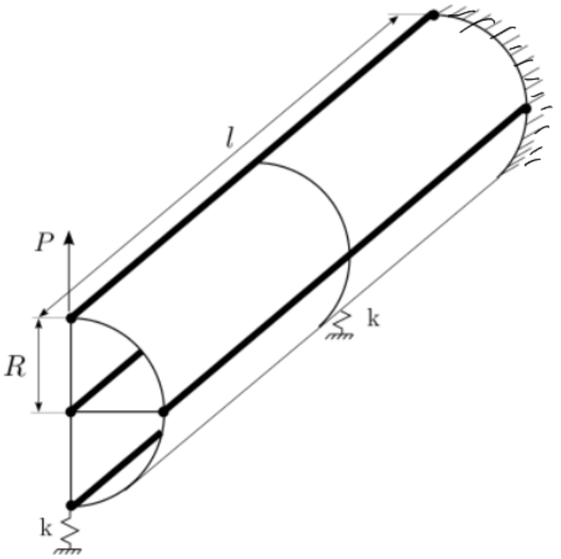
EXERCISE SESSION 8 – 18/11/22

Semi monocoque Approximation

Exercise 1

The thin-walled beam in the figure is characterized by a semi-circular section with five webs of thickness t and four stringers of area A (note: A is the equivalent stringer area, inclusive of the contribution due to the panels). The length of the beam is denoted with l. The material is homogeneous and isotropic with Young modulus E and Poisson's ratio 0.3. The beam is grounded with two linear elastic springs of stiffness k, as reported in the sketch, and is loaded with a concentrated force P applied at one of the two ends. Determine the reaction forces in correspondence of the springs by accounting for the beam shear deformability, and plot the internal actions in the beam.

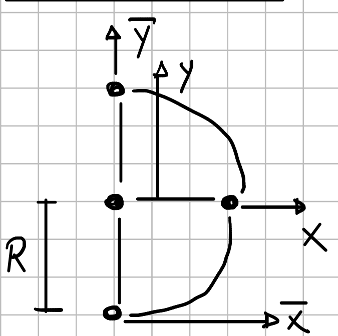


Data
$E = 70\ GPa$; $\nu = 0.3$;
$A = 200\ mm^2$; $t = 1\ mm$;
$R = 200\ mm$; $l = 2000\ mm$;
$k = 10^6\ N/mm$; $P = 1\ kN$;

COMPUTE G

$$G = \frac{E}{2(1+\nu)} = 26\,923\ MPa$$

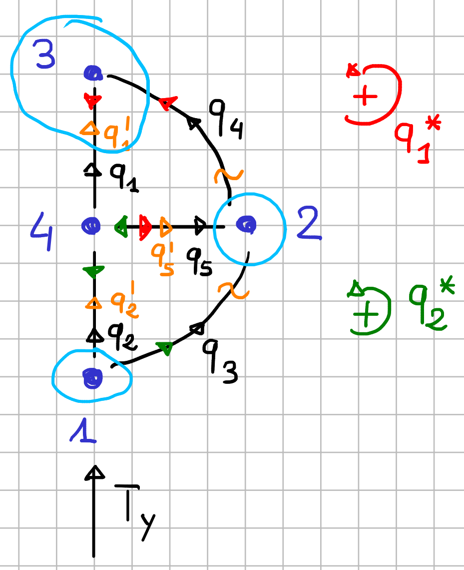
CENTROID



$\bar{y}_{CG} = R$ ~ SYMMETRY

$$\bar{x}_{CG} = \frac{RA}{4A} = \frac{1}{4}R$$

INERTIAS



$$J_{xx} = 2AR^2$$

$$S_{x_1} = -AR$$

$$S_{x_3} = AR$$

$$S_{x_2} = S_{x_4} = 0$$

OPEN CELL FLUXES

$$q_1' = +T_y \frac{S_{x_3}}{J_{xx}} = \frac{T_y}{2R} \qquad q_5' = +T_y \frac{S_{x_2}}{J_{xx}} = 0$$

$$q_2' = -T_y \frac{S_{x_1}}{J_{xx}} = \frac{T_y}{2R}$$

MOM EQUIVALENCE WRT 4

$$\text{LHS\_MOM} = 0$$

$$\text{RHS\_MOM} = 2q_1^* \cdot \Omega_{CELL_1} + 2q_2^* \cdot \Omega_{CELL_2}$$

WHERE $\Omega_{CELL_1} = \Omega_{CELL_2} = \frac{1}{4} R^2 \cdot \pi$

$\rightarrow$ LHS_MOM = RHS_MOM

COMPATIBILITY

$$\vartheta_1' = \frac{1}{2\Omega_{CELL_1} Gt} \left(q_1^* \cdot \left(2R + \frac{\pi R}{2}\right) - q_2^* \cdot R - q_1' \cdot R \right)$$

$$\vartheta_2' = \frac{1}{2\Omega_{CELL_2} Gt} \left(q_2^* \cdot \left(2R + \frac{\pi R}{2}\right) - q_1^* \cdot R - q_2' \cdot R \right)$$

