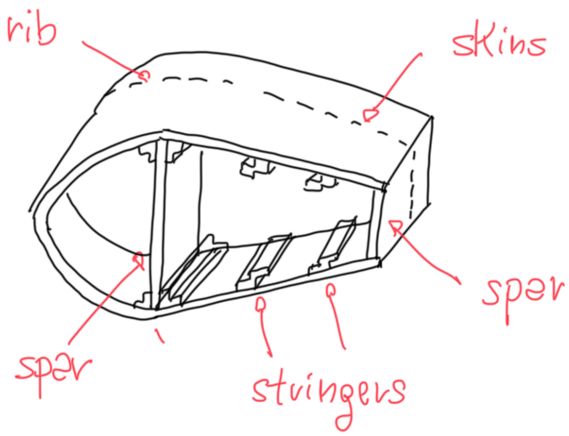
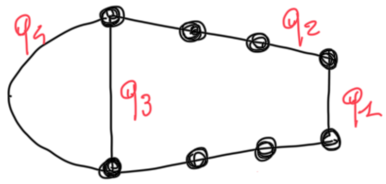
LAB 7 – Semi-Monocoque Approximation

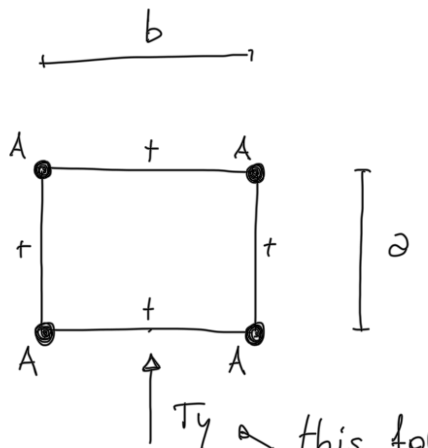




Semi-Monocoque Assumptions:

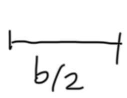
- panel thickness t is small → τ is constant through the thickness
- panels carry shear loads only → shear flux is constant between any couple of directly connected stringers
- Area of panels is concentrated in the stringers
- stringers must have an area which is smaller than the dimension of the section.
- Ribs (if present) have infinitely high stiffness in-plane

1)



Let's find the shear fluxes

← this force is applied to the section



- CENTROID

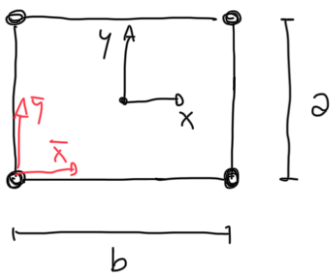
centroid = geometrical center

$$r_c = \frac{\int_A r \, dA}{\int_A dA}$$

center of gravity = mass center

$$r_{CG} = \frac{\int_V \rho(r) \, dV}{\int_V dV}$$

if the density is uniform $\rightarrow$ centroid = center of gravity



Areas

$$\bar{x}_c = \frac{\sum_i A_i \bar{x}_i}{\sum_i A_i} = \frac{2Ab + 2A \cdot \emptyset}{4A} = \frac{b}{2}$$

$$\bar{y}_c = \frac{\sum_i A_i \cdot \bar{y}_i}{\sum_i A_i} = \frac{2Aa + 2a \cdot \emptyset}{4A} = \frac{a}{2}$$

- Static Moments

Areas

$$S_x = \int_A y \, dA \rightarrow S_x = \sum_i A_i \cdot y_i$$

$$S_y = \int_A x \, dA \rightarrow S_y = \sum_i A_i \cdot x_i$$

- Inertias wrt Centroid

$$J_{xx} = \int_A y^2 \, dA \rightarrow J_{xx} = \sum_i A_i \cdot y_i^2 = 2A \cdot \left(\frac{a}{2}\right)^2 + 2A \cdot \left(-\frac{a}{2}\right)^2 = Aa^2$$

$$J_{yy} = \int_A x^2 \, dA \rightarrow J_{yy} = \sum_i A_i \cdot x_i^2 = Ab^2$$

$$J_{xy} = \int_A x \cdot y \, dA \rightarrow J_{xy} = \sum_i A_i \cdot x_i \cdot y_i = \emptyset \rightarrow X \text{ and } Y \text{ are}$$

