

Determine the axial force at  $x = L/2$

Data

$$L = 1200 (1 + D/10) \text{ mm}$$

$$EA_1 = 2.0 \cdot 10^4 \text{ N}$$

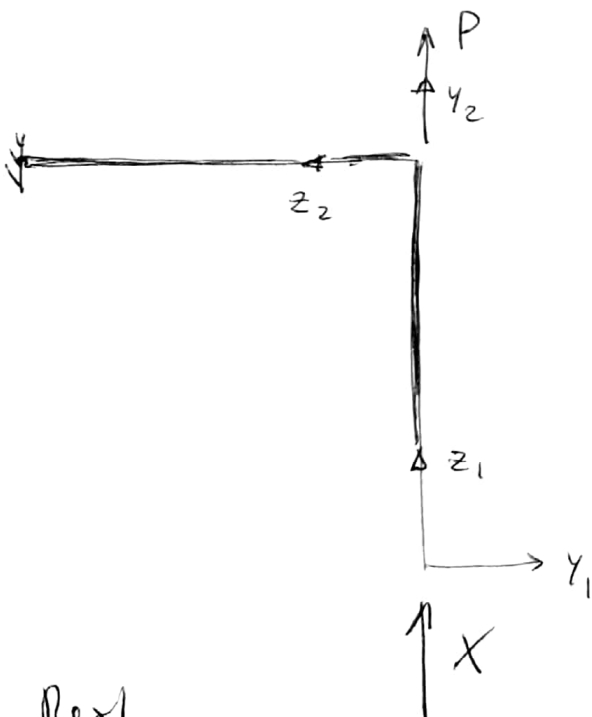
$$EA_2 = 1.0 \cdot 10^4 \text{ N}$$

$$EA_3 = 2.9 \cdot 10^4 \text{ N}$$

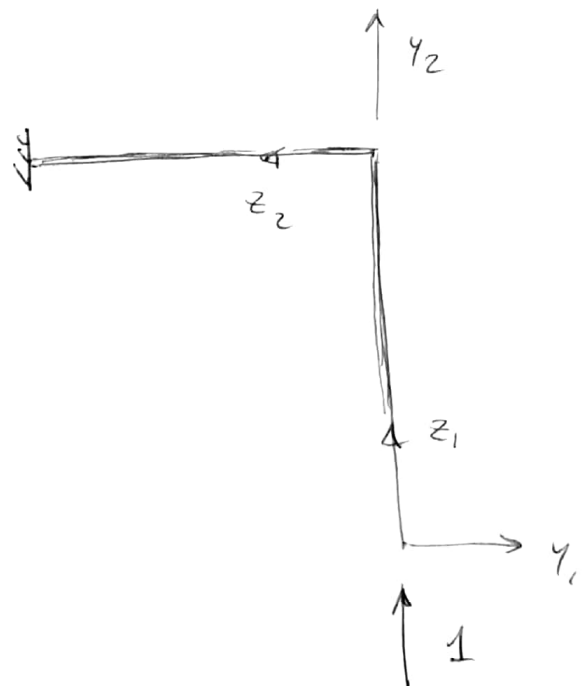
$$EJ = 1.5 \cdot 10^{10} \text{ N mm}^2$$

$$P = 1200 (1 + F/10) \text{ N}$$

Solution ( $D = F = 0$ )



Real



Dummy

$$N = -X$$

$$\delta N = -1$$

(beam 1)

