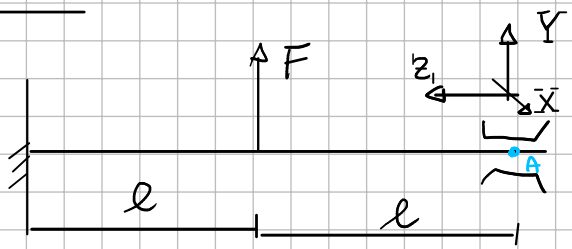
EXERCISE SESSION 4 — 14/10/2022

7) Hyperstatic beam systems — Displ. of beam syst.

Ex 1



A LOCKS: TRANSL Y

ROT. IN X

DATA:

$\ell = 1000$ mm

$F = 4000$ N

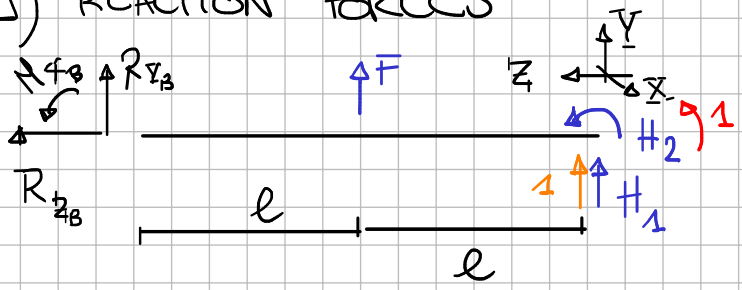
$E = 200000$ MPa

$J_{xx} = 500000$ mm^4

FIND:

REACTION FORCES IN A

1) REACTION FORCES



REAL

DUMMY 1

DUMMY 2

REAL

$$\begin{cases} R_{Z_B} = 0 \\ R_{Y_B} = -F - H_1 \\ H_{X_B} = -F \cdot \ell - H_1 \cdot 2\ell - H_2 \end{cases}$$

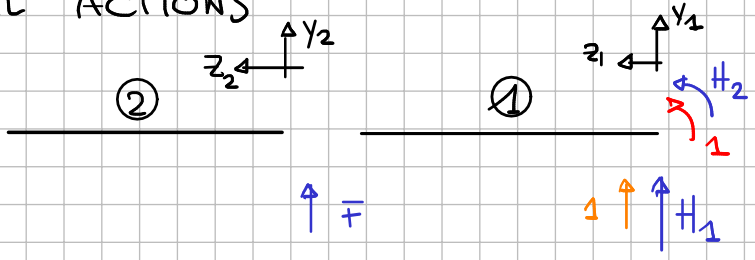
DUMMY 1

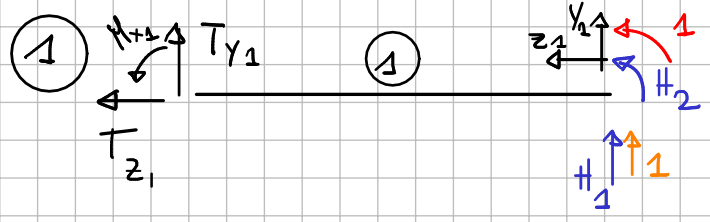
$$\begin{cases} R'_{Z_B} = 0 \\ R'_{Y_B} = -1 \\ H'_{X_B} = -1 \cdot 2\ell \end{cases}$$

DUMMY 2

$$\begin{cases} R''_{Z_B} = 0 \\ R''_{Y_B} = 0 \\ M''_{X_B} = -1 \end{cases}$$

2) INTERNAL ACTIONS





REAL

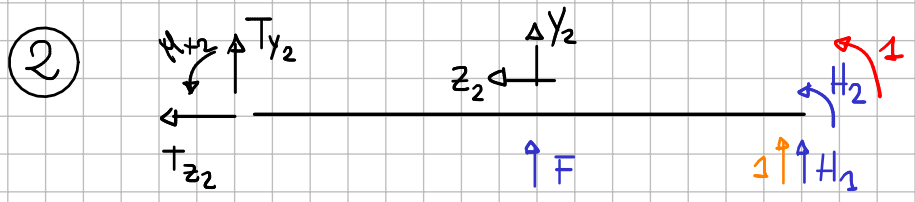
$$\begin{cases} T_{z_1}(z_1) = 0 \\ T_{y_1}(z_1) = -H_1 \\ M_{x_1}(z_1) = -H_2 - H_1 \cdot z_1 \end{cases}$$

