assignment1 part2

Yuchen Wu

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1 problem6

(a)
(1)f(a)=f(b)=f(c)=0
(2)f(a)=1,f(b)=0,f(c)=0
(3)f(a)=1,f(b)=1,f(c)=0
(4)f(a)=1,f(b)=1,f(c)=1
(5)f(a)=0,f(b)=1,f(c)=0
(6)f(a)=0,f(b)=1,f(c)=1
(7)f(a)=0,f(b)=0,f(c)=1
(8)f(a)=1,f(b)=0,f(c)=1
(b)we $can\ assume\ a\ function\ that\ for\ each\ set\ in\ pow(a,b,c), t \in \{a,b,c\}$
$if\ t \in this\ set\ then\ f(t) = 1,\ else\ f(t) = 0$
$than\ we\ would\ get\ result\ in\ (a)$
$from\ another\ perspective, we\ already\ know |Pow(X)|\ =\ 2^{|X|}$
$|Pow(\{a,b,c\})|\ =\ 2^3 = 8\ which\ is\ equals\ to\ the\ number\ in\ (a)$
$(c)\{w \in \{0,1\}* : length(w) = 3\}\ is\ equal\ to$
$\{000, 100, 110, 111, 010, 011, 001, 101\}$
$the\ connection\ could\ be : for\ each\ element\ in\ \{w \in \{0,1\}* : length(w) = 3\}$
$its\ first\ number\ is\ the\ value\ of\ a,\ second\ number\ is\ the\ value\ of\ b$
$third\ number\ is\ the\ value\ of\ c$

2 problem7

if $we\ need\ to\ proof\ bijection$
$we\ need\ to\ proof\ 1) If\ f(x) = f(y)\ then\ x = y \quad (Injective)$
$2) Im(f)\ =\ Codom(f) \qquad\qquad (surjective)$
$we\ need\ to\ have\ \varphi\ that\ \varphi : \left(A^B\right)^C \to A^{B\times C}$
$A^{B\times C}\ is\ set\ which\ take\ a\ pair\ of\ B\ and\ C\ then\ return\ an\ element\ of\ A$
$\left(A^B\right)^C\ \ would\ take\ an\ element\ of\ C\ and\ return\ a\ function\ from\ B\ into\ A$
$we\ can\ proof\ Injective\ by\ if\ x \neq y\ then\ f(x) \neq f(y)$

$suppose\ we\ have\ g, f \in \left(A^B\right)^C\ \ and\ \ g \neq f,$
$\varphi\ in\ f\ can\ have\ another\ expression:\ f(c)(b)\ (take\ c \in C\ as\ first\ parameter$
$and\ b \in B\ as\ second\ parameter)\ , and\ the\ same\ as\ g$

3 problem8

(a)(R1;R2)R3={(a,c):there $is\ a\ b\ with\ (a,b) \in R1\ and\ (b,c) \in R2\}; R3$
$= \{(a,d) : there\ is\ a\ b\ with\ (a,b) \in R1\ and\ (b,c) \in R2$
$and\ there\ is\ a\ c\ with\ (a,c) \in (R1; R2)\ and\ (c,d) \in R3\}$
$= \{(a,d) : there\ are\ b, c\ with\ (a,b) \in R1\ and\ (b,c) \in R2$
$and\ (c,d) \in R3\}$
$= \{(a,d) : there\ is\ a\ with\ (a,b) \in R1$
$and\ there\ is\ a\ c\ with\ (a,b) \in R1\ and\ (b,d) \in (R2; R3)\}$
$= R1; \{(b,d) : there\ is\ a\ c\ with\ (b,c) \in R2\ and\ (c,d) \in R3\}$
$= R1; (R2; R3)$
$(b) I; R1 = \{(a,c) : there\ is\ a\ b\ with(a,b) \in I\ and(b,c) \in R1\}\ (a,b,c \in S)$
$when\ I = \{(x,x) : x \in S\} : (a,b) \in I\ means\ a = b\ and\ a, b \in S$
$I; R1 = \{(a,c) : there\ is\ a\ b\ with\ b \in S\ and\ a = b\ and\ (b,c) \in R1\}\ (a,b,c \in S)$
$= \{(a,c) : there\ is\ a\ b\ with\ a = b\ and\ (b,c) \in R1\} \quad (a,b,c \in S)$
$= \{(b,c) : there\ is\ a\ b\ with\ (b,c) \in R1\} \quad (b,c \in S)$
$= \{(b,c) : (b,c) \in R1\}\ \ (b,c \in S)$
$= R1$
$= \{(a,b) : there\ is\ a\ b\ with\ (a,b) \in R1\}\ (a,b \in S)$
$= \{(a,c) : there\ is\ a\ b\ with\ (a,b) \in R1,\ b = c\}\ (a,b,c \in S)$
$= \{(a,c) : there\ is\ a\ b\ with\ (a,b) \in R1,\ b = c\ and\ b, c \in S\}\ (a,b,c \in S)$
$= \{(a,c) : there\ is\ a\ b\ with\ (a,b) \in R1,\ (b,c) \in I\}\ (a,b,c \in S)$
$= R1; I$
$(c)(R1 \cup R2);\ R3$
$= \{(a,c) : there\ is\ a\ with\ (a,\ b) \in (R1 \cup R2)$
$and\ (b,c) \in R3\} \qquad (a,b,c \in S)$
$= \{(a,c) : there\ is\ a\ b\ with\ (a,b) \in R1\ and\ (b,c) \in R3$
$or\ there\ is\ a\ b\ with\ (a,b) \in R2\ and\ (b,c) \in R3\}$
$= \{(a,c) : there\ is\ a\ b\ with\ (a,b) \in R1\ and\ (b,c) \in R3\} or$
$\{(a,c) : there\ is\ a\ b\ with\ (a,b) \in R2\ and\ (b,c) \in R3\}$
$= (a,c) \in R1; R3\ or\ (a,c) \in R2; R3$
$= (R1;\ R3) \cup (R2;\ R3)$
$(d) that\ is\ false,\ for\ example\ if\ R1 = \{(2,1),(2,3)\}, R2 = \{(1,5)\}, R3 = \{(3,5)\}$
$then\ R2 \cap R3 = \rightarrow R1; (R2 \cap R3) =$
$but\ R1; R2 = \{(2,5)\}; R1; R3 = \{(2,5)\}$
$thus\ (R1; R2) \cap (R1; R3) = \{(2,5)\}$