CPSC 532L – Assignment 1

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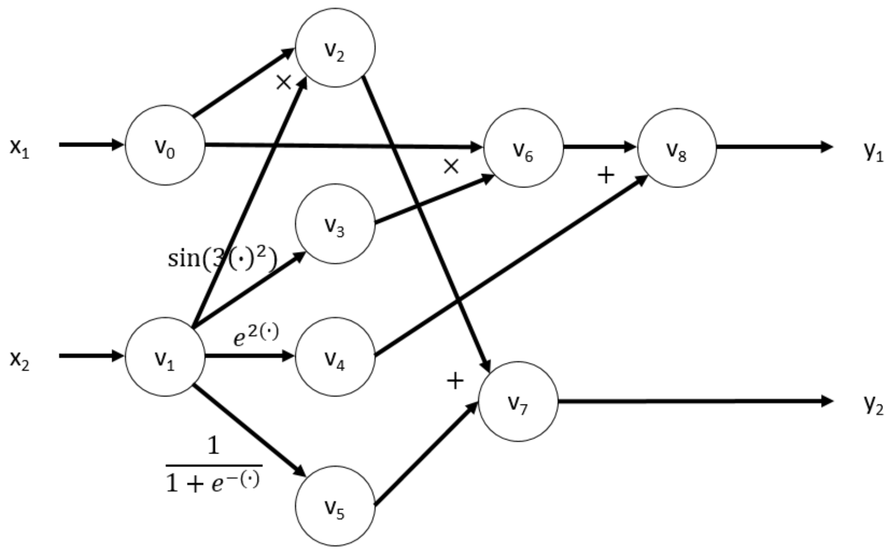
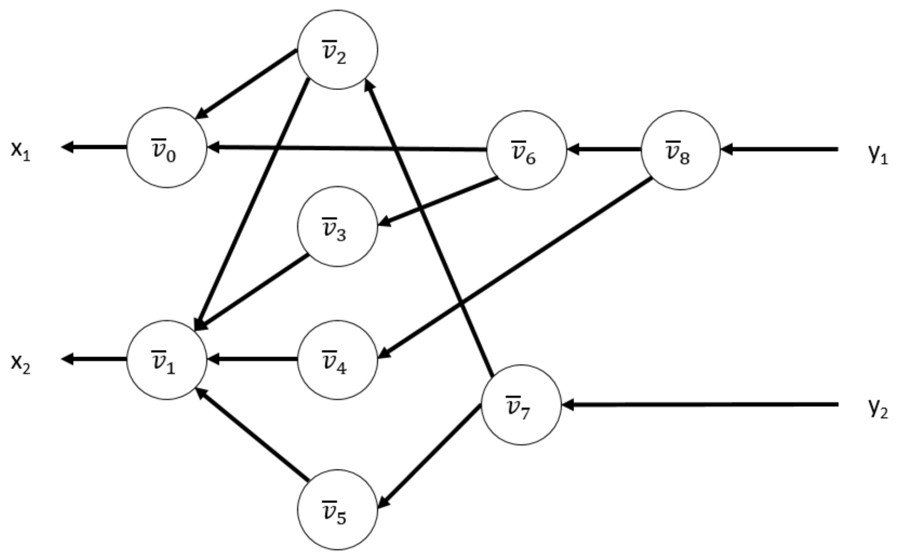
Question 1

1a. Computation Graph



1b. Backpropagation Graph, where $\bar{v}_i = \sum_{k \in pa(i)} \frac{\partial v_k}{\partial v_i} \bar{v}_k$



1c. Calculating $\mathbf{f}(\mathbf{x})$ at $x_1 = 2$, $x_2 = 1$.

$$\mathbf{f}(\mathbf{x}) = \begin{bmatrix} y_1 & y_2 \end{bmatrix}^T = \begin{bmatrix} e^2 + 2sin(3) & 2 + \frac{1}{1+e^{-1}} \end{bmatrix}^T$$
$$= \begin{bmatrix} 7.67 & 2.73 \end{bmatrix}^T$$

1d. Jacobian using forward mode at $x_1 = 2$, $x_2 = 1$.

Forward Evaluation

	$f(2,1)$
$v_0 = x_1$	2
$v_1 = x_2$	1
$v_2 = v_0 v_1$	$2 \cdot 1 = 2$
$v_3 = \sin(3{v_1}^2)$	$\sin(3) = 0.141$
$v_4 = e^{2v_1}$	$e^2 = 7.389$
$v_5 = \dfrac{1}{1+e^{-v_1}}$	$\dfrac{1}{1+e^{-1}} = 0.731$
$v_6 = v_0 v_3$	$2 \cdot 0.141 = 0.282$
$v_7 = v_2 + v_5$	$2 + 0.731 = 2.731$
$v_8 = v_4 + v_6$	$7.389 + 0.282 = 7.671$
$y_1 = v_8$	7.671
$y_2 = v_7$	2.731

