

Essentials of Calculus

Homework 3.1

Derivative formulas

1. Find the derivative and second derivative for the following functions.

a) $f(x) = x^5$

Numeric answer: $f'(x) = 5x^4$

$f''(x) = 20x^3$

b) $f(x) = \frac{1}{x^3}$

Numeric answer: $f'(x) = -3x^{-4}$

$f''(x) = 12x^{-5}$

c) $g(t) = 2t^3$

Numeric answer: $g'(t) = 6t^2$

$g''(t) = 12t$

d) $h(x) = 4/x^4$

Numeric answer: $h'(x) = -16x^{-5}$

$f''(x) = 80x^{-6}$

e) $f(x) = 3\sqrt{x}$

Numeric answer: $f'(x) = (3/2)x^{-1/2}$

$f''(x) = (-3/4)x^{-3/2}$

f) $h(t) = 2t^3 - 5t + 9$

Numeric answer: $h'(t) = 6t^2 - 5$

$h''(t) = 12t$

g) $f(x) = 3/\sqrt{x}$

Numeric answer: $f'(x) = -(3/2)x^{-3/2}$

$f''(x) = (9/4)x^{-5/2}$

h) $g(t) = (2t - 1)(t^2 + 1)$

Numeric answer: $g'(t) = 6t^2 - 2t + 2$

$g''(t) = 12t - 2$

i) $f(x) = 5x^3 - 6x^2 + 7x - 8$

Numeric answer: $f'(x) = 15x^2 - 12x + 7$

$f''(x) = 30x - 12$

j) $f(x) = \sqrt{x} - 1/\sqrt{x}$

Numeric answer: $f'(x) = (1/2)x^{-1/2} + (1/2)x^{-3/2}$
 $f''(x) = (-1/4)x^{-3/2} - (3/4)x^{-5/2}$

k) $f(x) = (2x - 1)(3x + 5)$

Numeric answer: $f'(x) = 12x + 7$
 $f''(x) = 12$

2. For each of the following functions, find $f(1)$, $f'(1)$, $f''(1)$, $f(3)$, $f'(3)$ and $f''(3)$.

a) $f(x) = 5x^2 + 8x$

Numeric answer: $f(1) = 13$, $f'(1) = 18$, $f''(1) = 10$
 $f(3) = 69$, $f'(3) = 38$, $f''(3) = 10$

b) $f(x) = 2/x^3$

Numeric answer: $f(1) = 2$, $f'(1) = -6$, $f''(1) = 24$
 $f(3) = 2/27$, $f'(3) = -2/27$, $f''(3) = 8/81$

c) $f(x) = (x + 1)(x - 1)$

Numeric answer: $f(1) = 0$, $f'(1) = 2$, $f''(1) = 2$
 $f(3) = 8$, $f'(3) = 6$, $f''(3) = 2$

3. Let $f(x) = 2x^2 + 4 + 2/x^2$. Find the rate of change of f at $x = 2$.

Numeric answer: $f'(2) = 7.5$

4. Find the tangent lines to the following graphs.

a) $y = x^2 + x$ at $x = 2$.

Numeric answer: $y = 6 + 5(x - 2)$

b) $y = \sqrt{x}$ at $x = 4$.

Numeric answer: $y = \sqrt{2} + 1/(2\sqrt{2})(x - 2)$

5. The number of fish in a pond in t years is $P(t) = 100t^{1/3}$ fish. Find the population and the rate of growth of the population in 8 years.

Numeric answer: $P(8) = 200$ fish
 $P'(8) = 25/3$ fish/year

6. It costs a company $C(q) = 0.1q^2 + 75q + 50$ dollars to make q doodads. Find the cost and marginal cost at $q = 20$ doodads.

Numeric answer: $C(20) = 1590$ dollars,
 $C'(20) = 79$ dollars/doodad.

7. A ball is thrown down from the top of a tall building. In t seconds, the ball's height will be $f(t) = 200 - 10t - 16t^2$ feet. What is the ball's height and velocity in $t = 2$ seconds?

Numeric answer: The height will be $f(2) = 16$ feet,
the velocity will be $f'(2) = -74$ feet/sec.