
Graphs

Problem 1

True or false?

- (a) The complete bipartite graph $K_{5,5}$ has no cycle of length five.
 - (b) If T is a tree with at least four edges, then $\chi(T) = 3$.
 - (c) Let C_n denote a cycle on n vertices. For all $n \geq 5$ it holds $\chi(C_n) \neq \chi(C_{n-1})$.
 - (d) It is possible to remove two edges from K_6 so that the resulting graph has a clique number of 4.
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Problem 2

What is the minimum number of edges that need to be removed from K_5 so that the resulting graph has a chromatic number of

- (a) 3?
 - (b) 2?
 - (c) 1?
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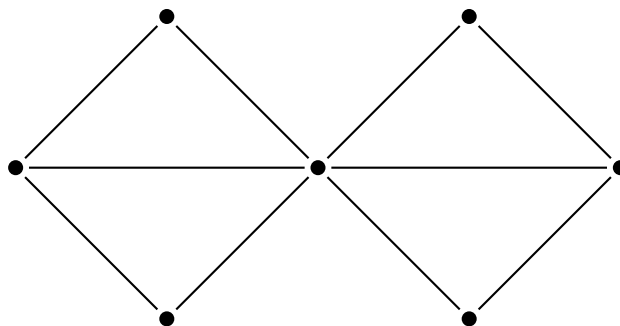
Problem 3

Consider the complete 3-partite graphs $K_{4,1,1}$, $K_{3,2,1}$, $K_{2,2,2}$.

- (a) What is the chromatic number of each of these graph?
 - (b) Which of these graphs are planar?
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Problem 4

Consider the following graph, G :



- (a) What is the chromatic number of G ?

- (b) What is the clique number of G ?
 - (c) Does G have a Hamiltonian path and/or a Hamiltonian cycle?
 - (d) Does G have an Eulerian path and/or an Eulerian cycle?
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Problem 5

Draw a single graph with 6 vertices and 10 edges that satisfies each of the following:

- (a) is planar,
- (b) contains a Hamiltonian circuit, and
- (c) does not contain an Eulerian path.