

EXERCISE SESSION 9 - 22/11/22

Ribs Frames & Junctions

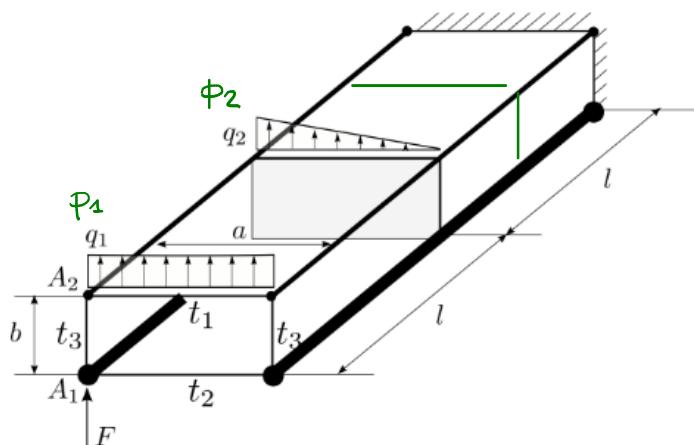
Ex 1

Course of Aerospace Structures Written test, June 26th, 2019

Exercise 1

Consider the single-cell, thin-walled beam in the figure. The beam length is equal to $2l$; the section has dimensions $a \times b$, and is stiffened by four stringers, each characterized by lumped area equal to A_1 (at the bottom) and A_2 (at the top). The thickness of the panels is denoted with t_1 , t_2 and t_3 , as illustrated in the figure. Referring to the loading conditions reported in the sketch:

- determine the shear stresses in the panels and the axial stresses in the stringers for the section at a distance $l/2$ from the constraint;
- plot the internal actions on the rib at the mid-span, assuming that the rib can be modeled as a beam.



Data

$l = 1500 \text{ mm}$; $a = 400 \text{ mm}$; $b = 250 \text{ mm}$;
 $t_1 = 1 \text{ mm}$; $t_2 = 2 \text{ mm}$; $t_3 = 1.5 \text{ mm}$;
 $A_1 = 2000 \text{ mm}^2$; $A_2 = 1000 \text{ mm}^2$;
 $q_1 = 90 \text{ N/mm}$; $q_2 = 300 \text{ N/mm}$;
 $F = 10 \text{ kN}$;

γ_1 & P_2

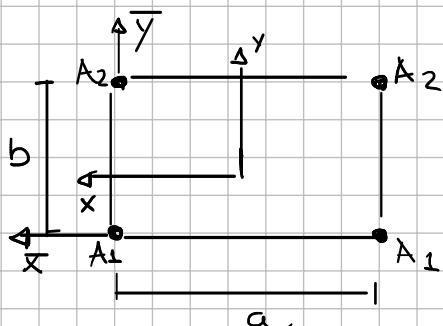
SECTION PROPERTIES

$$A_2 = A \quad A_1 = 2A$$

$$\bar{x}_{CG} = -\frac{a}{2}$$

$$\bar{y}_{CG} = \frac{1}{3}b$$

$$J_{xx} = \sum_i A_i y_i^2 = \frac{4}{3} A b^2$$



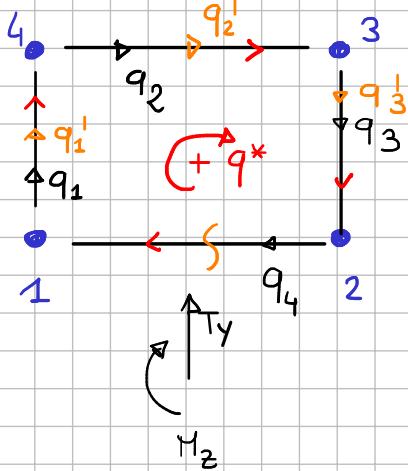
OPEN CELL FLUXES

$$S_{x_1} = S_{x_2} = -A_1 \frac{1}{3} b = -\frac{2}{3} Ab$$

$$S_{x_3} = S_{x_4} = A_2 \frac{2}{3} b = \frac{2}{3} Ab$$

$$q_1' = -q_3' = -T_y \cdot \frac{S_{x_1}}{J_{xx}} = \frac{1}{2} \frac{T_y}{b}$$

$$q_2' = 0$$



MOMENT EQUIVALENCE WRT ① ↗

$$\text{LHS}_{\text{Mom}} = M_z - T_y \cdot \frac{a}{2}$$

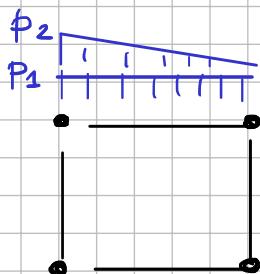
$$\text{RHS}_{\text{Mom}} = 2q^* \cdot \mathcal{L}_{\text{cell}} + 2q_3' \cdot \mathcal{L}_3$$

$$\text{WHERE } \mathcal{L}_{\text{cell}} = a \cdot b$$

$$\mathcal{L}_3 = \frac{1}{2} a \cdot b$$

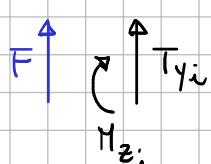
$$\text{SOLVE } \text{LHS}_{\text{Mom}} = \text{RHS}_{\text{Mom}} \rightarrow q^*(T_y, M_z)$$

