UNIVERSITY OF BOUMERDES
INSTITUTE OF ELECTRICAL AND ELECTRONIC ENGINEERING PHYSICS II (EE178)

1st Recitation: Electric Charges and Electric fields

Exercise #1:

Two identical conducting small spheres are placed with their centers 0.300 m apart. One is given a charge of 12.0 nC, and the other is given a charge of -18.0 nC.

1. Find the electric force exerted on one sphere by the other.
2. The spheres are connected by a conducting wire. Find the electric force between the two after equilibrium has occured.

Exercise #2:

Three charges, each of value q, are placed at the corners of an equilateral triangle. A fourth charge Q is placed at the centre of the triangle.

1. If Q = -q, will the charges at the corners move towards the centre or fly away from it?
2. For what value of Q will the charges remain stationary?

Exercice #3:

In the Bohr theory of the hydrogen atom, an electron moves in a circular orbit about a proton, where the radius of the orbit is 0.529×10^{-10} m.

1. Find the electric force between the two
2. If this force causes the centripetal acceleration of the electron, what is the speed of the electron?

Exercise #4:

An airplane is flying through a thundercloud at a height of 2000 m. (This is a very dangerous thing to do because of updrafts, turbulence, and the possibility of electric discharge).
If there are charge concentrations of +40.0 C at a height of 3000 m within the cloud and of -40.0 C at height of 1000 m, what is the electric field at the aircraft?

Exercise #5:

Four point charges are at the corners of a square of side a, as shown in figure 1.

1. Determine the magnitude and direction of the electric field at the location of charge q
2. What is the resultant force on q?

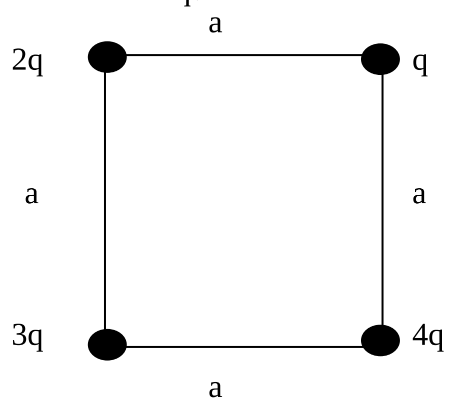


Figure 1

Exercise #6:

Consider an infinite number of identical charges (each of charge q) placed along the x axis at distances a, 2a, 3a, 4a,.... from the origin.
What is the electric field at the origin due to this distribution?
Hint: Use the fact that

$$1+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots=+\frac{\pi^2}{6}$$

Exercise #7:

A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle, as shown in figure 2. The rod has a total charge of -7.50 µC. Find the magnitude and direction of the electric field at O, the center of the semicircle.

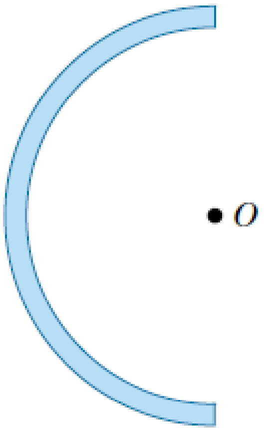


Figure 2

Exercise #8:
A uniformly charged disk of radius 35.0 cm carries a charge density of 7.90×10^{-3} C/m^2. Calculate the electric field on the axis of the disk at (a) 5.00 cm, (b) 10.0 cm, (c) 50.0 cm, and (d) 200 cm from the center of the disk.

Exercise #9:

A thin rod of length L and uniform charge per unit length λ lies along the x-axis, as shown in figure 3.

1. Show that the electric field at P, a distance y from the rod, along the perpendicular bisector has no x component and is given by $E = 2\, k_e\, \lambda\, \sin\theta_0/y$.
2. Using your result to part 1. , show that the field of a rod of infinite length is:
   $E = 2\, k_e\, \lambda\, /y$.
   (Hint: First calculate the field at P due to an element of length dx, which has a charge of λdx. Then change variables from x to θ using the fact that x = y tan θ and integrate over θ).

