Course of Aerospace Structures

Written test, July 8^{th}, 2024

Name ________________________________

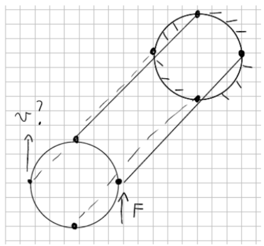
Surname ________________________________

Person code: | | | | | | | | |

Exercise 1

The semi-monocoque structure in the figure has a circular cross-section, four stringers with concentrated area equal to A and four panels with thickness t. The structure is clamped at one extremity and loaded by the vertical force F, applied to the rightmost stringer, at the other end. Compute the vertical displacement v of the leftmost stringer at the loaded extremity, as sketched.

(Unit for result: mm)



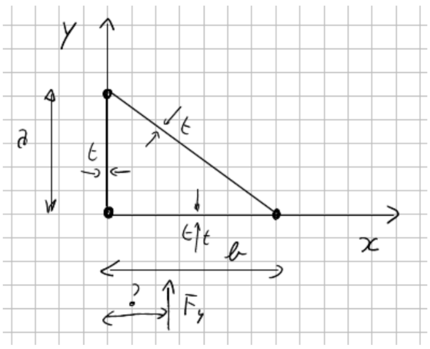
Data
$r = 100$ mm
$l = 2000$ mm
$t = 0.3$ mm
$E = 72000$ MPa
$\nu = 0.3$
$F = 1000$ N
$A = 400$ mm^2

Answer ________________________________

Exercise 2

Consider the semi-monocoque cross section sketched in the figure, with three panels (thickness t) and three stringers (concentrated area A). Compute the x position of the shear center.

(Unit for result: mm)

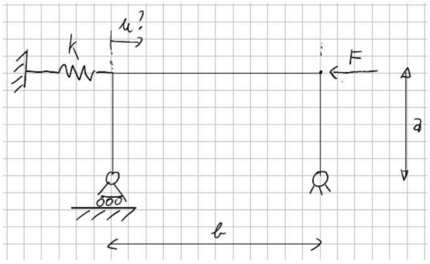


Data
$a = 100$ mm
$b = 150$ mm
$t = 1$ mm
$A = 100$ mm^2
$E = 72000$ MPa
$\nu = 0.3$

Answer ________________________________

Exercise 3
Consider the slender beam structure sketched in the figure, loaded by the concentrated force F. Compute the displacement u of the extremity of the spring. Neglect shear deformability.

(Unit for result: mm)

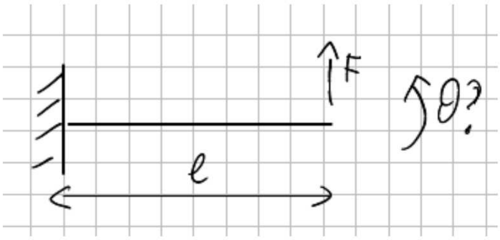


Answer

Data
$a = 1000$ mm
$b = 1500$ mm
$EA = 1 \times 10^{10}$ N
$EI_{xx} = EI_{yy} = 1 \times 10^{13}$ N mm^2
$GJ = 7 \times 10^{9}$ N mm^2
$K = 1 \times 10^{3}$ N/mm
$F = 1000$ N

Exercise 4
Consider the relatively short cantilever beam sketched in the figure, with length l. The beam is loaded by the concentrated vertical force F, as sketched. Compute the rotation θ at the free extremity. *Do account*, whenever required, for shear deformability.

