✓ **Congratulations! You passed!**
TO PASS 80% or higher

Keep Learning

GRADE
100%

Properties of inner products

LATEST SUBMISSION GRADE
100%

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1. The function

`1 / 1 point`

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

☐ not positive definite

☐ not an inner product

☑ positive definite

> ✓ **Correct**
> Yes, the matrix has only positive eigenvalues and $\beta(\mathbf{x}, \mathbf{x}) > 0$ for all $\mathbf{x} \neq \mathbf{0}$ and $\beta(\mathbf{x}, \mathbf{x}) = 0 \iff \mathbf{x} = \mathbf{0}$

☑ an inner product

> ✓ **Correct**

☑ symmetric

> ✓ **Correct**
> Yes: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

☐ not bilinear

☑ bilinear

> ✓ **Correct**
> Yes:
> - β is symmetric. Therefore, we only need to show linearity in one argument.
> - For any $\lambda \in \mathbb{R}$ it holds that $\beta(\mathbf{x} + \lambda\mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda\beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

☐ not symmetric

---

2. The function

`1 / 1 point`

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

☐ not symmetric

☑ not an inner product

> ✓ **Correct**
> Correct: Since β is not positive definite, it cannot be an inner product.

☐ an inner product

☐ positive definite

☑ bilinear

> ✓ **Correct**
> Correct:
> - β is symmetric. Therefore, we only need to show linearity in one argument.
> - $\beta(\mathbf{x} + \lambda\mathbf{z}, \mathbf{y}) = \beta(\mathbf{x}, \mathbf{y}) + \lambda\beta(\mathbf{z}, \mathbf{y})$. This holds because of the rules for vector-matrix multiplication and addition.

☑ symmetric

> ✓ **Correct**
> Correct: $\beta(\mathbf{x}, \mathbf{y}) = \beta(\mathbf{y}, \mathbf{x})$

☑ not positive definite

> ✓ **Correct**

---

3. The function

`1 / 1 point`

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 2 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{y}$$

is

☐ symmetric

☑ not symmetric

> ✓ **Correct**
> Correct: If we take $\mathbf{x} = [1, 1]^T$ and $\mathbf{y} = [2, -1]^T$ then $\beta(\mathbf{x}, \mathbf{y}) = 0$ but $\beta(\mathbf{y}, \mathbf{x}) = 6$. Therefore, β is not symmetric.

☑ bilinear

> ✓ **Correct**
> Correct.

☐ not bilinear

☐ an inner product

☑ not an inner product

> ✓ **Correct**

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4. The function

`1 / 1 point`

$$\beta(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \mathbf{y}$$

is

☑ an inner product

> ✓ **Correct**
> It is the dot product, which we know already. Therefore, it is also an inner product.

☐ not positive definite

☑ positive definite

> ✓ **Correct**
> It is the dot product, which we know already. Therefore, it is positive definite.

☑ symmetric

> ✓ **Correct**
> It is the dot product, which we know already. Therefore, it is symmetric.

☐ not bilinear

☐ not an inner product

☑ bilinear

> ✓ **Correct**
> It is the dot product, which we know already. Therefore, it is positive bilinear.

☐ not symmetric

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5. For any two vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$ write a short piece of code that defines a valid inner product.

`1 / 1 point`

```
1  import numpy as np
2
3  def dot(a, b):
4      """Compute dot product between a and b.
5      Args:
6          a, b: (2,) ndarray as R^2 vectors
7
8      Returns:
9          a number which is the dot product between a, b
10     """
11
12     dot_product = np.dot(a,b)
13
14     return dot_product
15
16  # Test your code before you submit.
17  a = np.array([1,0])
18  b = np.array([0,1])
19  print(dot(a,b))
```

Run

Reset

0

> ✓ **Correct**
> Good job!