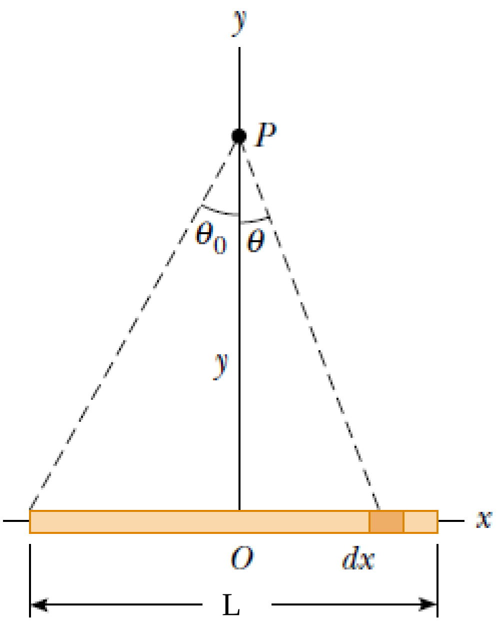


Figure 3

Exercise #10:

A line of charge starts at x = +x_0 and extends to positive infinity. If the linear charge density is $\lambda = \lambda_0 x_0/x$, determine the electric field at the origin.

Exercise #11:

Identical thin rods of length 2a carry equal charges, +Q, uniformly distributed along their lengths. The rods lie along the x axis with their centers separated by a distance of b > 2a (Figure 4). Show that the magnitude of the force exerted by the left rod on the right one is given by:

$$F = \left(\frac{k_e Q^2}{4a^2}\right) ln\left(\frac{b^2}{b^2 - 4a^2}\right)$$

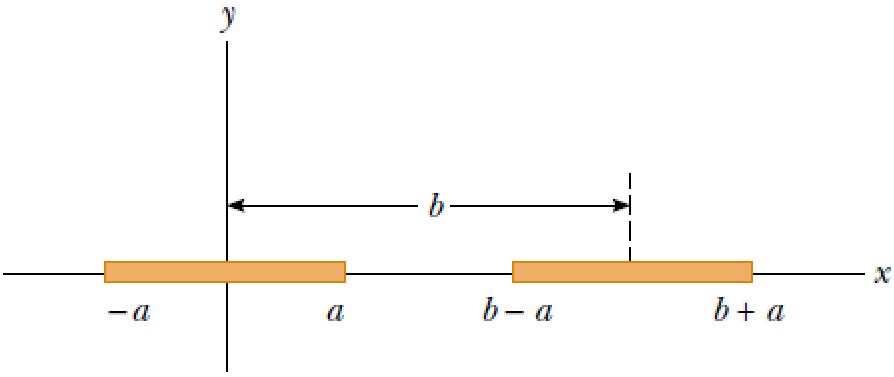


Figure 4

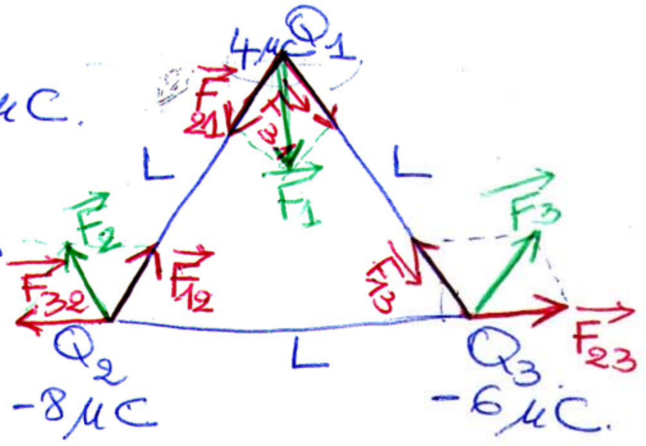
Solution of Recitation 1

Exercise 1:

$Q_1 = 4\,\mu C$, $Q_2 = -8\,\mu C$, $Q_3 = -6\,\mu C$.

$L = 1.2\,m$

Equilateral triangle: Angles in the triangle $60°$.



$$F_{12} = F_{21} = k\,\frac{Q_1 Q_2}{L^2}$$

$$= 9\times10^9 \cdot \frac{4\times10^{-6}\times 8\times10^{-6}}{(1.2)^2} = 0.20\,N.$$

$$F_{13} = F_{31} = k\,\frac{Q_1 Q_3}{L^2} = 9\times10^9\,\frac{4\times10^{-6}\times6\times10^{-6}}{(1.2)^2} = 0.15\,N.$$

$$F_{23} = F_{32} = k\,\frac{Q_2 Q_3}{L^2} = 9\times10^9\,\frac{8\times10^{-6}\times6\times10^{-6}}{(1.2)^2} = 0.30\,N$$

