Quadratic Curve Derivation (Hermite Approach)

$$Cubic\ Equation = \alpha_3\, t^3 + \alpha_2\, t^2 +\ \alpha_1 t +\ \alpha_0$$

$$Parametric\ Equation\ p(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

$$\dot{p}(t) = \begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \end{bmatrix} = \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{bmatrix} 3t^2 \\ 2t \\ 1 \\ 0 \end{bmatrix}$$

- You need four points to draw the curve
  - Start – End (x,y)
  - Slope of Start – Slope of End (u,v)

$$p(0) = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\dot{p}(0) = \begin{bmatrix} \dot{u}_1 \\ \dot{v}_1 \end{bmatrix} = \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

$$p(1) = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\dot{p}(1) = \begin{bmatrix} \dot{u}_2 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

- Combine the four equations

$$\rightarrow \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} x_1 & u_1 & x_2 & u_2 \\ y_1 & v_1 & y_2 & v_2 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} = \begin{bmatrix} x_1 & u_1 & x_2 & u_2 \\ y_1 & v_1 & y_2 & v_2 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}^{-1}$$

$$\rightarrow \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} = \begin{bmatrix} x_1 & u_1 & x_2 & u_2 \\ y_1 & v_1 & y_2 & v_2 \end{bmatrix} \begin{bmatrix} 2 & -3 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ -2 & 3 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$$

$$call \rightarrow\ p(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} \alpha_3 & \alpha_2 & \alpha_1 & \alpha_0 \\ \beta_3 & \beta_2 & \beta_1 & \beta_0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

$$p(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_1 & u_1 & x_2 & u_2 \\ y_1 & v_1 & y_2 & v_2 \end{bmatrix} \begin{bmatrix} 2 & -3 & 0 & 1 \\ 1 & -2 & 1 & 0 \\ -2 & 3 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

$$p(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 2x_1 + u_1 - 2x_2 + u_2 & -3x_1 - 2u_1 + 3x_2 - x_2 & u_1 & x_1 \\ 2y_1 + v_1 - 2y_2 + v_2 & -3y_1 - 2v_1 + 3y_2 - y_2 & v_1 & y_1 \end{bmatrix} \begin{bmatrix} t^3 \\ t^2 \\ t \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} (2x_1 + u_1 - 2x_2 + u_2)t^3 +\ (-3x_1 - 2u_1 + 3x_2 - x_2)t^2 +\ u_1 t + x_1 \\ (2y_1 + v_1 - 2y_2 + v_2)t^3 +\ (-3y_1 - 2v_1 + 3y_2 - y_2)t^2 +\ v_1 t + y_1 \end{bmatrix}$$

- Note that (x,y)1 is the start point and (x,y)2 is end point
- We need to transform (u,v) to (x,y)

We will set the four points as key points when $t = 0$, $t = \frac{1}{3}$, $t = \frac{2}{3}$, and $t = 1$ respectively.

$$u_1 = \frac{x_1 - x_0}{t_1 - t_0} = \frac{x_1 - x_0}{\frac{1}{3} - 0} = 3(x_2 - x_1), v_1 = 3(y_2 - y_1)$$

$$u_2 = \frac{dy}{dt}_{en} = \frac{x_3 - x_2}{1 - \frac{2}{3}} = 3(x_3 - x_4), v_2 = 3(y_3 - y_4)$$

- (x,y)1 is start point
- (x,y)2 is second point
- (x,y)3 is Third point
- (x,y)4 is end point

So final formula

int x = x1 + x2 * t + (-3 * x1 - 2 * x2 + 3 * x4 - x3) * t * t + (2 * x1 + x2 - 2 * x4 + x3) * t * t * t;

int y = y1 + y2 * t + (-3 * y1 - 2 * y2 + 3 * y4 - y3) * t * t + (2 * y1 + y2 - 2 * y4 + y3) * t * t * t;

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} (2x_1 + x_2 - 2x_4 + x_3)t^3 +\ (-3x_1 - 2x_2 + 3x_4 - x_3)t^2 +\ x_2 t +\ x_1 \\ (2y_1 + y_2 - 2y_4 + y_3)t^3 + (-3y_1 - 2y_2 + 3y_4 - y_3)t^2 +\ x_2 t +\ y_1 \end{bmatrix}$$

Clipping Algorithm

- Before calling SetPixel check whether the point is inside the circle or outside by substituting in $x^2 + y^2 = v \rightarrow\ if\ v < r^2 \rightarrow inside\ , ow \rightarrow outside\ or\ on\ circle$