PRINCIPAL INERTIAL AXIS

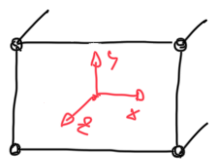
if $J_{xy} \neq 0$, M_x and M_y are coupled:

$$\begin{bmatrix} M_x \\ M_y \end{bmatrix}^T = \begin{bmatrix} EJ_{xx} & -EJ_{xy} \\ -EJ_{xy} & J_{yy} \end{bmatrix} \cdot \begin{bmatrix} \chi_x \\ \chi_y \end{bmatrix} \leftarrow \text{curvatures}$$

then we must rotate x and y of an angle α:

$$\alpha = \frac{1}{2} \operatorname{atan}\left(\frac{2 J_{xy}}{J_{yy} - J_{xx}}\right)$$

- Axial stress in the stringer

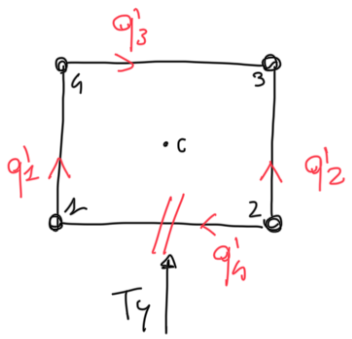


$$\sigma_{zz_i} = \frac{T_z}{\sum_i A_i} + \frac{M_x}{J_{xx}} \cdot y_i - \frac{M_y}{J_{yy}} x_i$$

i ← # Stringers

- SHEAR FLUXES IN THE PANEL

  - Open Cell Fluxes



Assumption: An open-section beam have a negligible torsional stiffness

→ we are isolating the shear fluxes given by the shear forces applied to the section

Let's define a conventional sign for the shear flux in each panel ARBITRARY!

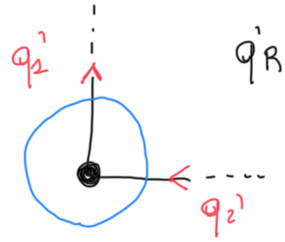
Then, for each region R of our section:

i = # Areas included R

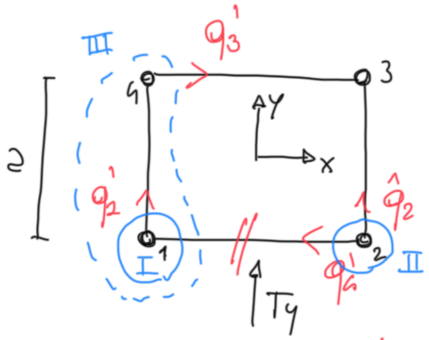
$$q'_R = -T_y \frac{\Sigma_i S_{xi}}{J_{xx}} - T_x \frac{\Sigma_i S_{yi}}{J_{yy}}$$

total shear flux passing through the boundary of R:

- OUTWARD → POSITIVE
- INWARD → Negative



$$q'_R = q'_1 - q'_2$$



$$S_{x1} = -A \cdot \frac{a}{2}$$

I) $q'_1 - \cancel{q'_4} = -T_y \cdot \frac{S_{x1}}{J_{xx}} = -T_y \cdot \frac{-\frac{1}{2}Aa}{Aa^2} = \frac{1}{2}\frac{T_y}{a}$ ($q'_4 \to \emptyset$)

II) $q'_2 = -T_y \cdot \frac{S_{x2}}{J_{xx}} = \frac{1}{2}\frac{T_y}{a}$

III) $q'_3 = -T_y \cdot \frac{(S_{x1} + S_{x4})}{J_{xx}} = -T_y \cdot \frac{\cancel{-\frac{1}{2}Aa} + \cancel{\frac{1}{2}Aa}}{Aa^2} = \emptyset$

- Moment Equivalence

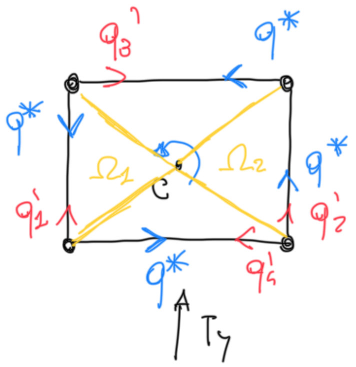
$\boxed{\text{these shear fluxes are EQUIVALENT to } T_y}$

Torsional moment given by the shear fluxes
↓
thus, from T_y

$$M_z = \Sigma_i \, q_i \cdot 2\Omega_i$$ wrt an arbitrary point, e.g. centroid

i → # Panels

Ω_i is the area formed by the chosen point and the panel



Let's define q^* as the shear flux given by the torsional moment, such that for each panel i :

$$q_i = \underbrace{q_i'}_{\text{SHEAR FORCES}} + \underbrace{q^*}_{\text{TORSIONAL MOMENT}}$$