WIMP FORCES @ $x=0$



$$T_{y_F} = F$$

$$M_{z_F} = F \cdot \frac{a}{2}$$



$$T_{y_{P1}} = P_1 \cdot a$$

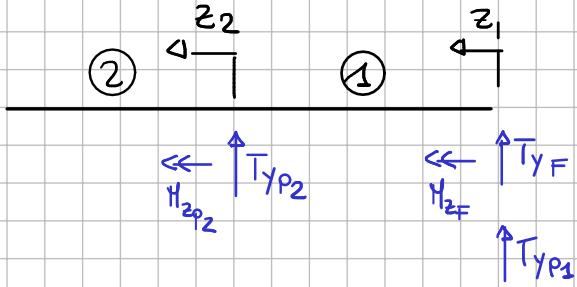
$$M_{z_{P1}} = 0$$

$$T_{y_{P2}} = \frac{1}{2} P_2 \cdot a$$

$$M_{z_{P2}} = \frac{1}{12} P_2 a^2$$

$$= \frac{1}{2} P_2 a \cdot \frac{1}{6} a$$

BEAM INTERNAL ACTIONS



$$\textcircled{1} \quad \bar{T}_{y_1} = -\bar{T}_{y_F} - \bar{T}_{y_{P_1}}$$

$$M_{x_1} = -(\bar{T}_{y_F} + \bar{T}_{y_{P_1}})z_1$$

$$M_{z_1} = -M_{z_F}$$

$$\textcircled{2} \quad \bar{T}_{y_2} = -\bar{T}_{y_{P_1}} - \bar{T}_{y_F} - \bar{T}_{y_{P_2}}$$

$$M_{x_2} = -(\bar{T}_{y_{P_1}} + \bar{T}_{y_F})(z_2 + l) - \bar{T}_{y_{P_2}} \cdot z_2$$

$$M_{z_2} = -M_{z_F} - M_{z_{P_2}}$$

SHEAR FLUXES @ $z_2 = \frac{l}{2}$ → EVALUATE FOR

$$\bar{T}_y = \bar{T}_{y_2} \left(\frac{l}{2} \right)$$

$$M_z = M_{z_2} \left(\frac{l}{2} \right)$$

$$q_1 = q_1' + q^* = -242 \frac{N}{mm}$$

$$q_2 = q_2' + q^* = -30 \frac{N}{mm}$$

$$q_3 = q_3' + q^* = -182 \frac{N}{mm}$$

$$q_4 = q^* = -30 \frac{N}{mm}$$

Axial stress in stringers @ $z_2 = \frac{e}{2}$ $M_x(e/2)$

$$\sigma_{zz_i} = \frac{T_z}{\sum_i A_i} + \frac{M_x}{J_{xx}} y_i - \frac{M_y}{J_{yy}} \cdot x_i$$

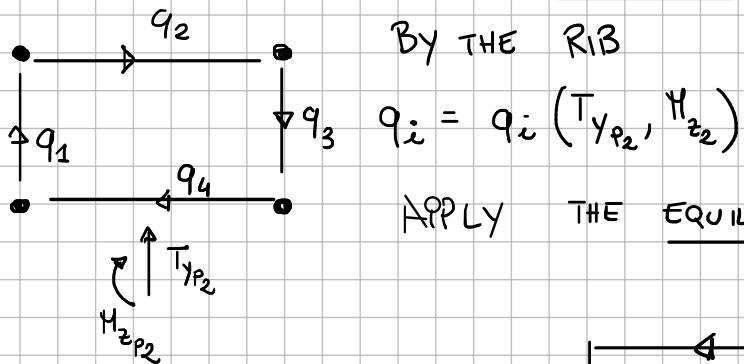
$$\sigma_{zz_1} = \sigma_{zz_2} = - \frac{M_x(e/2)}{J_{xx}} \cdot \frac{1}{3} b = 148,5 \text{ MPa}$$

$$\sigma_{zz_3} = \sigma_{zz_4} = + \frac{M_x(e/2)}{J_{xx}} \cdot \frac{2}{3} b = -297 \text{ MPa}$$

