

Solving Math Word Problems by Scoring Equations with Recursive Neural Networks

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Abstract

Solving math word problems is a cornerstone task in assessing language understanding and reasoning capabilities in NLP systems. Recent works use automatic extraction and ranking of candidate solution equations providing the answer to math word problems. In this work, we explore novel approaches to score such candidate solution equations using tree-structured recursive neural network (Tree-RNN) configurations. The advantage of this Tree-RNN approach over using more established sequential representations, is that it can naturally capture the structure of the equations. Our proposed method consists in transforming the mathematical expression of the equation into an expression tree. Further, we encode this tree into a Tree-RNN by using different Tree-LSTM architectures. Experimental results show that our proposed method (i) improves overall performance with more than 3% accuracy points compared to previous state-of-the-art, and with over 18% points on a subset of problems that require more complex reasoning, and (ii) outperforms sequential LSTMs by 4% accuracy points on such more complex problems.

Keywords: math word problems, recursive neural networks, information extraction, natural language processing

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1. Introduction

Natural language understanding often requires the ability to comprehend and reason with expressions involving numbers. This has produced a recent rise in interest to build applications to automatically solve math word problems (Kushman et al., 2014; Koncel-Kedziorski et al., 2015; Mitra & Baral, 2016; Wang et al., 2018b; Zhang et al., 2019). These math problems consist of a textual description comprising numbers with a question that will guide the reasoning process to get the numerical solution (see Fig. 1 for an example). This is a complex task because of (i) the large output space of the possible equations representing a given math problem, and (ii) reasoning required to understand the problem.

The research community has focused in solving mainly two types of mathematical word problems: *arithmetic word problems* (Hosseini et al., 2014; Mitra & Baral, 2016; Wang et al., 2017; Li et al., 2019; Chiang & Chen, 2019) and *algebraic word problems* (Kushman et al., 2014; Shi et al., 2015; Ling et al., 2017; Amini et al., 2019). Arithmetic word problems can be solved using basic mathematical operations ($+$ $-$ \times \div) and involve a single unknown variable. Algebraic word problems, on the other hand, involve more complex operators such as square root, exponential and logarithm with multiple unknown variables. In this work, we focus on solving *arithmetic word problems* such as the one illustrated in Fig. 1. This figure illustrates (a) *math word problem* statement, (b) the arithmetical formula of the *solution* to the problem, and (c) the *expression tree* representation of the solution formula where the leaves are connected to quantities and internal nodes represent operations.

The fact that arithmetic word problems involve only a single unknown makes it easy to represent their solution using a single tree structure (i.e., expression tree) as in Fig. 1(c). This way, several works have been proposed in the literature for solving math word problems using tree structured representations. Specifically, Koncel-Kedziorski et al. (2015) build a system called ALGES that, given a math word problem statement as input, uses Integer Linear Programming (ILP) constraint-based optimization to obtain possible expression trees of the candidate equations. To get the final solution, these candidates are jointly ranked by a (i) *local model* to link spans of text to operators, and

a (ii) *global model* to score the coherence of the entire expression tree. Similarly, Roy & Roth (2015, 2017) use manually engineered features to rank the candidate solutions. These features include lexicons (e.g., verbs like “give”, “add”, “receive”, etc., that get linked to specific operation types), as well as features that relate two quantity nodes in an expression tree with an operator (e.g., whether both quantities depend on the same verb, or one quantity is greater than another). Conversely, we propose a data-driven ranking method that doesn’t depend on previously defined features.

More recently, Wang et al. (2018b) applied expression trees in a reinforcement learning setting where the reward is positive whenever the mathematical operator between two quantities is correct. In this paper, we adopt tree structures to encode the expression tree directly in the neural network representation using tree-based Recursive Neural Networks (Tree-RNNs). This contrasts with predominantly sequential neural representations (Wang et al., 2017, 2018a; Chiang & Chen, 2019) that encode the problem statement from left to right/right to left. We demonstrate the deficiency of this sequential approach for capturing the logic necessary for solving more complex mathematical problems.

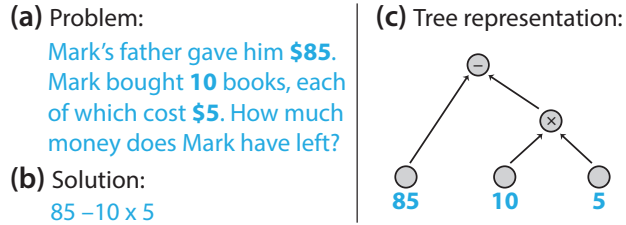


Figure 1: An example of arithmetic math word problem from the SingleEQ dataset. It illustrates the (a) *math word problem* statement, (b) the respective *solution* formula, and (c) the *expression tree* representing the solution.

By using Tree-RNN architectures, we can naturally embed the equation inside a tree structure such that the link structure directly reflects the various mathematical operations between operands selected from the sequential textual input. We hypothesize that this structured approach can efficiently capture the semantic representations of the candidate equations to solve more complex mathematical problems involving multiple and/or non-commutative operators. To test our results, we use the recently introduced

SingleEQ dataset (Koncel-Kedziorski et al., 2015). It contains a collection of 508 math word problems with varying degrees of complexity. This allows us to track the performance of the evaluated systems on subsets that require different reasoning capabilities. More concretely, we subdivide the initial dataset into different subsets of varying reasoning complexity (i.e., based on number of operators, commutative (symmetric) or non-commutative (asymmetric) operations), to investigate whether the performance of the proposed architecture remains consistent across problems of increasing complexity.

