# Section 07
# Motion

Suppose an object is moving along a line. (The line will be taken to be a number line so the position of the object can be given by a single number, and there will be a positive and negative direction.) Let $f(t)$ represent the position of the object at time $t$. Then $f'(t)$ will be the object's velocity at time $t$.

**Example**
A squirrel is running so that in $t$ seconds, it is $f(t) = -5t^2 + 25t + 100$ feet away. How fast is it going in $t = 2$ seconds?

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Since $f'(t) = -10t + 25$ feet/sec, in 2 seconds it will be going $f'(2) = -10 \cdot 2 + 25 = 5$ feet/sec. Also, in 3 seconds, it will be going $f'(3) = -10 \cdot 3 + 25 = -5$ feet/sec.

Notice that at $t = 2$ seconds, the squirrel's velocity is positive. This means that the squirrel is moving in the positive direction (away from us). At $t = 3$ seconds, the squirrel's velocity is negative, so the squirrel is then moving in the negative direction (toward us). Give the squirrel a peanut.

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**Remark**
The velocity of an object tells you how fast the object is traveling and also in which direction the object is traveling. Speed is the absolute value of velocity, and only tells you how fast the object is traveling.

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**Example**
A rock is thrown straight up so that in $t$ seconds, it is $f(t) = -16t^2 + 64t + 6$ feet high. How high does the rock go?

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We might notice that when the rock reaches the highest point, it will change direction, and so it will have to momentarily stop. Note that $f'(t) = -32t + 64 = -32(t - 2)$, and so the only place where the motion stops is at $t = 2$ seconds; i.e., the only time that the maximum could occur is at $t = 2$ seconds. Since, for physical considerations, there must be a maximum height, that

height must occur at $t = 2$ seconds, and so the maximum height is $f(2) = 70$ feet.

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If $f(t)$ represents the position of an object at time $t$, then $f'(t)$ will give the velocity of the object at time $t$, and $f''(t)$, the derivative of $f'(t)$, will be the rate of change of the velocity; i.e., the acceleration of the object at time $t$ .

**Example**
A fly flies so that in $t$ seconds, it is $f(t) = t^3 - 10t^2 + 50t + 1$ feet away. Since

$$f'(t) = 3t^2 - 20t + 50$$

and

$$f''(t) = 6t - 20,$$

in $t = 2$ seconds, the fly's velocity will be $f'(2) = 22$ feet/sec and the fly's acceleration will be $f''(2) = -8$ feet/sec$^2$.

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