Problem 1
Consider the following program with two unspecified lines.

```
for j = 1 to n :
  (*)
  while i > 1 :
    print i
    (**)
  end while
end for
```

Give an asymptotic upper bound on the running time, in terms of n for the given program when the missing lines are specified as follows:

(a) $(*) : i = n \qquad (**) : i = i - 1$

(b) $(*) : i = n \qquad (**) : i = i/2$

(c) $(*) : i = j \qquad (**) : i = i - 2$

(d) $(*) : i = j \qquad (**) : i = i/2$

Problem 2
Let $\Sigma = \{0, 1\}$

(a) Recursively define a function $\mathsf{str2num} : \Sigma^+ \to \mathbb{N}$ that converts a non-empty word over Σ to the number that one obtains by viewing the word as a binary number. For example $\mathsf{str2num}(1100) = 12$, $\mathsf{str2num}(0111) = 7$, $\mathsf{str2num}(0000) = 0$.

(b) Recursively define a function $\mathsf{num2str} : \mathbb{N} \to \Sigma^+$ that converts a number to its (shortest) binary representation. *Hint: you may want to use* div *and* %.

(c) Writing your functions as code in the natural way,

(i) Give an asymptotic upper bound in terms of $\text{length}(()w)$ on the running time to compute $\mathsf{str2num}(w)$.

(ii) Give an asymptotic upper bound in terms of n on the running time to compute $\mathsf{num2str}(n)$.

Problem 3
Consider the procedure given in lectures to simulate a die using a fair coin:

(A) Flip a coin 3 times.

(B) If the outcome was:

- HHH: Output 1

- HHT: Output 2
- HTH: Output 3
- HTT: Output 4
- THH: Output 5
- THT: Output 6
- TTH: Go to (A)
- TTT: Go to (A)

What is the expected number of coin flips to obtain an output?

Problem 4
We want to tile a $2 \times n$ rectangle with 2×1 tiles so that the rectangle is completely covered and no tiles are overlapping. For example, here are two different ways to tile a 2×3 rectangle:

How many different ways (ignoring symmetry) are there of tiling a $2 \times n$ rectangle with 2×1 tiles in this way?

Problem 5
A tennis doubles match consists of two teams of two players per team. Ordering between teams, and within teams is not considered.

(a) How many different tennis doubles matches can be made with 4 players?

(b) How many different tennis doubles matches can be made with 5 players?

(c) How many different tennis doubles matches can be made from n players?