chapter 2 voltage and current Laws

Introduction

Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concepts of network topology.

A node is a point in a circuit where two or more elements are joined such as a voltage source or a resistor.
A branch is a path in a circuit consisting of a single element.
A loop of a circuit is a closed path starting from a node and returning to the same node.
A mesh is a loop that does not contain another loop inside it.

- Two or more elements are in series if they exclusively share a single node and consequently carry the same current.
- Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

Example 2.1
Find all the nodes, loops, and meshes for the circuit shown in Figure 2.1.

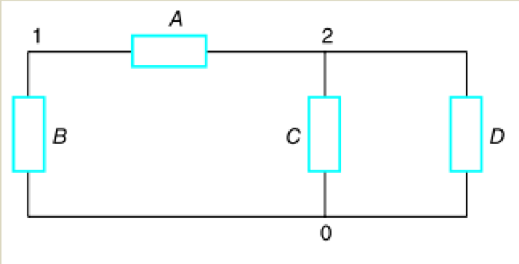
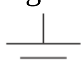


Figure 2.1.

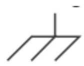
in the circuit shown in Figure 2.1, there are three nodes: **0, 1, and 2.** Elements **B, C, and D are joined at node 0**. Node 1
connects **elements A and B,** and node 2 connects elements **A, C, and** D. If node 0 is the ground node, then the potential is set to zero at node 0. The voltages at node 1 and node 2 are measured with respect to node 0.
There are four branches in the circuit shown in Figure 2.1: A, B, C, and D. There are three loops in the circuit shown in Figure 2.1:
0-B-1-A-2-D-0
0-B-1-A-2-C-0
0-C-2-D-0
There are two meshes in the circuit shown in Figure 2.1:
0-B-1-A-2-C-0
0-C-2-D-0
The loop 0-B-1-A-2-D-0 contains two meshes: **0-B-1-A-2-C-0 and 0-C-2-D-0.**

Circuit ground

- Usually, there is a ground for each electric circuit. Electrical circuit grounding is important because it is always at zero potential (0 V), and provides a reference voltage level in which all other voltages in a circuit are measured example negative terminal of the battery (voltage source) is connected to ground. There are two types of circuit grounds: one is the earth ground and another is the common ground (or chassis ground).
- Earth ground: Connecting one terminal of the voltage source to the earth. The symbol for it is:

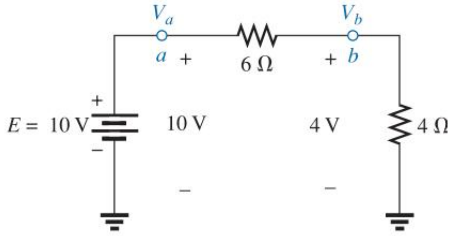


- Common ground or chassis ground: The common point for all elements in the circuit. All the common points are electrically connected together through metal plates or wires. The symbol for the common point is:



- In a circuit, the voltage with the single-subscript notation (such as $\boldsymbol{V_a}$) is the voltage drop from the point **a** with respect to ground. The single-subscript notation V a specifies the voltage at point a with respect to ground (zero volts). If the voltage is less than zero volts, a negative sign must be associated with the magnitude of V

In Figure below, V a is the voltage from point a to ground. In this case, it is obviously 10 V since it is right across the source voltage E. The voltage V b is the voltage from point b to ground. Because it is directly across the 4 Ω resistor, V b = 4 V.



Figure

- And the voltage with the double-subscripts notation (such as $\boldsymbol{V_{ab}}$) is the voltage drop across the two points a and b (each point is represented by a subscript).

