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EE 178: Physics II

Electricity & Magnetism

Chapter 2: Electric fields

2.1. The electric field:

The electric field is a vector field. It consists of a distribution of vectors, one for each point in the region around a charged object, such as a charged rod.

We first place a positive charge q_0, called a test charge, at the point. We then define the electric field $\vec{E}$ at point P due to the charged object as:

$$\vec{E} = \frac{\vec{F}}{q_0}$$

(Electric field)

The magnitude of the electric field $\vec{E}$ at point P is
$E = \frac{F}{q_0}$ and the direction of $\vec{E}$ is tha of the force $\vec{F}$ if the test charge is positive (and vice versa if q_0 is negative). The SI unit for electric field $\vec{E}$ is Newton per Coulonb (N/C).

2.2. The electric field lines:

The relation between the field lines and electric field vectors is:

- At any point, the direction of a straight field line or the direction of the tangent to a curved field line gives the direction of $\vec{E}$ at that point.

- The field lines are drawn so that the number of lines per unit area, measured in a place that is perpendicular to the lines, is proportional to the magnitude of $\vec{E}$

E large → field lines are close together

E small → field lines are far apart

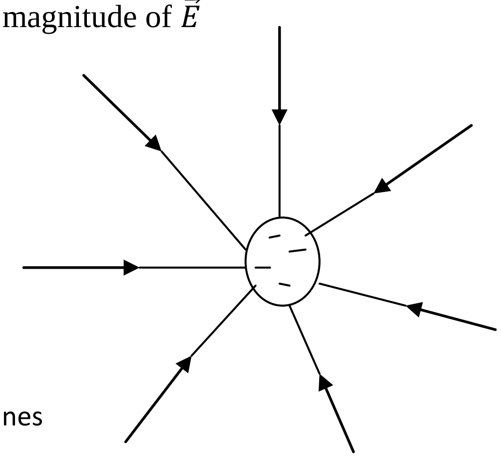


Figure 2.1: Electric field lines

Electric field lines

2.3. The electric field due to a point charge:

To find the electric field $\vec{E}$ due to a point charge q, we make use of Coulomb's Law. If a point charge q′ is placed at a distance r for the charge q, it will experience a force:

$$\overrightarrow{F_E} = \frac{1}{4\pi\varepsilon_0} \; \frac{q \; \times q'}{r^2} \; \overrightarrow{u_r} = q' \; \times \left(\frac{1}{4\pi\varepsilon_0} \; \frac{q}{r^2} \; \overrightarrow{u_r}\right)$$

But if a point charge q′ is placed at a position where the electric field $\vec{E}$, then the force on q′ is $\overrightarrow{F_E} = \grave{q} \, \vec{E}$

Comparing the two expressions for $\overrightarrow{F_E}$, we see that:

$$\overrightarrow{E} = \frac{1}{4\pi\varepsilon_0} \; \frac{q}{r^2} \; \overrightarrow{u_r}$$

This is the electric field at a distance r from a point charge q. The same relation applies at a point charge q. The same relation applies at points outside a finite spherical charge q.

$$q > 0, \vec{E} \; directed \; radially \; outward \; from \; q$$

$$q < 0, \vec{E} \; directed \; radially \; inward \; to \;\; q$$

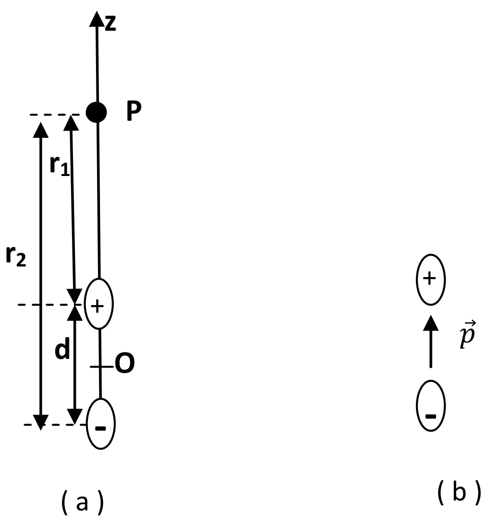
2.4. The superposition principle: The force experienced by a charge due to other charges is the vector sum of the Coulomb forces acting on it due to these other charges.

