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EE 178: Physics II

Chapter 4: Electric Potential

4.1: Introduction:

Scientists found that electric forces are conservative and thus has an associated electric potential energy. It is the purpose of this chapter.

4.2- Electric potential energy:

An electric potential energy U is assigned to a system of particles when an electrostatic force acts between two or more charged particles within a system of particles.

System: state i$\rightarrow$ state j

The electrostatic force $\rightarrow$ work W on the particles

$$\Delta U = U_f - U_i = -W$$

With ΔU: Resulting change in potential energy; W: The work done

4.3: Electric potential:

The potential energy of a charged particle in an electric field depends on the charge magnitude. So, the potential energy per unit charge, symbolized as U/q , is independent of the charge q of the particle and ***is characteristic only of the electric field.***

The electric potential V (or potential) is the potential energy per unit charge

$$V = \frac{U}{q}$$

V is a scalar, not a vertor

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q}$$

ΔV: The electric potential difference between two points i and f

$$\Delta V = V_f - V_i = \frac{-W}{q}$$

(By using the previous equation in paragraph 4.2)

If we set $U_i = 0$ at infinity as our reference potential energy, the electric potential V must also be zero.

Thus, the electric potential at any point in an electric field is:

$$V = -\frac{W_\infty}{q}$$

(Potential defined)

W_∞ is the work done by the electric field on a charged particle as that particle moves in from infinity to point f.

The SI unit for potential is $Volt = \frac{Joule}{Coulomb}$

Work done by an applied force:

Suppose we move a particle of charge q from point i to point f in an elctric field by applying a force to it. During the move, our applied force does work W_{app} on the charge while the electric field does work W:

Work-Kinetic energy theorem: $\Delta K = K_f - K_i = W_{app} + W$

We suppose the particle s stationary before and after move: K_f and K_i are zero.

$$W_{app} = -W$$

The work W_{app} done by our applied force during the move is equal to the negative of the work W done by the electric field.

$$\Delta U = U_f - U_i = W_{app} = q\Delta V$$

4.4- Equipotential surfaces:

Adjacent points that have the same electric potential form an ***equipotential surface,*** which can be either an imaginary surface or real, physical surface.

For the same equipotential, when the particle moves between two points i and f, the work W must be zero because $V_f = V_i$