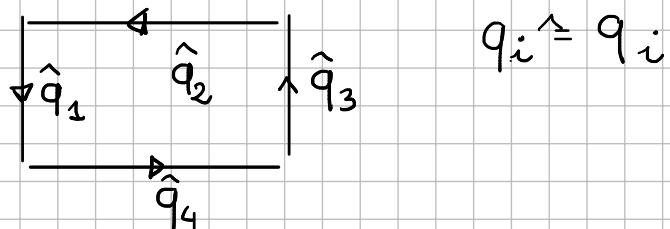
Internal action in RIB

EVALUATE EQUIVALENT FLOWS FOR THE LOAD APPLIED

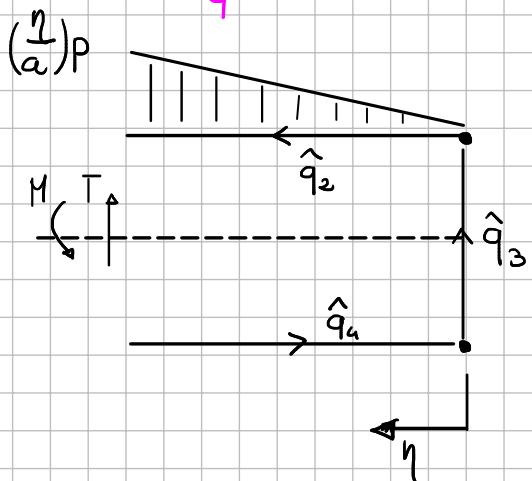
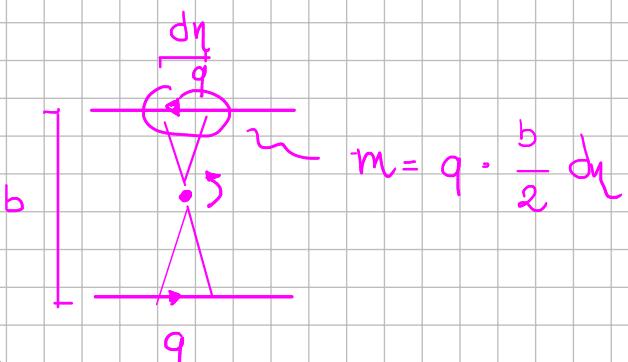
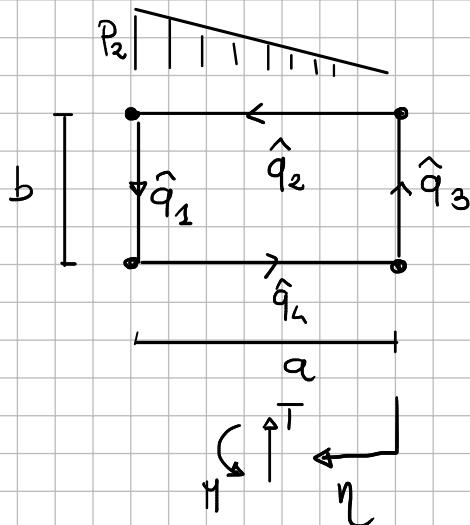
BY THE RIB



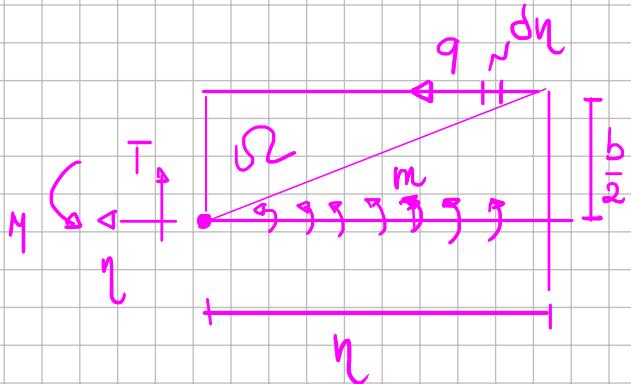
APPLY THE EQUILIBRATING FLOWS TO THE RIB



BEAM MODEL FOR RIB



TWO WAY TO COMPUTE INTERNAL MOMENT



WAY ①:
COLLAPSE IN ONE
LINE AS
DISTR. MOMENT

$$m = q \cdot \frac{b}{2} \quad \xrightarrow{\text{CONTRIB. FOR } d\eta}$$

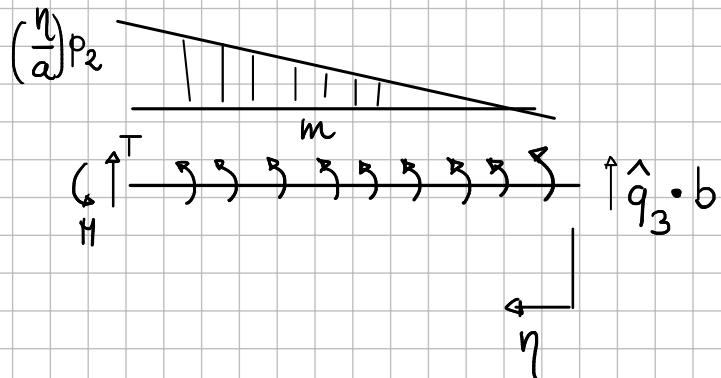
$$M = - \int_0^h m d\eta = - \frac{1}{2} qb \eta$$

WAY ② USE MOM EQUIVALENCE FORMULATION $2q \Delta L$

$$M = -2q \cdot \Delta L$$

$$= -2q \cdot \frac{1}{2} \cdot \frac{b}{2} \cdot \eta = -\frac{1}{2} qb \eta$$

Pay attention to the sign: beam internal action impose equilibrium not equivalence.



$$m = \hat{q}_2 \cdot \frac{b}{2} + \hat{q}_4 \cdot \frac{b}{2}$$

$$T(\eta) = -\hat{q}_3 \cdot b - \frac{1}{2} \left(\frac{n}{a} \right) P_2 \cdot \eta$$

$$M(\eta) = -m \cdot \eta - \hat{q}_3 \cdot b \cdot \eta - \frac{1}{2} \left(\frac{n}{a} \right) P_2 \cdot \eta \cdot \frac{1}{3} \eta$$

Contents

- Data
- Section properties
- Open cell fluxes
- Moment equivalence wrt 1
- Lump forces to centroid
- Beam internal actions
- Shear fluxes for $z_2 = l/2$
- Axial stress in stringers for $z_2 = l/2$
- Internal action in rib

```
clear variables
close all
home
```

Data

```
l = 1500; % mm
a = 400; % mm
b = 250; % mm
t_1 = 1; % mm
t_2 = 2; % mm
t_3 = 1.5; % mm
A_1 = 2000; % mm^2
A_2 = 1000; % mm^2
% On the given data this was defined as q_1 and q_2 renamed to avoid
% confusion with fluxes
p_1 = 90; % N/mm
p_2 = 300; % N/mm
F = 10000; % N
```

