Course of Aerospace Structures

Written test, January 23th, 2024

Name ______________________

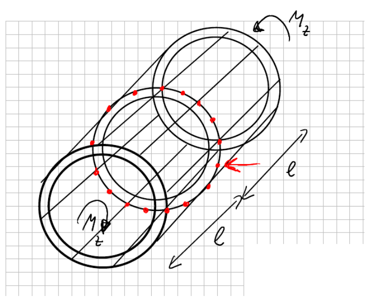
Surname ______________________

Person code: | | | | | | | | |

Exercise 1

The figure represents a semi-monocoque model of a fuselage trunk of length $2l$. It has three circular frames, each with outer radius r_1 and inner radius r_2, and eight uniformly spaced stringers, each with concentrated area A. The outer cylindrical thin panel thickness is equal to t. The panel is connected to the central frame by means of sixteen uniformly spaced rivets, each with shank diameter d, and represented by the red dots in the figure. The outer cylindrical thin panel in continuous across the central rib (there is not a junction between a "forward" and "rear" panel), thus the rivets do only connect the rib to the continuous panel. The fuselage is transmitting a constant torsional moment M_z. Compute the shear stress in the rivet shank highlighted by the red arrow.

(Unit for result: MPa)



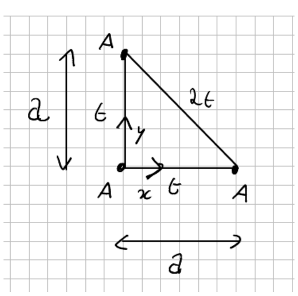
Data
$r_1 = 1.5$ m
$r_2 = 1.45$ m
$l = 1$ m
$d = 3$ mm
$A = 10$ cm^2
$t = 1.5$ mm
$M_z = 15000$ N m

Answer ______________________

Exercise 2

The semi-monocoque cross section sketched in the figure has three concentrated areas A, two panels with thickness t and one panel with thickness $2t$. Compute, with respect to the reference system sketched in the figure, the x coordinate of the shear center.

(Unit for result: mm)

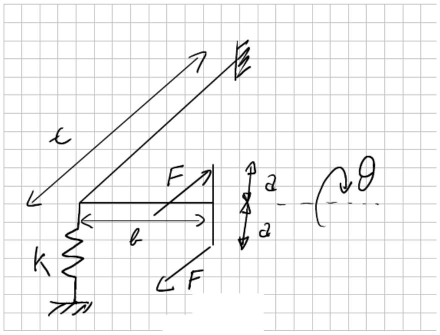


Data
$a = 1$ m
$A = 4$ cm^2
$E = 7 \times 10^4$ MPa
$\nu = 0.3$

Answer ______________________

Exercise 3
Consider the thin beam model sketched in the figure, and loaded by the two forces F. All the beams have the same cross-section. Compute the rotation θ of the point where the beam of length b is joined to the two beams of length a. Neglect shear deformability.

(Unit for result: rad)

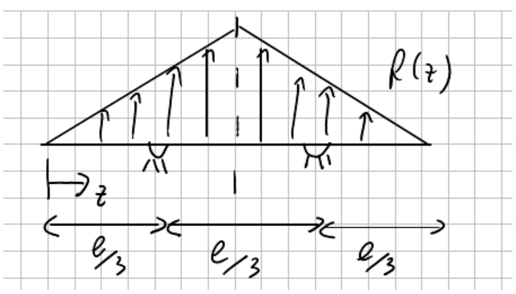


Data
$a = 1$ m
$b = 2$ m
$c = 3$ m
$EI_{xx} = EI_{yy} = 12 \times 10^{14}$ N mm^2
$EA = 6 \times 10^{10}$ N
$GJ = 7 \times 10^{14}$ N mm^2
$K = 1 \times 10^6$ N/mm
$F = 10000$ N

Answer

Exercise 4
Consider the simply supported thin beam model sketched in the figure, with overall length l, and loaded by the linearly varying force per unit of length $f(z)$, with maximum value equal to a. Compute the exact vertical displacement $v(l/2)$ in the middle of the beam. Neglect shear deformability.

(Unit for result: mm)

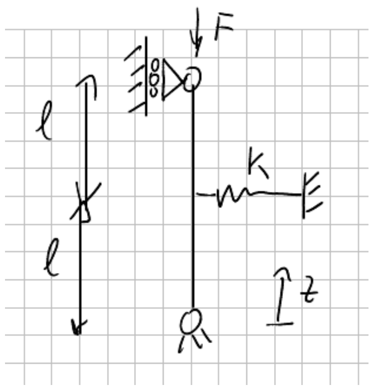


Data
$a = 1000$ N/mm
$l = 2000$ mm
$EI = 12 \times 10^{12}$ N mm^2

Answer

Exercise 5
The thin beam sketched in the figure, of length $2l$, is loaded by the compressive force F. A spring with stiffness K is connected in the middle of the beam, and helps increasing the beam buckling load. Compute the approximated value of critical buckling load by resorting to a suitable polynomial approximation of the transverse displacement truncated to the first non-null term.