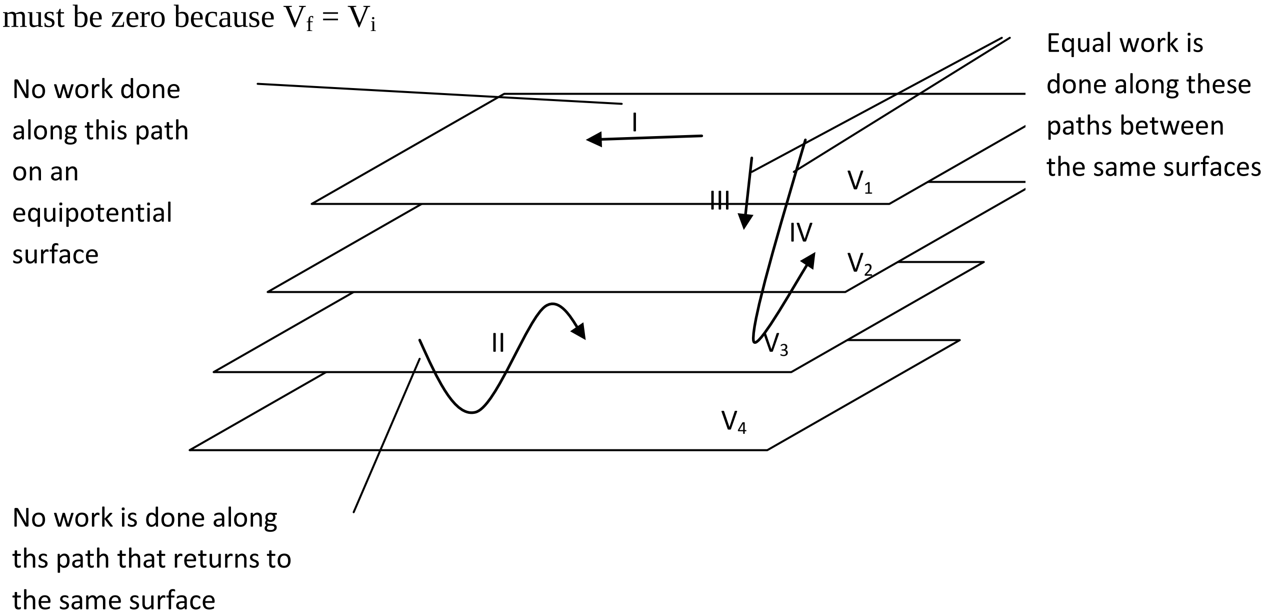
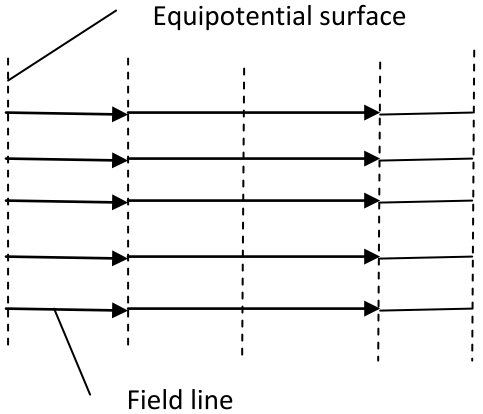
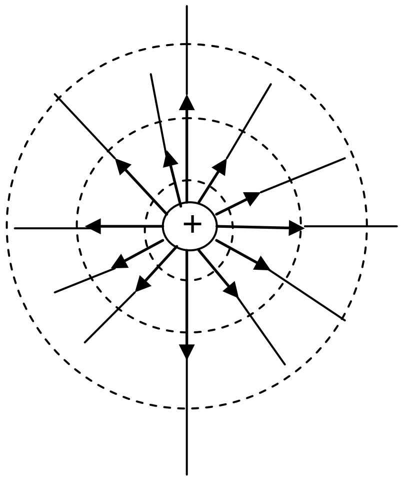


Figure 4.1: Portions of four equipotental surfaces at electric potentials V_1 = 100 V, V_2 = 80V, V_3 = 60V and V_4 = 40V.



(a)



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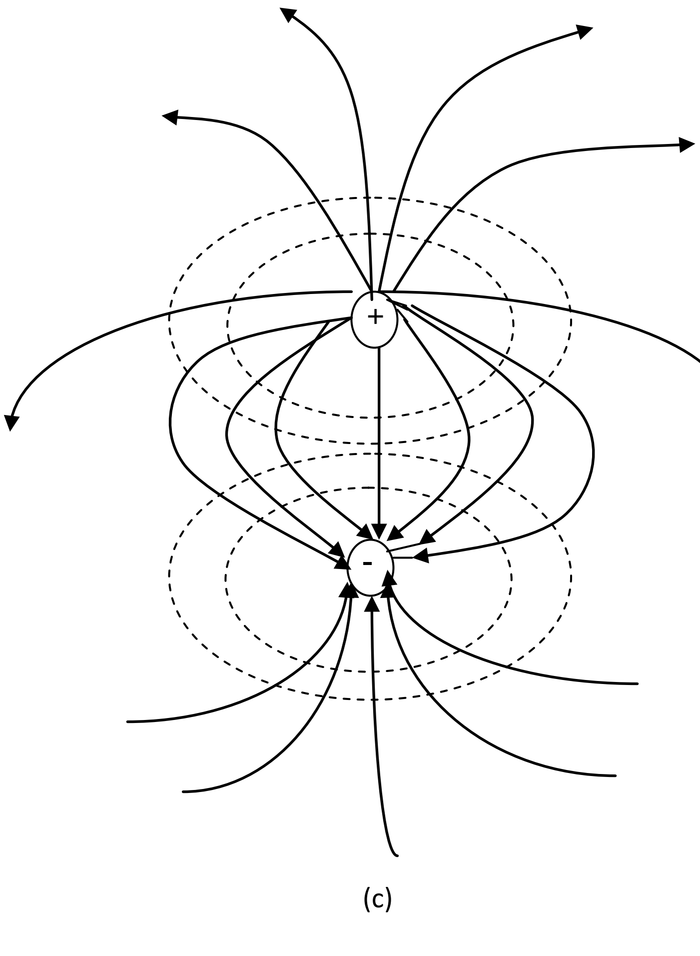


(c)

Figure 4.2: Electric field lines, and cross sections equipotential surfaces (dashed) for (a) uniform electric field, (b) The field due to a point charge, (c) The field due to an electric dipole

Equipotential surfaces are always perpendicular to electric field lines and thus to $\vec{E}$, which is always tangent to these lines.

