Sec. 2.2 36. Define $A \oplus B = \{x | x \in A \cup B \wedge\ x \notin A \cap B\} = (A \cup B) \cap (\overline{A \cap B})$.
Show $A \oplus B = (A - B) \cup (B - A)$.

$$
\begin{aligned}
&(A-B)\cup(B-A) & \\
&\quad = (A \cap \bar{B}) \cup (B \cap \bar{A}) && \text{definition of complement} \\
&\quad = [(A \cap \bar{B}) \cup B] \cap [(A \cap \bar{B}) \cup \bar{A}] && \text{distributive law} \\
&\quad = [(A \cup B) \cap (\bar{B} \cup B)] \cap [(A \cup \bar{A}) \cap (\bar{B} \cup \bar{A})] && \text{distributive law} \\
&\quad = [(A \cup B) \cap U] \cap [U \cap (\bar{B} \cup \bar{A})] && \text{complement law} \\
&\quad = (A \cup B) \cap (\bar{B} \cup \bar{A}) && \text{domination law} \\
&\quad = (A \cup B) \cap (\overline{A \cap B}) && \text{commutative law, de Morgan's law.}
\end{aligned}
$$

Sec. 2.2 40. We shall show that $(A \oplus B) \oplus C = A \oplus (B \oplus C)$ is true. Before we do so, we shall make use of two simplier identities:

Fact 1: $(A \cup B) - C = (A - C) \cup (B - C)$.
This is established in additional problem 2h.
Fact 2: $A - (B \oplus C) = (A \cap \bar{B} \cap \bar{C}) \cup (A \cap B \cap C)$.
Let us prove this one:

$$
\begin{aligned}
&A - (B \oplus C) & \\
&\ = A - (B \cup C - B \cap C) && \text{definition of } \oplus \\
&\ = A \cap \overline{B \cup C \cap \overline{B \cap C}} && \text{definition of complement} \\
&\ = A \cap [\overline{B \cup C} \cup (B \cap C)] && \text{de Morgan's Law} \\
&\ = (A \cap \bar{B} \cap \bar{C}) \cup (A \cap B \cap C) && \text{de Morgan's law and distributive law.}
\end{aligned}
$$

We are now ready to prove the associativity of $\oplus$.

$$
\begin{aligned}
&(A \oplus B) \oplus C & \\
&\ = [A \oplus B - C] \cup [C - A \oplus B] && \text{definition of } \oplus \\
&\ = [((A - B) \cup (B - A)) - C] \cup [(C \cap \bar{A} \cap \bar{B}) \cup (C \cap A \cap B)] && \text{definition of } \oplus\text{, fact 2} \\
&\ = [(A - B) - C \cup (B - A) - C] \cup [(C \cap \bar{A} \cap \bar{B}) \cup (C \cap A \cap B)] && \text{fact 1} \\
&\ = [(A \cap \bar{B} \cap \bar{C}) \cup (B \cap \bar{A} \cap \bar{C})] \cup [(C \cap \bar{A} \cap \bar{B}) \cup (C \cap A \cap B)] && \text{definition of complement}
\end{aligned}
$$

$$
\begin{aligned}
&A \oplus (B \oplus C) & \\
&\ = [A - B \oplus C] \cup [B \oplus C - A] && \text{definition of complement} \\
&\ = [(A \cap \bar{B} \cap \bar{C}) \cup (A \cap B \cap C)] \cup [((B - C) \cup (C - B)) - A] && \text{fact 2, defintion of } \oplus \\
&\ = [(A \cap \bar{B} \cap \bar{C}) \cup (A \cap B \cap C)] \cup [((B - C) - A) \cup ((C - B) - A)] && \text{fact 1} \\
&\ = [(A \cap \bar{B} \cap \bar{C}) \cup (A \cap B \cap C)] \cup [(B \cap \bar{C} \cap \bar{A}) \cup (C \cap \bar{B} \cap \bar{A})] && \text{definition of complement}
\end{aligned}
$$

It is straightforward to check now that the two sets are in fact equal.

Additional Problem 2: True or False.

a. $A - (B - C) = (A - B) - C$

False. Let $A = \{1, 2, 3, 4, 5\}, B = \{4, 5\}, C = \{5\}$. $B - C = \{4\}$ while $A - B = \{1, 2, 3\}$. Thus, $A - (B - C) = \{1, 2, 3, 5\}$ while $(A - B) - C = \{1, 2, 3\}$. The two sets are not equal.

b. If $A \cup C = B \cup C$ then $A = B$.

False. Let $A = \{a\}, B = \{b\}$ and $C = \{a, b\}$. $A \cup C = B \cup C = C$ but $A \neq B$.

c. If $A \cap C = B \cap C$ then $A = B$.

False. Let $A = \{a, b\}, B = \{a, c\}$ and $C = \{a\}$. $A \cap C = B \cap C = C$ but $A \neq B$.

d. If $A = B - C$ then $B = A \cup C$.

False. Let $B = \{0, 1, 2\}$ and $C = \{0, 3\}$ so that $A = B - C = \{1, 2\}$. Note, however, that $A \cup C = \{0, 1, 2, 3\} \neq B$.

e. $|A - B| = |A| - |B|$.

False. Let $A = \emptyset$, $B = \{b\}$. $|A - B| = |\emptyset| = 0$ but $|A| - |B| = 0 - 1 = -1$.

f. $(A - B) \cup B = A$.

False. Let $A = \{0, 1, 2\}, B = \{0, 3\}$. Thus, $A - B = \{1, 2\} \cup B = \{0, 1, 2, 3\} \neq A$.

g. $(A \cup B) - B = A$.

False. Use the same A and B above.

h. $(B - A) \cup (C - A) = (B \cup C) - A$.

True.

$$
\begin{aligned}
(B \cup C) - A &= (B \cup C) \cap \bar{A} && \text{definition of complement} \\
&= (B \cap \bar{A}) \cup (C \cap \bar{A}) && \text{distributive law} \\
&= (B - A) \cup (C - A) && \text{definition of complement}
\end{aligned}
$$