Forward Derivative Trace – Only showing non-zero terms

	$\left.\dfrac{\partial \mathbf{y}}{\partial \mathbf{x}}\right\|_{x_1=2,\, x_2=1}$
$\dfrac{\partial v_0}{\partial x_1}$	1
$\dfrac{\partial v_2}{\partial x_1} = \dfrac{\partial v_2}{\partial v_0}\dfrac{\partial v_0}{\partial x_1} + \dfrac{\partial v_2}{\partial v_1}\dfrac{\partial v_1}{\partial x_1}$	$1 \cdot 1 + 2 \cdot 0 = 1$
$\dfrac{\partial v_2}{\partial x_2} = v_0$	2
$\dfrac{\partial v_3}{\partial x_2} = \cos(3{v_1}^2) \cdot 6v_1$	$6\cos(3) = -5.940$
$\dfrac{\partial v_4}{\partial x_2} = 2e^{2v_1}$	$2e^2 = 14.778$
$\dfrac{\partial v_5}{\partial x_2} = \dfrac{e^{-v_1}}{(1+e^{-v_1})^2} = v_5(1 - v_5)$	$0.731(1 - 0.731) = 0.197$
$\dfrac{\partial v_6}{\partial x_1} = v_3 \dfrac{\partial v_0}{\partial x_1}$	0.141
$\dfrac{\partial v_6}{\partial x_2} = v_0 \dfrac{\partial v_3}{\partial x_2}$	$2 \cdot -5.940 = -11.88$
$\dfrac{\partial v_7}{\partial x_1} = \dfrac{\partial v_2}{\partial x_1}$	1
$\dfrac{\partial v_7}{\partial x_2} = \dfrac{\partial v_2}{\partial x_2} + \dfrac{\partial v_5}{\partial x_2}$	$2 + 0.197 = 2.197$
$\dfrac{\partial v_8}{\partial x_1} = \dfrac{\partial v_6}{\partial x_1}$	0.141
$\dfrac{\partial v_8}{\partial x_2} = \dfrac{\partial v_4}{\partial x_2} + \dfrac{\partial v_6}{\partial x_2}$	$14.778 - 11.88 = 2.898$
$\dfrac{\partial y_1}{\partial x_1} = \dfrac{\partial v_8}{\partial x_1}$	0.141
$\dfrac{\partial y_1}{\partial x_2} = \dfrac{\partial v_8}{\partial x_2}$	2.898
$\dfrac{\partial y_2}{\partial x_1} = \dfrac{\partial v_7}{\partial x_1}$	1
$\dfrac{\partial y_2}{\partial x_2} = \dfrac{\partial v_7}{\partial x_2}$	2.197

Jacobian

$$\mathbf{J} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0.141 & 2.898 \\ 1 & 2.197 \end{bmatrix}$$

1e. Jacobian using backward mode at $x_1 = 2$, $x_2 = 1$.
Forward Evaluation – Same as `1d.`
Backward Derivative Trace – Only showing non-zero terms
For y_1,

$\bar{v}_8 = \frac{\partial y_1}{\partial v_8}$	1
$\bar{v}_6 = \bar{v}_8 \frac{\partial v_8}{\partial v_6} = \bar{v}_8$	1
$\bar{v}_4 = \bar{v}_8 \frac{\partial v_8}{\partial v_4} = \bar{v}_8$	1
$\bar{v}_3 = \bar{v}_6 \frac{\partial v_6}{\partial x_3} = \bar{v}_6 \cdot v_0$	$1 \cdot 2 = 2$
$\bar{v}_1 = \bar{v}_4 \frac{\partial v_4}{\partial v_1} + \bar{v}_3 \frac{\partial v_3}{\partial v_1}$ $= \bar{v}_4 \cdot 2e^{v_1} + \bar{v}_3 \cdot 6v_1 \cos(3{v_1}^2)$	$2 \cdot 7.389 + 2 \cdot 6\cos(3) = 2.898$
$\bar{v}_0 = \bar{v}_6 \frac{\partial v_6}{\partial v_0} = v_3$	0.141

For y_2,

$\bar{v}_7 = \frac{\partial y_2}{\partial v_7}$	1
$\bar{v}_5 = \bar{v}_7 \frac{\partial v_7}{\partial v_5} = \bar{v}_7$	1
$\bar{v}_2 = \bar{v}_7 \frac{\partial v_7}{\partial v_2} = \bar{v}_7$	1
$\bar{v}_1 = \bar{v}_5 \frac{\partial v_5}{\partial v_1} + \bar{v}_2 \frac{\partial v_2}{\partial v_1}$ $= \bar{v}_5 v_5 (1 - v_5) + \bar{v}_2 v_0$	$0.731 \cdot (1 - 0.731) + 1 \cdot 2 = 2.197$
$\bar{v}_0 = \bar{v}_2 \frac{\partial v_2}{\partial v_0} = v_1$	1

Jacobian – As expected, result is the same as forward mode.

$$\mathbf{J} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 0.141 & 2.898 \\ 1 & 2.197 \end{bmatrix}$$