Moment of External Forces = Moment of the sher fluxes wrt Centroid

$$T_y \cdot \emptyset = \underbrace{\Sigma_i}_{\text{\# Panels}} (q_i' + q^*) \cdot 2\Omega_i$$

$$\emptyset = +q^* \cdot 2\Omega_{CELL} - q_1' \cdot 2\Omega_1 + q_2' \cdot 2\Omega_2$$

$$\emptyset = q^* \cdot 2\,a \cdot b - \cancel{\frac{T_y}{2a} \cdot 2\,\frac{1}{2} \cdot a \cdot \frac{b}{2}} + \cancel{\frac{T_y}{2a} \cdot 2\,\frac{1}{2} \cdot a \cdot \frac{b}{2}}$$

$$q^* \cdot 2ab = \emptyset \qquad q^* = \emptyset$$

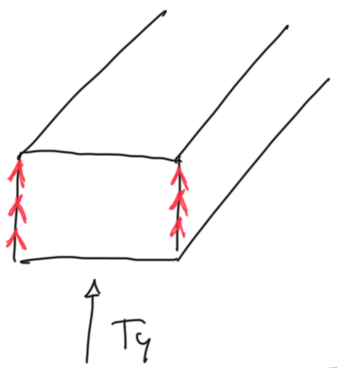
- Compute Total Fluxes

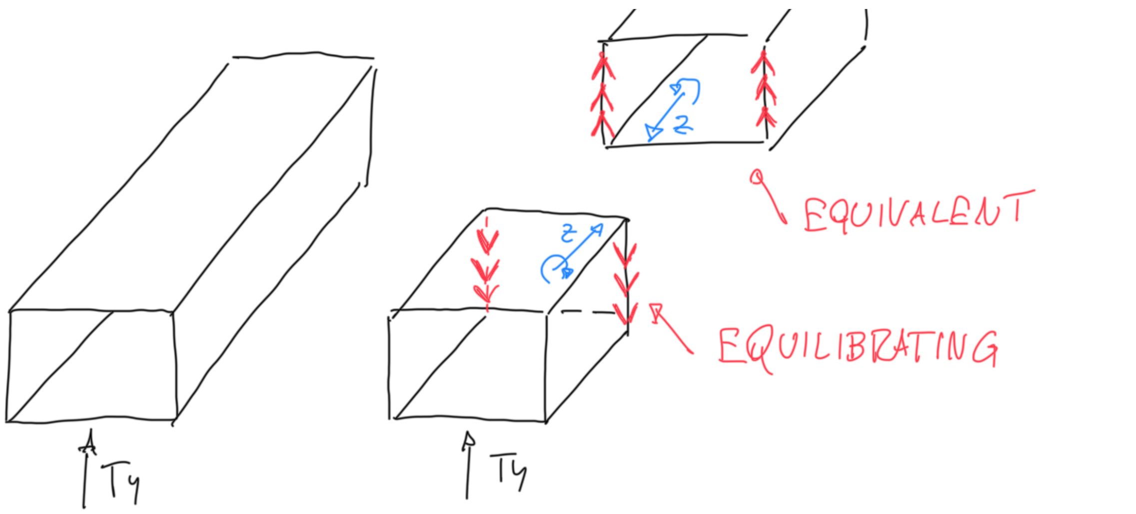
$$q_1 = q_1' - q^* = \frac{T_y}{2a}$$

$$q_2 = q_2' + q^* = \frac{T_y}{2a}$$

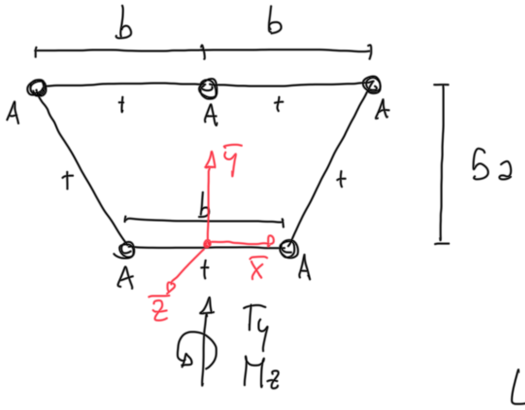
$$q_3 = q_3' - q^* = \emptyset$$

$$q_4 = q_4' - q^* = \emptyset$$





2)



