assignment2

October 2020

1 problem1

(a)if $R1 \cap R2 = empty : that\ is\ exactly\ an\ equivalence\ relation$
$else : R : assume\ a \in S$
$then\ for\ all\ possible\ a\ (a, a) \in R_1\ (reflexive)$
$and\ (a, a) \in R_2\quad (reflexive)$
$thus\ (a, a) \in (R1 \cap R2)$
$thus\ R1 \cap R2\ is\ reflexive$
$S :\ assume\ x,\ y \in S\ that\ (x, y) \in (R_1 \cap R_2)$
$it\ means\ (x, y) \in R1\ and\ \ (x, y) \in R2$
$if\ (x, y) \in R_1\ then\ (y, x) \in R_1\quad (symmetric)$
$if\ (x, y) \in R_2\ then\ (y, x) \in R_2\quad (symmetric)$
$thus\ (y, x) \in R1\ and\ (y, x) \in R2 \to (y, x) \in (R_1 \cap R_2)$
$thus\ R1 \cap R2\ is\ symmetric$
$T : assume\ x, y, z\ that\ (x, y)\ and\ (y, z) \in (R1 \cap R2)$
$it\ means\ there\ exists\ y \in S\ that\ makes$
$\to (x, y)\ and\ (y, z) \in R1$
$\to (x, y)\ and\ (y, z) \in R2$
$if\ (x, y)\ and\ (y, z) \in R1\ then\ (x, z) \in R1\quad (transitive)$
$if\ (x, y)\ and\ (y, z) \in R2\ then\ (x, z) \in R2\quad (transitive)$
$thus\ there\ exists\ y \in S\ that\ makes$
$(x, z) \in R1, and\ (x, z) \in R2 \to (x, z) \in R1 \cap R2$
$thus\ R1 \cap R2\ is\ transitive$
$thus\ R1 \cap R2\ is\ R, S, T \to it\ is\ a\ Equivalence\ relation$
$(b) assume\ we\ get\ (x, y) \in R1 \cap R2$
$\to (x, y) \in R1\ and\ (x, y) \in R2\ \ (definition)$
$by\ the\ definition\ of\ equivalence\ class$
$\to (x, y) \in [x]_1\ and\ (x, y) \in [x]_2$
$that\ means\ (x, y) \in [x]_1\ \cap [x]_2\ \ (definition)$
$thus\ [x]_1\ \cap [x]_2\ is\ a\ equivalence\ class\ of\ x\ under\ R1 \cap R2$
$[x] = [x]_1\ \cap [x]_2$
$c)\ it\ is\ false,\ for\ example\ assume\ S = \{1, 2, 3, 4, 5\}$
$R_1\ as\ the\ relation\ = (mod3)\ \ ;\ R_2\ as\ the\ relation\ = (mod4)$
$thus\ R_1 : \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 4), (4, 1), (2, 5), (5, 2)\}$
$R_2 : \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (1, 5), (5, 1)\}$

$elements\ in\ R1 \cup R2\ is$
$\{(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1),(2,5),(5,2),(1,5),(5,1)\}$
$if\ it\ satisfy\ Transitivity \rightarrow (2,5),(5,1) \in R1 \cup$
$R2 \rightarrow (2,1) \in \mathrm{R1} \cup \mathrm{R2}$
which is not true
thus R1 $\cup$ R2 is not an equivalence relation