DUMMY 1

$$\begin{cases} T'_{z_1}(z_1) = 0 \\ T'_{y_1}(z_1) = -1 \\ M'_{x_1}(z_1) = -1 \cdot z_1 \end{cases}$$

DUMMY 2

$$\begin{cases} T''_{z_1}(z_1) = 0 \\ T''_{y_1}(z_1) = 0 \\ M''_{x_1}(z_1) = -1 \end{cases}$$



REAL

$$\begin{cases} T_{z_2}(z_2) = 0 \\ T_{y_2}(y_2) = -F - H_1 \\ M_{x_2}(y_2) = -F \cdot z_2 - H_1(z_2 + \ell) - H_2 \end{cases}$$

DUMMY 1

$$\begin{cases} T'_{z_2}(z_2) = 0 \\ T'_{y_2}(z_2) = -1 \\ M'_{x_2}(z_2) = -1(z_2 + \ell) \end{cases}$$

DUMMY 2

$$\begin{cases} T''_{z_2} = 0 \\ T''_{y_2} = 0 \\ M''_{x_2} = -1 \end{cases}$$

3) PCVW

REAL — DUMMY 1

$$\delta W_e = 0$$

$$\delta W_i = \int_0^\ell M'_{x_1} \cdot \frac{M_{x_1}}{EJ_{xx}} dz_1 + \int_0^\ell M'_{x_2} \cdot \frac{M_{x_2}}{EJ_{xx}} dz_2$$

$$= \frac{1}{EJ_{xx}} \left[\int_0^\ell (-z_1) \cdot (-H_1 \cdot z_1 - H_2) dz_1 + \int_0^\ell (z_2 + \ell) \cdot (F z_2 + H_1 z_2 + H_1 \ell + H_2) dz_2 \right.$$

PCVW $\delta W_e = \delta W_i$

$$0 = \left[\frac{1}{2} H_2 z_1^2 + \frac{1}{3} H_1 z_1^3 \right]_0^\ell + \left[\frac{1}{3} F z_2^3 + \frac{1}{3} H_1 z_2^3 + \frac{1}{2} H_1 \ell z_2^2 + \frac{1}{2} H_2 z_2^2 + \dots \right.$$

$$\left. \dots + \frac{1}{2} F z_2^2 \ell + \frac{1}{2} H_1 \ell z_2^2 + H_1 z_2 \ell^2 + H_2 \ell z_2 \right]_0^\ell$$

$$\Rightarrow \frac{5}{6} F\ell + 2H_2 + \frac{8}{3} H_1 \ell = 0$$

REAL – DUMMY

$$\delta W_e = 0$$

$$\delta W_i = \int_0^\ell M_{x_1}^{II} \cdot \frac{M_{x_1}}{EJ_{xx}} dz_1 + \int_0^\ell M_{x_2}^{II} \frac{M_{x_2}}{EJ_{xx}} dz_2$$

