Matrix Algebra
Matrix Operations
Homework 4

1. For the following matrices A and B, find $A + B$, $2A - B$, $A \cdot B$ and $B \cdot A$ (or explain why they don't exist).

   a)

$$A = \begin{pmatrix} 2 & 5 & 4 \\ 8 & 2 & 9 \end{pmatrix}$$

$$B = \begin{pmatrix} 4 & 2 & 1 \\ 3 & 0 & 9 \end{pmatrix}$$

Result:

$$A + B = \begin{pmatrix} 6 & 7 & 5 \\ 11 & 2 & 18 \end{pmatrix}$$

$$2A - B = \begin{pmatrix} 0 & 8 & 7 \\ 13 & 4 & 9 \end{pmatrix}$$

   b)

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 7 & -2 & 3 \end{pmatrix}$$

$$B = \begin{pmatrix} 5 & 1 & 5 \\ 3 & 1 & 2 \end{pmatrix}$$

Result:

$$B \cdot A = \begin{pmatrix} 43 & 2 & 31 \\ 20 & 4 & 16 \end{pmatrix}$$

   c)

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 1 & 1 & 4 \\ -2 & 3 & 1 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 5 & 1 & 5 \\ 3 & 1 & 2 \\ 2 & 1 & 0 \\ 4 & 1 & 1 \end{pmatrix}$$

Result:

$$A \cdot B = \begin{pmatrix} 33 & 10 & 13 \\ 46 & 11 & 31 \\ 9 & 4 & -2 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 0 & 26 & 21 & 34 \\ 4 & 13 & 12 & 20 \\ 7 & 5 & 7 & 12 \\ 7 & 12 & 14 & 22 \end{pmatrix}$$

d)

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 1 & 1 & 4 \\ -2 & 3 & 1 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 5 & 1 & 5 \\ 3 & 1 & 2 & 2 \\ 2 & 2 & 1 & 0 \end{pmatrix}$$

Result:

$$A + B = \begin{pmatrix} 2 & 7 & 4 & 9 \\ 8 & 2 & 3 & 6 \\ 0 & 5 & 2 & 2 \end{pmatrix}$$

$$2A - B = \begin{pmatrix} 1 & -1 & 5 & 3 \\ 7 & 1 & 0 & 6 \\ -6 & 4 & 1 & 4 \end{pmatrix}$$

e)

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 2 \\ 0 & 3 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 2 & 2 & -1 \\ 0 & 1 & -9 \\ -2 & 2 & 1 \end{pmatrix}$$

Result:

$$A + B = \begin{pmatrix} 3 & 1 & 0 \\ -1 & 2 & -7 \\ -2 & 5 & 2 \end{pmatrix}$$

$$2A - B = \begin{pmatrix} 0 & -4 & 3 \\ -2 & 1 & 13 \\ 2 & 4 & 1 \end{pmatrix}$$

$$A \cdot B = \begin{pmatrix} 0 & 3 & 9 \\ -6 & 3 & -6 \\ -2 & 5 & -26 \end{pmatrix}$$

$$B \cdot A = \begin{pmatrix} 0 & -3 & 5 \\ -1 & -26 & -7 \\ -4 & 7 & 3 \end{pmatrix}$$

2. Write the following systems in the form $A\mathbf{x} = \mathbf{b}$.

a)

$$\begin{array}{rcrcrcl} 2x & + & 3y & - & 4z & = & 3 \\ 4x & - & y & + & 3z & = & 1 \\ x & + & y & + & 2z & = & 2 \end{array}$$

Result: This can be written $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 2 & 3 & -4 \\ 4 & -1 & 3 \\ 1 & 1 & 2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}.$$

b)

$$\begin{array}{rcrcrcrcl} 3x_1 & - & x_2 & - & 2x_3 & + & 2x_4 & = & 7 \\ 2x_1 & - & 2x_2 & + & x_3 & + & 4x_4 & = & 6 \\ 1x_1 & + & 3x_2 & + & 3x_3 & - & 8x_4 & = & 5 \end{array}$$

Result: This can be written $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{pmatrix} 3 & -1 & -2 & 2 \\ 2 & -2 & 1 & 4 \\ 1 & 3 & 3 & -8 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix}.$$

3. Given A and $\mathbf{b}$, write a system of equations equivalent to $A\mathbf{x} = \mathbf{b}$.

a)

$$A = \begin{pmatrix} 1 & 3 & 5 & 7 \\ 3 & -2 & -2 & 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$$

Result:

$$\begin{aligned} x_1 + 3x_2 + 5x_3 + 7x_4 &= 5 \\ 3x_1 - 2x_2 - 2x_3 + x_4 &= -2 \end{aligned}$$

b)

$$A = \begin{pmatrix} 3 & 1 & 5 \\ 2 & -1 & -1 \\ 5 & 5 & 5 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$$

Result:

$$\begin{aligned} 3x + y + 5z &= 1 \\ 2x - y - z &= 2 \\ 5x + 5y + 5z &= -3 \end{aligned}$$

c)

$$A = \begin{pmatrix} 1 & 2 & 3 & 4 & -5 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 10 \end{pmatrix}$$

Result:

$$x_1 + 2x_2 + 3x_3 + 4x_4 - 5x_5 = 10$$