EX1

$r(z)$

$1800 \text{ mm} \qquad 700 \text{ mm}$

$p = 10^{-3} \text{MPa}$

300 mm

$M \cdot g = Q = 500 \text{ N}$

$$r(z) = \int_{chord} p \, dx = 10^{-3} \cdot 300 = 0,3 \text{ N mm}^{-1}$$

$$T_y + \int_0^z r \, dz = 0$$

$$\Rightarrow T_y = - r z$$

moment eq w R T A:

$$M_{x_B} - \int_0^z r \cdot z \, dz - T_y \cdot z = 0$$

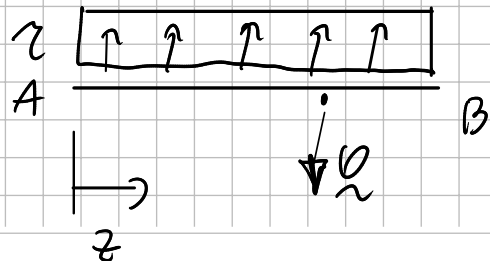$$M_{x_B} - \frac{1}{2} r z^2 + r z^2 = 0$$

$$M_{x_B} = -\frac{1}{2} r z^2$$

OR   moment eq   WRT pt B

$$M_{x_B} + r z \left(\frac{1}{2} z \right) = 0$$

$$\Rightarrow M_{x_B} = -\frac{1}{2} r z^2$$

$$0 \leq z \leq 1800 \, mm$$

$$1800 \leq z \leq 2500 \, mm$$

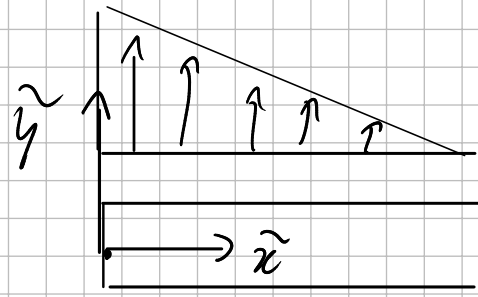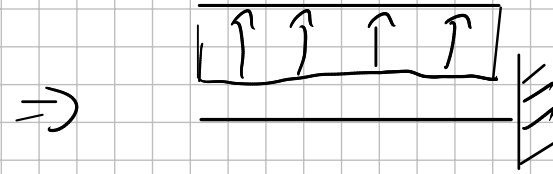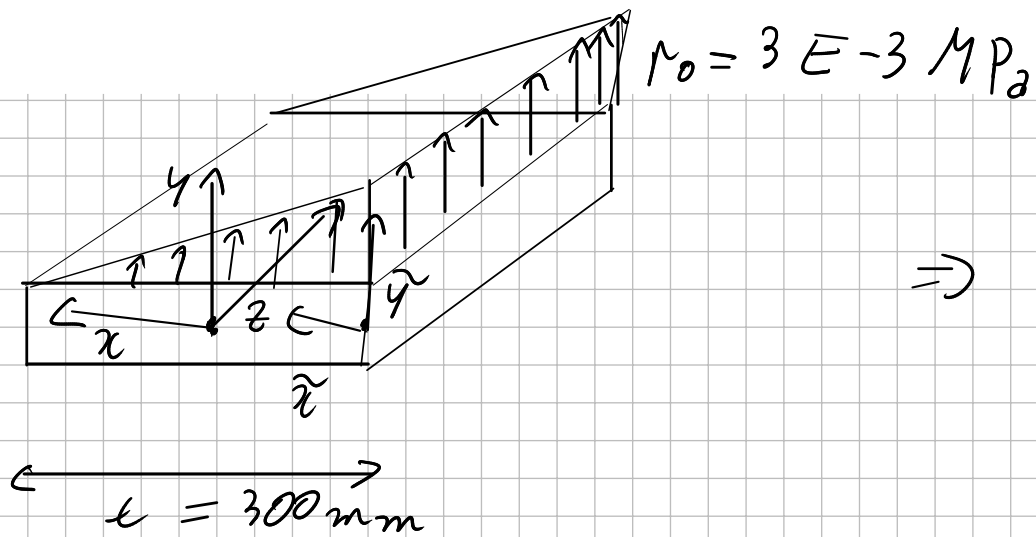$$T_y = - r z + Q$$

moment eq WRT pt A

$$M_{x_B} - \frac{1}{2} \tau z^2 - T_y z + \frac{Q}{2} 1800 = 0$$

$$M_x = \frac{1}{2} \tau z^2 + \left(-\tau z + \frac{Q}{2}\right) z - Q \cdot 1800$$