IMPOSE

$$\begin{cases} LHS\_MOM = RHS\_MOM \\ \vartheta_1' = \vartheta_2' \end{cases} \rightarrow \text{FIND } q_1^* \,\&\, q_2^* \text{ AS FUNCTION OF } T_y$$

$$q_1^* = q_2^* = 0$$

CLOSED CELL FLUXES

$$q_1 = q_1' - q_1^* \qquad q_3 = q_2^* \qquad q_5 = q_5' + q_1^* - q_2^*$$

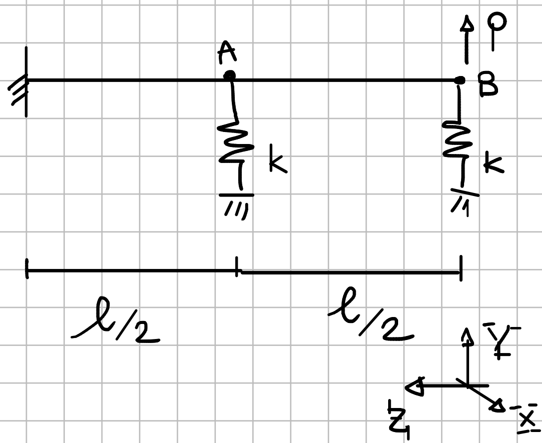
$$q_2 = q_2' - q_2^* \qquad q_4 = q_1^*$$

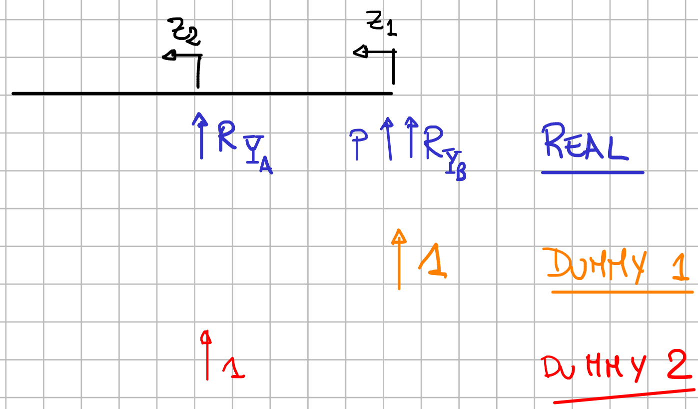
HYPERSTATIC REACTIONS → FIRST SOLUTION METHOD

Take into account deformation work due to shear in the panels

$$\text{PCVW}: \quad \delta W_i = \int_0^l M_x' \cdot \frac{M_x}{EJ_{xx}} dz + \int_V \frac{\delta\tau\,\tau}{G} dV$$

INTERNAL ACTION OF BEAM





INTERNAL ACTIONS IN 1

$$T_{y_1}(z_1) = -R_{Y_B} - P \qquad M_{x_1}(z_1) = -(R_{Y_B} + P) \cdot z_1$$

$$T_{y_1}'(z_1) = -1 \qquad M_{x_1}'(z_1) = -1 \cdot z_1$$

INTERN. ACTS IN ②

$$T_{y_2}(z_2) = -R_{Y_B} - P - R_{Y_A}$$

$$T'_{y_2}(z_2) = -1$$

$$T''_{y_2}(z_2) = -1$$

$$M_{x_2}(z_2) = -\left(z_2 + \frac{\ell}{2}\right)\left(R_{Y_B} + P\right) - z_2 \cdot R_{Y_A}$$

$$M'_{x_2}(z_2) = -\left(z_2 + \frac{\ell}{2}\right) \cdot 1$$

$$M''_{x_2}(z_2) = -z_2 \cdot 1$$

PCVW REAL DUMMY 1

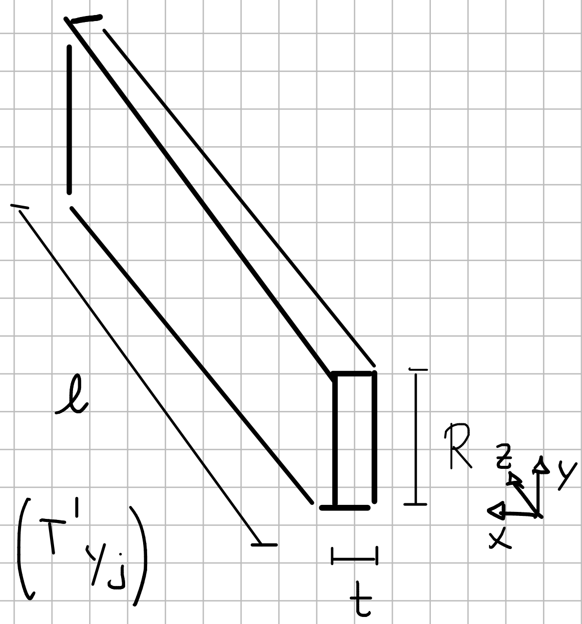
$$\delta W_e = \underbrace{-1}_{} \cdot \underbrace{\frac{R_{Y_B}}{k}}_{}$$

