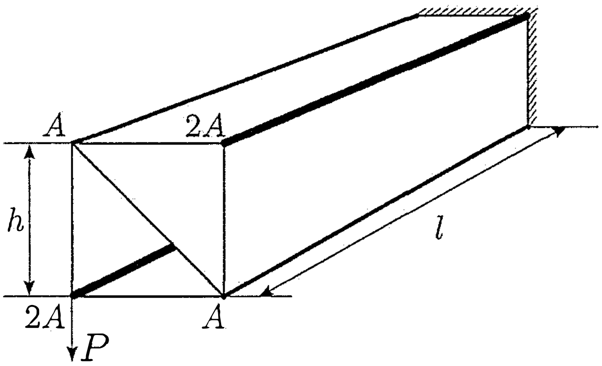
Course of Spacecraft Structures

Written test, February 11^{th}, 2019

Exercise 1

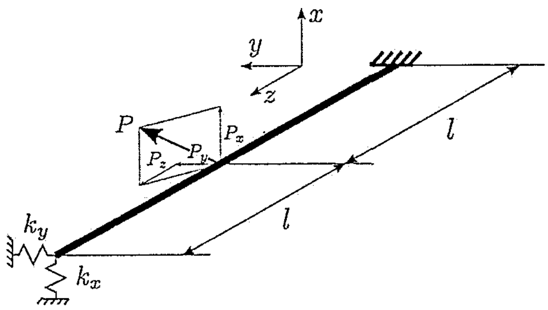
The two-cell beam in the figure is composed of five panels of thickness t, and four stringers of areas A and $2A$ (note: the area is the lumped one). The section is square and has dimension h.
The material is a light aluminum alloy, whose elastic properties are provided in terms of Young's modulus and Poisson's coefficient, denoted with E and ν, respectively. The beam is fixed at one end and free at the other one, and is loaded with a concentrated forces of magnitude P.
Determine the internal stresses at a distance $l/2$ from the free end.



Data
$h = 100\ mm$; $l = 1500\ mm$;
$t = 1.5\ mm$; $A = 150\ mm^2$;
$E = 72000\ MPa$; $\nu = 0.3$;
$P = 15\ kN$;

Exercise 2

The slender beam in the figure is fixed at one end, and is elastically supported by two springs of stiffness k_x and k_y at the other end. The beam axial stiffness is EA, while the bending stiffnesses are EJ_{xx} and EJ_{yy} (note: $EJ_{xy} = 0$). A concentrated force P, whose components are P_x, P_y and P_z, is applied at the middle.
Determine the unknown reactions forces and the strain energy stored in the structure (including the springs).



Data
$l = 1000\ mm$;
$EA = 7 \times 10^7\ N$;
$EJ_{xx} = 5 \times 10^9\ Nmm^2$;
$EJ_{yy} = 8 \times 10^9\ Nmm^2$;
$k_x = 5\ N/mm$; $k_y = 2\ N/mm$;
$P_x = 150\ N$; $P_y = 100\ N$; $P_z = 10000\ N$;

