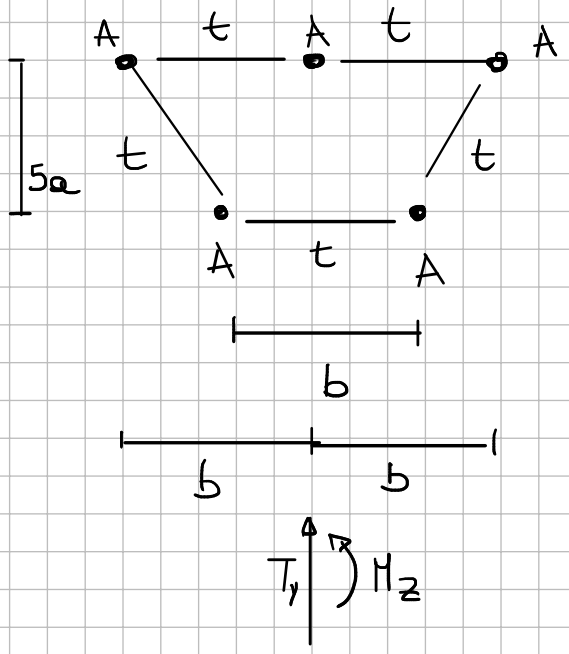
EXERCISE SESSION 7 - 28/10/22

Semimonocoque Approximation

Ex 1



$A = 250\ mm^2$

$a = 50\ mm$

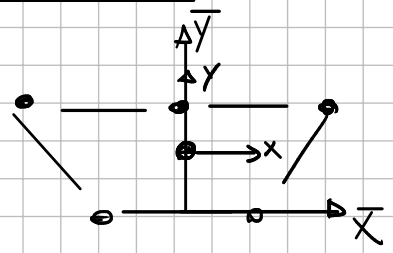
$b = 600\ mm$

$T_y = 3000\ N$

$M_z = 45000\ N mm$

FIND FLUXES

CENTROID



$$x_{CG} = 0$$

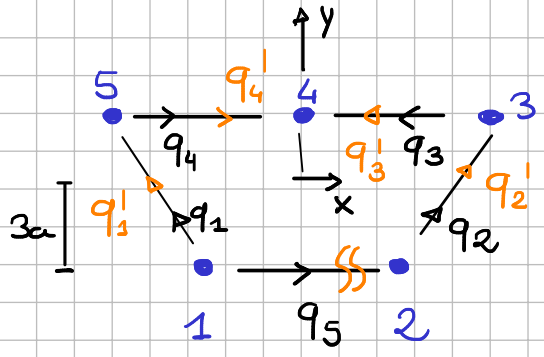
$$y_{CG} = \frac{3A \cdot 5a}{5A} = 3a$$

INERTIAS

$$J_{xx} = 3A \cdot (2a)^2 + 2A \cdot (3a)^2 = 30Aa^2$$

$$J_{yy} = [\dots] = \frac{5}{2} Ab^2 \quad \sim \text{NOT NEEDED}$$

$$J_{xy} = 0 \quad \text{ITS SYMMETRIC}$$

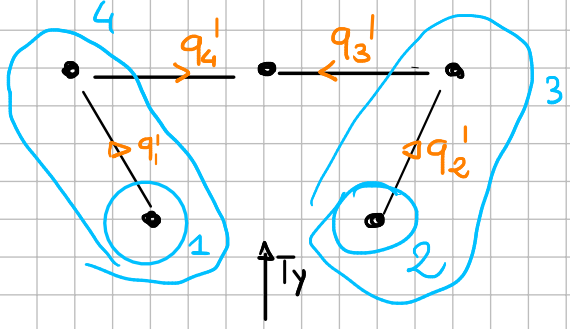


STATIC INERTIAS

$$S_{x_1} = S_{x_2} = -3Aa$$

$$S_{x_5} = S_{x_4} = S_{x_3} = 2Aa$$

OPEN SECTION FLUXES



1-2) $$q_1' = -T_y \cdot \frac{S_{x_1}}{J_{xx}} = q_2' \quad \text{as } S_{x_1} = S_{x_2}$$

