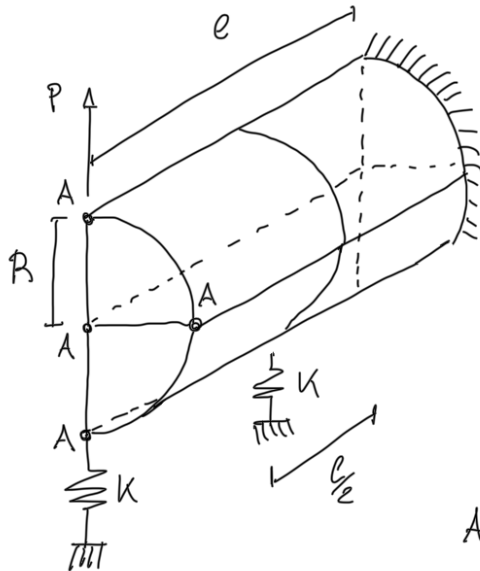


EX 09 - Semi-monocoque III

1)



DATA

$$E = 70 \text{ GPa}$$

$$\nu = 0.3$$

$$A = 200 \text{ mm}^2$$

$$t = 1 \text{ mm for all the panels}$$

$$R = 200 \text{ mm}$$

$$l = 2000 \text{ mm}$$

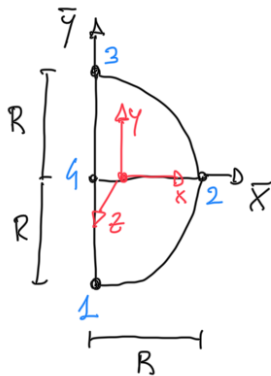
$$K = 10^6 \text{ N/mm}$$

$$P = 1 \text{ kN}$$

Accounting for beam shear deformability

Let's find the reaction forces at the base of the two springs.

- Let's find the shear fluxes q_i as a function of a generic shear load $T_y \Rightarrow q_i = q_i(T_y)$



- Centroid

$$\bar{y}_c = 0 \quad \bar{x}_c = \frac{R \cdot A}{4A} = \frac{R}{4}$$

- Inertias

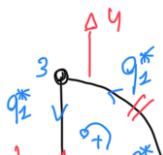
$$J_{xx} = 2AR^2$$

$$S_{x1} = -A \cdot R$$

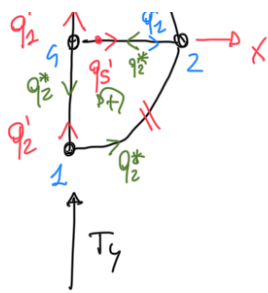
$$S_{x2} = S_{x3} = \phi$$

$$S_{x3} = AR$$

- Open Cell Fluxes



$$\textcircled{3} \quad q_2' = T_y \cdot \frac{S_{x3}}{J_{xx}} = T_y \cdot \frac{AR}{2AR^2} = \frac{T_y}{2R}$$



$$\textcircled{2} \quad q_s' = \phi$$

$$\textcircled{1} \quad q_2' = -T_y \frac{S_{x1}}{J_{xx}} = \frac{T_y}{2R}$$

$$q_1' = q_2' = q'$$

• Moment Equivalence wrt \textcircled{G}



$$\text{LHS} = \phi$$

$$\text{RHS} = 2 \cdot q_1^* \cdot \Omega_{\text{CELL1}} + 2 \cdot q_2^* \cdot \Omega_{\text{CELL2}}$$

$$\text{where } \Omega_{\text{CELL1}} = \Omega_{\text{CELL2}} = \frac{\pi R^2}{4} = \Omega$$

$$q_1^* = -q_2^* = q^*$$

• Compatibility

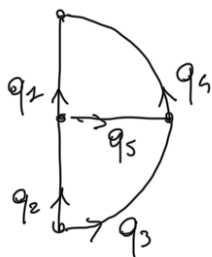
$$\theta_1' = \frac{1}{2\Omega G} \left(q_1^* (2R + \frac{2\pi R}{4}) - q_2^* R - q_2' \cdot R \right)$$

