

# Essentials of Calculus

## Homework 3.2

### Exponentials and logarithms

1. For each function  $f(x)$ , find  $f'(x)$ .

a)  $f(x) = 10^x$

**Numeric answer:**  $f'(x) = \ln(10)10^x$

b)  $f(x) = 2x^3 + 3 \cdot 2^x$

**Numeric answer:**  $f'(x) = 6x^2 + 3 \ln(2)2^x$

c)  $f(x) = 9x^4 - 5x + 3 - 5 \cdot 2^x$

**Numeric answer:**  $f'(x) = 36x^3 - 5 + 5 \ln(2)2^x$

d)  $f(x) = 5 \cdot 3^x + 3 \cdot 5^x$

**Numeric answer:**  $f'(x) = 5 \ln(3)3^x + 3 \ln(5)5^x$

e)  $f(x) = 2x^3 - 5x + 2e^x$

**Numeric answer:**  $f'(x) = 6x^2 - 5 + 2e^x$

f)  $f(x) = 6e^x + 6x^7 - 3 \ln(x)$

**Numeric answer:**  $f'(x) = 6e^x + 42x^6 - \frac{3}{x}$

g)  $f(x) = 5 \ln(x) - \frac{5}{x}$

**Numeric answer:**  $f'(x) = \frac{5}{x} + 5x^{-2}$

2. Let  $f(x) = 3x^2 + x - 2e^x$ .

a) Find  $f'(0)$ ,  $f'(1)$  and  $f'(2)$ .

**Numeric answer:**  $f'(0) = -1$

$f'(1) = 7 - 2e \approx 1.563$

b) Find an equation for the tangent line at  $x = 0$ .

**Numeric answer:**  $y = -2 - x$

3. Let  $f(x) = 3x^2 + 2 \ln(x)$ . Find an equation for the tangent line at  $x = 1$ .

**Numeric answer:**  $y = 3 + 8(x - 1)$

4. If \$100 is put in a bank at 5% yearly interest, compounded continuous, in  $t$  years there will be  $f(t) = 100e^{0.05t}$  dollars. Find  $f(5)$  and  $f'(5)$ , with units. Say what they represent.

**Numeric answer:**  $f(5) = 100e^{0.25} \approx 128.4$  dollars  
 $f'(5) = 5e^{0.25} \approx 6.42$  dollars/year

5. A certain car is worth  $f(t) = 5000e^{-0.05t}$  dollars in  $t$  years. Find  $f(10)$  and  $f'(10)$ , with units. Say what they represent. (Note: Apparently, a different problem was handed out. That problem and solution are at the end.)

**Numeric answer:**  $f(10) = 5000e^{-0.5} \approx 3032$  dollars  
 $f'(10) = -250e^{-0.5} \approx -151.63$  dollars/year

6. The human population of a certain region  $t$  years from now is given by  $P(t) = 10000(.995)^t$ . Find  $P(50)$  and  $P'(50)$ , with units, and give their interpretation.

**Numeric answer:**  $P(50) = 10000(0.995)^{50} \approx 7783$  people  
 $P'(50) = 10000(\ln(0.995))(0.995)^{50} \approx -39$  people/year

7. It costs a company  $C(q) = 500 + 200\ln(q)$  dollars to make  $q$  objects. Find the total cost and marginal cost at a production level of  $q = 100$  objects.

**Numeric answer:**  $C(100) = 500 + 200\ln(100) \approx 1421$  dollars  
 $MC(100) = 2$  dollars/object

A certain car is worth  $f(t) = 5000e^{0.85t}$  dollars in  $t$  years. Find  $f(10)$  and  $f'(10)$ , with units. Say what they represent.

**Numeric answer:**  $f(10) = 5000e^{8.5} \approx 24573844.20$  dollars. This is the amount the car will be worth in 10 years.  
 $f'(10) = 4250e^{8.5} \approx 20887767.57$  dollars/year, which means that the car is gaining about 20887767.57 dollars a year in 50 years.