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EE 178: Physics II

Electricity & Magnetism

Chapter 5: Capacitance

5.1- Introduction:

The purpose of this chapter is the study of capacitors. They are electric devices in which electrical energy can be stored. They are the basis of battery systems.

5.2- Capacitance:

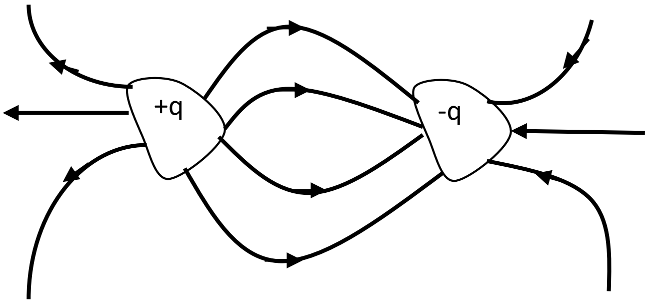


Figure 1: Two isolated conductors from each other, form a capacitor

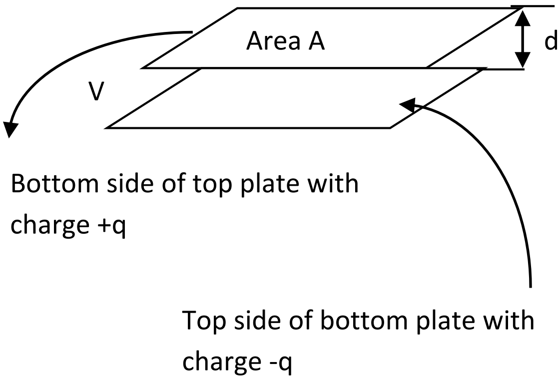


Figure 2: A parallel-plate capacitor, made up of 2 plates with area A, distance d and opposite charge q

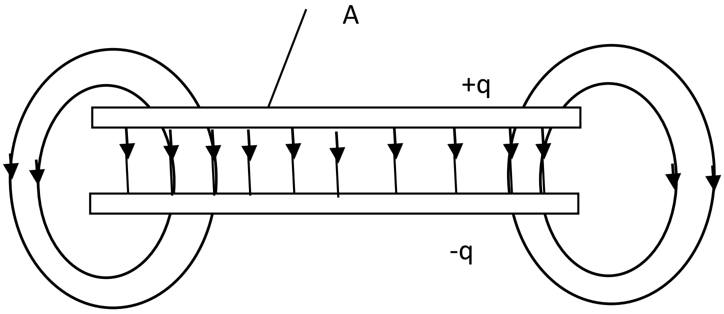


Figure 3 : Electric field lines in a capacitor

When a capacitor is **charged**, its plates have charges of equal magnitude but opposite signs: +q and –q.
Charge q: the absolute value of these charges.
The charge q and the potential difference V for a capacitor are proportional to each other,
$q = C\,.V$

The proportionality constant C is called the **capacitance** of the capacitor. Its value depends only on the geometry of the plates and not on their charge or potential difference.
SI unit of capacitance→ Coulomb/Volt = Farad
1 Farad= 1 F = 1 Coulomb per volt = 1 C/V
1 μF= 10^{-6} F; 1 pF = 10^{-12} F

5.3- Calculating the capacitance:

To deduce the capacitance, we must follow these steps:

1. Assume a charge q on the plates
2. Calculate the electric field $\vec{E}$ between the plates by using Gauss' Law
3. Calculate the potential difference V between the plates
4. Calculate C from : Q = C.V

1) Calculating the electric field:

Gauss' Law: $\varepsilon_0 \oint \vec{E}\,\overrightarrow{dA} = q$ with q is the enclosed charge

In Gauss' Law, generally $\vec{E}$ and $\vec{A}$ are parallel, so: $q = \varepsilon_0\, E\, A$

2) Calculating the potential difference:

$$V_f - V_i = -\int_i^f \vec{E}\,.\overrightarrow{ds}$$

$$\Delta V = V = V_f - V_i = \int_-^+ E\, ds$$

Parallel- plate capacitor: E constant

q = ε_0 E A

$$V = \int_-^+ E\, ds = E\int_0^d ds = E\, d = \frac{q}{C} = \frac{\varepsilon_0\, E\, A}{C}$$

$$C = \frac{\varepsilon_0\, A}{d}$$

(Parallel-plate capacitor)

ε_0= 8.85×10^{-12} F/m = 8.85 pF/m (permittivity constant)

A cylindrical capacitor:

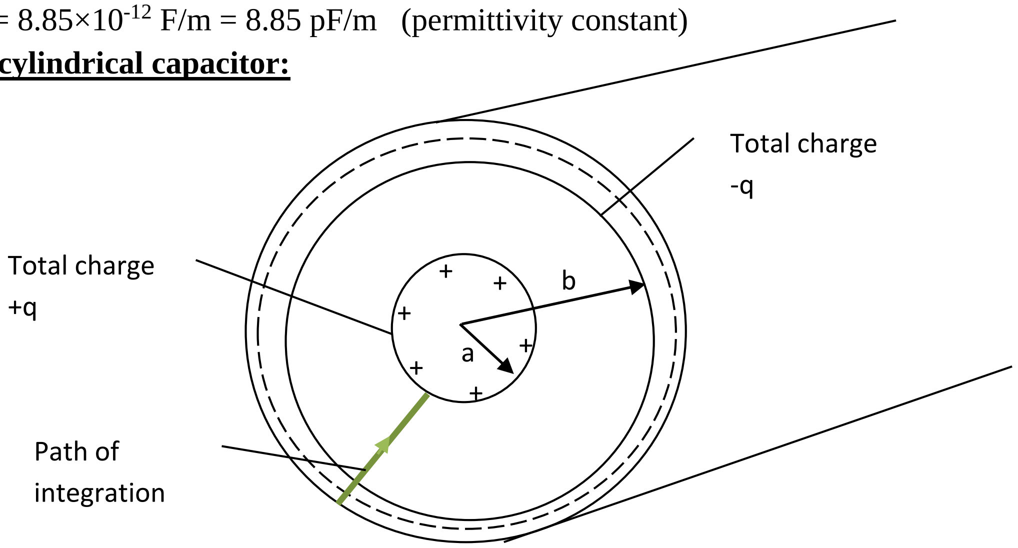


Figure 4: A cross section of a long cylindrical capacitor

So, as a Gaussian surface, we choose a cylinder of length L and radius r

$$q = \varepsilon_0 \, E \, A = \varepsilon_0 \, E \, (2\pi \, r \, l)$$

With $2\pi \, r \, l$ the area of the curved part

There is no flux through the end caps

$$V = \int_{-}^{+} E \, ds = -\frac{q}{2\pi\varepsilon_0 \, L} \int_{b}^{a} \frac{dr}{r} = \frac{q}{2\pi\varepsilon_0 \, L} \ln\left(\frac{b}{a}\right)$$

Here ds =-dr (we integrated radially inward from the negative to the positive plate)

$$C = 2\pi\varepsilon_0 \, \frac{L}{\ln\left({}^{b}/_{a}\right)}$$

(cylindrical capacitor)

A spherical capacitor:

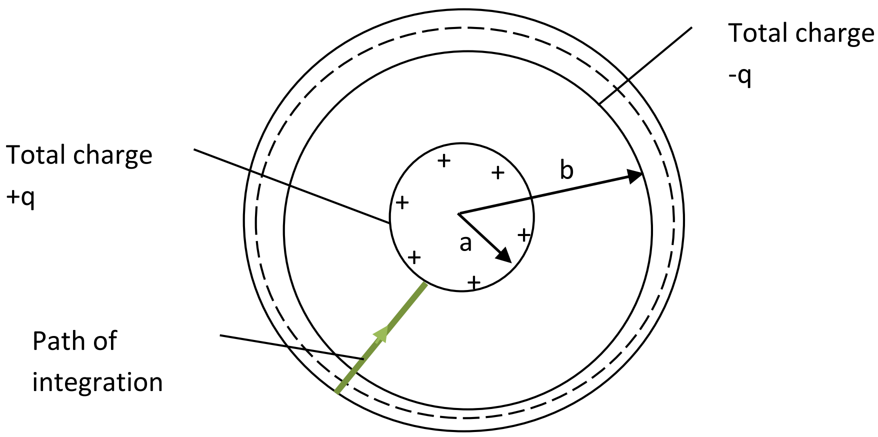


Figure 5: A cross section of a spherical capacitor

In this case, we have a capacitor with two concentric spherical shells, of radii a and b. The Gaussian surface is chosen as a sphere of radius r between the two shells.

$$q = \varepsilon_0 \, E \, A = \varepsilon_0 \, E \, (4\pi \, r^2 \,) \rightarrow E = \frac{1}{4\pi \, \varepsilon_0} \frac{q}{r^2}$$

$$V = \int_{-}^{+} E \, ds = -\frac{q}{4\pi\varepsilon_0} \int_{b}^{a} \frac{dr}{r^2} = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{q}{4\pi\varepsilon_0} \frac{b-a}{ab}$$

