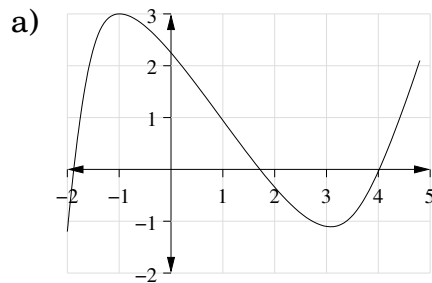


# Essentials of Calculus

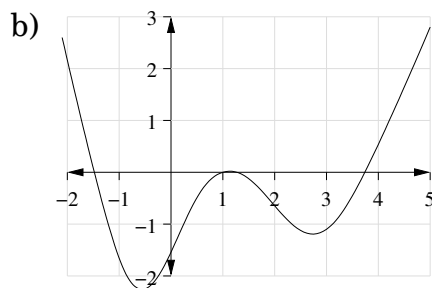
## Homework 4.1

### Local extrema

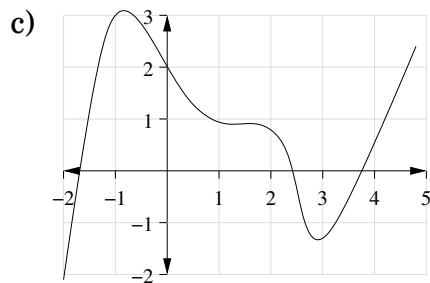
1. For each part, let  $f$  be the function with the given graph. Find all critical points, and classify each one as a local maximum, local minimum, or neither.



**Numeric answer:**  $f$  has critical points at  $x = -1$  and  $x = 3$   
 $x = -1$  is a local maximum.  
 $x = 3$  is a local minimum.



**Numeric answer:**  $f$  has critical points at  $x = -0.5$ ,  $x = 1$  and  $x = 2.5$   
 $x = -0.5$  is a local minimum  
 $x = 1$  is a local maximum  
 $x = 2.5$  is a local minimum

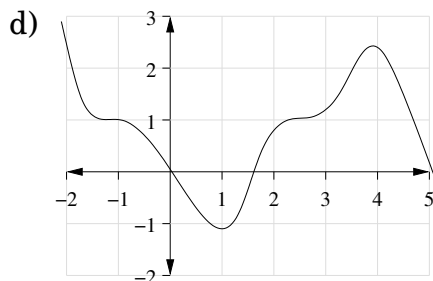


**Numeric answer:**  $f$  has critical points at  $x = -1$ ,  $x = 1.5$  and  $x = 3$ .

$x = -1$  is a local maximum

$x = 1.5$  is not a local extremum

$x = 3$  is a local minimum



**Numeric answer:**  $f$  has critical points at  $x = -1$ ,  $x = 1$ ,  $x = 2.5$  and  $x = 3$

$x = -1$  is not a local extremum

$x = 1$  is a local minimum

$x = 2.5$  is not a local extremum

$x = 3$  is a local maximum

2. For each part, find the critical points of the given function, and classify each one as a local maximum, local minimum, or neither.

a)  $f(x) = x^2 - 4x + 1$

**Numeric answer:**  $f$  has a critical point at  $x = 2$

$x = 2$  is a local minimum.

b)  $f(x) = 2x^3 - 3x^2 + 4$

**Numeric answer:**  $f$  has critical points at  $x = 0$  and  $x = 1$

$x = 0$  is a local maximum

$x = 1$  is a local minimum

c)  $f(x) = x^4 - 2x^2 + 1$

**Numeric answer:**  $f$  has critical points at  $x = -1, x = 0$   
and  $x = 1$

$x = -1$  is a local minimum.

$x = 0$  is a local maximum.

$x = 1$  is a local minimum.

d)  $f(x) = 3x^5 - 5x^3 + 9$

**Numeric answer:**  $f$  has critical points at  $x = -1, x = 0$   
and  $x = 1$

$x = -1$  is a local maximum.