Similarly, the electric intensity $\overrightarrow{E}$ at a point due to several charges is the vector sum of the intensities due to the individual charges.

2.5. The electric field due to an electric dipole:

Figure 2.2: (a) An electric dipole;

(b) The electric dipole moment



The figure 2 above represents two charged particles of magnitude q but of opposite sgn, separated by a distance d. This configuration is called **an electric dipole**.

Due to symmetry, the electric field $\vec{E}$ at point P- and also the fields $\vec{E}_1$ and $\vec{E}_2$ due to the separate charges that make up the dipole- must lie along the dipole axis, which we have taken to be a z-axis.

By applying the superposition principle for electric fields, we find that the electric field at P is:

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \left(\frac{1}{4\pi\varepsilon_0}\frac{q}{r_1^2}\,\vec{k} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_2^2}\,\vec{k}\right)$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\left(z-\frac{d}{2}\right)^2} - \frac{1}{\left(z+\frac{d}{2}\right)^2}\right)\vec{k}$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right)\vec{k}$$

After forming a common denominator and multiplying its terms, we obtain:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left(\frac{2d/z}{\left(1-\frac{d}{2z}^2\right)^2}\right)\vec{k} = \frac{q}{2\pi\epsilon_0 z^3}\,\frac{d}{\left(1-\left(\frac{d}{2z}\right)^2\right)^2}\,\vec{k}$$

Generally, we are interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole: i.e: z>>d which means that $\frac{d}{2z} \ll 1$. Thus, we can neglect the $\frac{d}{2z}$ term: $\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}\,\vec{k}$

The product qd, is the magnitude p of a vector quantity known as the **electric dipole moment $\vec{p}$ of the dipole** (its unit is Coulomb-meter)

$$\vec{E} = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}\,\vec{k}$$

The direction of $\vec{p}$ is taken to be from the negative to the positive end of the dipole as shown in figure 2.2b

2.6. The electric field due to a line of charge:

Now we consider charge distributions that consist of a great many closely spaced point charges that are spread along a line, over a surface, or within a volume. We call them **continuous distributions**

Name	Symbol	SI unit
Charge	q	C
Linear density of charge	$\lambda = \frac{dq}{dl}$ (l length)	C/m
Surface density of charge	$\sigma = \frac{dq}{dA}$ (A surface)	C/m^2
Volume density of charge	$\rho = \frac{dq}{dV}$ (V volume)	C/m^3

Let us consider a thin ring of radius R with a uniform positive linear charge density λ around its circumference. We suppose that the ring is made of plastic or some other insulator, so that the charges can be regarded as fixed in place. What is the electric field $\vec{E}$ at point P, a distance z from the plane of the ring along its central axis.0

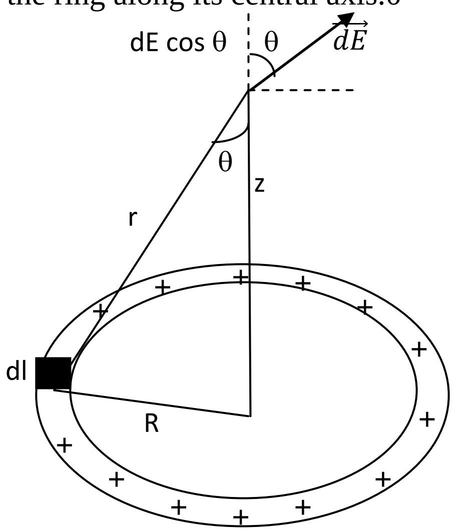


Figure 2.3: A ring of uniform positive charge

The element charge has a charge magnitude: dq = λ dl

This differential charge sets up a differential electric field $\overrightarrow{dE}$ at point P, which is a distance r from the element:

$$\overrightarrow{dE} = \frac{1}{4\pi\varepsilon_0} \frac{dq}{r^2} \overrightarrow{u_r} = \frac{1}{4\pi\epsilon_0} \frac{\lambda dl}{r^2} \overrightarrow{u_r}$$

$$\overrightarrow{dE} = \frac{\lambda dl}{(z^2 + R^2)} \overrightarrow{u_r}$$

Due to symmetry, the sum of all pairs of oppositely directed components, is zero. Thus, the perpendicular components cancel. This leaves only the parallel components; they all have the same direction, so the net electric field at P is their sum.