$$\delta W_i = \int_0^{\ell/2} M'_{x_1} \cdot \frac{M_{x_1}}{EJ_{xx}} dz_1 + \int_0^{\ell/2} M'_{x_2} \cdot \frac{M_{x_2}}{EJ_{xx}} dz_2 + $$

$$+ \int_V \frac{\delta\tau \cdot \tau}{G} dV$$

WHERE $\displaystyle\int_V \frac{\delta\tau \cdot \tau}{G} dV$

$$= \int_\ell \int_R \int_t \frac{\delta\tau \cdot \tau}{G} dx\,dy\,dz$$



WE KNOW $\underbrace{\tau = \frac{q}{t}}_{}$ $\underbrace{\delta\tau = \frac{\delta q}{t}}_{}$

$$q = q_i\left(T_{y_j}\right) \quad \& \quad \delta q = q_i\left(T'_{y_j}\right)$$

$$= \int_\ell \frac{\delta q \cdot q}{t^2 G} \cdot \int_R \int_t dx\,dy\,dz = \int_\ell \frac{R}{tG} \underbrace{\delta q}_{} \underbrace{q}_{} dz$$

$$\rightarrow \int_V \frac{\delta\tau \cdot \tau}{G} dV = \int_0^{\ell/2} \frac{R}{tG} \cdot \underline{q_1 (T_{y_1}')} \cdot \underline{q_1 (T_{y_1})} dz_1 +$$

$$+ \int_0^{\ell/2} \frac{R}{tG} \, q_2 (T_{y_1}') \cdot q_2 (T_{y_1}) dz_1 +$$

$$+ \int_0^{\ell/2} \frac{R}{tG} \, q_1 (T_{y_2}') \cdot q_1 (T_{y_2}) dz_2 +$$

$$+ \int_0^{\ell/2} \frac{R}{tG} \, q_2 (T_{y_2}') \cdot q_2 (T_{y_2}) dz_2$$

PCVW REAL DUMMY 2

$$\delta W_e = -1 \cdot \frac{R_{Y_A}}{K}$$

$$\delta W_i = \int_0^{\ell/2} M_{x_2}'' \frac{M_{x_2}}{EJ_{xx}} dz_2 + \int_0^{\ell/2} \frac{R}{tG} \, q_1 (T_{y_2}'') q_1 (T_{y_2}) dz_2 +$$

$$+ \int_0^{\ell/2} \frac{R}{tG} \, q_2 (T_{y_2}'') q_2 (T_{y_2}) dz_2$$

SOLVE

$$\begin{cases} \text{PCVW 1} \quad (\delta W_e = \delta W_i) \\ \text{PCVW 2} \quad (\delta W_e = \delta W_i) \end{cases} \rightarrow \begin{cases} R_{Y_A} = -2.7460 \text{ N} \\ R_{Y_B} = -998.7155 \text{ N} \end{cases}$$

SECOND METHOD (SECTION PROPERTIES)

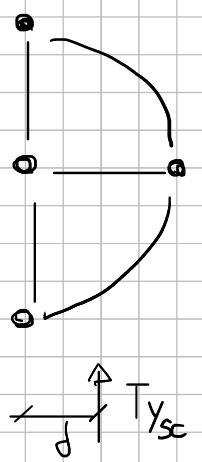
We aim to PCVW

$$\delta W_i = \int_\ell \frac{T_y' T_y}{GA^*} dz + \int_\ell \frac{M_z' M_z}{GJ} dz + \int_\ell \frac{M_x' M_x}{EJ_{xx}} dz$$

NOT AVAILABLE : GA^* GJ

M_z POSITION OF SHEAR CENTER

GA^* & SHEAR CNTR POSITION



$$\begin{cases} LHS\_MOM = RHS\_MOM & \text{WRT } 4 \\ \vartheta_1' = 0 & \rightarrow d, q_1^*, q_2^* \\ \vartheta_2' = 0 \end{cases}$$

$$T_{y_{sc}} = 1$$

$LHS\_MOM = T_{y_{sc}} \cdot d$

$RHS\_MOM =$ SAME AS BEFORE

ϑ_1' IS NOT CHANGED

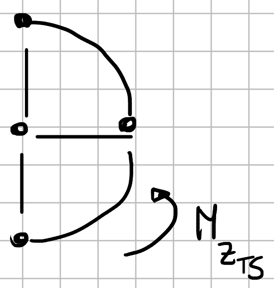
ϑ_2' IS NOT CHANGED

$$GA^* = G \cdot \frac{T_{y_{sc}}^2}{\sum q^2 \cdot L/t}$$

L=LENGTH OF PANEL IN SECTION

$R, \frac{1}{4}\pi 2R$

GJ



TAKE $M_{z_{TS}} = 1 \qquad T_{y_{TS}} = 0$

COMPUTE FLUXES WITH $T_y = T_{y_{TS}} = 0$

REDEFINE $LHS\_MOM = M_{z_{TS}}$

SOLVE $\begin{cases} LHS\_MOM = RHS\_MOM \\ \vartheta_1' = \vartheta_2' \end{cases} \rightarrow q_1^*,\ q_2^*$

