Matrix Algebra
Linear Transformations
Extra Homework 2

1. Determine whether the following functions are linear or not.
   a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+2 \\ 3y+4 \end{pmatrix}$.
   b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$.
   c) $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \\ 2x+2y \end{pmatrix}$.
   d) $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} z \\ y \\ x^2 \end{pmatrix}$.
   e) $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ xy \\ xyz \end{pmatrix}$.
   f) $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} y+2z \\ x+y \\ x-y \end{pmatrix}$.
   g) $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3z \\ 2x \\ 4/z \end{pmatrix}$.
2. For each linear transformation from part (1), find a matrix A such that $T(\boldsymbol{x}) = A\boldsymbol{x}$.
3. For the following parts, L will be a linear transformation.
   a) Given $L : \mathbb{R}^2 \to \mathbb{R}^3$ with $L\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ and $L\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$, find $L\begin{pmatrix} -3 \\ 5 \end{pmatrix}$.
   b) Given $L : \mathbb{R}^2 \to \mathbb{R}^3$ with $L\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$ and $L\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$, find $L\begin{pmatrix} 5 \\ 3 \end{pmatrix}$.
   c) Given $L : \mathbb{R}^3 \to \mathbb{R}^2$ with $L\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $L\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $L\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$, find $L\begin{pmatrix} 5 \\ 3 \\ 3 \end{pmatrix}$.
4. For each linear transformation from part (3), find a matrix A such that $T(\boldsymbol{x}) = A\boldsymbol{x}$.
5. Let $A = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ -3 & 7 \\ 0 & 5 \end{pmatrix}$, $\boldsymbol{x} = \begin{pmatrix} -3 \\ 9 \end{pmatrix}$ and $\boldsymbol{y} = \begin{pmatrix} 5 \\ -8 \\ 2 \end{pmatrix}$.
   Evaluate the following, or state why they don't exist.
   a) $A\boldsymbol{x}$
   b) $B\boldsymbol{x}$
   c) $A\boldsymbol{y}$
   d) $B\boldsymbol{y}$