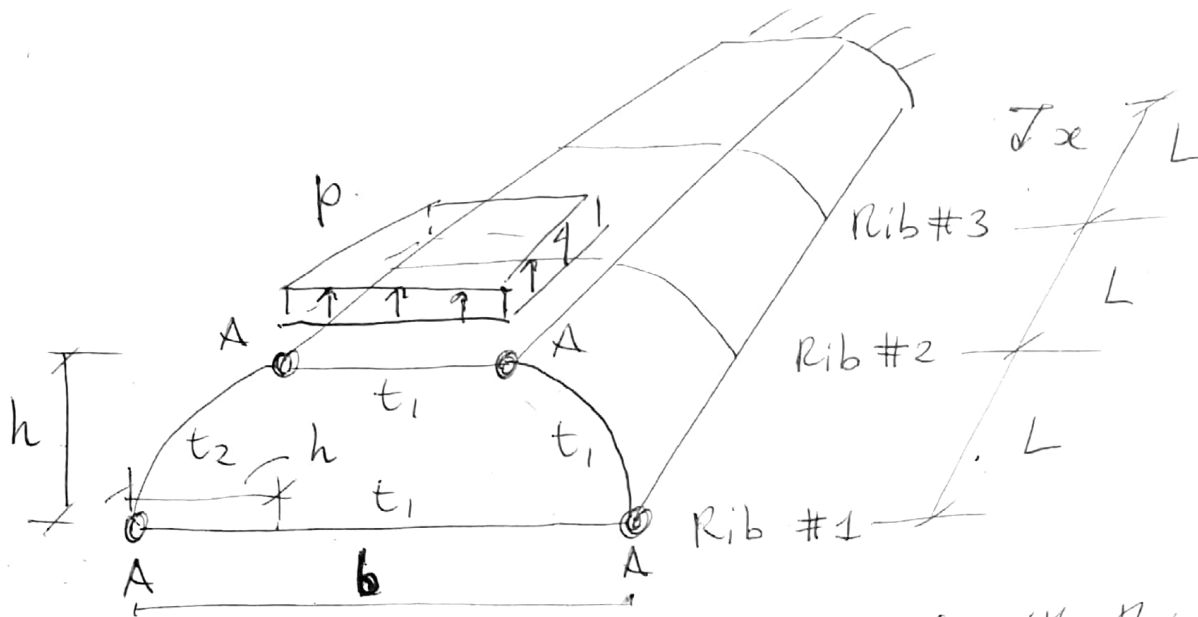
$$M_x = -(P+X)z_2$$

$$\delta M_x = -z_2$$

(beam 2)

$$\int_0^L \frac{N \delta N}{EA_3} dz_1 + \int_0^L \frac{M_x \delta M_x}{EI} dz_2 = 0$$

$$X = - \frac{PL^2/3}{\frac{EI}{EA_3} + L^2/3} = -577.59 \text{ N}$$



Evaluate the shear flow in the panel with thickness  $t_2$  at  $x=0$ .

Data

$$A = 700 \text{ mm}^2$$

$$t_2 = 1.5 \text{ mm}$$

$$b = 600 \text{ mm}$$

$$L = 1800 (1 + A/10) \text{ mm}$$

$$h = 150 \text{ mm}$$

$$p = 2.5 \cdot 10^{-2} (1 + B/10) \text{ N/mm}^2$$

$$t_1 = 1 \text{ mm}$$

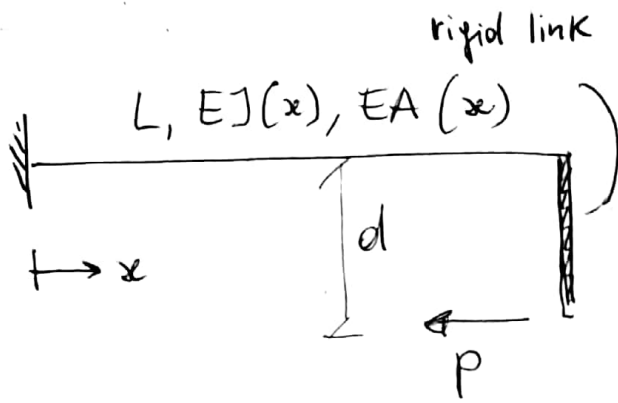
Solution ( $A=B=0$ )

The shear at  $x=0$  is

$$T = p \cdot L (b - 2h)$$

And from the symmetry of the section,

$$q = \frac{T}{2h} = 45 \text{ N/mm}$$



Using the Ritz method and a 1-dot polynomial approximation for any displacement

Component, determine the strain energy.

Data

$$L = 1200 \text{ mm}$$

$$EJ_0 = 1.5 \cdot 10^9 \text{ Nmm}^2$$

$$d = 45 \text{ mm}$$

$$EJ_1 = 7.5 \cdot 10^9 \text{ Nmm}^2$$

$$EA_0 = 1.15 \cdot 10^6 \text{ N}$$

$$P = 800 \text{ N}$$

$$EA_1 = 6.5 \cdot 10^5 (1 + B/10) \text{ N}$$

Note,  $EA$  and  $EJ$  vary linearly with  $x$  from  $EA_0/EJ_0$  to  $EA_1/EJ_1$ .