Section properties

```
x_cg = -a/2;
y_cg = (2*A_2*b)/(2*A_2+2*A_1);

J_xx = 2*A_1*(1/3*b)^2 + 2*A_2*(2/3*b)^2;

S_x1 = -A_1*(1/3)*b;
S_x2 = S_x1;
S_x3 = A_2*(2/3)*b;
S_x4 = S_x3;
```

Open cell fluxes

```
syms T_y
q_1_p = -T_y*(S_x1/J_xx);
q_2_p = -T_y*((S_x1+S_x4)/J_xx);
q_3_p = -q_1_p;
```

Moment equivalence wrt 1

```
syms M_z q_s
Omega_cell = a*b;
Omega_3 = .5*a*b;
LHS_Mom = M_z - T_y*(a/2);
RHS_Mom = 2*q_s*Omega_cell + 2*q_3*p*Omega_3;

q_s = solve(LHS_Mom == RHS_Mom, q_s);

q_1 = q_1_p + q_s;
q_2 = q_2_p + q_s;
q_3 = q_3_p + q_s;
q_4 = q_s;
```

Lump forces to centroid

```
T_y_F = F;
M_z_F = F*.5*a;

T_y_p1 = p_1*a;
M_z_p1 = 0;

T_y_p2 = .5*p_2*a;
M_z_p2 = (1/12)*p_2*a^2;
```

Beam internal actions

```
syms z_1 z_2
T_y_1 = -T_y_F-T_y_p1;
M_x_1 = -(T_y_F + T_y_p1)*z_1;
M_z_1 = -M_z_F-M_z_p1;

T_y_2 = -T_y_F-T_y_p1-T_y_p2;
M_x_2 = -(T_y_F + T_y_p1)*(1+z_2) - T_y_p2*z_2;
M_z_2 = -M_z_F-M_z_p1-M_z_p2;
```

Shear fluxes for z_2 = l/2

```
q_1_d = double(subs(q_1, [T_y, M_z], [T_y_2, M_z_2]))
q_2_d = double(subs(q_2, [T_y, M_z], [T_y_2, M_z_2]))
q_3_d = double(subs(q_3, [T_y, M_z], [T_y_2, M_z_2]))
q_4_d = double(subs(q_4, [T_y, M_z], [T_y_2, M_z_2]))
```

q_1_d =

-242

q_2_d =

-30

q_3_d =

```
q_4_d =
```

```
-30
```

Axial stress in stringers for z_2 = l/2

```
s_zz1 = double(-(subs(M_x_2, z_2, .5*l)/J_xx)*b/3)
s_zz2 = double(s_zz1)
s_zz3 = double((subs(M_x_2, z_2, .5*l)/J_xx)*2*b/3)
s_zz4 = double(s_zz3)
```

```
s_zz1 =
```

```
148.5000
```

```
s_zz2 =
```

```
148.5000
```

```
s_zz3 =
```

```
-297.0000
```

```
s_zz4 =
```

```
-297.0000
```