(Unit for result: N)

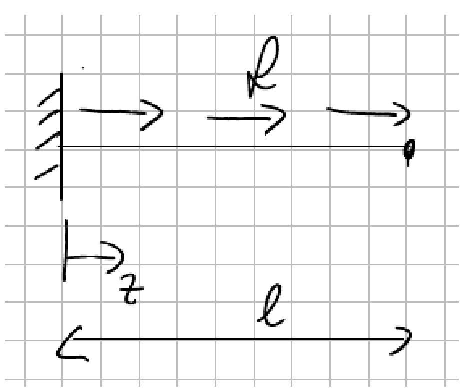


Data
$l = 2000$ mm
$EA = 6 \times 10^{10}$ N
$EI = 12 \times 10^{10}$ N mm^2
$K = 1 \times 10^3$ N/mm

Answer ______________________

Exercise 6
The clamped beam sketched in the figure has a varying axial stiffness $EA(z) = a + bz$, and is loaded by the constant force per unit of length f. By resorting to a polynomial approximation of the axial displacement, truncated to one term, estimate the axial displacement $w(l)$ at the free extremity of the beam.

(Unit for result: mm)



Data
$l = 4000$ mm
$f = 1000$ N/mm
$a = 6 \times 10^{10}$ N
$b = 2.5 \times 10^7$ N/mm

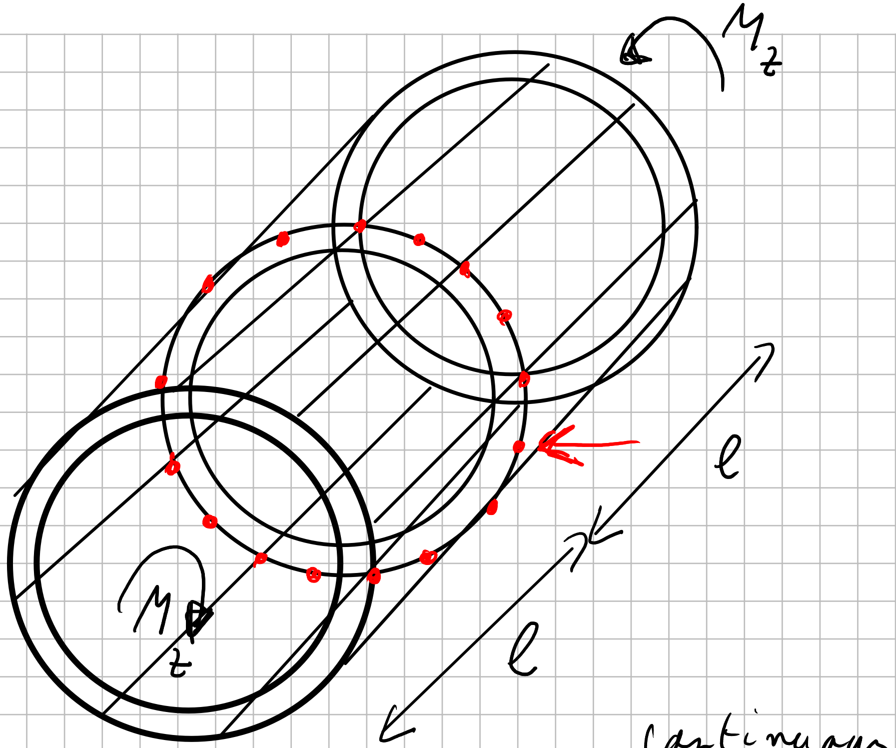
Answer ______________________

True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. The axial stress of beam that is transmitting a constant bending moment M_x does not depend on the material elastic modulus E
2. The Timoshenko model is used to compute the critical buckling stress of a simply supported compressed plate
3. Hermitian shape functions are C^2 (continuous, and with continuous first and second derivatives)

Multiple Choice questions
(Circle the correct answer)

1. "Crippling" is:
   (a) a failure mode of thin-walled compressed beam
   (b) a failure mode of compact compressed beam
   (c) a failure mode affecting the fuselage of Boeing 737 MAX
   (d) a failure mode of railways
   (e) a special design technique preventing the buckling of beams
   (f) none of the above
2. Assume that the solution of a given three dimensional elastic problem has a finite H_{10} norm; an approximated solution, obtained with quadratic finite elements with average dimension h:
   (a) has quadratic convergence of the stress with respect to h
   (b) has cubic convergence of the stress with respect to h
   (c) has linear convergence of the displacement with respect to h
   (d) has quadratic convergence of the displacement with respect to h
   (e) none of the above
3. A structure is modeled using finite elements. It is unconstrained and subjected to a set of loads in self equilibrium. The solution of the linear static problem:
   (a) is stress-free because the loads have null resultant and moment resultant
   (b) can be computed, up to a rigid body motion, after constraining the displacement of the structure all over its boundary
   (c) can be computed only if the loads are concentrated
   (d) can be computed, up to a rigid body motion, only if the loads are distributed
   (e) is defined up to a rigid body motion; thus, not being unique, it is not possible to compute the stress and strain fields
   (f) none of the above



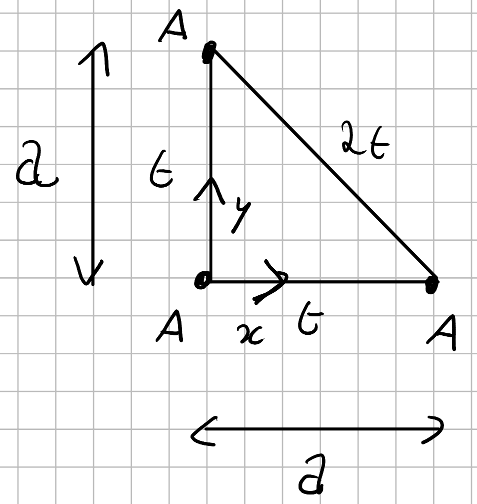
The figure represents a semi-monocoque model of a fuselage section, with three circular frames, with outer radius r_1 and inner radius r_2, eight equispaced stringers with concentrated area A and a thin lateral panel with thickness t. The panel is connected to the frames by means of sixteen equispaced rivets with shank diameter d, represented by the red dots in the figure. The fuselage is transmitting the torsional moment M_z. Compute the shear stress in the rivet shank.

