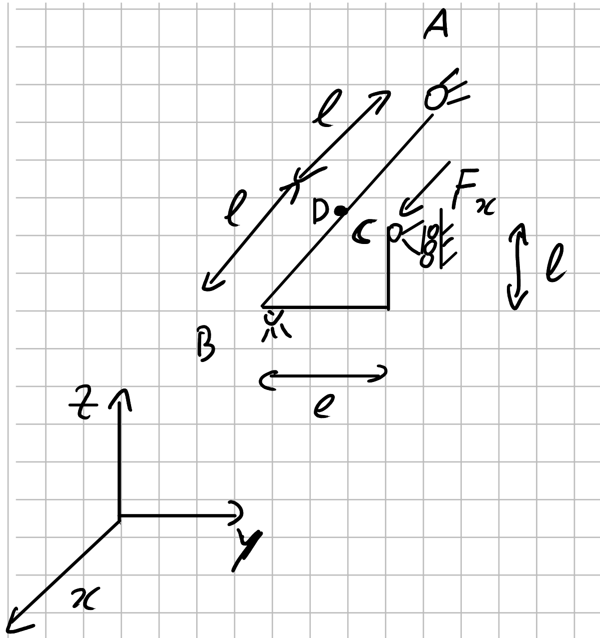


The hinges at points A an B constraint the three components of displacement, u, v and w. The cart at point B constraints only the displacement in the y direction, v. Compute the displacement in the z direction of point D.

Data: $EA = 7{,}2\,E7$ N mm^2

$EJ_{xx} = EJ_{yy} = 6\,E11$ N mm^2

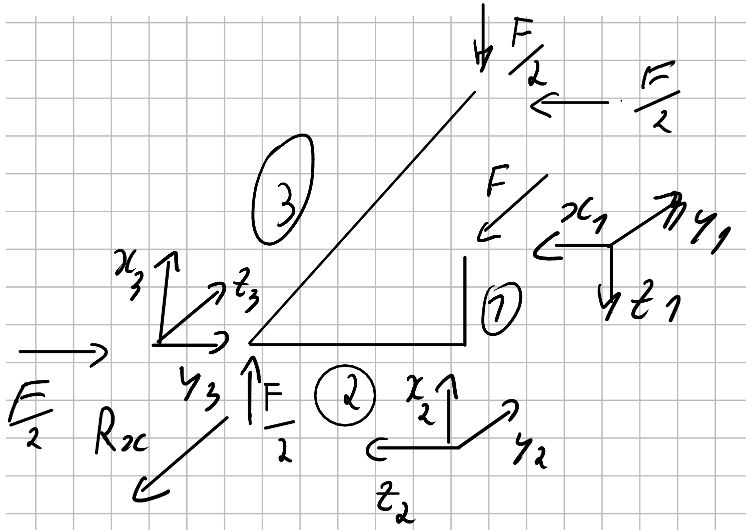
$GJ = 2\,EJ_{xx}$

Beam cross-section stiffness in a local reference system.

$l = 1200$ mm

$F = 2500$ N

Unit for result: mm



① : $M_x = Fz$

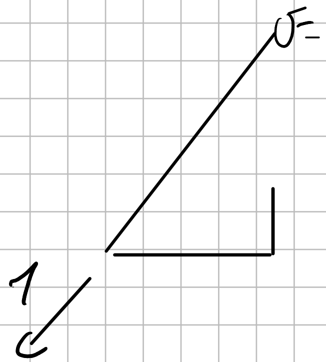
② : $M_z = Fl$

$M_x = -Fz$

③ : $N_z = R_x + F$

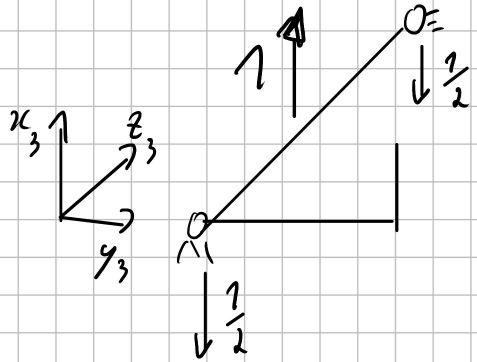
$M_x = +Fl - \frac{F}{2} z_3$

$M_y = -Fl + \frac{F}{2} z_3$



$$3: \quad \delta N_z = 1$$

$$\int_0^{2\ell} \frac{\delta N_z \, (R_x + F)}{EA} \, dz_3 = 0 \quad \Rightarrow \quad R_x = -F$$