$$= -\frac{1}{2} \tau z^2 + \frac{Q}{2}(z - 1800)$$

Do on your own: moment eq WRT pt B

$r_0 = 3E{-}3 \, MPa$

$\ell = 300 \, mm$

$r = r_0 - \dfrac{r_0}{\ell} \cdot \tilde{x}$

$= 3E{-}3 - 10^{-5} \, \tilde{x} \quad MPa$

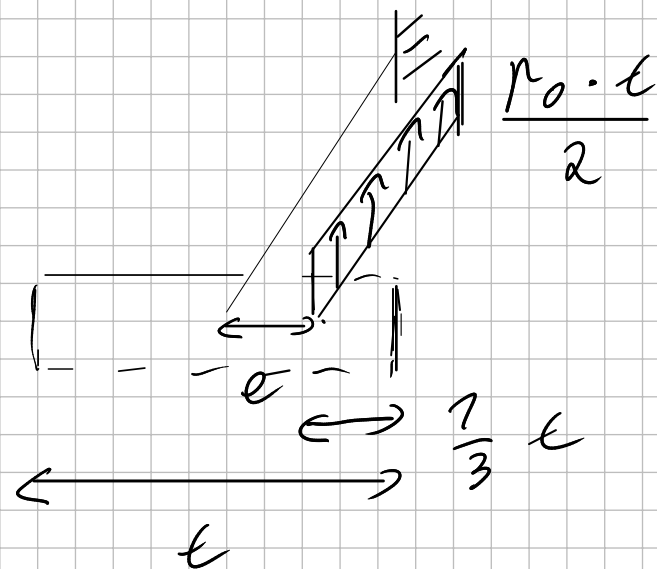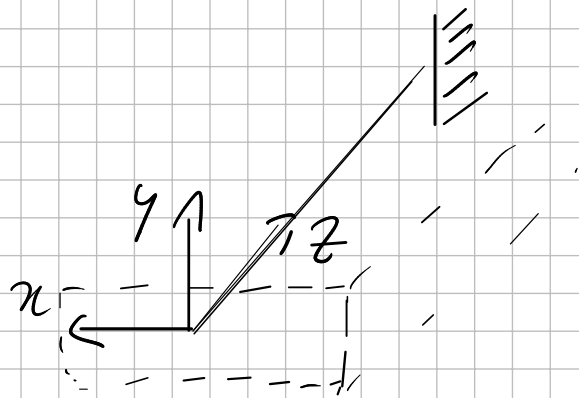$\displaystyle \int\limits_{chord} r(\tilde{x}) \, d\tilde{x} = 0{,}9 - 0{,}45 = 0{,}45 \quad N \, mm^{-1}$

$$r = \frac{1}{2} \rho_0 \cdot \ell$$

$$r\, d = \int_0^\ell \frac{\rho_0}{\ell} \cdot x \cdot x = \frac{1}{3} \frac{\rho_0\, \ell^{3}}{\ell}$$

