

COMP9020 Week 2 Recap and Administrivia

Assignment 1

Assignment 1: due **12 noon (Sydney time [daylight savings!])**,
Tues 13th October.

- Lateness penalty: 10% (of raw mark) every 12 hours or part thereof
- Unable to meet deadline through illness/injury/misadventure: apply for Special Consideration
- Submission via webCMS or give – single pdf file, 2Mb maximum size.
- Submissions should be typed, not handwritten.
- Typing math symbols: unicode, \LaTeX

Assignment 1: Marking

- Looking for **ability** and **understanding**
 - Don't do too little, don't do too much (if in doubt, ask!)
 - External resources can be used, but should not be necessary
 - Can (and do) use results from lectures/assignments/quizzes
- Marks do not necessarily reflect difficulty
- Marking generally done on a 4 or 5 point percentage scale:
 - 0%: No reasonable attempt
 - 20-25%: Shows promise
 - 50%: Major errors; 3+ minor errors – not demonstrating one of understanding or ability
 - 75-80%: Minor errors
 - 100%: Excellent answer, clearly demonstrating understanding and ability

Assignment 1: Comments

- Q1: Bézout's Identity
- Q2: Use (a) for (b) and use (b) for (c).
- Q3 & Q7: "Difficult" questions - though partial marks are available.
- Q4:
 - Laws of set operations; Principle of duality; Uniqueness of complement – ok
 - Limit use of derived rules to those proven (idempotence, double complement, any self-proven result)
 - Online tool (up soon)
- Q5 and Q8:
 - Proofs need to be general (true for arbitrary sets)
 - Counterexamples need to be concrete (specific example)
- Q6: (b) & (c) Any connection you observe; why if possible

Week 2 Recap

Sets:

- Set notation: $\in, \emptyset, \mathcal{U}, \subseteq, \{\dots\}, [\dots], (\dots)$
- Set operations: $\cap, \cup, ^c, \setminus, \oplus, \text{Pow}(), \times$
- Defining sets:
 - Explicitly listing elements
 - Subsets of a given set
 - Constructed using set operations
- Cardinality: $|X| = \#(X) = \text{card}(X)$
- Venn diagrams
- Laws of set operations

Formal languages:

- Symbols, words, languages
- Language definitions: Σ^* , $\text{length}()$, concatenation, Kleene star

Laws of Set Operations

For all sets A , B , C :

Commutativity

$$A \cup B = B \cup A$$

$$A \cap B = B \cap A$$

Associativity

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Distribution

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Identity

$$A \cup \emptyset = A$$

$$A \cap \mathcal{U} = A$$

Complementation

$$A \cup (A^c) = \mathcal{U}$$

$$A \cap (A^c) = \emptyset$$

Other useful set laws

Idempotence

$$A \cap A = A$$

$$A \cup A = A$$

Double complementation

$$(A^c)^c = A$$

Annihilation

$$A \cap \emptyset = \emptyset$$

$$A \cup \mathcal{U} = \mathcal{U}$$

de Morgan's Laws

$$(A \cap B)^c = A^c \cup B^c$$

$$(A \cup B)^c = A^c \cap B^c$$

Theorem (Principle of Duality)

If you can prove $A_1 = A_2$ using the Laws of Set Operations then you can prove $\text{dual}(A_1) = \text{dual}(A_2)$

Theorem (Uniqueness of complement)

$A \cap B = \emptyset$ and $A \cup B = \mathcal{U}$ if, and only if, $B = A^c$.

Exercises

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Show the following for all sets A , B , C :

- $B \cup (A \cap \emptyset) = B$
- $(C \cup A) \cap (B \cup A) = A \cup (B \cap C)$
- $(A \cap B) \cup (A \cup B^c)^c = B$

Exercises

Give counterexamples to show the following do not hold for all sets:

- $A \setminus (B \setminus C) = (A \setminus B) \setminus C$
- $(A \cup B) \setminus C = A \cup (B \setminus C)$
- $(A \setminus B) \cup B = A$

Need to know for this course

- How to define sets
- Cartesian product
- Cardinality computations
- Proofs using Laws of Set Operations
- How to define languages