$r_1 = 1,5$ m $\qquad \ell = 1$ m $\qquad d = 3$ mm

$r_2 = 1,45$ m $\qquad A = 10$ cm^2 $\qquad t = 1,5$ mm

$M_z = 15000$ Nm

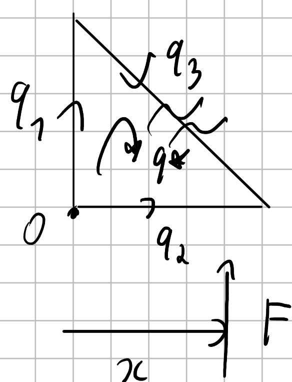
$\cancel{\sigma_{zz}} = 0 \quad MPa$



The semi-monocoque cross section sketched in the figure has thee concentrated areas A, two panels with thickness t and one panel with thickness 2t. Compute, considering the reference system sketched in the figure, the x coordinate of the shear center.

$a = 1 \text{ m} \qquad A = 4 \text{ cm}^2 \qquad t = 1 \text{ mm}$

$E = 70000 \text{ MPa} \qquad \nu = 0{,}3$



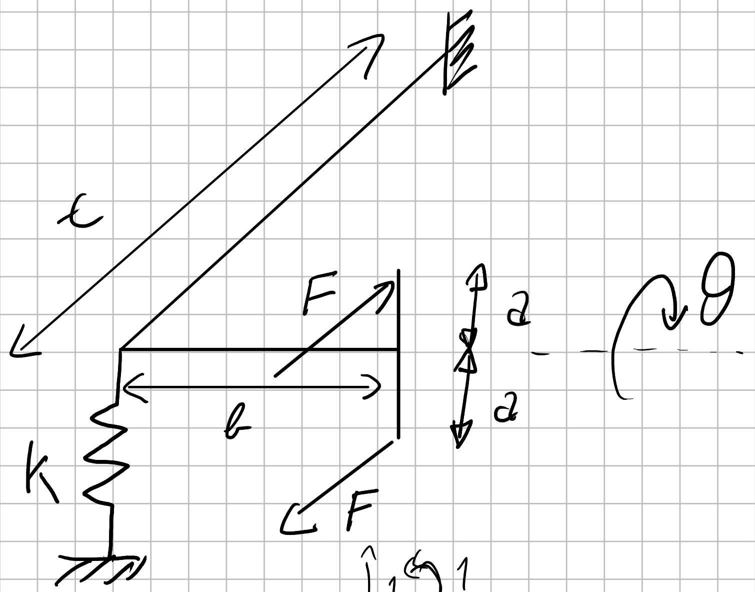
$$q_1' = \frac{F}{a} \qquad q_2' = \emptyset$$

$$M_z(O) = q^* \cdot a^2 = -Fx \qquad q^* = -\frac{Fx}{a^2}$$

$$q_1 = \frac{F}{a}\left(1 - \frac{x}{a}\right) \qquad q_2 = \frac{Fx}{a^2} \qquad q_3 = -\frac{Fx}{a^2}$$

$$\theta' = \frac{1}{a^2 G}\left(\frac{q_1 a}{t} + \frac{q_3 a\sqrt{2}}{2t} - \frac{q_2 a}{t}\right) = \frac{a}{a^2 t G}\frac{F}{a}\left(\left(1 - \frac{x}{a}\right) - \frac{x}{a} - \frac{\sqrt{2}\,x}{2a}\right) = 0$$

$$\frac{(-4-\sqrt{2})x}{2d} = -1 \qquad x = \frac{2d}{(4+\sqrt{2})} = 369{,}4 \text{ mm}$$



$a = 1\ m$

$b = 2\ m$

$c = 3\ m$

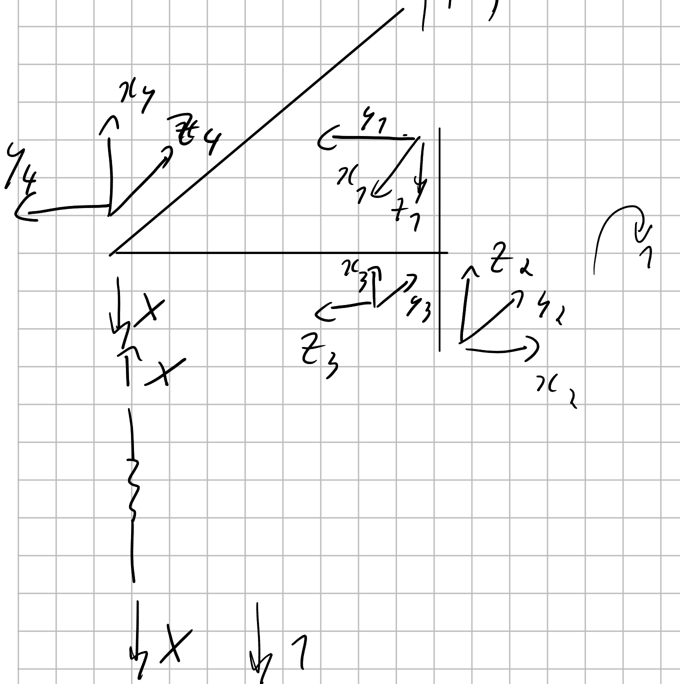
$k = 1E6\ N/mm$

$F = 10000\ N$

$EI = 12E14\ N\,mm^2$

$EA = 6E10\ N$

$GJ = 7E14\ N\,mm^2$



$M_{y_1} = -F z_1$

$M_{x_2} = +F z_2$

$M_{z_3} = -2Fa$

$M_{y_4} = X z_4 - 2Fa$

1

$M'_{y_1} = \emptyset$

$M'_{x_2} = \emptyset$

$M'_{z_3} = -1$

$M'_{y_4} = -1$

$X' = \emptyset$

1

$M''_{y_1} = \emptyset$

$M''_{x_2} = \emptyset$

$M''_{z_3} = \emptyset$

$M''_{y_4} = z_4$

Compute X

$$\int_0^l \frac{M_{y4} \cdot M''_{y4}}{E\,\cancel{I}_{y4}}\,dz + \frac{X}{K} = \cancel{0}$$