The forces applied on each charge:

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} \Leftrightarrow F_1 = \sqrt{F_{21}^2 + F_{31}^2 + 2\cdot F_{21}\cdot F_{31}\cdot \cos(\vec{F}_{21}, \vec{F}_{31})}$$

$$F_1 = \sqrt{0.20^2 + 0.15^2 + 2\times0.20\times0.15\times\cos 60°}$$

$$\boxed{F_1 = 0.304\,N}$$

$$F_{1x} = -F_{21}\cos60°\,\vec{i} + F_{31}\cos60°\,\vec{i} = (-0.20\cos60° + 0.15\cos60°)\,\vec{i}$$
$$= -0.025\,\vec{i}.$$

$$F_{1y} = -F_{21}\sin60°\,\vec{j} - F_{31}\sin60°\,\vec{j} = (-0.20\times\sin60° - 0.15\sin60°)\,\vec{j}$$
$$= -0.303\,\vec{j}$$

$$\tan\theta_1 = \frac{F_{1y}}{F_{1x}} = \frac{-0.303}{-0.025} = 12.12 \rightarrow \theta_1 = 85°.28 \text{ or}$$

$$\theta_1 = \pi + 85°.28$$
$$= 180 + 85.28$$

$$\boxed{F_1 = 0.304\,N} \quad \boxed{\theta_1 = 265°.28}$$

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2/ The forces applied on charge Q_2:

$$\vec{F_2} = \vec{F_{12}} + \vec{F_{32}} \rightarrow F_2 = \sqrt{F_{12}^2 + F_{32}^2 + 2\, F_{12}\, F_{32} \cos(\vec{F_{12}}, \vec{F_{32}})}$$

$$F_2 = \sqrt{0.20^2 + 0.30^2 + 2 \times 0.20 \times 0.30 \times \cos 120°}$$

$$\boxed{F_2 = 0.264\,N}$$

$$F_{2x} = F_{12} \cos 60° \vec{i} - F_{32} \vec{i} = (0.20 . \cos 60° - 0.30)\vec{i} = -0.20 \vec{i}.$$

$$F_{2y} = F_{12} \sin 60° \vec{j} = 0.20 \sin 60° \vec{j} = 0.173 \vec{j}.$$

$$\tan\theta_2 = \frac{0.173}{-0.20} = -0.865 \rightarrow \theta_2 = -40°.85 \text{ or } \underline{139°.14}$$

Choose this value

$$\boxed{F_2 = 0.264\,N , \theta_2 = 139°.14}$$

The forces applied on charge Q_3:

$$\vec{F_3} = \vec{F_{13}} + \vec{F_{23}} \rightarrow F_3 = \sqrt{F_{13}^2 + F_{32}^2 + 2\, F_{13} . F_{32} \cos 120°}$$

$$F_3 = \sqrt{0.15^2 + 0.30^2 + 2 \times 0.15 \times 0.30 \times \cos 120°}$$

$$\boxed{F_3 = 0.259\,N}$$

$$F_{3x} = F_{23} - F_{13} \cos 60° = 0.30 - 0.15 \cos 60° = 0.225.$$

$$F_{3y} = F_{13} \sin 60° = 0.15 \times \sin 60° = 0.129.$$

$$\tan\theta_3 = \frac{0.129}{0.225} = 0.573 \rightarrow \boxed{\theta_3 = 29°.82}$$

$$\boxed{F_3 = 0.259\,N , \theta_3 = 29°.82}$$

Exercise 2

Recitation 1 Physics II

Let a be the side of the equilateral triangle. The forces on the charge q placed at C due to the charges at A and B are repulsive and represented by CE and CD, respectively, each given by $q^2/4\pi\varepsilon_0 a^2$. The resultant of these two forces is given by CP, the diagonal of the parallelogram CDPE, Fig. 11.20

$$\mathrm{CP} = 2\mathrm{CD}\cos 30^\circ = \frac{2q^2}{4\pi\varepsilon_0 a^2}\frac{\sqrt{3}}{2} = \frac{\sqrt{3}q^2}{4\pi\varepsilon_0 a^2}$$