DATA

$A = 250 \ mm^2$

$a = 50 \ mm$

$b = 600 \ mm$

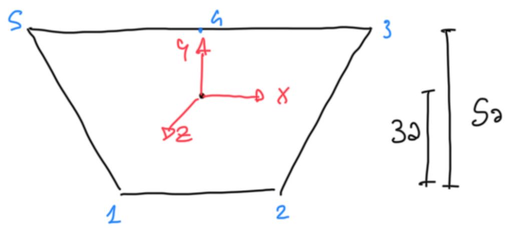
$T_y = 3000 \ N$

$M_z = 450\,000 \ Nmm$

Let's find the shear flaxes

- Centroid and Inertias

$$\bar{x}_c = \frac{\sum_i A_i \bar{x}_i}{\sum_i A_i} = \emptyset \qquad \bar{y}_c = \frac{3A \cdot 5a}{5A} = 3a$$

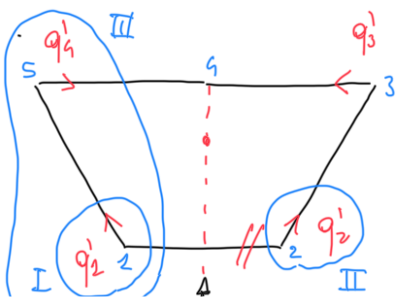


$$J_{xx} = \sum_i A_i y_i^2 = 3A \cdot (2a)^2 + 2A \cdot (3a)^2 = 30 A a^2$$

$$S_{x1} = S_{x2} = A \cdot (-3a) = -3Aa$$

$$S_{x3} = S_{x4} = S_{x5} = A \cdot (2a) = 2Aa$$

- Open Cell Flaxes



I) $q_1' = -T_y \cdot \dfrac{S_{x1}}{J_{xx}} = \dfrac{T_y}{10a}$

II) $q_2' = -T_y \cdot \dfrac{S_{x2}}{J_{xx}} = \dfrac{T_y}{10a}$

III) $q_4' = -T_y \cdot \dfrac{2Aa - 3Aa}{} = T_y$

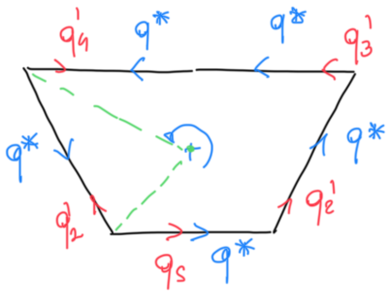
$| \; T_y$ $\qquad J_{xx} \qquad 30a$

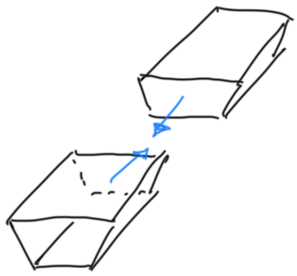
$q_3' = q_4'$ for symmetry

- Moment Equivalence (wrt Centroid)

Moment of external forces = Moment of shear fluxes

$$M_z + \cancel{T_y \cdot \phi} = q^* \cdot 2\Omega_{CELL} - \cancel{q_1'} \cdot 2\Omega_1 + \cancel{q_2'} \cdot 2\Omega_2 + \cancel{q_3'} \cdot 2\Omega_3 - \cancel{q_4'} \cdot 2\Omega_4$$





$$\Omega_{CELL} = \left(\frac{1}{2} b \cdot 5a\right) \cdot 3 = \frac{15}{2} ab$$

$$q^* = \frac{M_z}{2\Omega_{CELL}} = 1 \text{ N/mm}$$

$$q_1 = q_1' - q^* = 5 \text{ N/mm} \qquad q_5 = \cancel{q_5'} + q^* = 1 \text{ N/mm}$$

$$q_2 = q_2' + q^* = 7 \text{ N/mm}$$

$$q_3 = q_3' + q^* = 3 \text{ N/mm}$$

$$q_4 = q_4' - q^* = 1 \text{ N/mm}$$

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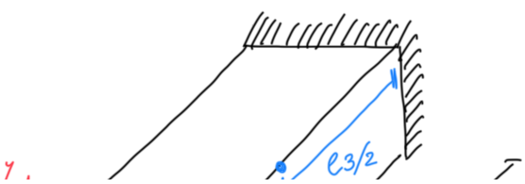
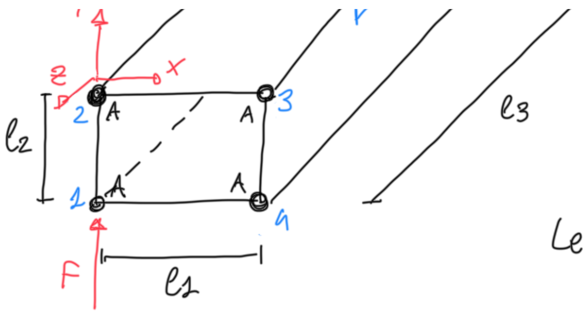
3) EXAM 16/06/2021



