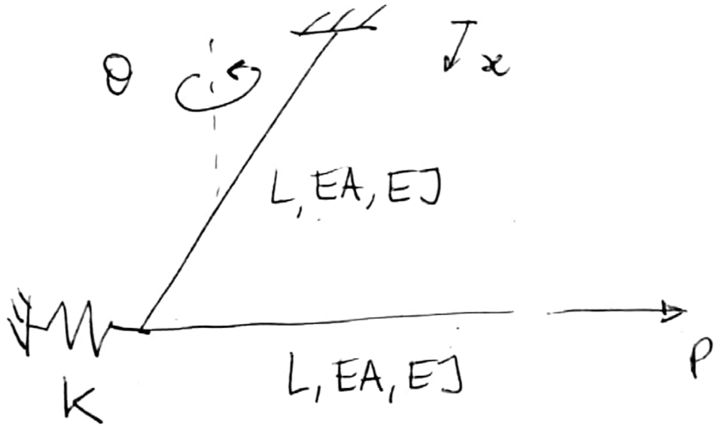


Find the rotation θ (in deg) at $x = L/2$

Data

$L = 400\,(1 + A/10)$ mm

$EA = 7.2 \cdot 10^4$ N

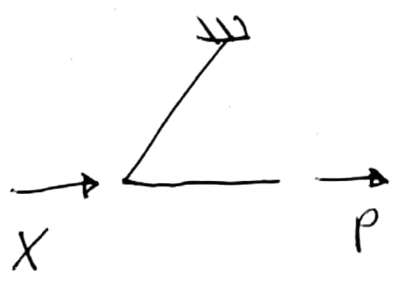
$EJ = 1.25 \cdot 10^8\,(1 + C/10)$ N mm^2

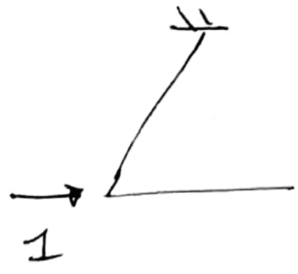
$K = 60$ N/mm

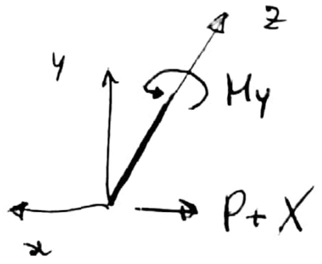
$P = 5000$ N

Solution $(A = C = 0)$

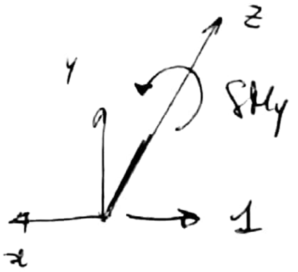
Evaluate the reaction force in correspondence of the spring







Real



Dummy

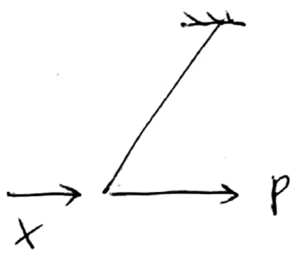
$$M_y = -(P+X)z \qquad \delta M_y = -z$$

$$\int_0^L \frac{M_y\,\delta M_y}{EJ}\,dz + \frac{X}{k} = 0$$

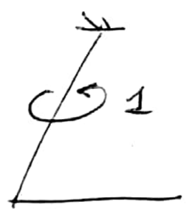
From which:

$$X = -\frac{P}{\left(1 + \frac{3EJ}{kL^3}\right)}$$

Evaluate now the rotation



Real



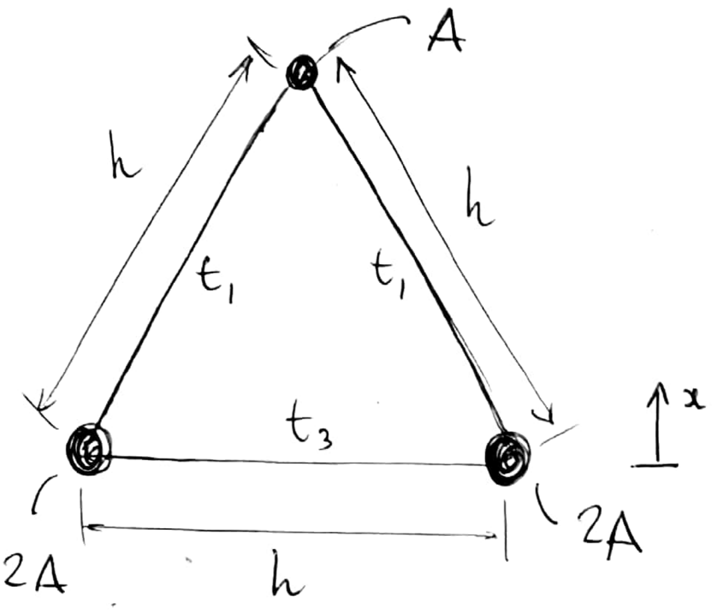
Dummy

As before $\qquad \delta M_y = -1 \quad z \geq L/2$

$$\int_{L/2}^{L} \frac{M_y \, \delta M_y}{EJ} \, dz = \theta$$

From which

$$\theta = \frac{3L^2 (P+X)}{8EJ} = 12.23^\circ$$



Find the vertical position of the shear center (measured from $x=0$).

Data

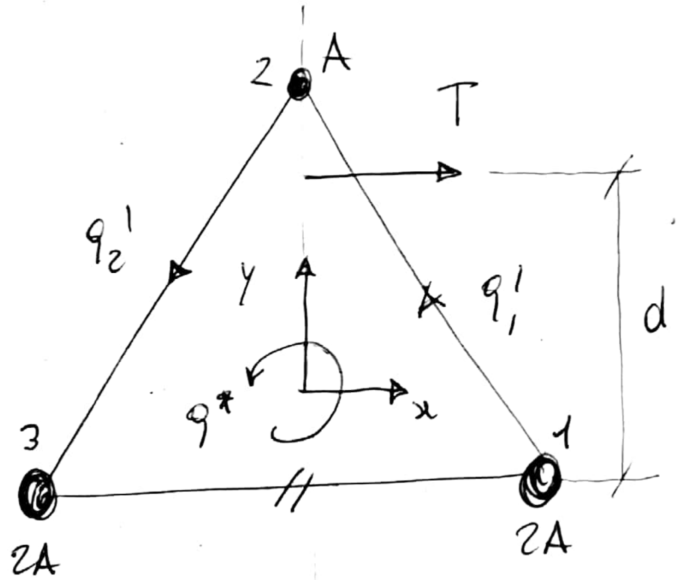
$A = 500 \ mm^2$

$h = 150 \ mm$

$t_1 = 1 \cdot (1 + F/10)$

$t_3 = 2 \ (1 + A/10)$

Solution $(A = F = 0)$



Apply T at an arbitrary distance d from the reference position

$(T = 1000 \ N)$

$$J_{yy} = 2A\left(\frac{h}{2}\right)^2 2 = Ah^2$$

$$S'_{y_1} = 2A\,\frac{h}{2} = Ah$$

$$S'_{y_2} = Ah$$

Applying the shear flow equation two times:

$$q'_1 = -T\,\frac{S'_{y_1}}{J_{yy}} = -T/h = -6.67 \ \ N/mm$$

$$q'_2 = -T\,\frac{S'_{y_2}}{J_{yy}} = -T/h = -6.67 \ \ N/mm$$

Equivalence wrt 1

$$2q_*\Omega + 2q'_2\Omega = -Td$$

Impose $\theta' = 0$

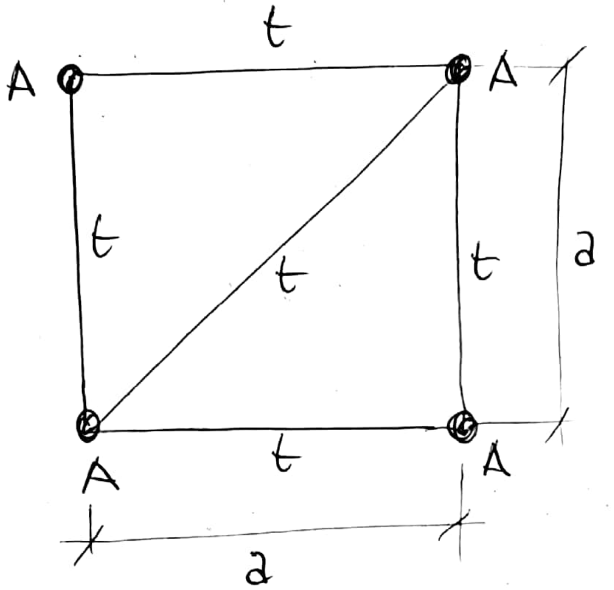
$$\theta' = \frac{1}{2G\Omega}\,\Sigma_i\,\frac{q_i l_i}{t_i} = 0$$

$$\frac{q_1 h}{t_1} + \frac{q_2 h}{t_1} + \frac{q_3 h}{t_3} = 0$$

$$(q'_1 + q^*) + (q'_2 + q^*) + \frac{t_1}{t_3}\,q^* = 0$$

$$\Rightarrow \quad q^* = \frac{2T}{h}\,\frac{1}{2 + t_1/t_3} = 5.33 \ \ N/mm$$

$$d = -\frac{2\Omega\,(q_* + q'_2)}{T} = 25.98 \ mm$$



Find the torsional stiffness of the section GJ/GJ_{ref}

Data

$A = 600 \text{ mm}^2$

$a = 400\,(1+B/10)$ mm

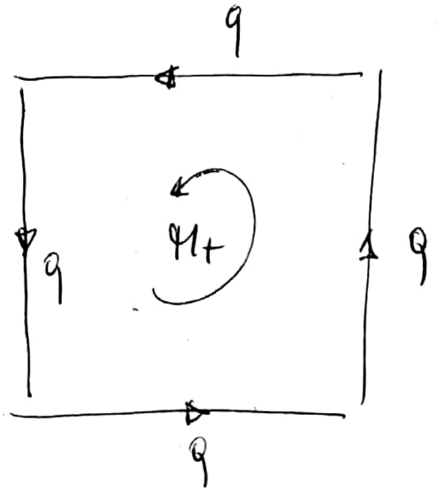
$t = 1\,(1+A/10)$ mm

$G = 2.7\,10^4$ MPa

$GJ_{ref} = 1 \cdot 10^{10} \text{ N mm}^2$

Solution ($A = B = 0$)

From the symmetries of the section it is immediate to note that the position of the shear center is in the middle of the section. Therefore the shear flow along the oblique panel does not provide any torsional resistance. Therefore, owing to the symmetry of the section, the flows on the four remaining panels is equal.



$$M_t = 2\Omega_c q \quad \text{with } \Omega_c = a^2$$

$$q = \frac{M_t}{2a^2}$$

from which

$GJ = G a^3 t$ and so:

