Final team Assignment

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Analytics for a Better World

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1.1 Linear model

The mathematical model:

Sets:

I: set of available ingredients

N: set of nutrients

R: set of combined ingredient rules

Parameters:

$\mathbf{C_i}$: cost of ingredient $i \in I$

$\mathbf{V_{ij}}$: amount of nutrient $j \in N$ per kg of ingredient $i \in I$

$\mathbf{B_i^{lb/ub}}$: lower and upper bound on each ingredient i ($\forall\ i \in I$)

$\mathbf{D_j^{lb/ub}}$: lower and upper bound on each nutrient j ($\forall\ j \in N$)

$\mathbf{U_{ir}}$: upper bound on mix of ingredients in $i \in R$ for rule $r \in R$

Variables:

$\mathbf{X_i}$: fraction of the feed mix that consists of ingredient $i \in I$

Objective:

$$min \sum_{i \in I} Ci \times Xi$$

Constraints:

$$\text{s. t. } D_j^{lb} \leq \sum_{i \in I} Vij \times Xj \leq D_j^{ub} \ (\forall\ j \in N)\ ;$$

$$B_i^{lb} \leq Xi \leq B_i^{ub} \ (\forall\ i \in I)\ ;$$

$$\sum_{i \in R} Xi \leq U_{ir} \ (\forall\ r \in R)\ ;$$

$$\sum_{i \in I} Xi = 1\ ; \qquad X_i \geq 0 \ (\forall\ i \in I)$$

Objective value:

The minimal cost of 1 kg of feed that will satisfy all of the requirements is 0.2650680604942 (~= 0.265).

Solution:

(Appendix: Image 1)

1.2 Minimize the number of ingredients used

The mathematical model:

Set and parameters remain the same as in 1.1

One more variable has to be added: $\mathbf{P_i}$ = **1** if ingredient number I is used, **0** otherwise

Objective:

$$\min \sum_{i \in I} Pi$$

Constraints:

s. t. $D_j^{lb} \leq \sum_{i \in I} Vij \times Xj \leq D_j^{ub}$ ($\forall$ j $\in$ N) ;

$B_i^{lb} \leq Xi \leq B_i^{ub}$ ($\forall$ i $\in$ I) ;

$\sum_{i \in R} Xi \leq U_{ir}$ ($\forall$ r $\in$ R) ;

$\sum_{i \in I} Xi = 1$; $X_i \geq 0$ ($\forall$ i $\in$ I)

$X_i \leq P_i$ ($\forall$ i $\in$ I) ;

$P_i = \{0;1\}$ ($\forall$ i $\in$ I)

Objective value:

The minimum number of ingredients that need to be used in order to satisfy all of the constraints is 5.

Solution:

(Appendix: Image 2)

1.3 Minimum cost for the minimum number of ingredients

The mathematical model:

Sets and variables remain the same as in 1.1

One more parameter has to be added: **P**: number of ingredients used

Objective:

$$\min \sum_{i \in I} \mathrm{C}i \times Xi$$

Constraints:

s. t. $D_j^{lb} \leq \sum_{i \in I} Vij \times Xj \leq D_j^{ub}$ ($\forall$ j $\in$ N) ;

$B_i^{lb} \leq Xi \leq B_i^{ub}$ ($\forall$ i $\in$ I) ;

$\sum_{i \in R} Xi \leq U_{ir}$ ($\forall$ r $\in$ R) ;

$\sum_{i \in I} Xi = 1$; $X_i \geq 0$ ($\forall$ i $\in$ I) ;

$\sum_{i \in I} Xi = P$

Objective value:

The minimal cost of 1 kg of feed that will satisfy all of the requirements is 0.341822823725 (~= 0.342).

Solution:

(Appendix: Image 3)

1.4 Navigating the trade-off

Graph:

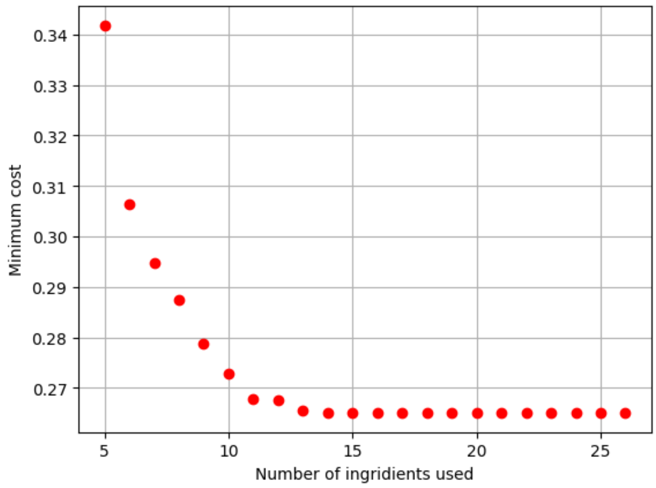

Table of solutions:

(Appendix: Image 4)

Optimal solution:

Let's begin by looking at our graph, which provides insights into various aspects. As you can see, it shows the relationship between the amount of ingredients used and the minimum cost. The dependence is not linear.

When analyzing farms, we place equal importance on minimizing both the quantity of ingredients used and the cost per kilogram. Consequently, the elbow method proves to be a valuable tool in this context. Beyond **11** ingredients, the increase in the number of ingredients becomes counterproductive as it leads to higher delivery costs without a reduction in the cost per kilogram. On the other hand, up to and including **7** ingredients, a notable drop in price occurs, making it feasible to contemplate larger ingredient sets. This underscores the optimal range, falling between **8** and **10** ingredients.

To determine the most suitable value within this range, a comprehensive analysis is essential. This analysis should consider factors such as the farm's production volume (a significant need for feed) and the feasibility of wholesale options (storage capacity and order frequency etc.) and other important factors. As far as we haven't got any data provided above, it is logical to take the middle value of **9** as our optimal choice.

As we choose **9** ingredients, the minimal cost of 1 kg of feed is approximately **0.279**. We consider this to be the optimal solution.

Appendix:

Ingredient	Kiligrams of ingredient per kg of feed
barley	0.1
blood	0
boneash	0
cotton	0.06
fish	0.020073385
fishlq	0
gnseeds	0.093696352
maize	0.014835407
maizebranhighq	0.25
maizebranlowq	0
mbmeal	0.046812217
sugars	0
soybeanexp	0.07
soybeanmeal	0
sunflower	0.09045313
sunflowerseeds	0
tapbran	0
caswhole	0.25
casfine	0
wheatbran	0
lysine	0.001140595
dl	0.00082781
ltryp	0
dicaph	0.000115784
shells	0
salt	0.002045321

Image 1. Graph showing kilograms of each ingredient needed per kg of feed to satisfy the requirements of task 1.1

Ingredient	Kiligrams of ingredient per kg of feed
barley	0
blood	0
boneash	0.02662492
cotton	0
fish	0
fishlq	0
gnseeds	0
maize	0.66196751
maizebranhighq	0
maizebranlowq	0
mbmeal	0
sugars	0
soybeanexp	0
soybeanmeal	0.30719143
sunflower	0
sunflowerseeds	0
tapbran	0
caswhole	0
casfine	0
wheatbran	0
lysine	0
dl	0.001216135
ltryp	0
dicaph	0
shells	0
salt	0.003

Image 2. Graph showing kilograms of each ingredient needed per kg of feed to satisfy the requirements of task 1.2

Ingredient	Kiligrams of ingredient per kg of feed
barley	0
blood	0
boneash	0.028172935
cotton	0
fish	0
fishlq	0
gnseeds	0
maize	0.71727804
maizebranhighq	0
maizebranlowq	0
mbmeal	0
sugars	0
soybeanexp	0
soybeanmeal	0.25091677
sunflower	0
sunflowerseeds	0
tapbran	0
caswhole	0
casfine	0
wheatbran	0
lysine	0
dl	0.00063225114
ltryp	0
dicaph	0
shells	0
salt	0.003

Image 3. Graph showing kilograms of each ingredient needed per kg of feed to satisfy the requirements of task 1.3

Number of ingedients used	Minimal cost of 1 kg of feed
5	0.341822824
6	0.3063974
7	0.294637914
8	0.28744429
9	0.278828274
10	0.272920426
11	0.267758425
12	0.267478748
13	0.265418542
14	0.26506806
15	0.265073049
16	0.26506806
17	0.265073049
18	0.26506806
19	0.265073049
20	0.26507304
21	0.265073049
22	0.265073049
23	0.265073049
24	0.26506806
25	0.26506806
26	0.26506806

Image 4. Graph showing the minimal cost of 1 kg of feed achieved for each number of ingredients used.