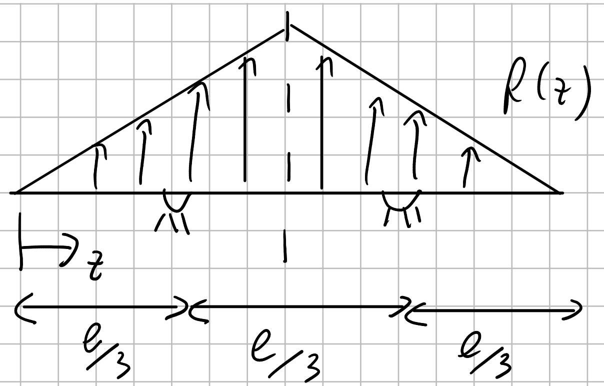
$$\int_0^l \frac{X z_4^2 - 2 F_a z_4}{E\cancel{I}}\,dz_4 + \frac{X}{K} = \frac{1}{3}\frac{X l^3}{E\cancel{I}} - \frac{F_a l^2}{E\cancel{I}} + \frac{X}{K} = \cancel{0}$$

$$X\left(\frac{l^3}{3\,E\cancel{I}} + \frac{1}{K}\right) = \frac{F_a l^2}{E\cancel{I}} \qquad X = 8823{,}5 \;N$$

Compute ϑ

$$\int_0^b \frac{M_{z3}\;M'_{z3}}{G\,\bar{J}}\,dz_3 + \int_0^l \frac{M_{y4}\;M'_{y4}}{E\cancel{I}}\,dz_4 = 1\cdot\vartheta$$

$$\frac{2F_a}{G\bar{J}}\,b - \frac{X l^2}{2E\cancel{I}} + \frac{2F a l}{E I} = \vartheta = 7{,}4E-5 \text{ rad}$$



$$p(z) = a\,\frac{2z}{l} \quad \forall z \in \left[0, \frac{l}{2}\right]$$

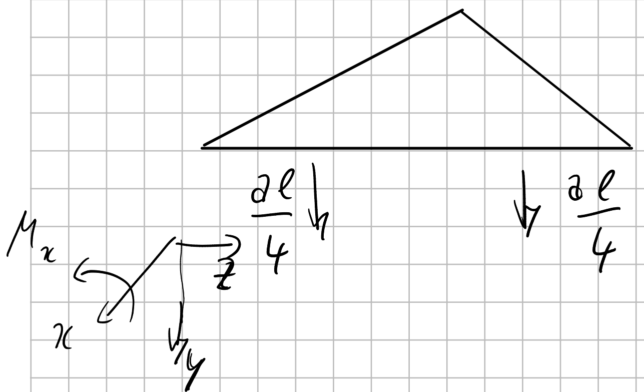
$$a\left(1 - \frac{2z}{l}\right) \quad \forall z \in \left[\frac{l}{2}, l\right]$$

$a = 1000$ N/mm

$l = 2000$ mm

$EJ = 12E12$ N mm^2

$$\int_0^l p(z) = \frac{al}{2}$$

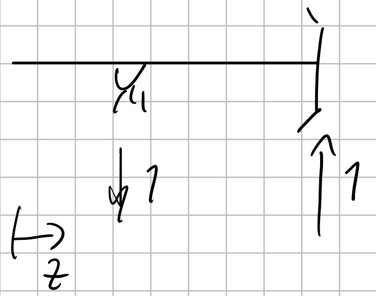


$$T = \frac{a z^2}{l}$$

$$\int_0^z p\, z\, dz = \int_0^z \frac{2a\, z^2}{l}\, dz = \frac{2}{3}\,\frac{a\, z^3}{l} \quad \forall z \in \left[0, \frac{l}{2}\right]$$

$$M_x = -\frac{2}{3}\,\frac{a z^3}{l} + \frac{a z^3}{l} = \frac{a z^3}{3l} \quad \forall z \in \left[0, \frac{l}{3}\right)$$

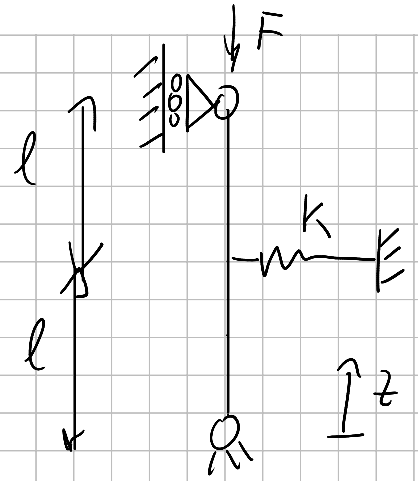
$$-\frac{2}{3}\,\frac{a z^3}{l} - \frac{al}{4}\left(z - \frac{l}{3}\right) + \frac{a z^3}{l} = \frac{a z^3}{3l} - \frac{al}{4}\left(z - \frac{l}{3}\right) \quad \forall z \in \left[\frac{l}{3}, \frac{l}{2}\right]$$