PCVW

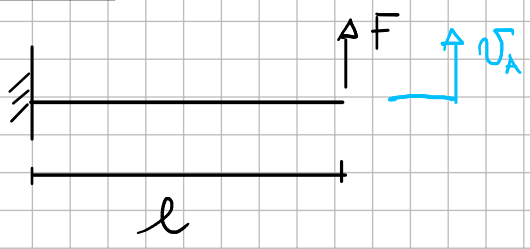
$$\rightarrow \quad 0 = \left[\frac{1}{2} H_1 z_1^2 + H_2 z_1\right]_0^\ell + \left[\frac{1}{2} F z_2^2 + \frac{1}{2} H_1 z_2^2 + H_1 \ell z_2 + H_2 z_2\right]_0^\ell$$

$$\rightarrow \quad 2H_1\ell + 2H_2 + \frac{1}{2} F\ell = 0$$

$$\begin{cases} \frac{5}{6} F\ell + 2H_2 + \frac{8}{3} H_1 \ell = 0 \rightarrow \frac{5}{6} F\ell - 2H_1\ell - \frac{1}{2} F\ell + \frac{8}{3} H_1 \ell = 0 \\ 2H_1\ell + 2H_2 + \frac{1}{2} F\ell = 0 \rightarrow H_2 = -H_1\ell - \frac{1}{4} F\ell \end{cases}$$

$$\rightarrow \begin{cases} H_1 = -\frac{1}{2} F = -2000 \text{ N} \\ H_2 = +\frac{1}{4} F\ell = 1000000 \text{ Nmm} \end{cases}$$

Ex 2



DATA

$\ell = 1000$ mm

$F = 4000$ N

$E = 200\,000$ MPa

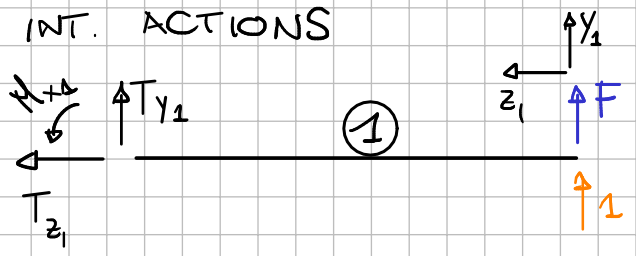
$J_{xx} = 50\,00\,00$ mm^4

FIND VERTICAL DISPL. A

$v_A = ?$

1) REACTION FRCS NOT NEEDED

2) INT. ACTIONS



REAL

$$\begin{cases} T_{z_1} = 0 \\ T_{y_1} = -F \\ M_{x_1} = -F \cdot z_1 \end{cases}$$

DUMMY

$$\begin{cases} T'_{z_1} = 0 \\ T'_{y_1} = -1 \\ M'_{x_1} = -1 \cdot z_1 \end{cases}$$

3) PCVW

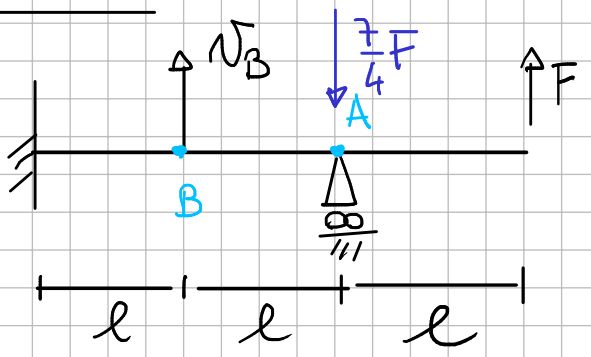
$$\delta W_e = \underline{1} \cdot \underline{v_A}$$

$$\delta W_i = \int_0^\ell \frac{M_{x_1} M'_{x_1}}{EJ_{xx}} dz_1 = \int_0^\ell -F z_1 \cdot \frac{-z_1}{EJ_{xx}} dz_1$$

$$= \frac{1}{3} \frac{F z_1^3}{EJ_{xx}} \Big|_0^\ell = \frac{1}{3} F \frac{\ell^3}{EJ_{xx}}$$

