

Matrix Algebra

Orthonormal Bases

Extra Homework 13

1. Which of the following sets are orthogonal? Which are orthonormal?

a) $\left\{ \begin{pmatrix} 1 \\ 2 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ -2 \\ -4 \\ 2 \end{pmatrix} \right\}$

b) $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} \right\}$

c) $\left\{ \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ 1/\sqrt{2} \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1/\sqrt{2} \\ 0 \\ -1/\sqrt{2}, 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

Answer: (b) and (c) are orthogonal; (c) is also orthonormal.

2. For each orthogonal basis from part 1, find the coefficients c_1, c_2, c_3, c_4 in the expansions $\mathbf{x} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + c_4\mathbf{v}_4$. Here, $\mathbf{x} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$

Answer: (b): $\mathbf{x} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + c_4\mathbf{v}_4$ for $c_1 = 1, c_2 = 0, c_3 = 5/2, c_4 = 1/2$.

(c): $\mathbf{x} = c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 + c_4\mathbf{v}_4$ for $c_1 = 2\sqrt{2}, c_2 = 2, c_3 = \sqrt{2}, c_4 = 1$.