2 problem2

(a) $assume\ R_1; R_2\ \ from\ the\ definition\ we\ know$
$R_1; R_2 = \{(a,c) : there\ is\ a,b \in S\ such\ that\ (a,b) \in R1\ and\ (b,c) \in R2\}(1)$
$if\ R1 \subseteq S \times S, R1\ is\ reflexive\ we\ can\ ensure\ that:$
$(a,a) \in R1\ thus\ a = b\ is\ acceptable\ in\ (1)$
$because\ R2\ is\ reflexive, R2 \subseteq S \times S$
$(a,a) \in R2$
$when\ a = b, there\ can\ be\ a = c\ that\ satisfy\ (b,c) = (a,c) = (a,a) \in R2$
$thus\ (a,a) \in R_1; R_2$
$(b)\ it\ is\ false, for\ example\ \ assume\ S = \{1,2,3,4,5\}$
$R_1\ as\ the\ relation\ = (mod3)\ \ ;\ R_2\ as\ the\ relation\ = (mod4)$
$thus\ R_1 : \{(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1),(2,5),(5,2)\}$
$R_2 : \{(1,1),(2,2),(3,3),(4,4),(5,5),(1,5),(5,1)\}\ \ \ R1, R2\ are\ symmetric$
$by\ definition\ we\ know\ (4,5) \in R1; R2\ \ \sin ce\ (4,1) \in R1, (1,5) \in R2$
$but\ (5,4) \notin R1; R2$
$(c)\ it\ is\ false, for\ example\ \ assume\ S = \{1,2,3,4,5\}$
$R_1\ as\ the\ relation\ = (mod3)\ \ ;\ R_2\ as\ the\ relation\ = (mod4)$
$thus\ R_1 : \{(1,1),(2,2),(3,3),(4,4),(5,5),(1,4),(4,1),(2,5),(5,2)\}$
$R_2 : \{(1,1),(2,2),(3,3),(4,4),(5,5),(1,5),(5,1)\}\ \ \ R1, R2\ are\ transitive$
$by\ definition\ we\ know\ (2,1) \in R1; R2\ \ \sin ce\ (2,5) \in R1, (5,1) \in R2$
$(1,4) \in R1; R2\ \ \sin ce\ (1,4) \in R1, (4,4) \in R2$
$but\ (2,4) \notin R1; R2$

3 problem3

(a)Let $P(x)\ be\ the\ proposition\ that\ R^i = R^j$
$we\ need\ to\ show$
1)$basic\ case(j = i):\ R^i = R^i\ obviously\ it\ is\ true$
$so\ P(i)\ holds$
2)$then\ we\ assume\ P(j)\ holds$
$we\ need\ to\ show\ P(j+1)\ holds$
$P(j)\ holds \rightarrow there\ is\ i\ that\ R^i = R^j$
$R^{j+1} = R^j \cup (R; R^j)\ \ (definition)$
$= R^i \cup (R; R^i)\qquad (R^i = R^j)$

$= R^{i+1}$ $\quad (definition)$
$= R^{i}$ $\quad (R^{i} = R^{i+1})$
$thus\ P(j+1)\ holds$
$thus\ P(j)\ implies\ P(j+1)$
$Therefore,\ by\ induction,\ P(j)\ holds\ for\ all\ j >= i$