COMPUTE ϑ_1' EQUAL ϑ_2'

$GJ = \dfrac{M_{z_{TS}}}{\vartheta_1'}$ COMING FROM $M_{z_{TS}} = GJ \cdot \vartheta_1'$

INTERNAL ACTIONS

T_y, T_y', M_x, M_x' ARE IDENTICAL AS THE ONES BEFORE

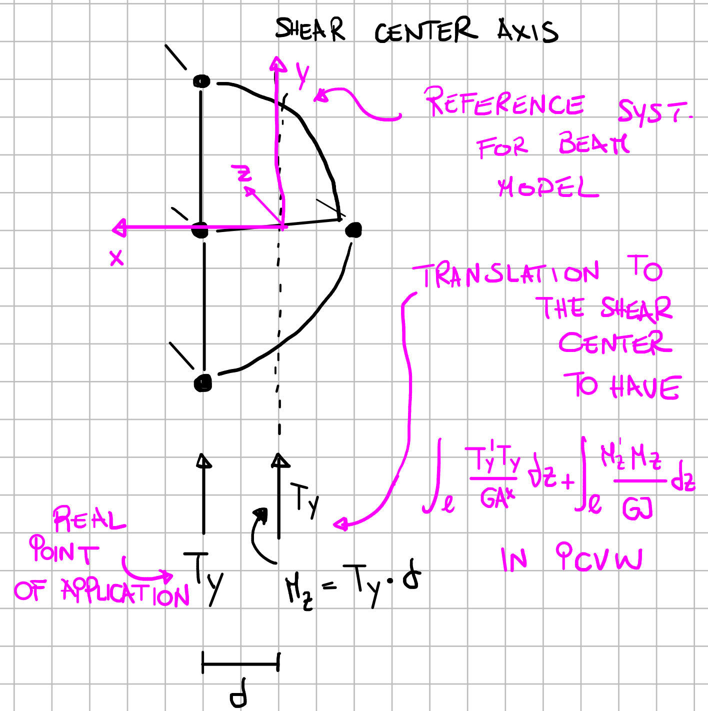
$$M_{z_1}(z_1) = -(R_{Y_B} + P) \cdot d$$

$$M_{z_1}'(z_1) = -1 \cdot d$$

$$M_{z_2}(z_2) = -(R_{Y_B} + P + R_{Y_A}) \cdot d$$

$$M_{z_2}'(z_2) = -1 \cdot d$$

$$M_{z_2}''(z_2) = -1 \cdot d$$



THIS TRANSLATION WAS NOT REQUIRED BEFORE SINCE IN METHOD 1 THE "INFORMATION" ABOUT THE APPLICATION POINT OF SHEAR IS CONTAINED IN $\int_V 1/G\, \delta\tau\, \tau\, dV$, WHICH IS FUNCTION OF THE FLUXES, WHICH THROUGH MOM. EQUIVALENCE AND COMPATIBILITY TAKE INTO ACCOUNT THE POSITION OF T_y

PCVW REAL DUMMY 1

$$\delta W_e = -\frac{R_{\bar{Y}_B}}{k}$$

$$\delta W_i = \int_0^{\ell/2} \left(M_{x_1}' \cdot \frac{M_{x_1}}{EJ_{xx}} + T_{y_1}' \cdot \frac{T_{y_1}}{GA^*} + M_{z_1}' \cdot \frac{M_{z_1}}{GJ} \right) dz_1 + $$
$$+ \int_0^{\ell/2} \left(M_{x_2}' \frac{M_{x_2}}{EJ_{xx}} + T_{y_2}' \cdot \frac{T_{y_2}}{GA^*} + M_{z_2}' \cdot \frac{M_{z_2}}{GJ} \right) dz_2$$

PCVW REAL DUMMY 2

$$\delta W_e = -\frac{R_{\bar{Y}_A}}{k}$$

$$\delta W_i = \int_0^{\ell/2} M_{x_2}'' \frac{M_{x_2}}{EJ_{xx}} + T_{y_2}'' \frac{T_{y_2}}{GA^*} + M_{z_2}'' \frac{M_{z_2}}{GJ} \Big) dz_2$$

SOLVE

$$\begin{cases} \text{PCVW } 1 \\ \text{PCVW } 2 \end{cases} \longrightarrow \text{GET } R_{\bar{Y}_A} \quad R_{\bar{Y}_B}$$

Contents


```
clear variables
close all
home
```

Data

```
E = 70000;  % MPa
nu = 0.3;
A = 200;    % mm
t = 1;      % mm
R = 200;    % mm
l = 2000;   % mm
k = 10^6;   % N/mm
P = 1000;   % N
```