Internal action in rib

```
syms eta
q_1_r = double(subs(q_1, [T_y, M_z], [T_y_p2, M_z_p2]));
q_2_r = double(subs(q_2, [T_y, M_z], [T_y_p2, M_z_p2]));
q_3_r = double(subs(q_3, [T_y, M_z], [T_y_p2, M_z_p2]));
q_4_r = double(subs(q_4, [T_y, M_z], [T_y_p2, M_z_p2]));

m = q_2_r*b/2 + q_4_r*b/2;
T = -q_3_r*b - .5*(eta/a)*p_2*eta
M = -m*eta - q_3_r*b*eta -.5*(eta/a)*p_2*eta*(1/3)*eta

syms x
figure(1)
hold on
title('Shear in rib', 'Interpreter', 'latex')
fplot(subs(T, eta, (x+a/2)), [-a/2, a/2], 'Color', '#0072BD')
xlabel('$x$ [mm]', 'Interpreter', 'latex')
ylabel('T [N]', 'Interpreter', 'latex')
grid on
ax = gca();
ax.XDir='reverse';
```

```

ax.TickLabelInterpreter='latex';
ax.Box = 1;

figure(2)
hold on
title('Bending moment in rib', 'Interpreter','latex')
fplot(subs(M, eta, (x+a/2)), [-a/2, a/2], 'Color', '#0072BD')
xlabel('$x$ [mm]', 'Interpreter','latex')
ylabel('M [Nmm]', 'Interpreter','latex')
grid on
ax = gca();
ax.XDir='reverse';
ax.TickLabelInterpreter='latex';
ax.Box = 1;

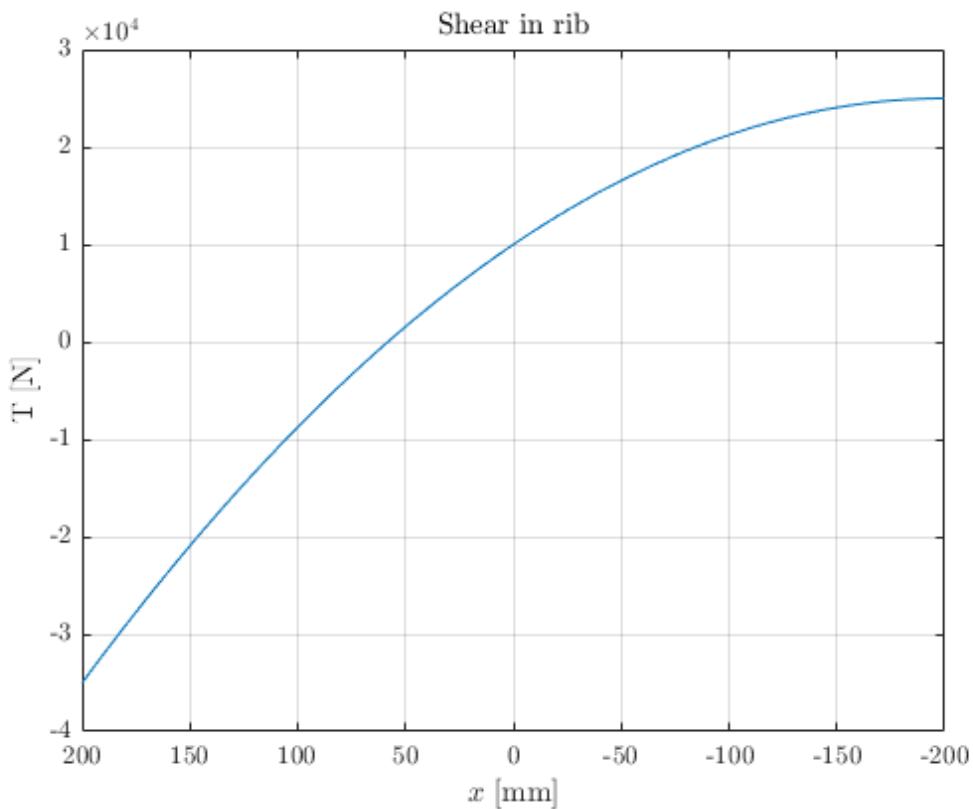
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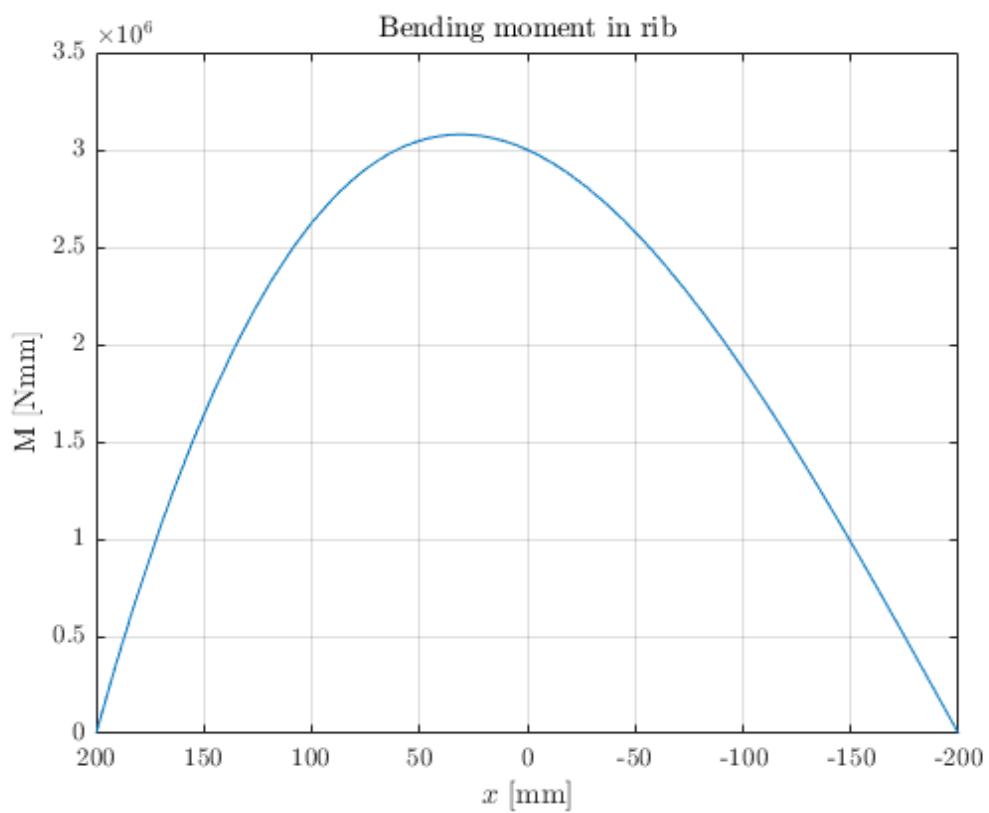
$T =$

$$25000 - (3\eta^2)/8$$

$M =$

$$- \eta^3/8 + 2000\eta$$



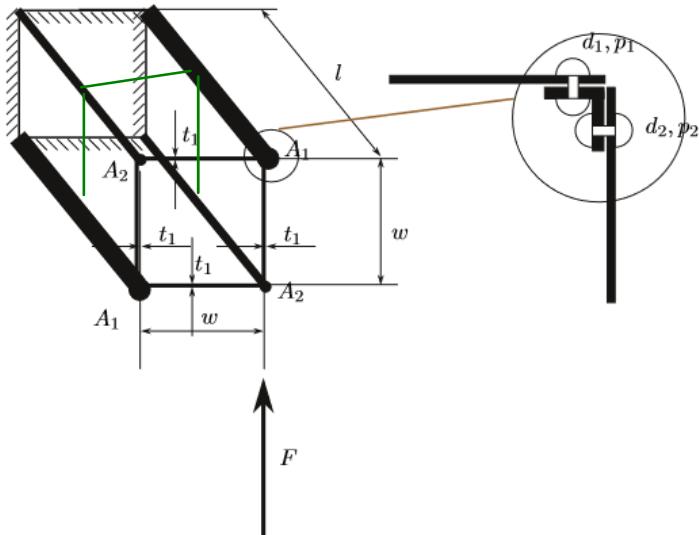


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AEROSPACE STRUCTURES

Written test February 11 2019

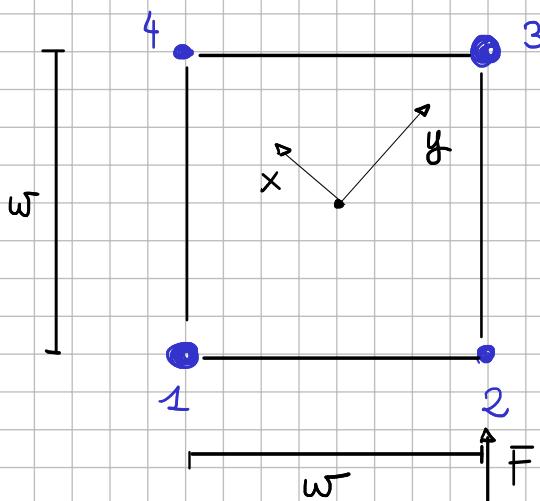
1. The semi-momocoque wing of Figure 1 is loaded by a concentrated force F . Refer to Table 1 for the problem data, and assume reasonable values for any missing constant. Compute
- the stress of the stringers at a distance $l/2$ from the loaded end;
 - the average shear stress of the two rivets with diameter d_1 and d_2 connecting the panels to the stringer. The rivets are spaced with pitch p_1 and p_2 , respectively. Assume the area A_1 to belong to the L-shaped stringer only.



A_1	300 mm^2	l	1500 mm
A_2	150 mm^2	d_1	2 mm
t_1	1.5 mm	d_2	3 mm
w	100 mm	p_1	10 mm
E	72000 MPa	p_2	12 mm
ν	0.3	F	15 kN

Table 1: Semi-monocoque wing data

