symm.T_y2_p = 1;
symm.M_x2_p = z_2 - b*h;

symm.T_z2_pp = 0;
symm.T_y2_pp = 0;
symm.M_x2_pp = -1;



k = .5*sqrt(2);
symm.T_z3 = -k*R_XB;
symm.T_y3 = k*R_XB;
symm.M_x3 = M_ZB + k*R_XB*z_3;

symm.T_z3_p = -k;
symm.T_y3_p = k;
symm.M_x3_p = k*z_3;

symm.T_z3_pp = 0;
symm.T_y3_pp = 0;
symm.M_x3_pp = 1;
```

PCVW

```
LHS_PCVW1 = 0;
RHS_PCVW1 = int(symm.T_z1_p*symm.T_z1/E/A, z_1, 0, b) + ...
    int(symm.M_x1_p*symm.M_x1/E/J, z_1, 0, b) + ...
    int(symm.T_z2_p*symm.T_z2/E/A, z_2, 0, h) + ...
    int(symm.M_x2_p*symm.M_x2/E/J, z_2, 0, h) + ...
    int(symm.T_z3_p*symm.T_z3/E/A, z_3, 0, 2*k*b) + ...
    int(symm.M_x3_p*symm.M_x3/E/J, z_3, 0, 2*k*b);

LHS_PCVW2 = 0;
RHS_PCVW2 = int(symm.T_z1_pp*symm.T_z1/E/A, z_1, 0, b) + ...
    int(symm.M_x1_pp*symm.M_x1/E/J, z_1, 0, b) + ...
    int(symm.T_z2_pp*symm.T_z2/E/A, z_2, 0, h) + ...
    int(symm.M_x2_pp*symm.M_x2/E/J, z_2, 0, h) + ...
    int(symm.T_z3_pp*symm.T_z3/E/A, z_3, 0, 2*k*b) + ...
    int(symm.M_x3_pp*symm.M_x3/E/J, z_3, 0, 2*k*b);

[R_XB_d, M_ZB_d] = solve([LHS_PCVW1 == RHS_PCVW1; LHS_PCVW2 == RHS_PCVW2], ...
    [R_XB, M_ZB]);

R_XB_d = double(R_XB_d)
M_ZB_d = double(M_ZB_d)
```

```
R_XB_d =

     0


M_ZB_d =

     0
```

Internal action antisymm

```
syms R_YB

R_YA = -R_YB - F + q_s*h + q_s*b;

asymm.T_z1 = -q_s*z_1;
asymm.T_y1 = -R_YA;
asymm.M_x1 = -R_YA*z_1;

asymm.T_z1_p = 0;
asymm.T_y1_p = 1;
asymm.M_x1_p = z_1;



asymm.T_z2 = F - q_s*z_2 + R_YA;
asymm.T_y2 = - q_s*b;
asymm.M_x2 = - R_YA*b - q_s*b*z_2;

asymm.T_z2_p = -1;
asymm.T_y2_p = 0;
asymm.M_x2_p = b;



asymm.T_z3 = -k*R_YB + q_s*z_3;
asymm.T_y3 = -k*R_YB;
asymm.M_x3 = -k*R_YB*z_3;

asymm.T_z3_p = -k;
asymm.T_y3_p = -k;
asymm.M_x3_p = -k*z_3;



LHS_PCVW1 = 0;
RHS_PCVW1 = int(asymm.T_z1_p*asymm.T_z1/E/A, z_1, 0, b) + ...
    int(asymm.M_x1_p*asymm.M_x1/E/J, z_1, 0, b) + ...
    int(asymm.T_z2_p*asymm.T_z2/E/A, z_2, 0, h) + ...
    int(asymm.M_x2_p*asymm.M_x2/E/J, z_2, 0, h) + ...
    int(asymm.T_z3_p*asymm.T_z3/E/A, z_3, 0, 2*k*b) + ...
    int(asymm.M_x3_p*asymm.M_x3/E/J, z_3, 0, 2*k*b);

R_YB_d = solve(LHS_PCVW1 == RHS_PCVW1, R_YB);
R_YB_d = double(R_YB_d)
```

```
R_YB_d =

 -758.8885
```