The force on q at C due to Q at the centre of the triangle is

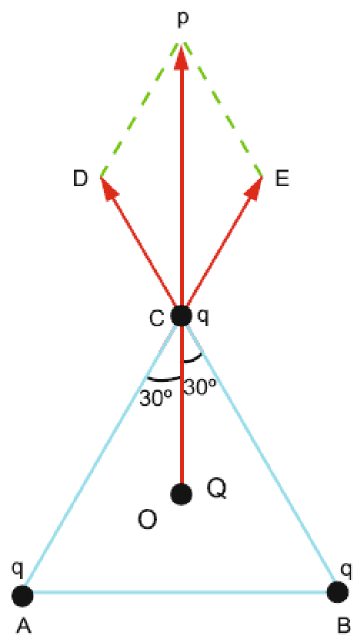
$$\frac{\mathrm{Q}q}{4\pi\varepsilon_0(\mathrm{OC})^2} = \frac{3\,\mathrm{Q}q}{4\pi\varepsilon_0 a^2}$$

i. If $Q = -q$, this force will be attractive and will be directed along CO. As the attractive force due to $-q$ is greater than the combined repulsive force due to charges $+q$ at A and B, the charge at C will be attracted towards O. Same is true for the charges placed at A and B.

ii. For equilibrium, the attractive force must balance the repulsive force:

$$\frac{\sqrt{3}q^2}{4\pi\varepsilon_0 a^2} - \frac{3Qq}{4\pi\varepsilon_0 a^2} = 0 \to Q = -q/\sqrt{3}$$

Fig. 11.20



Exercise 3:

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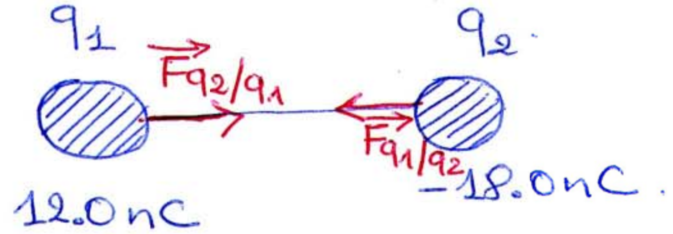


1/ Find the electric force exerted by one sphere on the other.

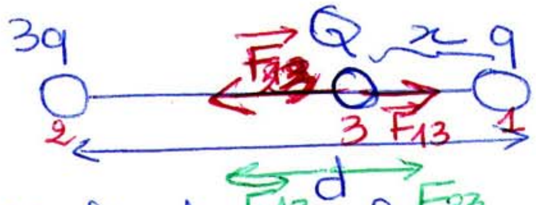
$$F_{q_1/q_2} = F_{q_2/q_1} = k\,\frac{q_1 q_2}{r^2} = 9\times10^{9}\times\frac{18\times10^{-9}\times12\times10^{-9}}{(0.3)^2}$$

$$\boxed{F = 2.16\times10^{-5}\,N}$$

2/ After the spheres come to equilibrium. The charge will be equitably distributed. $-18\,nC + 12\,nC = -6\,nC$

So. $-6\,nC$ on each sphere.

Exercise 2:



The forces applied on charge Q are:

$$\vec{F}_Q = \vec{F}_{qQ} + \vec{F}_{3qQ}$$

if the charge Q is negative: $\vec{F}_3 = \vec{F}_{13} + \vec{F}_{12}$

$$F_3 = F_{23} - F_{13} = k\cdot\frac{3qQ}{(d-x)^2} - k\frac{qQ}{x^2} = 0.$$

$$\cancel{k}\,\frac{3q\cancel{Q}}{(d-x)^2} = \cancel{k}\,\frac{q\cancel{Q}}{x^2} \Leftrightarrow \frac{3}{(d-x)^2} = \frac{1}{x^2}$$

$$\Leftrightarrow 3x^2 = (d-x)^2 \Leftrightarrow 3x^2 - (d-x)^2 = 0.$$

$$\left[\sqrt{3}\,x - (d-x)\right]\left[\sqrt{3}\,x + (d-x)\right] = 0.$$

$$\left[x(1+\sqrt{3}) - d\right]\left[(\sqrt{3}-1)x + d\right] = 0 \Leftrightarrow$$

$$x = \frac{d}{1+\sqrt{3}} = 0.36\,d$$

$$x = \frac{d}{1-\sqrt{3}} = -1.36\,d$$

The third bead is at equilibrium at x=0.36 d.

Stable equilibrium.

Exercise 4:

radius of the orbit. 0.529×10^{-10} m.

