Matrix Algebra
Projections
Extra Homework 14

1. In each part of this problem, V is the space with orthogonal basis $\mathcal{B}$. Find the projection of $\boldsymbol{x}$ onto V.

   a) $\mathcal{B} = \left\{\begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}1\\0\\-1\end{pmatrix}\right\}$, $\boldsymbol{x} = \begin{pmatrix}1\\2\\3\end{pmatrix}$

   Answer: The projection is $\boldsymbol{p} = \begin{pmatrix}1\\0\\3\end{pmatrix}$.

   b) $\mathcal{B} = \left\{\begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\1\\-1\\-1\end{pmatrix}, \begin{pmatrix}1\\-1\\1\\-1\end{pmatrix}\right\}$, $\boldsymbol{x} = \begin{pmatrix}4\\3\\2\\1\end{pmatrix}$

   Answer: The projection is $\boldsymbol{p} = \begin{pmatrix}4\\3\\2\\1\end{pmatrix}$.

2. In each part of this problem, V is the space with basis $\mathcal{B}$. Find the projection matrix P; i.e. the matrix P such that for each $\boldsymbol{x} \in \mathbb{R}^n$, $P\boldsymbol{x}$ is the projection of $\boldsymbol{x}$ onto V. Then find the projection of $\boldsymbol{b}$ onto V.

   a) $\mathcal{B} = \left\{\begin{pmatrix}1\\1\\1\end{pmatrix}, \begin{pmatrix}1\\0\\1\end{pmatrix}\right\}$, $\boldsymbol{b} = \begin{pmatrix}2\\2\\1\end{pmatrix}$

   Answer:

   $$P = \frac{1}{2}\begin{pmatrix}1 & 0 & 1\\0 & 2 & 0\\1 & 0 & 1\end{pmatrix},$$

   $$P\boldsymbol{b} =' \begin{pmatrix}3/2\\2\\3/2\end{pmatrix}.$$

   b) $\mathcal{B} = \left\{\begin{pmatrix}1\\0\\1\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\\0\end{pmatrix}\right\}$, $\boldsymbol{b} = \begin{pmatrix}1\\3\\2\\1\end{pmatrix}$

   Answer:

   $$P = \frac{1}{5}\begin{pmatrix}2 & -1 & 1 & 2\\-1 & 3 & 2 & -1\\1 & 2 & 3 & 1\\2 & -1 & 1 & 2\end{pmatrix},$$

   $$P\boldsymbol{b} = \begin{pmatrix}3/5\\11/5\\14/5\\3/5\end{pmatrix}.$$

3. In each part of this problem, find the line $y = mx + b$ which bests fits the data.

a)

x	1	3	5
y	1	2	1

Answer: $y = \frac{4}{3}$

b)

x	1	3	5
y	2	1	3

Answer: $y = \frac{1}{4}x + \frac{5}{4}$

c)

x	1	2	4	5
y	1	2	1	3

Answer: $y = \frac{3}{10}x + \frac{17}{20}$

4. In each part of this problem, find the parabola $y = ax^2 + bx + c$ which bests fits the data.

a)

x	1	2	3	4
y	1	4	4	5

Answer: $y = \frac{-1}{2}x^2 + \frac{37}{10}x - 2$.

b)

x	0	1	2	3
y	0	1	2	4

Answer: $y = \frac{1}{4}x^2 + \frac{11}{20}x + \frac{1}{20}$.