Defining the sign for double-subscript notation.

a . General comments

A particularly useful relationship can now be established that has extensive applications in the analysis of electronic circuits. For the above notational standards, the following relationship exists:

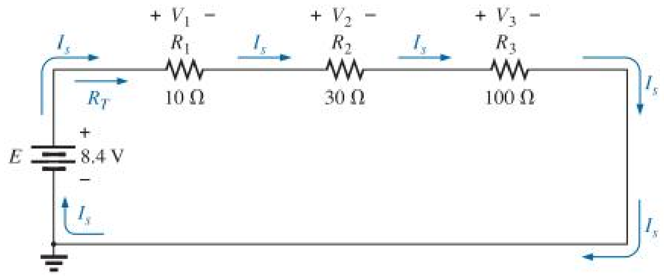
$$\boxed{V_{ab} = V_a - V_b}$$

In other words, if the voltage at points a and b is known with respect to ground, then the voltage Vab can be determined . In Figure above, for example,

$$V_{ab} = V_a - V_b = 10\text{ V} - 4\text{ V}$$
$$= 6\text{ V}$$

2.Series Circuit

is constructed by combining various elements is series. The direction of conventional current in a series DC circuit is such that it leaves the positive terminal of the supply and returns to the negative terminal, as shown in Figure.



In any configuration, if two elements are in series, the current must be the same. However, if the current is the same for two adjoining elements, the elements may or may not be in series.

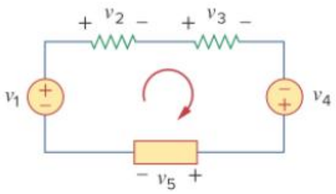
3. **Kirchhoff's Voltage Law**

(KVL) states that the algebraic sum of all voltages around a closed path (or loop) is zero.

Expressed mathematically, KVL states that

$$\sum_{m=1}^{M} v_m = 0$$

The algebraic sum used in ***KVL*** means that there are voltage polarities existing in a closed-loop circuit. It requires assigning a loop direction and it could be in either clockwise or counter-clockwise directions (usually clockwise).



$$-v1 + v2 + v3 - v4 + v5 = 0$$

Rearranging terms gives

$$v2 + v3 + v5 = v1 + v4$$

which may be interpreted as

Sum of voltage drops = Sum of voltage rises

- a plus sign is assigned to a potential rise(- to+); and a ,minus sign to a potential drop (+to-).

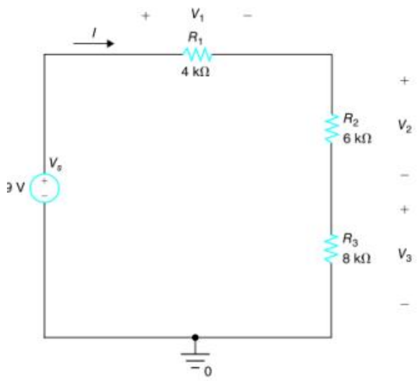
Consider a circuit shown in Figure 2.20. We are interested in finding the voltages across the resistors R 1 , R 2 , and R 3 and the current through them.

$$-v_s + v_1 + v_2 + v_3 = 0$$

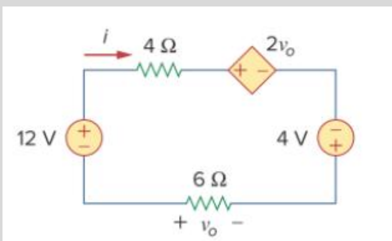
using ohm's law

$$v_s = v_1 + v_2 + v_3$$

$$v_s = R_1 I + R_2 I + R_3 I \; = IR_T \;\; and \; R_T = R_1 + R_2 + R_3$$



Example: Determine v o and i in the circuit shown in Figure(a).



We apply KVL around the loop as shown in Fig. 2.23(b). The result is

$$-12 + 4i + 2vo - 4 + 6i = 0$$

Applying Ohm's law to the 6-Ω resistor gives

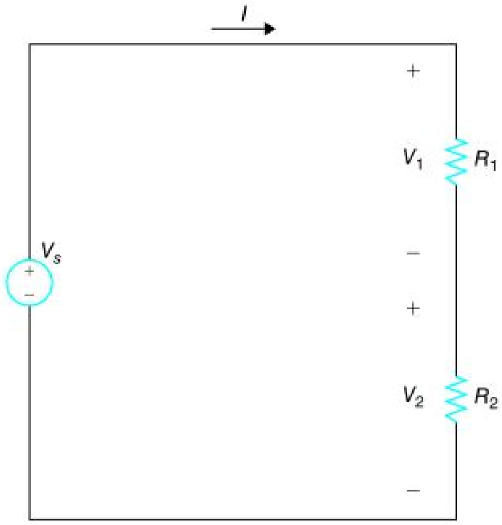
$$v_o = -6i$$

Substituting (1) into (2) yields $-16 + 10i - 12i = 0 \quad \Rightarrow \quad i = -8\,A$ and v o = 48 V.