$(b)(1) when\ j \geq i\ from\ (a)\ we\ know\ R^{j} = R^{i} \subseteq R^{i}$
$(2) when\ j < i$
$Let\ P(x)\ be\ the\ proposition\ that\ R^{j} \subseteq R^{j+x}\ x \in N$
$1) basic\ case(k = 0):$
$R^{j} \subseteq R^{j}\ so\ P(0)\ holds$
$2)\ we\ assume\ P(m)\ holds\ then\ we\ try\ to\ proof\ P(m+1) holds$
$when\ P(m)\ holds\ R^{j} \subseteq R^{j+m}$
$because\ R^{j+m+1} = R^{j+m} \cup (R; R^{j+m}) \to R^{j+m} \subseteq R^{j+m+1}$
$thus\ \ R^{j} \subseteq R^{j+m} \subseteq R^{j+m+1}\ thus\ P(m+1)\ holds$
$we\ can\ replace\ j + x\ by\ i\ \sin ce\ there\ must\ be\ a\ x\ that\ let\ j + x = i$
$thus\ R^{j} \subseteq R^{j+x} = R^{i}\ always\ holds\ when\ j < i$
$thus\ it\ is\ true$
$(c) we\ need\ to\ show\ R^{k^2} = R^{k^2+1}$
$from\ (b)\ we\ already\ know\ that\ R^{j} \subseteq R^{j+x}\ x \in N\ is\ true$
$let\ x = 1, j = k^2 \quad R^{k^2} \subseteq R^{k^2+1}$
$then\ we\ need\ to\ show\ R^{k^2} \supseteq R^{k^2+1}$
$assume\ there\ is\ (a,c) \in R^{k^2+1}, and\ (a,c) \notin R^{k^2}$
$because\ (a,c) \in R^{k^2+1} = R^{k^2} \cup (R; R^{k^2})$
$= R^{k^2} \cup \{(a,c) : there\ is\ a, b \in S\ such\ that (a,b) \in R\ and (b,c) \in R^{k^2})\}$
$thus\ (a,c)\ can\ only \in \{(a,c) : there\ is\ a, b \in S\ such\ that (a,b) \in R\ and (b,c) \in$
$R^{k^2})\}$
$thus\ there\ exist\ (a, b_{k^2}) \in R\ and\ (b_{k^2}, c) \in R^{k^2}$
$(b_{k^2}, c) \in R^{k^2} = R^{k^2-1} \cup (R; R^{k^2-1})$
$(b_{k^2}, c)\ can\ not\ belong\ to\ R^{k^2-1}\ because$
$if\ (b_{k^2}, c) \in R^{k^2-1}, (a, b_{k^2}) \in R\ then\ (a,c) \in R^{k^2}\ which\ violate\ our\ assumption$
$thus\ (b_{k^2}, c) \in (R; R^{k^2-1})$
$\to thus\ there\ exist\ (b_{k^2}, b_{k^2-1}) \in R\ and\ (b_{k^2-1}, c) \in R^{k^2-1}$
$(b_{k^2-1}, c)\ can\ not\ belong\ to\ R^{k^2-1}\ because$
$if\ (b_{k^2-1}, c) \in R^{k^2-1}, (b_{k^2}, b_{k^2-1}) \in R\ then\ (b_{k^2}, c) \in R^{k^2}\ which\ violate\ our\ presupposition$
$similarly, we\ can\ extend\ this \to for\ 0 \leq n \leq k^2{+}1, R^{n} would\ get\ one\ new\ element\ than\ R^{n-1}$
$however\ whole\ sizeof\ it\ \ could\ only\ be\ |s||s| = k^2\ while\ need\ k^2{+}1\ different\ elements$
$there\ must\ be\ some\ element\ violate\ the\ rule\ which\ make\ (a,b) \in R^{k^2}$
$or\ \ k = 1\ (R^{1} = R^{2}\ is\ true\ with\ only\ one\ element)$
$Therefore, (a,b) \in R^{k^2}\ , which\ means\ for\ every\ element\ in\ R^{k^2+1}\ , it\ is\ also\ in\ R^{k^2}\ ,$
$so\ R^{k^2+1} = R^{k^2}$
$(d) basic\ case:\ \ R^{0}; R^{m} = R^{0+m}$
$R^{0+m} = R^{m}$
$= I; R_{m} \quad (u \sin g\ ass1 - p8 - (b))$

$= R^0; R_m \quad (R^0 = I)$
$so\ P(0)\ holds$
$we\ assume\ P(n)\ holds\ then\ need\ to\ show\ P(n+1) holds$
$if\ P(n)\ holds\ then\ R^{n+m} = R^n; R^m$
$R^{(n+1)+m} = R^{n+m+1} = R^{n+m} \cup (R; R^{m+n}) \quad (definition)$
$= (R^n; R^m) \cup (R; (R^n; R^m)) \quad (it\ holds)$
$= (R^n; R^m) \cup ((R; R^n); R^m) \quad u\sin g\ ass1 - p8 - (a)$
$= (R^n \cup (R; R^n)); R^m \qquad u\sin g\ ass1 - p8 - (c)$
$= R^{n+1}; R^m \quad (definition)$
$thus\ P(n+1)\ holds$
$Therefore\ P(n)\ holds\ for\ all\ n \in N$
$(e) assume\ there\ is\ (a,b) \in R^{k^2}, (b,c) \in R^{k^2}$
$\rightarrow from\ (c) : R^{k^2} = R^{k^2+1}$
$\rightarrow from\ (d)\ R^{k^2}; R^{k^2} = R^{2k^2} = \{(a,c) : there\ is\ a, b \in S\ such\ that (a,b) \in R^{k^2}\ and (b,c) \in R^{k^2})\}$
$thus\ (a,c) \in R^{2k^2}$
$because\ R^{k^2} = R^{k^2+1}\ so\ K^2\ satisfy\ our\ definition\ of\ i$
$from\ (a)\ we\ know\ R^{2k^2} = R^{k^2}\ \sin ce\ 2k^2 > k^2$
$thus\ (a,c) \in R^{k^2}$
$thus\ R^{k^2}\ is\ transitive$
$(f)\ we\ need\ to\ make\ (R \cup R^{\leftarrow})^{k^2}\ reflexive,\ symmetric\ and\ transitive$
$R:\ from\ (b)\ and\ definition\ we\ know\ for\ all\ x \in S$
$(x,x) \in (R \cup R^{\leftarrow})^0 = I \subseteq\ (R \cup R^{\leftarrow})^{k^2}\ thus\ it\ is\ reflexive$
$T:\ we\ can\ consider\ \ (R \cup R^{\leftarrow})\ as\ a\ big\ R\ from\ (e)\ we\ know\ it\ is\ transitive$
$S:\ For\ all\ x,\ y\ \in\ S:\ If\ (x,\ y)\ \in\ R\, then\ (y,\ x)\ \in\ R$
(1) case1 : $(R \cup R^{\leftarrow})^0\ \ is\ true\ \sin ce\ (R \cup R^{\leftarrow})^0 = I$ and I is $symmetric$
(2)case 2 $(R \cup R^{\leftarrow})^1\ \ is\ true\ \sin ce (a,b) \in R, (b,a) \in R^{\leftarrow} and\ vice\ versa$
(3)$assume\ there\ exist\ (a,b) \in (R \cup R^{\leftarrow})^{k^2}, (b,a) \notin (R \cup R^{\leftarrow})^{k^2}\ \ a \neq b$
$from\ (b)\ we\ know\ (R \cup R^{\leftarrow})^1 \subseteq (R \cup R^{\leftarrow})^{k^2}$
$thus\ (a,b) \notin (R \cup R^{\leftarrow})^{k^2}\ which\ is\ contradictory\ to\ our\ precondition$
$thus\ there\ do\ not\ exist\ such\ (a,b)$
$(a,b) \in (R \cup R^{\leftarrow})^{k^2}, (b,a) \in (R \cup R^{\leftarrow})^{k^2}\ would\ always\ be\ synchronous$
$combine (1), (2)\ it\ is\ symmetric$

