

Iheb Belgacem
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EVOLUTIONARY GAME THEORY

Course PH2821JB
Final Project



CLASSICAL GAME THEORY

- branch of applied mathematics
- provides tools for analyzing situations in decisions that are interdependent.
- A **solution** to a game describes the **optimal decisions** of the players

What is Game Theory?



CLASSICAL GAME THEORY

Prisoner's Dilemma – Payoff Matrix

The diagram illustrates the Prisoner's Dilemma payoff matrix. It features a 2x2 grid of cells. The rows represent Player 1's strategies: 'Co-operate' and 'Defect'. The columns represent Player 2's strategies: 'Co-operate' and 'Defect'. Each cell contains a pair of payoffs (Player 1, Player 2). Red dashed arrows indicate Player 1's preference to move from 'Co-operate' to 'Defect' in both columns. Blue dashed arrows indicate Player 2's preference to move from 'Co-operate' to 'Defect' in both rows. The cell containing (1, 1) is highlighted with a yellow border and labeled 'Nash Equilibrium'. A legend on the right explains the dashed arrows: a red dashed arrow for 'Preference to Move Based on Higher Payoff' for Player 1, and a blue dashed arrow for Player 2. Labels 'Strategies For Player 1' and 'Strategies For Player 2' point to the row and column headers respectively.

ROW \ COL	Co-operate	Defect
Co-operate	(3, 3)	(0, 5)
Defect	(5, 0)	(1, 1)

Strategies For Player 1

Strategies For Player 2

Preference to Move Based on Higher Payoff

Nash Equilibrium

Source: <http://www.studycas.com/component/k2/revisiting-nash-equilibrium-in-prisoner-s-dilemma>

EVOLUTIONARY GAME THEORY

- Games played by groups of animals, each group share a gene to which correspond a strategy
- **Frequency** dependent fitness
- **Natural Selection** determines which strategy becomes predominant in the overall population.

Strategy



EVOLUTIONARY GAME THEORY

In the context of EGT:

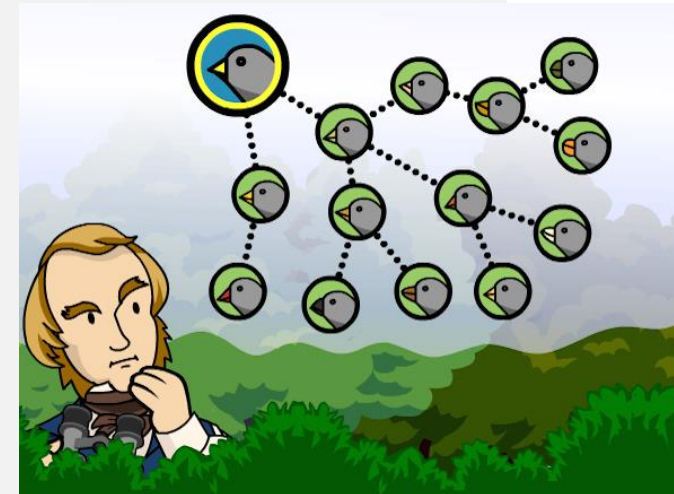
- Strategies **are not deliberate decisions** made by the players
- **Survival of the fittest**

Payoff



Darwinian fitness

Expected number of
offspring



Natural Selection

EVOLUTIONARY STABLE STRATEGIES

An evolutionarily stable strategy (ESS) is a strategy which, if adopted by a population in a given environment, is impenetrable, meaning that it **cannot be invaded** by any alternative strategy that is initially rare.