Compute G

```
G = E/(2*(1+nu));   % MPa
```

Centroid

```
x_cg = R*A/(4*A);   % mm
```

```
y_cg = R;            % mm
```

Inertia

```
J_xx = 2*A*R^2;      % mm^4
S_x1 = -A*R;         % mm^3
S_x3 = A*R;          % mm^3
S_x2 = 0;            % mm^3
S_x4 = 0;            % mm^3
```

Open fluxes

```
syms T_y q_1_s q_2_s

q_1_p = T_y*(S_x3/J_xx);     % N/mm
q_2_p = -T_y*(S_x1/J_xx);    % N/mm
q_5_p = T_y*(S_x2/J_xx);     % N/mm
```

Moment equivalence wrt 4

```
Omega_1_C = .25*R^2*pi;
Omega_2_C = Omega_1_C;

f_LHS_Mom = T_y*0;
f_RHS_Mom = q_1_s*2*Omega_1_C + q_2_s*2*Omega_2_C;
```

Compatibility

```
f_theta_1_p = 1/(2*G*t*Omega_1_C) * (q_1_s*(2*R+.5*R*pi) - q_2_s*R - q_1_p*R);
f_theta_2_p = 1/(2*G*t*Omega_2_C) * (q_2_s*(2*R+.5*R*pi) - q_1_s*R - q_2_p*R);
```

Closed cell fluxes

```
[q_1_s, q_2_s] = solve([f_LHS_Mom == f_RHS_Mom, f_theta_1_p == f_theta_2_p], [q_1_s, q_2_s])

q_1 = q_1_p - q_1_s                 % N/mm
q_2 = q_2_p - q_2_s                 % N/mm
q_3 = q_2_s                         % N/mm
q_4 = q_1_s                         % N/mm
q_5 = q_5_p + q_1_s - q_2_s         % N/mm
```

```
q_1_s =

0


q_2_s =

0


q_1 =
```

```
T_y/400
```

```
q_2 =
```

```
T_y/400
```

```
q_3 =
```

```
0
```

```
q_4 =
```

```
0
```

```
q_5 =
```

```
0
```

First solution method (shear deformation work in panels)

```
syms R_YB R_YA z_1 z_2
```

Internal actions in beam 1

```
T_y1 = -R_YB - P;
M_x1 = -(R_YB + P)*z_1;

T_y1_p = -1;
M_x1_p = -1*z_1;
```

Internal actions in beam 2

```
T_y2 = -R_YB - P - R_YA;
M_x2 = -(R_YB + P)*(z_2 + .5*l) - R_YA*z_2;

T_y2_p = -1;
M_x2_p = -1*(z_2 + .5*l);

T_y2_pp = -1;
M_x2_pp = -1*(z_2);
```

PCVW Real - Dummy 1

```
f_LHS_PCVW1 = -1*(R_YB/k);
f_RHS_PCVW1 = int((M_x1_p*M_x1/(E*J_xx)), z_1, 0, .5*l) + ...
    + int((M_x2_p*M_x2/(E*J_xx)),z_2,0,.5*l) + ...
    + int(((R/(t*G)) * subs(q_1, T_y, T_y1_p) * subs(q_1, T_y, T_y1)), z_1, 0, .5*l) + ...
    + int(((R/(t*G)) * subs(q_2, T_y, T_y1_p) * subs(q_2, T_y, T_y1)), z_1, 0, .5*l) + ...
    + int(((R/(t*G)) * subs(q_1, T_y, T_y2_p) * subs(q_1, T_y, T_y2)), z_2, 0, .5*l) + ...
    + int(((R/(t*G)) * subs(q_2, T_y, T_y2_p) * subs(q_2, T_y, T_y2)), z_2, 0, .5*l);
```

PCVW Real - Dummy 2

```
f_LHS_PCVW2 = -1*(R_YA/k);
f_RHS_PCVW2 = int((M_x2_pp*M_x2/(E*J_xx)),z_2,0,.5*l) + ...
    + int(((R/(t*G)) * subs(q_1, T_y, T_y2_pp) * subs(q_1, T_y, T_y2)), z_2, 0, .5*l) + ...
    + int(((R/(t*G)) * subs(q_2, T_y, T_y2_pp) * subs(q_2, T_y, T_y2)), z_2, 0, .5*l);
```

Hyperstatic reaction of the springs

```
[R_YA, R_YB] = solve([f_LHS_PCVW1 == f_RHS_PCVW1, f_LHS_PCVW2 == f_RHS_PCVW2], [R_YA, R_YB]);

R_YA = double(R_YA)     % N
R_YB = double(R_YB)     % N
```

```
R_YA =

   -2.7460


R_YB =

 -998.7155
```

