Course of Aerospace Structures

Written test, September 5^{th}, 2023

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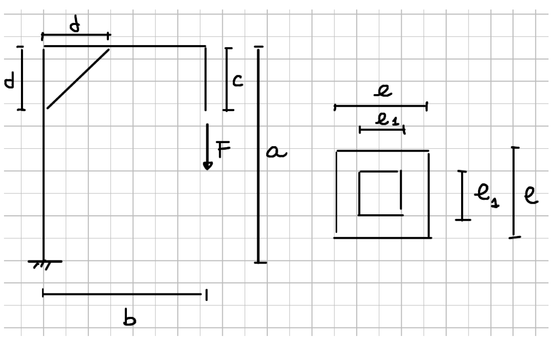
Person code: | | | | | | | | |

Exercise 1

The beam structure sketched in the figure is made with a thin-walled square cross section, and is designed to whitstand a maximum load equal to F. Compute the maximum value of the beam compressive stress at the clamp. Be careful with the measurement units.

(Unit for result: MPa)



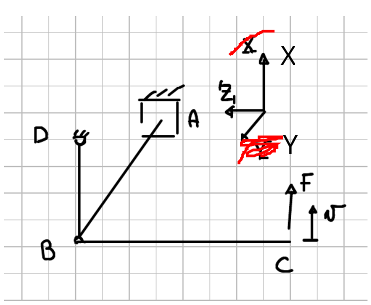
Data
$e = 0.3$ m
$e_1 = 0.29$ m
$a = 3.0$ m
$d = 0.4$ m
$b = 1.0$ m
$c = 0.8$ m
$F = 2300$ N

Answer
____

Exercise 2

The structure sketched in the figure is composed by two beams; the first one has a L shape, and is clamped at one extremity; the second one is hinged at both extremities, i.e. to the ground and to the corner of the first one. Compute the vertical displacement v at the point of application of the force F. Neglect shear deformation. The coordinates of the points are given, with respect to the sketched reference system, in the data. The bending and torsional stiffness of the beams are given with respect to local reference systems, with the z axis aligned with the beam axis.

(Unit for result: mm)



Data
$A : (0; 1000; 0)$ mm
$B : (1000; 1000; 0)$ mm
$C : (1000; 1000; -1000)$ mm
$D : (1000; 1500; 0000)$ mm
$EA = 1 \times 10^5$ N
$EI_{xx} = EI_{yy} = 1 \times 10^{10}$ N mm^2
$GJ = 2 \times 10^{11}$ N mm^2
$F = 1000$ N

Answer
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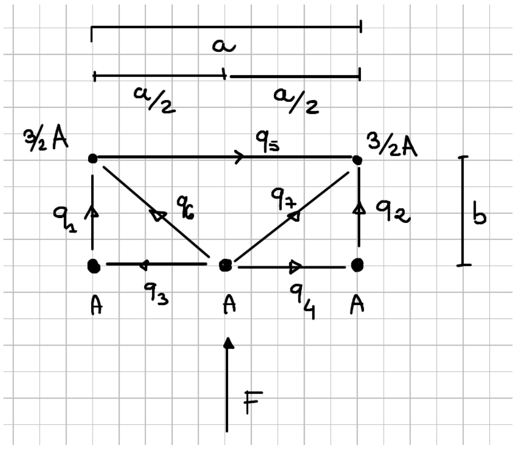
Exercise 3
The semi-monocoque structure sketched in the figure is loaded by the force F. All panels have thickness equal to t. Compute the flux q_1.

(Unit for result: N/mm)

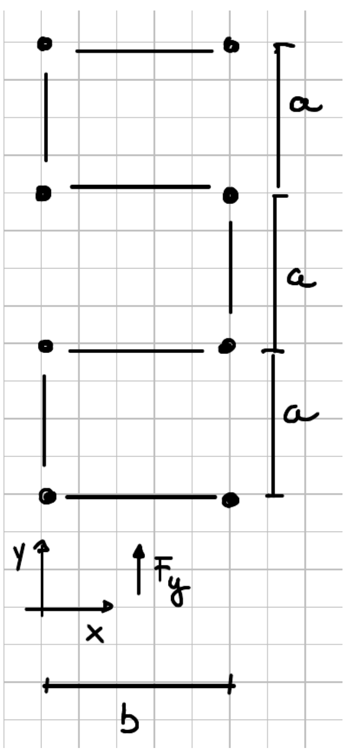


Data
$a = 3000$ mm
$b = 300$ mm
$A = 500$ mm^2
$t = 3$ mm
$F = 30000$ N

Answer

Exercise 4
Consider the semi-monocoque beam open cross section sketched in the figure; all panels have thickness equal to t; all the concentrated areas are equal to A. Compute the x position of the shear center.