Ds was substituted by –dr

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a}$$

(spherical capacitor)

An isolated sphere:

We can assign a capacitance to a single isolated spherical conductor of radius R by supposing that “the missing plate” is a conducting sphere of infinite radius

$$C = 4\pi\varepsilon_0 \frac{1}{a - {}^{a}/_{b}}$$

If $b \to \infty$ and R = a we find

$$C = 4\pi\varepsilon_0 R$$

(An isolated sphere)

Capacitors in parallel:

$$C_{eq} = \sum_{j=1}^{n} C_j$$

$$(\, n \; capacitors \; in \; parallel)$$

Capacitors in series:

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$$

$$(\, n \; capacitors \; in \; series)$$

5.5- Energy stored in electric field:

Suppose that, at instant t, a charge $\acute{q}$ has been transferred from one plate of a capacitor to the other. The potential difference $\acute{V}$ between the plates at that instant will be ${}^{\acute{q}}/_{C}$

If an additional increment of charge $d\acute{q}$ is then transferred, the increment of work required is $dW = V\, d\grave{q} = \frac{\acute{q}}{C}\; d\acute{q}$

The work required to bring the total charge up to a final value q is:

$$W = \int dW = \frac{1}{C}\int_0^q \acute{q}\, d\acute{q}$$

The work is stored as potential energy U in the capacitor

$$U = \frac{q^2}{2C}$$

$$U = \frac{1}{2}\, CV^2$$

$$(potential \; energy)$$

5.6- Capacitor with a dielectric:

The space between the plates of a capacitor is filled with **a dielectric**, which is an insulating material such as mineral oil or plastic, the capacitance is increased by a numerical factor κ, called the **dielectric constant** of the insulating material.

Table 1: Some properties of dielectrics

Material	Dielectric constant κ
Air	1.00054
Polystyrene	2.6
Paper	3.5
Transformer oil	4.5
Pyrex	4.7

Ruby mica	5.4
Porcelain	6.5
Silicon	12
Germanium	16
Ethanol	25 with
Water (20°C)	80.4
Water (25°C)	78.5
Titania ceramic	130
Strontium titanate	310
Vacuum	**1**

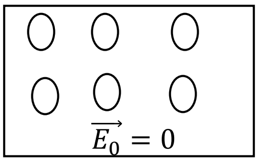
In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ε_0 are to be modified by replacing ε_0 with $\kappa\varepsilon_0$

5.7- Dielectrics: An atomic view

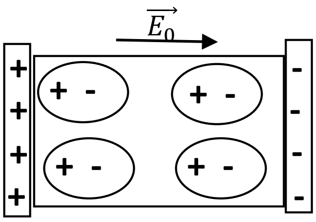
When we put a dielectric in an electric field, there are two possibilities depending on the type of molecule.

1. Polar dielectrics: for molecules with permanent electric dipole (like water), the electric dipoles tend to line up with an external electric field
2. Non-polar dielectrics. Molecules acquire dipole moment by induction when placed in an external electric field. The external field tends to **"stretch"** the molecules slightly separating the centers of negative and positive charge

The initial electric field inside this non polar dielectric slab is zero



The applied field aligns the atomic dipole moments



The field of the aligned atoms is opposite the applied field

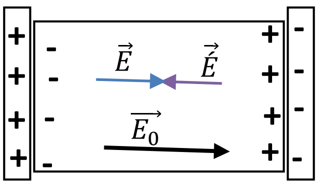


Fig 7: (a) A non polar dielectric slab; (b) An electric field is applied via charges capacitor plates; the field slightly stretches the atoms, separating the centers of the positive and negative charges, (c) This separation created surface charges on the slab faces and set up a field $\vec{E'}$ opposite to $\overrightarrow{E_0}$. The resultant electric field has the same direction of the initial $\overrightarrow{E_0}$ but with lower magnitude.

<u>5.8- Dielectrics and Gauss' Law:</u>

The Gauss' Law in vacuum: $\varepsilon_0 \oint \vec{E} \,. \overrightarrow{dA} = q$

The Gauss' Law in a dielectric:

$$\boldsymbol{\varepsilon_0} \oint \boldsymbol{\kappa} \vec{E} \,. \overrightarrow{dA} = q$$

$$(\,\boldsymbol{Gauss' Law\ with\ dielectric})$$