$$M'_x = -\left(z - \frac{l}{3}\right) \qquad \forall\, z \in \left[\frac{l}{3}, \frac{l}{2}\right]$$

$$\int_{l/3}^{l/2} \frac{\left[\frac{1}{3}\,\frac{a z^3}{l} - \frac{a l z}{4} + \frac{a l^2}{12}\right]\left(\frac{l}{3} - z\right)}{EI}\, dz = v = \frac{-7 a l^4}{233280} \quad \frac{1}{EI}$$

$$= -0,04$$



$l = 2000\ mm$

$EI = 12E10\ N\,mm^2$

$EA = 6E10\ N$

$k = 1E3\ N/mm$

$w = az + bz^2 \qquad 2al + 4bl^2 = 0 \qquad a = -2bl$

$\Rightarrow w = -2blz + bz^2 \qquad w' = -2bl + 2bz$

$w'' = 2b \qquad \delta w'' = 2\delta b \qquad w(l) = -2bl^2 + bl^2 = -bl^2$

$\int_0^{2l} \delta w'' EI w'' - \delta w' F w' \, dz + \delta b\, l^4 k b = 0 \qquad \delta w(l) = -\delta b\, l^2$

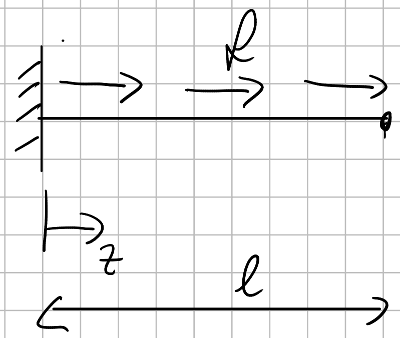
$\int_0^{2l} 4\delta b\, b\, EI - (-2\delta b l + 2\delta b z) F (-2bl + 2bz) dz + \delta b\, l^4 k b = 0$

$\delta b \int_0^{2l} (4EI - 4l^2F + 8lzF - 4z^2F)\, dz\, b + \delta b\, l^4 k b = 0$

$$\left(8EIl - 8l^3F + \frac{8}{2}Fl\,4l^2 - \frac{4}{3}8l^3F + kl^4\right)b = 0$$

$$F\left(-8 + 4 - \frac{32}{3}\right)l^3 = -8EIl - kl^4$$

$$F = \frac{8EIl + kl^4}{\left(8 - 4 + \frac{32}{3}\right)l^3} = 8{,}4\,E5$$



$EA = a + b z$

$p = 1000 \ N/mm$

$a = 6E10 \ N$

$b = 2{,}5E7 \ N/mm$

$l = 4000 \ mm$

1 term polynomial approx

$u(l) = ?$

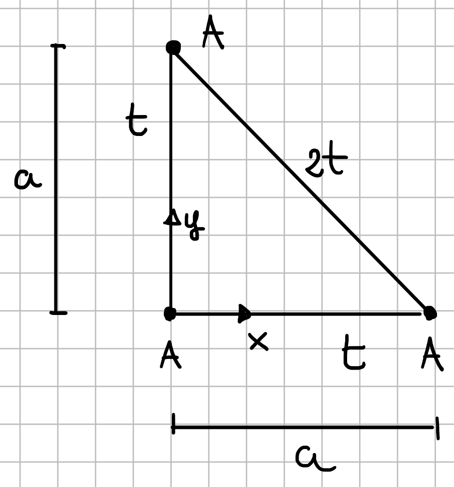
$u = c z \qquad u' = c \qquad \delta u' = \delta c$

$$\int_0^l \delta c \, (a + b z) \, c \, dz = \int_0^l \delta c \, p z \, dz$$

$$\left(a l + \frac{1}{2} b l^2\right) c = \frac{1}{2} p l^2 \qquad c = \frac{1}{2} p l^2 \Big/ \left(a l + \frac{1}{2} b l^2\right)$$

$$u(l) = c l = 0{,}07272 \ mm$$

Ex 2



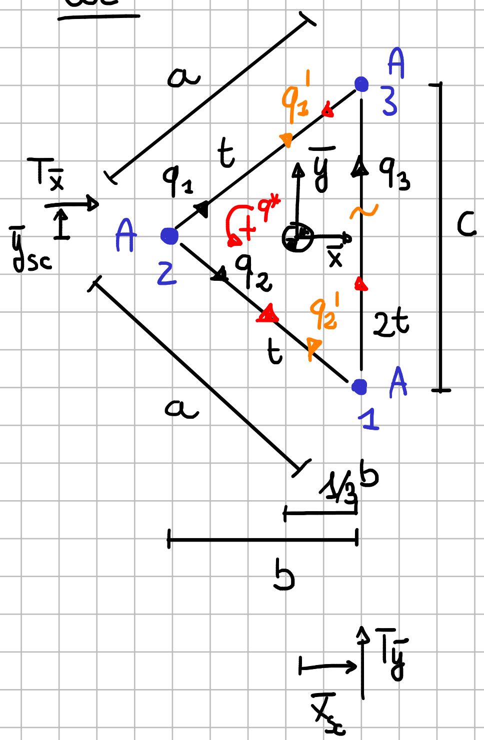
$a = 1000$ mm

$A = 400$ mm^2

$E = 10^4$ MPa

$\nu = 0.3$

SOL



$$b = \frac{\sqrt{2}}{2} a$$

$$c = 2b = \sqrt{2} a$$

$$J_{\bar{x}\bar{x}} = 2\left(\frac{c}{2}\right)^2 A = 2Ab^2$$

$$S_{\bar{x}_1} = -S_{\bar{x}_3} = -Ab$$

$$S_{\bar{x}_2} = 0$$

$$T_{\bar{y}} = 1$$

DUE TO SYMM.