$$\theta_2' = \frac{1}{2\Omega G} \left(q_2^* (2R + \frac{2\pi R}{4}) - q_1^* R - q_1' \cdot R \right)$$

$$\theta_1' = \theta_2' \quad q^* (2R + \frac{\pi R}{2}) + q^* R - \cancel{q_1' R} = -q^* (2R + \frac{\pi R}{2}) - q^* R - \cancel{q_1' R}$$

$$q^* = \phi$$

• Total Fluxes



$$q_1 = q_1' - q_2^* = \frac{T_y}{2R}$$

$$q_2 = q_2' - q_2^* = \frac{T_y}{2R}$$

$$q_3 = q_2^* = \phi$$

$$q_4 = q_1^* = \phi$$

$$q_5 = q_5' + q_1^* - q_2^* = \phi$$

• HYPERSTATIC REACTION

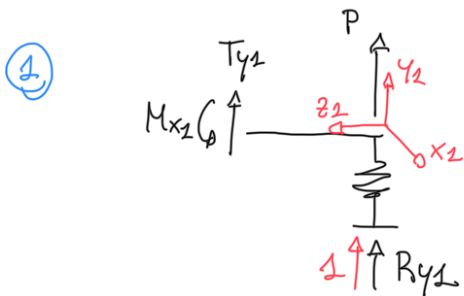
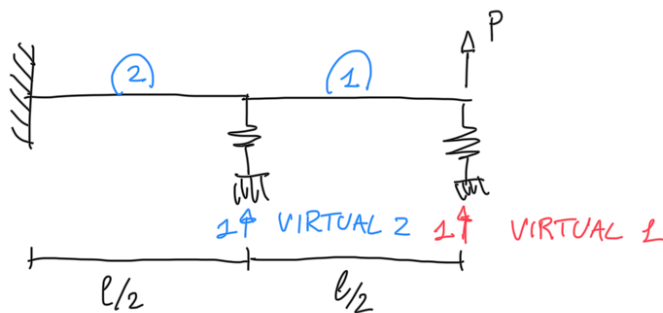
1st Method: DEFORMATION WORK OF PANELS

the internal work related to shear forces and torsional moments is computed directly inside panels

$$\delta W_i = \int_V \delta \sigma : \epsilon dV = \underbrace{\int_0^l T_z' \frac{T_z}{EA} dz + \int_0^l M_x' \frac{M_x}{EJ_{xx}} dz + \int_0^l M_y' \frac{M_y}{J_{yy}} dz}_{\text{stringers}} + \dots$$

$$\underbrace{\int_{V_p} \delta \sigma : \epsilon dV_p}_{\substack{\text{volume} \\ \text{of panels}}} + \underbrace{F \cdot \frac{F}{k}}_{\text{spring}}$$

• Internal Actions



REAL

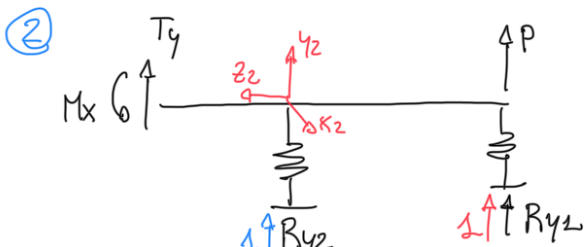
$$T_{y1} = -P - R_{y1}$$

$$M_{x1} = -(P + R_{y1}) \cdot z_1$$

VIRTUAL 1

$$T'_{y1} = -1$$

$$M'_{x1} = -z_1$$



REAL

$$T_y = -P - R_{y1} - R_{y2}$$

$$M_x = -(P + R_{y1}) \left(\frac{l}{2} + z_2 \right) - R_{y2} \cdot z_2$$

VIRTUAL 1

VIRTUAL 2

-1 -1

VIRTUAL 1

$$T_y' = -1$$

$$M_x' = -(\frac{l}{2} + z_2)$$

VIRTUAL 2

$$T_y'' = -1$$

$$M_x'' = -z_2$$

• Panels Internal Work

$$\delta w_{ip} = \int_{V_p} \delta \sigma : \epsilon \, dV_p = \int_{V_p} \delta \tau : \gamma \, dV_p \quad \tau = G \cdot \gamma \quad \gamma = \frac{\tau}{G}$$

