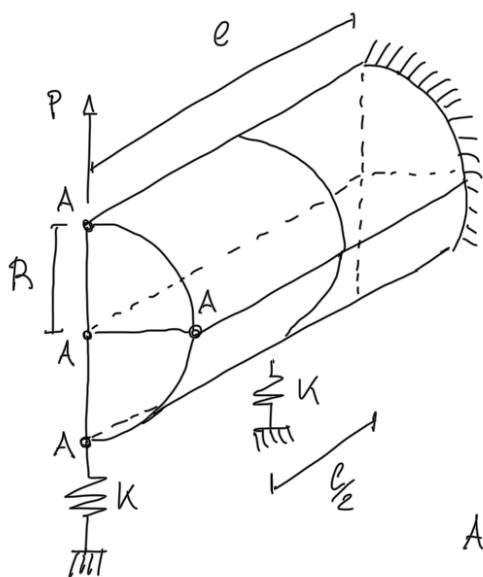


EX 09 - Semi-monocoque III

1)



DATA

$$E = 70 \text{ GPa}$$

$$\nu = 0.3$$

$$A = 200 \text{ mm}^2$$

$t = 1 \text{ mm}$ for all the panels

$$R = 200 \text{ mm}$$

$$l = 2000 \text{ mm}$$

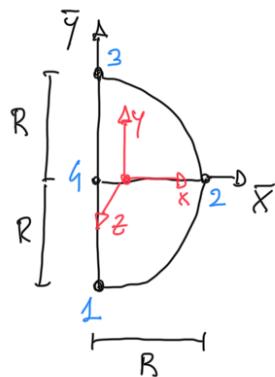
$$K = 10^6 \text{ N/mm}$$

$$P = 1 \text{ kN}$$

Accounting for beam shear deformability

Let's find the reaction forces at the base of the two springs.

- Let's find the shear fluxes q_i as a function of a generic shear load $T_y \Rightarrow q_i = q_i(T_y)$



• Centroid

$$\bar{y}_c = 0 \quad \bar{x}_c = \frac{RA}{GA} = \frac{R}{6}$$

• Inertias

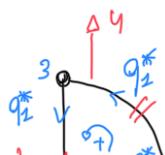
$$J_{xx} = 2AR^2$$

$$S_{x1} = -AR$$

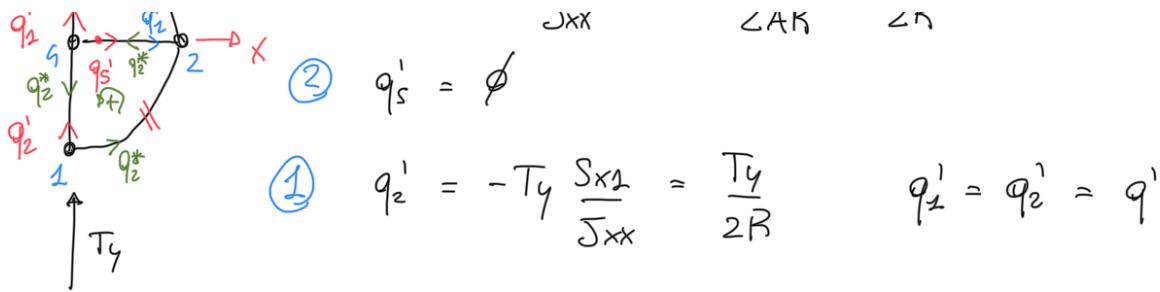
$$S_{x2} = S_{x3} = \phi$$

$$S_{x3} = AR$$

- Open Cell Fluxes



$$\textcircled{3} \quad q_2^* = T_y \cdot \frac{S_{x3}}{\tau} = T_y \cdot \frac{AR}{\tau_1 \tau_2} = \frac{T_y}{\tau_1 \tau_2}$$