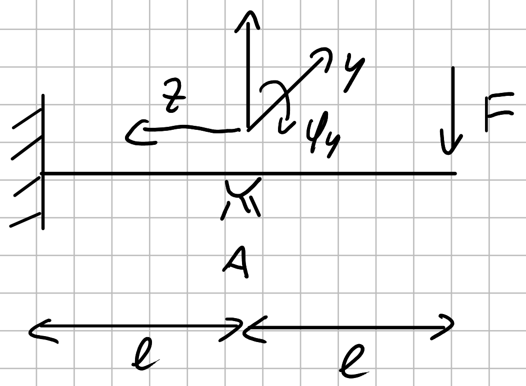


$$3: \delta M_y = -\frac{1}{2} z_3 \qquad 0 < z_3 < \ell$$

$$= -\frac{1}{2} z_3 + z_3 - \ell = \frac{1}{2} z_3 - \ell$$

$$\int_0^{l} \frac{\delta M_y \; M_y}{E I_{yy}} \, dz_3 + \int_{l}^{2l} \frac{\delta M_y \; M_y}{E I_{yy}} \, dz_3 =$$

$$= \frac{1}{E I_{yy}} F l^3 \left(\frac{1}{6} + \frac{1}{12} \right) = \frac{F l^3}{E I_{yy}} \cdot \frac{1}{4} = 1{,}8 \text{ mm}$$



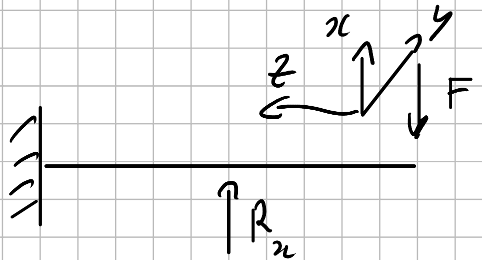
Compute the rotation φ_y of the beam at point A

Data: $l = 4\,000$ mm

$E\bar{J}_{yy} = 6\,E11$ N mm^2

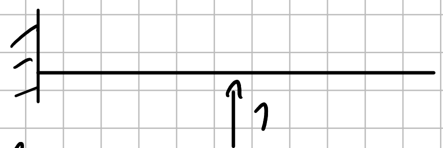
$F = 8000$ N

Unit for result: rad



$M_y = -Fz \qquad 0 < z < l$

$M_y = -Fz + R_x(z-l) \qquad l < z < 2l$



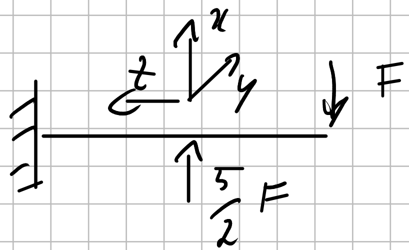
$$\delta M_y = \begin{cases} 0 & 0 < z < l \\ (z-l) & l < z < 2l \end{cases}$$

$$\frac{1}{E\bar{J}_{yy}} \int_{l}^{2l} \left[-Fz^2 + Fzl + R_x\left(z^2 + l^2 - 2lz\right)\right] dz =$$

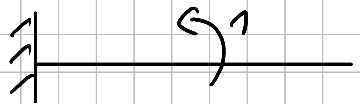
$$= \frac{1}{EJ_{yy}} \left(-\frac{1}{3} F z^3 + \frac{1}{2} F \ell z^2 + \frac{1}{3} R_x z^3 + R_x \ell^2 z - R \ell z^2 \right) \Big|_{\ell}^{2\ell}$$

$$= \frac{1}{EJ_{yy}} \left[F \ell^3 \left(-\frac{8}{3} + 2 + \frac{1}{3} - \frac{1}{2} \right) + R_x \ell^3 \left(\frac{8}{3} + 2 - 4 - \frac{1}{3} - 1 + 1 \right) \right] = 0$$

