Matrix Algebra
Gaussian Elimination
Homework 4

Show all steps and identify each row operation in working the following problems.

1. Use back-substitution to find all solutions of the following systems.
   a)
$$\begin{aligned} x + 3y + 5z &= 3 \\ y - 2z &= 2 \\ z &= 1 \end{aligned}$$
   Answer: $x = -14, y = 4, z = 1$
   b)
$$\begin{aligned} x_1 + 3x_2 + x_3 - x_4 &= 1 \\ x_2 - 4x_3 + 2x_4 &= 2 \\ x_4 &= 5 \end{aligned}$$
   Answer: $x_1 = 30 - 13x_3, x_2 = 4x_3 - 8, x_3$ is free, $x_4 = 5$
2. Use Gaussian elimination and back-substitution (if necessary) to find all solutions of the following systems.
   a)
$$\begin{aligned} 2x + 4y + 2z &= 10 \\ x + y - 3z &= 7 \\ 3x - 3y + 4z &= -1 \end{aligned}$$
   Answer: $x = 95/37, y = 62/37, z = -34/37$
   b)
$$\begin{aligned} x + 3y - 2z &= 5 \\ 2x - y + 2z &= 7 \\ x + 10y - 8z &= 3 \end{aligned}$$
   Answer: No solution.
   c)
$$\begin{aligned} 2x + 2y + 4z &= 6 \\ x + y + 3z &= 2 \\ 4x + 4y + 10z &= 12 \end{aligned}$$
   Answer: No solution.
3. Put the following matrices in row-echelon form.

a)

$$\begin{pmatrix} 2 & 4 & 6 & 2 \\ 1 & 3 & 8 & 1 \\ 2 & 2 & 3 & 4 \end{pmatrix}$$

Answer: One solution is

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 2/7 \end{pmatrix}$$

b)

$$\begin{pmatrix} 0 & 1 & 3 & 4 \\ 1 & 3 & 8 & 1 \\ 2 & 7 & 19 & 7 \end{pmatrix}$$

Answer: One solution is

$$\begin{pmatrix} 1 & 3 & 8 & 1 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

4. Find all solutions of the following system of equations.

$$\begin{array}{rcrcrcr} \frac{2}{x} & + & \frac{3}{y} & + & \frac{4}{z} & = & 13 \\ \frac{1}{x} & + & \frac{2}{y} & + & \frac{1}{z} & = & 6 \\ \frac{3}{x} & & & + & \frac{1}{z} & = & 10 \end{array}$$

Answer: $x = 1/3, y = 1, z = 1$