

Diagonalisation and applications

TOTAL POINTS 7

1. In this quiz you will diagonalise some matrices and apply this to simplify calculations.

1 / 1 point

Given the matrix $T = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix}$ and change of basis matrix $C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ (whose columns are eigenvectors of T), calculate the diagonal matrix $D = C^{-1}TC$.

- ☒ $\begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$
- ☐ $\begin{bmatrix} 6 & 0 \\ 0 & 3 \end{bmatrix}$
- ☐ $\begin{bmatrix} 9 & 0 \\ 0 & 20 \end{bmatrix}$
- ☐ $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

✓ **Correct**
Well done!

2. Given the matrix $T = \begin{bmatrix} 2 & 7 \\ 0 & -1 \end{bmatrix}$ and change of basis matrix $C = \begin{bmatrix} 7 & 1 \\ -3 & 0 \end{bmatrix}$ (whose columns are eigenvectors of T), calculate the diagonal matrix $D = C^{-1}TC$.

1 / 1 point

- ☒ $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix}$

✓ **Correct**
Well done! As it turns out, because T is a special type of diagonal matrix, where all entries on the diagonal are the same, this matrix is just a scalar multiple of the identity matrix. Hence, given any change of co-ordinates, this matrix remains the same.

5. Given that $T = \begin{bmatrix} 6 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$, calculate T^3 .

1 / 1 point

- ☐ $\begin{bmatrix} 3 & 122 \\ 186 & -61 \end{bmatrix}$
- ☒ $\begin{bmatrix} 186 & -61 \\ 122 & 3 \end{bmatrix}$
- ☐ $\begin{bmatrix} 61 & 2 \\ 2 & 1 \end{bmatrix}$
- ☒ $\begin{bmatrix} 8 & 21 \\ 0 & -1 \end{bmatrix}$

✓ **Correct**
Well done!

7. Given that $T = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$, calculate T^5 .

1 / 1 point

- ☐ $\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$
- ☐ $\begin{bmatrix} -1 & 0 \\ 2 & 1 \end{bmatrix}$
- ☐ $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$
- ☒ $\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$

✓ **Correct**
Well done!