(Unit for result: rad)



Answer

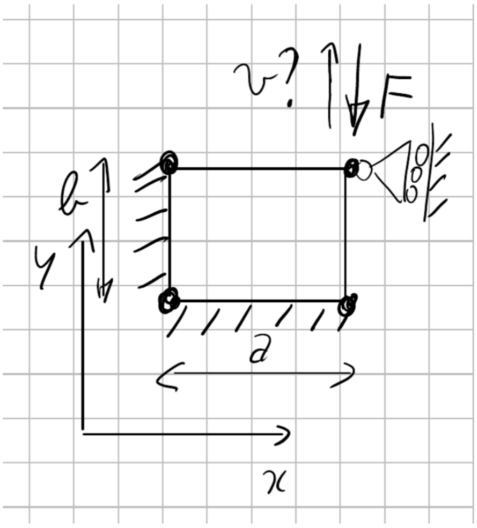
Data
$l = 1000$ mm
$GA^{*} = 1 \times 10^{10}$ N
$EI = 12 \times 10^{9}$ N mm^2
$EA = 1 \times 10^{10}$ N
$F = 1500$ N

Exercise 5
The single bilinear finite element sketched in the figure has the displacement of the top left, bottom left and bottom right nodes completely constrained to zero. The top right node, where the force is applied, can move only in the vertical direction (the horizontal displacement component is constrained to zero). The element has unit thickness, and the material works in a state of plane stress, so that

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \tag{1}$$

The top right node is loaded by the vertical force F, as sketched. Compute the vertical displacement v of the top right node.

(Unit for result: mm)

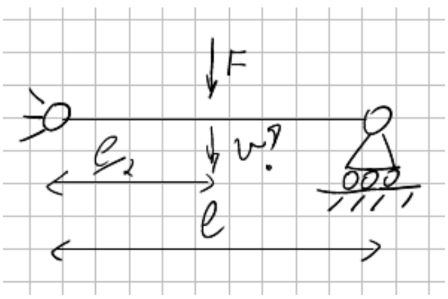


Data
$t = 1$ mm
$a = 4$ mm
$b = 3$ mm
$E = 72000$ MPa
$\nu = 0.3$
$F = 100$ N

Answer

Exercise 6
The slender beam sketched in the figure has length l. It is loaded in the middle by the vertical force F. Resorting to a quadratic polynomial approximation of the transverse displacement compute the vertical displacement v at the point of application of the force. Neglect shear deformability..

(Unit for result: mm)



Data
$l = 2000$ mm
$EI_{xx} = EI_{yy} = 1{\times}10^{12}$ N mm^2
$F = 200$ N

Answer

True/False Questions
(Put a T (true) or F (false) at the end of the sentence)

1. Linear finite elements have linear convergence of the displacement wrt the dimension of the elements.
2. The small strain tensor components are equal to zero for arbitrary rigid body motions.
3. The critical compressive buckling stress of a simply supported plate is inversely proportional to the second power of plate thickness.

Multiple Choice questions
(Circle the correct answer)

An open cross section semi-monocoque beam is clamped...

1. An ~~open semi-monocoque cross section~~ is clamped at one extremity and loaded by a torsional moment; the torsional rotation angle is:
   (a) infinite, because the torsional stiffness is null
   (b) polynomial (linear) because of differential bending
   (c) polynomial (linear) because of differential torsion
   (d) polynomial (quadratic) because of differential bending
   (e) polynomial (quadratic) because of differential torsion
   (f) polynomial (cubic) because of differential bending
   (g) polynomial (cubic) because of differential torsion
   (h) none of the above

2. Isoparamentric elements means that:
   (a) the stress components have the same value all over the element
   (b) the strain components have the same value all over the element
   (c) the displacement components have the same value all over the element
   (d) the strain components are interpolated using the same shape functions used for the undeformed position components
   (e) the displacement components are interpolated using the same shape functions used for the undeformed position components
   (f) none of the above

3. A system of slender beams can be modeled by beams finite elements:
   (a) false
   (b) true only if the structure can sustain the loads through an internal axial load path
   (c) true only if shear deformability is not negligible
   (d) true
   (e) none of the above

