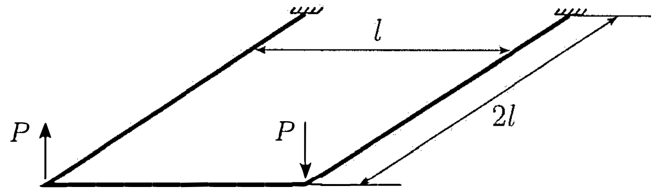
Course of Spacecraft Structures

Written test, July 10^{th}, 2017

Exercise 1

Consider the structure in the figure. The three beams are characterized by axial stiffness EA, bending stiffness EJ and torsional stiffness GJ. The two ends are fixed, while two forces of intensity P are applied as illustrated in the sketch. Determine the reaction forces at the two fixed ends and plot the internal actions.



Data

$EA = 1.8e6\ N$

$EJ_{xx} = EJ_{yy} = EJ = 3.75e6\ N\,mm^2$

$GJ = 3.00e6\ N\,mm^2$

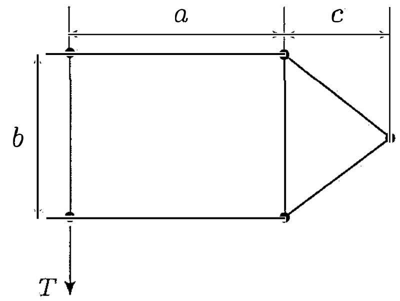
$l = 1000\ mm$

$P = 1.5\ kN$

Exercise 2

A thin-walled beam section is modeled according to the semi-monocoque scheme, as reported in the figure. All the panels have thickness equal to 1 mm, while the lumped area of the stringers, viz. including the contribution of the panels, is equal to A; the dimensions a, b and c are reported below. An internal shear force T is considered as reported in the figure.

Determine: 1. the state of internal shear stress; 2. the position of the shear center of the section.



Data

$A = 200\ mm^2$

$a = 300\ mm$

$b = 200\ mm$

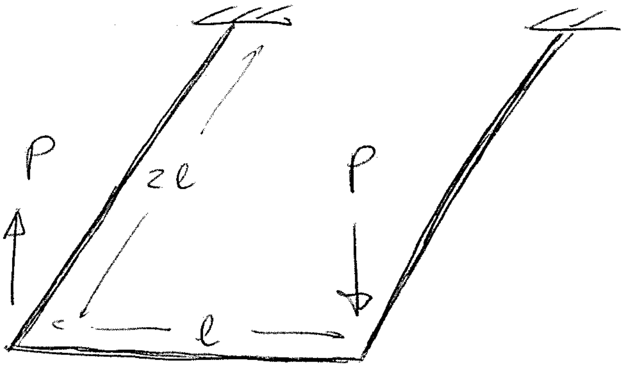
$c = 100\ mm$

$T = 3000\ N$

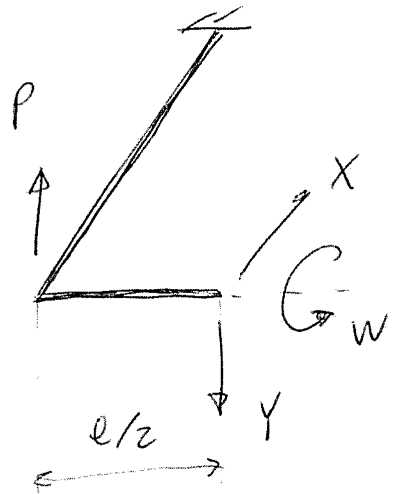
Question 1

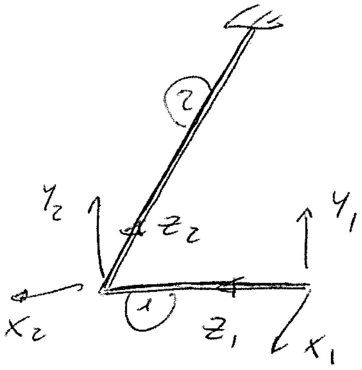
Discuss and illustrate mathematically the relation between the Principle of Complementary Virtual Works (PCVW) and the compatibility conditions.

