Matrix Algebra
Linear Transformations
Homework 2

1. Determine whether the following functions are linear or not.

   a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x+2y \\ 3y+1 \end{pmatrix}$.

   b) $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ x+3y \end{pmatrix}$.

   c) $T : \mathbb{R}^2 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2xy \\ 2x+y \\ x \end{pmatrix}$.

   d) $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+2y \\ y+2z \\ z+2x \end{pmatrix}$.

   e) $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x \\ 3y \\ 1 \end{pmatrix}$.

   f) $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+2y+z \\ x+z+2 \\ 4z \end{pmatrix}$.

   g) $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x+3z \\ 3y+z \\ z^2 \end{pmatrix}$.

   Answer: The only linear functions are (b) and (d).

2. For each linear transformation from part (1), find a matrix A such that $T(\boldsymbol{x}) = A\boldsymbol{x}$.

   Answer:

   (b) $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$.

   (d) $A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$.

3. For the following parts, L will be a linear transformation.

   a) Given $L : \mathbb{R}^2 \to \mathbb{R}^2$ with $L\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $L\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$, find $L\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

   Answer: $L\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 13 \end{pmatrix}$.

   b) Given $L : \mathbb{R}^2 \to \mathbb{R}^3$ with $L\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ and $L\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, find $L\begin{pmatrix} 2 \\ 5 \end{pmatrix}$.

   Answer: $L\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \\ 15 \end{pmatrix}$.

c) Given $L : \mathbb{R}^3 \to \mathbb{R}^2$ with $L\begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}2\\1\end{pmatrix}$, $L\begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}2\\2\end{pmatrix}$ and $L\begin{pmatrix}0\\0\\1\end{pmatrix} = \begin{pmatrix}2\\3\end{pmatrix}$, find $L\begin{pmatrix}1\\2\\3\end{pmatrix}$.

Answer: $L\begin{pmatrix}1\\2\\3\end{pmatrix} = \begin{pmatrix}12\\14\end{pmatrix}$.

4. For each linear transformation from part (3), find a matrix A such that $T(\boldsymbol{x}) = A\boldsymbol{x}$.

Answer:

a) $A = \begin{pmatrix}2 & -2\\3 & 2\end{pmatrix}$

b) $A = \begin{pmatrix}2 & 1\\1 & 2\\0 & 3\end{pmatrix}$

c) $A = \begin{pmatrix}2 & 2 & 2\\1 & 2 & 3\end{pmatrix}$

5. Let $A = \begin{pmatrix}2 & 4\\7 & -6\\2 & 0\end{pmatrix}$, $B = \begin{pmatrix}4 & 1 & 1\\2 & 5 & 2\\7 & 6 & 5\end{pmatrix}$, $\boldsymbol{x} = \begin{pmatrix}2\\4\\-1\end{pmatrix}$ and $\boldsymbol{y} = \begin{pmatrix}3\\9\end{pmatrix}$. Evaluate the following, or state why they don't exist.

a) $A\boldsymbol{x}$

Answer: It doesn't exist, since the number of columns of A doesn't equal the number of coordinates of $\boldsymbol{x}$

b) $B\boldsymbol{x}$

Answer: $B\boldsymbol{x} = \begin{pmatrix}11\\22\\33\end{pmatrix}$

c) $A\boldsymbol{y}$

Answer: $A\boldsymbol{y} = \begin{pmatrix}42\\-33\\6\end{pmatrix}$

d) $B\boldsymbol{y}$

Answer: It doesn't exist, since the number of columns of B doesn't equal the number of coordinates of $\boldsymbol{y}$