4.Voltage Division

- **Series Circuit**

In a series circuit, the larger the resistance the larger the voltage across the resistance. Suppose that two resistors with resistances R 1 and R 2 , respectively, are connected in series to a voltage source with voltage Vs volts, as shown in Figure. The equivalent resistance is R 1+ R 2 .



According to Ohm's law, the current through the mesh is

$$I = \frac{V_S}{R_1 + R_2}$$

Thus, the voltage V_1 across the first resistor with resistance R_1 is

$$V_1 = IR_1 = R_1 \frac{V_S}{R_1 + R_2}$$

and the voltage V 2 across the second resistor with resistance R 2 is

$$V_2 = IR_2 = R_2 \frac{V_S}{R_1 + R_2}$$

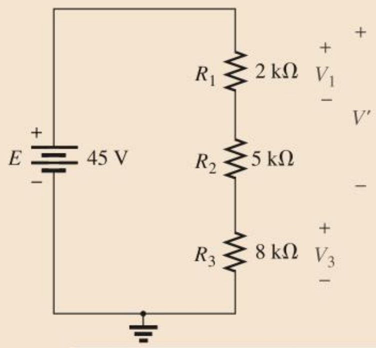
The voltage divider rule (VDR) permits the determination of the voltage across a series resistor without first having to determine the current of the circuit. The voltage divider rule states that

the voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series configuration.

$$\boldsymbol{V_x = R_x \frac{V}{R_T}}$$

where $\boldsymbol{V_x}$ is the voltage across the resistor $\boldsymbol{R_x}$, V is the impressed voltage across the series elements, and $\boldsymbol{R_T}$ is the total resistance of the series circuit.

Example Using the voltage divider rule, determine voltages V 1 and V 3 for the series circuit in Figure. and Determine the voltage (denoted V′) across the series combination of resistors R 1 and R 2



Solution:

$$R_T = R_1 + R_2 + R_3$$
$$= 2\text{ k}\Omega + 5\text{ k}\Omega + 8\text{ k}\Omega$$
$$R_T = 15\text{ k}\Omega$$
$$V_1 = R_1\frac{E}{R_T} = 2\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{ k}\Omega}\right) = \mathbf{6\ V}$$

and $$V_3 = R_3\frac{E}{R_T} = 8\text{ k}\Omega\left(\frac{45\text{ V}}{15\ \Omega}\right) = \mathbf{24\ V}$$

Solution: Since the voltage desired is across both R_1 and R_2, the sum of R_1 and R_2 will be substituted as R' in Eq. (5.12). The result is

$$R' = R_1 + R_2 = 2\text{ k}\Omega + 5\text{ k}\Omega = 7\text{ k}\Omega$$

and $$V' = R'\frac{E}{R_T} = 7\text{ k}\Omega\left(\frac{45\text{ V}}{15\text{ k}\Omega}\right) = \mathbf{21\ V}$$

As seen from above, the main characteristics of a series circuit are :

1. same current flows through all parts of the circuit.
2. different resistors have their individual voltage drops.
3. voltage drops are additive.
4. applied voltage equals the sum of different voltage drops.

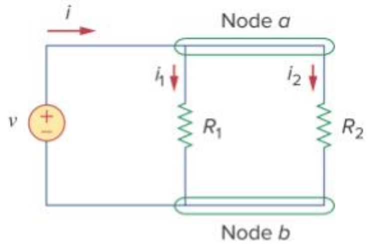
5. resistances are additive.
6. powers are additive.