$because\ (R \cup R^{\leftarrow})^{k^2}\ is\ R, T, S\ \rightarrow it\ is\ a\ an\ equivalence\ relation$

$(g) we\ know\ R \subseteq (R \cup R^{\leftarrow})^1 \subseteq (R \cup R^{\leftarrow})^{k^2}\ \ from\ (c) induction$
$thus\ for\ all\ a \in R\ , a \in (R \cup R^{\leftarrow})^{k^2}$
$for\ (f)\ we\ know\ (R \cup R^{\leftarrow})^{k^2}\ is\ a\ equivalence\ relation$
$the\ definition\ of\ minimum : x \prec= y\ for\ all\ y \in S$
$assume\ there\ is\ T^{k^2-1}\ \subseteq (R \cup R^{\leftarrow})^{k^2}\ which\ is\ a\ equivalence\ relation$
$contain\ R,\ and\ it\ have\ less\ elements,\ at\ least\ one \rightarrow it\ can\ only\ have\ k^2 -$

$1\ elements$
$if(x,y) \in T^{k^2-1}, (y,z) \in T^{k^2-1} \to (x,z) \in T^{k^2-1}; T^{k^2-1}$
$from\ (d) : T^{k^2-1}; T^{k^2-1} = T^{2k^2-2}$
$if\ it\ is\ transitive,\ (x,z) \in T^{k^2-1}\ thus\ T^{2k^2-2} should\ be\ equal\ to\ T^{k^2-1}\ or\ contain\ more\ elements$
$thus \to 2k^2 - 2 \geq k^2 - 1 \to k^2 \geq 1$
$actually\ in\ order\ to\ form\ a\ closure : 2k^2 - 2 \geq k^2, k^2 \geq 2$
$thus\ it\ did\ not\ contain\ k = 0, and\ k = 1$
$smiliarly : for\ all\ T^{k^2-n}\ n \in N^+ if\ it\ is\ a\ equivalence\ relation\ contain\ R$
$2k^2 - 2n \geq k^2 - n\ k^2 \geq n\ there\ most\ be\ some\ k\ don't\ satisfy$
$n\ should\ be\ no\ bigger\ than\ 0,\ in\ other\ words\ only\ T^{k^2}\ or\ bigger\ satisfy$
$T^{k^2}\ would\ have\ more\ than\ k^2{-}1\ elements,\ it\ could\ not\ be\ subsets\ of\ (R \cup R^{\leftarrow})^{k^2}$
$thus\ (R \cup R^{\leftarrow})^{k^2}\ is\ the\ least\ upper\ bound$