- Moment Equivalence wrt ⑤



$$LHS = \emptyset$$

$$RHS = 2 \cdot q_2^* \cdot \Omega_{\text{sector}} + 2 \cdot q_2^* \cdot \Omega_{\text{sector}}$$

where $\Omega_{\text{sector}} = \Omega_{\text{sector}} = \frac{\pi R^2}{2} = \Omega$

$$q_2^* = -q_2^* = q^*$$

- Compatibility

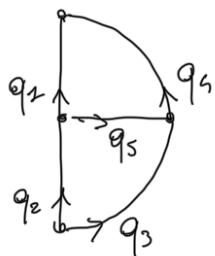
$$\theta_2^1 = \frac{1}{2\Omega G +} \left(q_2^* \left(2R + \frac{2\pi R}{G} \right) - q_2^* R - q_2^1 \cdot R \right)$$

$$\theta_2^1 = \frac{1}{2\Omega G +} \left(q_2^* \left(2R + \frac{2\pi R}{G} \right) - q_2^* R - q_2^1 \cdot R \right)$$

$$\theta_2^1 = \theta_2^1 \quad q^* \left(2R + \frac{\pi R}{2} \right) + q^* R - \cancel{q^1 R} = -q^* \left(2R + \frac{\pi R}{2} \right) - q^* R - \cancel{q^1 R}$$

$$q^* = \emptyset$$

- Total Fluxes



$$q_2 = q_2^1 - q_2^* = \frac{T_y}{2R}$$

$$q_2 = q_2^1 - q_2^* = \frac{T_y}{2R}$$

$$q_3 = q_2^* = \emptyset$$

$$q_4 = q_2^* = \emptyset$$

$$q_5 = q_5^1 + q_2^* - q_2^* = \emptyset$$

• HYPERSTATIC REACTION

1st Method: DEFORMATION WORK OF PANELS

the internal work related to shear forces and torsional moments is computed directly inside panels

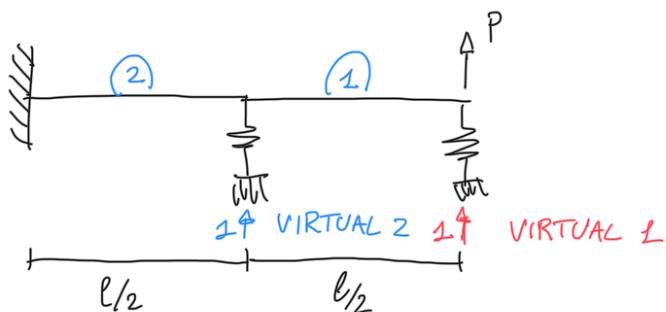
$$\delta w_i = \int_V \delta \sigma \cdot \epsilon \, dV = \underbrace{\int_0^l T_z^i \frac{T_z}{EA} \, dz + \int_0^l M_x^i \frac{M_x}{EJ_{xx}} \, dz + \int_0^l M_y^i \frac{M_y}{J_{yy}} \, dz}_{\text{stringers}} + \dots$$

Stringers

$$\int_{V_p} \delta \sigma \cdot \epsilon \, dV_p + \underbrace{F \cdot \frac{F}{k}}_{\text{spring}}$$

volume of panels *panels*

• Internal Actions



①

REAL

$$T_{y1} = -P - R_{y1}$$

$$M_{x1} = -(P + R_{y1}) \cdot z_1$$

VIRTUAL 1

$$T_{y1}^i = -1$$

$$M_{x1}^i = -z_1$$

②

REAL

$$T_y = -P - R_{y1} - R_{y2}$$

$$M_x = -(P + R_{y1}) \left(\frac{l}{2} + z_2 \right) - R_{y2} \cdot z_2$$

111D TRIAL 1 111D + 111 -

-1 -1

VIRTUAL 1

$$T_y^1 = -1$$

$$M_x^1 = -\left(\frac{\ell}{2} + z_2\right)$$

VIRTUAL 2

$$T_y^2 = -1$$

$$M_x^2 = -z_2$$

• Panels Interval Work

$$\delta w_{ip} = \int_{V_p} \delta \sigma : \epsilon \, dV_p = \int_{V_p} \delta \tau : \gamma \, dV_p, \quad \tau = G \cdot \gamma \quad \gamma = \frac{I}{G}$$