For Example : **Prisoner's Dilemma**

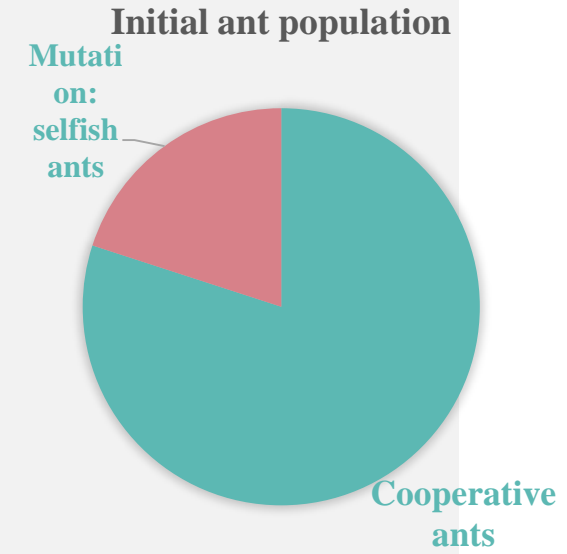
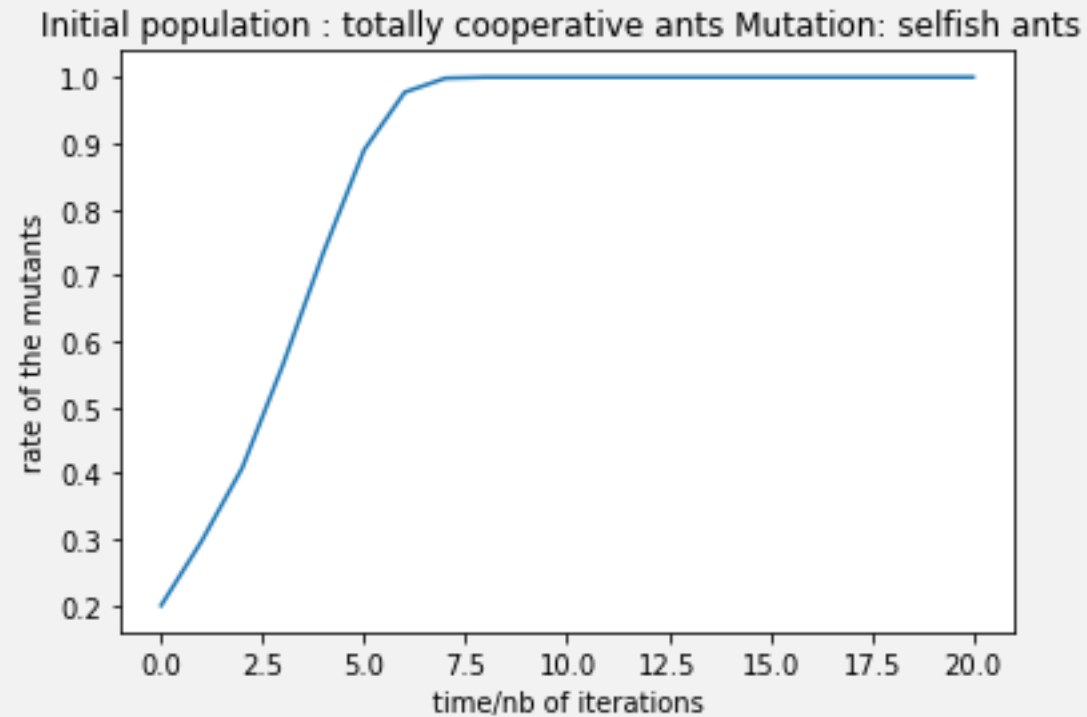
We have a population of **N cooperative** ants, if there is a **mutation** that creates ants that are selfish. How will evolve the group of the mutant ants?

Payoff Matrix

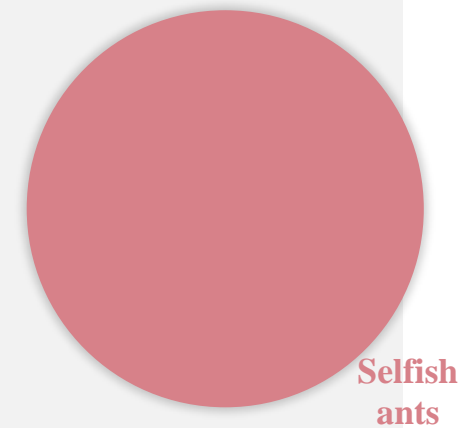
	C	D
C	(2,2)	(0,3)
D	(3,0)	(1,1)