① The electric force between the two:

$$F = k\frac{e^2}{R^2} = 9\times10^9 \times \frac{(1.6\times10^{-19})^2}{(0.529\times10^{-10})^2}$$

$$\boxed{F = 8.23\times10^{-8}\,N}$$

② If this force causes the centripetal acceleration of the electron, what is the speed of the electron

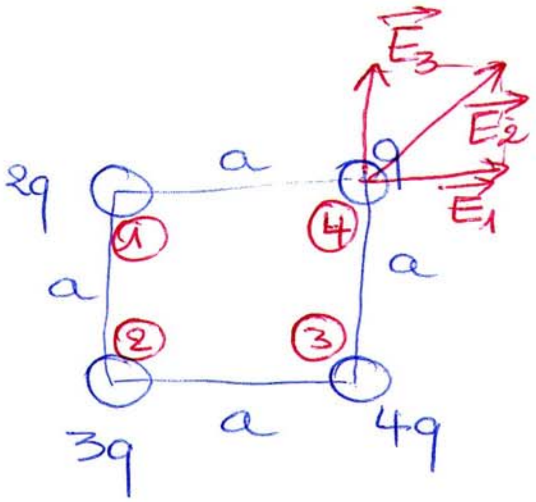
$$a = \frac{v^2}{R} \quad , \quad F = m_e a = m_e \frac{v^2}{R}$$

$$v^2 = \frac{F.R}{m_e} \Rightarrow v = \sqrt{\frac{F.R}{m_e}}$$

$$v = \sqrt{\frac{8.23\times10^{-8}\times0.529\times10^{-10}}{9.11\times10^{-31}}} = 2.18\times10^{6}\ m/s.$$

$$\boxed{v = 2.18\times10^{6}\ m/s.}$$

Exercise 5.



$\vec{E_4}$ = Electric field at charge q.

$$\vec{E_4} = E_1\vec{i} + E_2\cos45^\circ\vec{i} + E_2\sin45^\circ\vec{j} + E_3\vec{j}$$

$$\vec{E_4} = (E_1 + E_2\cos45^\circ)\vec{i} + (E_2\sin45^\circ + E_3)\vec{j}$$

$$E_1 = k\frac{2q}{a^2},\quad E_2 = k\frac{3q}{(a\sqrt{2})^2},\quad E_3 = k\frac{4q}{a^2}$$

$$\vec{E_4} = \frac{kq}{a^2}\left\{\left(2+\frac{3}{2}\cos45^\circ\right)\vec{i} + \left(\frac{3}{2}\cos45^\circ+4\right)\vec{j}\right\}.$$

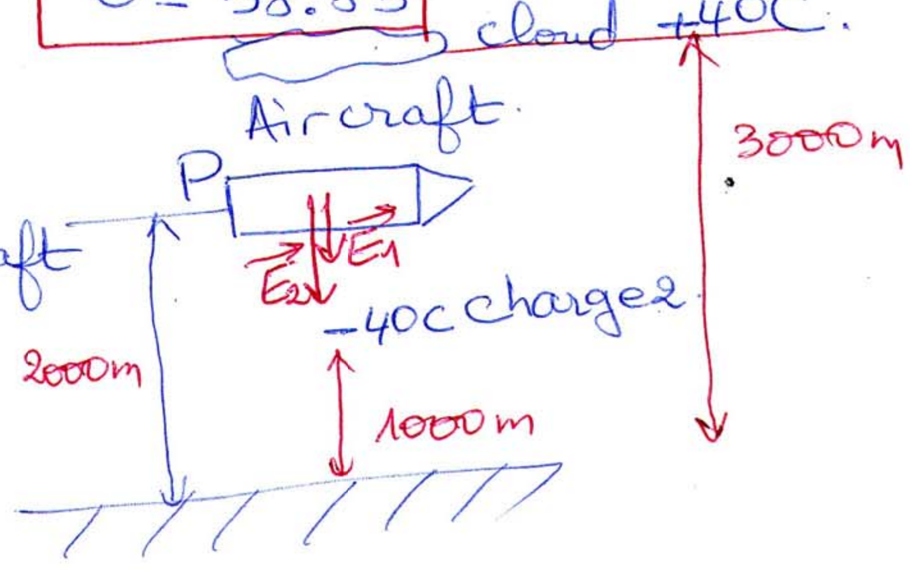
$$\vec{E_4} = \frac{kq}{a^2}(3.06)\vec{i} + \frac{kq}{a^2}(5.06)\vec{j}.$$

Its magnitude: $\boxed{E_4 = \frac{kq}{a^2}(5.91).}$

Its direction: $\tan\theta = \frac{E_y}{E_x} = \frac{5.06}{3.06} = 1.65$

$$\boxed{\theta = 58.83^\circ}$$

Exercise 6.



