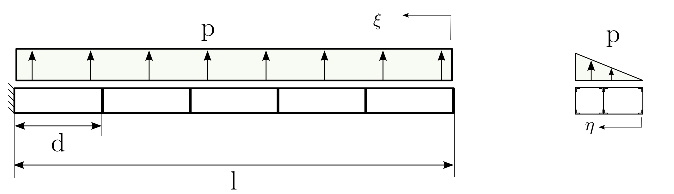
Course of Spacecraft Structures

Written test, February 10th, 2017

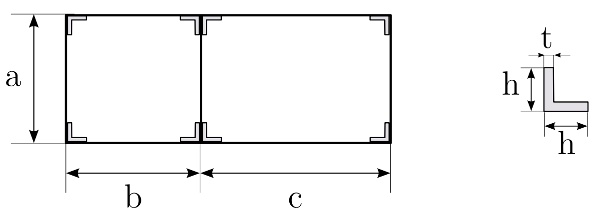
Exercise 1

Consider the thin-walled beam reported in the figure. The beam has a total length equal to l, while the ribs are equally spaced with pitch d. The load is a pressure distribution $p = p_0 \dfrac{\eta}{b+c}$, uniform along the beam axis and linear along the chord.



side view *front view*

The cross section of the beam is sketched here below.



By making use of the semi-monocoque scheme, determine: the shear flows and the shear stresses acting on the panels, and the axial force carried by the stringers for the section at ξ=900 mm.

Data

d = 400 mm

l = 2000 mm

p_0 = 2e-2 N/mm^2

a = 200 mm

b = 200 mm

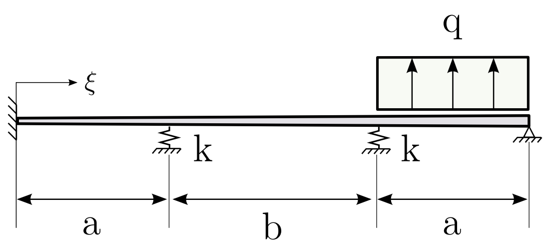
c = 300 mm

t = 4 mm

h = 20 mm

All the panels have thickness 0.8 mm.

Exercise 2
Consider the beam reported in the figure. It is fixed at one end, and hinged at the other end. The bending stiffness of the beam varies linearly from EJ_0 at $\xi = 0$ to EJ_1 at $\xi = 2a + b$. Two linear springs of stiffness k restrain the transverse displacement at $\xi{=}a$ and $\xi{=}a + b$. A uniform force per unit length of magnitude q is applied in the interval $a + b \le \xi \le 2a + b$.
Assume a Euler-Bernoulli beam model and apply the method of Ritz to obtain an approximation of the displacement field. Consider a single-degree-of-freedom description for the bending displacement.



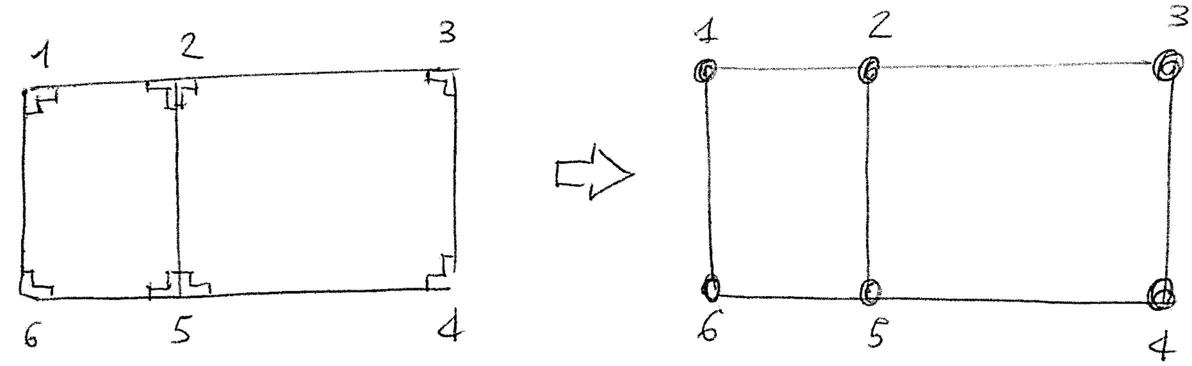
Data
$a = 300$ mm
$b = 400$ mm
$l = 2a + b = 1000$ mm
$q = 20$ N/mm
$EJ_0 = $ 1e9 Nmm^2
$EJ_1 = 2\ EJ_0$
$k = 100\ \dfrac{EJ_0}{l^3} = 100$ N/mm

Question 1
Derive the pre-buckling and buckling equations for an axially compressed beam. Discuss the differences between the two sets of equations and illustrate how the buckling load can be determined for a generic set of boundary conditions.