$$\frac{1}{2} \rho_0 \ell\, d = \frac{1}{3} \rho_0 \ell^{2}$$

$$d = \frac{2}{3} \ell$$

$$\frac{r_0 \cdot t}{2}$$

$$e = \left(\frac{1}{2} - \frac{1}{3}\right)t = \frac{1}{6}t$$

$$\frac{1}{6}r \cdot t$$
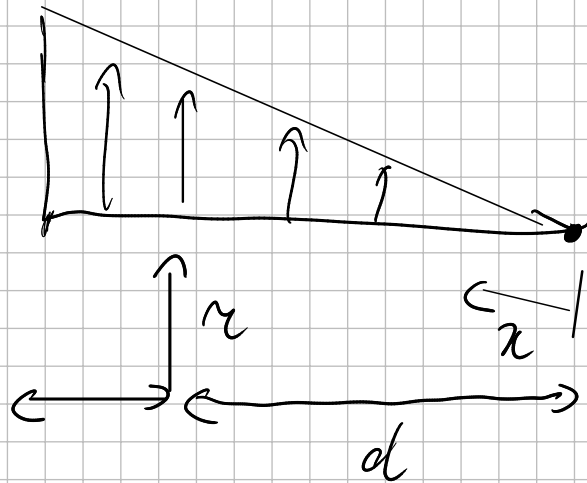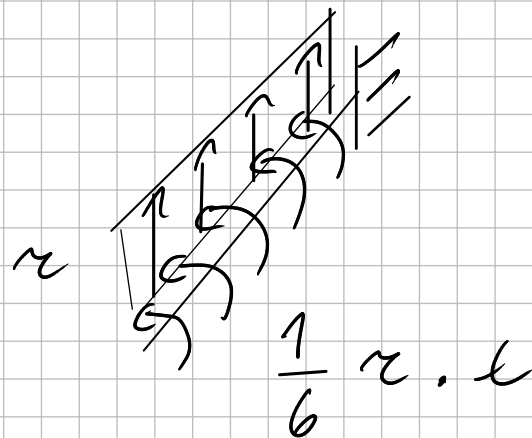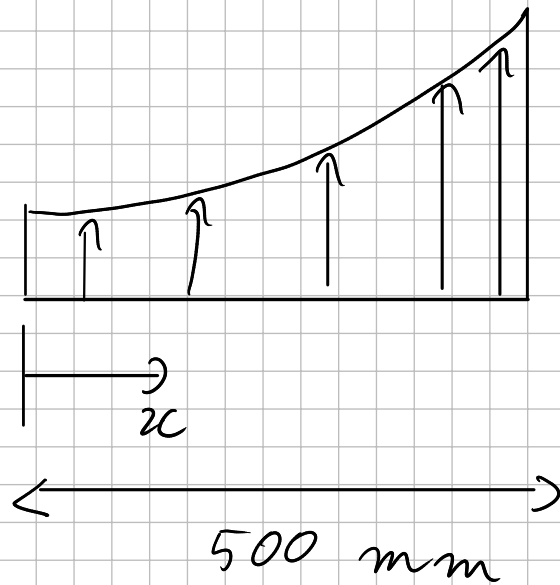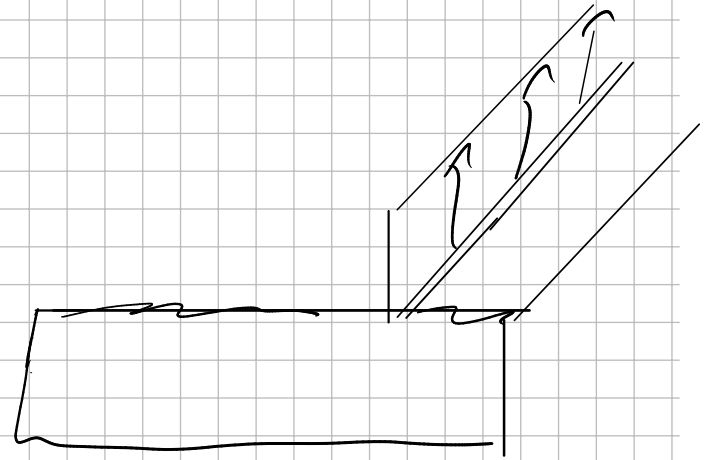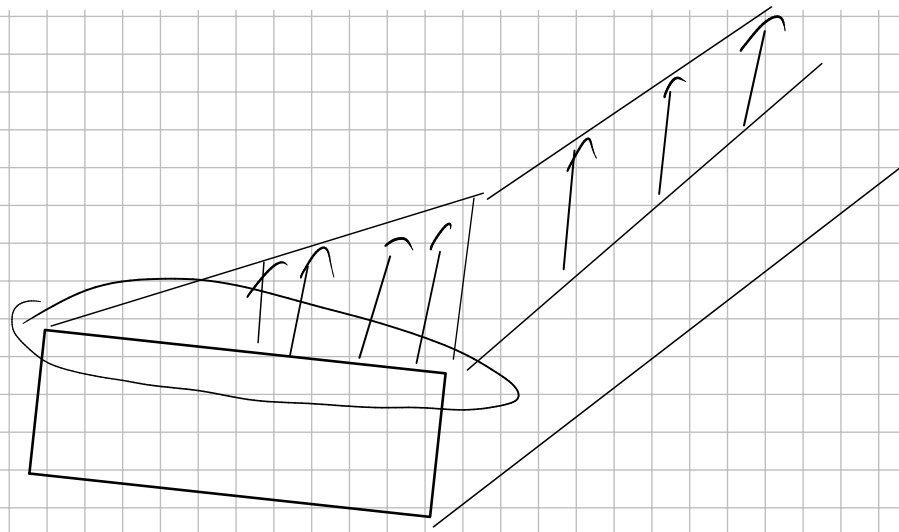
$$p(x) = p_0 + a\,x + b\,x^2$$