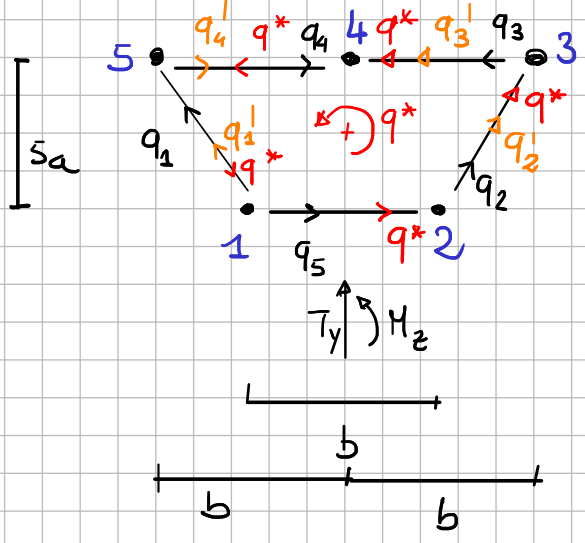
$$q_1' = \frac{1}{10}\frac{T_y}{a} = q_2'$$

3-4) $$q_3' = -T_y \frac{S_{x_2} + S_{x_3}}{J_{xx}} = q_4'$$

$$S_{x_2} = S_1 \quad S_{x_3} = S_{x_4} \rightarrow q_4' = -T_y \frac{S_{x_1} + S_{x_4}}{J_{xx}}$$

$$q_3' = \frac{1}{30}\frac{T_y}{a} = q_4'$$

MOMENT EQUIVALENCE



WRT (4)

$$M_z + T_y \cdot 0 = +q^* \cdot 2\Omega_{CELL} - q_1' \cdot 2\Omega_1 + q_2' \cdot 2\Omega_2$$

$= 0$ $\quad q_1' = q_2'$ $\quad \Omega_1 = \Omega_2$

$$\Omega_{CELL} = \left(\frac{1}{2} b \cdot 5a\right) \cdot 3 = \frac{15}{2} ab$$

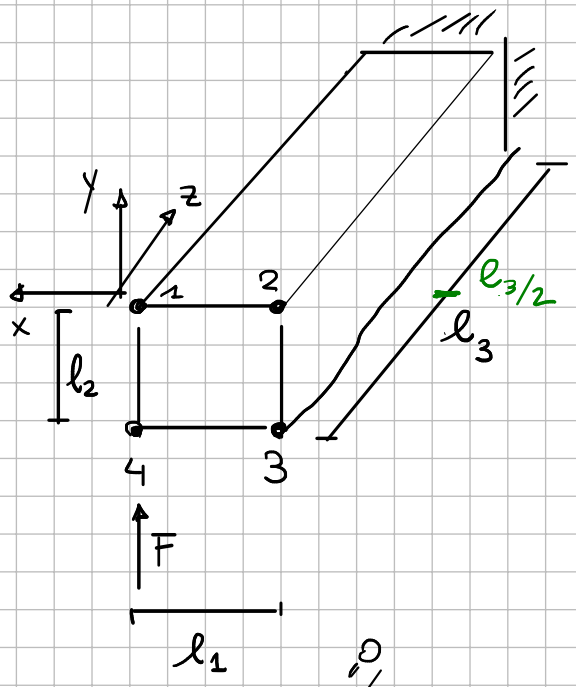
$$q^* = \frac{M_z}{2\Omega_{CELL}} = \frac{M_z}{15ab} = 1 \ N/mm$$

COMBINE

$$q_1 = q_1' - q^* = 6 - 1 = 5 \ N/mm \qquad q_3 = q_3' + q^* = 2 + 1 = 3 \ N/mm$$

$$q_2 = q_2' + q^* = 7 \ N/mm \qquad q_4 = +q_4' - q^* = 2 - 1 = 1 \ N/mm \qquad q_5 = q^* = 1 \ N/mm$$

Ex 2 (EXAM 26/07/2021)