4 problem4

from $week5's\ lecture, we\ can\ just\ ignore\ \lfloor\rfloor$
$f(n) = f(\frac{n}{3}) + 3f(\frac{n}{5}) + n$
$f(\frac{n}{3}) = f(\frac{n}{9}) + 3f(\frac{n}{15}) + \frac{n}{3}$
$f(\frac{n}{5}) = f(\frac{n}{15}) + 3f(\frac{n}{25}) + \frac{n}{5}$
$f(n) = f(\frac{n}{9}) + 3f(\frac{n}{15}) + \frac{n}{3} + 3f(\frac{n}{15}) + 9f(\frac{n}{25}) + 3 * \frac{n}{5} + n$
$f(n) = f(\frac{n}{9}) + 3f(\frac{n}{15}) + \frac{n}{3} + 3f(\frac{n}{15}) + 9f(\frac{n}{25}) + 3 * \frac{n}{5} + n$
$= f(\frac{n}{9}) + 6f(\frac{n}{15}) + 9f(\frac{n}{25}) + \frac{14n}{15} + n$
$f(\frac{n}{9}) = f(\frac{n}{27}) + 3f(\frac{n}{45}) + \frac{n}{9}$
$f(\frac{n}{15}) = f(\frac{n}{45}) + 3f(\frac{n}{75}) + \frac{n}{15}$
$f(\frac{n}{25}) = f(\frac{n}{75}) + 3f(\frac{n}{125}) + \frac{n}{25}$
$put\ the\ above\ 3\ to\ our\ f(n)\ equation$
$f(n) = f(\frac{n}{9}) + 6f(\frac{n}{15}) + 9f(\frac{n}{25}) + \frac{14n}{15} + n$
$= f(\frac{n}{27}) + 3f(\frac{n}{45}) + \frac{n}{9} + 6f(\frac{n}{45}) + 18f(\frac{n}{75}) + \frac{6n}{15} + 9f(\frac{n}{75}) + 27f(\frac{n}{125}) + 9\frac{n}{25}$
$+ \frac{14n}{15} + n = f(\frac{n}{27}) + 9f(\frac{n}{45}) + 27f(\frac{n}{75}) + 27f(\frac{n}{125}) + \frac{196n}{225} + \frac{14n}{15} + n$
$.......(i\ failed\ to\ u\sin g\ induction, but\ plz\ wait,\ maybe\ i\ worth\ more\ mark...)$
$seeing\ our\ foundation : f(n) = f(\frac{n}{3}) + 3f(\frac{n}{5}) + n, it\ add\ a\ n\ to\ equation$
$thus\ our\ added\ an = sumof(\frac{n}{t}\ inside\ those\ messy\ f(\frac{n}{t})) \times n$
$our\ sumof(\frac{n}{t}\ into\ those\ messy\ f(\frac{n}{t}))\ \ downsize\ to\ (\frac{n}{3} + 3\frac{n}{5}) = (\frac{14}{15}n)$
$each\ time\ we\ u\sin g\ foundation\ to\ unwrap\ it,$
$we\ can\ found\ when\ we\ unwrap\ it\ ,\ it\ always\ add\ \ (\frac{14}{15})^t n\ after\ it (t \in N)$
$because\ f(0) = 0, we\ can\ just\ ignore\ those\ messy\ f(\frac{n}{t})\ when\ they\ become\ extreme\ small$
$f(n)\ would\ apporximate\ to$
$= 0 + n(1 + \frac{14}{15} + (\frac{14}{15})^2 + (\frac{14}{15})^3 .....)$
$u\sin g\ sum\ of\ Geometric\ sequence$
$\frac{1-(\frac{14}{15})^n}{1-\frac{14}{15}}\ \ when\ n \to \infty\ (\frac{14}{15})^n \to 0$

$$\frac{1-(\frac{14}{15})^n}{1-\frac{14}{15}} \to 15$$
$f(n) \to 15n \in O(n)$

5 problem5

(a)when $it\ is\ empty, count\ is\ 0, else\ we\ count\ its\ left-tree\ and\ its\ right-tree$

$$count = \begin{cases} 0 & T = \tau \ \ (base\ case 1) \\ count(T_{left}) + count(T_{right}) + 1 & T \neq \tau\ (recursive) \end{cases}$$