Solution ( $B=0$ )

$$u = q_1 x ; \quad w = q_2 x^2$$

$$EJ(x) = \alpha + \beta x$$

$$EA(x) = \delta + \gamma x$$

$$\alpha = EJ_0 \quad \beta = \frac{EJ_1 - EJ_0}{L}$$

$$\delta = EA_0 \quad \gamma = \frac{EA_1 - EA_0}{L}$$

$$\int_0^L (\delta u_{xx} EA(x) u_{xx} + \delta w_{xx} EJ(x) w_{xx}) dx =$$

$$= - \delta u(L) P - \delta w(L) Pd$$

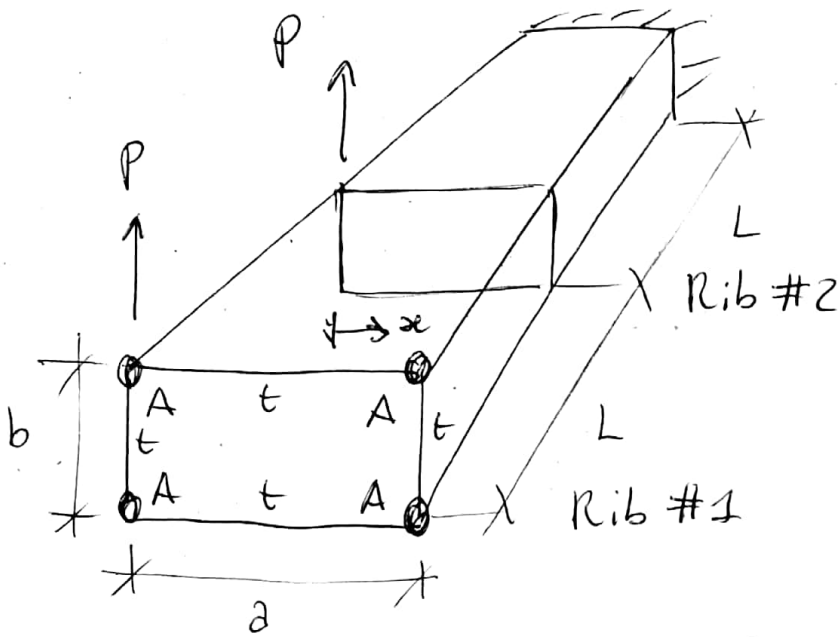
After substitution of the approximations:

$$\begin{bmatrix} \delta L + \delta L^2/2 & 0 \\ 0 & 4(\alpha L + \beta L^2/2) \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = \begin{Bmatrix} -PL \\ -2LPd \end{Bmatrix}$$

$$\underline{\underline{K}} \quad \underline{\underline{q}} = \underline{\underline{f}}$$

Solving for  $\underline{\underline{q}}$  and evaluating  $U$  provides:

$$U = \frac{1}{2} \underline{\underline{q}}^T \underline{\underline{K}} \underline{\underline{q}} = 599.47 \text{ Nmm}$$