DATA

$l_1 = 400$ mm

$l_2 = 250$ mm

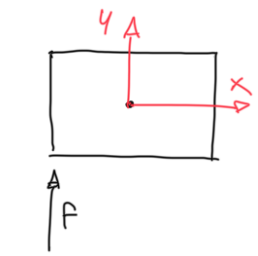


$l_3 = 2000$ mm
$A = 500$ mm^2
$F = 8000$ N

Let's find $\sigma_{zz4}\left(\frac{l_3}{2}\right)$

$$\sigma_{zz4}(z) = \cancel{\frac{T_z(z)}{\sum_i A_i}} + \frac{M_x(z)}{J_{xx}} \cdot y_4 - \cancel{\frac{M_y(z)}{J_{yy}}} \cdot x_4$$

- Centroid

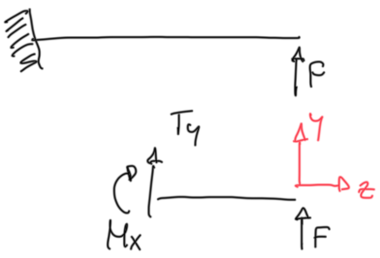


$x_c = l_1/2$
$y_c = l_2/2$

$$J_{xx} = 4A \cdot \left(\frac{l_2}{2}\right)^2 = A l_2^2$$

$$J_{yy} = 4A \cdot \left(\frac{l_1}{2}\right)^2 = A l_1^2$$

- Internal Action



$T_y = -F$
$M_x = F \cdot (-z) = -Fz$

$$\sigma_{zz4}\left(\frac{l_3}{2}\right) = \frac{M_x(l_3/2)}{J_{xx}} \cdot y_4 =$$

$$= \frac{-F \cdot \frac{l_3}{2}}{A l_2^2} \left(-\frac{l_2}{2}\right) = \frac{1}{4} \frac{F l_3}{A l_2} = 32 \text{ MPa}$$

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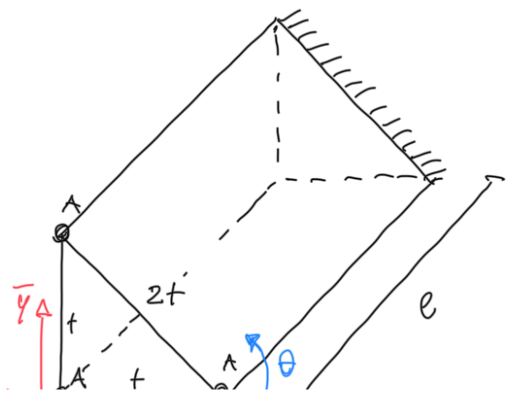
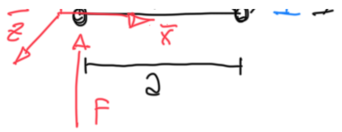
4) EXAM 13/02/2024



DATA

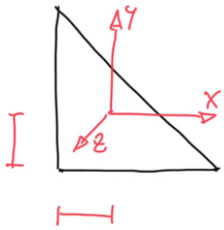
$a = 500$ mm
$A = 400$ mm^2
$t = 1$ mm
$l = 4000$ mm
$E = 70$ GPa
$\nu = 0.3$
$F = 10$ kN



Let's find θ, the rotation of the free extremity

- Centroid

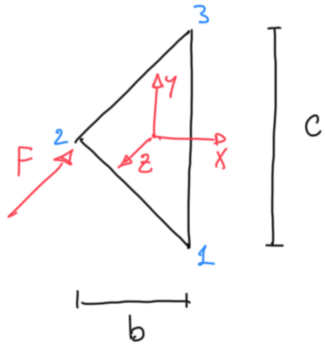
$$\bar{x}_c = -\frac{Aa}{3A} = \frac{a}{3} \qquad \bar{y}_c = \frac{Aa}{3a} = \frac{a}{3}$$