We define two Tree-RNN models inspired by the work of Tai et al. (2015): (i) T-LSTM (Child-Sum Tree-LSTM), and (ii) NT-LSTM (N-ary Tree-LSTM). The main difference between the two is that, while in T-LSTM the child node representations are summed up, in NT-LSTM they are concatenated. We show that NT-LSTM is more suitable to deal with equations that involve non-commutative operators because this architecture is able to capture the order of the operands. We also compare our Tree-RNN models with a bi-directional Long Short Term Memory (LSTM; Hochreiter & Schmidhuber (1997)) model (BiLSTM). After conducting a thorough multi-fold experimentation phase involving multiple random weight re-initializations in order to ensure the validity of our results, we will show that the main added value of our Tree-LSTM-based models compared to state-of-the-art methods lays in an increased performance for more complex math word problems.

More concretely, our contribution is three-fold: (i) we propose several neural baseline architectures (e.g., using sequential LSTMs) for solving math word problems, (ii) propose using Tree-LSTMs to embed structural information of the equation, and (iii) we perform an extensive experimental study on the SingleEQ dataset, showing an overall accuracy improvement of 3%, including an increase $>18\%$ for more complex problems (i.e., requiring multiple and non-commutative operations), compared to previous state-of-the-art results.

2. Related work

Over the last few years, there has been an increasing interest in building systems to solve *arithmetic word problems*. The adopted approaches can be grouped in three main

categories: (i) Rule-based systems, (ii) Statistical systems, and (iii) Neural network systems.

Rule-based systems: The first attempts to solve arithmetic problems date back to the
85 1960s with the work by Bobrow (1964), who proposed and implemented STUDENT,
a rule-based parsing system to extract numbers and operations between them by using
pattern matching techniques. Charniak (1968, 1969) extended STUDENT by including
basic coreference resolution and capability to work with rate expressions (e.g., “kms
per hour”). On the other hand, Fletcher (1985) designed and implemented a system
90 that given a propositional representation of a math problem¹, applies a set of rules to
calculate the final solution. The disadvantage of this system is that it needs a parsed
propositional representation of a problem as input and cannot operate directly on raw
text. This issue was tackled by Bakman (2007), who developed a schema-based system
that consisted of six main reasoning schemas, each one with slots to fill in. After
95 instantiating the schemas for a particular math problem using lexical verb-based rules,
the system could derive the corresponding mathematical equation to solve the problem.

The main disadvantages of such rule-based approaches are that they (i) rely on
hard-coded lexico-grammar rules, and (ii) lack an integrated view of the problem to
be solved, extracting operations one by one. We address these issues by proposing a
100 model that integrates the mathematical representation of a problem in a single struc-
tured expression tree. This way, we are able to capture the operator-operator and
number-operator relations involved in a particular mathematical expression in a uni-
fied manner. Furthermore, we are able to avoid the use of lexico-grammar hard-coded
rules (e.g., the use of pattern-based matching) when connecting numbers with the oper-
105 ators, replacing them by composition-semantic representations that link the arithmetic
operations with parameters (numbers or other operations) in a recursive tree. In conse-
quence, our solution is more generalizable by not depending on explicit hand-crafted

¹With propositions such as *GIVE Y X P9*, where entity Y gives to entity X the object defined in P9. This proposition in particular can be linked to the first sentence of example in Fig. 1: “Mark’s father gave him \$85”, where Y represents “Mark’s father”, X represents “him” which is coreferenced to “Mark”, and P9 represents “\$85” that are being given.

logic.

Statistical systems: Recently, there has been a shift towards statistical feature-driven systems that automatically produce models by capturing patterns present in math word problem datasets. For example, Hosseini et al. (2014) presented an inductive model that links specific lexicon-based features (e.g., verb categories) to equation operators. The mathematical solution to the problem is built sequentially using state transitions related to operators that are triggered by different verb categories found in the problem statement. On the other hand, Mitra & Baral (2016) connected carefully designed features to equation templates in order to solve specific problem types. While these techniques produced competitive results, they were limited to summation (+) and subtraction (−) operations on a very narrow problem set domain. In order to solve more diverse types of problems that also involve multiplication and division operators, the community shifted towards more integrated approaches involving tree structure representations. Koncel-Kedziorski et al. (2015) proposed to rank candidate expression trees by training jointly a *local* model to link spans of text with operator tree nodes, and a *global* model that is used to score the consistency of an entire tree. The list of candidates to these two models is generated by an ILP constraint optimization component that, given a set of extracted numbers from a math word problem text as input, produces a set of candidate solution equations. Conversely, Roy & Roth (2015, 2017) introduced the concept of *monotonic expression tree* to generate candidates. It defines a set of conditions (e.g., two division and subtraction nodes cannot be connected to each other) that considerably restricts the expression tree search space. The authors propose to score the resulting monotonic expression trees jointly by summing up the scores of different classifiers related to a specific expression tree (e.g., the mathematical operator between two numbers in the tree, whether a particular number is related to a rate such as “kms per hour”, etc). Recently, the same authors (Roy & Roth, 2018) included additional latent declarative rules (e.g., $[\text{Verb1} \in \text{HAVE}] \wedge [\text{Verb2} \in \text{GIVE}] \wedge [\text{Coref}(\text{Subj1}, \text{Subj2})] \implies \text{Subtraction}$) to link textual expression patterns (derived from preliminary dependency parsing) to specific operations. While these statistical approaches rely on tree structures to evaluate the mathematical expressions, on one hand, they require high manual effort to engineer

the features and, on the other hand, it is hard to scale the features to capture operations
 140 between more than two numbers. This makes it challenging to apply such models to
 more complex equations that involve multiple operators. We tackle this problem by
 defining a single Tree-RNN structure that evaluates an entire mathematical expression
 at once. This is done by recursively combining the information from the child nodes
 in the expression tree and then using a backpropagation mechanism to correspondingly
 145 adjust the weights of our model. Furthermore, our equation ranking architecture does
 not depend on hand-crafted features and parsing-dependent rules, making it easier to
 integrate in different domains.

