Logic

Problem 1
Let F be the set of well-formed formulas with propositional variables from Prop. Define a relation, $R \subseteq F \times F$ by $(\varphi, \psi) \in R$ if $\varphi \models \psi$. Prove or give a counter-example to disprove:

(a) R is a partial order.

(b) $R \cup R^{\leftarrow}$ is an equivalence relation.

(c) $R \cap R^{\leftarrow}$ is an equivalence relation.

Problem 2
Prove that $\neg N$ follows logically from $H \wedge \neg R$ and $(H \wedge N) \rightarrow R$.

Problem 3
Consider the formulae $\phi_1 = (r \rightarrow p)$ and $\phi_2 = (p \rightarrow (q \vee \neg r))$. Transform the formula $\phi = (\neg q \rightarrow (\phi_1 \wedge \phi_2))$ into

(a) **DNF**, and

(b) **CNF**.

Simplify the result as much as possible.

Problem 4
Let $(T, \wedge, \vee, ', 0, 1)$ be a Boolean Algebra. Define $\oplus : T \times T \rightarrow T$ as follows:

$$x \oplus y = (x \wedge y') \vee (x' \wedge y)$$

(a) Prove using the laws of Boolean Algebra that for all $x \in T$, $x \oplus 1 = x'$.

(b) Prove using the laws of Boolean Algebra that $x \wedge (y \oplus z) = (x \wedge y) \oplus (x \wedge z)$.

(c) Find a Boolean Algebra (and x, y, z) which demonstrates that $x \oplus (y \wedge z) \neq (x \oplus y) \wedge (x \oplus z)$

Problem 5

(a) How many well-formed formulas can be constructed from one $\vee$; one $\wedge$; two parenthesis pairs (,); and the three literals p, $\neg p$, and q?

(b) Under the equivalence relation defined by **logical equivalence**, how many equivalence classes do the formulas in part (a) form?