Matrix Algebra
Eigenvectors and Eigenvalues
Extra Homework 15

For each of the following matrices A, find all eigenvalues and eigenvectors. If A can be written in the form PDP^{-1} for a diagonal matrix D, find P and D.

1.

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 2 \end{pmatrix}$$

Answer: The only eigenvalue is $\lambda = 2$. The eigenvectors corresponding to it are of the form $\begin{pmatrix} x \\ 0 \end{pmatrix} = x \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

2.

$$A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

Answer: The eigenvalues are $\lambda = 2$ and $\lambda = 0$. The eigenvectors corresponding to $\lambda = 2$ are of the form $y \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. The eigenvectors corresponding to $\lambda = 0$ are of the form $y \begin{pmatrix} 1 \\ -1 \end{pmatrix}$. A is diagonalizable; $P = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $D = \begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$.

3.

$$A = \begin{pmatrix} -1 & 3 & -1 \\ -3 & 5 & -1 \\ -3 & 3 & 1 \end{pmatrix}$$

Answer: The eigenvalues are $\lambda = 1$ and $\lambda = 2$. The eigenvectors corresponding to $\lambda = 1$ are of the form $z \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$. The eigenvectors corresponding to $\lambda = 2$ are of the form $y \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + z \begin{pmatrix} -1 \\ 0 \\ -3 \end{pmatrix}$. A is diagonalizable; $P = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 3 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.