- **Parallel Circuit**

Two elements, branches, or circuits are in parallel if they have two points in common.
Consider the circuit in Figure, where two resistors are connected in parallel and therefore have the same voltage across them.

The main characteristics of a parallel circuit are :

1. same voltage acts across all parts of the circuit
2. different resistors have their individual current.
3. branch currents are additive.
4. conductance's are additive.
5. powers are additive.

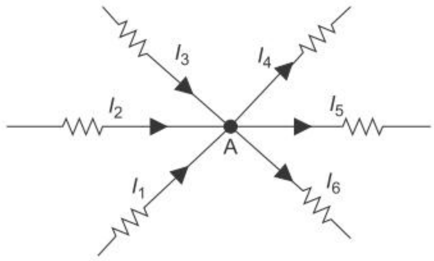


5. 3. **Kirchhoff's current Law**

Kirchhoff's current Law states that the algebraic sum of the total currents entering and exiting(leaving) a node or junction of the circuit is equal to zero, i.e.

$$\sum I = 0$$

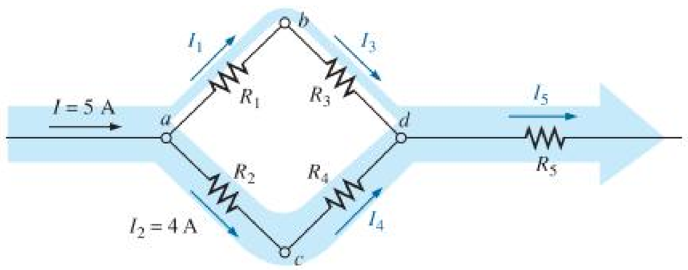
- Assign a positive sign (+) to the current in the equation if current is entering the node.
- Assign a negative sign (-) to the current in the equation if current is exiting the node.



KCL can also be expressed in another way: the total current flowing into a node is equal to the total current flowing out of the node,

$$\sum I_{in} = \sum I_{out}$$

Example 6.17 Determine currents I 1 , I 3 , I 4 , and I 5 for the network in



In this configuration, four nodes are defined. Nodes a and c have only one unknown current at the junction, so Kirchhoff's current law can be applied at either junction.

At node a

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$

and $I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\ A}$

At node c

$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$

and $I_4 = I_2 = \mathbf{4\ A}$

Using the above results at the other junctions results in the following.

At node b

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$

and $I_3 = I_1 = \mathbf{1\ A}$

At node d

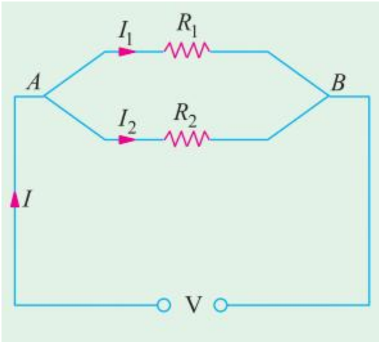
$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5 = \mathbf{5\ A}$$

If we enclose the entire network, we find that the current entering from the far left is I = 5 A, while the current leaving from the far right is I 5 = 5 A. The two must be equal since the net current entering any system must equal the net current leaving.

Current Divider Rule

$$I_1 = \frac{V}{R_1} \quad \text{and} \quad I_2 = \frac{V}{R_2} \Rightarrow \frac{I_2}{I_1} = \frac{R_1}{R_2}$$

$$\frac{I_1}{R_1} = G_1 \quad \text{and} \quad \frac{I_2}{R_2} = G_2 \Rightarrow \frac{I_2}{I_1} = \frac{G_1}{G_2}$$



Hence, the division of current in the branches of a parallel circuit is directly proportional to the conductance of the branches or inversely proportional to their resistances. We may also express the branch currents in terms of the total circuit current

$$I = I_1 + I_2 \ \ and \ \ V = \frac{I}{R_{eq}} = \frac{R_1R_2I}{R_1 + R_2}$$

$$V = I_1R_1 = \frac{R_1R_2I}{R_1 + R_2} \Rightarrow I_1 = \frac{R_2I}{R_1 + R_2} \quad and \ \ I_2 = \frac{R_1I}{R_1 + R_2}$$

in other word

$$I_1 = \frac{V}{R_1} = \frac{R_{eq}I}{R_1}$$

so

The current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistance of the resistor of interest and multiplied by the total current entering the parallel configuration.