- Inertias

$$J_{xy} = \sum_i A_i \cdot x_i \cdot y_i = A \cdot \frac{a}{3} \cdot \left(-\frac{a}{3}\right) + A\left(-\frac{2}{3}a\right) \cdot \left(-\frac{a}{3}\right) + A\left(\frac{a}{3}\right) \cdot \left(\frac{2}{3}a\right) = \frac{Aa^2}{3}$$

we must rotate the section



$$b = \frac{\sqrt{2}}{2}a \qquad c = 2b = \sqrt{2} \cdot a$$

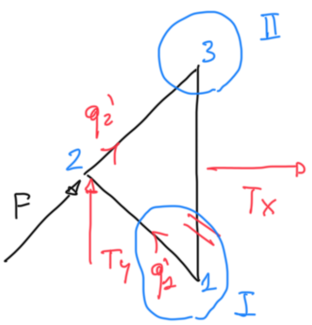
$$J_{xy} = A\left(\frac{2}{3}b\right) \cdot \emptyset + \cancel{A\left(-\frac{b}{3}\right)\left(\frac{c}{2}\right)} + \cancel{A\left(-\frac{b}{3}\right)}\left(-\frac{c}{2}\right) = \emptyset$$

$$J_{xx} = 2A\left(\frac{c}{2}\right)^2 = 2Ab^2$$

$$J_{yy} = 2A\left(\frac{b}{3}\right)^2 + A\left(\frac{2}{3}b\right)^2 = \frac{2}{3}Ab^2$$

$$S_{x1} = -S_{x3} = -A \cdot \frac{c}{2} = -Ab \qquad S_{x2} = \emptyset$$

$$S_{y1} = S_{y3} = +\frac{Ab}{3} \qquad S_{y2} = -\frac{2}{3}Ab$$



$$T_y = \frac{\sqrt{2}}{2}F \qquad T_x = \frac{\sqrt{2}}{2}F$$

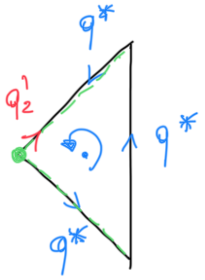
- Open Cell Fluxes

I) $$q_1' = -T_y \cdot \frac{S_{x1}}{J_{xx}} - T_x \cdot \frac{S_{y1}}{J_{yy}} = \frac{\sqrt{2}}{2}F\frac{Ab}{2Ab^2} - \frac{\sqrt{2}}{2}F\frac{Ab}{2Ab^2} = \emptyset$$

III) $+q_2' = +T_y \frac{\partial x_3}{J_{xx}} + T_x \frac{\partial y_3}{J_{yy}} = \frac{\sqrt{2}}{2} F \frac{\cancel{A}b}{2\cancel{A}b^{\cancel{2}}} + \frac{\sqrt{2}}{2} F \cdot \frac{\cancel{A}b}{\cancel{3}} \cdot \frac{\cancel{3}}{2\cancel{A}b^{\cancel{2}}} =$

$$= \frac{\sqrt{2}}{2} F \cdot \left(\frac{1}{2b} + \frac{1}{2b} \right) = \frac{\sqrt{2}}{2} \frac{F}{b}$$

- Moment Equivalence wrt ②



$$\phi = q^* \cdot 2\,\Omega_{cell} \qquad q^* = \phi$$

$$q_2 = q_2' - q^* = \frac{\sqrt{2}}{2} \frac{F}{b}$$

- Angle Computation

$$\theta' = \frac{d\theta}{dz} = \frac{1}{2\Omega_{cell} \cdot G} \cdot \Sigma_i \frac{q_i \cdot l_i}{t_i} = \qquad G = \frac{E}{2(1+\nu)}$$

length

panels

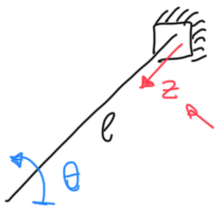
thickness

$$\Omega_{cell} = \frac{1}{2} a^2$$

$$= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{a^{\cancel{2}}} \cdot \frac{2(1+\nu)}{E} \cdot \frac{\sqrt{2}}{2} \frac{F}{b} \cdot \frac{\cancel{a}}{t} =$$

$$= \frac{2(1+\nu)}{E a t} \cdot \frac{\cancel{\sqrt{2}}F}{\cancel{2}} \cdot \frac{\cancel{2}}{\cancel{\sqrt{2}}a} =$$