$$-\frac{5}{6} F + \frac{1}{3} R_x = 0 \qquad R_x = \frac{5}{2} F$$



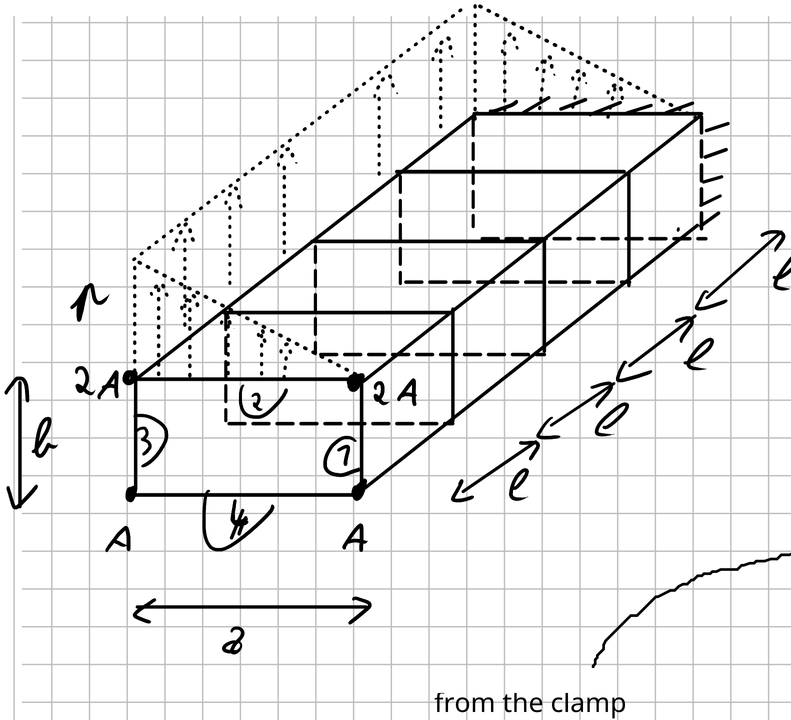
$$M_y = -F(z+\ell) + \frac{5}{2} F z \qquad 0 < z < \ell$$



$$\delta M_y = 1$$

$$\int_0^{\ell} \frac{-F(z+\ell)}{EJ_{yy}} + \frac{5}{2} \frac{Fz}{EJ_{yy}} = (-1) \cdot \varphi_y$$

$$\varphi_y = \frac{1}{4} \frac{F \ell^2}{EJ_{yy}} = 0{,}05\overline{3} \quad rad$$



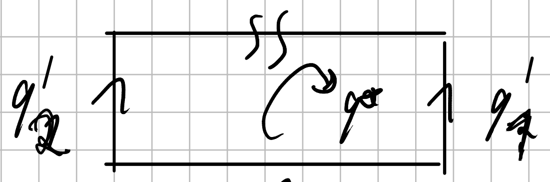
The semi-monocoque beam model in the figure is loaded by a pressure distribution that is constant along the beam span and varies linearly from $\emptyset$ to p along the upper side with dimension a. Compute the shear stress in panel 1 at a distance $2l - \varepsilon$, with $0 < \varepsilon < l$, accounting for the fact that the external load is introduced by the ribs.

Data:

$l = 2000$ mm
$p = 0{,}2$ MPa
$a = 800$ mm
$b = 150$ mm
$A = 500$ mm^2
$t = 4$ mm

