

Matrix Algebra
Inverses
More Homework 6

1. Find the inverses (if they exist) of the following matrices.

a)

$$\begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix}$$

Answer: The inverse is

$$\begin{pmatrix} 1 & -2 \\ -2 & 3 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & 5 \\ 7 & 1 \end{pmatrix}$$

Answer: The inverse is

$$\begin{pmatrix} \frac{-1}{34} & \frac{5}{34} \\ \frac{7}{34} & \frac{-1}{34} \end{pmatrix}$$

c)

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$$

Answer: The inverse is

$$\begin{pmatrix} -24 & 18 & 5 \\ 20 & -15 & -4 \\ -5 & 4 & 1 \end{pmatrix}$$

d)

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{pmatrix}$$

Answer: The inverse is

$$\begin{pmatrix} \frac{12}{11} & -\frac{6}{11} & -\frac{1}{11} \\ -\frac{5}{22} & \frac{3}{22} & -\frac{5}{22} \\ -\frac{2}{11} & \frac{1}{11} & \frac{2}{11} \end{pmatrix}$$

e)

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 2 & 3 & 1 & 0 \\ 1 & 3 & 2 & 2 \end{pmatrix}$$

Answer: The inverse is

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ -1/2 & 1/4 & 0 & 0 \\ -1/2 & -3/4 & 1 & 0 \\ 3/4 & 3/8 & -1 & 1/2 \end{pmatrix}$$

f)

$$\begin{pmatrix} 0 & 0 & 6 & 5 \\ 0 & 0 & 7 & 6 \\ 4 & 3 & 0 & 0 \\ 3 & 2 & 0 & 0 \end{pmatrix}$$

Answer: The inverse is

$$\begin{pmatrix} 0 & 0 & -2 & 3 \\ 0 & 0 & 3 & -4 \\ 6 & -5 & 0 & 0 \\ -7 & 6 & 0 & 0 \end{pmatrix}$$

2. For each of the following systems of equations, write it in the form $Ax = b$. Then find A^{-1} and use it to solve the system.

a)

$$\begin{aligned} 2x + 3y &= 4 \\ x + 2y &= 5 \end{aligned}$$

Answer: This is $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix}$$

b)

The solution of the system is $x = -7, y = 6$.

$$\begin{aligned} 2x + y &= 3 \\ x + 2y + z &= 2 \\ y + 2z &= 1 \end{aligned}$$

Answer: This is $Ax = b$, where

$$A = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix} \quad b = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 3/4 & -1/2 & 1/4 \\ -1/2 & 1 & -1/2 \\ 1/4 & -1/2 & 3/4 \end{pmatrix}$$

The solution of the system is $x = 3/2, y = 0, z = 1/2$.