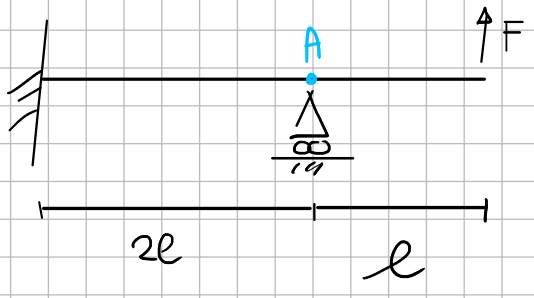
EXERCISE SESSION 3 – 07/10/22

Hyperstatic beam systems

Ex 1



DATA:

$\ell = 1000\ mm$

$F = 4000\ N$

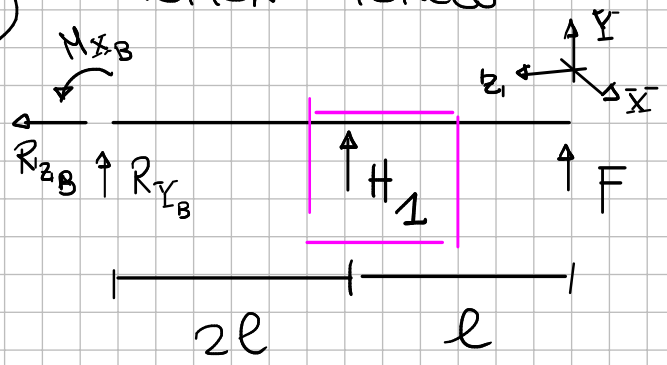
$E = 200000\ MPa$

$J_{xx} = 500000\ mm^4$

FIND

REACT. FORCE IN A

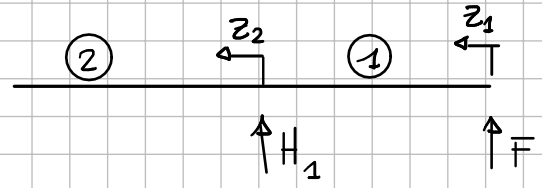
1) REACTION FORCES

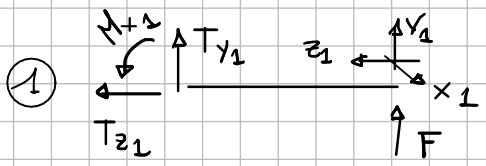


EQ EQS

$$\begin{cases} R_{z_B} = 0 \\ R_{y_B} + H_1 + F = 0 \\ M_{x_B} = -H_1 \cdot 2\ell - F \cdot 3\ell \end{cases}$$

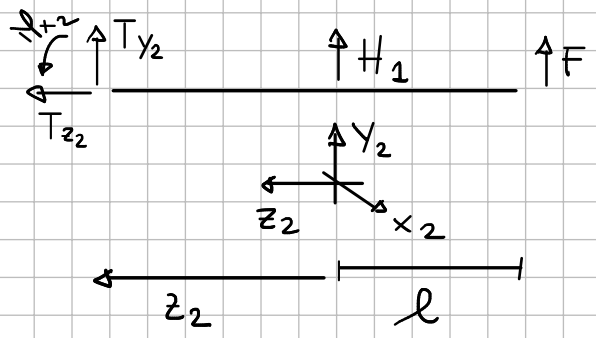
2) TRUE INTERNAL FORCES





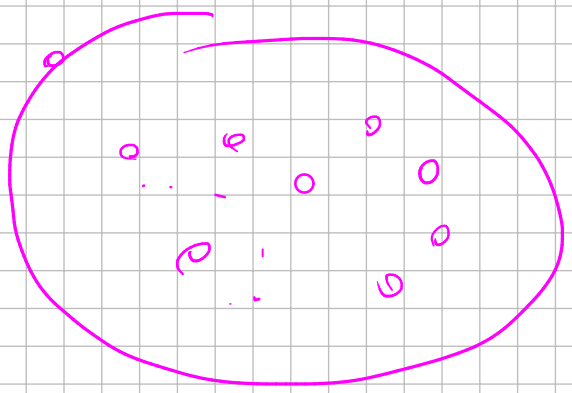
EQ EQS

$$\begin{cases} T_{z_1}(z_1) = 0 \\ T_{y_1}(z_1) = -F \\ M_{x_1}(z_1) = -F \cdot z_1 \end{cases}$$

