Matrix Algebra

$\mathbb{R}n$

Homework 1

1. For each pair of points $\mathbf{x}$ and $\mathbf{y}$, draw $\mathbf{x}$, $\mathbf{y}$ and the vector from $\mathbf{x}$ to $\mathbf{y}$. Find the vector from $\mathbf{x}$ to $\mathbf{y}$, and use it to find the distance from $\mathbf{x}$ to $\mathbf{y}$.

   (a) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

   (b) $\mathbf{x} = \begin{pmatrix} -2 \\ 1 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

   (c) $\mathbf{x} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

   (d) $\mathbf{x} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

2. For each set of vectors $\mathbf{x_1}$, $\mathbf{y_1}$, $\mathbf{x_2}$ and $\mathbf{y_2}$, determine whether or not the vector from $\mathbf{x_1}$ to $\mathbf{y_1}$ is equal to the vector from $\mathbf{x_2}$ to $\mathbf{y_2}$.

   (a) $\mathbf{x_1} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \mathbf{y_1} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \mathbf{y_2} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

   (b) $\mathbf{x_1} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{y_1} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \mathbf{y_2} = \begin{pmatrix} 5 \\ 0 \end{pmatrix}$

   (c) $\mathbf{x_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{y_1} = \begin{pmatrix} -3 \\ 1 \\ 7 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 2 \\ 5 \\ 1 \end{pmatrix}, \mathbf{y_2} = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$

   (d) $\mathbf{x_1} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}, \mathbf{y_1} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \mathbf{x_2} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}, \mathbf{y_2} = \begin{pmatrix} -2 \\ 3 \\ -3 \end{pmatrix}$

3. For each pair of vectors $\mathbf{x}$ and $\mathbf{y}$, find the dot product $\mathbf{x} \cdot \mathbf{y}$. Determine whether or not $\mathbf{x}$ and $\mathbf{y}$ are orthogonal.

   (a) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

   (b) $\mathbf{x} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

   (c) $\mathbf{x} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

   (d) $\mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

4. For each pair of vectors $\mathbf{x}$ and $\mathbf{y}$, find a value of c which makes $\mathbf{x}$ and $\mathbf{y}$ orthogonal.

(a) $\mathbf{x} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 4 \\ c \end{pmatrix}$

(b) $\mathbf{x} = \begin{pmatrix} -2 \\ 2c \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

(c) $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 3c \\ 2 \\ 1 \end{pmatrix}$

(d) $\mathbf{x} = \begin{pmatrix} 2 \\ 1 \\ 3c \end{pmatrix}, \mathbf{y} = \begin{pmatrix} 4c \\ 2 \\ -2 \end{pmatrix}$