Figure 1: Loaded wing

SECTION PROPERTIES

$$A_1 = 2A \quad A_2 = A$$

$$J_{xx} = 2A\omega^2$$

$$J_{yy} = A\omega$$

$$S_{x_1} = -2A \frac{\sqrt{2}}{2} \omega$$

$$S_{y_1} = 0$$

$$S_{x_2} = 0$$

$$S_{y_2} = -A \frac{\sqrt{2}}{2} \omega$$

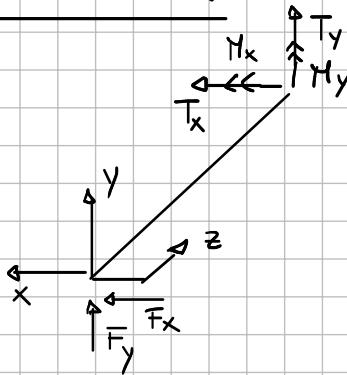
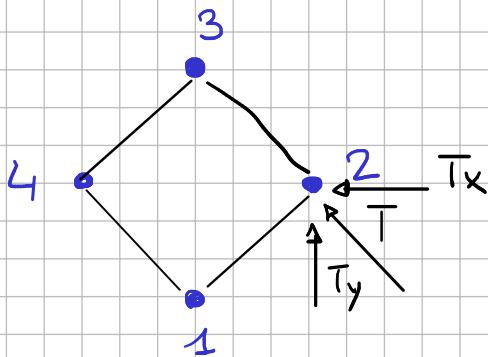
$$S_{x_3} = 2A \frac{\sqrt{2}}{2} \omega$$

$$S_{y_3} = 0$$

$$S_{x_4} = 0$$

$$S_{y_4} = A \frac{\sqrt{2}}{2} \omega$$

INTERNAL ACTION FOR THE BEAM



$$F_x = \frac{\sqrt{2}}{2} F = F_y$$

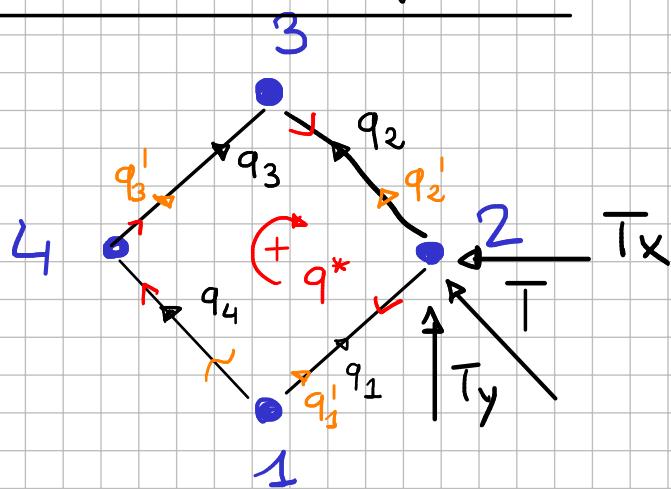
$$T_y = -F_y$$

$$M_x = -F_y \cdot z$$

$$T_x = -F_x$$

$$M_y = F_x \cdot z$$

OPEN CELL FLUXES



$$q_1^1 = -T_y \frac{S_{x_1}}{J_{xx}} - T_x \frac{S_{y_1}}{J_{yy}}$$

$$q_2^1 = -T_y \frac{S_{x_1} + S_{x_2}}{J_{xx}} - T_x \frac{S_{y_1} + S_{y_2}}{J_{yy}}$$

$$q_3^1 = -T_y \frac{S_{x_4}}{J_{xx}} - T_x \frac{S_{y_4}}{J_{yy}}$$

MOT. EQUIVALENCE WRT (2) (+)

$$\text{LHS-Flow} = 0$$

$$\text{RHS-Flow} = 2q^* \cdot \mathcal{Q}_{\text{CELL}} + 2q_3^1 \cdot \mathcal{Q}_3$$

WHERE $\mathcal{Q}_{\text{CELL}} = \omega^2$

$$\mathcal{Q}_3 = \frac{1}{2} \omega^2$$

SOLVE FOR q^*

STRESS IN STRINGSERS @ $z = \frac{l}{2}$

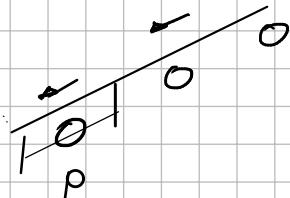
$$\sigma_{zz_1} = -\sigma_{zz_3} = -\frac{M_x}{J_{xx}} \cdot \frac{\sqrt{2}}{2} \omega = 187.5 \text{ MPa}$$

$$\sigma_{zz_2} = -\sigma_{zz_4} = +\frac{M_y}{J_{yy}} \cdot \frac{\sqrt{2}}{2} \omega = 375.0 \text{ MPa}$$

SHEAR STRESS IN RIVETS

EACH RIVET CARRIES:

$$F_i = q_i \cdot p_i$$



$$\tau_i = \frac{F_i}{\pi \cdot (\frac{d_i}{2})^2}$$

$$\tau_1 = \frac{q_3 \cdot p_1}{\pi \left(\frac{d_1}{2}\right)^2} = 119.37 \text{ MPa} \quad \text{WHERE } q_3 = q_3^1 + q^*$$

$$\tau_2 = \frac{q_2 \cdot p_2}{\pi \left(\frac{d_2}{2}\right)^2} = 190.99 \text{ MPa} \quad q_2 = q_2^1 - q^*$$

Contents

- [Data](#)
- [Section properties](#)
- [Internal actions](#)
- [Open cell fluxes](#)
- [Moment equivalence wrt 2](#)
- [Closed cell fluxes](#)
- [Axial stresses in stringers](#)
- [Rivet shear stress](#)

