The University of New South Wales

Mid-Session Test

2012-08-20

COMP9020

Foundations of Computer Science

Time allowed: **1 hour**

Total number of questions: **9**

Maximum number of marks: **25**

Not all questions are worth the same.

Answer all questions

Textbooks, lecture notes, etc. are not permitted.

Calculators may not be used.

Answers must be written in ink. Use a pencil or the back of the booklet for rough work. Your rough work will not be marked.

You can answer the questions in any order.

You may take this question paper out of the exam.

Write your answers into the answer booklet provided.

Question 1 (1 mark)

How many integers that are not divided by 3 are there between 123 and 66789?

Question 2 (1 mark)

What is $\gcd(9876543213, 9876543210)$?

Question 3 (4 marks)

Let S, T, U, and V be sets. Prove that $(S \cap T) \times (U \cap V) = (S \times U) \cap (T \times V)$.

Question 4 (3 marks)

Let $\phi_1 = (p \Rightarrow (q \vee r))$, $\phi_2 = (s \Rightarrow (q \vee p))$, and $\phi = \phi_1 \wedge \phi_2$.

  (a) Draw Karnaugh maps for the three formulae, ϕ_1, ϕ_2, and ϕ.

  (b) Read off a minimal DNF for ϕ.

  (c) Give a minimal CNF for $\neg \phi_1$.

Question 5 (4 marks)

Suppose Portia puts a dagger in one of three caskets and places the following inscriptions on the caskets:

Gold casket: The dagger is in this casket.

Silver casket: The dagger is not in this casket.

Lead casket: At most one of the caskets has a true inscription.

Portia tells her suitor to pick a casket that does not contain the dagger. Which casket should the suitor choose? Formalise the problem in propositional logic and calculate an answer.

Question 6 (1 mark)

In $\mathbb{B}^5$, what is the value of $(0, 0, 1, 1, 1) \wedge (0, 1, 0, 1, 0)$?

Question 7 (3 marks)

How many Boolean algebra isomorphisms of $\mathcal{P}(\{a, b, c\})$ onto $\mathbb{B}^3$ are there? Explain your answer briefly.

Question 8 (4 marks)

Let S be a finite set and let $n \in \mathbb{N}$. How many

(a) functions,

(b) onto functions,

(c) binary relations, and

(d) n-ary relations

are there on S? Explain your answers briefly.

Question 9 (4 marks)

Let S, T, and U be sets. Let $R_1 \subseteq S \times T$ and $R_2 \subseteq T \times U$. Prove that $(R_1; R_2)^{\leftarrow} = (R_2^{\leftarrow}); (R_1^{\leftarrow})$.

Appendix: Non-Textbook Notation

Let S, T, and U be sets. Let $R_1 \subseteq S \times T$ and $R_2 \subseteq T \times U$. Their *sequential composition* $R_1; R_2$ is defined as $\{ (s, u) : \exists t \, ((s, t) \in R_1 \wedge (t, u) \in R_2) \}$.