Neural network systems: Recently, as in all sub-domains of natural language pro-
 cessing, neural network architectures have been applied to tackle math word problems.
 150 The first contribution was made by Wang et al. (2017), who introduced a model trained
 to map problem statements to equation templates. Their model was expanded upon
 by Huang et al. (2018), who introduced an attention-based copy mechanism for to-
 kens representing numbers. They used a reinforcement learning setting, where positive
 rewards were assigned when the predicted mathematical expression resulted in a cor-
 155 rect answer. Recently, Chiang & Chen (2019) used stack structures inside a sequential
 encoder-decoder setting where the encoder captures the semantics of a math word prob-
 lem in a vector that is used by decoder to generate the equation to solve the problem.
 Moreover, Wang et al. (2018b) proposed the use of Q-Networks in order to generate
 expression trees, by giving positive reward whenever the operator between two num-
 160 bers is correct. The aforementioned studies, while showing promising results, were not
 designed to naturally capture the structural form of mathematical expressions when
 multiple operators are involved (e.g., $1 + (2/3)$ vs. $(1 + 2)/3$). We propose encoding
 equations with Tree-LSTMs (Tai et al., 2015), (i.e., recursive neural sequence models),
 which allow to naturally reflect the execution order of operations in an expression tree
 165 by recursively combining the children nodes’ semantic representations.

Tree-RNN models (Socher et al., 2011) have been shown to perform better for
 modeling data on tasks that have an inherently hierarchical structure. For example,
 Socher et al. (2011) proposed to use recursive models in order to model the com-
 positional structure of scene images (e.g., a scene image of a house can be split in

170 composing regions such as doors, windows, walls, etc.). The authors show that a Tree-RNN-based architecture outperforms previous methods in prediction of hierarchical structure of scene images and in scene image classification. Later, Socher et al. (2013) also showed how recursive structures can be used to encode the inherently hierarchical phrase structural grammar (e.g., the sentence “riding a bike” can be decomposed in
175 the verb “riding” and the noun phrase “a bike”, which itself can be decomposed into determiner “a” and the noun “bike”). This way, the authors achieved state-of-the-art performance in grammatical parsing of the sentences. More recently, Tai et al. (2015); Chen et al. (2017) showed how encoding the syntactic parsing trees of the sentence with Tree-RNN models can improve the performance in tasks such as sentiment clas-
180 sification and semantic relatedness (e.g., natural language inference). Similarly, we propose to take advantage of the inherently hierarchical representation of mathematical expression trees by encoding them using Tree-RNN architectures. Our experiments demonstrate that this representation can be helpful in capturing the semantic relations between operators needed in order to solve more complex mathematical problems con-
185 sisting of multiple and/or non-commutative operations.

3. Model

Shortly stated, our task at hand is to identify the correct arithmetic equation, corresponding to a mathematical problem expressed in natural language text. We follow a two-step approach similar to the work of Koncel-Kedziorski et al. (2015), which formalizes solving multi-sentence math word problems as the generation and ranking of
190 expression trees. The *first step* consists in generating candidate equations using the ILP optimization solver proposed in Koncel-Kedziorski et al. (2015). This solver uses a set of constraints (e.g., syntactic validity of the candidate equations, type consistency between the operands, etc.) in order to produce candidate equations. The *second step*
195 ranks these candidates and selects the top ranked one as the final answer to the mathematical word problem. We replace this equation scoring step with our proposed model whose architecture is sketched in Fig. 2 and comprises: (i) a word embedding layer, (ii) a bidirectional LSTM layer (BiLSTM), and (iii) an additional layer that encodes

the equation, using either BiLSTM- or Tree-LSTM- based approaches detailed below.

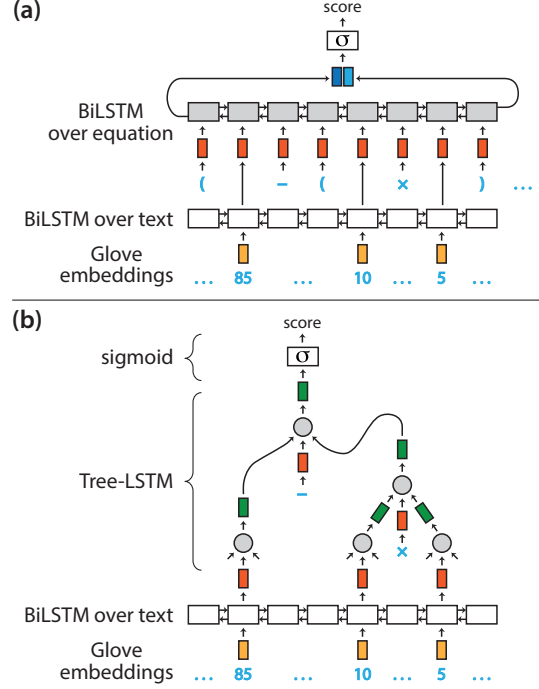


Figure 2: Models for scoring equations, taking the text and the equation from Fig. 1 to score (e.g., $85 - (10 \times 5)$) as input: (i) a word embedding layer at the bottom, (ii) a BiLSTM layer, and (iii) a top layer that encodes the equation. For the latter we consider either **(a)** a sequential BiLSTM, or **(b)** a structured Tree-LSTM.