Exercise 1

Stringer area: $A_{St} = th + t(h-t)$

$= 144 \text{ mm}^2$



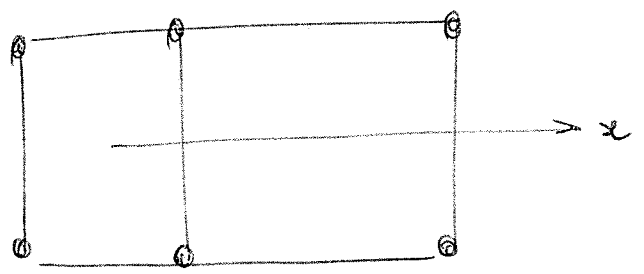
Lumped areas:

$$A_1 = A_6 = A_{St} + \frac{1}{2}(b+a)t = 304 \text{ mm}^2$$

$$A_3 = A_4 = A_{St} + \frac{1}{2}(a+c)t = 344 \text{ mm}^2$$

$$A_2 = A_5 = 2A_{St} + \frac{1}{2}(a+b+c)t = 568 \text{ mm}^2$$

Section properties



Neutral axis x from the symmetry of the section

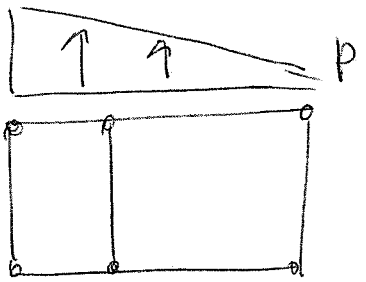
$$S_{x_1} = -S_{x_6} = A_1 \frac{a}{2} = 30400 \text{ mm}^3$$

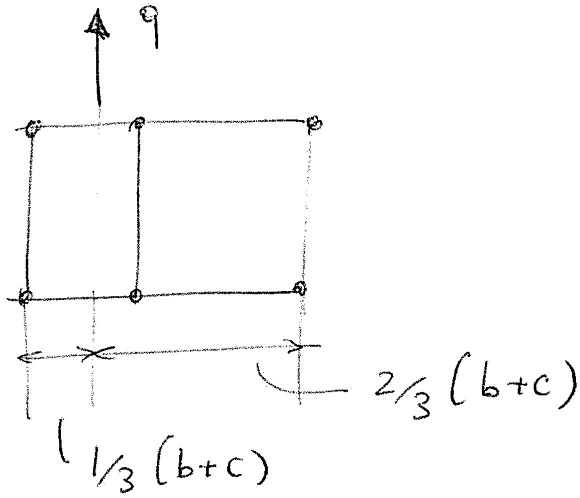
$$S_{x_2} = -S_{x_5} = A_2 \frac{a}{2} = 56800 \text{ mm}^3$$

$$S_{x_3} = -S_{x_4} = A_3 \frac{a}{2} = 34400 \text{ mm}^3$$

$$J_{xx} = 2(A_1 + A_2 + A_3)\left(\frac{a}{2}\right)^2 = 24320000 \text{ mm}^4$$

