

Matrix Algebra

Basis

Extra Homework 10

1. Explain why each of the following sets is not a basis for \mathbb{R}^2 .

a) $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$

b) $\left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \end{pmatrix} \right\}$

c) $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$

2. Explain why each of the following sets is not a basis for \mathbb{R}^3 .

a) $\left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$

b) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\}$

c) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 6 \end{pmatrix} \right\}$

3. For each of the following, determine whether or not the given set S is a basis for the corresponding \mathbb{R}^n .

a) $S = \left\{ \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \end{pmatrix} \right\}$ for \mathbb{R}^2 .

Answer: It is a basis.

b) $S = \left\{ \begin{pmatrix} 3 \\ 6 \end{pmatrix}, \begin{pmatrix} 5 \\ 10 \end{pmatrix} \right\}$ for \mathbb{R}^2 .

Answer: It is not a basis.

c) $S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$ for \mathbb{R}^3 .

Answer: It is a basis.

d) $S = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} \right\}$ for \mathbb{R}^3 .

Answer: It is not a basis.