In general for,

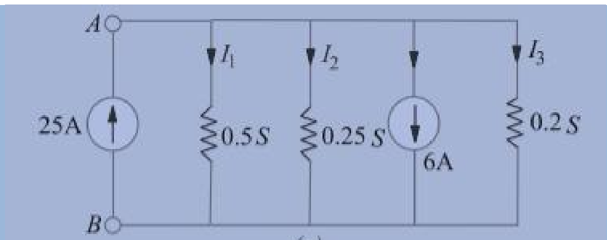
$$I_x = \frac{R_{eq}I}{R_x} \quad or \ \ I_x = \frac{G_xI}{G_{eq}}$$

where

I_x refers to the current through a parallel branch of resistance R_x

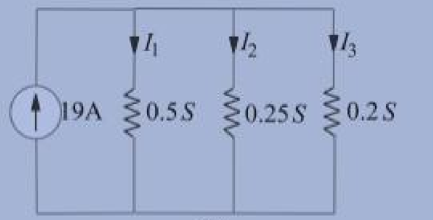
- For two parallel elements of equal value, the current will divide equally.
- For parallel elements with different values, the smaller the resistance, the greater is the share of input current.
- For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistance values.

Example. Compute the values of three branch currents for the circuits of Fig. 1.49 (a).What is the potential difference between points A and B ?



(a)

Solution. The two given current sources may be combined together as shown in Fig. 1.49 (b).
Net current = 25 − 6 = 19 A because the two currents flow in opposite directions.



(b)

Now, $G = 0.5 + 0.25 + 0.2 = 0.95\text{ S}; \quad I_1 = I\dfrac{G_1}{G} = 19 \times \dfrac{0.5}{0.95} = \mathbf{10\ A}$

$I_2 = I\dfrac{G_2}{G} = 19 \times \dfrac{0.5}{0.95} = \mathbf{5\ A}; \; I_3 = I\dfrac{G_3}{G} = 19 \times \dfrac{0.2}{0.95} = \mathbf{4\ A}$

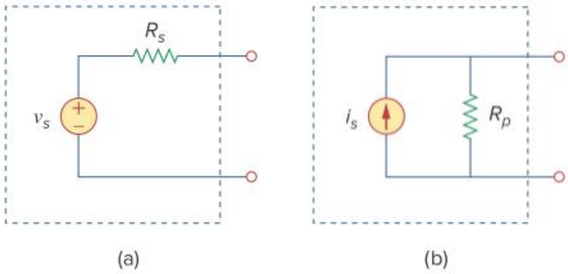
$V_{AB} = I_1 R_1 = \dfrac{I_1}{G_1} = \dfrac{I_2}{G_2} = \dfrac{I_3}{G_3} \quad \therefore \; V_{AB} = \dfrac{10}{0.5} = \mathbf{20\ A}$

The same voltage acts across the three conductances.

Internal resistance

Practical voltage and current sources are not ideal, due to their internal resistances or source resistances Rs and Rp . They become ideal as R s → 0 and R p → ∞.

Every practical (source) supply ;whether battery or generator has an internal resistance in series with the idealized voltage source as shown in figure



(a) Practical voltage source, (b) practical current source.

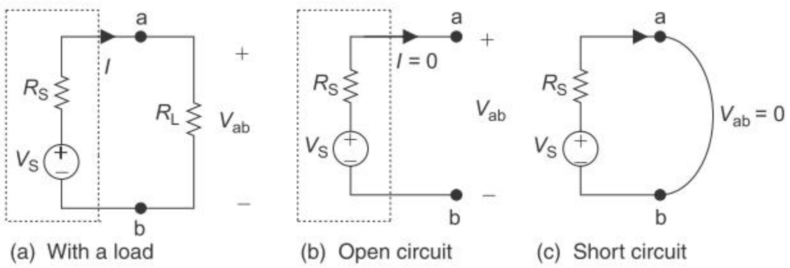
$$V_{ab} = V_S - I R_S, \quad I = (V_S / R_S + R_L).$$