② 

$$\begin{cases} T_{z_2}(z_2) = 0 \\ T_{y_2}(z_2) = -H_1 - F \\ M_{x_2}(z_2) = -H_1 \cdot z_2 - F(z_2 + \ell) \end{cases}$$

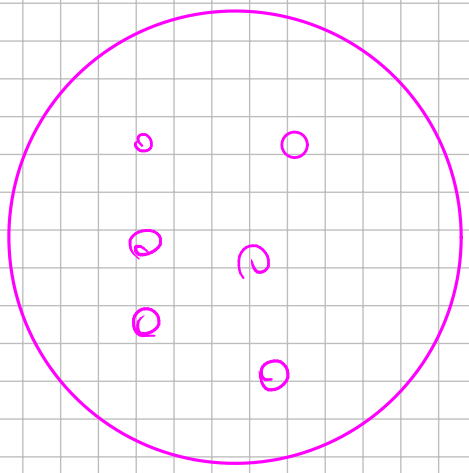
PVW



INFINITE COMPATIBLE SOLUTIONS

SOLUTION WHICH SATISFIES PVW → IS ALSO EQUILIBR

PCVW



INFINITE EQUILIBRATED SOLUTIONS

SOLUTION WHICH SATISFIES PCVW → IS ALSO COMPATIBLE

PCVW

$$\delta W_i = \delta W_e$$

WHERE δW_e = EXTERNAL VIRT. WORK.

δW_i = INT. VIRT WORK (DEFORM WORK)

FOR BEAMS

$$\delta W_e = \sum \left(\delta F_i \cdot u_i + \delta M_i \varphi_i \right)$$

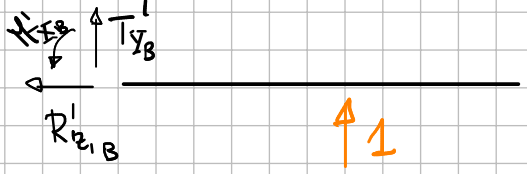
$$\delta W_i = \int_L \frac{T_{z_i} T'_{z_i}}{EA} + \frac{T_{y_i} T'_{y_i}}{\cancel{GA}^{\,0}} + \frac{T_{x_i} T'_{x_i}}{\cancel{GA}^{\,0}} + \frac{M_{x_i} M'_{x_i}}{EJ_{xx}} + \frac{M_{y_i} M'_{y_i}}{EJ_{yy}} + \frac{M_{z_i} M'_{z_i}}{GJ_p} \, dz$$

$$\delta W_i = \int_L \left(\frac{T_{z_i} T'_{z_i}}{EA} + \frac{T_{y_i} T'_{y_i}}{0 \cdot GA} + \frac{T_{x_i} T'_{x_i}}{0 \cdot GA} + \frac{M_{x_i} M'_{x_i}}{EJ_{xx}} + \frac{M_{y_i} M'_{y_i}}{EJ_{yy}} + \frac{M_{z_i} M'_{z_i}}{GJ_p} \right) dz$$

FORCE VIRT. — DISPL

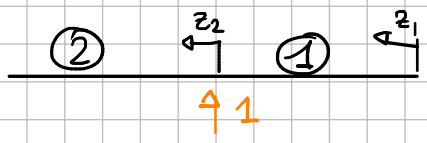
WE WILL STUDY A DUMMY SYST. EQUILIBRATED

