EXo n°= 1

- Show that if a is rational and b is irrational then $a+b$ is irrational
- Show that $\sqrt[3]{2}$ is irrational
- Show that

$$1^3 + 2^3 + \ldots n^3 = \left(1+2+3+\ldots n\right)^2 \quad \forall n \geq 1 \ n \in \mathbb{N}$$

Hint (use the formula $1+2+\ldots+n = \dfrac{n(n+1)}{2}$)

EXo n°= 2

a) let f be a function defined as $f(x)= \begin{cases} x & x<1 \\ x^2 & 1 \leq x \leq 4 \\ 8\sqrt{x} & x>4 \end{cases}$

① is f invertible ? ② Find f^{-1}

③ is f Continuous ?

b) Let $g(x)$ is a function defined by $g(x)= \begin{cases} 0 & x \in \mathbb{Q} \\ x & x \in \mathbb{R}\backslash\mathbb{Q} \end{cases}$

① For what value of x g is Continuous ?

② For what value of x g is discontinuous ? precise the type of discontinuity (essential or removeable).

EXo n°= 3

● Show that $\displaystyle\lim_{x \to 0} \sin\left(\dfrac{2}{x}\right)$ Does not exist

● Using E-δ Show that $\displaystyle\lim_{x \to 0} \dfrac{2x+3}{3x-1} = -3$ and $\displaystyle\lim_{x \to c} |x| = |c|$

● Compute these limits : $\displaystyle\lim_{x \to 0} \dfrac{x^2-2x}{\sin 3x}$, $\displaystyle\lim_{x \to 0} \dfrac{x^2}{\sec x - 1}$

$\displaystyle\lim_{x \to \frac{\pi}{2}} \dfrac{\cos x}{x - \frac{\pi}{2}}$, $\displaystyle\lim_{x \to \pi} \dfrac{\sin x}{x - \pi}$.

$x \overset{v}{=} \dfrac{?}{\cos t}$

Problem 1

① using the $\varepsilon - \delta$ definition of limits show that

$$\underset{x \to 1}{Lim} \quad \frac{2x^4 - 6x^3 + x^2 + 3}{x - 1} = -8$$

② Find the tangent line of the function $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point (x_0, y_0) where $a, b \in \mathbb{R}^*$

③ Find the critical points of the function
$f(x) = 2\cos x + \sin^2 x$, $g(x) = x^{1/3} - x^{-2/3}$

④ Find local and absolute extrema for $R(x) = 1 - \sqrt{x}$ over it's Dom

⑤ Show that $\sin x = \frac{1}{x}$ has at least one solution in the Intervals
$\left[2k\pi, 2k\pi + \frac{\pi}{2} \right]$ and $\left[\frac{\pi}{2} + 2k\pi, (2k+1)\pi \right]$

⑥ Let $0 < a < b$
use the mean value theorem to show that $\sqrt{b} - \sqrt{a} < \frac{b-a}{2\sqrt{a}}$

⑦ Let $f(x) = \frac{1}{x}$ Find $f^{(n)}(x)$ the nth derivative of f

Problem 2

① Find the area of the region R between the Graph of $f(x) = 16 - x^2$ and the x-axis over $[1, 3]$ using lower or upper Sums with equal width

② Evaluate the Integrals

$$\int_{e}^{e^4} \frac{1}{x\sqrt{\ln x}} \, dx \quad , \quad \int \sin \sqrt{x} \, dx \quad , \quad \int \cos(\ln x) \, dx$$

$$\int e^{ax} \sin bx \, dx \quad , \quad \int \frac{x e^{2x}}{(2x+1)^2} \, dx$$

Good luck