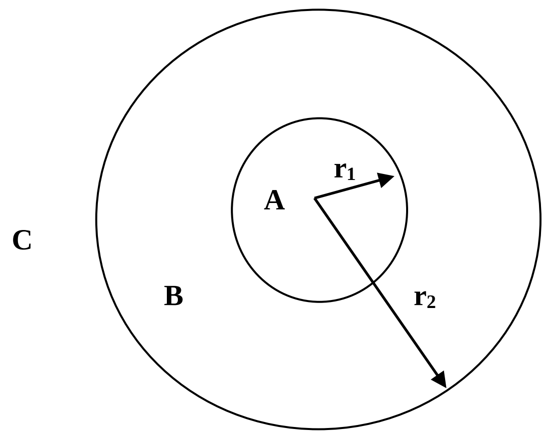
EE178 Third Recitation
Electric potential

Exercise 1 :

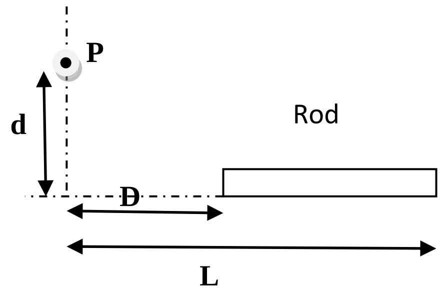
Consider two thin, conducting, spherical shells as shown in the figure below. The inner shell has a radius r_1 = 15.0 cm and a charge 10.0 nC. The outer shell has a radius r_1 = 30.0 cm and a charge -15.0 nC. Find:

1. The electric field $\vec{E}$
2. The electric potential V in regions A, B and C, with V = 0 at r = ∞



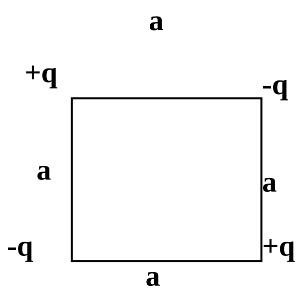
Exercise 2:

The figure below shows a thin rod with a uniform charge density of 1.00 µC/m. Evaluate the electric potential at P if d = D = L/4



Exercise 3:

How much work is required to set up the arrangement of the figure if q = 2.30 pC, a = 64.0 cm and the particles are infinitely far apart and at rest?



Exercice 4:

A total charge of 5.4×10^{-6} C is uniformly distributed along a ring of radius 0.89 m.

1. What is the potential at the center of the ring?
2. What is the potential at a point on the axis of the ring at a distance of 0.98 m from the plane of the ring?

Exercise 5:

Two parallel plates carry a surface charge density of $+\sigma$ and $-\sigma$ respectively, and are separated by a small distance d. Assume that the size of the plates is always large compared with the distance to the plates.

1. What is the electric field in the region between the plates?
2. What is the potential difference between a point on one plate and a point on the other plate?
3. Which plate, the positive or the negative plate, is at the higher potential?

Exercise 6:

What is the magnitude of the electric field at the point $(3\vec{\imath} - 2\vec{\jmath} + 4\vec{k})m$ if the electric potential is given by $V = 2xyz^2$, where V is in volts and x, y and z are in meters?

Exercise 7:

A coaxial cable consists of a long, conducting wire, of radius $\mathbf{R_1}$ with a linear charge density of λ, and a long conducting coaxial spherical cylindrical shell, with an inner radius $\mathbf{R_2}$ and an outer radius $\mathbf{R_3}$, and with a symmetric linear density $-\lambda$. We assume the length to be much greater than any of the radial distances of interest.

1. What is the potential due to the cable at a point at a radial distance from the axis r, such that $\mathbf{r > R_3}$
2. What is the potential at a point within the outer cylindrical shell at $\mathbf{R_2 < r < R_3}$?
3. What is the potential at a point between the wire and the cylinder at $\mathbf{R_1 < r < R_2}$?
4. What is the potential at a point within the wire $\mathbf{r < R_1}$?