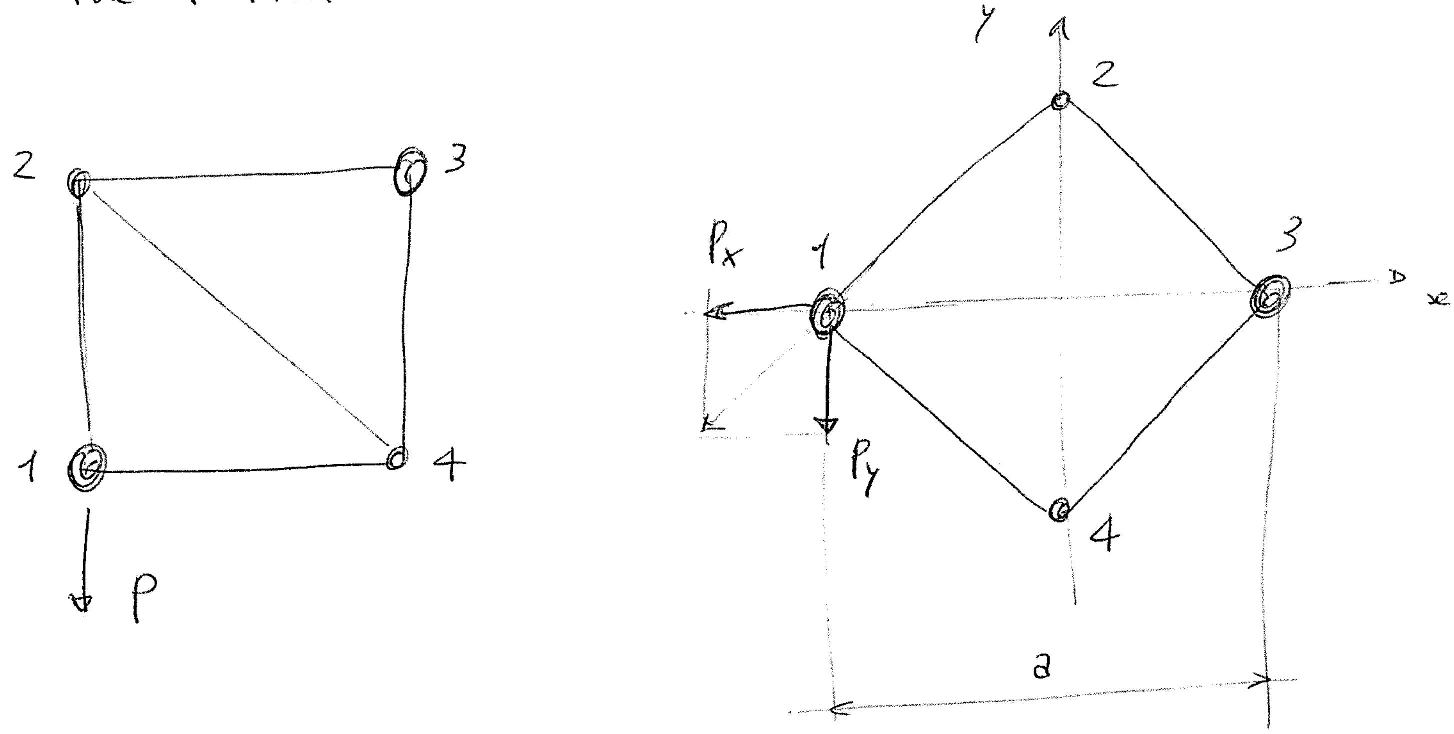
Question 1

Illustrate how to determine the torsional and transverse shear stiffnesses for a thin-walled beam in the context of the semi-monocoque approximation.

