Matrix Algebra
LU Factorization
Homework 7

1. Find an LU factorization for the following matrices.

a)

$$\begin{pmatrix} 2 & 5 \\ -3 & -4 \end{pmatrix}$$

Answer: One solution is $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 \\ -3/2 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 2 & 5 \\ 0 & 7/2 \end{pmatrix}$$

b)

$$\begin{pmatrix} 3 & 0 & 1 \\ 6 & 1 & 1 \\ -3 & 1 & 0 \end{pmatrix}$$

Answer: One solution is $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

c)

$$\begin{pmatrix} 2 & -4 & 2 \\ 1 & 5 & -4 \\ -6 & -2 & 4 \end{pmatrix}$$

Answer: One solution is $A = LU$, where

$$L = \begin{pmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -3 & -2 & 1 \end{pmatrix}$$

$$U = \begin{pmatrix} 2 & -4 & 2 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{pmatrix}$$

2. Use the LU factorizations found above to solve the following systems.

a)

$$\begin{aligned} 2x + 5y &= 1 \\ -3x - 4y &= 2 \end{aligned}$$

Answer: $x = -2, y = 1$

b)

$$\begin{aligned} 3x + z &= -2 \\ 6x + y + z &= -3 \\ -3x + y &= 5 \end{aligned}$$

Answer: $x = -1$,$y = 2$,$z = 1$

c)

$$\begin{aligned} 2x - 4y + 2z &= -4 \\ x + 5y - 4z &= 15 \\ -6x - 2y + 4z &= -22 \end{aligned}$$

Answer: $x = (20 + 3z)/7$, $y = (17 + 5z)/7$, z is free.