External loads

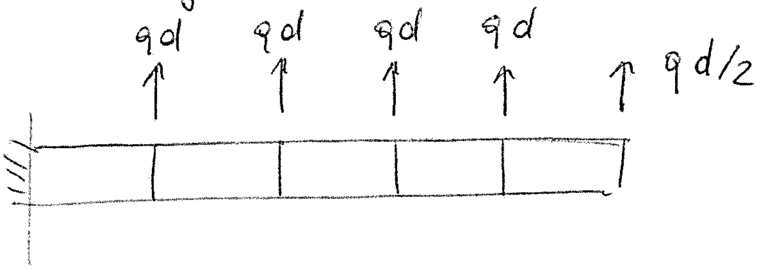




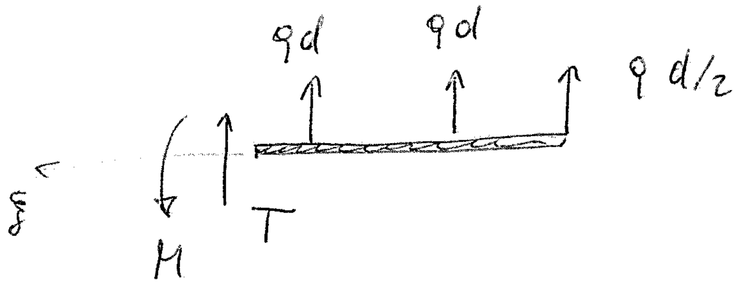
$$q = p\,\frac{b+c}{2} = 5\ N/mm$$

Internal actions

Assuming that loads are introduced by ribs:



Internal actions at $\xi = 900$ mm

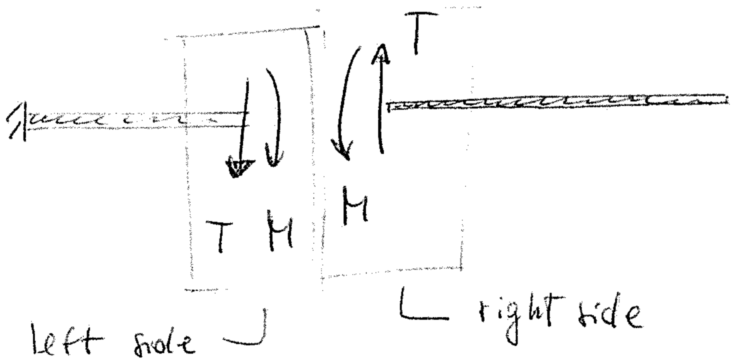


$$T = -q\,d\left(2+\frac{1}{2}\right) = -\frac{5}{2}\,q\,d = -5000\ N$$

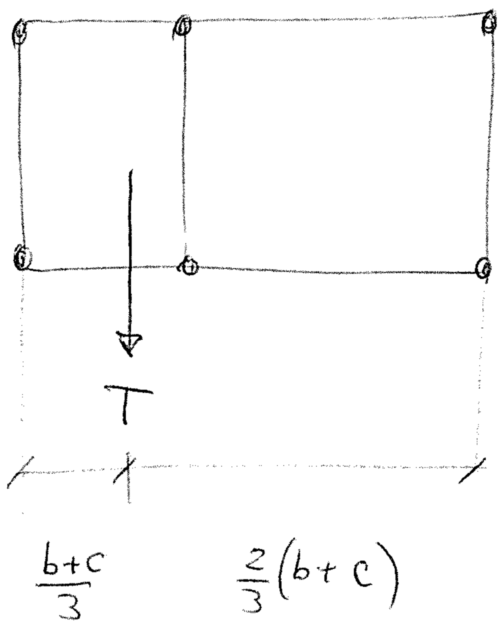
$$M = -q\,d \cdot (1050) = -2\,100\,000\ Nmm$$

Evaluation of shear stresses

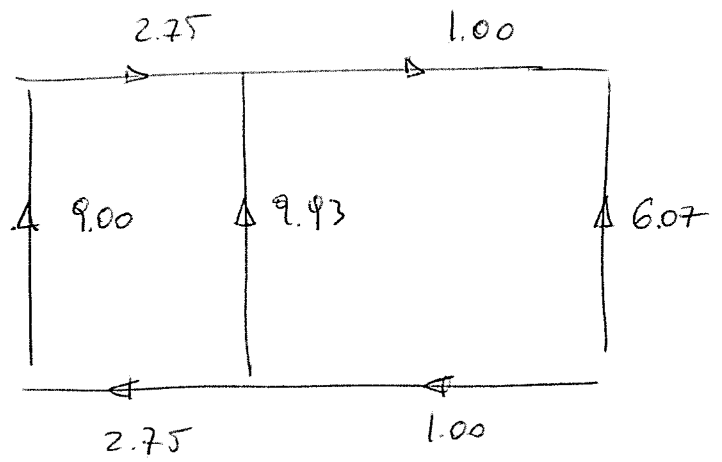
A preliminary remark: once the internal actions at $\xi = 900$ mm are available, one can arbitrarily consider the section at the left or right side of the "cut"