$$p_0 = 10 \; P_a$$

$$a = \frac{1}{50} \; P_a \, m^{-1}$$

$$b = \frac{1}{(25)^2} \; P_a \, m^{-2}$$
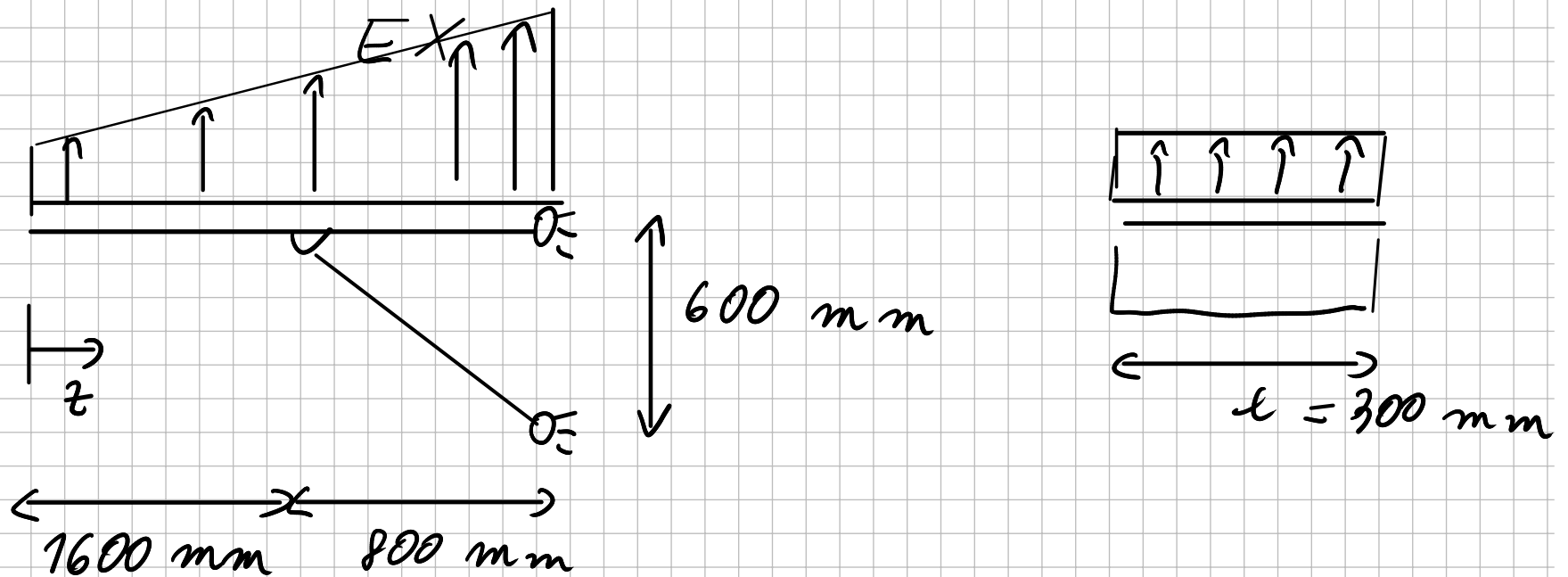
500 mm

$$\rho(z) = \rho_0 + \frac{\rho_1}{L} \cdot z$$

$$\rho_0 = 10^{-3} \, MPa$$

$$\rho_1 = 6E-4 \, MPa$$

$$\tau(z) = \rho(z) \cdot t = 0{,}3 + 7{,}5E-5 \, N \, mm^{-1}$$

$\uparrow V_1$

$O_A \rightarrow H_1$

$\circlearrowleft M$

$\vec{z}$

$\uparrow V_2$

$O \rightarrow H_2$
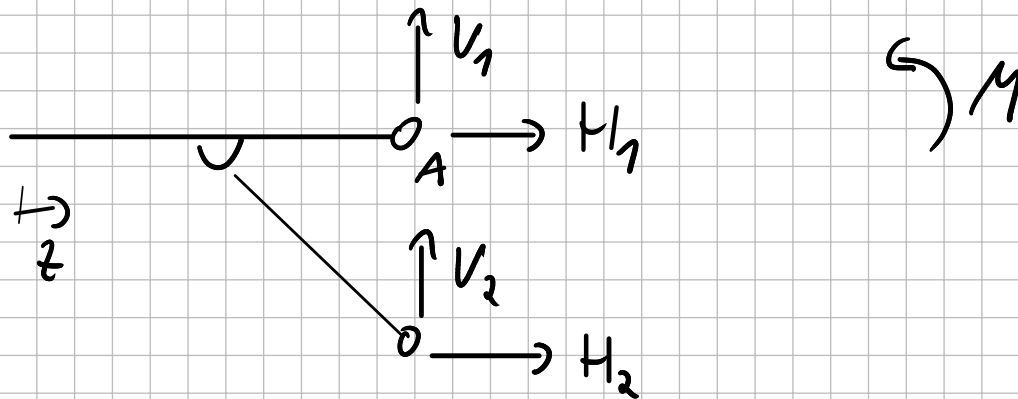
1) $\quad V_1 + V_2 + \int_0^{2400} r(z)\, dz = \emptyset$

2) $\quad H_1 + H_2 = \emptyset$
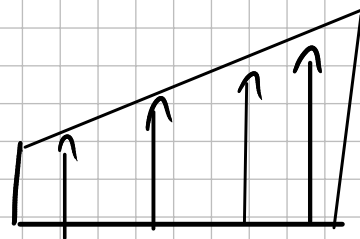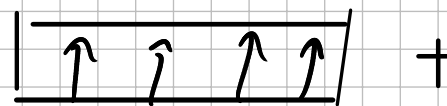
3) $\quad H_2 \cdot 600 + M_{ext} = \emptyset$