$\ell_1 = 400$ mm $\quad F = 8000$ N

$\ell_2 = 250$ mm

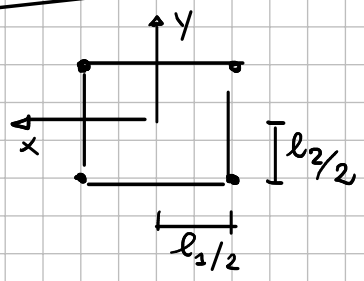
$\ell_3 = 2000$ mm

$A = 500$ mm^2

FIND $\sigma_{zz}\left(z=\frac{\ell_3}{2}\right)$ FOR STRINGER (4)

$$\sigma_{zz_4}(z) = \frac{T_z(z)}{\Sigma_i A_i} + \frac{M_x(z)}{J_{xx}} \cdot y_4 - \frac{M_y(z)}{J_{yy}} \cdot x_4$$

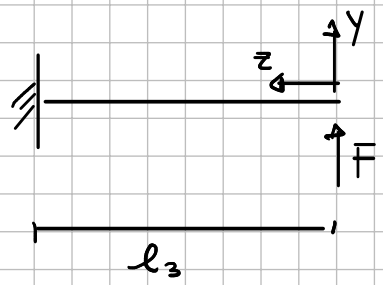
CENTROID

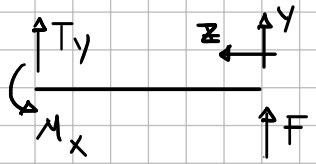


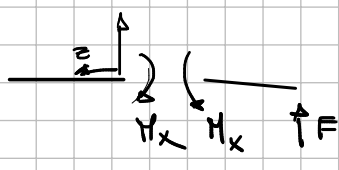
INERTIA

$$J_{xx} = 4A \cdot \left(\frac{\ell_2}{2}\right)^2 = \Sigma_i A_i y_i^2 = A\ell_2^2$$

BEAM MODEL







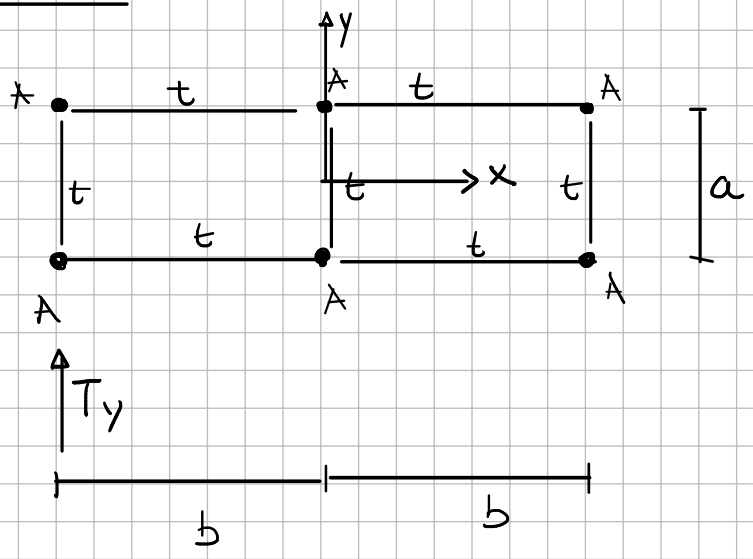
$$M_x(z) = -F \cdot z$$

$$M_x\left(\frac{\ell_3}{2}\right) = -\frac{1}{2} F \ell_3$$

STRESS IN STRINGER

$$\sigma_{zz_4}\left(\frac{\ell_3}{2}\right) = \frac{M_x(\ell_3/2)}{J_{xx}} \cdot y_4 = +\frac{1}{4}\frac{F\ell_3}{A\ell_2} = 32 \text{ MPa}$$