$$= \int_{V_p} \delta \tau \cdot \frac{\tau}{G} \, dV_p = \int_{\ell} \int_R \int_t \delta \tau \frac{\tau}{G} \, dx \cdot dy \cdot dz$$

we know $\tau = \frac{Q}{t}$ and $\delta \tau = \frac{\delta Q}{t}$

and $q_i = f(T_y)$ and $\delta q_i = f'(T_y')$

In our case, T_y depends only on z , thus q does too.

$$= \int_{\ell} \frac{\delta q}{t} \cdot \frac{q}{Gt} \int_R \int_t dx \, dy \, dz = \frac{R}{tG} \int_{\ell} \delta q \cdot q \, dz$$

• PCVW

VIRTUAL 1

$$\delta w_e = \emptyset$$

$$\delta w_i = \int_0^{\frac{l}{2}} M_{x1}' \cdot \frac{M_{x1}}{E J_{xx}} \, dz_1 + \int_0^{\frac{l}{2}} M_{x2}' \cdot \frac{M_{x2}}{E J_{xx}} \, dz_2 + \dots \quad] \text{ stringers}$$

$$+ \frac{B}{tG} \left(\int_0^{\frac{l}{2}} \delta q_1(T_{y2}') \cdot q_1(T_{y1}) \, dz_1 + \int_0^{\frac{l}{2}} \delta q_1(T_{y2}') \cdot q_1(T_{y2}) \, dz_2 \quad] \text{ panel 1} \right.$$

$$+ \int_0^{\frac{l}{2}} \delta q_2(T_{y1}') \cdot q_2(T_{y1}) \, dz_1 + \int_0^{\frac{l}{2}} \delta q_2(T_{y2}') \cdot q_2(T_{y2}) \, dz_2 \quad] \text{ panel 2}$$

$$+ 1 \cdot \frac{R_{y1}}{k}$$

VIRTUAL 2

$$\delta w_{e2} = \emptyset$$

$$\delta W_{i2} = \int_0^{\frac{l}{2}} M_{x2}'' \cdot \frac{M_{x2}}{EI_{xx}} dz_2 + \frac{R}{tG} \left(\int_0^{\frac{l}{2}} q_1(T_{y2}) \cdot q_1(T_{y2}) dz_2 + \dots \right. \\ \left. + \int_0^{\frac{l}{2}} q_2(T_{y2}) \cdot q_2(T_{y2}) dz_2 + 1 \cdot \frac{R_{y2}}{K} \right)$$

$$\text{solve } \begin{cases} \delta W_{i1} = \delta W_{e1} \\ \delta W_{i2} = \delta W_{e2} \end{cases} \rightarrow R_{y1}, R_{y2}$$

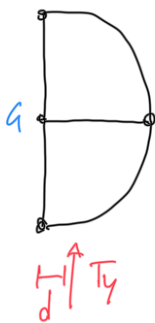
2nd Method: SECTION PROPERTIES

Shear fluxes are used to compute the section properties GA^* , GJ , shear center. Knowing them, all the internal work can be computed without directly compute the internal work in the panels.

$$\delta W_i = \int_0^l T_z' \cdot \frac{T_z}{EA} dz + \int_0^l M_x' \cdot \frac{M_x}{EI_{xx}} dz + \int_0^l M_y' \cdot \frac{M_y}{EI_{yy}} dz + \int_0^l T_y' \cdot \frac{T_y}{GA^*} dz + \\ + \int_0^l T_x' \cdot \frac{T_x}{GA^*} dz + \int_0^l M_z' \cdot \frac{M_z}{GJ} dz$$