Under a no-load condition ($R_L = \infty\Omega$), there is no current in the circuit and so the terminal voltage will be equal to the voltage appearing across the ideal voltage source.

$$V_{ab} = V_S, \quad I = 0.$$

If the output terminals are shorted together (R_L=0Ω), the current in the circuit will be a maximum and the terminal voltage will be equal to approximately zero. In such a situation, the voltage dropped across the internal resistance will be equal to the voltage of the ideal source.

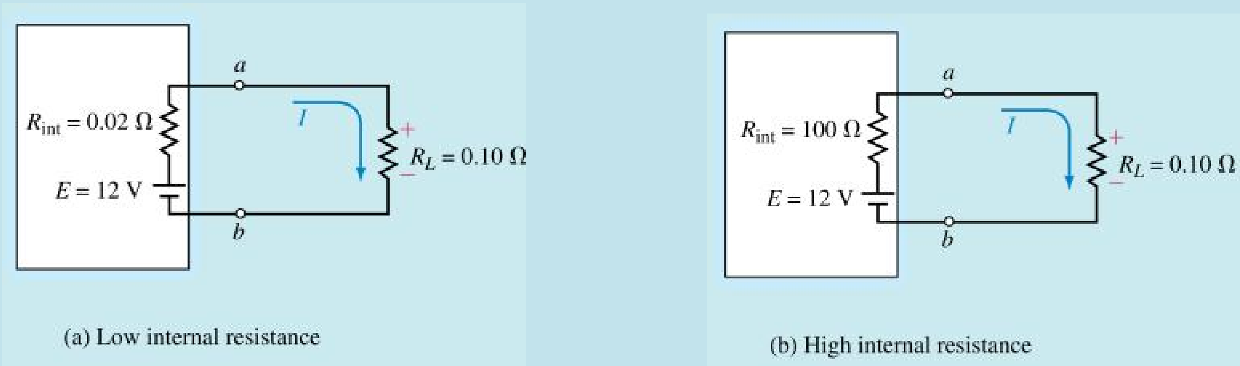
$V_{ab} = 0, \quad I = (V_S/R_S)$



Three states of a voltage source

Example

Two batteries having an open-terminal voltage of 12 V are used to provide current to the starter of a car having a resistance of $0.10\ \Omega$. If one battery has an internal resistance of 0.02Ω and the second battery has an internal resistance of $100\,\Omega$, calculate the current through the load and the resulting terminal voltage for each of the batteries.