EE 178 3rd Recitation. Electric Potential & Potential Energy

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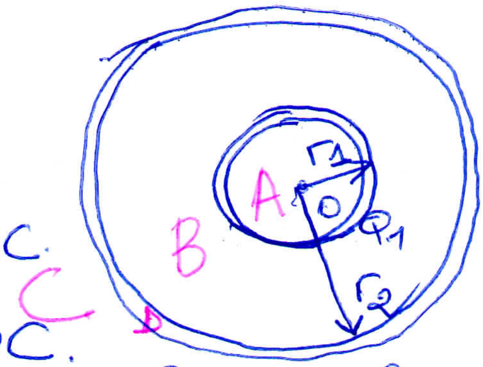
Exercise 1:

2 thin conducting spherical shells:



$r_1 = 15.0\,cm$, $Q_1 = 10.0\,nC$.

$r_2 = 30.0\,cm$, $Q_2 = -15\,nC$.

1 _ The electric field $\vec{E}$ in the regions A, B,

Gauss' Law, Region A $r<r_1$: Conducting sphere, no charge inside, $E=0$.

$$\boxed{E(r) = 0 \quad , \; r<r_1 \qquad \text{Region A}}$$

Region B: $r_1 < r < r_2$. $\oint \vec{E}.\vec{dA} = \frac{q_{enc}}{\varepsilon_0}$.

$$E.A = E.4\pi r^2 = \frac{Q_1}{\varepsilon_0}$$

$$\boxed{E(r) = \frac{Q_1}{4\pi\varepsilon_0 r^2} \quad , r_1 < r < r_2 \qquad \text{Region B}}$$

Region C: The inner charge of the 2nd shell is $-Q_1$, So the outer charge of the 2nd shell is $Q_2 + Q_1$.

$$\oint \vec{E}.\vec{dA} = \frac{q_{enc}}{\varepsilon_0}.$$

$$E.A = E.4\pi r^2 = \frac{Q_1+Q_2}{\varepsilon_0}$$

$$\boxed{E(r) = \frac{Q_1+Q_2}{4\pi\varepsilon_0 r^2} \qquad r>r_2.} \qquad \text{Region C}$$

2 - The electric potential V in the regio[ns]
A, B, C

with $V=0$ at $r=\infty$.

Region C:

$$V_\infty - V(r) = -\int_r^\infty \vec{E}\cdot\vec{dl} = -\int_r^\infty E(r)\,dr = -\int_r^\infty \frac{(Q_2+Q_1)}{4\pi\varepsilon_0 r^2}\,dr$$

$$0 - V(r) = -\frac{(Q_2+Q_1)}{4\pi\varepsilon_0}\left.\frac{r^{-1}}{-1}\right|_r^\infty = \frac{Q_2}{4\pi\varepsilon_0}\left.\frac{1}{r}\right|_r^\infty$$

$$-V(r) = \frac{(Q_2+Q_1)}{4\pi\varepsilon_0}\left(0-\frac{1}{r}\right)$$

$$\boxed{V(r) = \frac{(Q_2+Q_1)}{4\pi\varepsilon_0 r}}$$

Region B: $V(r) = -\int \frac{Q_1}{4\pi\varepsilon_0 r^2}\,dr$

$$= \frac{-Q_1}{4\pi\varepsilon_0}\left(\frac{r^{-1}}{-1}\right) + C_1$$

$$= \frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r} + C_1.$$

Due to continuity $V(r_2) = \underset{\text{(region C)}}{\frac{Q_1+Q_2}{4\pi\varepsilon_0 r_2}} = \underset{\text{(regi[on B])}}{\frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r_2}}$

so: $C_1 = \frac{Q_2}{4\pi\varepsilon_0 r_2} = \frac{-15\times10^{-9}\times 9\times10^9}{30\cdot10^{-2}}$

$$= 450\text{ V}.$$

Region B: $\boxed{V(r) = \frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r} + 450\text{ V}}$