Second solution method (section properties)

```
syms R_YB R_YA z_1 z_2
```

GA* evaluation

```
syms q_1_s q_2_s d
T_y_sc = 1;

f_LHS_Mom = T_y_sc*d;

% Closed cell fluxes
[q_1_s, q_2_s, d] = solve([f_LHS_Mom == subs(f_RHS_Mom, T_y, T_y_sc), ...
    subs(f_theta_1_p, T_y, T_y_sc) == 0, ...
    subs(f_theta_2_p, T_y, T_y_sc) == 0], ...
    [q_1_s, q_2_s, d]);

d = double(d);

q_1 = double(subs(q_1_p, T_y, T_y_sc) - q_1_s);                  % N/mm
q_2 = double(subs(q_2_p, T_y, T_y_sc) - q_2_s);                  % N/mm
q_3 = double(q_2_s);                                              % N/mm
q_4 = double(q_1_s);                                              % N/mm
q_5 = double(subs(q_5_p, T_y, T_y_sc) + q_1_s - q_2_s);          % N/mm

GA_st = G * (T_y_sc^2 / ...
    (((q_1^2)*R/t) + ((q_2^2)*R/t) + ((q_3^2)*.25*pi*2*R/t) + ((q_4^2)*.25*pi*2*R/t) + ((q_5^2)*R/t)))
```

```
GA_st =

   1.7625e+07
```

GJ evaluation

```
syms q_1_s q_2_s
T_y_ts = 0;
M_z_ts = 1;

f_LHS_Mom = M_z_ts;

% Closed cell fluxes
[q_1_s, q_2_s] = solve([f_LHS_Mom == subs(f_RHS_Mom, T_y, T_y_ts), ...
    subs(f_theta_1_p, T_y, T_y_ts) == subs(f_theta_2_p, T_y, T_y_ts)], ...
    [q_1_s, q_2_s]);

q_1_s_d = double(q_1_s);
q_2_s_d = double(q_2_s);

q_1 = double(subs(q_1_p, T_y, T_y_ts) - q_1_s);                  % N/mm
q_2 = double(subs(q_2_p, T_y, T_y_ts) - q_2_s);                  % N/mm
q_3 = double(q_2_s);                                             % N/mm
q_4 = double(q_1_s);                                             % N/mm
q_5 = double(subs(q_5_p, T_y, T_y_ts) + q_1_s - q_2_s);          % N/mm

syms q_1_s q_2_s

GJ = double(M_z_ts/subs(f_theta_1_p, [T_y, q_1_s, q_2_s], [T_y_ts, q_1_s_d, q_2_s_d]))
```

```
GJ =

   4.1344e+11
```

Internal actions in beam 1

```
T_y1 = -R_YB - P;
M_x1 = -(R_YB + P)*z_1;
M_z1 = -(R_YB + P)*d;

T_y1_p = -1;
M_x1_p = -1*z_1;
M_z1_p = -1*d;
```

Internal actions in beam 2

```
T_y2 = -R_YB - P - R_YA;
M_x2 = -(R_YB + P)*(z_2 + .5*l) - R_YA*z_2;
M_z2 = -(R_YB + P + R_YA)*d;

T_y2_p = -1;
M_x2_p = -1*(z_2 + .5*l);
M_z2_p = -1*d;

T_y2_pp = -1;
M_x2_pp = -1*(z_2);
M_z2_pp = -1*d;
```

PCVW Real - Dummy 1

```
f_LHS_PCVW1 = -1*(R_YB/k);
f_RHS_PCVW1 = int(( (M_x1_p*M_x1/(E*J_xx)) + ...
    + (T_y1_p*T_y1/GA_st) + ...
    + (M_z1_p*M_z1/GJ) ), z_1, 0, .5*l) + ...
    int(( (M_x2_p*M_x2/(E*J_xx)) + ...
    + (T_y2_p*T_y2/GA_st) + ...
    + (M_z2_p*M_z2/GJ) ), z_2, 0, .5*l);
```

PCVW Real - Dummy 2

```
f_LHS_PCVW2 = -1*(R_YA/k);
f_RHS_PCVW2 = int(( (M_x2_pp*M_x2/(E*J_xx)) + ...
    + (T_y2_pp*T_y2/GA_st) + ...
    + (M_z2_pp*M_z2/GJ) ), z_2, 0, .5*l);
```

Hyperstatic reaction of the springs

```
[R_YA, R_YB] = solve([f_LHS_PCVW1 == f_RHS_PCVW1, f_LHS_PCVW2 == f_RHS_PCVW2], [R_YA, R_YB]);

R_YA = double(R_YA)     % N
R_YB = double(R_YB)     % N
```

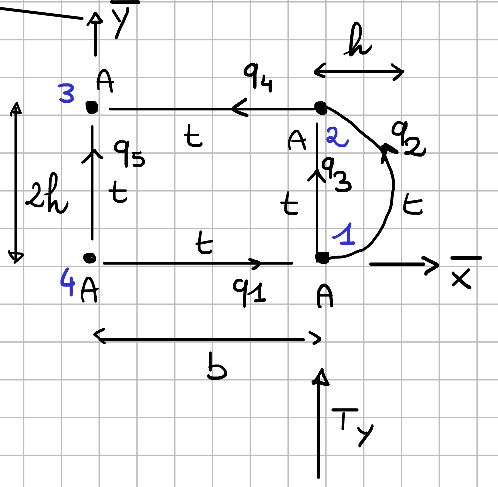
```
R_YA =

   -2.7460


R_YB =

 -998.7155
```

