LinearRegression

Sunday, August 9, 2020 6:01 PM

Data:

	Lot.Area	Bedroom.AbvGr	SalePrice
0	31770	3	215.0
1	11622	2	105.0
2	14267	3	172.0
3	11160	3	244.0
4	13830	3	189.9
...	...	...	...
96	3182	2	151.0
97	2544	2	149.5
98	2544	2	152.0
99	4403	2	222.0
100	2117	3	177.5

101 rows × 3 columns

Notation: Lot Area $= x_1$, Bedroo.AbvGr $= x_2$

Sale price $= y$

Training set: $(x^{(i)}, y^{(i)})$ where $i = 0:100$

i.e (X, Y)

hypothesis: $h(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$

$$= \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2$$

$$= \sum_{i=0}^{n} \theta_i x_i = \theta^T x$$

n = no. of training examples

① Iterative: $\theta = \theta + \alpha \sum_{i=1}^{n} (y^{(i)} - h_\theta(x^{(i)}))\, x^{(i)}$

Since, $\frac{\partial}{\partial \theta} J(\theta) = \left(h_\theta(x^{(i)}) - y^{(i)} \right) x^{(i)}$

where, $J(\theta)$ is mean squared error
(Cost function / loss)

② Implementation of Normal Equation (Closed form)

$$X = \begin{array}{c} \begin{array}{ccc} x_0 & x_1 & x_2 \end{array} \\ \begin{bmatrix} 1 & * & * \\ \vdots & * & * \\ \vdots & \vdots & \vdots \\ 1 & \vdots & \vdots \end{bmatrix} \end{array} \left.\vphantom{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}\right\} \rightsquigarrow n \text{ examples}$$

$$\rightsquigarrow \begin{bmatrix} - (x^{(1)})^T - \\ - (x^{(2)})^T - \\ \vdots \\ - (x^{(n)})^T - \end{bmatrix}$$

$$Y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(n)} \end{bmatrix}$$

$$h_\theta(x^{(i)}) = X\theta - Y$$

$$J(\theta) = \frac{1}{2} (X\theta - Y)(X\theta - Y)$$

$$\nabla_\theta J(\theta) = \nabla_\theta \frac{1}{2} (X\theta - Y)(X\theta - Y)$$

$$= \frac{1}{2} \nabla_\theta \left((X\theta)^T X\theta - (X\theta)^T Y - Y(X\theta) + Y^T Y \right.$$

$$- Y(X\theta) + Y^T Y$$

$$= \frac{1}{2} \nabla_\theta \left(\theta^T (X^T X)\theta - 2 (X^T Y)^T \theta \right)$$

$$= \frac{1}{2} \left(2 X^T X \theta - 2 X^T Y \right)$$

$$= X^T X \theta - X^T Y$$

To find min arg θ,

$$\nabla_\theta J(\theta) = 0 \Rightarrow X^T X \theta - X^T Y = 0$$

$$\theta = (X^T X)^{-1} X^T Y$$

[Using, $a^T b = b^T a$, $\nabla_x b^T x = b$ $\nabla_x x^T A x = 2Ax$ for a symmetric matrix 'A']

Code Implementation Note

Define X and Y, then use normal equation to find 'θ' parameter directly.