Matrix Algebra
Eigenvectors and Eigenvalues
Homework 15

For each of the following matrices A, find all eigenvalues and eigenvectors. If A can be written in the form PDP^{-1} for a diagonal matrix D, find P and D.

1.
$$A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

Answer: The only eigenvalue is $\lambda = 1$. The eigenvectors corresponding to it are of the form $\begin{pmatrix} x \\ 0 \end{pmatrix} = x \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

2.
$$A = \begin{pmatrix} -5 & 4 \\ -8 & 7 \end{pmatrix}$$

Answer: The eigenvalues are $\lambda = 3$ and $\lambda = -1$. The eigenvectors corresponding to $\lambda = 3$ are of the form $y \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. The eigenvectors corresponding to $\lambda = -1$ are of the form $y \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. A is diagonalizable; $P = \begin{pmatrix} 1 & 1 \\ 2 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$.

3.
$$A = \begin{pmatrix} -5 & 3 \\ -6 & 4 \end{pmatrix}$$

Answer: The eigenvalues are $\lambda = -2$ and $\lambda = 1$. The eigenvectors corresponding to $\lambda = -2$ are of the form $y \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. The eigenvectors corresponding to $\lambda = 1$ are of the form $y \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. A is diagonalizable; $P = \begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}$, $D = \begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$.

4.
$$A = \begin{pmatrix} 9 & -7 & 7 \\ 3 & -1 & 3 \\ -5 & 5 & -3 \end{pmatrix}$$

Answer: The eigenvalues are $\lambda = 1$ and $\lambda = 2$. The eigenvectors corresponding to $\lambda = 1$ are of the form $z \begin{pmatrix} -7 \\ -3 \\ 5 \end{pmatrix}$. The eigenvectors corresponding to $\lambda = 2$ are of the form $y \begin{pmatrix} -5 \\ 0 \\ 5 \end{pmatrix} + z \begin{pmatrix} 0 \\ -5 \\ -5 \end{pmatrix}$. A is diagonalizable; $P = \begin{pmatrix} -7 & -5 & 0 \\ -3 & 0 & -5 \\ 5 & 5 & -5 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.