PCVW $\rightarrow$ $v_A = \frac{1}{3} \frac{F\ell^3}{EJ_{xx}} = \frac{40}{3}$ mm

Ex 3



DATA

$\ell = 1000\ mm$

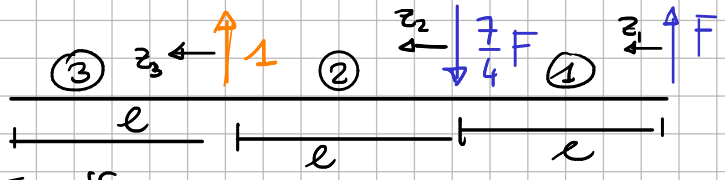
$F = 4000\ N$

$E = 200000\ MPa$

$J_{xx} = 500000\ mm^4$

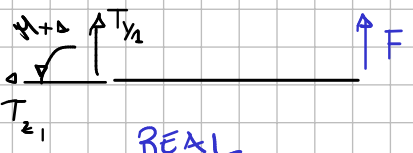
FIND VALUE OF N_B

1) REACTION FORCES



2) INT. ACTIONS

①



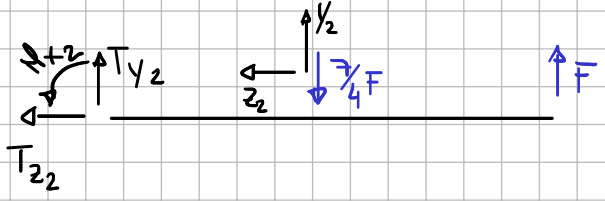
REAL

DUMMY

UNLOADED

$$\begin{cases} T_{z_1} = 0 \\ T_{y_1} = -F \\ M_{x_1} = -F \cdot z_1 \end{cases}$$

②



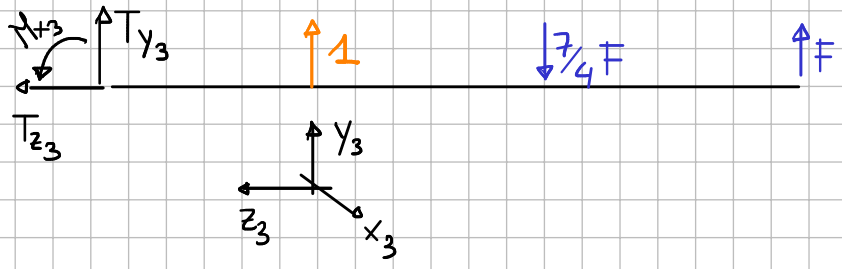
REAL

DUMMY

UNLOADED

$$\begin{cases} T_{z_2} = 0 \\ T_{y_2} = \frac{7}{4}F - F \\ M_{x_2} = -F(\ell + z_2) + \frac{7}{4}F \cdot z_2 \end{cases}$$

③



REAL

$$\begin{cases} T_{z_3} = 0 \\ T_{y_3} = \frac{7}{4}F - F \\ M_{x_3} = \frac{7}{4}F(\ell + z_3) - F(z_3 + 2\ell) \end{cases}$$

DUMMY

$$\begin{cases} T'_{z_3} = 0 \\ T'_{y_3} = -1 \\ M'_{x_3} = -1 \cdot z_3 \end{cases}$$

PCVW

$$\delta W_e = 1 \cdot v_B$$

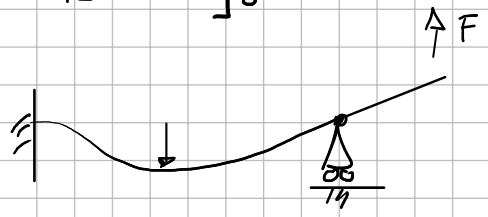
$$\delta W_i = \int_0^\ell M_{x_1} \frac{M'_{x_1}=0}{EJ_{xx}} dz_1 + \int_0^\ell M'_{x_2} \frac{M_{x_2}=0}{EJ_{xx}} dz_2 + \int_0^\ell M'_{x_3} \frac{M_{x_3}}{EJ_{xx}} dz_3$$

$$= \frac{1}{EJ_{xx}} \int_0^\ell \left[2F\ell z_3 + F z_3^2 - \frac{7}{4}F\ell z_3 - \frac{7}{4}F z_3^2 \right] dz_3$$