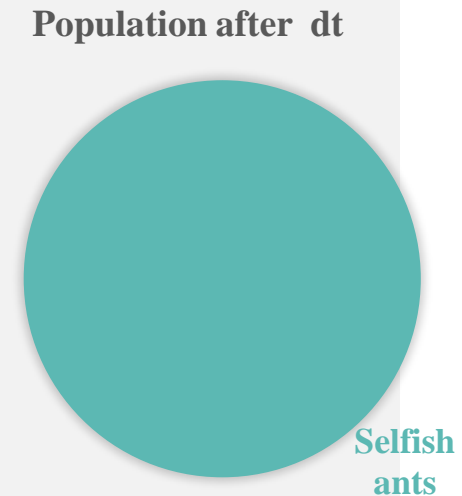
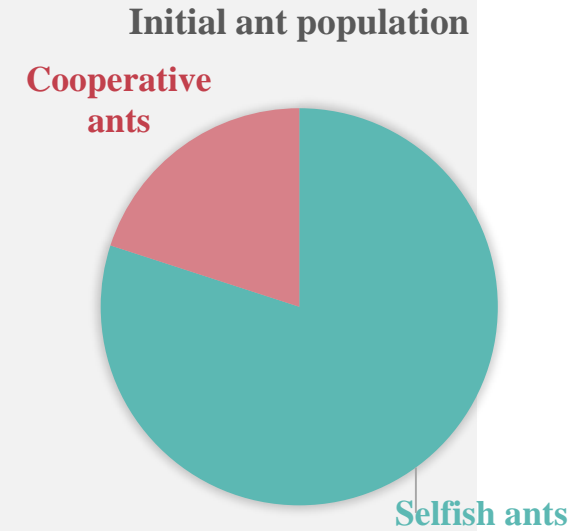
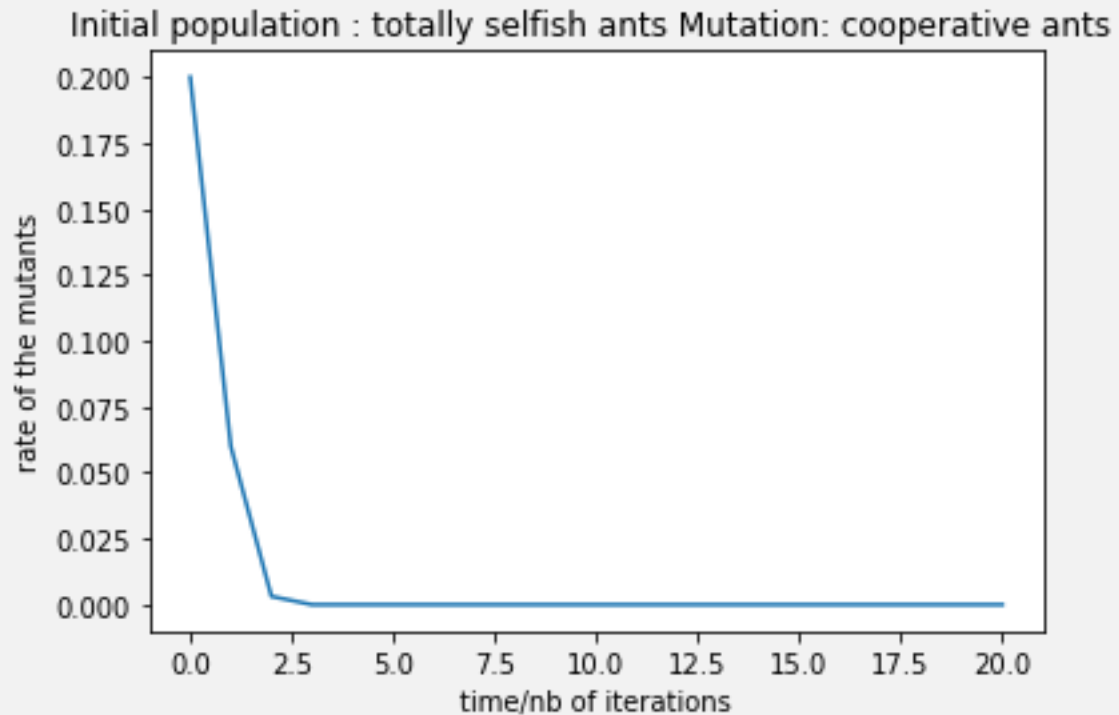
EVOLUTIONARY STABLE STRATEGIES







Population after dt



EVOLUTIONARY STABLE STRATEGIES



A CLASSICAL EXAMPLE

Hawk-Dove Model: Costs and Benefits of Fighting over Resources		
Payoff* to...	...in fights against:	
	hawk	dove
hawk	 Hawk wins 50% of fights; is injured in 50% of fights. Payoff: $(V-D)/2$	 Hawk always wins; dove flees. Payoff: V
dove	 Dove never wins; is never injured. Payoff: 0	 Dove wins 50% of fights; is never injured; wastes time. Payoff: $V/2 - T$

