Matrix Algebra
Rank
Homework 11

1. For each of the following sets S, find a basis for the subspace of $\mathbb{R}^4$ spanned by S.
   a) $S = \{(1,2,1,3),(2,2,3,3),(6,7,5,9),(2,1,0,0)\}$
      Answer: $\{(1,2,1,3),(0,1,-1/2,3/2),(0,0,1,3/7)\}$
   b) $S = \{(2,3,4,5),(5,4,3,2),(1,2,3,4),(1,1,1,1)\}$
      Answer: $\{(1,1,1,1),(0,1,2,3)\}$
2. For each of the following matrices, find bases for their row space, column space and null space.
   a)
   $$\begin{pmatrix} 1 & 2 & 3 \\ 5 & 1 & 8 \\ 7 & 5 & 14 \end{pmatrix}$$
   Answer: A basis for the row space is $\{(1,2,3),(0,1,7/9)\}$.

   A basis for the column space is
   $$\left\{\begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}\right\}$$
   A basis for the null space is
   $$\left\{\begin{pmatrix} -13/9 \\ -7/9 \\ 1 \end{pmatrix}\right\}$$
   b)
   $$\begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$
   Answer: A basis for the row space is $\{(1,2,1),(0,1,1/3),(0,0,1)\}$.

   A basis for the column space is
   $$\left\{\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}\}, \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}\right\}$$
   The null space is trivial (it only consists of the zero vector).
   c)
   $$\begin{pmatrix} 2 & 4 & -4 & 2 \\ 1 & 3 & 4 & 1 \\ 5 & 13 & 8 & 55 \end{pmatrix}$$

Answer: A basis for the row space is $\{(1,2,-2,1),(0,1,6,0),(0,0,0,1)\}$.

A basis for the column space is

$$\left\{\begin{pmatrix}2\\1\\5\end{pmatrix},\begin{pmatrix}4\\3\\13\end{pmatrix},\begin{pmatrix}2\\1\\55\end{pmatrix}\right\}$$

A basis for the null space is

$$\left\{\begin{pmatrix}14\\-6\\1\\0\end{pmatrix}\right\}$$

d)

$$\begin{pmatrix}2&1&2&1\\1&2&1&2\\7&8&7&8\\1&1&1&1\end{pmatrix}$$

Answer: A basis for the row space is $\{(1,1/2,1,1/2),(0,1,0,1)\}$.

A basis for the column space is

$$\left\{\begin{pmatrix}1\\1/2\\7/2\\1/2\end{pmatrix},\begin{pmatrix}0\\1\\3\\1/3\end{pmatrix}\right\}$$

A basis for the null space is

$$\left\{\begin{pmatrix}-1\\0\\1\\0\end{pmatrix},\begin{pmatrix}0\\-1\\0\\1\end{pmatrix}\right\}$$

3. For each of the following matrices, find their rank and nullity.

a)

$$\begin{pmatrix}1&2&1&1\\3&4&7&8\\5&8&9&10\end{pmatrix}$$

Answer: The rank is 2, the nullity is 2.

b)

$$\begin{pmatrix}3&2&1&0\\2&1&0&3\\1&0&3&2\end{pmatrix}$$

Answer: The rank is 3, the nullity is 1.

c)

$$\begin{pmatrix} 2 & 4 & 6 & 8 \\ 3 & 1 & 2 & 1 \\ 5 & 4 & 6 & 6 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Answer: The rank is 3, the nullity is 1.