1) REACTION FORCES DUMMY

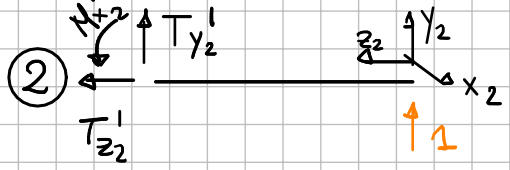


$$\begin{cases} R'_{z_1 B} = 0 \\ R'_{y_B} = -1 \\ M'_{x_B} = -1 \cdot 2\ell \end{cases}$$

2) INTERN. ACTIONS DUMMY



① UNLOADED



$$\begin{cases} T'_{z_2}(z_2) = 0 \\ T'_{y_2}(z_2) = -1 \\ M'_{x_2}(z_2) = -1 \cdot z_2 \end{cases}$$

3) PCVW

$$\delta W_e = \overset{\text{FORCE}}{1} \cdot 0 = 0$$

← FORCES FROM DUMMY
DISPL FROM REAL

$$\delta W_i = \int_0^{2\ell} \frac{M_{x_2}}{EJ_{xx}} \cdot M'_{x_2} \, dz_2$$

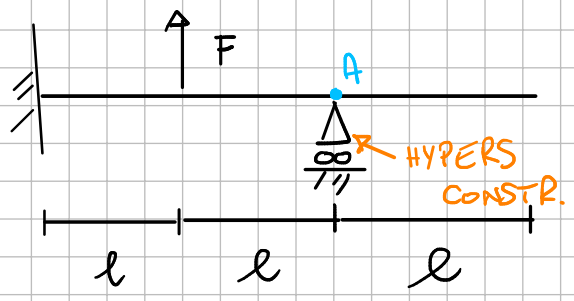
$$= \int_0^{2\ell} \left(-H_1 \cdot z_2 - F(z_2 + \ell) \right) \frac{1}{EJ_{xx}} \cdot -1 \cdot z_2 \, dz_2$$

$$= \frac{1}{EJ_{xx}} \left[\frac{1}{3} H_1 z_2^3 + \frac{1}{3} F z_2^3 + \frac{1}{2} F \ell z_2^2 \right]_0^{2\ell}$$

PCVW → $\delta W_e = \delta W_i \rightarrow \frac{8}{3} H_1 \ell^3 = -\frac{8}{3} F\ell^3 - \frac{4}{2} F\ell^3 \rightarrow$

$$H_1 = -\frac{7}{4} F = -7000 \text{ N}$$

Ex 2



DATA:

$\ell = 1000$ mm

$F = 4000$ N

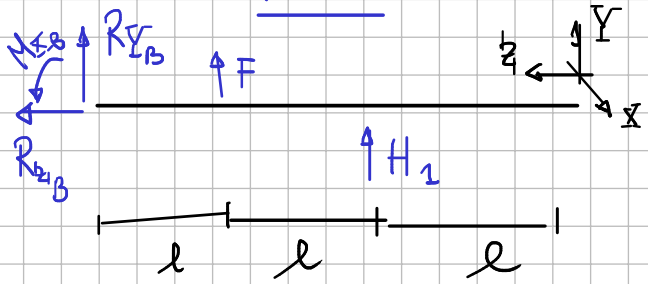
$E = 200000$ MPa

$J_{xx} = 500000$ mm^4

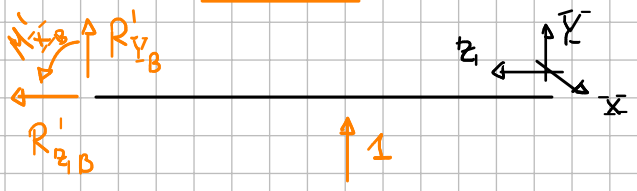
FIND REACT. FORCE IN A

1) REACTION FORCES

REAL



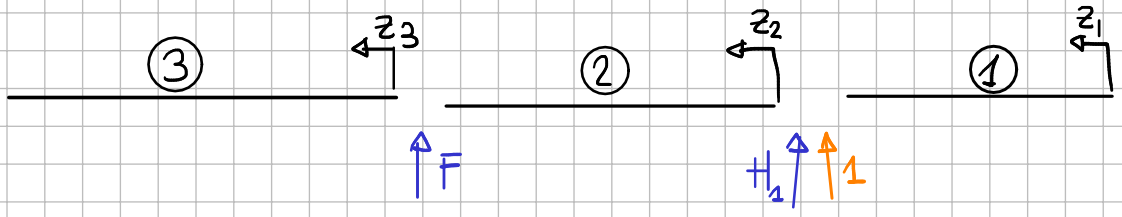
DUMMY

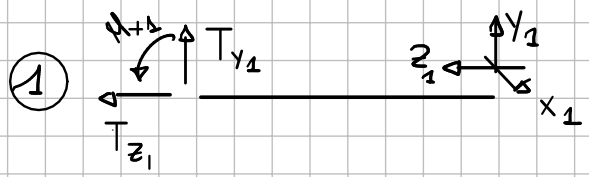


$$\begin{cases} R_{z_{1_B}} = 0 \\ R_{Y_B} = -H_1 - F \\ M_{X_B} = -H_1 \cdot 2\ell - F \cdot \ell \end{cases}$$

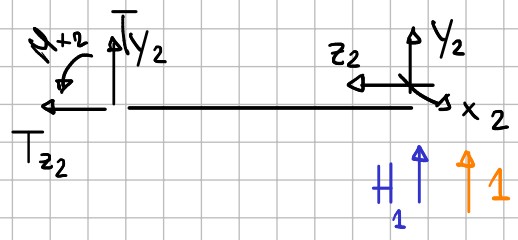
$$\begin{cases} R'_{z_{1_B}} = 0 \\ R'_{Y_B} = -1 \\ M'_{X_B} = -1 \cdot 2\ell \end{cases}$$

2) INTERNAL ACTIONS





UNLOADED FOR REAL DUMMY

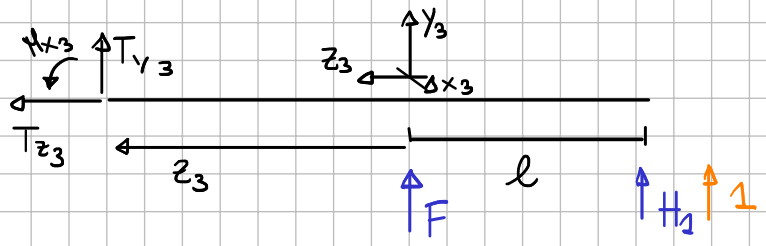
② 

REAL

$$\begin{cases} T_{z_2} = 0 \\ T_{y_2}(z_2) = -H_1 \\ M_{x_2}(z_2) = -H_1 \cdot z_2 \end{cases}$$

DUMMY

$$\begin{cases} T'_{z_2}(z_2) = 0 \\ T'_{y_2}(z_2) = -1 \\ M'_{x_2}(z_2) = -1 \cdot z_2 \end{cases}$$

③ 

REAL

$$\begin{cases} T_{z_3} = 0 \\ T_{y_3} = -F - H_1 \\ M_{x_3} = -F \cdot z_3 - H_1 (z_3 + \ell) \end{cases}$$

DUMMY

$$\begin{cases} T'_{z_3} = 0 \\ T'_{y_3} = -1 \\ M'_{x_3} = -1 (z_3 + \ell) \end{cases}$$

3) PCVW

$$\delta W_e = \overset{\text{DUMMY}}{1} \cdot \overset{\text{REAL DISPL.}}{0} = 0$$

$$\delta W_i = \int_0^\ell \frac{M_{x_2}}{EJ_{xx}} \cdot M'_{x_2} dz_2 + \int_0^\ell \frac{M_{x_3}}{EJ_{xx}} \cdot M'_{x_3} dz_3$$

