

IGEE-UMBB
EE 174 : Recitations set 3

1. Check the linear independence of the following vectors of R^3
 - a) $\{(0, 0, -1); (1, 0, 4)\}$
 - b) $\{(1, 1, 0); (-1, 2, 0)\}$
 - c) $\{(1, 1, 0); (-1, 1, 2); (0, 2, 2)\}$
2. Let P_2 be the space of real polynomials of degree ≤ 2 ; Check the linear independence of the following sets
 - a) $\{1, x, x^2\}$
 - b) $\{-1 + 4x - x^2, 1 + 7x, 2 + 3x + x^2\}$
 - c) Can we select a basis for P_2 from these sets?
3. Let $S = \{v_1, v_2, \dots, v_k\}$ be a subset of the vector space $V(F)$; Show that $[S] = [v_1, v_2, \dots, v_k] = [v_1, v_2, \dots, v_k, v_{k+1}]$ **if and only if** $v_{k+1} \in [v_1, v_2, \dots, v_k]$ for any $v_{k+1} \in V(F)$. What conclusions can you draw from this result?
4. Let S, T be linearly independent subsets of vectors of the vector space $V(F)$
 - a) Is $S \cap T$ linearly independent? Justify.
 - b) Are the complements \bar{S} and \bar{T} linearly independent? Justify.
5. Let P_3 be the space of real polynomials of degree ≤ 3 ; Is the subset $\{1, 1 + x, 1 + x + x^2, 1 + x + x^2 + x^3\}$ a basis for P_3 ? If so, express $p(x) = x^2 + x^3$ in this basis.
6. Find a basis and the dimension of the solution space of the following system of linear equations:
$$\begin{aligned}x_1 - 4x_2 + 3x_3 - x_4 &= 0 \\ 2x_1 - 8x_2 + 6x_3 - 2x_4 &= 0\end{aligned}$$