The input to our model is a **sequence of tokens** of length N , $W = \{w_1, \dots, w_N\}$ of the math word problem, which we pass through an **embedding layer** to obtain embedded representations $X = \{x_1, \dots, x_N\}$ where $x_i \in \mathbb{R}^{d_1}$. We adopt a BiLSTM to obtain **contextual representations** of the tokens. The following is the formal representation

of the first LSTM layer:

$$i_t = \sigma(W_i x_t + U_i h_{t-1} + b_i) \quad (1)$$

$$o_t = \sigma(W_o x_t + U_o h_{t-1} + b_o) \quad (2)$$

$$f_t = \sigma(W_f x_t + U_f h_{t-1} + b_f) \quad (3)$$

$$u_t = \tanh(W_u x_t + U_u h_{t-1} + b_u) \quad (4)$$

$$c_t = f_t \odot c_{t-1} + i_t \odot u_t \quad (5)$$

$$h_t = o_t \odot \tanh(c_t) \quad (6)$$

where $t \in \{1, \dots, N\}$ represents a particular recursive execution time step and $h_t \in \mathbb{R}^{d_2}$ is the LSTM hidden state. $W_i, W_f, W_o, W_c \in \mathbb{R}^{d_2 \times d_1}$ and $U_i, U_f, U_o, U_c \in \mathbb{R}^{d_2 \times d_2}$ are the weight matrices related to different LSTM gates, and $b_i, b_f, b_o, b_c \in \mathbb{R}^{d_2}$ are the respective biases. In our experiments we initialize x_t with GloVe word embeddings (Pennington et al., 2014) and keep them static during training. In order to obtain the BiLSTM representation, we run two LSTMs in different directions and concatenate the respective hidden states. This results in N hidden state representations $H = \{h_1^{(b)}, \dots, h_N^{(b)}\}$ where $h_i^{(b)} \in \mathbb{R}^{d_3}$ and $d_3 = 2 \cdot d_2$. Using the input in H , we propose two different models to encode the candidate equations:

(a) Sequential LSTM: We perform an in-order traversal of the expression tree to obtain a sequential representation of the equation (e.g., $(85 - (10 \times 5))$) that is encoded using a second BiLSTM (see **(a)** in Fig. 2). We use as input the hidden state representations H calculated above for the numbers and (trainable) embeddings O for the operators $(+, -, \div, \times)$ and opening/closing parentheses. More formally, the input to BiLSTM is represented by $E = \{e_1^{(s)}, \dots, e_K^{(s)}\}$ where $e_i^{(s)} \in \{H \cup O\}$ and K is the number of tokens in the equation including parenthesis and operations. Thus, an equation like $(85 - (10 \times 5))$ contains 9 tokens. In order to predict, we concatenate the last (left and right) hidden states of the BiLSTM producing a vector of dimensionality d_4 , and then apply a linear transformation followed by a *sigmoid* function to obtain the equation’s score.

(b) Tree-LSTM: We base our Tree-RNN implementation on the Tree-LSTM architecture proposed by Tai et al. (2015). This architecture is based on the LSTM formulation

described in equations 1-6, but instead of being linearly linked, the input to a particular LSTM cell can come from different child step LSTM executions. More formally, we can describe the T-LSTM structure as follows:

$$\tilde{h}_t = \sum_{k \in C(t)} h_k \quad (7)$$

$$i_t = \sigma(W_i x_t + U_i \tilde{h}_t + b_i) \quad (8)$$

$$o_t = \sigma(W_o x_t + U_o \tilde{h}_t + b_o) \quad (9)$$

$$f_{tk} = \sigma(W_f x_t + U_f h_k + b_f) \quad (10)$$

$$u_t = \tanh(W_u x_t + U_u \tilde{h}_t + b_u) \quad (11)$$

$$c_t = i_t \odot u_t + \sum_{k \in C(t)} f_{tk} \odot c_k \quad (12)$$

$$h_t = o_t \odot \tanh(c_t) \quad (13)$$

220 where $C(t)$ is the set of child nodes for the current execution node at step t . In our case, $C(t)$ always consists of 2 children since the mathematical operations we are dealing with are binary (i.e., they take two arguments). $W_i, W_o, W_f, W_u \in \mathbb{R}^{d_4 \times d_3}$ together with $U_i, U_o, U_f, U_u \in \mathbb{R}^{d_4 \times d_4}$ are the weight matrices that transform the inputs $x_t \in \mathbb{R}^{d_3}$, the current hidden state $\tilde{h}_t \in \mathbb{R}^{d_4}$ and the children's hidden states $h_k \forall k \in C(t)$ correspond to the Tree-LSTM gate representations. As depicted in Fig. 2(b), the inputs x_t to the leaf nodes are the hidden state representations in H on the positions where the numbers occur in the problem statement. The input x_t to the inner nodes, on the other hand, are one of the randomly initialized operation embeddings $O \in \{o_-, o_+, o_\div, o_\times\}$ depending on the operation represented by the node. This contrasts with the original
230 setup proposed in Tai et al. (2015) where the input x_t always comes from the word representation in the sentence. By using a separate operation embeddings set O as input, we expect our model to be able to capture a semantic representation for each of the different operations $o \in O$. The Tree-LSTM model finally outputs the hidden state for the root of the expression tree (i.e., the last executed operation), which is then
235 passed through a sigmoid to deliver the equation score.