$\longleftrightarrow 1200\,mm$

$\uparrow 0{,}3 \cdot 2400$

$\uparrow 7{,}5 E\!-\!5$

$\cdot 2400 \cdot \frac{1}{2} \cdot \frac{1}{3}\, 240$
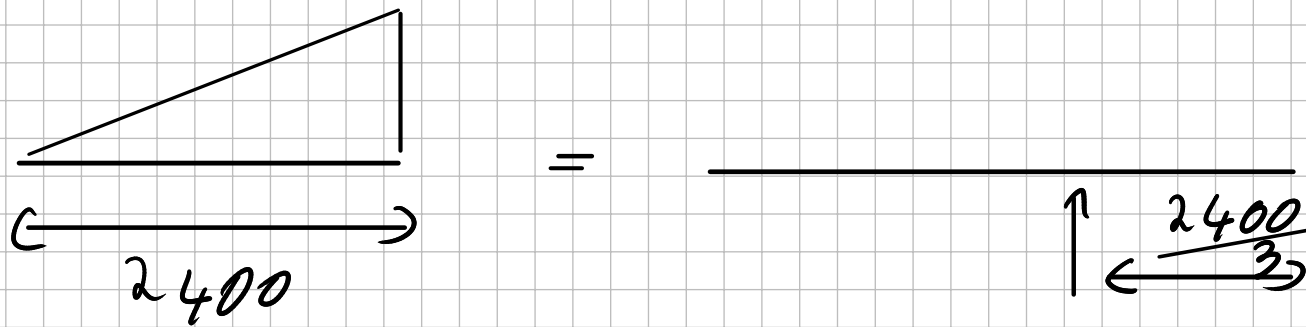
$$M_{ext} = -1036800 \ N \ mm$$

$$H_2 \cdot 600 = - M_{ext} \quad \Rightarrow \quad H_2 = 1728 \ N$$

$$\frac{-V_2}{600} = \frac{H_2}{800}$$
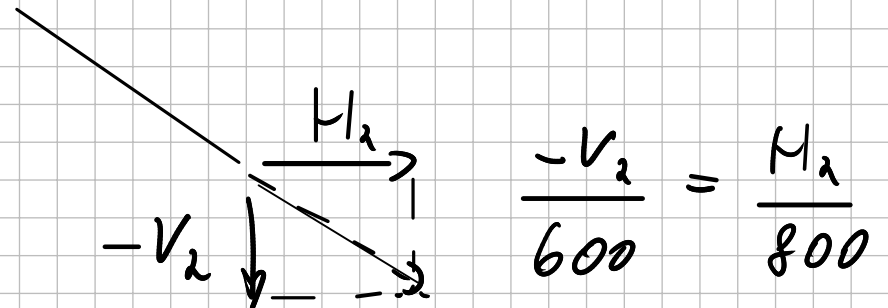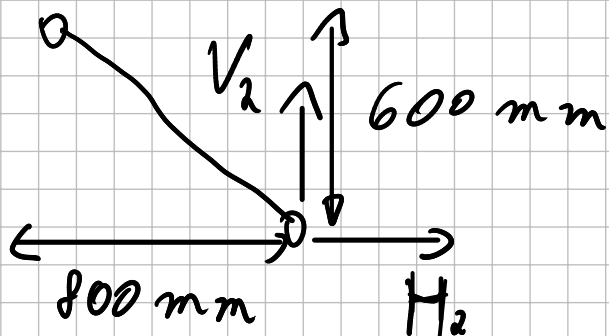
$$V_2 = - H_2 \cdot \frac{6}{8} = -1296 \ N$$

$$V_1 = 360 \, N$$
$$H_1 = -H_2 = -1728 \, N$$

$$T_2 = \sqrt{H_2^2 + V_2^2}$$

$$T_z$$

$$V_2 \uparrow \; \overrightarrow{H_2} \qquad \uparrow V_1 \quad \overrightarrow{H_1}$$