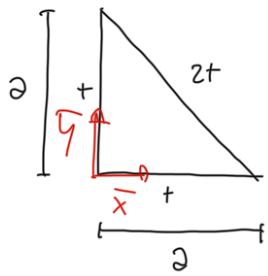
$$= \frac{2(1+\nu)}{E a^2 t} \cdot F = 1.49 \times 10^{-6} \ \frac{rad}{mm}$$



$$\theta(l) - \theta(\phi) = \int_0^l \theta' \, dz \qquad \theta(\phi) = \phi \quad \text{CLAMP}$$

$$\theta(l) = \theta' \cdot l = 5.9 \times 10^{-3} \ rad$$

5) EXAM 23/01/2024



DATA

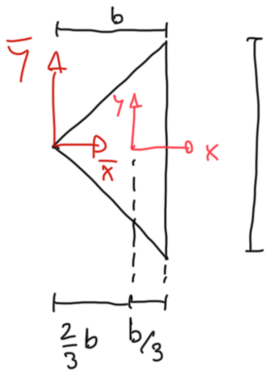
$a = 1000$ mm

$A = 4000$ mm^2

$E = 10^5$ MPa

$\nu = 0.3$

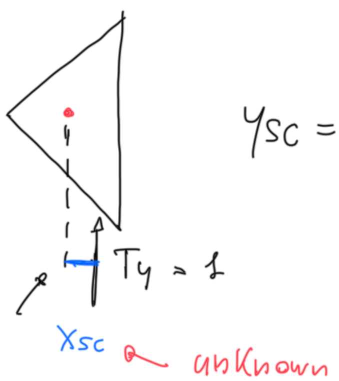
Let's find the shear center



$b = \frac{\sqrt{2}}{2} a$

centroid

$\bar{x}_c = \frac{2}{3} b \quad \bar{y}_c = \emptyset$



$y_{sc} = \emptyset$ for symmetry

- Inertias

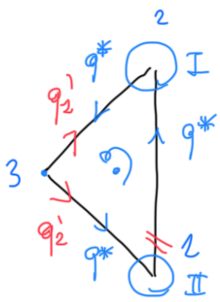
$J_{xx} = 2Ab^2$

$S_{x1} = -Ab \quad S_{x2} = Ab \quad S_{x3} = \emptyset$

We have to find x_{sc}. Let's define a shear force $T_y = 1$ passing for the shear center.

The rotation of the sectione due to T_y must be $\emptyset$

- Open Cell Fluxes



$$q_2' = T_y \frac{S_{x2}}{J_{xx}} = \frac{Ab}{2Ab^2} = \frac{1}{2b}$$

$$q_2' = T_y \frac{S_{x1}}{J_{xx}} = -\frac{Ab}{2Ab^2} = -\frac{1}{2b}$$

- Moment Equivalence wrt 2

$$T_y \cdot \left(\frac{2}{3}b + x_{sc}\right) = 2\,\Omega_{cell} \cdot q^*$$

$$q^* = \frac{\frac{2}{3}b + x_{sc}}{2b^2} \quad *$$

- Rotation Angle

$$\theta' = \frac{d\theta}{dz} = \frac{1}{2\,\Omega_{cell} \cdot G} \cdot \left(\frac{q^* \cdot 2a}{t} + \frac{q^* \cdot 2b}{2t} - \frac{q_1' \cdot a}{t} + \frac{q_2' \cdot a}{t}\right)$$

We impose $\theta' = \emptyset \rightarrow q^* = \dfrac{a}{b(2a+b)}$ $**$

$* + **$
$$\frac{\frac{2}{3}b + X_{sc}}{2b^2} = \frac{a}{b(2a+b)}$$

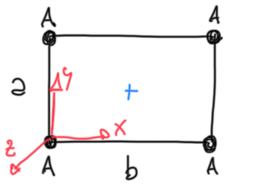
$$X_{sc} = \frac{2ab^2}{b(2a+b)} - \frac{2}{3}b = \leftarrow b = \frac{\sqrt{2}}{2}a$$

$$= \frac{2a \cdot \frac{1}{2}a^2}{\frac{\sqrt{2}}{2}a\left(2a + \frac{\sqrt{2}}{2}a\right)} - \frac{\sqrt{2}}{3}a = \frac{a^3}{a^2\left(\sqrt{2}+\frac{1}{2}\right)} - \frac{\sqrt{2}}{3}a =$$

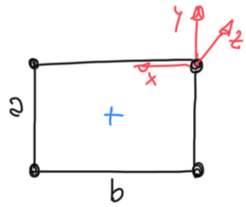
$$= \left(\frac{1}{\sqrt{2}+\frac{1}{2}} - \frac{\sqrt{2}}{3}\right)a = 51.00 \text{ mm}$$