$E = 72000$ MPa
$\nu = 0{,}3$

Unit for result: MPa



$$R = \frac{p\,a}{2} \cdot l \cdot \frac{5}{2}$$

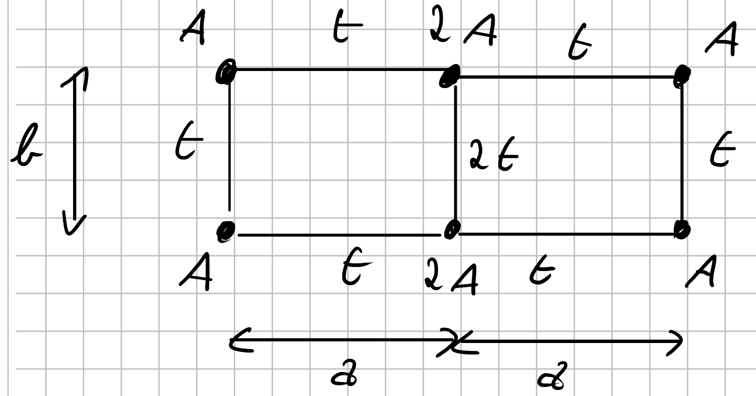
$$M = \frac{p\,a}{2} \cdot \frac{5}{2} l \cdot \left(\frac{a}{2} - \frac{a}{3}\right) = \frac{5}{24} p\, a^2 l$$

$$q'_1 = q'_2 = \frac{R}{2l}$$

$$q^* = \frac{M}{2al}$$

$$q_1 = q'_1 - q^*$$

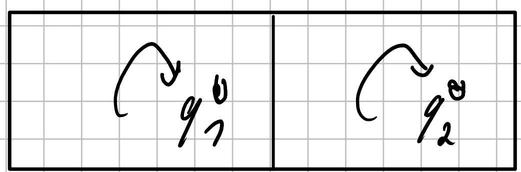
$$\tau_1 = \frac{q_1}{t_1} = 277{,}78 \ \text{MPa}$$



Consider the two-cells semi-monocoque cross section model sketched in the figure.
Compute its torsional stiffness

Data: $a = 200$ mm
$b = 100$ mm
$t = 1$ mm
$A = 1000$ mm^2
$E = 72\,000$ MPa
$\nu = 0{,}3$

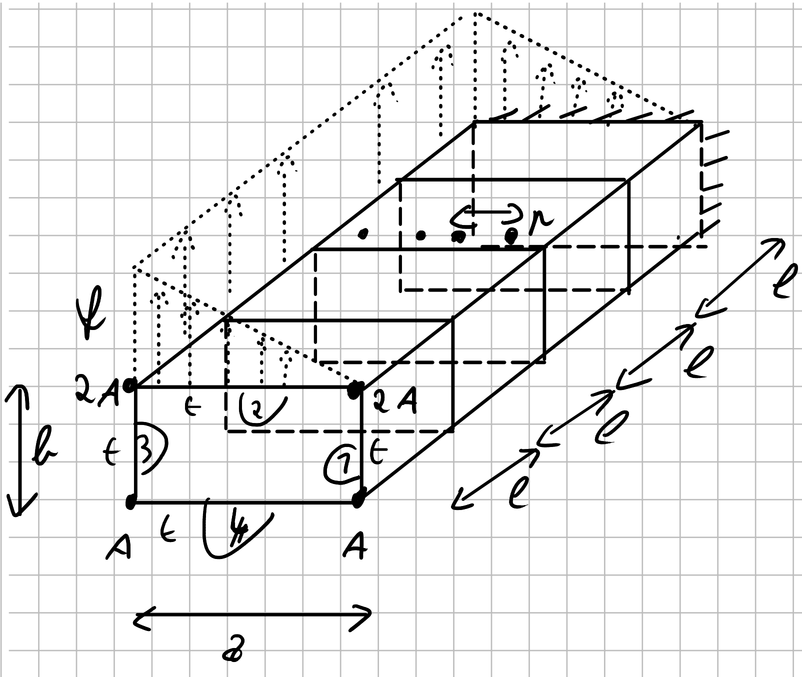
Unit for result: N mm^2/rad



$$q_1^0 = q_2^0 \qquad q_1^0 = \frac{M_z}{4ab}$$

$$\theta_1' = \frac{1}{2abG} \frac{(2a+b)\, q_1^0}{t} = \left[\frac{1}{2ab} \frac{(2a+b)}{t} \cdot \frac{1}{4ab}\right] \cdot M_z$$

$$K_\theta = \frac{1 \quad G}{\left[\frac{1}{2ah} \quad \frac{(2a+h)}{t} \cdot \frac{1}{4ah}\right]} \overset{1{,}77E11}{=\cancel{6{,}??E6}} \ \text{Nmm / rad}$$