$$\overrightarrow{z}$$

$$\xleftarrow{\hspace{3cm}} 1600 \, mm$$

$$0 \leq z \leq 1600 \, mm$$

$$\overline{T_z} = \emptyset \, N$$

$$T_y = -\int_0^z z = \left(-0{,}3 \, z + 7{,}5 \, E{-}5 \, \frac{z^2}{2} \right)$$
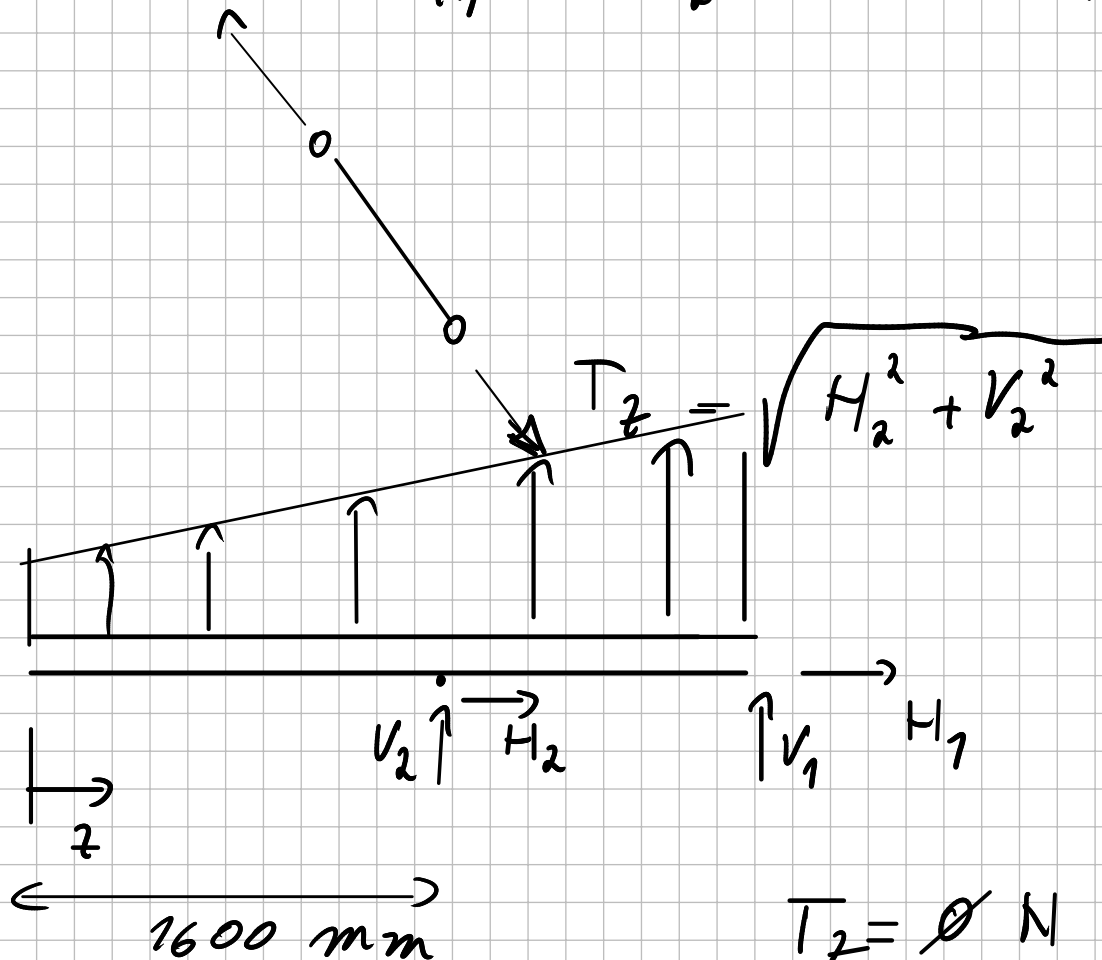
$$M_x = \ldots\ldots = \left(-0,3\,\frac{z^2}{2} - 7,5\,E\text{-}5\,\frac{z^3}{6} \right) \quad N\,mm$$
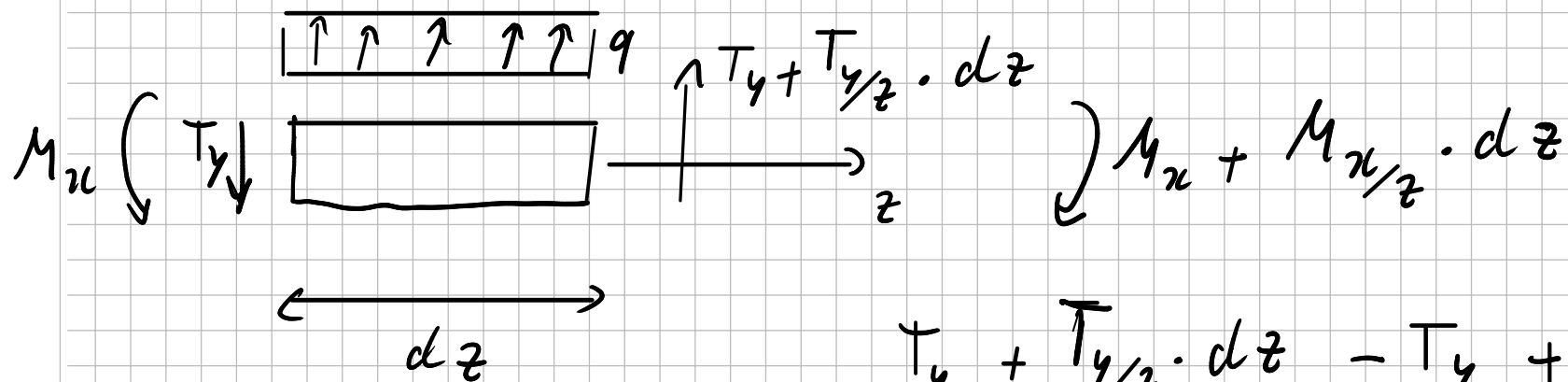
$$1600 \leq z \leq 2400 \; mm$$