$x = 0$  is not a local extremum

$x = 1$  is a local minimum

e)  $f(x) = x^5 - 10x^4 + 9$

**Numeric answer:**  $f$  has critical points at  $x = 0$  and  
 $x = 8$

$x = 0$  is a local maximum

$x = 8$  is a local minimum

f)  $f(x) = 4x^3 + 6x^2 - 24x + 10$

**Numeric answer:**  $f$  has critical points at  $x = -2$  and  
 $x = 1$

$x = -2$  is a local maximum

$x = 1$  is a local minimum

g)  $f(x) = x^3 + 9x^2 + 24x + 2$

**Numeric answer:**  $f$  has critical points at  $x = -2$  and  
 $x = -4$

$x = -2$  is a local maximum.

$x = -4$  is a local minimum.

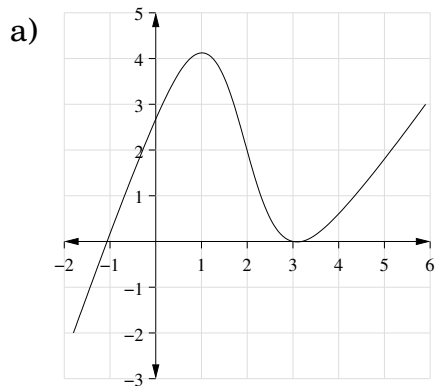
h)  $f(x) = 2x^3 + 9x^2 + 12x + 8$

**Numeric answer:**  $f$  has critical points at  $x = -1$  and  
 $x = -2$

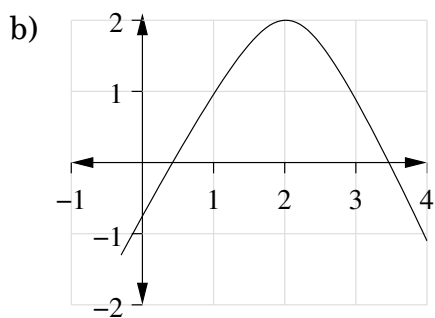
$x = -1$  is a local minimum

$x = -2$  is a local maximum

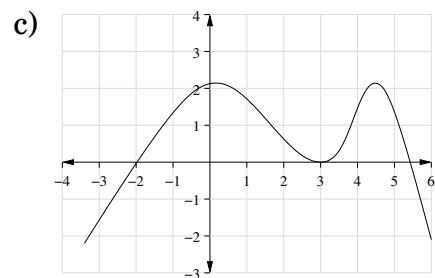
3. For each part, the graph is the graph of  $f'(x)$ . Find all critical points, and classify each one as a local maximum, local minimum, or neither.



**Numeric answer:**  $f$  has critical points at  $x = -1$  and  $x = 3$   
 $x = -1$  is a local minimum.  
 $x = 3$  is not a local extremum



**Numeric answer:**  $f$  has critical points at  $x = 0.5$  and  $x = 3.5$   
 $x = 0.5$  is a local minimum.  
 $x = 3.5$  is a local maximum.

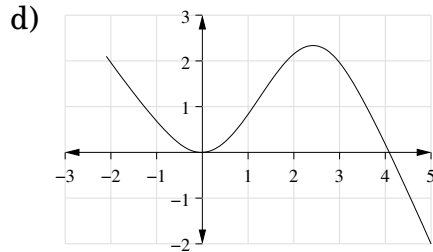


**Numeric answer:**  $f$  has critical points at  $x = -2$ ,  $x = 3$  and  $x = 5.5$

$x = -2$  is a local minimum

$x = 3$  is not a local extremum

$x = 5.5$  is a local maximum



**Numeric answer:**  $f$  has critical points at  $x = 0$  and  $x = 4$

$x = 0$  is not a local extremum.

$x = 4$  is a local maximum

4. For each part, the given function has the given critical point. Use the second derivative test to determine if the critical point is a local maximum or a local minimum.

a)  $f(x) = 6x^5 - 15x^4 + 20x^3 - 45x^2 - 60x + 4$  has a critical point at  $x = 2$ .

**Numeric answer:**  $x = 2$  is a local minimum.

b)  $f(x) = 5x^6 + 12x^5 + 15x^4 + 40x^3 + 15x^2 + 60x + 20$  has a critical point at  $x = -2$ .

**Numeric answer:**  $x = -2$  is a local maximum.