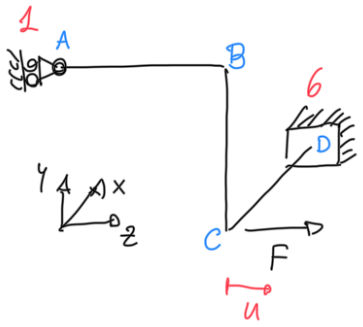
LAB 6

Displacement of Beams Systems II

1) EXAM 13/06/2023

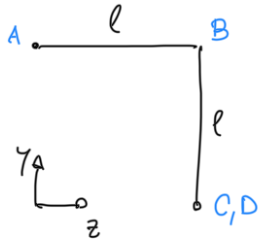
3D

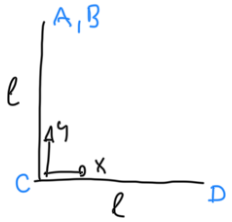


A: 0 , 1000 , 0 mm
B: 0 , 1000 , 1000 mm
C: 0 , 0 , 1000 mm
D: 1000 , 0 , 1000 mm

Let's find u

Hyperstatic





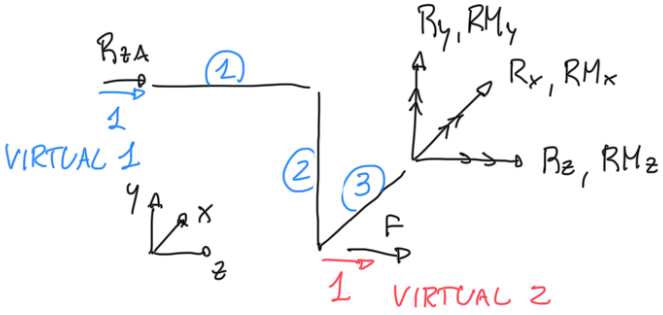
DATA

$F = 1000\ N$

$EA = 10^6\ N$

$EJ_{xx} = EJ_{yy} = GJ = 10^{10}\ Nmm^2$

$\ell = 1000\ mm$



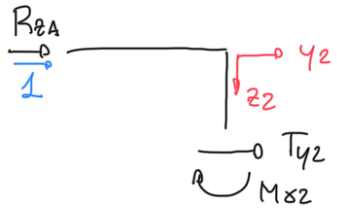
If we are smart, we can avoid computing all the RFs

- Internal Actions

REAL $T_{z1} = -R_{zA}$

VIRTUAL 1 $T'_{z1} = -1$

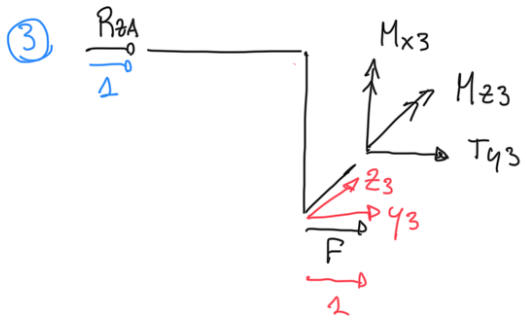
VIRTUAL 2 unloaded

② 

REAL $\begin{cases} T_{y2} = -R_{zA} \\ M_{x2} = -R_{zA} \cdot z_2 \end{cases}$

VIRTUAL 1 $\begin{cases} T'_{y2} = -1 \\ M'_{x2} = -z_2 \end{cases}$

VIRTUAL 2 unloaded

③ 

REAL
$\begin{cases} T_{y3} = -R_{zA} - F \\ M_{x3} = -(R_{zA} + F) \cdot z_3 \\ M_{z3} = -R_{zA} \cdot \ell \end{cases}$

VIRTUAL 1
$\begin{cases} T'_{y3} = -1 \\ M'_{x3} = -z_3 \\ M'_{z3} = -\ell \end{cases}$

VIRTUAL 2
$\begin{cases} T''_{y3} = -1 \\ M''_{x3} = -z_3 \\ M''_{z3} = 0 \end{cases}$