(Unit for result: mm)

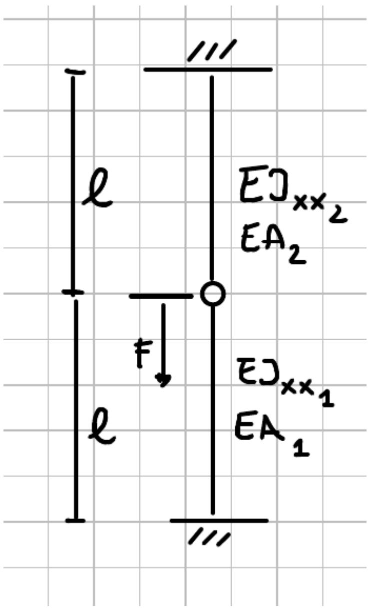


Data
$a = 1000$ mm
$b = 500$ mm
$t = 1$ mm
$A = 300$ mm^2

Answer

Exercise 5
The two beams sketched in the figure are constrained by means of two clamps and one hinge connecting them together. A force F is applied to the hinge in the middle. By resorting to a parabolic approximation for each beam (two different parabolic functions, at least in principle), compute the approximated value of the critical buckling load F.

(Unit for result: N)

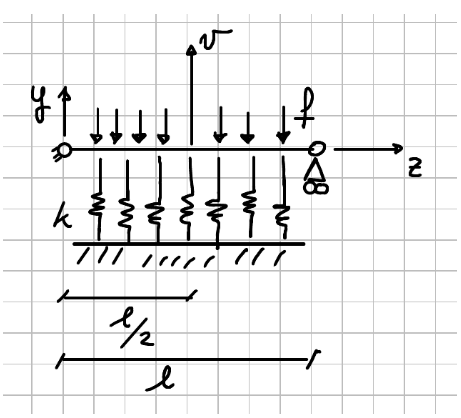


Data
$EA_1 = 3.5 \times 10^7$ N
$EA_2 = 1.4 \times 10^7$ N
$EI_{xx1} = 1.5 \times 10^{11}$ N mm^2
$EI_{xx2} = 1.5 \times 10^{11}$ N mm^2
$l = 3500$ mm

Answer

Exercise 6
The beam sketched in the figure is hinged at both extremities, *and* has a continuous elastic support, with stiffness k, underneath (you see different springs in the figure, but they are meant to represent a contiunuous support, not a set of discrete springs). The beam is loaded by the vertical force per unit of length f. By resorting to a trigonometric approximation of the transverse displacement, truncated to a single term, compute the approximated vertical displacement v in the middle of the beam.

(Unit for result: mm)



Data
$l = 1000$ mm
$EI = 1 \times 10^{12}$ N mm^2
$k = 8$ MPa
$f = 100$ N/mm

Answer

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. The internal forces in a statically determined beam structure depend on the material elastic properties
2. The Timoshenko beam model does not account for transverse shear deformation
3. The position of the shear center of a thin-walled beam depends on the loading conditions

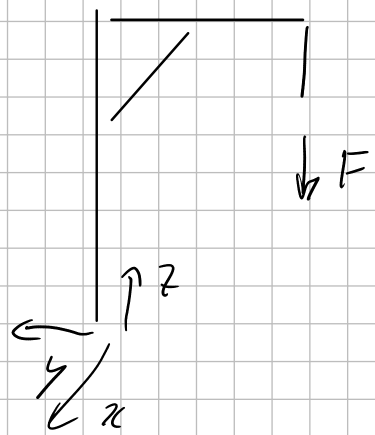
Multiple Choice questions

(Circle the correct answer)

