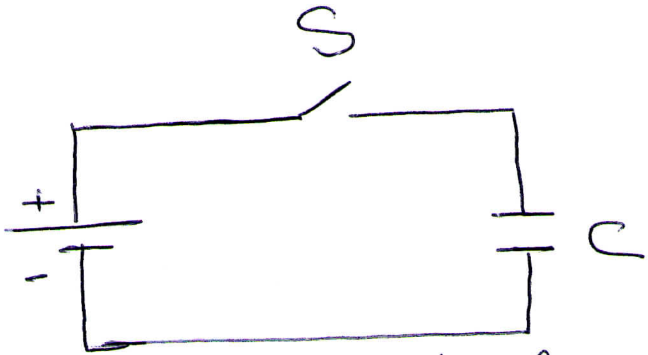
EE 178 4th Recitation
Capacitors & Resistors

Part I. Capacitors.

Ex # 1:



Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery.
The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery.

Thus: $q = (2.5 \times 10^{-6}\,F)(120\,V) = 3.0 \times 10^{-3}\,C$

Ex # 2: We use the formula for spherical capacitor:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40 \times 10^{-3})(38 \times 10^{-3})}{9 \times 10^{9}\ (40-38) \times 10^{-3}}$$

$$C = 84.5\ pF$$

(b) Let the area required be A.

$$C = \varepsilon_0 A / (b-a)$$

So: $A = \dfrac{C(b-a)}{\varepsilon_0} = \dfrac{(84.5\ pF)(40-38) \times 10^{-3}}{8.85 \times 10^{-12}} = 191\ cm^2$

Ex # 3:

We assume there is charge q on one plate and charge (−q) on the other. The electric field in the lower half of the region between the plates is:

$$E_1 = \frac{q}{K_1 \varepsilon_0 A}$$

The electric field in the upper half is:

$$E_2 = \frac{q}{K_2 \varepsilon_0 A}$$

let $\frac{d}{2}$ be the thickness of each dielectric.

Since the field is uniform in each region, the potential difference between the plates is:

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{q d}{2 \varepsilon_0 A}\left[\frac{1}{K_1} + \frac{1}{K_2}\right]$$

$$= \frac{q d}{2 \varepsilon_0 A} \quad \frac{K_1 + K_2}{K_1 K_2}$$

$$\boxed{C = \frac{q}{V} = 2 \frac{\varepsilon_0 A}{d} \quad \frac{K_1 K_2}{K_1 + K_2}}$$

This formula ≡ C_{eq} of 2 capacitors in series.

$A = 7.89 \times 10^{-4}\, m^2$, $d = 4.62 \times 10^{-3}\, m$, $K_1 = 11.0$
$K_2 = 12.0$

$$C = \frac{2(8.85 \times 10^{-12})(7.89 \times 10^{-4})(11)(12)}{4.62 \times 10^{-3} \quad (11 + 12)}$$

$$\boxed{C = 0.17\,pF}$$

Ex 5: The length d is effectively shortened by b:

$$C' = \frac{\varepsilon_0 A}{d-b} = 0.708\,pF.$$

2- The energy before, divided by the energy after inserting the slab is:

$$\frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d} = \frac{d}{d-b}$$

$$= \frac{5.00}{5.00 - 2.00}$$

$$\boxed{\frac{U}{U'} = 1.67}$$

3- The work done:

$$W = U' - U = \frac{q^2}{2}\left(\frac{1}{C'} - \frac{1}{C}\right) = \frac{q^2}{2\varepsilon_0 A}(d-b) -$$

$$= -\frac{q^2 b}{2\varepsilon_0 A} = -5.44\,J.$$

4- $W < 0$ the slab is sucked in.

Ex4: 1 $C = \frac{\varepsilon A}{d} = \frac{K \varepsilon_0 A}{d}$

$$C = \frac{2.0 \times 8.85 \times 10^{-12} \times 80.(10^{-2})^2}{0.50 \times 10^{-2}} = 28.3\,pF.$$

2 $V = 100\,V$, $Q = CV = 100 \times 28.3\,pF = 2.83\,nC$

Energy stored: $U = \frac{1}{2}CV^2 = 1.41 \times 10^{-7}\,J.$

Part II - Resistors

Ex7: ① The current in each strand is:

$$i = 0.750\,A / 125 = 6.00 \times 10^{-3}\,A.$$

② The potential difference:

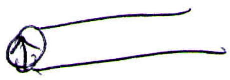
$$V = R\,i = (6.00 \times 10^{-3}\,A)(2.65 \times 10^{-6}\,\Omega).$$

$$V = 1.59 \times 10^{-8}\,V.$$

③ The resistance $R_{Total} = 2.65 \times 10^{-6}\,\Omega / 125$.

$$\boxed{R_{total} = 2.12 \times 10^{-8}\,\Omega}$$

Ex 6: $\ell = 4.00\,m$



Diameter: $6.00\,mm$

$R = 15.0\,m\Omega$, $V = 23.0\,V$.

① The current wire: $V = Ri \Rightarrow i = \frac{V}{R} = \frac{23.0\,V}{15.0 \times 10^{-3}}$ V.

$$i = 1.53 \times 10^{-3}\,A.$$

② The current density:

$$j = \frac{i}{A} = \frac{i}{\pi\left(\frac{D}{2}\right)^2} = 54.11\,A/m^2$$

③ Resistivity: $R = \rho\frac{L}{A} \rightarrow \rho = \frac{R.A}{L}$

$$\rho = \frac{15.0 \times 10^{-3} \times \pi(3 \times 10^{-3})^2}{4.00}$$

$$\boxed{\rho = 1.06 \times 10^{-7}\,\Omega m}$$