IGEE-UMBB

EE 174 : Recitations set 4

1. Let W be a subspace of dimension m of the vector space V of dimension n ; Show that:
   (a) $m \leq n$
   (b) if $m = n$ then $W = V$

2. Consider $T : R^2 \to R^2$ defined by $T(x,y) = (x+y, x+y)$
   (a) Is T linear ?
   (b) Is $v_1 = (1,0)$ and $v_2 = (1,1)$ a basis for R^2? Are their images also a basis for R^2?What can you conclude?

3. Consider $L : R^3 \to R^2$ defined by $T(x,y,z) = (x,y)$
   (a) Is L linear? What does it represent geometrically?
   (b) Is $v_1 = (1,0,0)$ and $v_2 = (0,1,0)$ a basis for R^3? Are their images a basis for the codomain? What do you conclude?

4. Let $L : V \to W$ be a linear mapping ; Show that:
   a) L is **onto** if and only if $r(L) = \dim W$
   b) If $\dim V = \dim W$, then L is **one-to-one** if and only if L is **onto.**

5. Find the nullity of the linear mapping $L : P_n \to R$ defined by $L[p(x)] = \int_0^1 p(x)dx$ where $p(x) = a_0 + a_1x + \cdots + a_nx^n$; Deduce the rank of L

6. Consider the linear transformation $T : R^3 \to R^3$ defined by $T(x,y,z) = (x+y+z, y-z, x-y+z)$; Is T nonsingular ? If so, determine $T^{-1}(x,y,z)$.