Exercise 1

Section properties

The principal axis are available from the symmetry of the section



$$a = h\sqrt{2} = 141.42 \text{ mm}$$

$$J_{xx} = 2A\left(\frac{a}{2}\right)^2 = 1.5 \cdot 10^6 \text{ mm}^4$$

$$J_{yy} = 4A\left(\frac{a}{2}\right)^2 = 3.0 \cdot 10^6 \text{ mm}^4$$

$S_{x_1} = 0$ | $S_{y_1} = -Aa$

$S_{x_2} = A\,a/2$ | $S_{y_2} = -Aa$

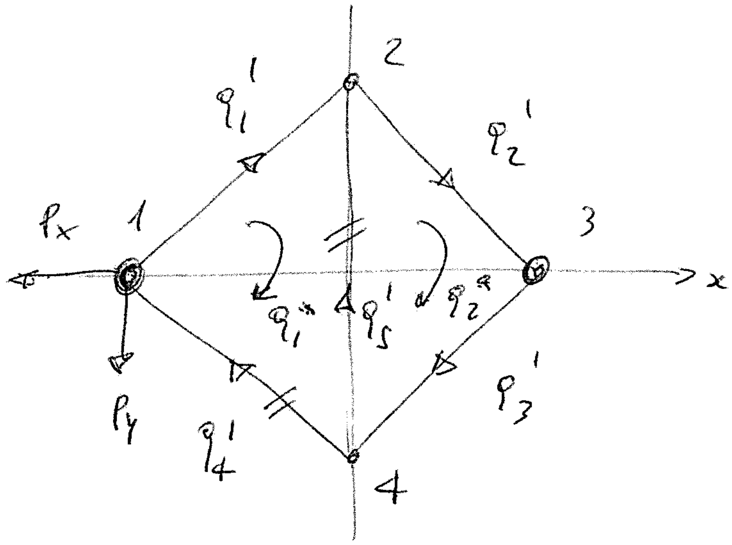
$S_{x_3} = A\,a/2$ | $S_{y_3} = 0$

$S_{x_4} = 0$ | $S_{y_4} = 0$

Shear flows

$$q_1' = + \frac{P_x S_{y1}}{J_{yy}} + \frac{P_y S_{x1}}{J_{xx}}$$

$$= - \frac{P_x}{a} = -75 \ N/mm$$



$$q_2' = 0 \quad N/mm$$

$$q_3' = \frac{P_y}{a} = 75 \quad N/mm \qquad\qquad q_4' = q_5' = 0$$

- Equivalence (wrt 1)

$$2\Omega_2 q_2' + 2\Omega_3 q_3' + 2\Omega_{c_1} q_1^* + 2\Omega_{c_2} q_2^* = 0$$

$$\Omega_2 = \Omega_3 = \Omega_{c_1} = \Omega_{c_2} = a^2/2 \quad , \text{ So:}$$

$$q_2' + q_3' + q_1^* + q_2^* = 0$$

$$\boxed{q_1^* + q_2^* = -75}$$

- Compatibility

$$q_1^* - q_2^* = \frac{(q_3' - q_1')\, h}{2(h+a)}$$

$$\boxed{q_1^* - q_2^* = 31.07}$$

The q^* flows are then:

$$\begin{cases} q_1^* + q_2^* = -75 \\ q_1^* - q_2^* = 31.07 \end{cases} \Rightarrow \begin{array}{l} q_1^* = -21.97 \ N/mm \\ q_2^* = -53.04 \ N/mm \end{array}$$

And so:

$q_1 = q_1' + q_1^* = -96.97 \ N/mm$ $\qquad \tau_1 = 64.65 \ MPa$

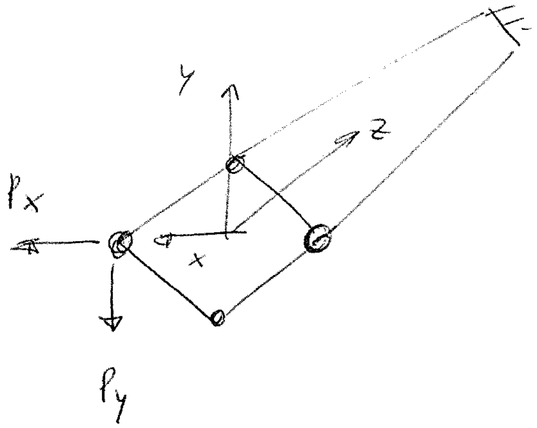
$q_2 = q_2^* = -53.04 \ N/mm$ $\qquad \tau_2 = 35.36 \ MPa$

$q_3 = q_3' + q_2^* = 21.96 \ N/mm$ $\qquad \tau_3 = 14.64 \ MPa$

$q_4 = q_1^* = -21.97 \ N/mm$ $\qquad \tau_4 = 14.65 \ MPa$

$q_5 = q_2^* - q_1^* = -31.08 \ N/mm$ $\qquad \tau_5 = 20.72 \ MPa$

Stresses σ_{zz}



Referring to the principal axis.

$M_x = P_y \, l/2$

$M_y = P_x \, l/2$

The stresses are:

$$\sigma_{zz} = \frac{M_x}{J_{xx}} y - \frac{M_y}{J_{yy}} x$$

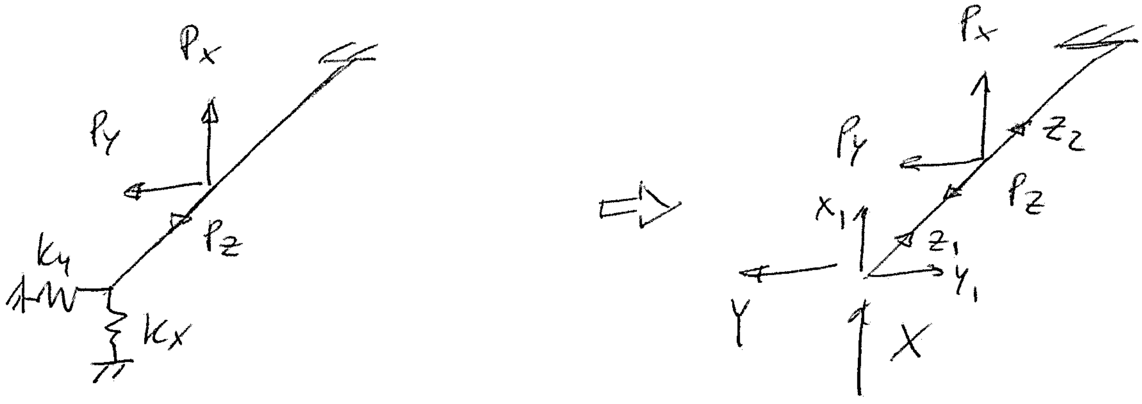
Stringers 2,4: $x=0, \ y=\pm a/2$

$$\sigma_{zz} = \frac{M_x}{I_{xx}} \left(\pm a/2 \right) = \pm 375 \text{ MPa}$$

Stringers 1,3: $x = \pm a/2 \quad y=0$

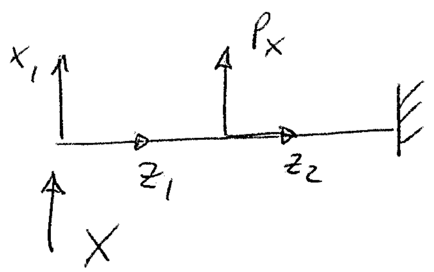
$$\sigma_{zz} = - \frac{M_y}{I_{yy}} \left(\pm a/2 \right) = \mp 187.5 \text{ MPa}$$

Exercise 2

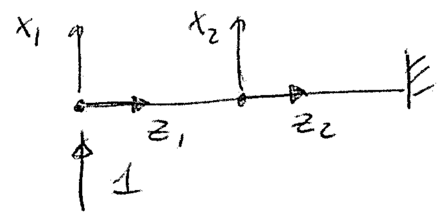


(2 times statically indetermined)

XZ – Plane



Real



Dummy

$$M_y^1 = X z_1 \qquad \delta M_y^1 = z_1$$

$$M_y^2 = X\ell + (X + P_x) z_2 \qquad \delta M_y^2 = z_2 + \ell$$

- PCVW

$$\int_0^\ell \frac{M_y^1\, \delta M_y^1}{EJ_{yy}}\, dz_1 + \int_0^\ell \frac{M_y^2\, \delta M_y^2}{EJ_{yy}}\, dz_2 + \frac{X}{k_x} = 0$$

$$X\left(\frac{8}{3}\ell^3 + \frac{EJ_{yy}}{k_x}\right) + \frac{5}{6} P_x \ell^3 = 0$$

yz-plane

By analogy with the xz-plane, it is immediate to obtain:

$$Y\left(\frac{8}{3}\ell^3+\frac{EJ_{xx}}{k_y}\right)+\frac{5}{6}P_y\ell^3=0$$

Solution: $X=-29.27\ N$

$Y=-16.13\ N$

Internal energy

$$U=U_{b_{xz}}+U_{b_{yz}}+U_{ax}+U_{springs}$$

axial

bending in yz plane

bending in xz plane

$$U_{b_{xz}}=\frac{1}{2}\int_0^\ell \frac{M_y^1 M_y^1}{EJ_{yy}}\,dz_1$$

$$=\frac{1}{EJ_{yy}}\left(\frac{2}{3}X^2\ell^3+\frac{1}{6}(X+P_x)^2\ell^3+\frac{1}{2}X(X+P_x)\ell^3\right)$$

$$=0.154\ J$$

$$U_{b_{yz}}=\frac{1}{EJ_{xx}}\left(\frac{2}{3}Y^2\ell^3+\frac{1}{6}(Y+P_y)^2\ell^3+\frac{1}{2}Y(Y+P_y)\ell^3\right)$$

$$=0.133\ J$$

$$U_{ax} = \frac{1}{2} \int_0^{\ell} \frac{P_z P_z}{EA} \, dz_z = \frac{1}{2} \frac{P_z^2}{EA} \ell = 0.714 \text{ J}$$

$$U_{springs} = \frac{1}{2} \frac{X^2}{k_x} + \frac{1}{2} \frac{Y^2}{K_y} = 0.150 \text{ J}$$