Matrix Algebra
Span
Homework 9

For each of the following, determine if $\boldsymbol{w}$ is in the span of $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$. Also determine if $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_n$ are linearly dependent or linearly independent.

1. $\boldsymbol{v}_1 = \begin{pmatrix} 3 \\ -1 \\ 2 \\ 0 \end{pmatrix}, \boldsymbol{v}_2 = \begin{pmatrix} -4 \\ 0 \\ -2 \\ 2 \end{pmatrix}, \boldsymbol{v}_3 = \begin{pmatrix} 1 \\ 5 \\ 4 \\ -2 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} -2 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

   Answer: $\boldsymbol{w}$ is in the span of $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$. The vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ are linearly independent.

2. $\boldsymbol{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 0 \\ -1 \end{pmatrix}, \boldsymbol{v}_2 = \begin{pmatrix} -1 \\ 0 \\ 1 \\ 3 \end{pmatrix}, \boldsymbol{v}_3 = \begin{pmatrix} 1 \\ -3 \\ 2 \\ 3 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 2 \\ -5 \\ 3 \\ 4 \end{pmatrix}$

   Answer: $\boldsymbol{w}$ is in the span of $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$. The vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ are not linearly independent.

3. $\boldsymbol{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \boldsymbol{v}_2 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \boldsymbol{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}$.

   Answer: $\boldsymbol{w}$ is in the span of $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$. The vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ are not linearly independent.

4. $\boldsymbol{v}_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \\ 0 \end{pmatrix}, \boldsymbol{v}_2 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 8 \end{pmatrix}, \boldsymbol{v}_3 = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}, \boldsymbol{w} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ -4 \end{pmatrix}$.

   Answer: $\boldsymbol{w}$ is not in the span of $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$. The vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$ are linearly independent.