✓ **Congratulations! You passed!**

**TO PASS** 80% or higher

Keep Learning

**GRADE**
100%

# Fitting a non-linear function

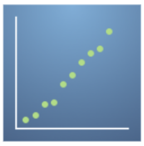
**TOTAL POINTS 5**

1. The previous quiz tested our knowledge of linear regression, and how we can begin to model sets of data. In the last video, we developed on this idea further, looking at the case for data that cannot be effectively modelled by linear approximations. As such, we were introduced to the nonlinear least squares method, as a way of fitting nonlinear curves to data.

**1 / 1 point**

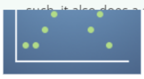
In this question, we have a set of graphs highlighting different distributions of data. Select the appropriate graphs where the nonlinear least squares method can be adapted to provide an effective fit to this data.

- [x] Option A



✓ **Correct**

The nonlinear least squares method is very similar to the linear regression method highlighted previously. As



✓ **Correct**

This data looks similar to a Gaussian and should be able to be fitted through the nonlinear regression technique.

✓ **Correct**

This data looks like the ln(x) relation and should be able to be fitted through the nonlinear regression technique.

2. In the previous lecture, you were taken through the example of $\chi^2$ and how it is important in utilising the sum of the differences and the least squares method. We were also introduced to the expression $\chi^2 = \sum_{i=1}^{n} \frac{[y_i - y(x_i;a_k)]^2}{\sigma^2}$. For the parameter $\chi^2$, select all the statements below that are true.

**1 / 1 point**

- [ ] The parameter $\chi^2$ is the uncertainty value of our variables.

Here we will define the matrix $[Z_j] = \frac{\partial f(x_i, \mathbf{a})}{\partial a_j}$

Assuming $f(x_i, \boldsymbol{a}) = a_1 x^3 - a_2 x^2 + e^{-a_3 x}$, select the option that correctly shows the partial differentiation for this function.

(◉)

$$\frac{\partial f}{\partial a_1} = x^3, \frac{\partial f}{\partial a_2} = -x^2, \frac{\partial f}{\partial a_3} = -xe^{-a_3 x}$$

$$\frac{\quad}{\partial a_1} = -2\sum_{i=1}[y_i - a_1(1 - e \quad)](e \quad)$$

$$\frac{\partial(\chi^2)}{\partial a_2} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](a_1 x_i^2 e^{-a_2 x_i^2})$$

( )

$$\frac{\partial(\chi^2)}{\partial a_1} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](1 + e^{-a_2 x_i^2})$$

$$\frac{\partial(\chi^2)}{\partial a_2} = -2\sum_{i=1}^{n}[y_i - a_1(1 - e^{-a_2 x_i^2})](a_1 x_i e^{-a_2 x_i^2})$$

In the lectures, we also showed how to find $\chi^2$ and how this forms the Jacobian shown below: $\boldsymbol{J} = \left[\frac{\partial(\chi^2)}{\partial a_k}\right] = \left[\frac{\partial(\chi^2)}{\partial \sigma}, \quad \frac{\partial(\chi^2)}{\partial x_p}, \quad \frac{\partial(\chi^2)}{\partial I}, \quad \frac{\partial(\chi^2)}{\partial b}\right]$.

where

$$\frac{\partial \chi^2}{\partial a_j} = -2\sum_{i=1}^{n} \frac{\boldsymbol{y_i} - y(x_i;\boldsymbol{a})}{\sigma_i^2} \frac{\partial y(x_i;\boldsymbol{a})}{\partial a_j} \, for \; j = 1......n$$

For the Gaussian function above, determine the partial differential

$$\frac{\partial y}{\partial x_p}$$

( )

$$\frac{\partial y}{\partial x_p} = -\frac{I}{\sqrt{2\pi}} \frac{(x - x_p)}{2\sigma^3} \exp\left\{\frac{-(x - x_p)^2}{2\sigma^2}\right\}$$

( )

$$\frac{\partial y}{\partial x_p} = \frac{I}{\sqrt{2\pi}} \frac{2(x - x_p)}{\sigma} \exp\left\{\frac{-(x - x_p)^2}{2\sigma^2}\right\}$$

(◉)

$$\frac{\partial y}{\partial x_p} = \frac{I}{\sqrt{2\pi}} \frac{(x - x_p)}{\sigma^3} \exp\left\{\frac{-(x - x_p)^2}{2\sigma^2}\right\}$$

( )

$$\frac{\partial y}{\partial x_p} = \frac{I}{\sqrt{2\pi}} \frac{2x}{\sigma} \exp\left\{\frac{-(x - x_p)^2}{2\sigma^2}\right\}$$

✓ **Correct**

Here we are only evaluating one partial derivative that forms part of the Jacobian. In order to correctly fit the Gaussian to a specific set of data, we will need to evaluate all the partial derivatives mentioned previously.