Ex 3



$A = 250\ mm^2$

$a = 100\ mm$

$b = 150\ mm$

$T_y = 1200\ N$

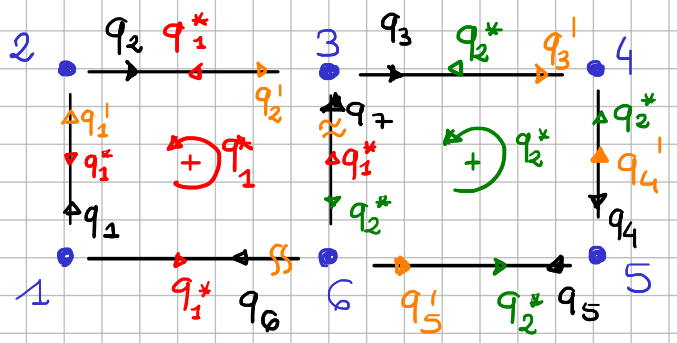
FIND FLUXES

CENTROID ← ITS SYMMETRIC

INERTIAS $J_{xx} = 6 \cdot A\left(\frac{a}{2}\right)^2 = \frac{3}{2}Aa^2$

AVOID J_{yy} = NOT NEEDED

J_{xy} = 0 SYM.

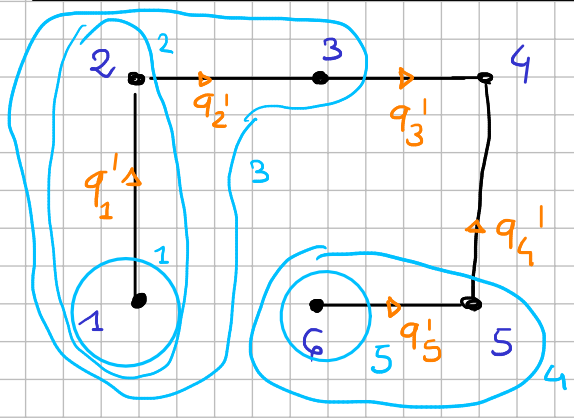


STATIC INERTIA

$$S_{x_1} = S_{x_6} = S_{x_5} = -\frac{1}{2}Aa$$

$$S_{x_2} = S_{x_3} = S_{x_4} = \frac{1}{2}Aa$$

FLUXES FOR OPEN SECTION



1) $q_1' = q_5' = -T_y \frac{S_{x_1}}{J_{xx}} = -\frac{T_y}{3a}$

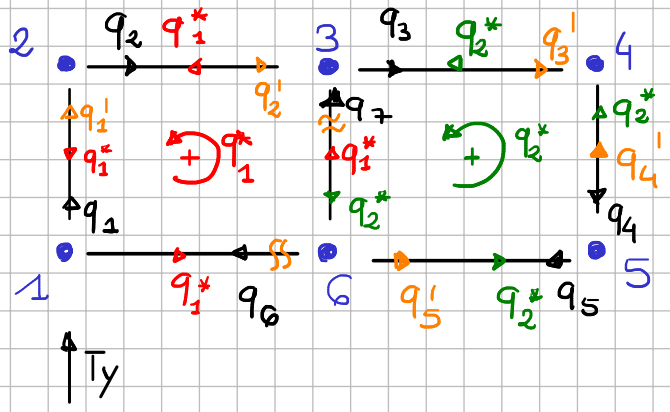
2) $q_2' = -T_y \frac{S_{x_1} + S_{x_2} = 0}{J_{xx}} = 0$

3) $q_3' = -T_y \cdot \frac{S_{x_3} + S_{x_1} + S_{x_2}}{J_{xx}} = -\frac{T_y}{3a}$

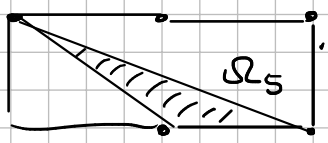
4) $q_4' = -T_y \frac{S_{x_6} + S_{x_5}}{J_{xx}} = \frac{2T_y}{3a}$