Let's consider the position of the aircraft as P.

The electric field at the aircraft.

$$\vec{E_P} = \vec{E_1} + \vec{E_2}$$

Because they are at the same axis y.

$$E_p = E_1 + E_2 = k\frac{Q_1}{R_1} + k\frac{Q_2}{R_2} = 9\times10^9 \times \frac{40}{(3000-2000)}$$

$$+ 9\times10^9 \cdot \frac{40}{(2000-1000)}$$

$$\boxed{E_p = 7.2\times10^5\ N/C}$$

Exercise 7:

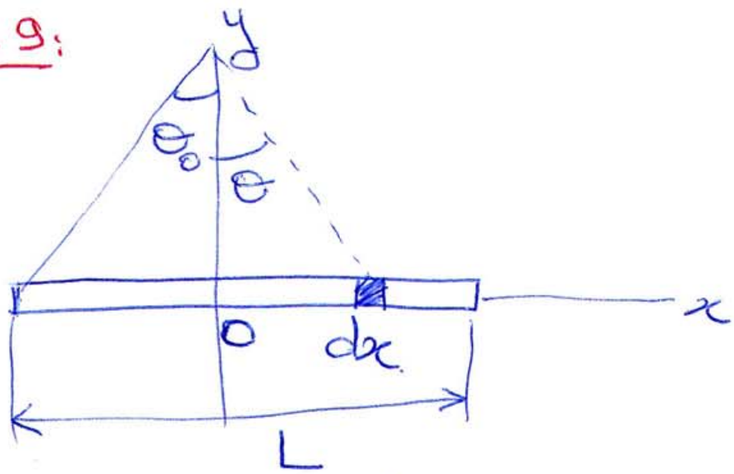
$$P = aq = (6000\ m)(45C) = 2.7\times10^4\ C.m.$$

Exercise 8:

$$\tau = |\vec{p}\wedge\vec{E}| = pE\sin\theta = (3.4\times10^{-30}\ C.m)(4\times10^6\ N/C)\sin 30°.$$

$$\boxed{\tau = 6.8\times10^{-24}\ N.m}$$

Exercise 9:



$$d\vec{E} = k\frac{dq}{r^2}\vec{u_r} = k\frac{\lambda\, dx}{x^2+y^2}\vec{u_r}.$$

The x component of E cancel each other. Only the y component stay.

$$dE_y = dE.\cos\theta = dE\frac{y}{\sqrt{x^2+y^2}}$$

θ varies from $-\theta_0$ to θ_0 with $\sin\theta = \frac{L/2}{y}$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{x}{y} \Leftrightarrow \frac{1}{\cos^2\theta}\, d\theta = \frac{dx}{y}.$$

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$$\cos\theta = \frac{y}{\sqrt{x^2+y^2}} \Leftrightarrow \sqrt{x^2+y^2} = \frac{y}{\cos\theta}$$

$$dE_y = k \frac{\lambda\, dx}{(x^2+y^2)^{3/2}}\, y$$

$$dE_y = k \frac{\lambda\, dx}{\frac{y^3}{\cos^3\theta}}\, y = k\lambda \frac{\frac{y}{\cos^2\theta}}{\frac{y^2}{\cos^3\theta}}\, y \; d\theta.$$

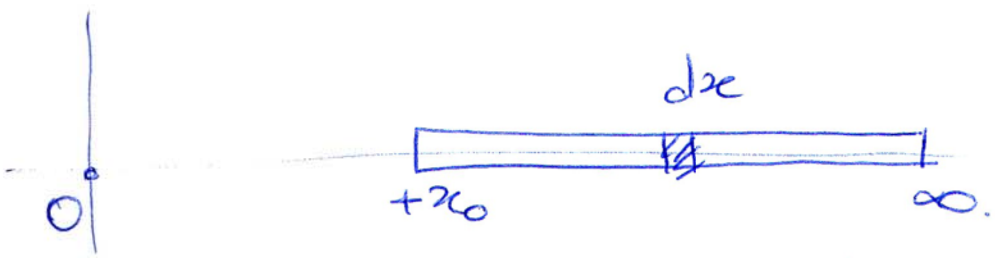
$$dE_y = k\lambda \frac{\cos\theta}{\lambda}\, d\theta.$$

$$E = E_y = \int dE_y = \frac{k\lambda}{y} \int_{-\theta_0}^{\theta_0} \cos\theta\, d\theta = \frac{2k\lambda}{y} \sin\theta_0.$$