$$= \int_{V_p} \delta \tau \cdot \frac{\gamma}{G} \, dV_p = \int_0^l \int_{R_i} \int_{-z}^z \delta \tau \frac{\gamma}{G} \, dx \cdot dy \cdot dz$$

we know $\tau = \frac{q}{t}$ and $\delta \tau = \frac{\delta q}{t}$

and $q_i = f(T_y)$ and $\delta q_i = f'(T_y) \delta T_y$

In our case, T_y depends only on \bar{z} , thus q does too.

$$= \int_0^l \frac{\delta q}{t} \cdot \frac{q}{Gt} \int_{R_i} \int_{-z}^z dx \cdot dy \cdot dz = \frac{R_i}{Gt} \int_0^l \delta q \cdot q \, dz$$

• PCVW

VIRTUAL 1

$$\delta w_e = \emptyset$$

$$\delta w_i = \int_0^{\frac{l}{2}} M_{xz}^1 \cdot \frac{M_{xz}}{EJ_{xz}} \, dz_2 + \int_0^{\frac{l}{2}} M_{xz}^1 \cdot \frac{M_{xz}}{EJ_{xz}} + \dots] \text{ stringers}$$

$$+ \frac{B}{tG} \left(\int_0^{\frac{l}{2}} \delta q_2(T_y^1) \cdot q_2(T_y) \, dz_2 + \int_0^{\frac{l}{2}} \delta q_1(T_y^1) \cdot q_1(T_y) \, dz_2 \right)] \text{ panel 1}$$

$$+ \int_0^{\frac{l}{2}} \delta q_2(T_y^1) \cdot q_2(T_y) \, dz_2 + \int_0^{\frac{l}{2}} \delta q_2(T_y^1) \cdot q_2(T_y) \, dz_2] \text{ panel 2}$$

$$+ 1 \cdot \frac{R_{yz}}{K}$$

VIRTUAL 2

$$\delta w_{e2} = \emptyset$$

$$\delta W_{i2} = \int_0^{\frac{l}{2}} M_{xz}'' \cdot \frac{M_{xz}}{EJ_{xx}} dz_2 + \frac{R}{tG} \left(\int_0^{\frac{l}{2}} \delta q_1(T_{y2}'') \cdot q_2(T_{y2}) dz_2 + \dots \right. \\ \left. + \int_0^{\frac{l}{2}} \delta q_2(T_{y2}'') \cdot q_2(T_{y2}) dz_2 + 1 \cdot \frac{R_{y2}}{K} \right)$$

$$\text{solve } \begin{cases} \delta W_{i2} = \delta W_{e2} \\ \delta W_{i2} = \delta W_{e2} \end{cases} \rightarrow R_{y1}, R_{y2}$$

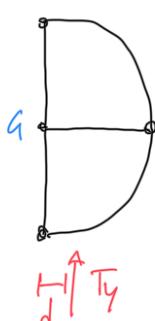
2nd Method: SECTION PROPERTIES

Shear fluxes are used to compute the section properties GA^* , GJ , shear center. Knowing them, all the internal work can be computed without directly compute the internal work in the panels.

$$\delta W_i = \int_0^l T_z' \frac{T_z}{EA} + \int_0^l M_x' \cdot \frac{M_x}{EJ_{xx}} dz + \int_0^l M_y' \cdot \frac{M_y}{EJ_{yy}} dz + \int_0^l T_y' \cdot \frac{T_y}{GA^*} dz + \\ + \int_0^l T_x' \frac{T_x}{GA^*} dz + \int_0^l M_z' \cdot \frac{M_z}{GJ} dz$$