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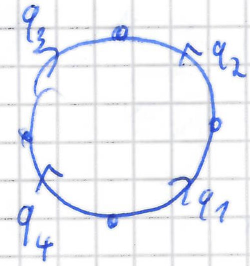


$$q_1 = \frac{F}{4r} + \frac{F\cancel{r}}{2\pi r^{\cancel{2}}}$$

$$q_2 = q_1$$

$$q_3 = \frac{F}{4r} - \frac{F}{2\pi r}$$

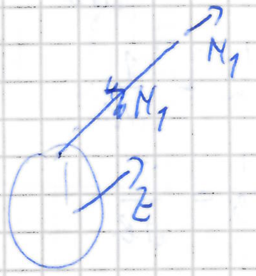
$$q_4 = q_3$$

$$q_1' = \frac{1}{4r} - \frac{1}{2\pi r}$$

$$q_2' = q_1$$

$$q_3' = \frac{1}{4r} + \frac{1}{2\pi r}$$

$$q_4' = q_3'$$



$$N_1 = \frac{-Fz}{2r}$$

$$M_2 = \frac{Fz}{2r}$$

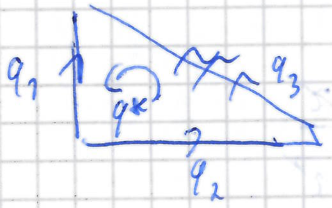
$$M_1' = \frac{-z}{2r}$$

$$M_2' = \frac{z}{2r}$$

$$\frac{\pi}{2} \, l r \left(\frac{q_1 q_1'}{tG} + \frac{q_2 q_2'}{tG} + \frac{q_3 q_3'}{tG} + \frac{q_4 q_4'}{EG} \right) + \int_0^l \frac{M_1 M_1'}{EA} + \frac{M_2 M_2'}{EA} \, dz$$

$$= 1 \cdot v$$

(2)