Ex 2



$A = 300\ mm^2$

$b = 300\ mm$

$h = 75\ mm$

$T_y = 1500\ N$

$t = 1\ mm$

FIND :

> SHEAR FLUXES q_i

> SHEAR CENTER $\bar{x}_{sc}$, $\bar{y}_{sc}$

CENTROID

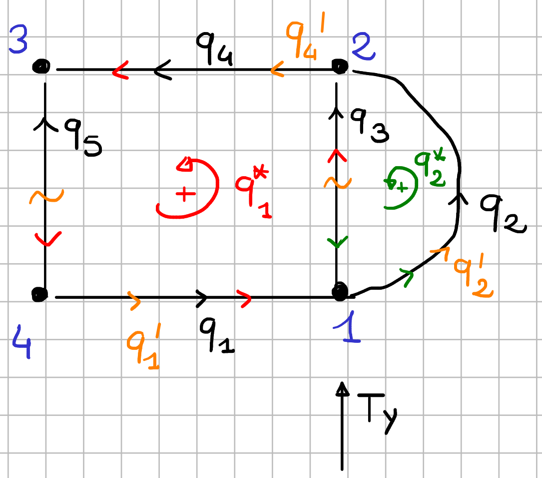
FROM SYMMETRY (COUNT ONLY THE STRINGERS)

$$\bar{x}_{CG} = b/2 \qquad \bar{y}_{CG} = h$$

INERTIAS

$$J_{xx} = 4 \cdot A h^2$$

$$S_{x_1} = S_{x_4} = -Ah \qquad S_{x_2} = S_{x_3} = Ah$$



OPEN CELL FLUXES

$$q_1' = -T_y \frac{S_{x_4}}{J_{xx}} = \frac{T_y}{4h}$$

$$q_2' = -T_y \frac{S_{x_1}+S_{x_4}}{J_{xx}} = \frac{T_y}{2h}$$

$$q_4' = -\left(-T_y \frac{S_{x_3}}{J_{xx}}\right) = -T_y \frac{S_{x_1}+\cancel{S_{x_4}}+\cancel{S_{x_2}}}{J_{xx}}$$

$$= \frac{T_y}{4h}$$

MOMENT EQUIVALENCE WRT 1 (+)

$$\overbrace{0}^{LHS\_Mom} = \overbrace{q_4' \cdot 2\Omega_4 + q_1^* \cdot 2\Omega_{CELL_1} + q_2^* \cdot 2\Omega_{CELL_2} + q_2' \cdot 2\Omega_2}^{RHS\_Mom}$$

WITH $\Omega_4 = \frac{1}{2} b \cdot 2h \qquad \Omega_2 = \frac{1}{2} \pi \cdot h^2$

$$\Omega_{CELL_1} = b \cdot 2h \qquad \Omega_{CELL_2} = \frac{1}{2} \pi h^2$$

COMPATIBILITY

$$\vartheta_1' = \frac{1}{2Gt\Omega_{cell_1}}\left(q_1'\cdot b + q_4'\cdot b + q_1^*\cdot(2b+4h) - q_2^*\cdot(2h)\right)$$

$$\vartheta_2' = \frac{1}{2G\Omega_{cell_2}t}\left(-q_1^* 2h + q_2'\pi h + q_2^*(2h+\pi h)\right)$$

WITH $\Omega_{cell_1} = 2bh$ & $\Omega_{cell_2} = \frac{\pi h^2}{2}$

SOLVE

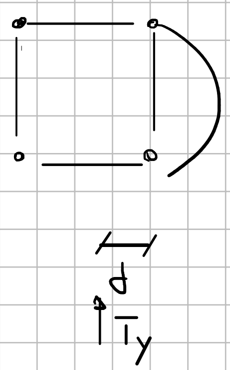
$$\begin{cases} LHS\_Mom = RHS\_Mom \\ \vartheta_1' = \vartheta_2' \end{cases} \longrightarrow \text{FIND } q_1^*,\ q_2^* \qquad q_1^* = -3.14\ N/mm \quad q_2^* = -6.73\ N/mm$$

CLOSED CELL FLUXES

$q_1 = q_1' + q_1^* = 1.85\ N/mm$ $\quad q_3 = q_1^* - q_2^* = 3.59\ N/mm$ $\quad q_5 = -q_1^* = 3.14\ N/mm$