NOTES ON THE COORDINATE SYSTEMS

- Centroid



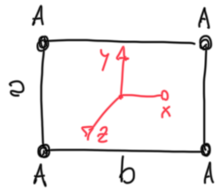
$$x = \frac{2Ab}{4A} = 2b \qquad y = \frac{2Aa}{4A} = 2a$$



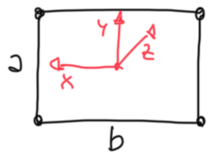
$$x = \frac{-2Ab}{4A} = -2b \qquad y = \frac{-2Aa}{4A} = -2a$$

The position of the centroid does <u>NOT</u> depend on the coordinate system. It is a section property

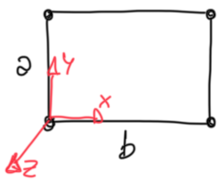
- Inertia



$$J_{xx} = 2A \cdot \left(-\frac{a}{2}\right)^2 + 2A \cdot \left(\frac{a}{2}\right)^2 = Aa^2$$



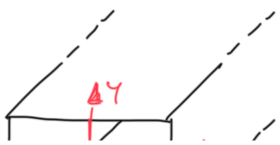
$$J_{xx} = 2A \cdot \left(-\frac{a}{2}\right)^2 + 2A \cdot \left(\frac{a}{2}\right)^2 = Aa^2$$

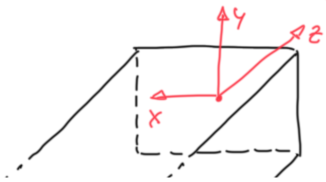


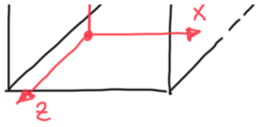
$$J_{xx} = 2A \cdot \left(\frac{a}{2}\right)^2 = \frac{Aa^2}{2}$$

Inertias depends on the position of the reference axes, but does <u>NOT</u> depend on its direction.

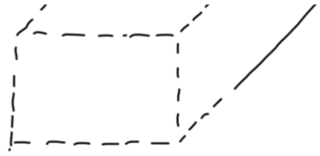
For static moments and fluxes, we must select a surface, defined by its normal vector, and keep it!





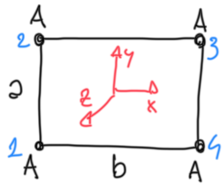


OUTWARD SYSTEM



INWARD SYSTEM

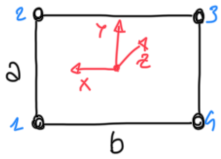
• Static Moments



OUTWARD

$S_{x1} = A\cdot\left(-\frac{a}{2}\right) = -\frac{Aa}{2}$

$S_{y1} = A\cdot\left(-\frac{b}{2}\right) = -\frac{Ab}{2}$



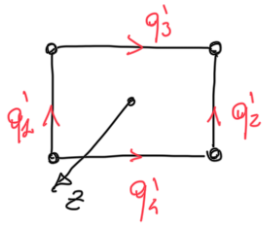
INWARD

$S_{x1} = A\left(\frac{a}{2}\right) = \frac{Aa}{2}$

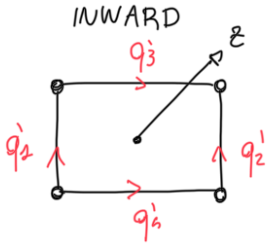
$S_{y1} = A\left(\frac{b}{2}\right) = \frac{Ab}{2}$

• Open Cell Fluxes

OUTWARD

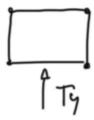


INWARD

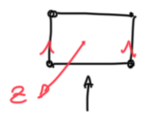


The fluxes orientation is ARBITRARY and does not depend on $\vec{z}$, but we must keep the same normal

eg.



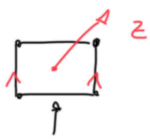
OUTWARD



$q_1' > 0$

$q_2' > 0$

INWARD

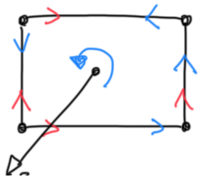


$q_1' < 0$

$q_2' < 0$

• Closed Cell Fluxes

OUTWARD

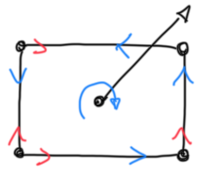


The positive rotation is given by the normal axis z

OUTWARD → ANTI - CLOCKWISE

z

INWARD



INWARD $\rightarrow$ CLOCKWISE

The direction of q^* is ARBITRARY

- Angle

The sign of θ' depends on the normal axis z

Concluding, the GOOD PRACTICE is to choose a surface and always keep it.