REPLICATOR DYNAMICS

- Symmetric Game with payoff Matrix A , Population of n types

- $x_i = f(t)$:frequency of strategy i

- State of the population is represented by the vector $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$

- Expected payoff for an individual of type i : $(A * x)_i$

- Average payoff in the population state \mathbf{x} : $x^T * A * x$

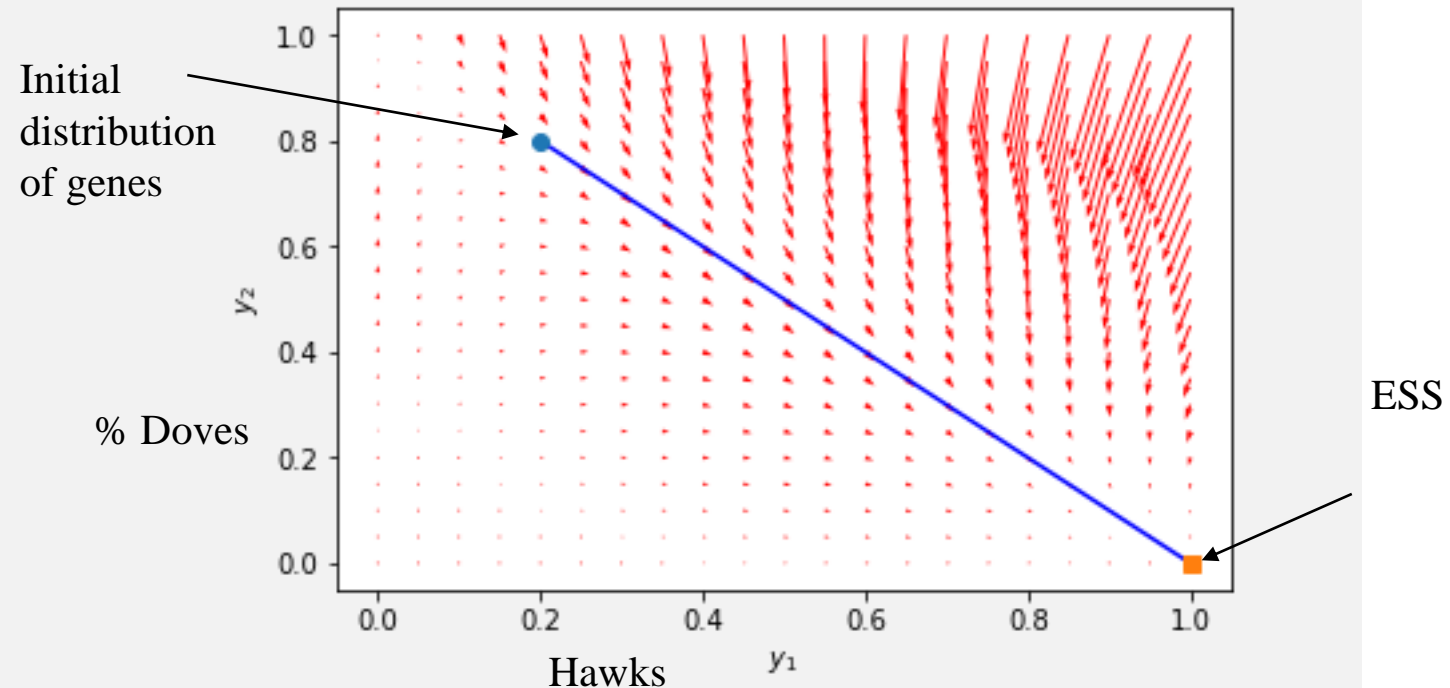
- Replicator dynamics equation:
$$\frac{dx_i}{dt} = x_i * ((A * x)_i - x^T * A * x)$$
$$\forall i \in \{1..n\}$$