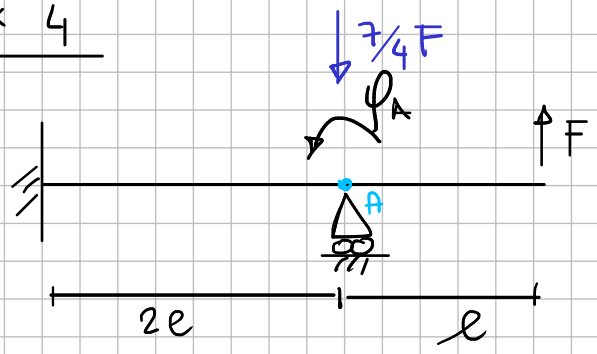
$$= \frac{1}{EJ_{xx}} \left[F\ell z_3^2 + \frac{1}{3}F z_3^3 - \frac{7}{8}F\ell z_3^2 - \frac{7}{12}F z_3^3 \right]_0^\ell$$

PCVW $\delta W_e = \delta W_i$

$$v_B = -\frac{3}{24} F\ell^3 \cdot \frac{1}{EJ_{xx}}$$



Ex 4



DATA

$\ell = 1000$ mm

$F = 4000$ N

$E = 200000$ MPa

$J_{xx} = 500000$ mm^4

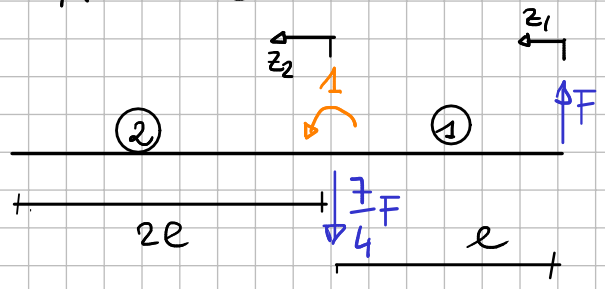
FIND

ROTATION IN A

$\varphi_A = ?$

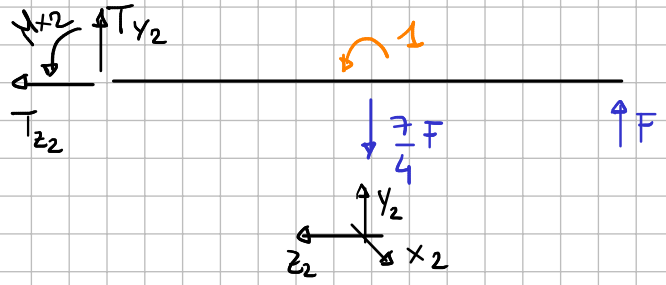
1) REACTION FORCES

2) INT. ACTIONS



(1) DUMMY ← UNLOADED.

(2)



REAL

$$\begin{cases} T_{z_2} = 0 \\ T_{y_2} = \frac{7}{4}F - F \\ M_{x_2} = \frac{7}{4}F\,(z_2) - F\,(z_2 + \ell) \end{cases}$$

DUMMY

$$\begin{cases} T'_{z_2} = 0 \\ T'_{y_2} = 0 \\ M'_{x_2} = -1 \end{cases}$$

3) PCVW

$$\delta W_e = 1 \cdot \varphi_A$$

$$\delta W_{\sim} = \int_0^{2\ell} M'_{x_2} \frac{M_{x_2}}{EJ_{xx}} \, dz_2$$

$$= \int_0^{2\ell} -1 \cdot \left(-F z_2 - F\ell + \frac{7}{4} F z_2 \right) dz_2 \cdot \frac{1}{EJ_{xx}}$$

$$= \frac{1}{EJ_{xx}} \left[\frac{1}{2} F z_2^2 + F\ell z_2 - \frac{7}{8} F z_2^2 \right]_0^{2\ell}$$

$$= \frac{1}{EJ_{xx}} \cdot \frac{1}{2} F\ell^2$$

PCVW $\rightarrow$ $\varphi_A = \frac{1}{2} \frac{F\ell^2}{EJ_{xx}}$