$q_2 = q_2' + q_2^* = 3.27\ N/mm$ $\quad q_4 = q_4' + q_1^* = 1.86\ N/mm$

SHEAR CENTER EVALUATION (DEFINED AS THE POINT AT WHICH A FORCE APPLICATION GIVES $\vartheta_i' = 0$)



q_i' SAME DEFINITION AS BEFORE

$LHS\_Mom = -T_y \cdot d$

$RHS\_Mom,\ \vartheta_1',\ \vartheta_2'$ SAME AS BEFORE

SOLVE

$$\begin{cases} LHS\_Mom = RHS\_Mom \\ \vartheta_1' = 0 \\ \vartheta_2' = 0 \end{cases} \longrightarrow \text{FIND } q_1^*,\ q_2^*,\ d$$

$\bar{X}_{sc} = b - d = 195.30$ mm $\quad \bar{Y}_{sc} = h$ (SYMMETRIC (BOTH STRINGERS & PANELS))

Contents


```
clear variables
close all
home
```

Data

```
A = 300;    % mm^2
b = 300;    % mm
h = 75;     % mm
T_y = 1500; % N
```

Centroid

```
x_cg = b/2; % mm
y_cg = h;   % mm
```

Inertias

```
J_xx = 4*A*h^2;     % mm^4
S_x1 = -A*h;        % mm^3
S_x2 = A*h;         % mm^3
S_x3 = A*h;         % mm^3
S_x4 = -A*h;        % mm^3
```

Open fluxes

```
q_1_p = -T_y*(S_x4)/J_xx;           % N/mm
q_2_p = -T_y*(S_x4+S_x1)/J_xx;      % N/mm
q_4_p = +T_y*(S_x3)/J_xx;           % N/mm
```

Moment equivalence wrt 1

```
Omega_4 = .5*b*2*h;         % mm^2
Omega_1_C = b*2*h;          % mm^2
Omega_2_C = .5*pi*h^2;      % mm^2
Omega_2 = .5*pi*h^2;        % mm^2

syms q_1_s q_2_s G t

f_LHS_Mom = 0;
f_RHS_Mom = q_4_p*2*Omega_4 + q_1_s*2*Omega_1_C + q_2_s*2*Omega_2_C + q_2_p*2*Omega_2;
```

Compatibility

```
f_theta_1_p = 1/(2*G*t*Omega_1_C) * (q_1_p*b - q_2_s*2*h + q_4_p*b + q_1_s*(2*b+4*h));
f_theta_2_p = 1/(2*G*t*Omega_2_C) * (-q_1_s*2*h + q_2_p*pi*h + q_2_s*(2*h+pi*h));

% Remove dependence from G and t to avoid errors
f_theta_1_p = subs(f_theta_1_p, G, 1);
f_theta_1_p = subs(f_theta_1_p, t, 1);

f_theta_2_p = subs(f_theta_2_p, G, 1);
f_theta_2_p = subs(f_theta_2_p, t, 1);
```

Closed cell fluxes

```
[q_1_s, q_2_s] = solve([f_LHS_Mom == f_RHS_Mom, f_theta_1_p == f_theta_2_p], [q_1_s, q_2_s]);

% Convert from symbolic
q_1_s = double(q_1_s)               % N/mm
q_2_s = double(q_2_s)               % N/mm

q_1 = q_1_p + q_1_s                 % N/mm
q_2 = q_2_p + q_2_s                 % N/mm
q_3 = q_1_s - q_2_s                 % N/mm
q_4 = q_4_p + q_1_s                 % N/mm
q_5 = -q_1_s                        % N/mm
```

```
q_1_s =

   -3.1420


q_2_s =

   -6.7306


q_1 =

    1.8580


q_2 =

    3.2694


q_3 =

    3.5886


q_4 =

    1.8580


q_5 =

    3.1420
```

Shear center evaluation

```
syms d q_1_s q_2_s

% Moment
f_LHS_Mom = -T_y*d;
f_RHS_Mom = q_4_p*2*Omega_4 + q_1_s*2*Omega_1_C + q_2_s*2*Omega_2_C + q_2_p*2*Omega_2;

% Compatibility
f_theta_1_p = 1/(2*G*t*Omega_1_C) * (q_1_p*b - q_2_s*2*h + q_4_p*b + q_1_s*(2*b+4*h));
f_theta_2_p = 1/(2*G*t*Omega_2_C) * (-q_1_s*2*h + q_2_p*pi*h + q_2_s*(2*h+pi*h));

% Remove dependence from G and t to avoid errors
f_theta_1_p = subs(f_theta_1_p, G, 1);
f_theta_1_p = subs(f_theta_1_p, t, 1);

f_theta_2_p = subs(f_theta_2_p, G, 1);
f_theta_2_p = subs(f_theta_2_p, t, 1);

[q_1_s, q_2_s, d] = solve([f_LHS_Mom == f_RHS_Mom, f_theta_1_p == 0, f_theta_2_p == 0], [q_1_s, q_2_s, d]);
% Convert from symbolic
d = double(d);                    % mm

x_sc = b-d                        % mm
```

```
x_sc =

  195.2991
```

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