

Fundamentals (Numbers, Sets, Words, Functions, and Relations)

Problem 1

How many numbers are there between 100 and 1000 that are

- (a) divisible by 3?
 - (b) divisible by 5?
 - (c) divisible by 15?
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Problem 2

Let $\Sigma = \{a, b, c\}$ and $\Phi = \{a, c, e\}$.

- (a) How many words are in the set Σ^2 ?
 - (b) What are the elements of $\Sigma^2 \setminus \Phi^*$?
 - (c) Is it true that $\Sigma^* \setminus \Phi^* = (\Sigma \setminus \Phi)^*$? Why?
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Problem 3

Prove that $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$

Problem 4

Consider the relation $R \subseteq \mathbb{R} \times \mathbb{R}$ defined by aRb if, and only if, $b + 0.5 \geq a \geq b - 0.5$. Is R

- (a) reflexive?
 - (b) antireflexive?
 - (c) symmetric?
 - (d) antisymmetric?
 - (e) transitive?
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Problem 5

For each of the following statements, provide a valid proof if it is true for all sets S and all relations $R_1 \subseteq S \times S$ and $R_2 \subseteq S \times S$. If the statement is not always true, provide a counterexample.

- (a) If R_1 and R_2 are symmetric, then $R_1 \cap R_2$ is symmetric.
- (b) If R_1 and R_2 are antisymmetric, then $R_1 \cup R_2$ is antisymmetric.