$$\tilde{q}_1' = \frac{F_y}{a}$$

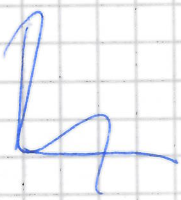
$$F_y \cdot x = ab\, q^*$$

$$q^* = \frac{F_y \cdot x}{ab}$$

$$q_1 = \frac{F_y}{a} - q^* = F_y\left(\frac{1}{a} - \frac{x}{ab}\right)$$

$$q_2 = q^* = \frac{F_y \cdot x}{ab}$$

$$q_3 = \frac{F_y x}{ab}$$



$$q^{*\prime} = \frac{1}{ab} \qquad q_1' = -\frac{1}{ab}$$

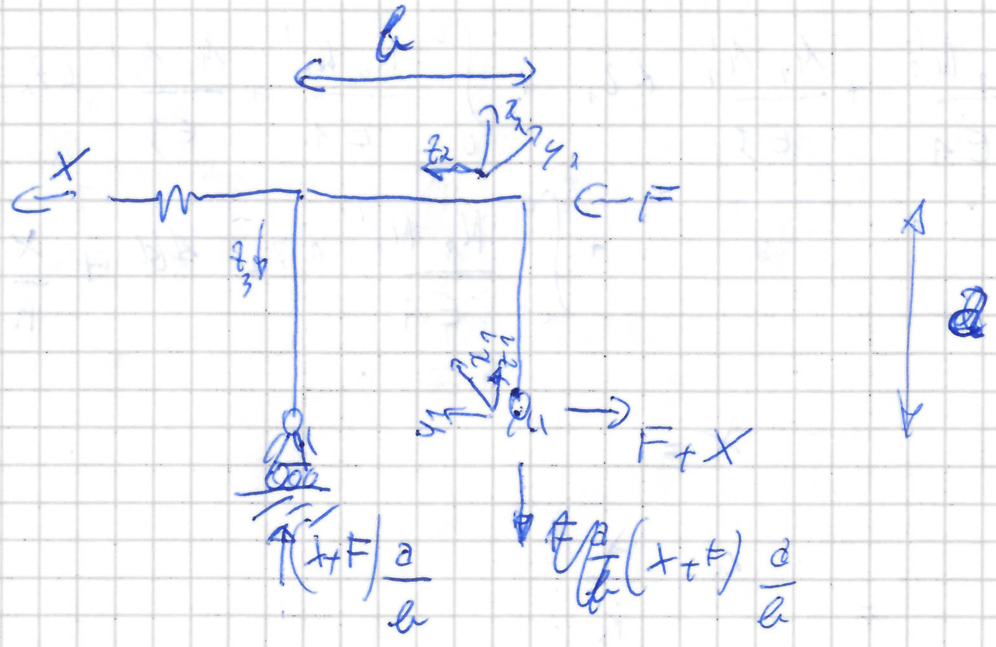
$$q_2' = \frac{1}{ab}$$

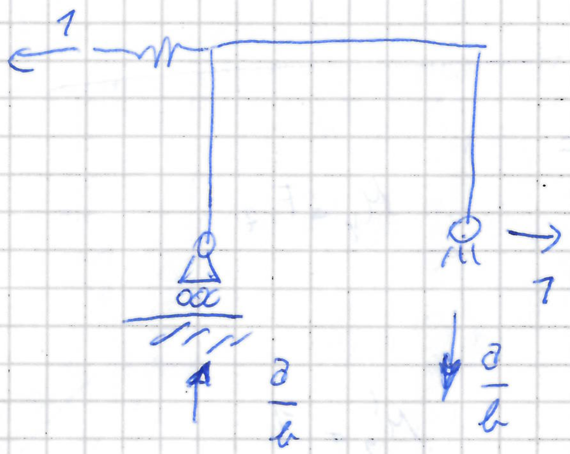
$$q_3' = \frac{1}{ab}$$

$$\frac{q_1 q_1'}{tG} \cdot a + \frac{q_2 q_2'}{tG} b + \frac{q_3 q_3' \sqrt{a^2+b^2}}{tG} = 0$$

$x \dots$

(3)





$$N_1 = (X+F)\frac{a}{b} \qquad N'_1 = \frac{a}{b}$$

$$T_{y_1} = F+X \qquad T'_{y_1} = 1$$

$$M_{z_1} = -(F+X)\cdot z \qquad M'_{z_1} = -z$$

$$T_{z_2} = (X+F)\frac{a}{b} \qquad T'_{z_2} = \frac{a}{b}$$

$$N_{z_2} = X \qquad N'_{z_2} = 1$$

$$M_{y_2} = -(X+F)\frac{a z}{b} + (X+F)\,a \qquad M'_{y_2} = -\frac{a}{b}\cdot z + a$$

$$N_{z_3} = -(X+F)\frac{a}{b} \qquad N'_{z_3} = -\frac{a}{b}$$

$$\int_0^a \left(\frac{M_1 M_1'}{EA} + \frac{M_1 M_1'}{EI} \right) dz_1 + \int_0^b \left(\frac{M_2 M_2'}{EA} + \frac{M_2 M_2'}{EI} \right) dz_2$$

$$+ \int_0^a \frac{M_3 M_3'}{EA} \, dz_3 + \frac{X}{K} = \emptyset$$

$$X = \ldots$$

$$u = \frac{X}{K}$$

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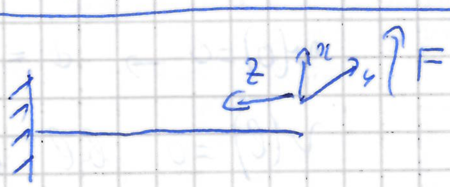
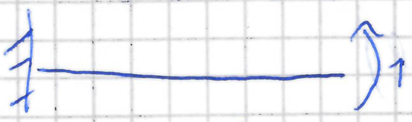
(4)



$$M_y = F \cdot z$$



$$M'_y = 1$$

$$\int_0^l \frac{Fz}{EI} \, dz = \theta$$

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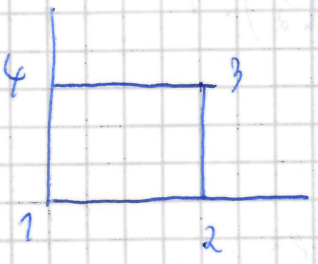
(5)



