Essentials of Calculus

Homework 3.2

Exponentials and logarithms

1. For each function $f(x)$, find $f'(x)$.

   a) $f(x) = 10^x$

   Numeric answer: $f'(x) = \ln(10)\,10^x$

   b) $f(x) = 2x^3 + 3 \cdot 2^x$

   Numeric answer: $f'(x) = 6x^2 + 3\ln(2)\,2^x$

   c) $f(x) = 9x^4 - 5x + 3 - 5 \cdot 2^x$

   Numeric answer: $f'(x) = 36x^3 - 5 + 5\ln(2)\,2^x$

   d) $f(x) = 5 \cdot 3^x + 3 \cdot 5^x$

   Numeric answer: $f'(x) = 5\ln(3)\,3^{x+3\ln(5)5^x}$

   e) $f(x) = 2x^3 - 5x + 2e^x$

   Numeric answer: $f'(x) = 6x^2 - 5 + 2e^x$

   f) $f(x) = 6e^x + 6x^7 - 3\ln(x)$

   Numeric answer: $f'(x) = 6e^x + 42x^6 - \frac{3}{x}$

   g) $f(x) = 5\ln(x) - \frac{5}{x}$

   Numeric answer: $f'(x) = \frac{5}{x} + 5x^{-2}$

2. Let $f(x) = 3x^2 + x - 2e^x$.

   a) Find $f'(0), f'(1)$ and $f'(2)$.

   Numeric answer: $f'(0) = -1$
   $f'(1) = 7 - 2e \approx 1.563$

   b) Find an equation for the tangent line at $x = 0$.

   Numeric answer: $y = -2 - x$

3. Let $f(x) = 3x^2 + 2\ln(x)$. Find an equation for the tangent line at $x = 1$.

   Numeric answer: $y = 3 + 8(x - 1)$

4. If \$100 is put in a bank at 5% yearly interest, compounded continuous, in t years there will be $f(t) = 100e^{0.05t}$ dollars. Find $f(5)$ and $f'(5)$, with units. Say what they represent.

   Numeric answer: $f(5) = 100e^{0.25} \approx 128.4$ dollars
   $f'(5) = 5e^{0.25} \approx 6.42$dollars/year

5. A certain car is worth $f(t) = 5000e^{-0.05t}$ dollars in t years. Find $f(10)$ and $f'(10)$, with units. Say what they represent.

   Numeric answer: $f(10) = 5000e^{-0.5} \approx 3032$ dollars
   $f'(10) = -250e^{-0.5} \approx -151.63$ dollars/year

6. The human population of a certain region t years from now is given by $P(t) = 10000(.995)^t$. Find $P(50)$ and $P'(50)$, with units, and give their interpretation.

   Numeric answer: $P(50) = 10000(0.995)^{50} \approx 7783$ people
   $P'(50) = 10000(\ln(0.995)(0.995)^{50} \approx -39$ people/year

7. It costs a company $C(q) = 500 + 200\ln(q)$ dollars to make q objects. Find the total cost and marginal cost at a production level of $q = 100$ objects.

   Numeric answer: $C(100) = 500 + 200\ln(100) \approx 1421$ dollars
   $MC(100) = 2$ dollars/object