Model Rib #2 as a beam and evaluate the bending moment at  $x = a/3$

Data

$$a = 500 (1 + A/10) \text{ mm}$$

$$A = 500 \text{ mm}^2$$

$$b = 250 \text{ mm}$$

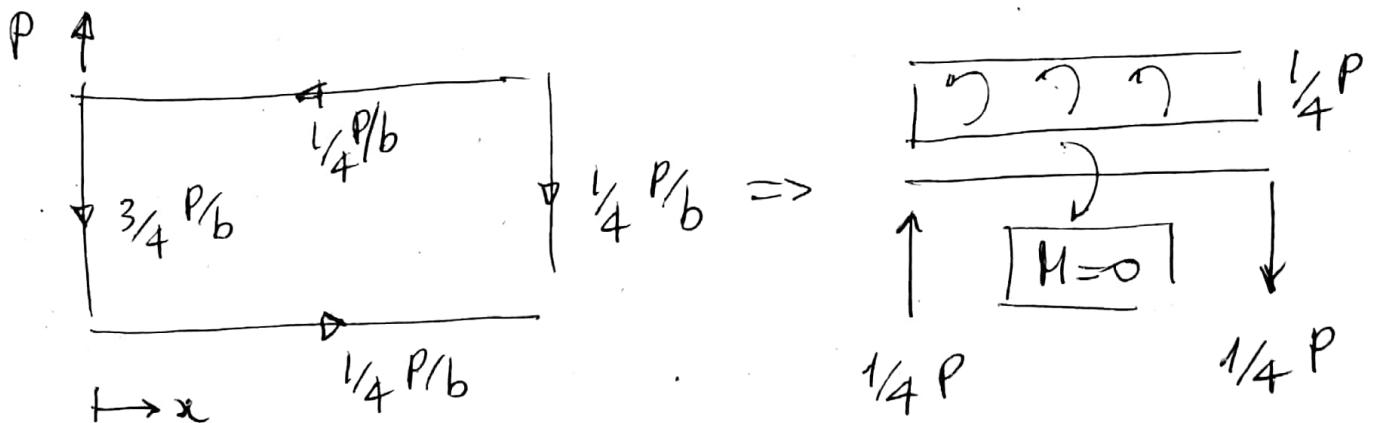
$$L = 2000 \text{ mm}$$

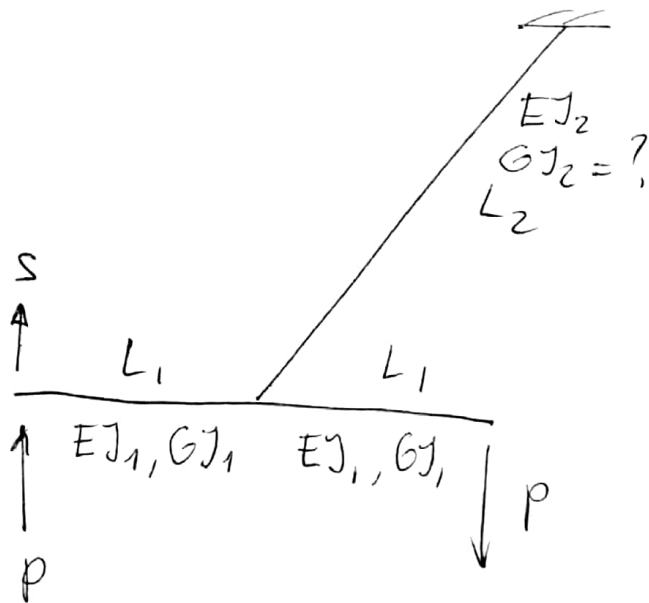
$$t = 0.6 \text{ mm}$$

$$P = 1000 \text{ N}$$

Solution ( $A = 0$ )

The equilibrating shear flows are found as:





Find the value of  $GJ_2/GJ_1$  such that  $S = S_{max}$ .

Data

$$L_1 = 300 \text{ mm}$$

$$L_2 = 750 \text{ mm}$$

$$EI_1 = 2 \cdot 10^9 \text{ Nmm}^2$$

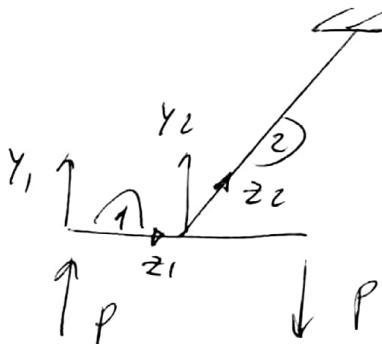
$$EI_2 = 3 \cdot 10^9 \text{ Nmm}^2$$

$$GJ_1 = 9 \cdot 10^7 \text{ Nmm}^2$$

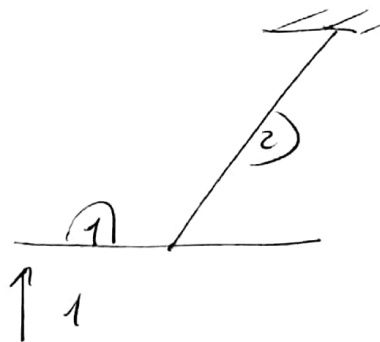
$$P = 1200 \text{ N}$$

$$S_{max} = 15 (1 + F/10)$$

Solution ( $F=0$ )