$$\frac{GJ}{GJ_{ref}} = \frac{G a^3 t}{GJ_{ref}} = 172.80$$

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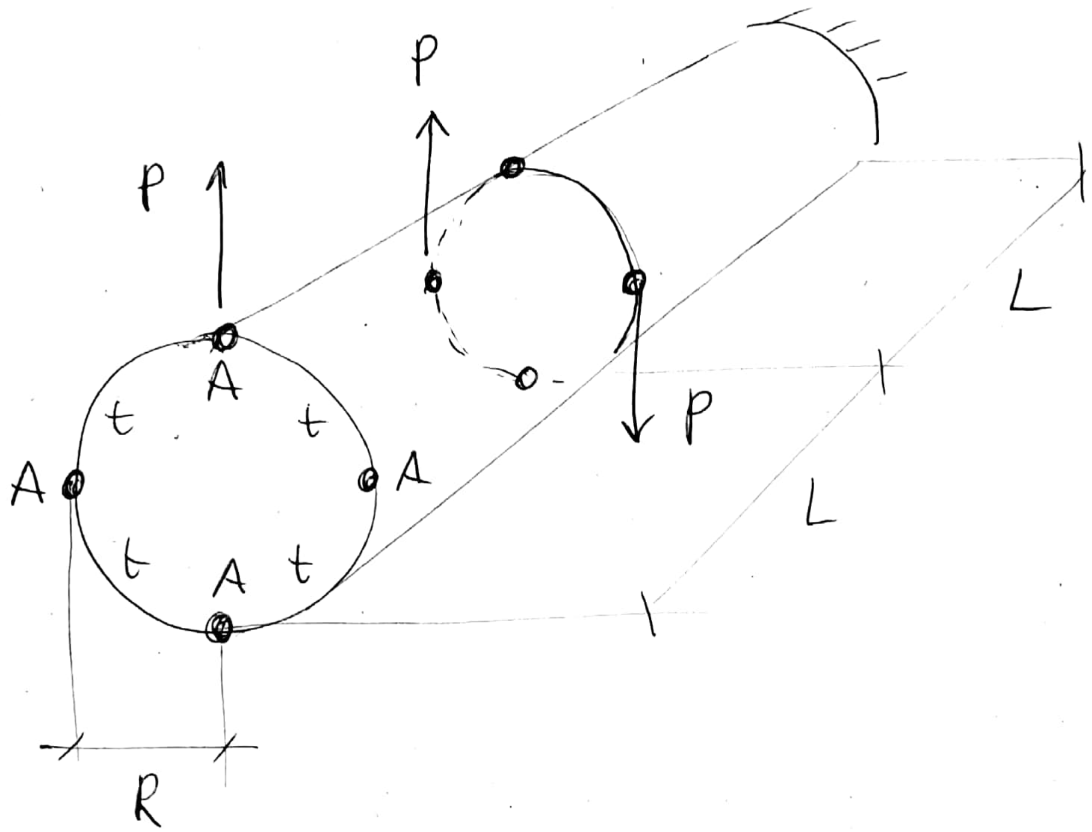


Evaluate the rotation θ (in deg) at the free end.

Data

$A = 500\ mm^2$

$R = 80\ mm$

$t = 1.2\ (1 + A/10)\ mm$

$L = 2500\ (1 + F/10)\ mm$

$G = 27000\ MPa$

$P = 7500\ N$

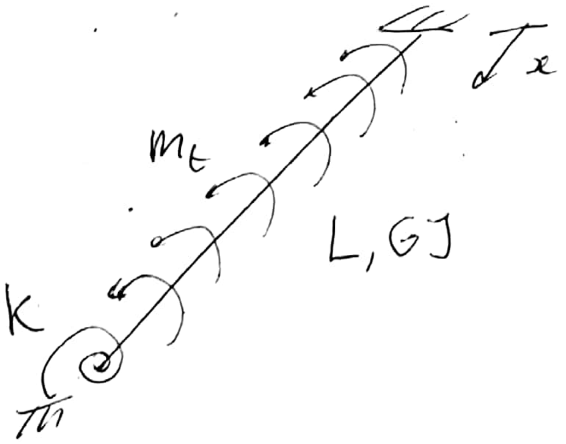
Solution $(A = F = 0)$

From Bredt's formula: $GJ = G\, 2\pi R^3 t$

and so:

$$\theta' = \frac{M_t}{G\, 2\pi R^3 t} = \frac{2PR}{G\, 2\pi R^3 t} = \frac{P}{G \pi R^2 t} \Rightarrow \theta' L = \theta$$

$$\theta = 1.65°$$



Use the Ritz method and a 1-dof polynomial approximation to determine the torsional moment $M_t / M_{t\,ref}$ at $x = L/2$