While T-LSTM allows to encode the equation information in a tree structure, it is

symmetric in its child nodes. This is because the hidden states of the children are first summed up in Eq. (7) before applying the linear transformation and the gate activation functions. This could be problematic for non-commutative operations ($-$ and \div) where the result depends on the order of the operands. Therefore, we introduce a second model called NT-LSTM that uses distinct weight matrices to transform each of the children’s hidden states. More formally, the gate definition in NT-LSTM is as follows:

$$i_t = \sigma \left(W_i x_t + \sum_{l=1}^N U_i^{(l)} h_t^{(l)} + b_i \right) \quad (14)$$

$$o_t = \sigma \left(W_o x_t + \sum_{l=1}^N U_o^{(l)} h_t^{(l)} + b_o \right) \quad (15)$$

$$f_{tk} = \sigma \left(W_f x_t + \sum_{l=1}^N U_f^{(kl)} h_t^{(l)} + b_f \right) \quad (16)$$

$$u_t = \tanh \left(W_u x_t + \sum_{l=1}^N U_t^{(l)} h_t^{(l)} + b_u \right) \quad (17)$$

$$c_t = i_t \odot u_t + \sum_{l=1}^N f_{tl} \odot c_l \quad (18)$$

$$h_t = o_t \odot \tanh(c_t) \quad (19)$$

where N is the number of child nodes, which in our case is always 2, and $k \in \{1, 2\}$ in Eq. (16). By introducing different weights U for each of the child node states, we make sure that the model can differentiate between the order of the operands. As we will show in Section 5, this change makes a big difference for equations involving

240 non-commutative operations.

4. Experimental setup

We evaluate the proposed models (code available upon acceptance) on the SingleEQ dataset introduced by Koncel-Kedziorski et al. (2015), which includes 508 mathematical problems of varying complexity (i.e., equations with single or multiple operators).

245 To obtain results comparable to previous work, we perform 5-fold cross-validation using the original splits defined in Koncel-Kedziorski et al. (2015). Furthermore, we partition the dataset into several subsets to investigate the effect of varying problem

Subset	Equation types	# Problems
Full	All operators	508
Single	Single operator	390
Multi	Multiple operators	118
Single _{sym}	Single symmetric operators	208
Multi _{sym}	Multiple symmetric operators	68
Single _{asym}	Single asymmetric operators	182
Multi _{asym}	Multiple asymmetric operators	50

Table 1: The defined subsets of the SingleEQ dataset with varying degrees of complexity.

complexity on the models’ performances. These different subsets are characterized in Table 1. We form three main categories: (i) **Full**: the whole dataset is included in this setting, (ii) **Complexity**: two subsets (i.e., Single, Multi) are formed based on the number of operators in the solution’s equation, and (iii) **Symmetry**: four main subsets, namely Single_{sym}, Single_{asym}, Multi_{sym}, and Multi_{asym} are formed to indicate whether the solution’s equation contains single/multiple symmetric (\times and $+$) or asymmetric (\div and $-$) operations.

We hypothesize that our Tree-LSTM models will exhibit stronger performance on subsets involving multiple and/or non-commutative operations (Multi, Multi_{sym}, Multi_{asym}), since they should be able to better capture the semantic relationships between operator nodes encoded in a tree structure. We also expect a significant difference between T-LSTM and NT-LSTM architectures on subsets involving non-commutative operations (Single_{asym} and Multi_{asym}). By using different weight matrices to transform each of the children’s states, the NT-LSTM model should be able to capture the order of the operands and link the resulting structural information of a particular non-commutative mathematical expression to the semantic representation of the problem statement.

We obtain the top-100 equation-trees using the ILP solver of Koncel-Kedziorski et al. (2015), which we rank using scores provided by our proposed model (see Sec-

tion 3). Training of our model is performed using the Adam optimizer (Kingma & Ba, 2015). As a bottom token representation layer, we use pre-trained 100-dimensional ($d_1 = 100$) GloVe embeddings (Pennington et al., 2014)² which we keep static during the training process. We use a single set of hyperparameters for all folds in the cross-validation setting. The hyperparameter optimization procedure, together with the final hyperparameters values (Table A.9) is detailed in Appendix A. We perform cross-validation on the original 5 folds of the SingleEQ dataset introduced in Koncel-Kedziorski et al. (2015). Similar to previous work (Koncel-Kedziorski et al., 2015; Wang et al., 2018b), we report performance using the overall accuracy metric. Moreover, we run the 5-fold cross-validation 5 times, with different random model initializations. Average and standard deviation over the 5 runs’ accuracies (calculated over the 5 folds together) are reported in Tables 2–3 and 5.

5. Results

In this section, we evaluate the performance of our proposed models on the SingleEQ dataset. Besides the performance on the full dataset, we are particularly interested in evaluating how each architecture behaves when evaluated on mathematical problems of varying complexity. We assume that the problems become more complex (i) as the number of needed mathematical operators grows, and (ii) when the used operators are non-commutative (asymmetric). We hypothesize that our structured Tree-LSTM-based approach is better suited to solve the aforementioned complex problems. In order to demonstrate this, we perform an extensive evaluation (Tables 2–3 and 5) of our models on subsets of different degree of complexity as defined in Table 1.

Comparison on the Full dataset: Table 2 shows the results of the evaluated systems on the Full SingleEQ dataset. The proposed models are the (i) LSTM, (ii) T-LSTM, and (iii) NT-LSTM as presented in Section 3. Clearly, all newly proposed architectures outperform previous methods. Concretely, our methods are able to outperform strong baselines on the task, reporting an accuracy improvement of more than 3% without

² <https://nlp.stanford.edu/projects/glove/>

Model	Features	Trees	Accuracy (%)
Hosseini et al. (2014)	✓	✗	48.00
Wang et al. (2018b)	✓	✓	52.96
Roy & Roth (2015)	✓	✓	66.38
Roy & Roth (2017)	✓	✓	72.25
ALGES	✓	✓	72.39
LSTM	✗	✗	75.90±1.90
T-LSTM	✗	✓	74.84±1.74 [‡]
NT-LSTM	✗	✓	75.47±0.51