$\boldsymbol{R_{int} = 0.02\ \Omega}$**:**

$$I = \frac{12\text{ V}}{0.02\ \Omega + 0.10\ \Omega} = 100.\text{ A}$$

$$V_{ab} = (100\text{ A})(0.10\ \Omega) = 10.0\text{ V}$$

$\boldsymbol{R_{int} = 100\ \Omega}$**:**

$$I = \frac{12\text{ V}}{100\ \Omega + 0.10\ \Omega} = 0.120\text{ A}$$

$$V_{ab} = (0.120\text{ A})(0.10\ \Omega) = 0.0120\text{ V}$$

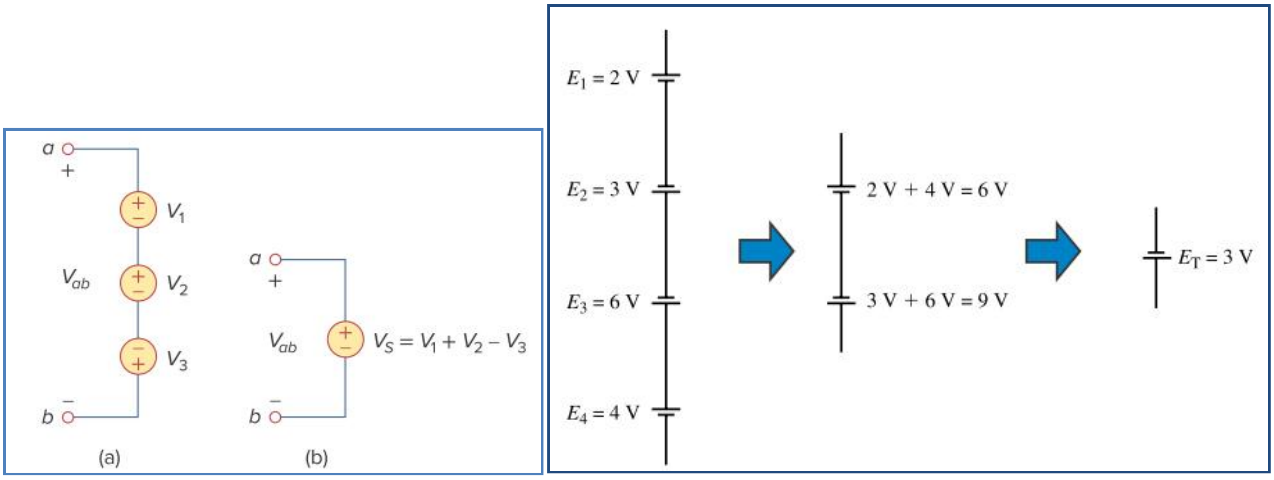
This simple example helps to illustrate why a 12-V automobile battery (which is actually 14.4 V) is able to start a car while eight 1.5 V-flashlight batteries connected in series will have virtually no measurable effect when connected to the same circuit.

Sources in Series and Parallel

Voltage sources in series

If a circuit has more than one voltage source in series, then the voltage sources may effectively be replaced by a single source having a value that is the sum or difference of the individual sources. Since the sources may have different polarities, it is necessary to consider polarities in determining the resulting magnitude and polarity of the equivalent voltage source.

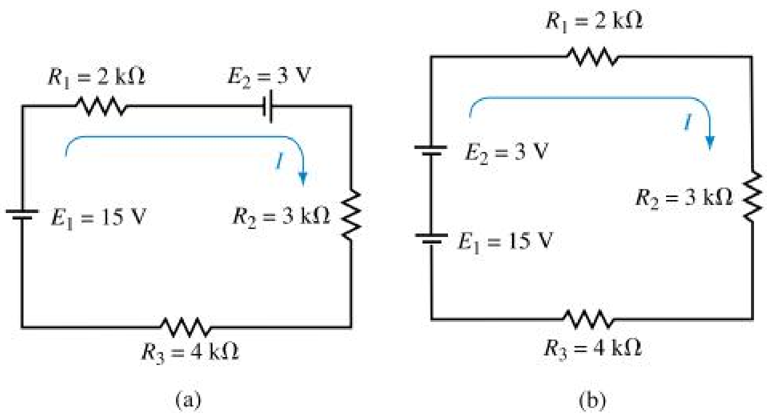
$$V_S = V_1 + V_2 + \cdots + V_n$$



Voltage sources in series

a positive sign (+) if the individual voltage has the same polarity as the equivalent voltage E (or V S); assign a negative sign (-) if the individual voltage has a different polarity from the equivalent voltage E.

Interchanging Series Components The order of series components may be changed without affecting the operation of the circuit. For example

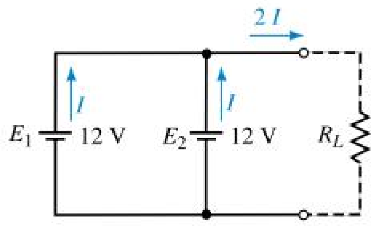


Therefore, we will regularly use the technique of interchanging components to simplify circuits before we analyze them.