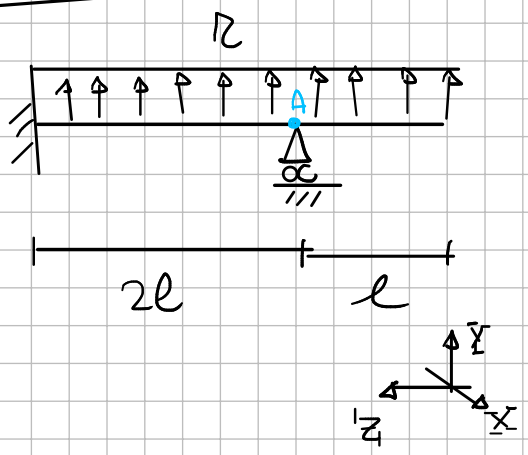
$$= \int_0^\ell \frac{1}{EJ_{xx}} \cdot H_1 z_2^2 dz_2 + \int_0^\ell \frac{1}{EJ_{xx}} (z_3 + \ell) \cdot (F z_3 + H_1 z_3 + H_1 \ell) dz_3$$

$$= \frac{1}{EJ_{xx}} \left[\frac{1}{3} H_1 z_2^3 \right]_0^\ell + \frac{1}{EJ_{xx}} \left[\frac{1}{3} F z_3^3 + \frac{1}{3} H_1 z_3^3 + \frac{1}{2} H_1 \ell z_3^2 + \frac{1}{2} F z_3^2 \ell + \frac{1}{2} H_1 \ell z_3^2 + H_1 \ell^2 z_3 \right]_0^\ell$$

PCVW $\longrightarrow$ $\delta W_i = \delta W_e \Rightarrow \frac{5}{6} F \ell^3 + \frac{8}{3} H_1 \ell^3 = 0$

$$\rightarrow H_1 = -\frac{5}{16} F = -1250 \text{ N}$$

Ex 3



DATA:

$\ell = 1000\ mm$

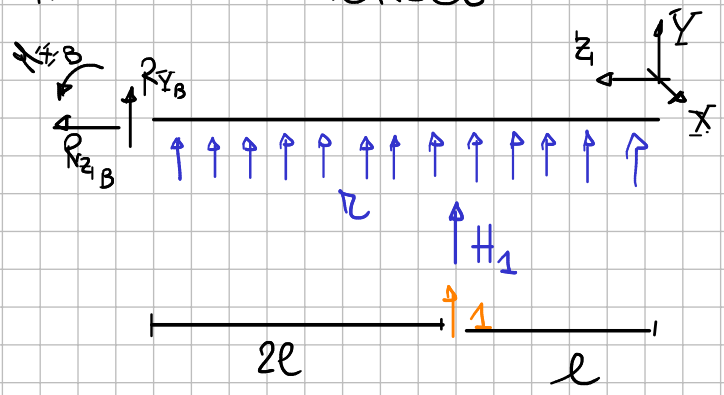
$r = 1\ N/mm$

$E = 200\ 000\ MPa$

$J_{xx} = 50\ 00\ 00\ mm^4$

FIND REACTION IN A

1) REACTION FORCES



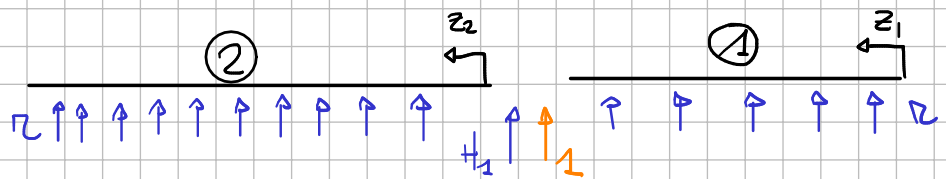
REAL

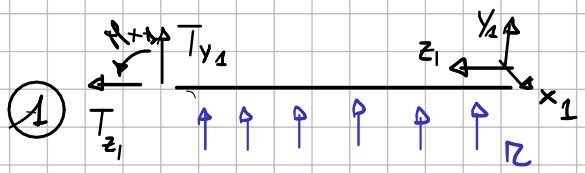
$$\begin{cases} R_{z_B} = 0 \\ R_{Y_B} = -H_1 - r \cdot 3\ell \\ M_{X_B} = -H_1 \cdot 2\ell - r\,3\ell \cdot \frac{3}{2}\ell \end{cases}$$

DUMMY

$$\begin{cases} R'_{z_B} = 0 \\ R'_{Y_B} = -1 \\ M'_{X_B} = -1 \cdot 2\ell \end{cases}$$

2) INTERNAL ACTIONS



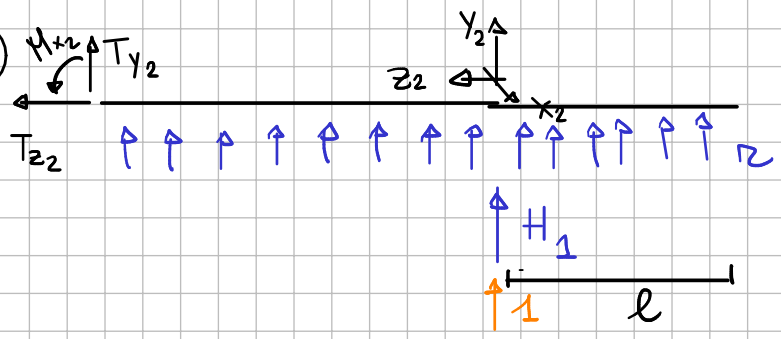


REAL

[· -]

DUMMY

WILL NOT CONTRIBUTE TO PCUW

② 

REAL

$$T_{z_2}(z_2) = 0$$

$$T_{y_2}(z_2) = -r \cdot (l + z_2) - H_1$$

$$M_{x_2}(z_2) = -H_1 \cdot z_2 - \underbrace{\frac{1}{2} r z_2^2}_{\text{PART } ②} - \underbrace{(r \cdot l) \cdot \left(z_2 + \frac{1}{2} l\right)}_{\text{PART } ①}$$

$$= -H_1 z_2 - \frac{1}{2} r z_2^2 - r l z_2 - \frac{1}{2} r l^2$$

DUMMY

$$\begin{cases} T'_{z_2} = 0 \\ T'_{y_2} = -1 \\ M'_{x_2} = -1 \cdot z_2 \end{cases}$$

3) PCVW

$$\delta W_e = 1 \cdot 0$$

$$\delta W_i = \int_0^{2\ell} M'_{x_2} \cdot \frac{M_{x_2}}{EJ_{xx}} \, dz_2$$

$$= \frac{1}{EJ_{xx}} \int_0^{2\ell} -z_2 \cdot \left(-H_1 z_2 - \frac{1}{2} q z_2^2 - q\ell z_2 - \frac{1}{2} q\ell^2 \right) dz_2$$

PCVW $\longrightarrow$ $\delta W_i = 0$

$$\rightarrow \left[\frac{1}{3} H_1 z_2^3 + \frac{1}{8} q z_2^4 + \frac{1}{3} q\ell z_2^3 + \frac{1}{4} q\ell^2 z_2^2 \right]_0^{2\ell} = 0$$

$$\frac{8}{3} H_1 \ell^3 + \frac{16}{8} q\ell^4 + \frac{8}{3} q\ell^4 + \frac{4}{4} q\ell^4 = 0$$

$$H_1 = -\frac{3}{8} \cdot q\ell \left(2 + \frac{8}{3} + 1 \right) = -\frac{17}{8} q\ell = -2125\,N$$