Region A: $V(r) = -\int \overset{0}{\overset{\shortparallel}{E}}(r)\,dr = C_2 = 1050\text{ V}.$

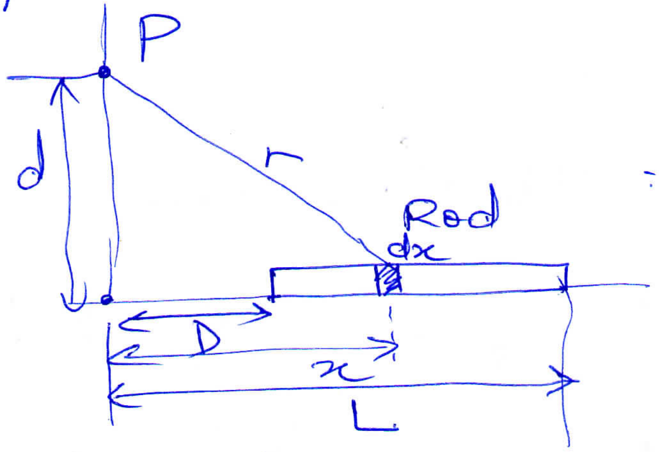
Due to Continuity $V(r_1) = C_2 = \frac{Q_1}{4\pi\varepsilon_0}\frac{1}{r_1} + 450$

$C_2 = 600+450 = 1050\text{V}$. region A $= \frac{10\times10^{-9}\times9\times10^9}{15\times10^{-2}} + 45[0]$

Exercise2:

Thin rod with $\lambda = 1.00\,\mu C/m$

$V = ?$ at P if $d = D = \frac{L}{4}$



$dq = \lambda\, dx$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{r}$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{\sqrt{d^2+x^2}}$$

$$V_p = \int dV = \frac{\lambda}{4\pi\varepsilon_0}\int_D^L \frac{dx}{\sqrt{d^2+x^2}} = \frac{1}{4\pi\varepsilon_0}\ln\left(x+\sqrt{x^2+d^2}\right)\Big|$$

$$\boxed{\int \frac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)}$$

$$V_p = \frac{\lambda}{4\pi\varepsilon_0}\left[\ln\left(L+\sqrt{L^2+d^2}\right) - \ln\left(D+\sqrt{D^2+d^2}\right)\right]$$

$$V_p = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(L+\sqrt{L^2+d^2}\right) - \ln\left(D-\sqrt{D^2+d^2}\right)$$

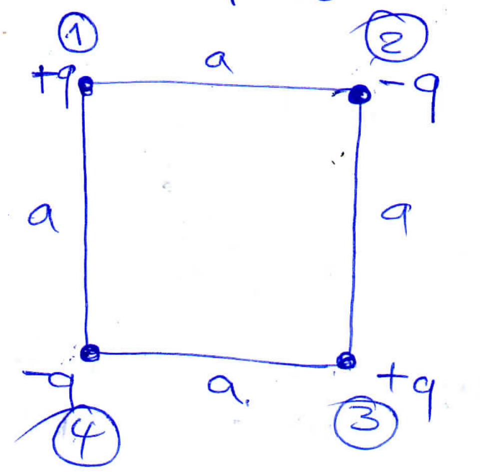
if $d = D = \frac{L}{4}$

$$V_p = \frac{\lambda}{4\pi\varepsilon_0}\ln\left(\frac{1+\sqrt{\frac{17}{16}}}{1+\sqrt{\frac{1}{8}}}\right) = 36$$

Exercise 3:

$q_2 = q_4 = -q$

$q_1 = q_3 = q$



$$U = \cancel{U_{12}} + \cancel{U_{13}} + \cancel{U_{14}} + \cancel{U_{21}} + \cancel{U_{23}} + \cancel{U_{24}} + \cancel{U_{31}} + \cancel{U_{32}} + \cancel{U_{34}} + \cancel{U_{41}} + \cancel{U_{42}} + \cancel{U_{43}}.$$

$$U = k\frac{q_1 q_2}{a} + k\frac{q_1 q_3}{a\sqrt{2}} + k\frac{q_1 q_4}{a} + k\frac{q_1 q_2}{a} + k\frac{q_2 q_3}{a} + k\frac{q_2 q_4}{a\sqrt{2}} + k\frac{q_1 q_3}{a\sqrt{2}} + k\frac{q_2 q_3}{a} + k.\frac{q_3 q_4}{a} + k.\frac{q_1 q_4}{a} + k.\frac{q_2 q_4}{a\sqrt{2}} + k\frac{q_4 q_3}{a}$$

$$U = \frac{k}{a}\left(-q^2 + \frac{q^2}{\sqrt{2}} - q^2 - q^2 - q^2 + \frac{q^2}{\sqrt{2}} + \frac{q^2}{\sqrt{2}} - q^2 - q^2 - q^2 + \frac{q^2}{\sqrt{2}} - q^2\right)$$

$$U = \frac{kq^2}{a}\left(-1 + \frac{1}{\sqrt{2}} - 1 - 1 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 - 1 - 1 + \frac{1}{\sqrt{2}} - 1\right)$$

$$U = \frac{kq^2}{a}\left(-8 + \frac{4}{\sqrt{2}}\right)$$

$$U = \frac{9\times10^{9} \times (2.30\times10^{-12})^2}{64\times10^{-2}}\left(-8 + \frac{4}{\sqrt{2}}\right)$$

$$\boxed{U = 3.84 \times 10^{-13}\ J}$$

Exercise 4:

$r = 0.89\ m$

$Q = 5.4 \times 10^{-6}\ C$.

Q is uniformly distributed.

ⓐ The potential at the center of the ring.

$$dV = k\frac{dq}{r} \quad ; \quad dq = \lambda\, d\ell$$

$$dV = k\frac{\lambda\, d\ell}{R} = k\frac{\lambda R\, d\theta}{R}$$

$$dV = k\lambda\, d\theta.$$

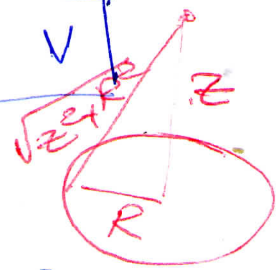
$$V = \int_0^{2\pi} k\lambda\, d\theta = k\lambda\int_0^{2\pi} d\theta = k\lambda . 2\pi.$$

$$\lambda = \frac{Q}{2\pi R}$$

$$V = k.\,2\pi\lambda = k.\cancel{2\pi}\frac{Q}{\cancel{2\pi}R} = k\frac{Q}{R}.$$

$$\boxed{V = 9\times10^9 .\ \frac{5.4\times10^{-6}}{0.89} = 5.46\times10^4\ V}$$

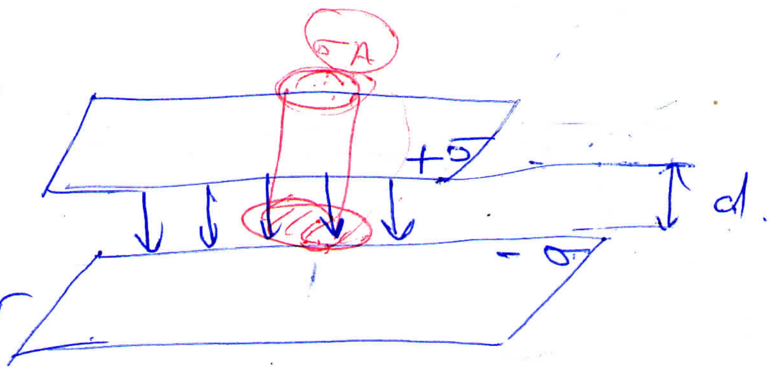
$$V = \frac{kQ}{\sqrt{z^2+R^2}} = 3.6\times10^4\ \text{volts}$$



