

## Solving linear equations using the inverse matrix

TOTAL POINTS 14

1. You go to the shops on Monday and buy 1 apple, 1 banana, and 1 carrot; the whole transaction totals €15. On Tuesday you buy 3 apples, 2 bananas, 1 carrot, all for €28. Then on Wednesday 2 apples, 1 banana, 2 carrots, for €23.

1 / 1 point

Construct a matrix and vector for this linear algebra system. That is, for

$$A \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} s_{\text{Mon}} \\ s_{\text{Tue}} \\ s_{\text{Wed}} \end{bmatrix}$$

Where  $a$ ,  $b$ ,  $c$ , are the prices of apples, bananas, and carrots. And each  $s$  is the total for that day.

Fill in the components of  $A$  and  $s$ .

1	# Replace A and s with the correct values below:	
2	A = [[1, 1, 1],	
3	[3, 2, 1],	
4	[2, 1, 2]]	
5	s = [15, 28, 23]	
6		
7		

[15, 28, 23]

Run  
Reset

✓ Correct

Correct! Well done.

$$\textcircled{2}: \begin{bmatrix} 0 & 1 & 1 \\ 2 & 8 & 13 \end{bmatrix} \begin{bmatrix} b \\ c \end{bmatrix} = \begin{bmatrix} -1/2 \\ 2 \end{bmatrix}$$

What steps did we take?

- ☐ The new second row,  $\textcircled{2}'$  is the old second row minus two times the old first row, i.e.,  $\textcircled{2}' = [\textcircled{2}' - 2\textcircled{1}']$ .
- ☐ The new second row,  $\textcircled{2}'$  is the old second row minus three, i.e.,  $\textcircled{2}' = \textcircled{2}' - 3$ .
- ☐ The new second row,  $\textcircled{2}'$  is the old second row divided by four minus the old first row, i.e.,  $\textcircled{2}' = \textcircled{2}'/4 - \textcircled{1}'$ .
- ☒ The new second row,  $\textcircled{2}'$  is the old second row minus three times the old first row, then all multiplied by -2, i.e.,  $\textcircled{2}' = [\textcircled{2}' - 3\textcircled{1}'] \times (-2)$ .

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

☐  $\begin{bmatrix} 1 & 3/2 & 1/2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/4 \\ -1/2 \\ 1/2 \end{bmatrix}$

☐  $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 9/4 \\ -1/2 \\ -1/4 \end{bmatrix}$

✓ Correct

This system is now in echelon form.

$$r = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

✓ Correct

Well done!

5. Let's return to the apples and bananas from Question 1.

3 / 3 points

Take your answer to Question 1 and convert the system to echelon form, i.e.,

$$r_1 \quad r_2 \quad r_3 \quad r_4 \quad r_5 \quad r_6 \quad r_7 \quad r_8 \quad r_9$$

6. Following on from the previous question; now let's solve the system using back substitution.

3 / 3 points

What is the price of apples, bananas, and carrots?

1	# Replace a, b, and c with the correct values below:	
2	s = [3, 7, 5]	
3		

[3, 7, 5]

Run  
Reset

✓ Correct

Correct! Well done.

8. In practice, for larger systems, one never solves a linear system by hand as there are software packages that can do this for you - such as *numpy* in Python.

1 / 1 point

Use this code block to see *numpy* invert a matrix.

You can try to invert any matrix you like. Try it out on your answers to the previous question.

1	import numpy as np	
2		
3	A = [[1, 1, 1],	
4	[3, 2, 1],	
5	[2, 1, 2]]	
6	s = [15, 28, 23]	
7		
8	r = np.linalg.solve(A, s)	
9		

[ 3. -0.5 0. ]

Run  
Reset

✓ Correct

In cases when you don't need the inverse matrix itself, linear algebra routines are quicker to solve the system for each case.