Matrix Algebra
Determinants
Homework 8

1. Find the determinants of the following matrices.
   a)
   $$\begin{pmatrix} 1 & 2 \\ 4 & 6 \end{pmatrix}$$
   Answer: The determinant is -2.
   b)
   $$\begin{pmatrix} 2 & 4 \\ 8 & 16 \end{pmatrix}$$
   Answer: The determinant is 0.
   c)
   $$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$
   Answer: The determinant is 0.
   d)
   $$\begin{pmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{pmatrix}$$
   Answer: The determinant is 0.
   e)
   $$\begin{pmatrix} 1 & 2 & 4 & 8 \\ 1 & 4 & 16 & 64 \\ 1 & 3 & 9 & 27 \\ 1 & -2 & 4 & -8 \end{pmatrix}$$
   Answer: The determinant is 240.
2. Which matrices from problem 1 are invertible?
   Answer: The matrices from parts (a) and (e) are invertible.
3. Let A and B be $n \times n$ matrices.
   a) If A and B are invertible, show that AB is invertible.
   b) If AB is invertible, show that A and B are both invertible.
4. Show that if A is an invertible matrix, then $\det(A^{-1}) = 1/\det(A)$.
   (Hint: Notice that $A \cdot A^{-1} = I$. A quick computation shows that $\det(I) = 1$.)
5. If A and B are $n \times n$ matrices, show that $\det(AB) = \det(BA)$.