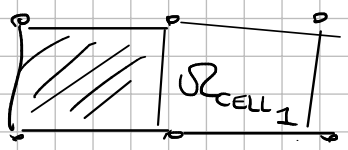
MOMENT EQUIVALENCE

WRT (2) (+)



$$T_y \cdot 0 = q_5' \cdot 2\Omega_5 + q_4' \cdot 2\Omega_4 + q_1^* \cdot 2\Omega_{CELL_1} + q_2^* \cdot 2\Omega_{CELL_2}$$





$$\Omega_5 = \frac{1}{2} ab$$

$$\Omega_4 = \frac{1}{2} a \cdot 2b = ab$$

$$\Omega_{CELL_1} = \Omega_{CELL_2} = ab$$

$$\frac{T_y}{3a} \cdot ab + \frac{2T_y}{3a} \cdot 2ab + q_1^* 2ab + q_2^* \cdot 2ab = 0$$

$$\Rightarrow \frac{5}{3} T_y + 2q_1^* \cdot a + 2q_2^* \cdot a = 0$$

COMPATIBILITY

$\vartheta_1' = \vartheta_2'$ $\qquad \vartheta' = \frac{1}{2\Omega_{cell} \cdot G} \sum_k \frac{q_i l_i}{t_i}$

$$\vartheta_1' = \frac{1}{2\Omega_{cell_1} \cdot G} \cdot \frac{1}{t} \left(q_1^* (2a+2b) - q_1' \cdot (a) - q_2' \cdot (b) - q_2^* \cdot a \right)$$

$$\vartheta_2' = \frac{1}{2\Omega_{cell_2} \cdot G} \cdot \frac{1}{t} \left(q_2^* (2a+2b) + q_5' \cdot (b) + q_4' \cdot (a) - q_3' \cdot (b) - q_1^* \cdot a \right)$$

$\vartheta_1' = \vartheta_2'$ WE KNOW $\Omega_{CELL_1} = \Omega_{CELL_2}$ G SAME t SAME

$$\begin{cases} \text{COMP} & 3q_1^* a + 2q_1^* b - \frac{1}{3} T_y = 3q_2^* a + 2q_2^* b + \frac{1}{3} T_y \frac{b}{a} + \frac{2}{3} T_y + \frac{1}{3} T_y \frac{b}{a} \\ \text{MOM} & \frac{5}{3} T_y + 2q_1^* \cdot a + 2q_2^* \cdot a = 0 \end{cases}$$

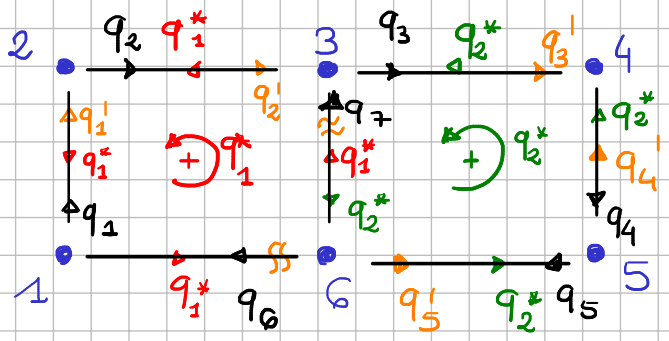
FROM MOM $\quad q_1^* = -q_2^* - \frac{5}{6} T_y \cdot \frac{1}{a}$

IN COMP $\Rightarrow \quad -3q_2^* a - \frac{5}{2} T_y - 2q_2^* b - \frac{5}{3} T_y \frac{b}{a} = 3q_2^* a + 2q_2^* b + T_y + \frac{2}{3} T_y \frac{b}{a}$

$$\rightarrow q_2^* = -\left(\frac{7}{2} T_y + \frac{7}{3} T_y \cdot \frac{b}{a} \right) \cdot \frac{1}{6a+4b} = -7 \ N/mm$$

FROM MOM $\rightarrow q_1^* = -3 \ N/mm$

COMBINE



$q_1 = q_1' - q_1^* = 4+3 = 7 \ N/mm$

$q_2 = q_2' - q_1^* = 0+3 = 3 \ N/mm$

$q_3 = q_3' - q_2^* = -4+7 = 3 \ N/mm$

$q_4 = -q_4' - q_2^* = -8+7 = -1 \ N/mm$

$q_5 = -q_5' - q_2^* = -4+7 = +3 \ N/mm$

$q_6 = -q_1^* = 3 \ N/mm$

$q_7 = q_1^* - q_2^* = 4 \ N/mm$