Table 2: Accuracy attained by the proposed and state-of-the-art methods on the *Full* SingleEQ dataset. The ✓ and ✗ symbols indicate whether or not a model adopts hand-crafted features (i.e., Features) or tree-structured encoding of the equations (i.e., Trees). The best result is typeset in **bold**. The markers *, †, ‡ indicate the achieved bootstrap significance levels $\alpha < 0.1$, < 0.05 and < 0.01 (of T-LSTM and NT-LSTM with respect to the LSTM model).

relying on hand-crafted features (Hosseini et al., 2014; Koncel-Kedziorski et al., 2015;
295 Roy & Roth, 2015, 2017). The hand-crafted features, used in previous works, are usually related to terms indicating specific operations and thus if they are not detected in the data, the system cannot generalize well on out-of-domain mathematical descriptions. This also applies to recent neural-based methods (see, e.g., Wang et al. (2018b)) where explicitly defined features are encoded in the neural structure. We were surprised
300 by the overall good performance of our sequential LSTM model. Furthermore, in order to ensure the validity of the differences between our proposed approaches, we carried out a bootstrap significance analysis (Efron & Tibshirani, 1994) by sampling with replacement the results of LSTM, T-LSTM, and NT-LSTM models 10,000 times. We compare the performance with respect to the LSTM model in Table 2. We indicated
305 significant differences with p-values below the 1%, 5%, and 10% level (respectively denoted with ‡, †, and *) for indicating models performing significantly different from the sequential LSTM model. This way, from Table 2 we can observe that, while our LSTM model significantly outperforms the T-LSTM model, the difference with the NT-LSTM model is not significant. We also observe that the NT-LSTM model is more

Model	Complexity		Symmetric		Asymmetric	
	Single	Multi	Single _{sym}	Multi _{sym}	Single _{asym}	Multi _{asym}
ALGES	77.69 [‡]	54.70 [‡]	89.90	72.06	63.74 [‡]	30.64 [‡]
LSTM	80.72±1.80	60.34±3.32*	82.21±0.53 [‡]	69.71±1.50 [‡]	79.01±3.95	47.60±9.07
T-LSTM	79.08±2.03 [†]	60.85±1.36*	80.19±2.00 [‡]	69.71±1.50 [‡]	77.80±2.32	48.80±2.71
NT-LSTM	79.33±0.75 [†]	62.71±1.93	80.00±1.96 [‡]	73.24±2.53	78.57±2.46	48.40±2.33

Table 3: Comparison of the proposed methods with the state-of-the-art on the SingleEQ dataset in terms of accuracy. **Bold** font indicates the best results for each subset of SingleEQ (see Table 1). The markers *, †, ‡ respectively indicate the achieved bootstrap significance levels $\alpha < 0.1$, < 0.05 and < 0.01 with respect to the best performing model in each of the subsets.

310 consistent (lower standard deviation) in its predictions than the LSTM and T-LSTM models.

Comparison for different problem complexity: Table 3 compares our models with ALGES (Koncel-Kedziorski et al., 2015) (i.e., the best performing state-of-the-art model of Table 2), for subsets of different complexity levels (defined in Table 1). Similarly as
315 in Table 2, we use bootstrap significance testing to estimate the degree of certainty between the lower performing models and the best performing one in each of the subsets.

We observe that our LSTM model significantly outperforms the NT-LSTM model on the subset of problems requiring a single operation to be solved (Single subset). Conversely, on the subsets requiring multiple operations to be solved (Multi, Multi_{sym}
320 and Multi_{asym}), the tree-based NT-LSTM model achieves higher accuracy scores than LSTM with a significant difference on Multi and Multi_{sym} subsets. This is in accordance with our a priori expectations and demonstrates that tree-based equation encoding is better suited to represent mathematical expressions that require more complex reasoning (i.e., multiple operations). The results in Table 3 further show that the
325 feature-based ALGES model has competitive performance on equations requiring single and/or non-commutative operators to be solved. In fact, it significantly outperforms all our models on Single_{sym} dataset and is only outperformed marginally by our tree-based NT-LSTM model on Multi_{sym}. This suggests that the feature-based method is able to explicitly capture symmetric operations by focusing on carefully engineered
330 features. However, we observe a large drop in performance of ALGES on problems

Candidates	Metric	Subsets						
		Full	Single	Multi	Single _{sym}	Multi _{sym}	Single _{asym}	Multi _{asym}
ILP	Correct	2.53	1.44	6.13	1.89	0.92	7.72	3.96
	Incorrect	12	2.9	42.08	2.48	3.38	28.43	60.64
ILP + Asym	Correct	2.41	1.44	5.62	1.89	0.92	7.66	2.84
	Incorrect	15.08	4.06	51.5	3.57	4.62	35.43	73.36
	Δ Correct	-4.74%	0.00%	-8.32%	0.00%	0.00%	-0.78%	-28.28%
	Δ Incorrect	25.67%	40.00%	22.39%	43.95%	36.69%	24.62%	20.98%

Table 4: This table illustrates the difference in average number of *Correct* and *Incorrect* candidate equations per problem between the original *ILP* candidate generation process and the one obtained by adding noisy equations with asymmetric operators (*ILP + Asym*).

that require non-commutative operations to be solved. This is showcased by a difference of more than 15% accuracy points on Single_{asym} and 18% accuracy points on Multi_{asym} subset in Table 3. This validates our initial intuition that feature-based models fall short to capture the reasoning necessary to address problems that require more complex (non-commutative and multiple) operators.

