Matrix Algebra
LU Factorization
More Homework 7

1. Find an LU factorization for the following matrices.

a)

$$\begin{pmatrix} 3 & 6 \\ 2 & 5 \end{pmatrix}$$

Answer: One solution is $A = LU$, where

$$L = \begin{pmatrix} 3 & 0 \\ 2 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$$

b)

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

Answer: One solution is $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 3 & -4 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & -16 \end{pmatrix}$$

c)

$$\begin{pmatrix} 1 & -2 & 3 \\ 2 & -5 & 12 \\ 0 & 2 & -10 \end{pmatrix}$$

Answer: One solution is $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 1 & -2 & 3 \\ 0 & 1 & -6 \\ 0 & 0 & 2 \end{pmatrix}$$

2. Use the LU factorizations found above to solve the following systems.

a)

$$\begin{aligned} 3x + 6y &= -6 \\ 2x + 5y &= -6 \end{aligned}$$

Answer: $x = 2, y = -2$

b)

$$\begin{aligned} x + 2y + 3z &= -6 \\ x + 3y + z &= -6 \\ 3x + 2y + z &= -2 \end{aligned}$$

Answer: $x = 1$,$y = -2$,$z = -1$

c)

$$\begin{aligned} x - 2y + 3z &= -4 \\ 2x - 5y + 12z &= 15 \\ 2y - 10z &= -22 \end{aligned}$$

Answer: $x = 58, y = 49, z = 12.$