DISCUSSING THE RESULTS

- *First Case : $V \geq C$*

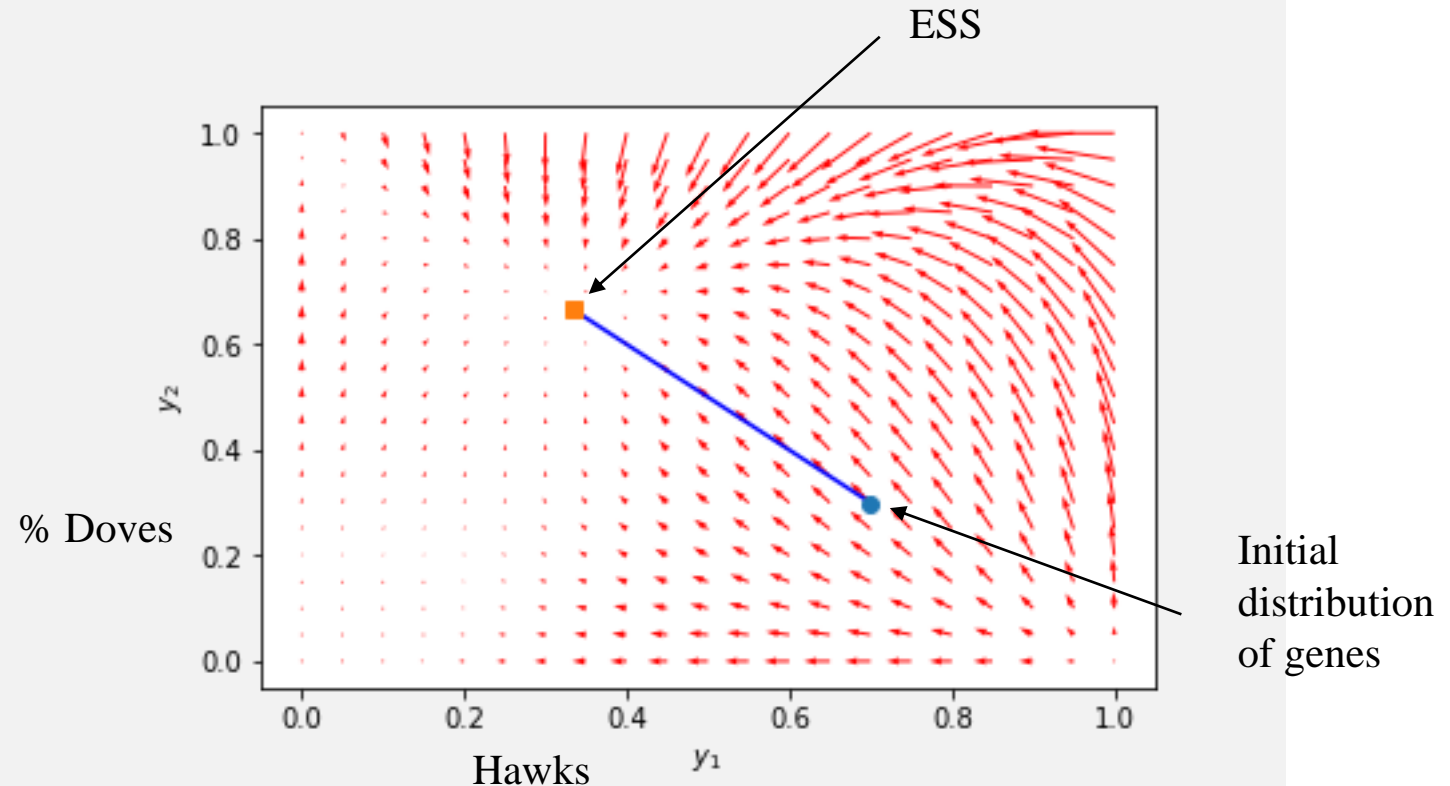
- We have one evolutionary stable situation, a situation where there are **only Hawks** in the population,

In fact, if the value of the resource is high and the cost of fighting low, it's logical to observe fights between animals