Exercise 5:

① The electric field in the region between the plates.

by applying Gauss' Law, for one plate; we consider a cylinder.



$$\oint \vec{E}.\vec{dA} = \frac{q_{enc}}{\varepsilon_0} = \frac{\sigma A}{\varepsilon_0} = E.A$$

$$\boxed{E = \frac{\sigma}{\varepsilon_0}}$$

b) $V = -\int_{+}^{-} \vec{E}.\vec{dl} = \int_{-}^{+} E.dx = \int_{0}^{d} E.dx.$

$V = E.d = \frac{\sigma}{\varepsilon_0} d$

The field points away from positive charge and towards negative charge, the potential decrea[ses] as we move away from positive or toward[s] negative charge.

Exercise 6:

pt P : $(3\vec{i} - 2\vec{j} + 4\vec{k})$ m.

$V = 2xyz^2$

$\vec{E} = -\nabla V$;

$E_x = -\left.\frac{\partial V}{\partial x}\right)_{y,z} = -2yz^2$

$E_y = -\left.\frac{\partial V}{\partial y}\right)_{x,z} = -2xz^2$

$E_z = -\left.\frac{\partial V}{\partial z}\right)_{x,y} = -2xy.2z$

$E_z = -4xyz.$

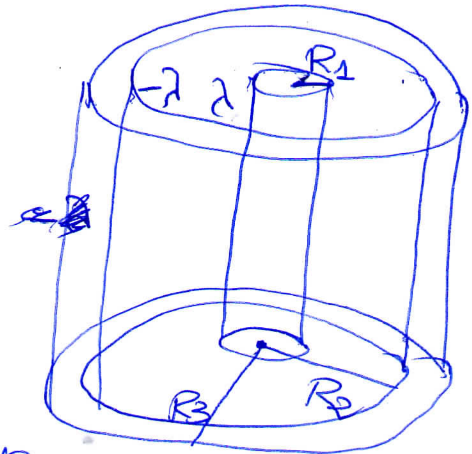
$\vec{E}(x,y,z) = -2yz^2\,\vec{i} - 2xz^2\,\vec{j} - 4xyz\,\vec{k}$

$E_p(3,-2,4) = -2.(-2).4^2\,\vec{i} - 2.3.4^2\,\vec{j} - 4.3\times(-2)\times 4$

$$\boxed{E_p(3,-2,4) = +64\vec{i} - 96\vec{j} + 96\vec{k}}$$

Exercise 7:

(a) $r > R_3$: outside the two cylinders by applying Gauss' Law.



$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}.$$

We choose a cylinder.

$$q_{enc} = (\lambda - \lambda)\, \ell = 0.$$

So $\boxed{E = 0.} \longrightarrow \boxed{V = 0}$.

(b) $R_2 < r < R_3$. We are in the cylindrical conducting shell: so according to Gauss' Law, no charge inside, no electric field. $E = 0 \longrightarrow V = 0$.

(c) $R_1 < r < R_2$: $\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$$E \cdot 2\pi r \ell = \frac{\lambda \ell}{\varepsilon_0}.$$

$$E(r) = \frac{\lambda}{2\pi\varepsilon_0 r}.$$

$$V(r) = -\int_{R_2}^{r} E(r)\, dr = -\frac{\lambda}{2\pi\varepsilon_0} \int_{R_2}^{r} \frac{dr}{r}$$

$$\boxed{V(r) = -\frac{\lambda}{2\pi\varepsilon_0} \ln \frac{r}{R_2}}$$

(d) $r < R_1$: Inside the conducting wire, no charge, no electric field $E = 0$ so $V = cst$

$$\boxed{V = cst = V(R_1) = -\frac{\lambda}{2\pi\varepsilon_0} \ln \frac{R_1}{R_2}}$$