IGEE-UMBB
EE 174 : Recitations set 2

1. Which of the following is a vector space ?
   a) $S = \{(x,y,z,w)/x+y-z+w=0\}$
   b) $S = \{ae^{x}+be^{-x}/a,b \text{ real}\}$
   c) $S = \{(x,y,z)/x+y+z=1\}$
   d) $S = \{f : R \to R/\ \frac{d^2f}{dx^2}+f=0\}$

2. Let $v \neq \theta$ be a vector of the vector space $V(\mathcal{F})$; Show that:
   (a) $0v = \theta$
   b) $\alpha v = \theta$ if and only if $\alpha = 0$
   c) $\alpha v = \beta v$ if and only if $\alpha = \beta$

3. Let $V(\mathcal{R})$ be the space of real-valued functions
   a) Is the set U of **even** real-valued functions a subspace ?
   b) Is the set W of **odd** real-valued functions a subspace ?
   c) From the fact that $f(x) = \frac{1}{2}[f(x)+f(-x)] + \frac{1}{2}[f(x)-f(-x)]$, what can you conclude on the relationship between $V(\mathcal{R}), U$ and W?

4. Consider again $S = \{(x,y,z)/x+y+z=1\}$
   a) Is S a subpace of $\mathcal{R}^3$?
   b) Suppose we define on S the following binary operations:
   $(x_1,y_1,z_1)+(x_2,y_2,z_2) = (x_1+x_2-1, y_1+y_2, z_1+z_2)$ and
   $\alpha(x,y,z) = (\alpha x-\alpha+1, \alpha y, \alpha z)$; Is S a vector space ?
   c) Comment your result.

5. Let S, T be subspaces of $V(\mathcal{F})$
   a) Are $\overline{S}$ and $\overline{T}$ subspaces of $V(\mathcal{F})$?
   b) Show that a necessary condition for $S \cup T$ to be a subspace is that one is contained in the other.
   Hint: Use contradiction to show that if the condition is not satisfied, $S \cup T$ is not closed under addition.