Real



Dummy

$$M = -Pz_1 \quad \delta M = -z_1 \quad (\text{beam 1})$$

$$M_2 = -2PL_1 \quad \delta M_2 = -L_1 \quad (\text{beam 2})$$

$$\int_0^{L_1} \frac{\delta M M}{EI_1} dz_1 + \int_0^{L_2} \frac{\delta M_2 M_2}{GI_2} dz_2 = S = S_{\max}$$

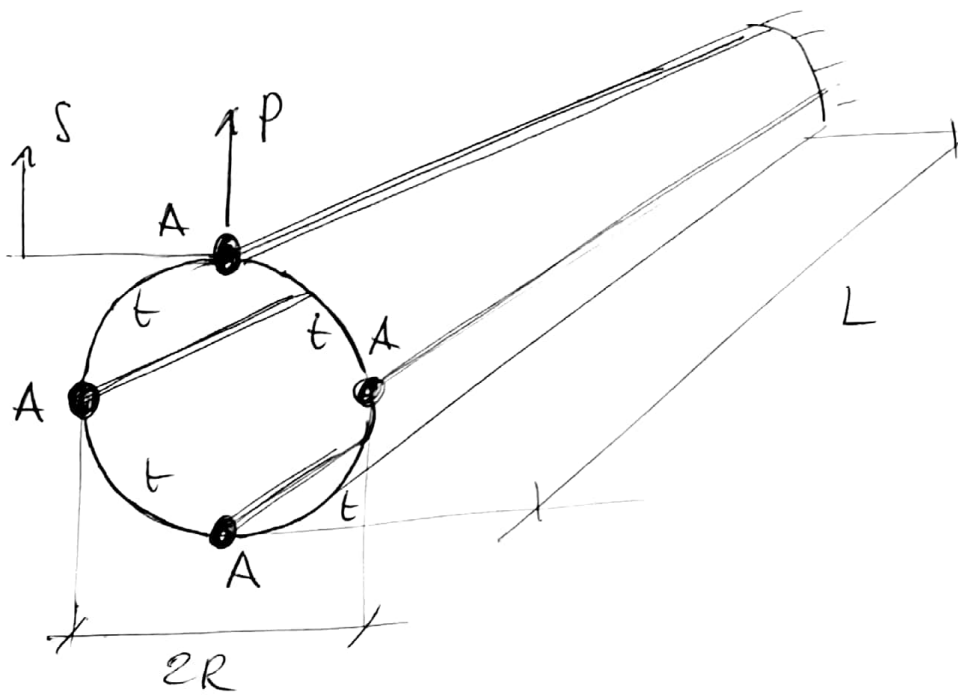
From which:

$$GI_2 = \frac{2PL_1^2 L_2}{S_{\max} - PL_1^3 / 3EI_1}$$

and

$$\frac{GI_2}{GI_1} = 187.5$$





Evaluate the contribution of shear deformability to the displacement  $S$ .

Data

$$L = 2300 \text{ mm}$$

$$R = 150 \text{ mm}$$

$$t = 0.8 (1 + A/10) \text{ mm}$$

$$P = 1700 (1 + B/10) \text{ N}$$

$$G = 27000 \text{ MPa}$$

Solution ( $A=B=0$ )

$$A^* = \frac{\theta t R}{\pi}$$

$$S_{\text{shear}} = \frac{PL}{GA^*} = 0.4739 \text{ mm}$$

- A beam model cannot be used for evaluating local effects due to load introduction.

True

- The semi-monocoque approximation provides the exact shear stress distribution along the panels' thickness.

False

- Essential boundary conditions are more important than natural ones.

False

- The semi-inverse approach for the De Saint Venant solution for isotropic, homogeneous beams leads to the exact solution of the problem.

- The shear center of beam section with one closed cell requires application of the compatibility equation  $\theta' = 0$

- The trial functions in the Ritz method must be part of a complete set