Robustness against asymmetric noise: The results analyzed so far are based on scoring the candidates generated by the ILP component introduced in Koncel-Kedziorski et al. (2015). However, this component already significantly reduces the number of incorrect candidates, particularly those involving asymmetric operators. In order to evaluate the robustness of the proposed models, we train and evaluate them on a noisy asymmetric candidate set where we add all possible permutations to the equations involving non-commutative operators. For example, if a particular candidate equation is $x = 8/2$, we would also add $x = 2/8$ to the candidate set. Table 4 shows the statistics of the noisy dataset (ILP + Asym) with respective deltas that indicate the percentage points (%) of increase/decrease in the average number of correct/incorrect candidate equations per problem with respect to the original ILP-generated candidate set. We observe a significant increase in the number of incorrect candidates for all subsets, as well as a drop in average number of correct equations for the subsets involving asymmetric operations (Multi and Multi_{asym}). This is because, similarly as in the original *ILP* setup, we only consider the first 100 generated candidates, which in *ILP + Asym*

Model	Full	Complexity		Symmetric		Asymmetric	
		Single	Multi	Single _{sym}	Multi _{sym}	Single _{asym}	Multi _{asym}
ALGES	68.44 [‡]	75.90*	43.59 [‡]	85.58	61.76 [‡]	64.83 [‡]	18.36 [‡]
LSTM	72.60±1.02*	77.23±0.66	57.29±2.82 [‡]	82.12±2.12 [‡]	72.06±3.22 [‡]	71.65±1.50	37.20±5.74*
T-LSTM	58.35±0.99 [‡]	61.74±1.09 [‡]	47.12±0.68 [‡]	80.00±1.16 [‡]	75.29±1.71	40.88±1.65 [‡]	8.80±0.98 [‡]
NT-LSTM	73.54±1.13	77.54±1.20	60.34±0.99	82.12±2.00 [‡]	74.41±0.72	72.31±2.21	41.20±1.60

Table 5: Comparison of the proposed methods with the state-of-the-art model (i.e., ALGES) on the SingleEQ dataset in terms of accuracy evaluated on candidate equations generated using *ILP* + *Asym* procedure (see Table 4). **Bold** font indicates the best results for each subset of SingleEQ (see Table 1). The markers *, †, ‡ respectively indicate the achieved bootstrap significance levels $\alpha < 0.1$, < 0.05 and < 0.01 with respect to the best performing model in each of the subsets.

include more incorrect equations, leaving many correct ones out. This results in a lower correct/incorrect ratio that makes it more challenging for the evaluated models to find the right mathematical expression to solve a particular problem. Table 5 compares our models with the best performing state-of-the-art model (i.e., ALGES) on candidates
355 generated in the *ILP* + *Asym* setting. Compared to the results presented in Table 3, we observe a sharp decrease in performance of the ALGES model on subsets involving multiple operations (Multi, Multi_{sym} and Multi_{asym}). This demonstrates once more the weakness of this feature-based model in capturing the reasoning necessary to distinguish the order of the operands involved in equations containing multiple and non-
360 commutative operators. Furthermore, we observe that the sequential LSTM model is now significantly outperformed by the tree-based NT-LSTM on the Full dataset. This difference is mainly influenced by a superior performance of NT-LSTM model on subsets involving multiple and/or non-commutative operations (Multi, Multi_{sym} and Multi_{asym}). This again supports our initial hypothesis that tree-structured approach is
365 better suited to capture more complex reasoning which is necessary to solve mathematical problems. In the *ILP* + *Asym* candidate generation setting this is even more important because of the additional noise introduced with the incorrect candidates that involve multiple and asymmetric operations. Additionally, we observe an important drop in performance of T-LSTM model which is mainly influenced by low accuracy
370 scores on asymmetric subsets (Single_{asym} and Multi_{asym}). This is in line with our initial intuition that by using a single weight matrix to transform each of the children’s

Type	Problem Text	NT-LSTM
Complex reasoning (57%)	Seth bought 20 cartons of ice cream and 2 cartons of yogurt. Each carton of ice cream cost \$6 and each carton of yogurt cost \$1. How much more did Seth spend on ice cream than on yogurt?	$20/2 - 1 \times 6$
Parsing and counting (22%)	Jane’s dad brought home 24 marble potatoes. If Jane’s mom made potato salad for lunch and served an equal amount of potatoes to Jane, herself and her husband, how many potatoes did each of them have?	n/a
World Knowledge (21%)	Bert runs 2 miles every day. How many miles will Bert run in 3 weeks?	3×2
	The sum of three consecutive odd numbers is 69. What is the smallest of the three numbers?	n/a

Table 6: Examples of problems where our NT-LSTM model fails.

states, the T-LSTM model is unable to distinguish the order of the operands involved in asymmetric equations. This difference was not observed in Table 3 because most of the incorrect candidates involving non-commutative operations were already filtered out by the ILP component. However, in our *ILP + Asym* candidate generation setup, we make sure that for each candidate involving non-commutative operation, we also include noisy candidates with all the possible asymmetric permutations. This makes it necessary not only to detect the right operation, but also to distinguish the order of the operands, where the T-LSTM model fails. Finally, we observe that our tree-based NT-LSTM model exhibits less variance among the different bootstrap results, compared to the linear LSTM model on subsets involving multiple operations. This indicates that NT-LSTM model is less susceptible to different seed initialization during the training process, making it more robust and better suitable for problems requiring more complex multi-step reasoning (i.e., multiple operations).