$$\boxed{\bar{y}_{sc} = 0}$$

$$\downarrow$$

$$\boxed{T_{\bar{x}} = 0}$$

OPEN CELL FLUX.

$$q_2' = -T_{\bar{y}} \frac{S_{x_1}}{J_{xx}}, \quad q_1' = T_{\bar{y}} \frac{S_{x_3}}{J_{xx}} \longrightarrow = \frac{1}{2} \frac{T_y}{b} = q'$$

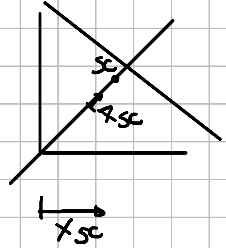
MOM. EQ. WRT 2 (+)

$$T_{\bar{y}} \cdot \left(\frac{2}{3} b + \bar{x}_{sc}\right) = 2\,\Omega_{cell}\, q^* \quad \text{WITH} \quad \Omega_{cell} = \frac{1}{2} cb$$

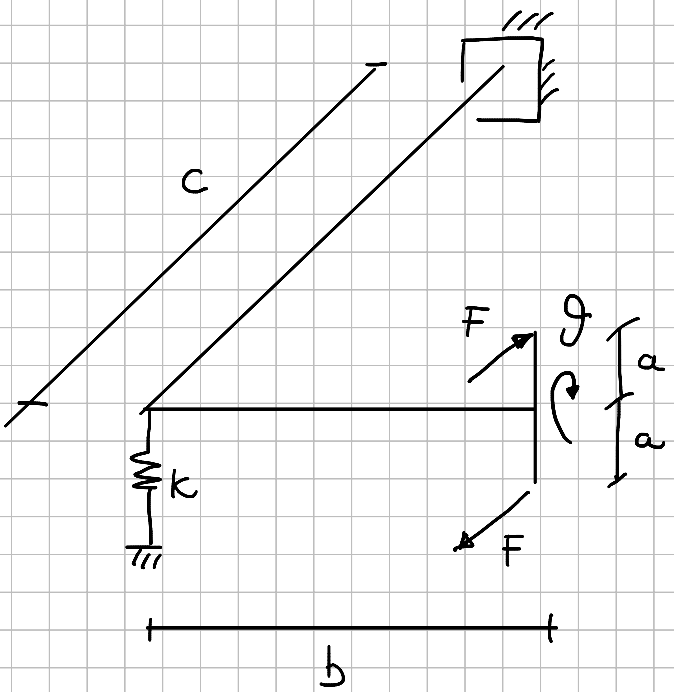
ROT

$$\vartheta' = \cancel{\frac{1}{G 2 \Omega_{cell}}} \cdot \left(\frac{q^* \cdot 2a}{t} + \frac{q^* \cdot c}{2t} - \frac{2 q' \cdot a}{t}\right) = 0 \longrightarrow \bar{x}_{sc}$$

$$\boxed{x_{sc} = \frac{2}{3} \frac{\sqrt{2}}{2} b + \frac{\sqrt{2}}{2} \bar{x}_{sc} = 369.398 \text{ mm}}$$