Voltage Source In Parallel

Voltage sources of different potentials should never be connected in parallel, However, when two equal potential sources are connected in parallel, each source will deliver half the required circuit current. For this reason automobile batteries are sometimes connected in parallel to assist in starting a car with a “weak” battery. Figure below illustrates this principle.

$$V_S = V_{S1} = V_{S2} = \cdots = V_{Sn}$$



Voltage sources in parallel

The equivalent internal resistance R_S is the individual internal resistances in parallel:

$$R_S = R_{S1} \; // \; R_{S2} \; // \cdots // \; R_{Sn}$$

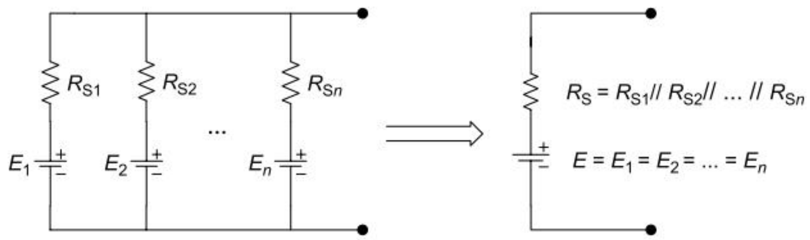
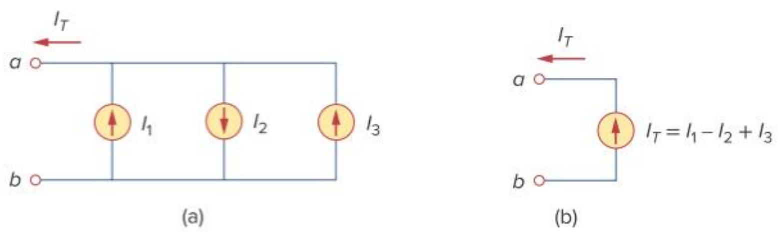


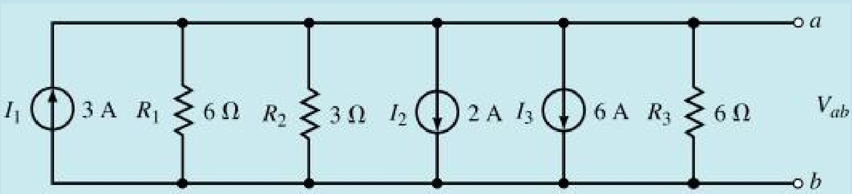
Figure 4.6 Voltage sources in parallel

When several current sources are placed in parallel, the circuit may be simplified by combining the current sources into a single current source. The magnitude and direction of this resultant source is determined by adding the currents in one direction and then subtracting the currents in the opposite direction.



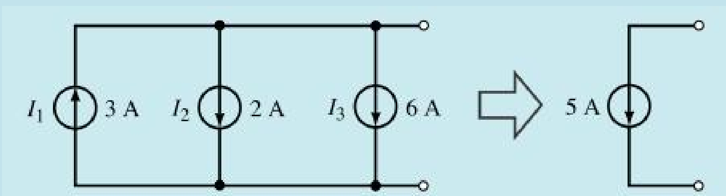
Current sources in parallel: (a) original circuit, (b) equivalent circuit.

Example: Simplify the circuit of Figure 2 and determine the voltage V_{ab} .



Since all of the current sources are in parallel, they can be replaced by a single current source. The equivalent current source will have a direction which is the same as both $I\,2$ and $I\,3$, since the magnitude of current in the downward direction is greater than the current in the upward direction.
The equivalent current source has a magnitude of

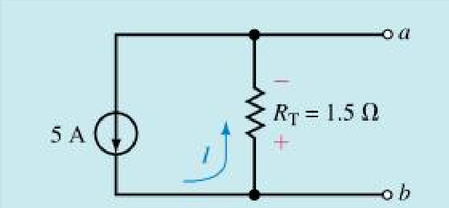
$$I = 2\text{ A} + 6\text{ A} - 3\text{ A} = 5\text{ A}$$



The circuit is further simplified by combining the resistors into a single value:

$$R_T = 6\ \Omega \| 3\ \Omega \| 6\ \Omega = 1.5\ \Omega$$

The equivalent circuit is shown in Figure below



The voltage V_{ab} is found as

$$V_{ab} = -(5\ \text{A})(1.5\ \Omega) = -7.5\ \text{V}$$

Current sources should never be placed in series.