```
clear variables
close all
home
```

Data

```
A_1 = 300; %mm^2
A_2 = 150; %mm^2
t_1 = 1.5; %mm
w = 100; %mm
E = 72000; %MPa
nu = 0.3;
l = 1500; %mm
d_1 = 2; %mm
d_2 = 3; %mm
p_1 = 10; %mm
p_2 = 12; %mm
F = 15000; %N
```

Section properties

```
k = sqrt(2)/2;
J_xx = 2*A_1*(k*w)^2;
J_yy = 2*A_2*(k*w)^2;

S_x1 = -A_1*k*w;
S_x2 = 0;
S_x3 = -S_x1;
S_x4 = 0;

S_y1 = 0;
S_y2 = -A_2*k*w;
S_y3 = 0;
S_y4 = -S_y2;
```

Internal actions

```
syms z

F_x = k*F;
F_y = F_x;
```

```
T_x = -F_x;
T_y = -F_y;

M_x = -F_y*z;
M_y = F_x*z;
```

Open cell fluxes

```
q_1_p = -T_x*(S_y1/J_yy) - T_y*(S_x1/J_xx);
q_2_p = -T_x*((S_y1+S_y2)/J_yy) - T_y*((S_x1+S_x2)/J_xx);
q_3_p = -T_x*(S_y4/J_yy) - T_y*(S_x4/J_xx);
```

Moment equivalence wrt 2

```
syms q_s

Omega_cell = w^2;
Omega_3 = .5*w^2;

LHS_mom = 0;
RHS_mom = 2*q_s*Omega_cell + 2*q_3_p*Omega_3;

[q_s] = solve(LHS_mom == RHS_mom, q_s);
```

Closed cell fluxes

```
q_1 = q_1_p - q_s;
q_2 = q_2_p - q_s;
q_3 = q_3_p + q_s;
q_4 = + q_s;
```

Axial stresses in stringers

```
s_zz1 = double(-(subs(M_x, z, 1/2)/J_xx)*k*w)
s_zz2 = double(+ (subs(M_y, z, 1/2)/J_yy)*k*w)
s_zz3 = double(-s_zz1)
s_zz4 = double(-s_zz2)
```

```
s_zz1 =
187.5000
```

```
s_zz2 =
375.0000
```

```
s_zz3 =
-187.5000
```

```
s_zz4 =
```

-375.0000

Rivet shear stress

```
tau_1 = double((q_3*p_1)/(pi*(.5*d_1)^2))
tau_2 = double((q_2*p_2)/(pi*(.5*d_2)^2))
```