- PCVV

VIRTUAL 1

$$\delta W_e = 1 \cdot \phi = \phi$$

$$\delta W_i = \int_0^\ell T'_{z1} \cdot \frac{T_{z1}}{EA} dz_1 + \int_0^\ell M'_{x2} \cdot \frac{M_{x2}}{EJ} dz_2 + \int_0^\ell \left(M'_{x3} \frac{M_{x3}}{EJ} + M'_{z3} \frac{M_{z3}}{GJ} \right) dz_3$$

$$= \frac{1}{EA} \cdot \int_0^\ell R_{zA} \cdot dz_1 + \frac{1}{EJ} \int_0^\ell R_{zA} \cdot z_1^2 \, dz_1 + \frac{1}{EJ} \int_0^\ell (R_{zA} + F) z_3^2 \, dz_3 + \frac{1}{GJ} \int_0^\ell R_{zA} \ell^2 \, dz_3 =$$

$$= \frac{R_{zA} \ell}{EA} + \frac{R_{zA} \ell^3}{3EJ} + \frac{(R_{zA} + F) \ell^3}{3EJ} + \frac{R_{zA} \ell^3}{GJ}$$

$$\delta W_e = \delta W_i \quad \rightarrow \quad R_{zA} = -125 \ N$$

VIRTUAL 2

$$\delta W_e = 1 \cdot u$$

$$\delta W_i = \int_0^{\ell} M_{x3}^{II} \cdot \frac{M_{x3}}{EJ}\, dz_3 = \frac{1}{EJ}\int_0^{\ell} (R_{zA} + F)\cdot z_3^2\, dz_3 =$$

$$= \frac{1}{EJ}\left[\frac{1}{3}(R_{zA} + F)\cdot z_3^3\right]_0^{\ell} = \frac{(R_{zA} + F)\,\ell^3}{3EJ} \qquad u = 29.1667 \text{ mm}$$

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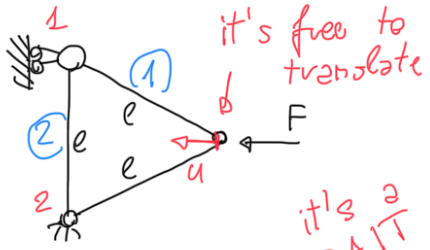
2) EXAM 09/09/2024 Let's compute u



DATA

$\ell = 1000$ mm

$EA = 10^8$ N

$EJ_{xx} = EJ_{yy} = 10^{13}$ N mm^2

$GJ = 7 \cdot 10^9$ N mm^2

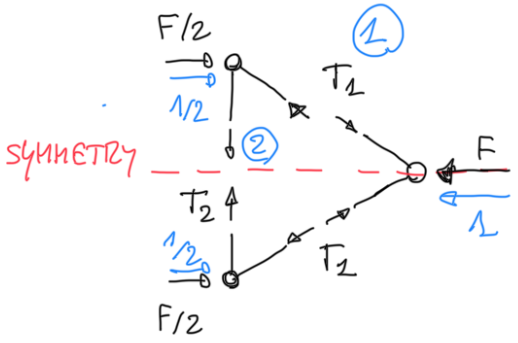
$F = 1000$ N

ALL TRUSS

- ONLY AXIAL LOADS
- NO MOMENTS, NO SHEARS

ISOSTATIC

- Internal Actions



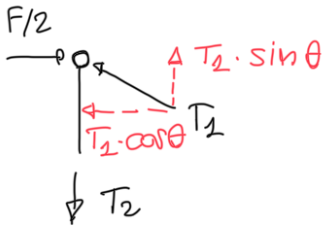
(1) $\theta = 30°$

REAL

$T_1 \cdot \cos\theta = \frac{F}{2}$ $\quad T_1 = \frac{F}{2}\cdot\frac{2}{\cdot} = \frac{F}{\cdot}$

VIRTUAL

$T_1' \cdot \cos\theta = \frac{1}{2}$ $\qquad T_1' = \frac{1}{\sqrt{3}}$

② 

REAL

horizontal eq. $T_1 \cdot \cos\theta = \frac{F}{2}$ TRIVIAL

vertical eq. $T_2 = T_1 \sin\theta = \frac{F}{\sqrt{3}} \cdot \frac{1}{2} = \frac{F}{2\sqrt{3}}$

VIRTUAL $\qquad T_2' = \frac{1}{2\sqrt{3}}$

- PCVW

$$\delta W_e = 1 \cdot u$$

$$\delta W_i = 2 \cdot \frac{1}{EA}\int_0^{\ell} T_2' T_2 \, dz_1 + \frac{1}{EA}\int_0^{\ell} T_2' T_2 \, dz_2 =$$

$$= \frac{2}{EA}\int_0^{\ell} \frac{F}{3} dz_1 + \frac{1}{EA}\int_0^{\ell} \frac{F}{12} dz_2 = \frac{2F\ell}{EA} + \frac{F\ell}{12EA}$$

$\delta W_e = \delta W_i \rightarrow u = 0{,}00775$ mm