$$N_3 = \frac{x \cdot y}{12}$$

$$v = \ldots + N_3 v_3 + \ldots$$

$$\varepsilon_{xx} = \emptyset$$

$$\varepsilon_{yy} = \frac{x}{12} v_3 \qquad \gamma_{xy} = \frac{y}{12} v_3$$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} = \underbrace{\begin{bmatrix} \emptyset \\ \frac{x}{12} \\ \frac{y}{12} \end{bmatrix}}_{B} \cdot v_3$$

$$K = \int_0^4 \int_0^3 B^T E B \, dy \, dx$$

$$K \cdot \{v_3\} = \{-F\} \qquad v_3 = K^{-1} \cdot (-F)$$

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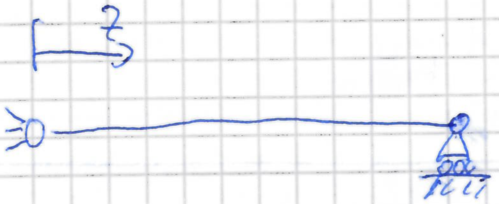
(6)



$$v = a + bz + cz^2$$

$$v(0) = 0 \rightarrow a = 0$$

$$v(l) = 0 \quad bl + cl^2 = 0$$

$$b = -cl$$

$$v = c(z^2 - zl)$$

$$v' = c(2z - l)$$

$$v'' = 2c$$

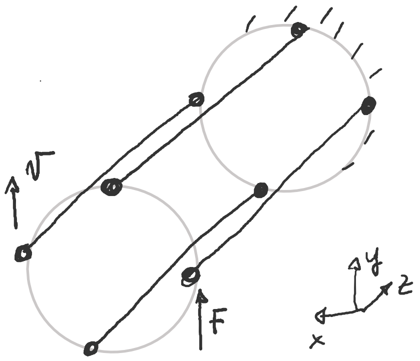
$$\int_0^l \delta v'' \, EI \, v'' \, dz = \delta v\left(\frac{l}{2}\right) \cdot F$$

$$c = \ldots$$

$$v = \left(\frac{l^2}{4} - \frac{l^2}{2}\right) \cdot c$$

2024-07-08 Ex1

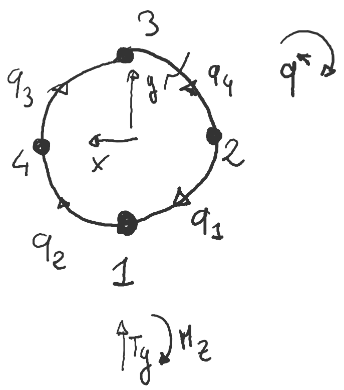
Wednesday, July 3, 2024 10:53 AM



$J_{xx} = 2 A r^2$

$S_{x_3} = -S_{x_1} =$

$= A r$



$q_1' = 0$

$q_2' = -T_y \frac{S_{x_1}}{J_{xx}} = T_y \frac{Ar}{2Ar^2}$

$= \frac{1}{2} \frac{T_y}{r}$

$q_3' = q_2' = \frac{1}{2} T_y / r$

$M_z = 2q^* \cdot \Omega_{CELL} + \left(\cancel{2} q_3' \frac{1}{\cancel{4}} \Omega_{CELL}\right) \cdot \cancel{2}$

$M_z = 2q^* \cdot \pi r^2 + \frac{1}{2} \frac{T_y}{\cancel{r}} \pi r^{\cancel{2}}$

$q^* = \dfrac{M_z - \frac{1}{2} T_y \pi r}{2 \pi r^2}$

<u>REAL</u>

$T_y = -F$

$M_z = F \cdot r$

$M_x = -F \cdot z$

DUMMY

$T_y' = -1$

$M_z' = -1 \cdot r$

$M_x' = -z$

$M_z = 1 \cdot R$ $\quad$ $M'_z = -1 \cdot R$

$M_x = -F \cdot z$ $\quad$ $M'_x = -z$

$$\delta W_e = 1 \cdot v$$

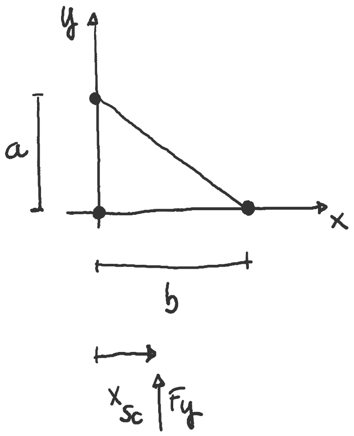
$$\delta W_i = \int_V \delta\tau \cdot \frac{\tau}{G} dV + \int_0^l M_x' \cdot \frac{M_x}{EJ} dz$$