Ex 3



$a = 1000\ mm$

$b = 2000\ mm$

$c = 3000\ mm$

$EJ_{xx} = EJ_{yy} = 12 \cdot 10^{14}\ N mm^2$

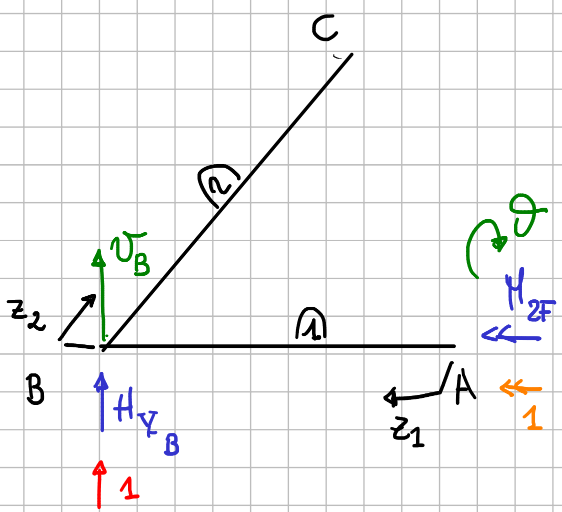
$EA = 6 \cdot 10^{10}\ N$

$GJ = 7 \cdot 10^{9}\ N mm^2$

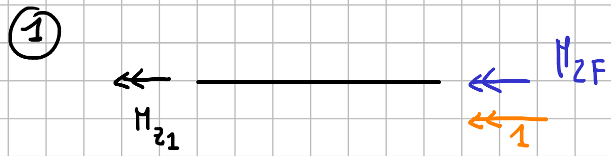
$k = 1 \cdot 10^{6}\ N/mm$

$F = 10^{4}\ N$

Sol

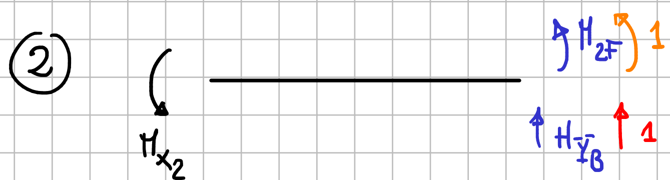


$M_{2F} = 2 F \cdot a$



$M_{z_1} = -M_{2F}$

$M'_{z_1} = -1$



$k v_B = -H_{Y_B} \longrightarrow v_B = -\frac{H_{Y_B}}{k}$

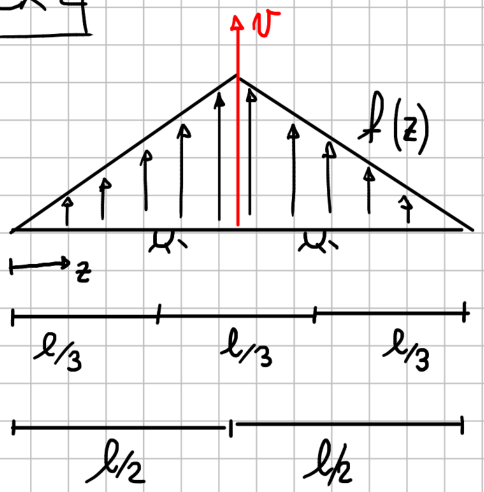
$M_{x_2}(z_2) = -M_{2F} - H_{Y_B} \cdot z_2$

$M'_{x_2}(z_2) = -1 \qquad M''_{x_2}(z_2) = -1 \cdot z_2$

PCVW

$$\begin{cases} 1 \cdot \vartheta = \int_0^b M_{z_1}' \cdot \frac{M_{z_1}}{GJ} \, dz_1 + \int_0^c M_{x_2}' \cdot \frac{M_{x_2}}{EJ} \, dz_2 \\ 1 \cdot v_B = \int_0^c M_{x_2}'' \cdot \frac{M_{x_2}}{EJ} \, dz_2 \end{cases}$$

Ex 4



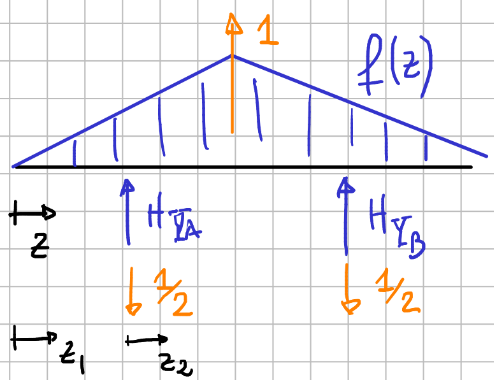
$a = 1000 \ N/mm$

$l = 2000 \ mm$

$EJ = 12 \cdot 10^{12} \ Nmm^2$

SOL $f(z) = \dfrac{2a}{l} \cdot z$

EQUILIBRIUM EQ.

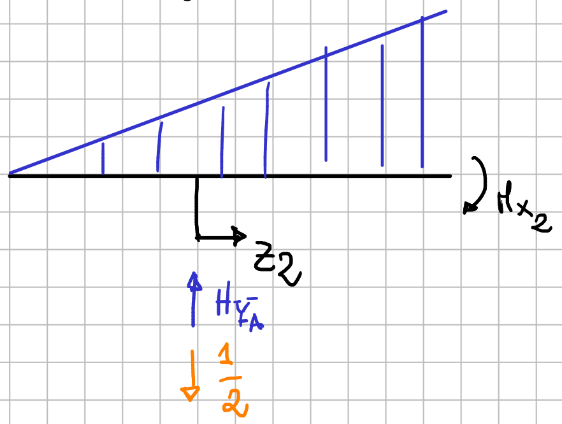


DUE TO SYMMETRY

$$H_{Y_A} = -\int_0^{l/2} f(z)\,dz = H_{Y_B}$$

WHERE $f(z) = \dfrac{2a}{l} z$

IN z_1 DUMMY SYST UNLOADED



IN z_2:

$$M_{x_2}(z_2) = -H_{Y_A} \cdot z_2 + - \frac{2a}{l}\left(z_2 + \frac{l}{3}\right) \cdot \frac{\left(z_2 + l/3\right)}{3} \cdot \left(z_2 + \frac{l}{3}\right) \cdot \frac{1}{2}$$

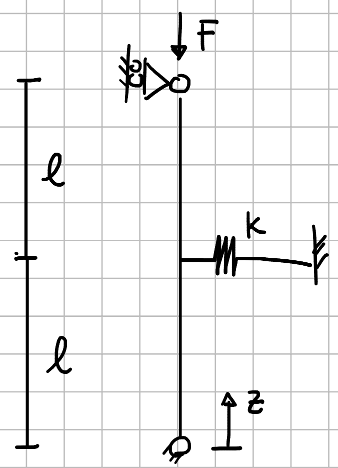
$$M'_{x_2}(z_2) = +\tfrac{1}{2} z_2$$

→ PCVW

$$\boxed{v = 2\int_0^{l/6} M'_{x_2} \cdot \frac{M_{x_2}}{EJ}\, dz_2 = -0.04 \ mm}$$

