

## Project 3D data onto a 2D subspace

TOTAL POINTS 6

1. For a vector  $\mathbf{x} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$  and the subspace  $U$  spanned by the basis vectors  $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$  and  $\mathbf{b}_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ , which of the following statements are true?

4 / 4 points

You can use the formula slide that comes with the corresponding lecture.

- ☐ The projection of  $\mathbf{x}$  onto  $U$  is  $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ .
- ☒ The coordinates of the projected point with respect to  $\mathbf{b}_1, \mathbf{b}_2$  are  $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$ .

✔ Correct  
Excellent job!

- ☒ The projection of  $\mathbf{x}$  onto  $U$  is  $\begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$ .

✔ Correct  
Well done.

- ☒ The projection matrix is symmetric.

✔ Correct  
Projection matrices are always symmetric.

- ☐ The coordinates of the projected point with respect to  $\mathbf{b}_1, \mathbf{b}_2$  are  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ .
- ☐ The rank of the projection matrix is 1.
- ☐ The projection matrix is not symmetric.

- ☒ The projection matrix is  $\frac{1}{6} \begin{bmatrix} 5 & 2 & -1 \\ 2 & 2 & 2 \\ -1 & 2 & 5 \end{bmatrix}$ .

✔ Correct  
Well done!

- ☐ The projection matrix is  $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{bmatrix}$ .

2. Project  $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$  onto the subspace spanned by  $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ .

1 / 1 point

You can use the formula slide that comes with the corresponding lecture.

- ☐  $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$
- ☒  $\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$
- ☐  $\begin{bmatrix} 6 \\ 4 \\ 4 \end{bmatrix}$
- ☐  $\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$

✔ Correct  
Absolutely! The original vector is already in the subspace, so the projection has no effect.

3. 1. Project  $\begin{bmatrix} 12 \\ 0 \\ 0 \end{bmatrix}$  onto the subspace  $U_1$  spanned by  $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ .

1 / 1 point

2. Project the result from 1. onto the subspace spanned by  $\begin{bmatrix} -10\sqrt{6} \\ -4\sqrt{6} \\ 2\sqrt{6} \end{bmatrix}$ . What is the final projection?

Hint: For step 2, you do not necessarily need to compute anything.

You can use the formula slide that comes with the corresponding lecture.

- ☐  $\begin{bmatrix} 5 \\ 2\sqrt{6} \\ -1\sqrt{6} \end{bmatrix}$
- ☐  $\begin{bmatrix} 5 \\ 2\sqrt{6} + 1 \\ -\sqrt{6} + 2 \end{bmatrix}$
- ☒  $\begin{bmatrix} 10 \\ 4 \\ -2 \end{bmatrix}$

✔ Correct  
Good job! The first projection already lies in the second subspace. Therefore, the second projection does not do anything.