1. The shear flows acting on a rib are:
   (a) the flows equivalent to the load introduced by the rib
   (b) the flows equivalent to the load applied to the whole structure
   (c) the flows computed from the torsional equivalence equation
   (d) the flows computed by resorting to the bending equations
   (e) the flows equilibrating the load introduced by the rib
   (f) none of the above
2. The Principle of Virtual Work:
   (a) is used to impose the equilibrium
   (b) is used to impose the equilibrium and the compatibility
   (c) is used to impose the compatibility
   (d) none of the above
3. A structure is modeled using finite elements. It is unconstrained and subjected to a set of loads in self equilibrium. The solution of the linear static problem:
   (a) is stress-free
   (b) can be obtained, up to a rigid body motion, after constraining the structure isostatically
   (c) can be obtained only if the loads are concentrated
   (d) can be obtained, up to a rigid body motion, only if the loads are distribuited
   (e) is defined up to a rigid body motion; thus, not being unique, ~~can never be obtained stress and displacement fields~~ the stress and displacement fields can never be computed
   (f) none of the above

(1)

$$\sigma_{zz} = \frac{T_z}{A} + \frac{M_x}{I_{xx}} \cdot y$$

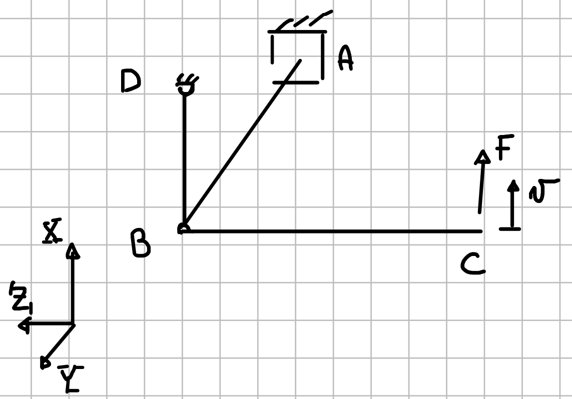


$$I_{xx} = \frac{1}{12}\left(e^4 - e_1^4\right)$$

$$y_{max} = \frac{e}{2}$$

$$\sigma_{zz} = -4{,}4202 \text{ MPa}$$

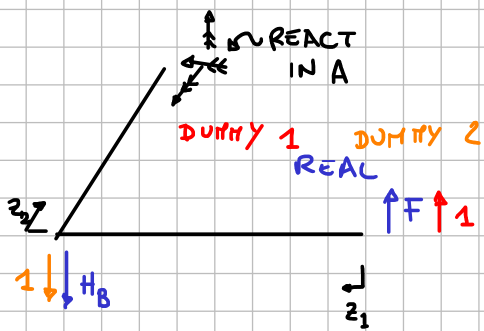
Ex 2



DATA

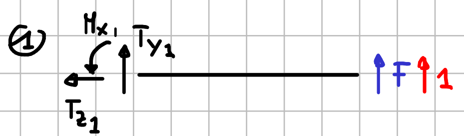
$$A = (0, 1000, 0) \qquad EA = 1 \cdot 10^5\ N$$
$$B = (1000, 1000, 0) \qquad EJ_{xx} = EJ_{yy} = 1 \cdot 10^{10}\ N\,mm^2$$
$$C = (1000, 1000, -1000) \qquad GJ = 2 \cdot 10^{11}\ N\,mm^2$$
$$D = (1000, 1500, 0) \qquad F = 1000\ N$$

SOL



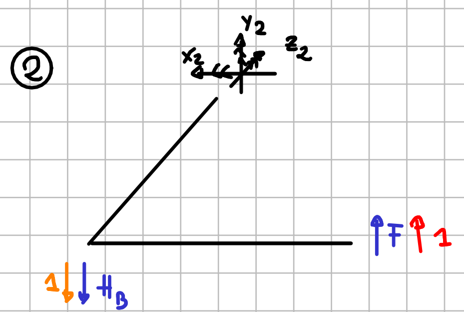
$$l_1 = 1000\ mm = l \qquad l_3 = 500\ mm = \frac{l}{2}$$
$$l_2 = 1000\ mm = l$$