$$= \sum_i \int_0^l \frac{1}{2}\pi R \cdot t \cdot \frac{\delta q_i \cdot q_i}{G t^2} dz + \int_0^l M_x' \cdot \frac{M_x}{EJ} dz$$

$$\boxed{v = 5.1919 \text{ mm}}$$

2024-07-08 Ex 2

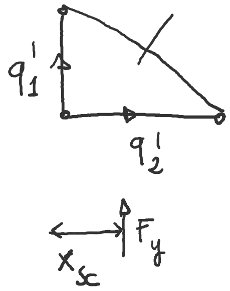
Wednesday, July 3, 2024 11:21 AM



DO NOT CONSIDER REFERENCE SYSTEM

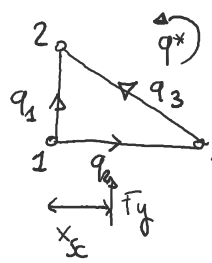
ONLY EQUIVALENCE OF INTERNAL ACTIONS

WHEN PANEL OPEN



$$q_1' = \frac{F}{a}$$

$$q_2' = 0$$



MOM EQUIV. WRT 1) +

$$F_y \cdot x_{sc} = \cancel{2} q^* \cdot \frac{ab}{\cancel{2}}$$

$$q^* = \frac{F_y \cdot x_{sc}}{ab}$$

ROTATION: $\vartheta' = 0$ FOR x_{sc}

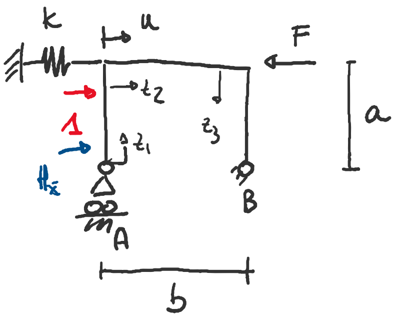
$$\vartheta' = \frac{1}{2Gt \cdot \Omega_{cell}} \cdot \left(-q_1' \cdot a + q_2' b + q^* \left(a + b + \sqrt{a^2 + b^2}\right)\right) = 0$$

WITH $G = \dfrac{E}{2(1+\nu)}$

$$\boxed{x_{sc} = 34.86 \text{ mm}}$$

2024-07-08 Ex 3

Wednesday, July 3, 2024 11:25 AM



Sol

EQUIL.

$X: \quad H_x - F + R_{x_B} = 0$

$Y: \quad R_{Y_A} + R_{Y_B} = 0$

$M_B^{\circlearrowleft}: \quad +F\cdot a - H_X \cdot a - R_{Y_A}\cdot b = 0$

$\boxed{R_{Y_A} = (F - H_x)\frac{a}{b}}$

$1 + R'_{XB} = 0$

$R'_{Y_A} + R'_{Y_B} = 0$

$-1\cdot a - R'_{Y_A}\cdot b = 0$

$\boxed{R'_{Y_A} = -\frac{a}{b}}$

$z_1: \quad T_{z_1} = -(F - H_x)\frac{a}{b}$

$T'_{z_1} = +\frac{a}{b}$

$z_2: \quad T_{z_2} = -H_x$

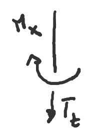
$T'_{z_2} = -1$

$M_{x_2} = -(F - H_x)\frac{a}{b}\cdot z_2$

$M'_{x_2} = +\frac{a}{b} z_2$

$z_3: \quad T_{z_3} = (F - H_x)\frac{a}{b}$

$T'_{z_3} = -\frac{a}{b}$



$M_{x_3} = -H_x\cdot z_3 + F\cdot z_3 - (F - H_x)\frac{a}{\cancel{b}}\cdot \cancel{b}$