Let's find GA^* , GJ , shear center

- GA^* , shear center



$$T_y = f$$

Moment Equivalence wrt G

$$LHS = T_y \cdot d$$

$$RHS = 2q_2^* \Omega + 2q_2^* \Omega \quad (\text{as before})$$

$$\theta_1^*, \theta_2^* \rightarrow \Omega \text{ as before}$$

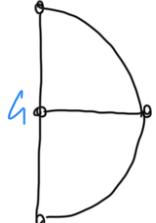
$$\text{solve } LHS = RHS$$

$$\begin{cases} \theta_1' = \theta_2' \\ \theta_1' = \theta_2' = \phi \end{cases} \rightarrow d, q_2^*, q_2^*$$

$$GA^* = G \frac{T_y^2}{\sum q_i^2 \cdot \frac{L_i}{t_i}} \quad \begin{matrix} \text{in the} \\ \text{shear center} \end{matrix}$$

↓
panels

- GJ



$$M_z = 1 \quad T_y \approx T_x = \phi \quad \begin{matrix} \text{in the} \\ \text{shear center} \end{matrix}$$

Mom EQ wrt G

$$LHS = M_z$$

$$RHS = \text{as before} \quad q_i' = \phi$$

⇒ M_z

solve $\begin{cases} LHS = RHS \\ \theta_1' = \theta_2' \end{cases} \rightarrow q_2^*, q_2^* \rightarrow \theta'$

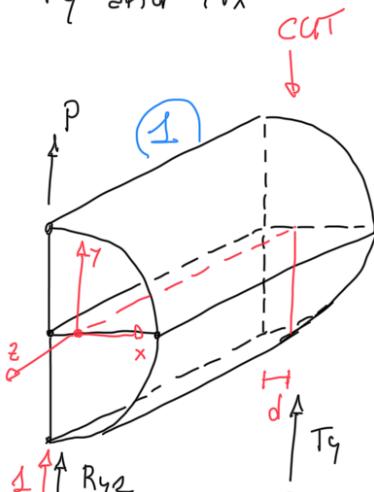
$$M_z = GJ \cdot \theta' \quad GJ = \frac{M_z}{\theta'}$$

- Internal Actions

We have already computed T_y and M_x

① $\begin{cases} M_{z1} = (P + R_{y2}) \cdot d \\ M_{z1} = d \end{cases}$

② $\begin{cases} M_{z2} = (P + R_{y2} + R_{z2}) \cdot d \\ M_{z2} = d \\ M_{z2}'' = d \end{cases}$



• PCVW

VIRTUAL 1

$$\delta w_{e1} = \phi$$

$$\delta w_{i1} = \int_0^{\frac{l}{2}} \left(T_{y2}^1 \cdot \frac{T_{yz}}{GA^*} + M_{z2}^1 \frac{M_{z1}}{GJ} + M_{x2}^1 \frac{M_{x1}}{EJ_{xx}} \right) dz_1 + \dots$$

$$\dots \int_0^{\frac{l}{2}} \left(T_{y2}^1 \cdot \frac{T_{yz}}{GA^*} + M_{z2}^1 \frac{M_{z2}}{GJ} + M_{x2}^1 \frac{M_{x2}}{EJ_{xx}} \right) dz_2 + \text{L} \cdot \frac{R_{y2}}{K}$$

VIRTUAL 2

$$\delta w_{e2} = \phi$$

$$\delta w_{i2} = \int_0^{\frac{l}{2}} \left(T_{y2}^1 \cdot \frac{T_{yz}}{GA^*} + M_{z2}^1 \frac{M_{z2}}{GJ} + M_{x2}^1 \frac{M_{x1}}{EJ_{xx}} \right) dz_2 + \text{L} \cdot \frac{R_{y2}}{K}$$

Solve $\begin{cases} \delta w_{e1} = \delta w_{i1} \\ \delta w_{e2} = \delta w_{i2} \end{cases} \rightarrow R_{y2}, R_{yz}$

$$R_{y1} = -998.7155 \text{ N}$$

$$R_{y2} = -2.766 \text{ N}$$