$$T_y = T_{y_1} - V_2$$

$$M_x = M_{x_1} + \boxed{V_2\,(z - 1600)}$$

$$T_z = -H_2$$

Distributed load

T_y

y

M_y

M_x

x → T_x

M_z z

T_z

q

$\uparrow T_y + T_{y/z} \cdot dz$

M_x $\bigg($ $T_y \downarrow$

z

$\bigg) M_x + M_{x/z} \cdot dz$

dz

$T_y + T_{y/z} \cdot dz - T_y + q\, dz = \emptyset$

$$\Rightarrow T_{y/z} = -q$$

$$T_y(z) = \int_0^z -q\, dz + \boxed{T_y(\emptyset)}$$

M_x ⤵ T_y↓   ↑q

↑$T_y + T_{y/z} \cdot dz$ ⟩ $M_x + M_{x/z}\, dz$

A
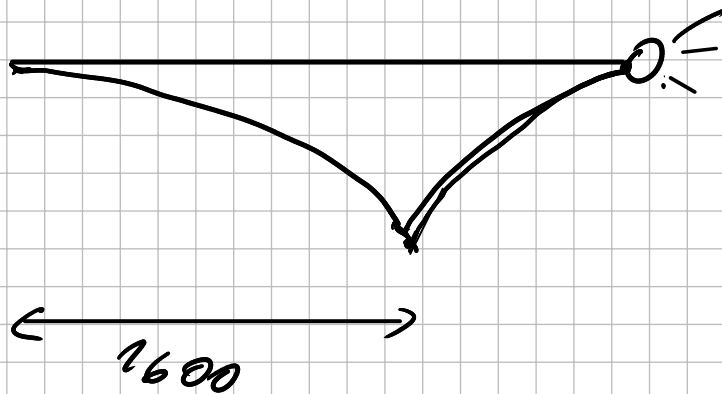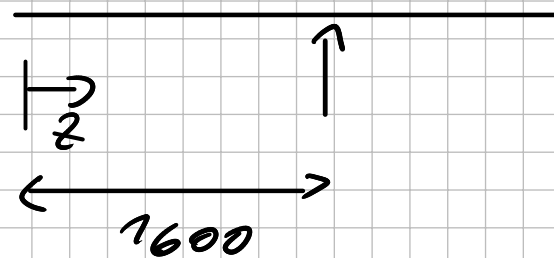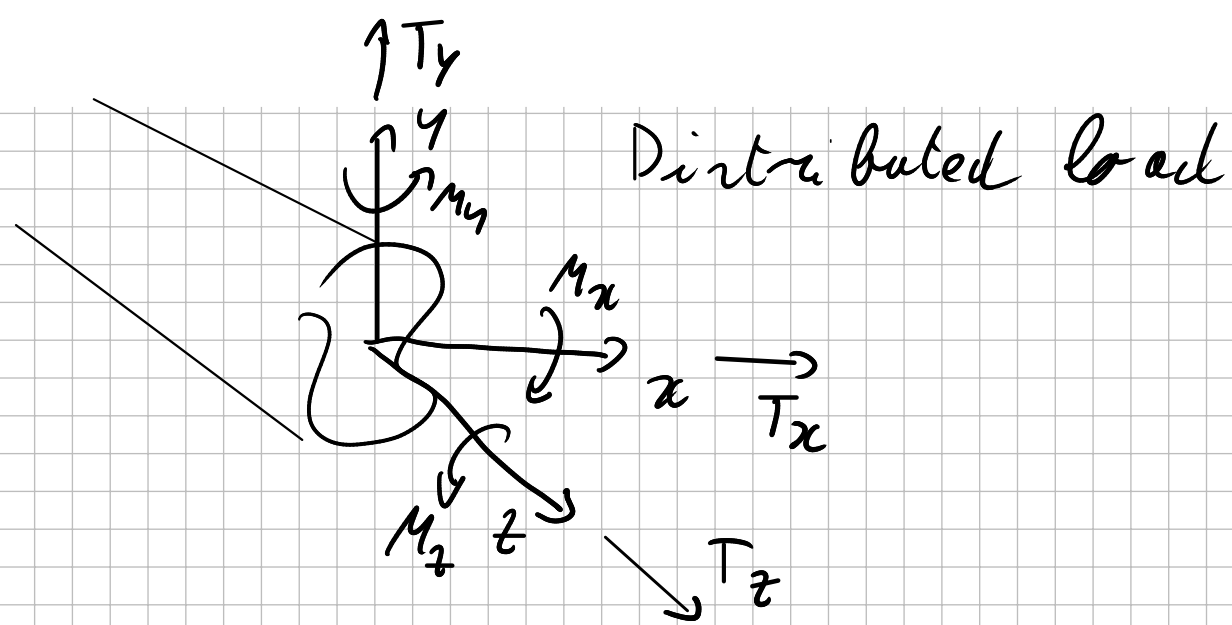
$\color{red}{: \text{2nd order terms} }$
$\color{red}{\text{w.r.t } dz}$
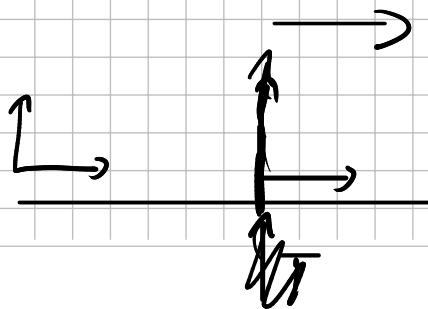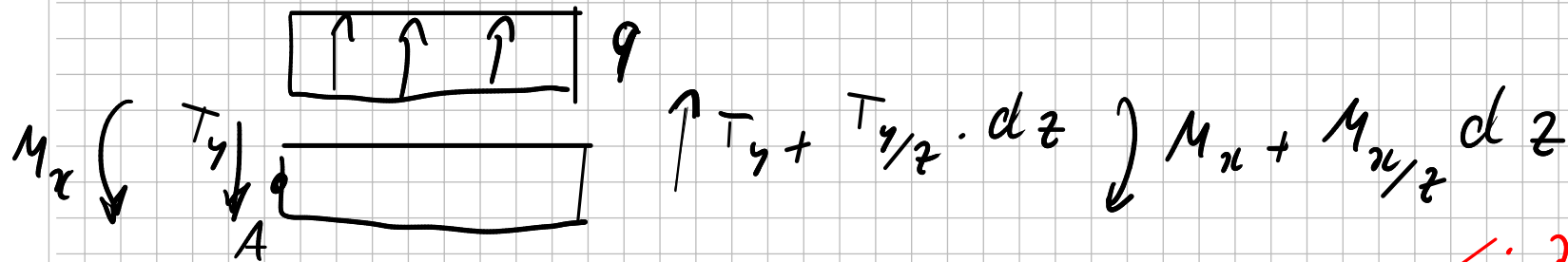
$\cancel{M_x} + M_{x/z}\, dz - \cancel{M_x} - \cancel{(T_y + T_{y/z}\, dz) \cdot dz} - \cancel{q\, dz \dfrac{dz}{2}} = \emptyset$