$M'_{x_3} = -1\cdot z_3 + \frac{a}{\cancel{b}}\cdot \cancel{b}$

PCVW

$$1\cdot -\frac{H_x}{k} = \int_0^a -\frac{F - H_x}{EA}\cdot\frac{a^2}{b^2}dz_1 +$$

$$+\int_0^b \frac{H_x}{EA} - \frac{F - H_x}{EJ}\cdot\frac{a^2}{b^2}\cdot\frac{z_2^2}{2}dz_2 +$$

$$+\int_0^a \frac{F - H_x}{\ldots}\,a^2 \ldots$$

$$+\int_0 \frac{\quad}{EA} - \frac{\quad}{EJ} \cdot \frac{\quad}{b^2} \cdot z_2 \, dz_2 +$$

$$+\int_0^a -\frac{F-H_x}{EA}\frac{a^2}{b^2}\, dz_3 +$$

$$+\int_0^a \frac{(F-H_x)z_3-(F-H_x)a}{EJ} \cdot (-z_3+a)\, dz_3$$

$$-\frac{H_x}{k} = \frac{H_x a^3}{EAb^2} - \frac{Fa^3}{EAb^2} + \frac{H_x}{EA}\cdot b + \frac{1}{3}\frac{H_x a^2 b^{\cancel{3}}}{EJ\cancel{b^2}} +$$

$$-\frac{Fa^2 b^{\cancel{3}}}{3EJ b^{\cancel{2}}} + \frac{H_x a^3}{EAb^2} - \frac{Fa^3}{EAb^2} +$$

$$+\frac{1}{3}\frac{H_x a^3}{EJ} - \frac{1}{3}\frac{Fa^3}{EJ} - \frac{1}{2}\frac{H_x a^3}{EJ} + \frac{1}{2}\frac{Fa^3}{EJ} +$$

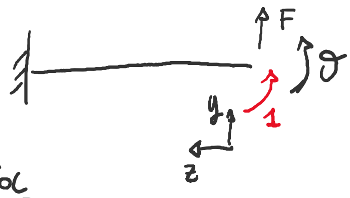
$$+\frac{1}{2}\frac{Fa^3}{EJ} - \frac{1}{2}\frac{H_x a^3}{EJ} - \frac{Fa^3}{EJ} + \frac{H_x a^3}{EJ}$$

SOLVE FOR H_x

$$\longrightarrow \boxed{u = -\frac{H_x}{k} = -7.699 \cdot 10^{-2}\ mm}$$

2024-07-08 Ex 4

Wednesday, July 3, 2024 11:47 AM



Sol

$M_x = -F \cdot z \qquad M_x' = -1$

$T_y = -F \qquad T_y' = 0$

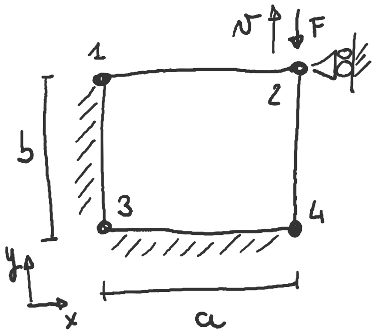
$$1 \cdot \vartheta = \int_0^{\ell} \frac{Fz}{EJ}\, dz$$

$$\vartheta = \frac{1}{2}\frac{F\ell^2}{EJ}$$

$$\boxed{\vartheta = 0.0625 \quad \text{rad}}$$

2024-07-08 Ex 5

Wednesday, July 3, 2024 11:54 AM



Sol

$u_1, u_3, u_4, u_2 = 0$

$v_1, v_3, v_4 = 0$

$$v(x,y) = \frac{xy}{ab} \cdot v_2 = N(x,y)\, q$$

$$\underline{\varepsilon} = \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \gamma_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \frac{x}{ab} \\ \frac{y}{ab} \end{Bmatrix} v_2 = \underline{B}(x,y)\, q$$

$$\delta W_e = \delta q^T \cdot -F = -\delta v_2 \cdot F$$

$$\delta W_i = \int_A \delta \underline{\varepsilon}^T \cdot \underline{\sigma}\, dA$$

$$= \int_b \int_a \delta q^T \cdot \underbrace{\underline{B}^T}_{1\times 3} \underbrace{\underline{\underline{D}}}_{3\times 3} \cdot \underbrace{\underline{B}}_{3\times 1} \cdot q \, dx\, dy$$

