Matrix Algebra

$\mathbb{R}^n$

Homework 1

1. Let $\mathbf{x} = \left(\begin{smallmatrix}2\\-4\end{smallmatrix}\right)$ and $\mathbf{y} = \left(\begin{smallmatrix}3\\5\end{smallmatrix}\right)$. Evaluate
   a) $\mathbf{x} + \mathbf{y}$

      Answer: $\begin{pmatrix}5\\1\end{pmatrix}$

   b) $-3\mathbf{x}$

      Answer: $\begin{pmatrix}-6\\12\end{pmatrix}$

   c) $2\mathbf{x} + 4\mathbf{y}$

      Answer: $\begin{pmatrix}16\\12\end{pmatrix}$

   d) $\mathbf{x} \cdot \mathbf{y}$

      Answer: -14

   e) $\|\mathbf{y}\|$

      Answer: $\sqrt{34}$

2. Let $\mathbf{x} = \left(\begin{smallmatrix}2\\3\\-1\end{smallmatrix}\right)$ and $\mathbf{y} = \left(\begin{smallmatrix}4\\2\\1\end{smallmatrix}\right)$. Evaluate
   a) $\mathbf{x} + \mathbf{y}$

      Answer: $\begin{pmatrix}6\\5\\0\end{pmatrix}$

   b) $4\mathbf{x}$

      Answer: $\begin{pmatrix}8\\12\\-4\end{pmatrix}$

   c) $3\mathbf{x} - 2\mathbf{y}$

      Answer: $\begin{pmatrix}-2\\5\\-5\end{pmatrix}$

   d) $\mathbf{x} \cdot \mathbf{y}$

      Answer: 13

   e) $\|\mathbf{x}\|$

      Answer: $\sqrt{14}$

3. Which of the following are subspaces of $\mathbb{R}^3$? (For the ones which aren't subspaces, explain why they aren't.)
   a) The set of $\left(\begin{smallmatrix}0\\y\\z\end{smallmatrix}\right)$ where y and z can be anything.

      Answer: This is a subspace.

   b) The set of $\left(\begin{smallmatrix}1\\y\\z\end{smallmatrix}\right)$ where y and z can be anything.

      Answer: This is not a subspace.

   c) The set of $\left(\begin{smallmatrix}x\\y\\z\end{smallmatrix}\right)$ with $x + y + z = 0$.

      Answer: This is a subspace.

d) The set of $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ with $x + y + z = 1$.

Answer: This is not a subspace.

e) The set of $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ with $x + 2y + 3z = 0$.

Answer: This is a subspace.

f) The set of $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ with $x + y = 0$.

Answer: This is a subspace.

g) The set of $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ with $x = y = 0$.

Answer: This is a subspace.

h) The set of $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ with $x = 0, y \geq 0$.

Answer: This is not a subspace.

4. Which of the following are subspaces of $\mathbb{R}^4$? (For the ones which aren't subspaces, explain why they aren't.)

a) The set of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \end{pmatrix}$ where x_1, x_2, x_3 can be anything.

Answer: This is a subspace.

b) The set of $\begin{pmatrix} 2 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ where x_2, x_3, x_4 can be anything.

Answer: This is not a subspace.

c) The set of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ with $x_1 = 0$ or $x_4 = 0$.

Answer: This is not a subspace.

d) The set of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ with $x_1 = 0$ and $x_4 = 0$.

Answer: This is a subspace.

e) The set of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ with $x_1 = 1$ or $x_4 = 1$.

Answer: This is not a subspace.

f) The set of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ with $x_1 = 1$ and $x_4 = 1$.

Answer: This is not a subspace.

g) The set of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ with $x_1 + x_2 + x_3 + x_4 = 0$.

Answer: This is a subspace.

h) The set of $\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$ with $x_1 + x_2 + x_3 + x_4 = 1$.

Answer: This is not a subspace.