$$M_{x/z} = T_y \qquad\qquad M_x = \int_0^z T_y \, dz + M_x(\emptyset)$$

Other axis:

M_y ⤵ T_x↓ ↑x ↑T_x →z ⟩ $M_y + M_{y/z} \cdot dz$

y

$T_{x/z} = -q \quad\Rightarrow\quad T_x = \int_0^z -q\, dz + T_x(\emptyset)$

$M_{y/z} = -T_x \quad\Rightarrow\quad M_y = -\int_0^z T_x \, dz + M_y(\emptyset)$
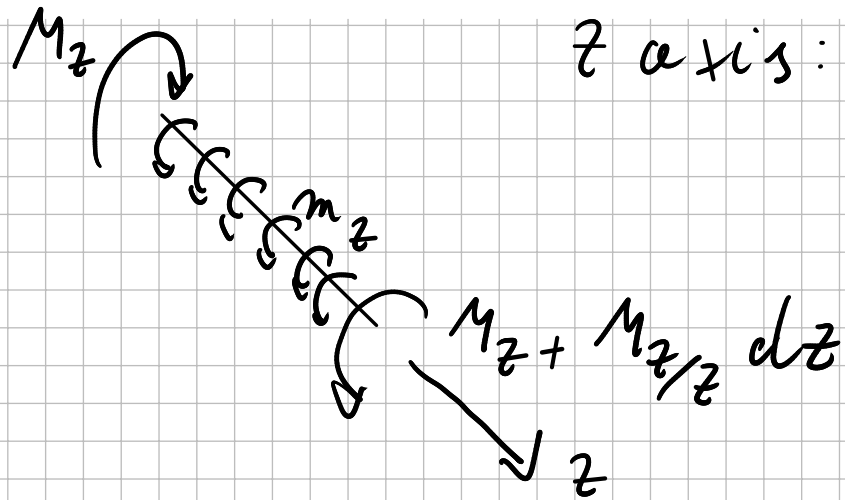
z axis: torsional moment.

$$M_{z,z} = -m_z$$

$$M_z = -\int_0^z m_z \, dz + M_z(\emptyset)$$