② For infinite length: θ_0 tends to $\pi/2$ So.

$$E = 2\frac{k\lambda}{y} \sin \pi/2 = \frac{2k\lambda}{y}.$$

Exercise 11:



Determine the electric field at the origin.

$$dE = k\frac{dq}{x^2} = k\frac{\lambda\, dx}{x^2} = k \frac{\lambda_0 x_0}{x} \frac{dx}{x^2}$$

$$dE = k\lambda_0 x_0 \frac{dx}{x^3} \rightarrow E = \int_{x_0}^{\infty} k\lambda_0 x_0 \frac{dx}{x^3}$$

$$E = k\lambda_0 x_0 \int_{x_0}^{\infty} x^{-3} dx = k\lambda_0 x_0 \left.\frac{x^{-2}}{-2}\right|_{x_0}^{\infty}$$

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2/ $$E_{Tot_{5.00cm}} = \int_0^5 dE_y = \int_0^5 \frac{1}{4\pi\varepsilon_0} 2\pi\sigma y \frac{r\,dr}{(y^2+r^2)^{3/2}}$$

$$= \frac{1}{2\varepsilon_0}\sigma_{,} y \int_0^5 \frac{r\,dr}{(y^2+r^2)^{3/2}} = \frac{\sigma y}{4\varepsilon_0}\int_0^5 \frac{d(r^2+y^2)}{(r^2+y^2)^{3/2}}$$

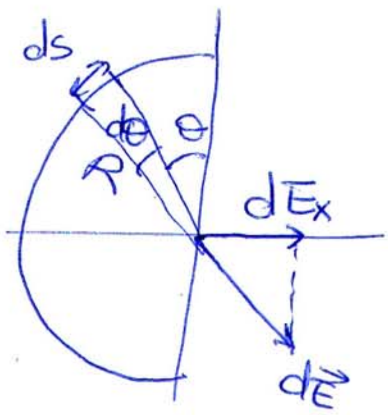
$$= \frac{\sigma y}{4\varepsilon_0}\frac{U^{-1/2}}{-1/2} = -\frac{\sigma y}{2\varepsilon_0}(r^2+y^2)^{-1/2}\Big|_0^5$$

$$E_{Tot_5} = -\frac{\sigma y}{2\varepsilon_0}\left[(y^2+5^2)^{-1/2} - y^{-1}\right] = \frac{\sigma}{2\varepsilon_0}\left[1-\frac{y}{\sqrt{y^2+5^2}}\right]$$

$$\boxed{E_{Tot_5} = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{y}{\sqrt{y^2+5^2}}\right)}$$

Exercise 10:

By symmetry the y components of the field due to each dq cancel. So: $E = E_x$.



$$dq = \lambda\, ds = \lambda R\, d\theta.$$

where $\lambda = \dfrac{Q}{\pi R}$

Thus $$E_x = \int k\frac{dq}{R^2}\sin\theta = \frac{k\lambda R}{R^2}\int_0^\pi \sin\theta\, d\theta = \frac{k\lambda}{R}(-\cos\theta)_0^\pi$$

$$E = E_x = \frac{kQ}{\pi R^2}(1+1) = \frac{2kQ}{\pi R^2}$$

$$\boxed{E = \frac{2kQ}{\pi R^2}}$$

Exercise 13:

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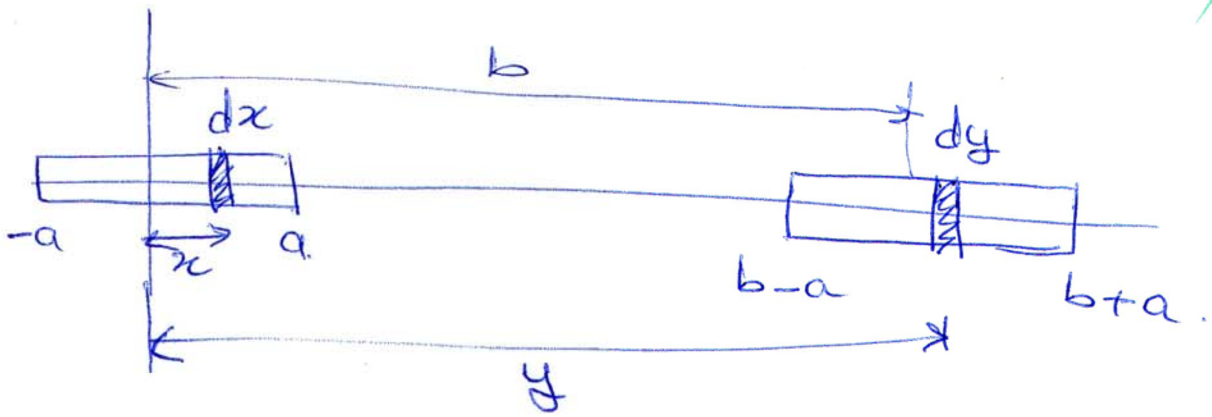


$$F = k\,\frac{dq\,dq'}{(y-x)^2} = k\,\lambda^2\,\frac{dx\,dy}{(y-x)^2}$$

$$F = k\lambda^2 \int_{-a}^{a}\int_{b-a}^{b+a} \frac{dx\,dy}{(y-x)^2} = k\lambda^2 \int_{-a}^{a} \frac{(y-x)^{-1}}{-1}\Bigg|_{b-a}^{b+a} dx$$

$$F = k\lambda^2 \int_{-a}^{a} \left\{ \frac{-1}{y-x}\Bigg|_{b-a}^{b+a} \right\} dx = k\lambda^2 \int_{-a}^{a} \left\{ \frac{-1}{b+a-x} + \frac{1}{b-a-x} \right\} dx$$

$$F = k\lambda^2 \left\{ \int_{-a}^{a} \frac{-1}{b+a-x}\,dx + \int_{-a}^{a} \frac{1}{b-a-x}\,dx \right\}$$

$$= k\lambda^2\ \mathrm{Ln}(b+a-x)\Big|_{-a}^{a} - \mathrm{Ln}(b-a-x)\Big|_{-a}^{a}$$

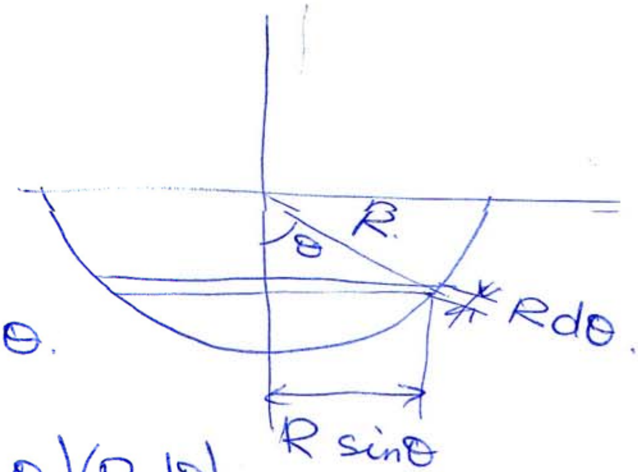
$$= k\lambda^2 \left\{ \ln b - \ln(b+2a) - \ln(b-2a) + \ln b \right\}$$

$$= k\lambda^2\ \mathrm{Ln}\,\frac{b^2}{b^2-4a^2} = k\,\frac{Q^2}{4a^2}\ \mathrm{Ln}\,\frac{b^2}{b^2-4a^2}$$

$$\boxed{F = k\,\frac{Q^2}{4a^2}\ \mathrm{Ln}\,\frac{b^2}{b^2-4a^2}}$$

Exercise 14:

If we break the surface into small rings of radius $2\pi R\sin\theta$ and width $R\,d\theta$.



$$dq = \sigma\, dA = \sigma \cdot (2\pi R\sin\theta)(R\,d\theta)$$

$E = E_x$ by symmetry.

$$= \int \frac{k\,dq}{R^2}\cos\theta = \frac{k}{R^2}\int \sigma\, dA\cos\theta$$

$$= \frac{k\sigma}{R^2}\int_0^{\pi/2} 2\pi R^2 \sin\theta\cos\theta\, d\theta$$

$$= 2\pi k\sigma\left(\frac{1}{2}\sin^2\theta\right)_0^{\pi/2} = \pi k\sigma$$

$$\boxed{E = \pi k\sigma}$$