①



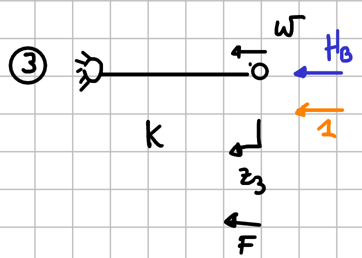
$$T_{z_1} = 0 \qquad T_{z_1}' = 0$$
$$T_{y_1} = -F \qquad T_{y_1}' = -1$$
$$M_{x_1} = -F z_1 \qquad M_{x_1}' = -z_1$$

②



$$T_{y_2} = H_B - F \qquad T_{y_2}' = -1 \qquad T_{y_2}'' = 1$$
$$M_{x_2} = (H_B - F) z_2 \qquad M_{x_2}' = -1 z_2 \qquad M_{x_2}'' = +z_2$$
$$M_{z_2} = F \cdot l \qquad M_{z_2}' = 1 \cdot l$$

③



$$w = \int_0^{l/2} \frac{F}{EA}\, dz_3 = + \frac{F \cdot l/2}{EA} = \frac{F l}{2EA}$$

$$\rightarrow k = \frac{2EA}{l}$$

PCVW REAL DUMMY 1

$$\delta W_{e_1} = 1 \cdot v$$

$$\delta W_{i_1} = \int_0^{l} M_{x_1}' \cdot \frac{M_{x_1}}{EJ_{xx}} \, dz_1 + \int_0^{l} M_{x_2}' \cdot \frac{M_{x_2}}{EJ_{xx}} \, dz_2 + \int_0^{l} M_{z_2}' \cdot \frac{M_{z_2}}{GJ} \, dz_2$$

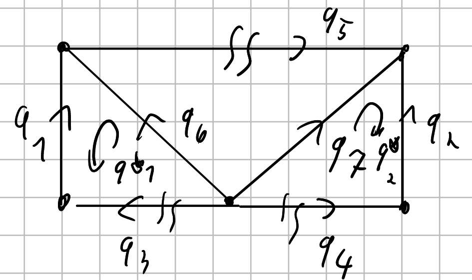
PCVW REAL DUMMY 2

$$\delta W_{e_2} = -1 \, \frac{H_B \, l}{2EA}$$

$$\delta W_{i_2} = \int_0^{l} M_{x_2}'' \, \frac{M_{x_2}}{EJ_{xx}} \, dz_2$$

$$\begin{cases} \delta W_{e_1} = \delta W_{i_1} \\ \delta W_{e_2} = \delta W_{i_2} \end{cases} \rightarrow v, H_B \qquad \boxed{v = 42.6812 \text{ mm}}$$

(3)



Symm:

$q_1 = q_2$

$q_3 = q_4$

$q_6 = q_7$

$q_5 = 0$

$b(2q_1' + 2q_6') = F$

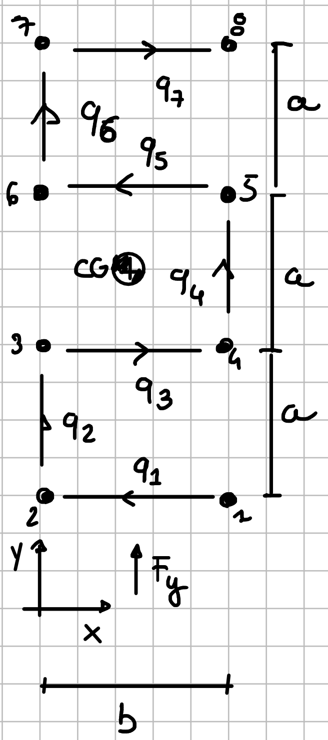
$l_6 = \sqrt{a^2/4 + b^2}$

$\theta' = 0$

$q_1^0 = (-q_6' l_6 + q_1' b) / (b + \frac{a}{2} + l_6)$

$q_1 = q_1' - q_1^0 = 37,987$ N/mm

Ex 4



DATA

$a = 1000$ mm $\qquad t = 1$ mm

$b = 500$ mm $\qquad A = 300$ mm^2

SOL

$$J_{xx} = 4A\left(\frac{a}{2}\right)^2 + 4A\left(\frac{3a}{2}\right)^2$$

$$S_{x_{12}} = -A\frac{3}{2}a \qquad S_{x_{78}} = A\frac{3}{2}a$$

$$S_{x_{34}} = -A\frac{1}{2}a \qquad S_{x_{56}} = A\frac{1}{2}a$$

$$q_1 = -F\cdot\frac{S_{x_1}}{J_{xx}} = +q_7$$

$$q_2 = q_1 - F\frac{S_{x_2}}{J_{xx}} = q_6$$

$$q_3 = q_2 - F\frac{S_{x_3}}{J_{xx}} = q_5$$

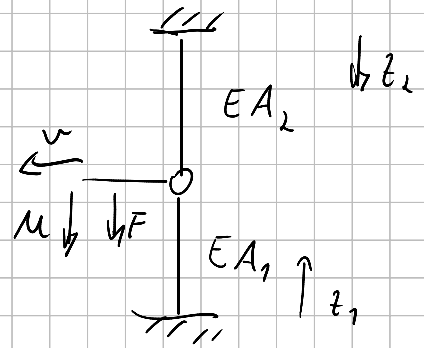
$$q_4 = q_3 - F\frac{S_{x_4}}{J_{xx}}$$

ROT ② $\curvearrowleft +$

$$F\cdot x_{sc} = -q_3\cdot 2\left(\frac{ab}{2}\right) + q_4 2\left(\frac{ab}{2}\right) + q_5 2\left(\frac{b\cdot 2a}{2}\right) - q_7 2\left(\frac{b\cdot 3a}{2}\right)$$

$$\boxed{x_{sc} = 150 \text{ mm}}$$

5



$$u = \frac{F l}{EA_1 + EA_2}$$

$$N_1 = - u \, EA_1 = \frac{EA_1 l}{EA_1 + EA_2} \cdot F = d_1 \cdot F$$

$$N_2 = u \, EA_2 = \frac{EA_2 l}{EA_1 + EA_2} \cdot F = d_2 \cdot F$$

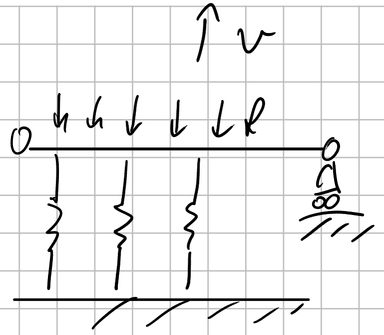
$$v_1 = a_1 z_1^2 \qquad v_2 = a_2 z_2^2$$

$$v_1(l) = v_2(l) \Rightarrow \quad a_1 = a_2 = a$$

$$\int_0^l \delta v_1'' \, EI_1 \, v_1'' + \delta v_1' \, N_1 \, v_1' \, dz_1 + \int_0^l \delta v_2'' \, EI_2 \, v_2'' + \delta v_2' \, N_2 \, v_2' \, dz_2 = 0$$

$$F = -8 \, l \, E I_{11} \Big/ \left((d_1 + d_2) \cdot \frac{4}{3} l^3 \right) = 1{,}7143 E5 \text{ N}$$

6



$$v = a \sin\left(\frac{\pi z}{l}\right)$$

$$v_{/z} = \frac{\pi}{l}\, a \cos\left(\frac{\pi z}{l}\right)$$

$$v_{/zz} = -\frac{\pi^2}{l^2}\, a \sin\left(\frac{\pi z}{l}\right)$$

$$\int_0^l \left(\delta v''\, EI\, v'' + \delta v\, k\, v\right) dz = \int_0^l \delta v\, (-p)\, dz$$

integrate e solve for a

$$a = \frac{-2pl}{\pi} \cdot \frac{1}{\frac{1}{2}\pi^4/l^3 \cdot EI + \frac{1}{2} kl}$$

$$v = a = -1{,}2079 \text{ mm}$$

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. The internal forces in a statically determined beam structure depend on the material elastic properties
   - False
2. The Timoshenko beam model does not account for transverse shear deformation
   - False
3. The position of the shear center of a thin-walled beam depends on the loading conditions
   - False

Multiple Choice questions

(Circle the correct answer)

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