Let's find GA^* , GJ , shear center

• GA^* , shear center



$$T_y = 1$$

Moment Equivalence wrt q

$$LHS = T_y \cdot d$$

$$RHS = 2q_2^* \Omega + 2q_2^* \Omega \quad (\text{as before})$$

$$\theta_1', \theta_2' \rightarrow \text{as before}$$

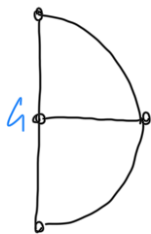
$$\text{solve } LHS = RHS$$

$$\begin{cases} \theta_1' = \theta_2' \\ \theta_1 = \theta_2 = \emptyset \end{cases} \rightarrow d, q_2^*, q_2^*$$

$$GA^* = G \frac{I_y^2}{\sum q_i^2 \cdot \frac{L_i}{t_i}} \quad \begin{array}{l} \text{in the} \\ \text{shear center} \end{array}$$

\downarrow
 # panels

8 GJ



$M_z = 1$ $T_y = T_x = 0$

Mem EQ wrt G

$$\text{LHS} = M_2$$

RHS = as before $q_i' = \emptyset$

⑦ M_2

solve $\begin{cases} \text{LHS} = \text{RHS} \\ \theta_2' = \theta_2' \end{cases} \rightarrow q_2^*, q_2^* \rightarrow \theta'$

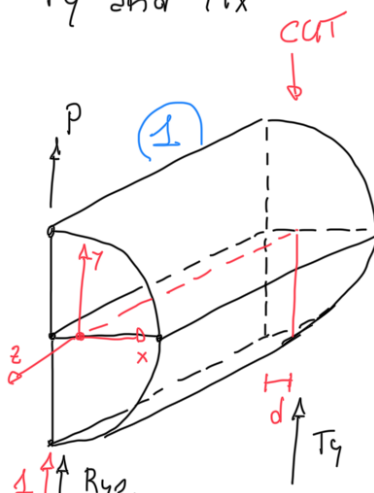
$$M_z = GJ \cdot \theta' \quad GJ = \frac{M_z}{\theta'}$$

- Internal Actions

We have already computed T_y and M_x

$$\textcircled{1} \begin{cases} M_{z1} = (P + R_{y2}) \cdot d \\ M'_{z1} = d \end{cases}$$

$$\textcircled{2} \quad \begin{cases} M_{z2} = (P + R_{y2} + R_{z2}) \cdot d \\ M'_{z2} = d \\ M''_{z2} = d \end{cases}$$



-11 -

↷ M_x

• PCVV

VIRTUAL 1

$$\delta W_{e1} = 0$$

$$\delta W_{i1} = \int_0^{\frac{\ell}{2}} \left(T_{y1}' \cdot \frac{T_{y1}}{GA^*} + M_{z1}' \cdot \frac{M_{z1}}{GJ} + M_{x1}' \cdot \frac{M_{x1}}{EJ_{xx}} \right) dz_1 + \dots$$

$$\dots \int_0^{\frac{\ell}{2}} \left(T_{y2}' \cdot \frac{T_{y2}}{GA^*} + M_{z2}' \cdot \frac{M_{z2}}{GJ} + M_{x2}' \cdot \frac{M_{x2}}{EJ_{xx}} \right) dz_2 + \textcolor{red}{1} \cdot \frac{R_{y1}}{K}$$

VIRTUAL 2

$$\delta W_{e2} = 0$$

$$\delta W_{i2} = \int_0^{\frac{\ell}{2}} \left(T_{y2}'' \cdot \frac{T_{y2}}{GA^*} + M_{z2}'' \cdot \frac{M_{z2}}{GJ} + M_{x2}'' \cdot \frac{M_{x2}}{EJ_{xx}} \right) dz_2 + \textcolor{blue}{1} \cdot \frac{R_{y2}}{K}$$

$$\text{solve } \begin{cases} \delta W_{e1} = \delta W_{i1} \\ \delta W_{e2} = \delta W_{i2} \end{cases} \rightarrow R_{y1}, R_{y2}$$

$$R_{y1} = -998.7155 \text{ N}$$

$$R_{y2} = -2.766 \text{ N}$$