The semi-monocoque beam model in the figure is loaded by a pressure distribution that is constant along the beam span and varies linearly from ∅ to p along the side with dimension a

The rib at 2l from the clamp is connected to the upper panel by means of a row of rivets, with pitch p and diameter d. Compute the force transmitted by one of those rivets

Data

$l = 700$ mm

$E = 72000$ MPa

$\nu = 0,3$

$t = 2$ mm

$p = 30$ mm

$A = 800$ mm^2

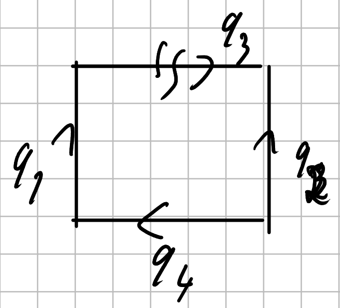
$a = 400$ mm

$b = 200$ mm

$p = 0,3$ MPa

$d = 3$ mm

Unit for result : N



$$R = \frac{\ell a l}{2}$$

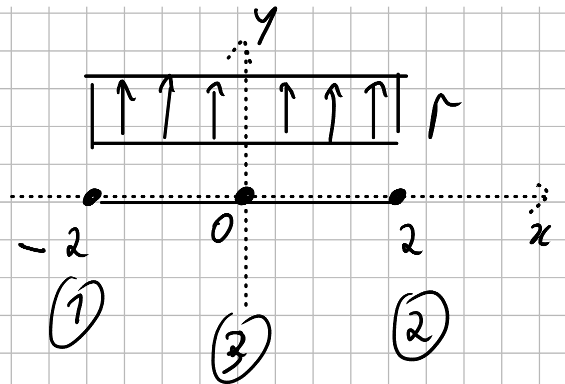
$$M = \frac{\ell a l}{2}\left(\frac{a}{2} - \frac{a}{3}\right)$$

$$q_1' = q_2' = \frac{R}{2h}$$

$$q^{*} = \frac{M}{2ah}$$

$$q_3 = q^{*}$$

$$\ell_i = q_3 \cdot \mu \approx 525\ M$$



The three-nodes parabolic element stetched inthe figure is loaded by a constant force per unit of length. Compute the virtual work of the load for a unit virtual displacement in the y direction of node 1 (the not at $x = -2$)

Data: $p = 0{,}56$ N/mm

Unit for result: N mm

$$N_1 = \frac{x \cdot (x-2)}{8}$$

$$\int_{-2}^{2} \left(\frac{x^2}{8} - \frac{x}{4} \right) p \, dx = \left(\frac{x^3}{24} - \frac{x^2}{8} \right) \Bigg|_{-2}^{2} p = \frac{2}{3} p = 0{,}37\bar{3} \text{ N mm}$$

- The assumption of plane strain implies that a component of strain is null
  - True
- The Newmann boundary conditions are satisfied in a weak sense by the Principle of Virtual Work:
  - True
- According to the semi-monocoque model, the shear stress in the panels can be computed from the axial derivative of the axial stress σ_{zz}:
  - False

1. The shear flux in a thin panel is equal to
   (a) the shear stress divided by the panel thickness
   (b) the shear stress multiplied by the panel thickness
   (c) the shear stress
   (d) the derivative of the axial stress in the panel
   (e) none of the above
2. The critical buckling compression force for the Euler instability of a beam is function of:
   (a) only the beam length and the constraints
   (b) only the beam bending stiffness and the constraints
   (c) only the beam torsional stiffness and the constraints
   (d) only the beam axial stiffness and the constraints
   (e) the beam length, the axial stiffness and the constraints
   (f) the beam length, the bending stiffness and the constraints
   (g) the beam length, the bending stiffness, the cross-section area and the constraints
   (h) the beam length, the torsional stiffness and the constraints
   (i) none of the above
3. The axial stiffness for a thin-walled beam:
   (a) is null
   (b) is generally larger with respect to a corresponding (same material and cross-section area of the material) compact section
   (c) is generally smaller with respect to a corresponding (same material and cross-section area of the material) compact section
   (d) is equal to that of a corresponding (same material and cross-section area of the material) compact section
   (e) can be neglected
   (f) none of the above