<u>Data</u>

$L = 3700$ mm

$GJ = 2.2 \cdot 10^6$ Nmm^2

$k = 1 \cdot 10^4 \; (1 + \Delta/10)$ Nmm

$m_t = 520 \; (1 + A/10)$ N

$M_{t\,ref} = 3 \cdot 10^4$ Nmm

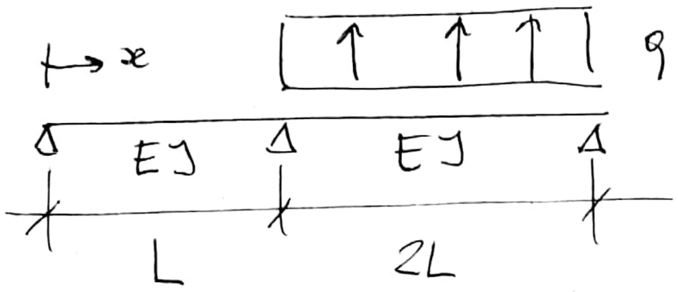
<u>Solution</u> $(A = \Delta = 0)$

$\theta = x q_1 \; ; \; \theta' = q_1$

$$\int_0^L \delta\theta' GJ \theta' \, dx + \delta\theta(L) k \theta(L) = \int_0^L \delta\theta \, m_t \, dx$$

$$\delta q_1 \left(GJL + kL^2 \right) q_1 = \delta q_1 \frac{L^2}{2} m_t$$ so:

$$M_t = GJ\theta' = GJ q_1 = GJ \frac{L^2 m_t}{2(GJL + kL^2)} \; ; \; \frac{M_t}{M_{t\,ref}} = 1.8$$



Use the Ritz method and a 1-dof trigonometric approximation to determine the vertical displacement at $x = L/2$

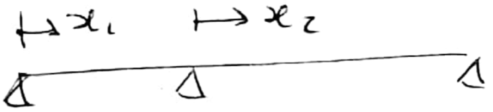
<u>Data</u>

$L = 300\,(1 + B/10)$ mm

$EJ = 1.25 \cdot 10^{8}$ N mm^2

$q = 0.2$ N/mm

<u>Solution</u> ($B = 0$)



$$w_1 = \sin\frac{\pi x_1}{L}\, q_1$$

$$w_2 = \sin\frac{\pi x_2}{2L}\, q_2$$

$w_1(L) = w_2(0) \rightarrow$ identically satisfied

$w_{1/x}(L) = w_{2/x}(0) \rightarrow$ to be imposed, so:

$$w_1 = \sin\frac{\pi x_1}{L}\, q_1$$

$$w_2 = -2\sin\frac{\pi x_2}{2L}\, q_1$$

$$\int_0^L \delta w_{1/xx} EJ\, w_{1/xx}\, dx_1 + \int_0^{2L} \delta w_{2/xx} EJ\, w_{2/xx}\, dx_2 =$$

$$= \int_0^{2L} \delta w_2\, q\, dx_2$$, from which:

$$q_1 = -\frac{8L}{\pi} q \frac{1}{\left(\frac{\pi}{L}\right)^4 EJ \, \frac{3}{4} L}$$

The displacement in $x = L/2$ is

$$w_1(L/2) = \sin \frac{\pi}{2} q_1 = q_1 = -0.452 \text{ mm}$$

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Q The Principle of Complementary Virtual Energy is used to find the equilibrium solution.

A False

Q De Saint Venant solution should not be used for evaluation of stresses near abrupt changes of section geometry.

A True

Q The shear flow equation is derived from an equilibrium condition. Therefore, the shear flow equation does not account for compatibility requirements.

A True

Q The solution of the elastic problem

A must guarantee equilibrium and compatibility

Q The semi-monocoque approximation provides

A an approximate solution for the shear stresses

Q The linear static response of simply-supported beam with bending stiffness EJ and loaded with a uniform load

A can be analyzed by imposing symmetry conditions

Q The Principle of Complementary Virtual Work is used to find the equilibrium solution.

A False

Q De Saint Venant solution should not be used for evaluation of stresses near abrupt changes of section geometry.

A True

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