```
tau_1 =
```

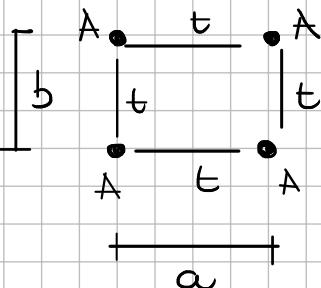
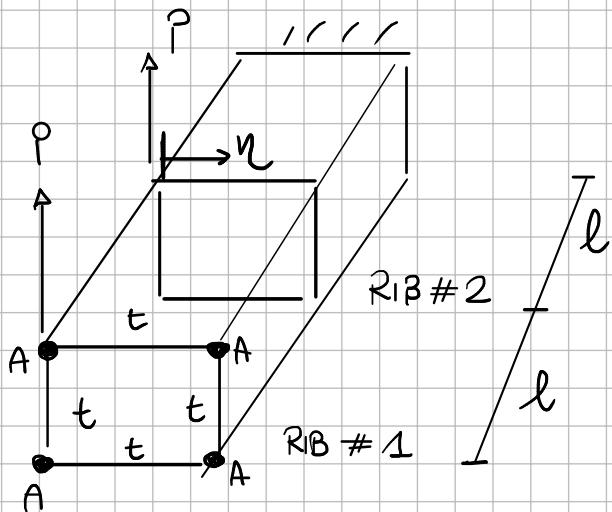
```
119.3662
```

```
tau_2 =
```

```
-190.9859
```

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Ex 3



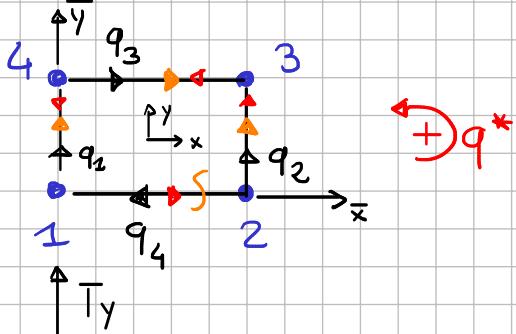
$$a = 500 \text{ mm} \quad A = 500 \text{ mm}^2$$

$$b = 250 \text{ mm} \quad l = 2000 \text{ mm}$$

$$t = 0.6 \text{ mm} \quad P = 1000 \text{ N}$$

FIND BENDING MOM OF THE RIB FOR $\eta = \frac{a}{3}$ IN RIB #2

OPEN CELL FLUX



$$q_1^1 = q_2^1 = \frac{T_y}{2b}$$

$$q_3^1 = 0$$

MOM EQ. WRT ① +

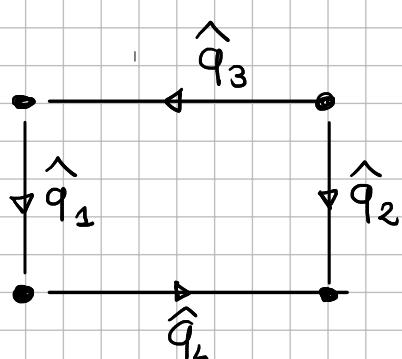
$$0 = 2q^* \cdot \Delta_{\text{CELL}} + 2q_2^1 \cdot \Delta_2$$

$$0 = 2q^* \cdot ab + 2q_2^1 \cdot \frac{1}{2}ab \rightarrow q^* = -\frac{1}{4b} T_y$$

INTERNAL ACTIONS IN RIB

WE COMPUTE EQUIVALENT FLUXES FOR $T_y = P$

IMPOSE EQUILIBRATING FLUXES TO RIB

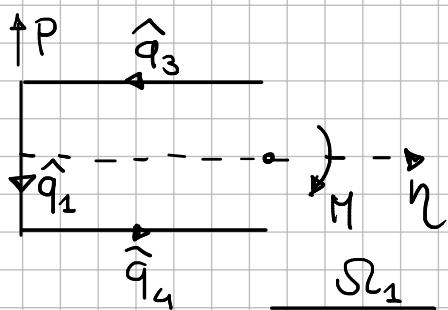


$$\hat{q}_1 = q_1(P) = +q_1^I - q^* = +\frac{3}{4b}P$$

$$\hat{q}_2 = q_2(P) = q_2^I + q^* = \frac{1}{4b}P$$

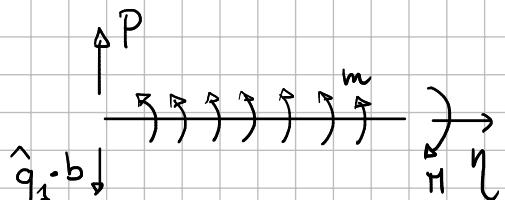
$$\hat{q}_3 = \hat{q}_4 = q_3 = q_4 = -q^*(P) = \frac{1}{4b}P$$

MODEL AS BEAM THE RIB



$$\begin{aligned} M &= -P \cdot \eta + 2\hat{q}_1 \cdot b \cdot \frac{1}{2}\eta + 2\hat{q}_4 \cdot \frac{b}{2} \cdot \frac{1}{2}\eta + 2\hat{q}_3 \cdot \frac{b}{2} \cdot \frac{1}{2}\eta \\ &= -P \cdot \eta + \frac{3}{4} \frac{P}{b} \cdot b \cdot \eta + \frac{1}{8} \frac{P}{b} b \cdot \eta + \frac{1}{8} \frac{P}{b} \cdot b \cdot \eta \\ &= 0 \end{aligned}$$

If we collapse to one line



WHERE

$$m = \frac{1}{4}P = \hat{q}_3 \cdot \frac{b}{2} + \hat{q}_4 \cdot \frac{b}{2}$$

$$M = -P \cdot \eta + \hat{q}_1 \cdot b \cdot \eta + m \cdot \eta = -P \cdot \eta + \frac{3}{4}P\eta + \frac{1}{4}P\eta = 0$$