Exercise 1



Anti-symmetry conditions can be used for simplifying the solution procedure:





Real system

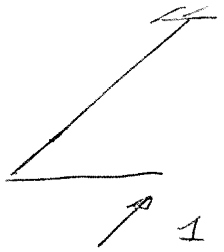
$$M_x^{(1)} = Y z_1$$
$$M_y^{(1)} = -X z_1$$
$$M_z^{(1)} = W$$

Beam 1

$$M_x^{(2)} = W + (Y - P) z$$
$$M_y^{(2)} = -X\, l/2$$
$$M_z^{(2)} = -Y\, l/2$$
$$T_z^{(2)} = -X$$

Beam 2

Dummy #1

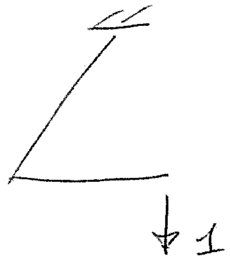


$${}^1\delta M_y^{(1)} = -z_1$$

$${}^1\delta M_y^{(2)} = -\ell/2$$

$${}^1\delta T_z^{(1)} = -1$$

Dummy #2

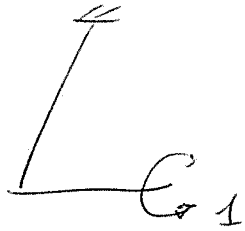


$${}^2\delta M_x^{(1)} = z_1$$

$${}^2\delta M_x^{(2)} = z_2$$

$${}^2\delta M_z^{(2)} = -\ell/2$$

Dummy #3



$${}^3\delta M_z^{(1)} = 1$$

$${}^3\delta M_x^{(2)} = 1$$

PCVW

$$\begin{cases} X = 0 \\ \left(\dfrac{65\ell^3}{24EJ} + \dfrac{\ell^3}{2GJ}\right) Y + \dfrac{2\ell^2}{EJ} W = \dfrac{8}{3}\dfrac{P\ell^3}{EJ} \\ \dfrac{2\ell^2}{EJ} Y + \left(\dfrac{\ell}{2GJ} + \dfrac{2\ell}{EJ}\right) W = \dfrac{2P\ell^2}{EJ} \end{cases}$$

From which

$Y = 947.36$ N

$W = 421052$ Nmm

The reaction forces are obtained by evaluating the internal actions of beam #2 at $z_2 = 2l$

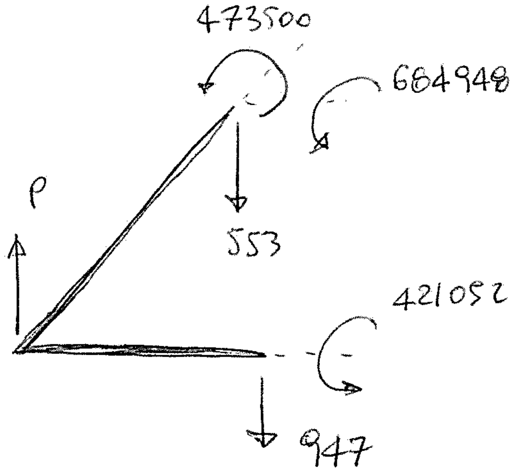
$$M_x^{(2)}(2l) = W + (Y-P)2l = -684948 \text{ Nmm}$$

$$M_y^{(2)}(2l) = -Xl/2 = 0 \text{ Nmm}$$

$$M_z^{(2)}(2l) = -Yl/2 = -473500 \text{ Nmm}$$

$$T_y^{(2)}(2) = Y - P = -553 \text{ N}$$

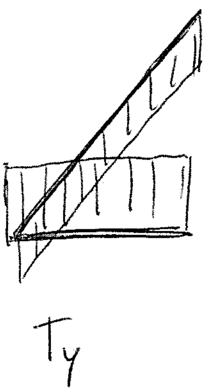
The free body diagram is then:



(Forces are N

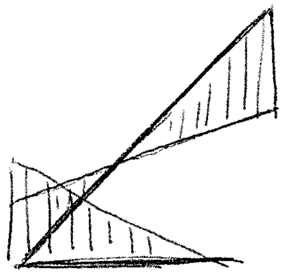
Moments are Nmm)

