Game Tree Searching by Min / Max Approximation summary

The paper's goal is to introduce a new iterative method for searching game trees that is alternative to traditional minimax search with alpha beta pruning.
Traditional minimax algorithm determines next move (next node expansion in the game tree) by picking node with max/min value if on max/min level of game tree. Values are backed up from the values of ancestor nodes at certain depth that were evaluated by static heuristic function. The method proposed in this paper relies on penalty based iterative heuristic instead of static ones. In this method each node expansion is assigned with certain weight. Next expanded node is determined by searching the node that has the least penalty, which is sum of all weights that lead from root to this node. After the node with least penalty is expanded, the values of nodes are updated accordingly to expanded node children values. In the method described algorithm traces a path from the root s down to the best expandable tip, adds all the successors of this tip, and updates the values where necessary by a traversal back up the tree from tip to the root.

Another difference of this method from traditional minimax is that it uses estimate heurisitc values of nodes instead of picking standard max/min values of its ancestors. Approximation is based on generalized mean-value operators with formula that looks like this:

$$M_p(\boldsymbol{a}) = \left(\frac{1}{n}\sum_{i=1}^{n} a_i^p\right)^{1/p}$$

, where "a" is the vector with values of children nodes.

Using approximation instead of traditional max/min operators has an important advantage - we can take its partial derivatives. These partial derivatives values determine the sensitivity of root value to its children. Based on sensitivity the weight of node expansion is computed - the node that contributes the most to value of tree root has the least weight. Knowing the weights the method expands the node with the least penalty.

Experimental results based on Connect-four game show that this method does not perform well when the time is the limiting power to make a move in the game. The fact is due to complex due to the computer power needed to approximate values of nodes with complex formula and traversing tree back and forth to define the best choice to expand. However if the limiting power is the total number of predicted moves then the introduced is superior to traditional minimax search with alpha beta pruning and iterative deepening.

The results are considered to be encouraging and further research is needed to improve the newly proposed method.