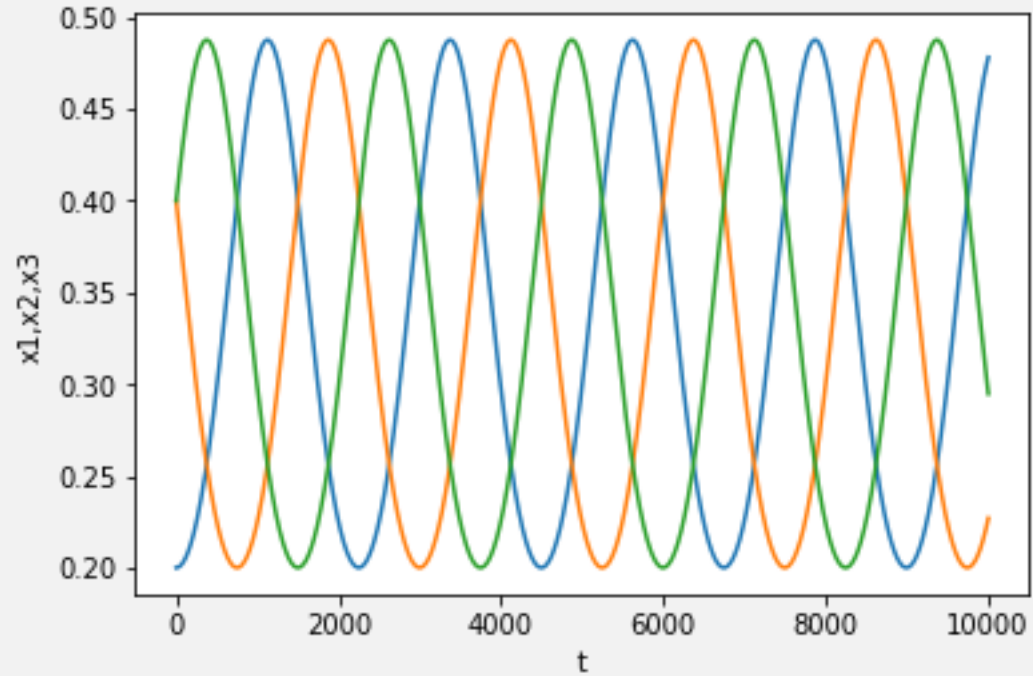
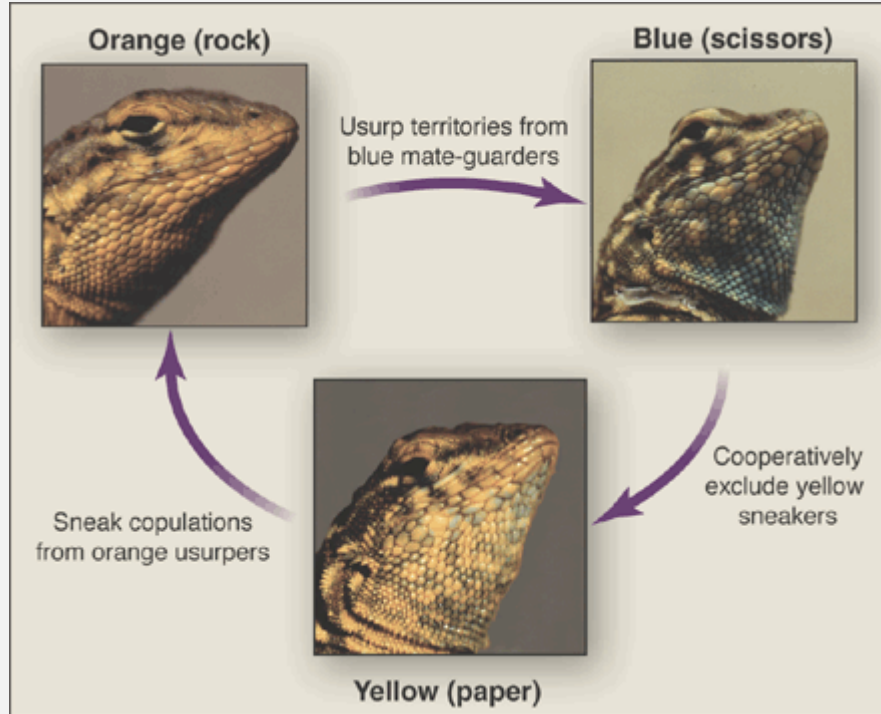


DISCUSSING THE RESULTS

- *Second Case : $C \geq V$*
- We have one evolutionary stable situation, a situation where there is V/C Hawk (in this case $1/3$) and the rest of the Population is Dove This situation is the most commun in nature.



THE SIDE-BLOTCHED LIZARD



- *There is no evolutionary stable strategy !*

BIBLIOGRAPHY

- *An Introduction To Evolutionary Game Theory* - Tim Rees : UBC Department of Computer Science
- *Game Theory with Engineering Applications* : Asu Ozdaglar- MIT
- *Evolutionary game Theory* Karl Sigmund and Martin A. Nowak
- *Evolutionary game Theory*, Jörgen Weibull, Toulouse School of Economics,

**THANK YOU FOR
YOU ATTENTION**