WE HAVE TO CONSIDER THE WHOLE STRUCTURE

Ex 5



DATA

$l = 2000\,mm$

$EA = 6 \cdot 10^{10}\ N$

$EJ = 12 \cdot 10^{10}\ N\,mm^2$

$k = 1 \cdot 10^3\ N/mm$

SOL WITH EB BEAM & NON LINEAR GL STRAIN TENSOR FOR INFINITESIMAL DISPL. IN Z & SMALL IN y

$$\varepsilon_{zz} = w_{/z} - y\, v_{/zz} + \frac{1}{2} v_{/z}^2$$

$$\delta W_i = \int_V \delta\varepsilon_{zz}\, \sigma_{zz}\, dV$$

WHERE $\delta\varepsilon_{zz} = \delta w_{/z} - y\,\delta v_{/zz} + \delta v_{/z} \cdot v_{/z}$

$$= \delta\varepsilon_0 - y\,\delta v_{/zz}$$

$$\delta W_i = \int_V \delta\varepsilon_0 \sigma_{zz}\, dV - \int_V \delta v_{/zz}\, y\, \sigma_{zz}\, dV$$

$$= \int_{2l} \delta\varepsilon_0\, N \cdot dz + \int_{2l} \delta v_{/zz}\, EJ\, v_{/zz}\, dz$$

$$\int_V y\sigma_{zz}\, dV = \int_{2l} \underbrace{M_x}_{-EJ\, v_{/zz}} dz$$

$$\delta W_e = -F\,\delta w(2l) - k \cdot v(l) \cdot \delta v(l)$$

$$= -\int_0^{2l} \delta w_{/z}\, F dz - \delta v(l)\, k \cdot v(l)$$

PVW $\delta w: \int_0^{2l} \delta w_{/z} \cdot N dz = -\int_0^{2l} \delta w_{/z} \cdot F\, dz \longrightarrow N = -F$

$\delta v: \int_0^{2l} \delta v_{/zz}\, EJ\, v_{/zz} + \delta v_{/z} \underbrace{N}_{-F} v_{/z}\, dz = -\delta v(l)\, k \cdot v(l)$

$v = Az^2 + Bz + C$

BC : $v(0) = 0 \rightarrow C = 0$

$v(2\ell) = 0 \rightarrow B = -A \cdot 2\ell$

$v = c\,\phi(z) \rightarrow$ $c = [A \quad -A \cdot 2\ell]$

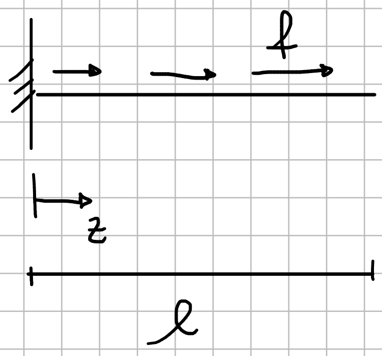
$\phi = [z^2 \quad z]^T$

$$\delta\cancel{A} \int_0^{2\ell} \frac{\partial c}{\partial A}\phi_{/zz} EJ\, c\,\phi_{/zz} - \frac{\partial c}{\partial A}\phi_{/z} F \cdot c\,\phi_{/z}\, dz = -\cancel{\delta A} \cdot \frac{\partial c}{\partial A}\phi(\ell)\, k\, c\,\phi(\ell)$$

TWO SOLUTIONS $A = 0$

$F = 840000$ N

Ex 6



DATA

$l = 4000 \text{ mm}$

$f = 1000 \text{ N/mm}$

$a = 6 \cdot 10^{10} \text{ N}$

$b = 2.5 \cdot 10^{7} \text{ N/mm}$

SOL $\quad w = Az + B$

$BC: \quad w(0) = 0 \longrightarrow B = 0$

$\delta W_i = \int_0^l \delta w_{/z} \cdot N \, dz \qquad$ WHERE $N = EA \cdot w_{/z} \qquad$ WITH $EA = a + bz$

$\delta W_e = \int_0^l \delta w \cdot f \, dz$

PVW $\quad \cancel{\delta A} \cdot \int_0^l EA \cdot A \, dz = \int_0^l \cancel{\delta A} \cdot z \cdot f \, dz$

$\rightarrow \int_0^l (Aa + Abz - zf) \, dz = 0$

$\rightarrow Aal + \frac{1}{2} Abl^2 - \frac{1}{2} fl^2 = 0$

$A = \frac{1}{2} \frac{fl^{\cancel{2}}}{a\cancel{l} + \frac{1}{2} bl^{\cancel{2}}} = \frac{1}{2} \frac{fl}{a + \frac{1}{2} bl}$

$w(l) = A \cdot l = \frac{1}{2} \frac{fl^2}{a + \frac{1}{2} bl} = 0.0727 \text{ mm}$

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. The axial stress of beam transmitting a constant bending moment M_x does not depend on the material elastic modulus E

   - True

2. The Timoshenko model is used to compute the critical buckling stress of a simply supported compressed plate

   - False

3. Hermitian shape functions are C^2 (continuous, and with continuous first and second derivatives)

   - False

Multiple Choice questions

(Circle the correct answer)

1. "Crippling" is:

   (a) a failure mode of thin-walled compressed beam
   (b) a failure mode of compact compressed beam
   (c) a failure mode affecting the fuselage of Boing 737 MAX
   (d) a failure mode of railways
   (e) a special design technique preventing the buckling of beams
   (f) none of the above

2. Assume that the solution of a given three dimensional elastic problem has a finite H_{10} norm; an approximated solution, obtained with quadratic finite elements with average dimension h:

   (a) has quadratic convergence of the stress with respect to h
   (b) has cubic convergence of the stress with respect to h
   (c) has linear convergence of the displacements with respect to h
   (d) has quadratic convergence of the displacements with respect to h
   (e) none of the above

3. A structure is modeled using finite elements. It is unconstrained and subjected to a set of loads in self equilibrium. The solution of the linear static problem:

   (a) is stress-free because the loads have null resultant and moment resultant
   (b) can be computed, up to a rigid body motion, after preventing the displacement of the structure all over its boundary
   (c) can be computed only if the loads are concentrated
   (d) can be computed, up to a rigid body motion, only if the loads are distributed
   (e) is defined up to a rigid body motion; thus, not being unique, it is not possible to compute the stress and strain fields
   (f) none of the above