We consider here the left side (but considering the right one would be correct as well)

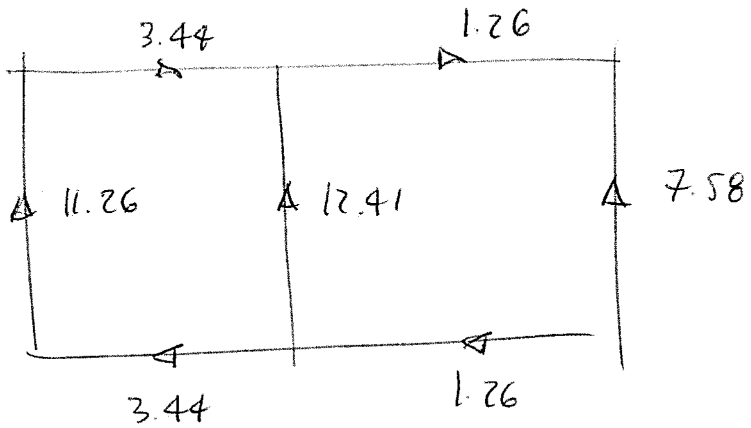


The solution is



(All units in N/mm)

The shear stress are obtained as $q_i / t = \tau_i$



(All units in MPa)

<u>Axial forces in the stringers</u>

$$F_1 = A_1 \frac{M}{J_{xx}} \left(\frac{a}{2} \right) = -2625 \text{ N}$$

$$F_2 = A_2 \frac{M}{J_{xx}} \left(\frac{a}{2} \right) = -4904 \text{ N}$$

$$F_3 = A_3 \frac{M}{J_{xx}} \left(\frac{a}{2} \right) = -2970 \text{ N}$$

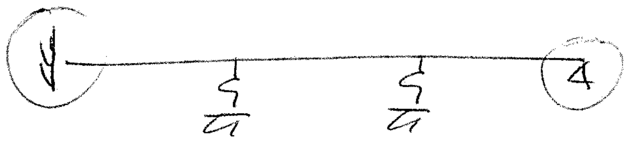
$F_i < 0$ compression

$F_6 = -F_1$ $\quad$ $F_4 = -F_3$

$F_5 = -F_2$

Exercise 2

A polynomial function is considered



2 essential conditions

1 essential condition

3 essential conditions + 1 dof approx = 4 terms in the polynomial expansion:

$$w = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

Imposing:

$$\begin{cases} w(0) = 0 \\ w_{,x}(0) = 0 \\ w(\ell) = 0 \end{cases} \Rightarrow w = a_3 \left(x^3 - \ell x^2\right)$$

The expression can be re-arranged (this is <u>not</u> strictly necessary!) to have a non-dimensional trial function:

$$w = \underbrace{C}_{[\ell]} \underbrace{\frac{1}{\ell^3}\left(x^3 - \ell x^2\right)}_{\text{non dimensional}}$$

$$w = C\phi \qquad \phi = \left(\frac{x}{\ell}\right)^3 - \left(\frac{x}{\ell}\right)^2$$

Stiffness matrix

$$\delta W_i = \int_0^{l} \delta w_{/xx} \, EJ \, w_{/xx} \, dx + \delta w(a) \, k \, w(a) + \delta w(b) \, k \, w(b) \qquad l = 2a + b$$

$$= \delta c \, K \, c$$

with $$K = \int_0^{l} \phi_{/xx} \, EJ \, \phi_{/xx} \, dx + \phi(a) \, k \, \phi(a) + \phi(b) \, k \, \phi(b)$$

$$= 9.56 \ N/mm$$

External load

$$\delta W_e = \int_{a+b}^{2a+b} \delta w \, q \, dx$$

$$= \delta c \, f$$

with $$f = \int_{a+b}^{2a+b} \phi \, q \, dx = -580.50 \ N$$

Solution

$$c = f/K = -60.74 \ mm$$

the displacement field is approximated as:

$$w = -60.74 \left[\left(\frac{x}{l}\right)^3 - \left(\frac{x}{l}\right)^2 \right]$$