Congratulations! You passed!

TO PASS 80% or higher

Keep Learning

GRADE
100%

Eigenvalues and eigenvectors

LATEST SUBMISSION GRADE

100%

1. This assessment will test your ability to apply your knowledge of eigenvalues and eigenvectors to some special cases.

1 / 1 point

Use the following code blocks to assist you in this quiz. They calculate eigenvectors and eigenvalues respectively:

```
# Eigenvalues
# M = np.array([[1, 0, 0],
#               [0, 2, 0],
#               [0, 0, 3]])
# vals, vecs = np.linalg.eig(M)
# vals

M = np.array([[4, -5, 6],
              [7, -8, 6],
              [3/2, -1/2, -2]])
vals, vecs = np.linalg.eig(M)
vals
```

Run

Reset

```
[ 1. -4. -3.]
```

```
# # Eigenvectors - Note, the eigenvectors are the columns of the output.
# M = np.array([[1, 0, 0],
#               [0, 2, 0],
#               [0, 0, 3]])
# vals, vecs = np.linalg.eig(M)
# vecs

M = np.array([[4, -5, 6],
              [7, -8, 6],
              [3/2, -1/2, -2]])
vals, vecs = np.linalg.eig(M)
vecs

```

Run

Reset

```
[[ 3. -2.  1.]
 [ 3. -2. -1.]
 [ 1.  1. -2.]]
```

To practice, select all eigenvectors of the matrix, $A = \begin{bmatrix} 4 & -5 & 6 \\ 7 & -8 & 6 \\ 3/2 & -1/2 & -2 \end{bmatrix}$.

- [ ] $\begin{bmatrix} 1/\sqrt{6} \\ -1/\sqrt{6} \\ 2/\sqrt{6} \end{bmatrix}$
- [ ] $\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$
- [x] $\begin{bmatrix} -3 \\ -3 \\ -1 \end{bmatrix}$

Correct

This is one of the eigenvectors.

- [x] $\begin{bmatrix} -2/\sqrt{9} \\ -2/\sqrt{9} \\ 1/\sqrt{9} \end{bmatrix}$

Correct

This is one of the eigenvectors. Note eigenvectors are only defined upto a scale factor.

- [x] $\begin{bmatrix} 1/2 \\ -1/2 \\ -1 \end{bmatrix}$

Correct

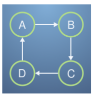
This is one of the eigenvectors. Note eigenvectors are only defined upto a scale factor.

- [ ] None of the other options.
- [ ] $\begin{bmatrix} -3 \\ -2 \\ 1 \end{bmatrix}$

2. Recall from the *PageRank* notebook, that in PageRank, we care about the eigenvector of the link matrix, L, that has eigenvalue 1, and that we can find this using *power iteration method* as this will be the largest eigenvalue.

1 / 1 point

PageRank can sometimes get into trouble if closed-loop structures appear. A simplified example might look like this,



With link matrix, $L = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

Use the calculator in Q1 to check the eigenvalues and vectors for this system.

What might be going wrong? Select all that apply.

- [x] Because of the loop, *Procrastinating Pats* that are browsing will go around in a cycle rather than settling on a webpage.

Correct

If all sites started out populated equally, then the incoming pats would equal the outgoing, but in general the system will not converge to this result by applying power iteration.

- [ ] Some of the eigenvectors are complex.
- [ ] None of the other options.
- [x] Other eigenvalues are not small compared to 1, and so do not decay away with each power iteration.

Correct

The other eigenvectors have the same size as 1 (they are -1, i, $-i$)

- [ ] The system is too small.

3. The loop in the previous question is a situation that can be remedied by damping.

1 / 1 point

If we replace the link matrix with the damped, $L' = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.7 \\ 0.7 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.7 & 0.1 \end{bmatrix}$, how does this help?

- [ ] The complex number disappear.
- [ ] It makes the eigenvalue we want bigger.
- [x] The other eigenvalues get smaller.

Correct

So their eigenvectors will decay away on power iteration.

- [x] There is now a probability to move to any website.

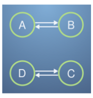
Correct

This helps the power iteration settle down as it will spread out the distribution of Pats

- [ ] None of the other options.

4. Another issue that may come up, is if there are disconnected parts to the internet. Take this example,

1 / 1 point



with link matrix, $L = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$.

This form is known as block diagonal, as it can be split into square blocks along the main diagonal, i.e., $L = \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}$, with $A = B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ in this case.

What is happening in this system?

- [x] There isn't a unique PageRank.

Correct

The power iteration algorithm could settle to multiple values, depending on its starting conditions.

- [x] There are two eigenvalues of 1.

Correct

The eigensystem is degenerate. Any linear combination of eigenvectors with the same eigenvalue is also an eigenvector.