$(b) when\ it\ is\ empty, leave\ is\ 0, when\ T = [\tau, \tau]\ it\ have\ no\ successor\ so\ leave\ is\ 1, else\ we\ try\ to\ count\ its\ left\ and\ right$

$$leaves = \begin{cases} 0 & T = \tau (base\ case 1) \\ 1 & T = [\tau, \tau] (base\ case 2) \\ leaves(T_{left}) + leaves(T_{right}) & T \neq \tau\ and\ T \neq [\tau, \tau]\ (recursive) \end{cases}$$

$(c) when\ it\ is\ empty\ it\ has\ 1\ half-leaves, else\ we\ recursive\ it$

$$half-leaves = \begin{cases} 1 & T = \tau \\ 0 & T = [\tau, \tau] \\ half - leaves(T_{left}) + half - leaves(T_{right}) & T \neq \tau, T \neq [\tau, \tau] \end{cases}$$

$(d) basic\ case:\ T = \tau$
$count(T) = 0 = 2 \times 0 + 1 - 1 = 2 \times$
$leaves(T)\ +\ half - leaves(T)\ -\ 1$
$T = [\tau, \tau]$
$count(T) = 1 = 2 \times 1 - 0 - 1 = 2 \times leaves(T)\ +\ half - leaves(T)\ -\ 1$
$thus\ two\ basic\ case\ holds$
$Inductive\ case:$
$(1) let\ us\ assume\ P(T1)\ holds, P(T2)\ holds$
$and\ not\ both\ of\ them\ are\ \tau$
$we\ need\ proof\ P([T1, T2])\ holds$
$count([T1, T2]) = count(T1) + count(T2) + 1\ \ \ (definition\ in\ (a)$
$=\ [2 \times leaves(T1) + half - leaves(T1) - 1]$
$+ 2 \times leaves(T2) + half - leaves(T2) - 1 + 1\ (T1, T2\ holds\ respectively)$
$= 2[leaves(T1) + leaves(T2)] + [half - leaves(T1) + half - leaves(T2)] - 1$
$= 2 \times leaves([T1, T2]) + half - leaves([T1, T2]) - 1 (definition\ in\ (b)\ and\ (c)$
$thus\ P([T1, T2])\ holds\ when\ not\ both\ of\ them\ are\ \tau$
$thus\ it\ holds\ for\ all\ cases$

6 problem6

(a)analyse $the\ sum\ function\ line\ by\ line$
$sum(A, B):$
$for\ i\ \in\ [0,\ n):\qquad O(1)\qquad |$
$for\ j\ \in\ [0,\ n):\qquad O(1)|\qquad |$
$C[i,\ j]\ =\ A[i,\ j]\ +\ B[i,\ j]\qquad O(1)|\quad O(n)\ times\ |\quad O(n)\ times$
$end\ for$
$end\ for$
$return\ C\qquad O(1)$
$running\ time: O(n^2)\ +\ O(n) + O(1)\ =\ O(n^2)$
$thus\ big\ O\ could\ be\ n^2$

$(b) analyse\ the\ product\ function\ line\ by\ line$
$function(A, B):$
$for\ i\ \in\ [0,\ n):\qquad O(1)\qquad |$
$for\ j\ \in\ [0,\ n):\qquad O(1)|\qquad |$
$C[i,\ j]\ = addA[i, k] * B[k, j] : k \in [0, n)\ O(1)|O(n)\ times\quad |O(n)\ times\quad |O(n)\ times$
$end\ for$
$end\ for$
$return\ C\qquad O(1)$
$running\ time: O(n^3)\ +\ O(n^2) + O(n) + O(1)\ =\ O(n^3)$
$thus\ big\ O\ could\ be\ n^3$

$(c) justification\ the\ equation, we\ can\ see\ we\ need\ to\ multipy\ 8times$
$aftering\ breaking\ the\ matrices\ into\ smaller\ submatrices\ ,$
$each\ submatrice\ is\ \frac{n}{2}\ long\ , except\ n = 1 : which\ is\ O(1)$
$then\ we\ need\ to\ add\ it\ up\ four\ times$
$from\ (a) we\ know\ the\ running\ time\ is\ O(n^2)$

$$T(n) \begin{cases} 1 & n = 1 \\ 8T(\frac{n}{2}) + O(n^2) & n > 1 \end{cases}$$

$(d)\ we\ can\ solving\ this\ by\ master\ theroem$
$b = 2, a = 8, c = 2$
$d = \log_2 8 = 3 > c$
$thus\ T(n) = O(n^d)$
$= O(n^3)$