Plot of internal actions



$T_y^{(1)} = 947$ N

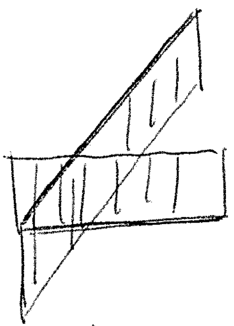
$T_y^{(2)} = -553$ N



M_x

$M_x^{①} = 947\, z_1$ Nmm

$M_x^{②} = 421052 - 553\, z_2$ Nmm

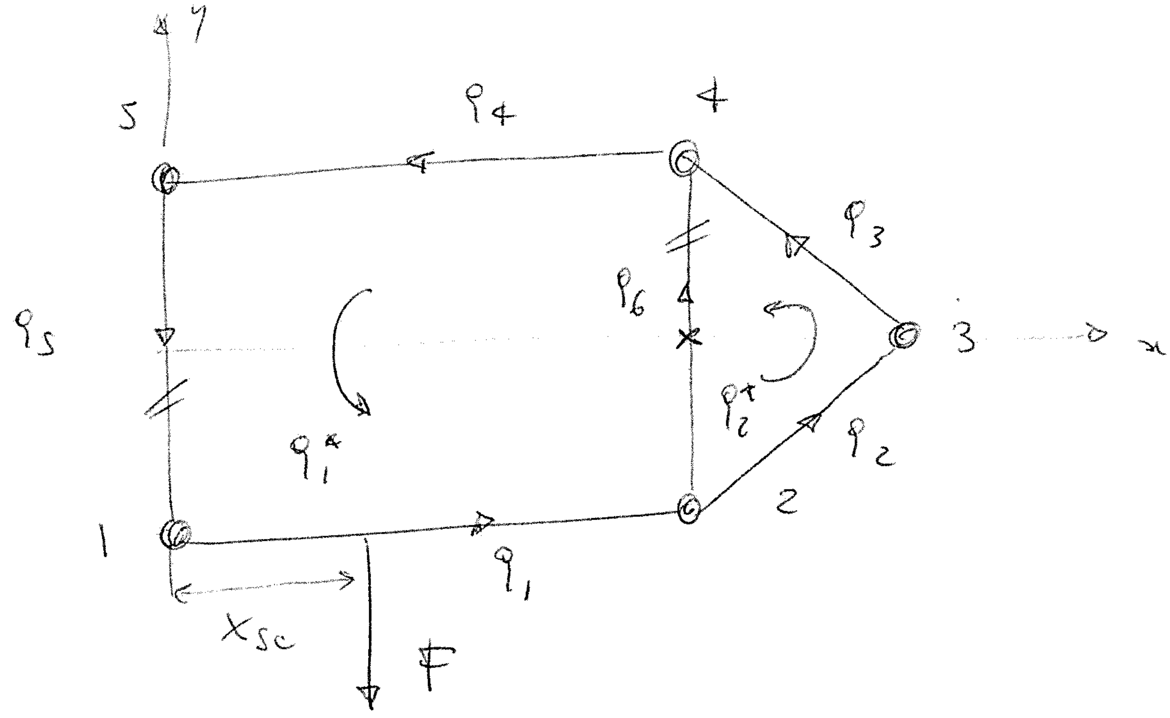


M_z

$M_z^{①} = 421052$ Nmm

$M_z^{②} = -473500$ Nmm

Exercise 2



Solution procedure

1. Evaluate x_{sc}
2. Determine the shear flows by setting $x_{sc} = 0$

Section properties

$J_{xx} = Ab^2$

$S_{x_1} = -Ab/2$ $\quad$ $S_{x_3} = -Ab$

$S_{x_2} = -Ab$ $\quad$ $S_{x_4} = -Ab/2$

~~Shear flows~~ Shear center position

$q_1' = -\dfrac{F}{2b}$ $\quad$ $q_2' = -F/b$ $\quad$ $q_3' = -F/b$

$q_4' = -F/2b$

- Equivalence wrt to x

$$2ab q_1^* + bc q_2^* - Fe = F a/2 + Fc$$

where $e = a - x_{sc}$

- Compatibility $(\theta_1' = \theta_2')$

$$\left[2(a+b) + \bar{\Omega} b\right] q_1^* - \left[b + \bar{\Omega}(2d+b)\right] q_2^* = F\left(\frac{a}{b} - 2\bar{\Omega}\frac{d}{b}\right)$$

where $\bar{\Omega} = \dfrac{\Omega_{cell\,1}}{\Omega_{cell\,2}}$

- Compatibility $(\theta_1' = 0)$

$$2 q_1^* (a+b) - q_2^* b = F a/b$$

The solving equations are (dividing by $1 \cdot 10^5$)

$$\begin{cases} 1.2\, q_1^* + 0.2\, q_2^* - 0.03\, e = 7.5 \\ 0.022\, q_1^* - 0.031\, q_2^* = -0.2096 \\ 0.01\, q_1^* - 0.002\, q_2^* = 0.045 \end{cases}$$

From which

$e = 100.32$ mm

So

$$\boxed{x_{sc} = 199.68 \text{ mm}}$$

From symmetry : $y_{sc} = 0$

Shear flows

The solving equations are:

$$\begin{cases} 2ab\, q_1^* + bc\, q_2^* = \frac{3}{2} Fa + Fc \\ \left[2(a+b) + \bar{\Omega} b\right] q_1^* - \left[b + \bar{\Omega}(2d+b)\right] q_2^* = F\left(\frac{a}{b} - 2\bar{\Omega}\frac{d}{b}\right) \end{cases}$$

From which:

$q_1^* = 11.29$ Nmm

$q_2^* = 14.78$ Nmm

And so:

$q_1 = q_1' + q_1^* = 3.79$ Nmm

$q_2 = q_2' + q_2^* = -0.22$ Nmm

$q_3 = q_3' + q_2^* = -0.22$ Nmm

$q_4 = q_4' + q_1^* = 3.79$ Nmm

$q_5 = q_1^* = 11.29$ Nmm

$q_6 = q_1^* - q_2^* = -3.49$ Nmm