Problem Text	ALGES	NT-LSTM
Nancy bought 615 crayons that came in packs of 15. How many packs of crayons did Nancy buy?	$615 - 15$	$615/15$
Joan paid \$8.77 on a cat toy, and a cage cost her \$10.97 with a \$20 bill. How much change did Joan receive?	$20 + 8.77 - 10.97$	$20 - 8.77 - 10.97$
Melanie had 19 dimes in her bank. Her dad gave her 39 dimes and her mother gave her 25 dimes. How many dimes does Melanie have now ?	$19 - 39 + 25$	$19 + 39 + 25$
On Saturday, Sara spent \$10.62 each on 2 tickets to a movie theater. Sara also rented a movie for \$1.59, and bought a movie for \$13.95. How much money in total did Sara spend on movies?	$10.62+2\times 1.59+13.95$	$10.62\times 2+13.95+1.59$

Table 7: Examples of problems that NT-LSTM gets correct, but current state-of-the-art ALGES Koncel-Kedziorski et al. (2015) fails.

385 **Error Analysis:** In order to understand our system’s weaknesses, we manually analyzed the errors that it consistently makes across different training seed instances. We grouped them into three main categories represented in Table 6: *complex reasoning*, *parsing and counting* and *world knowledge* errors. We observe that more than half (57%) of our system’s errors are due to problems requiring *complex reasoning* while

390 the numbers have been correctly extracted from the text. This reflects the results from Tables 3 and 5 that show lower performance of our models on problems requiring multiple and/or non-commutative operations. As future work to alleviate this type of problems we can complement the tree-structures using additional information such as the entities inside the sentence. For instance, in the first example illustrated in Table 6,

395 if the system would know that “ice cream” from the second sentence represents the same concept as in the first one, it would be easier to link numbers 6 and 20. A second

consistent type of error is related to *parsing and counting*. It mainly happens when there are several entities involved in a problem statement and the system has to count them correctly. For instance, in the second example presented in Table 6, our current
 400 system is unable to produce the correct candidate mathematical expression since it can only extract the number 24 from text. Further work in improving aspects related to parsing and entity identification in the problem statement should significantly reduce this kind of mistakes. Finally, the *world knowledge* related errors account for 21% of the total mistakes. Most of these errors are due to the fact that the system is unable
 405 to capture the units correctly (i.e., there are 7 days in a week, or one dime equals 0.1 dollars). However, as in the second example, some of the problems require a more advanced conceptual world understanding, such as the notion of odd numbers. Future work can be directed towards methods that are able to capture and represent this kind of world knowledge.

410 Furthermore, we performed an empirical study on the predicted results to understand better where our proposed model outperforms the current state-of-the-art model, ALGES (Koncel-Kedziorski et al., 2015). Table 7 illustrates some examples of the problems where our model gets consistently correct predictions on different training initialization weights (Section 4). Most of the gains came from improving on problems
 415 requiring multiple and/or asymmetric operations, corroborating our previous findings.

6. Conclusion

In this work we addressed the reasoning component involved in solving mathematical word problems. We proposed a recursive tree architecture to encode the underlying equations for solving arithmetic word problems. More concretely, we proposed to use
 420 two different Tree-LSTM architectures for the task of scoring candidate equations. We performed an extensive experimental study on the SingleEQ dataset and demonstrated consistent effectiveness (i.e., more than 3% increase in accuracy on the Full dataset and more than 18% for a subset of complex reasoning tasks) of our models compared to current state-of-the-art.

425 We observed that, while very strong on simple instances involving single oper-

ations, the current feature-based state-of-the-art model exhibits a significant gap in performance for mathematical problems whose solution comprises non-commutative and/or multiple operations. This reveals the weakness of this method to capture the intricate nature of reasoning necessary to solve more complex arithmetic problems. Furthermore, our experiments show that, while a traditional sequential approach based on recurrent encoding implemented using LSTMs proves to be a robust baseline, it is consistently outperformed by our recursive Tree-LSTM architecture on more complicated problems that require multiple operations to be solved. This difference in performance becomes more significant as we introduce additional noise in our set of candidates by adding incorrect equations that contain non-commutative operations.

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Appendix A. Hyperparameter Optimization

Our preliminary experiments show a significant performance bias on the 5 folds of SingleEQ dataset. As a result, in each of the tuning iterations, we perform a reshuffling of the whole dataset. Then, we take out 80% as the train set, and 20% as the development set on which the best hyperparameter combination is chosen. Due to limited resources to conduct a complete grid search, we conduct the tuning in steps. More specifically, in each step we perform a grid search on two hyperparameters that we identified as most correlated with each other. Table A.8 summarizes our hyperparameter search space for each of the sequential tuning steps. Besides the usual learning rate, batch size

Step	Hyperparameters				
	Learning Rate	Batch Size	Dim LSTM	Dim Encoder	Dropout
1	{5e−4, 3e−4, 1e−4}	{32, 64, 128}	-	-	-
2	-	-	{128, 256, 512}	-	{0.3, 0.4, 0.5}
3	-	-	-	{128, 256, 512}	{0.3, 0.4, 0.5}

Table A.8: The range of the hyperparameter search space for each of the hyperparameter tuning steps.

and dropout tuning, we also adjust the dimensionalities d_3 (Dim LSTM) of the first BiLSTM layer and d_4 (Dim Encoder) of either the sequential LSTM or the tree-based NT-LSTM models’ encoder layers. This process is repeated 10 times (iterations) in each of which the best hyperparameters are chosen on the development set after training for 75 epochs. These values are averaged to get the hyperparameters that will be used in a cross-validation setting (see Table A.9). For T-LSTM model we use the same hyperparameters as those used for NT-LSTM.

Model	Hyperparameters				
	Learning Rate	Batch Size	Dim LSTM	Dim Encoder	Dropout
LSTM	3.2e−4	61	282	282	0.37
NT-LSTM	3e−4	74	294	282	0.38

Table A.9: Final hyperparameter values used for LSTM and NT-LSTM models. The hyperparameters for T-LSTM model are the same as the ones used for NT-LSTM.