The parallel component of $\overrightarrow{dE}$ is dEcosθ

$cos\theta = \frac{z}{r} = \frac{z}{\sqrt{z^2+R^2}}$ is

So,

$$dE\ cos\theta = \frac{1}{4\pi\varepsilon_0} \frac{\lambda dl}{(z^2+R^2)} \frac{z}{(z^2+R^2)^{1/2}}$$

$$dE\ cos\theta = \frac{z\lambda}{4\pi\varepsilon_0} \frac{dl}{(z^2+R^2)^{3/2}}$$

The length varies from 0 to 2πR

The total electric field is obtained by integration

$$E = \int dE \cos\theta = \frac{z\lambda}{4\pi\varepsilon_0 \left(z^2+R^2\right)^{3/2}} \int_0^{2\pi R} dl$$

$$E = \frac{z\lambda\,(2\pi R)}{4\pi\epsilon_0\,(z^2+R^2)^{3/2}}$$

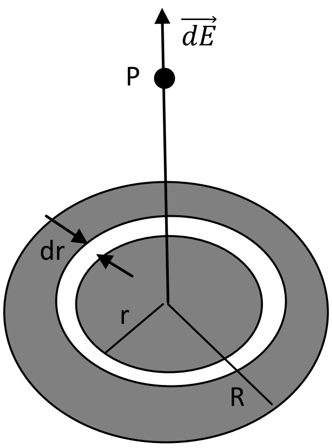
Since λ is the charge per length of the ring; λ (2πR) is the total charge q

$$E = \frac{qz}{4\pi\varepsilon_0\,(z^2+R^2)^{3/2}}$$

(charged disk)

2.7. The electric field due to a charged disk:

Figure 2.4: A disk with radius R and uniform positive charge



The figure 2.4 shows a circular plastic disk of radius R that has positive surface charge of uniform density σ on its upper surface. What is the electric field at point P, a distance z from the disk along its axis?

We divide the disk into concentric flat rings: dq = σ dA = σ (2πr dr)

dA: differential surface of the ring

We have already solved the problem of the electric field due to a ring of charge, and replacing R of the equation with r, we obtain

$$dE = \frac{z\sigma 2\pi r\, dr}{4\pi\epsilon_0\,(z^2 + r^2)^{3/2}} = \frac{z\sigma}{4\pi\epsilon_0}\,\frac{2r dr}{(z^2 + r^2)^{3/2}}$$

We can now find E by integrating the previous equation over the surface of the disk- that is by integrating with respect to the variable r from r = 0 to r = R

Remark that z remain constant during this process

$$E = \int dE = \frac{\sigma z}{4\epsilon_0}\int_0^R (z^2 + r^2)^{3/2}\ (2r)\ dr$$

To solve this integral, we cast it in the form $\int X^m\, dX$ by setting X=z^2+r^2 , m = -3/2 and dX = (2r)dr

$$\int X^m\, dX = \frac{X^{m+1}}{m+1}$$

$$E = \frac{\sigma z}{4\epsilon_0}\left[\frac{(z^2 + r^2)^{-1/2}}{-1/2}\right]_0^R$$

$$E = \frac{\sigma z}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)$$

(Charged disk)

If we let $R \to \infty$ while keeping z finite, the second term in the parentheses in the previous equation approaches zero, and this equation reduces to

$$E = \frac{\sigma}{2\varepsilon_0}$$

(Infinite sheet of charge)

2.8. A point charge in an electric field:

The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q is positive and has the opposite direction if q is negative

$$\vec{F} = q\vec{E}$$