Matrix Algebra
Rank
Extra Homework 11

1. For each of the following sets S, find a basis for the subspace of $\mathbb{R}^4$ spanned by S.
   a) $S = \{(1,1,1,1), (1,2,1,2), (1,-1,1,-1), (1,0,0,0)\}$
      Answer: One basis is $\{(1,1,1,1), (0,1,0,1), (0,0,1,0)\}$
   b) $S = \{(1,2,3,4), (4,3,2,1), (1,1,1,1), (1,0,1,0)\}$
      Answer: One basis is $\{(1,2,3,4), (0,1,2,3), (0,0,1,1)\}$
2. For each of the following matrices, find bases for their row space, column space and null space.
   a)
   $$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$
   Answer: A basis for the row space is $\{(1,2,3), (0,1,2), (0,0,1\}$.

   A basis for the column space is
   $$\left\{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}\right\}$$
   The null space is trivial.

   b)
   $$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 2 & 2 & 2 \end{pmatrix}$$
   Answer: A basis for the row space is $\{(1,2,3), (0,1,1), (0,0,1)\}$.

   A basis for the column space is
   $$\left\{\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix}\right\}$$
   The null space is trivial.

   c)
   $$\begin{pmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \end{pmatrix}$$

Answer: A basis for the row space is $\{(1,1,2,2),(0,1,-1,0),(0,0,1,1)\}$.

A basis for the column space is

$$\left\{\begin{pmatrix}1\\2\\1\\1\end{pmatrix},\begin{pmatrix}1\\2\\1\\2\end{pmatrix},\begin{pmatrix}2\\2\\1\\1\end{pmatrix}\right\}$$

A basis for the null space is

$$\left\{\begin{pmatrix}1\\-1\\-1\\1\end{pmatrix}\right\}$$

3. For each of the following matrices, find their rank and nullity.

a)

$$\begin{pmatrix}1&1&1&1\\3&4&5&6\\2&2&4&4\end{pmatrix}$$

Answer: The rank is 3, the nullity is 1.

b)

$$\begin{pmatrix}2&4&4&4\\3&2&1&0\\0&1&2&3\\1&1&1&1\end{pmatrix}$$

Answer: The rank is 3, the nullity is 1.