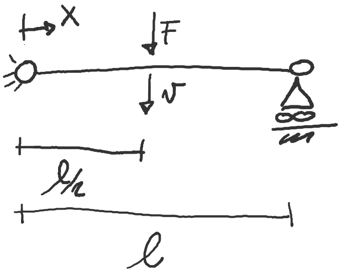
WHERE $\underline{\underline{D}} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{bmatrix}$

SOLVE PVW FOR v_2:

$$\boxed{v_2 = -2.3760 \cdot 10^{-3}\ \mathrm{mm}}$$

2024-07-08 Ex 6

Wednesday, July 3, 2024 2:29 PM



$$v(x) = Ax^2 + Bx + C$$

bc: $v(0) = 0 \rightarrow C = 0$

$v(\ell) = 0 \rightarrow B = -A\ell$ $\Big\}\ v(x) = A(x^2 - \ell x)$

$$\delta v = \delta A\,(x^2 - \ell x)$$

$$v'' = 2A$$

$$\delta v'' = 2\,\delta A$$

PVW

$$\delta w_e = F \cdot \delta v\left(\frac{\ell}{2}\right)$$

$$\delta w_i = \int_0^\ell \delta v''\, EJ\, v''\, dx$$

$$\cancel{\delta A} \cdot F\left(\frac{\ell^2}{4} - \frac{\ell^2}{2}\right) = \cancel{\delta A} \int_0^\ell 4EJ\, A\, dx$$

$$A = \frac{F\left(\ell^2/4 - \ell^2/2\right)}{\int_0^\ell 4EJ\,dx}$$

$$A = -\frac{F\ell}{16EJ}$$

$$v\left(\frac{\ell}{2}\right) = -\frac{F\ell}{16EJ} \cdot \left(\frac{\ell^2}{4} - \frac{\ell^2}{2}\right)$$

$$\boxed{v\left(\frac{\ell}{2}\right) = \frac{F\ell^3}{64EJ} = 0.025\ \text{mm}}$$

True/False Questions

(Put a T (true) or F (false) at the end of the sentence)

1. Linear finite elements have linear convergence of the displacement wrt the dimension of the elements.
    - False
2. The small strain tensor components are equal to zero for arbitrary rigid body motions.
    - False
3. The critical compressive buckling stress of a simply supported plate is inversely proportional to the second power of plate thickness.
    - False

Multiple Choice questions

(Circle the correct answer)

1. An open cross section semi-monocoque beam is clamped at one extremity and loaded by a torsional moment; the torsional rotation angle is:
    (a) infinite, because the torsional stiffness is null
    (b) polynomial (linear) because of differential bending
    (c) polynomial (linear) because of differential torsion
    (d) polynomial (quadratic) because of differential bending
    (e) polynomial (quadratic) because of differential torsion
    (f) polynomial (cubic) because of differential bending
    (g) polynomial (cubic) because of differential torsion
    (h) none of the above
2. Isoparamentric elements means that:
    (a) the stress components have the same value all over the element
    (b) the strain components have the same value all over the element
    (c) the displacement components have the same value all over the element
    (d) the strain components are interpolated using the same shape functions used for the undeformed position components
    (e) the displacement components are interpolated using the same shape functions used for the undeformed position components
    (f) none of the above
3. A system of slender beams can be modeled by beams finite elements:
    (a) false
    (b) true only if the structure can sustain the loads through an internal axial load path
    (c) true only if shear deformability is not negligible
    (d) true
    (e) none of the above