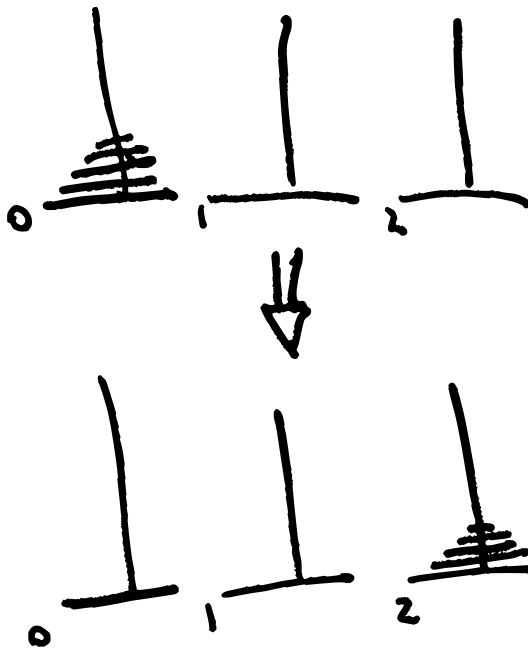# TORRI DI HANOI

LA SERIE RISOLUTIVA

(mossa) M :

(da) $a \rightarrow$ Indice torre partenza

(a) $b \rightarrow$ Indice torre arrivo

(soluzione) S : (n° dischi) $\longrightarrow$ Elenco mosse

N° dischi	Mosse
1	0 2
2	0 0 1 / 1 2 2
3	...

OPERAZIONI

a: indice

$(\text{simmetrico}) \ \bar{a}:$
$$
\begin{vmatrix}
\bar{0} = 2 \\
\bar{1} = 1 \\
\bar{2} = 0
\end{vmatrix}
$$

$(\text{complementare}) \ a^*:$
$$
\begin{vmatrix}
0^* = 0 \\
1^* = 2 \\
2^* = 1
\end{vmatrix}
$$

OPERAZIONI

$M = \dfrac{a}{b}$: mossa

(inverso) $M^1 = \dfrac{b}{a}$

(complementare) $M^* = \dfrac{a^*}{b^*}$

(simmetrico) $M^- = \left(\dfrac{a^-}{b^-}\right)^1 = \dfrac{b^-}{a^-}$

OPERAZIONI

$$S = (M \ldots N) = \begin{pmatrix} a & & c \\ b & \ldots & d \end{pmatrix} : \text{Sequenza di mosse}$$

$$(\text{complementare}) \; S^* = (M^* \ldots N^*)$$

$$(\text{simmetrica}) \; S^- = (N^- \ldots M^-)$$

SOLUZIONE

$M_\bullet = \begin{smallmatrix} 0 \\ 2 \end{smallmatrix}$: mossa mediana

$$S_{M+1} = S_M^* \, M_\bullet \, \left(S_M^*\right)^-$$

$S_1 = M_\bullet = \begin{smallmatrix} 0 \\ 2 \end{smallmatrix}$

$S_2 = S_1^* \, M_\bullet \, (S_1^*)^- = M_\bullet^* \, M_\bullet \, (M_\bullet^*)^- = \begin{smallmatrix} 0 & 0 \\ 1 & 2 \end{smallmatrix} \left(\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right)^-$

$\quad = \begin{smallmatrix} 0 & 0 \\ 1 & 2 \end{smallmatrix} \left(\begin{smallmatrix} 0^- \\ 1^- \end{smallmatrix}\right)' = \begin{smallmatrix} 0 & 0 \\ 1 & 2 \end{smallmatrix} \left(\begin{smallmatrix} 2 \\ 1 \end{smallmatrix}\right)' = \begin{smallmatrix} 0 & 0 & 1 \\ 1 & 2 & 2 \end{smallmatrix}$

$S_3 = S_2^* \, M_\bullet \, \left(S_2^*\right)^- = \dots$

$\dots$