4.5- Calculating the potential from the field:

Let us consider an electric field, represented in the figure below, and a positive charge q_0 that moves along the path shown from point i to point f.

At any point on the path, an electrostatic force $q_0\vec{E}$ acts on the charge as it moves through a differential displacement $\overrightarrow{ds}$

$$dW = \vec{F}\,.\overrightarrow{ds} \qquad ; \qquad \vec{F} = q_0\,\vec{E}$$

$$dW = q_0\,\vec{E}.\overrightarrow{ds} \qquad \text{so:} \qquad W = q_0\,\int_i^f \vec{E}\,.\overrightarrow{ds}$$

$$V_f - V_i = -\frac{W}{q_0} = -\int_i^f \vec{E}.\overrightarrow{ds}$$

Knowing that the electric force is conservative, we don't take care about the path. All paths give us the same result.

If we set $V_i = 0$, then $V = -\int_i^f \vec{E}.\overrightarrow{ds}$

If $V_i = 0$ at infinity: $V = -\int_\infty^f \vec{E}.\overrightarrow{ds}$

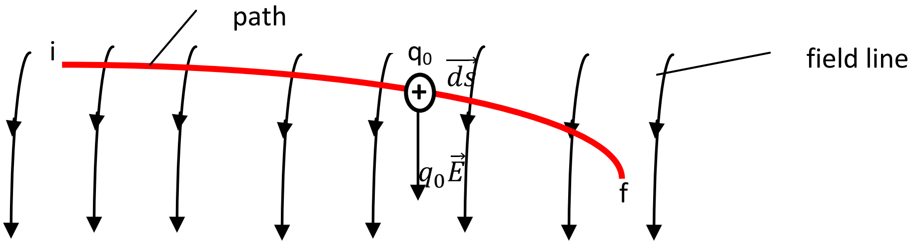


Figure 4.3: A test charge q_0 moves from point i to point f along the path shown in an electric field.

4.6- Potential due to a point charge:

In this paragraph, we want to deduce the electric potential V relative to the zero potential at infinity.

At point P at distance R from a fixed particle of positive charge q. To find the potential, we move a test charge q_0 from P to infinity (see figure 4.4)

$$\vec{E}\,.\,\overrightarrow{ds} = E\cos\theta\,.\,ds$$

$\vec{E}$ and $\overrightarrow{ds}$ have the same direction.

$$V_f - V_i = -\int_R^\infty E.dr$$

We set $V_f = 0$ (at ∞) and $V_i = V$ (at R). Then:

$$-V = -\int_R^\infty \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$

$$V = -\frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

So, V at r, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \qquad \text{(point charge)}$$

if $q>0 \rightarrow V>0$

If $q<0 \rightarrow V<0$

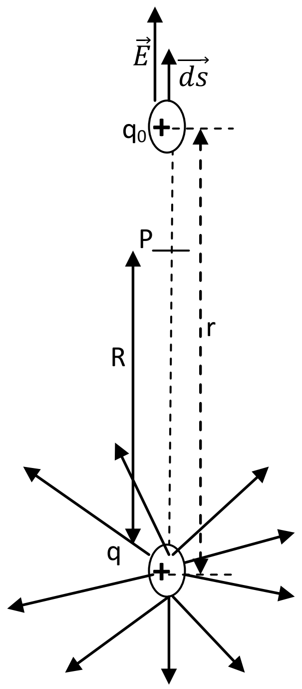


Figure 4.4: The charge produces an electric field $\vec{E}$ and potential V at point P

4.7. Potential due to a group of point charges:

By using the superposition principle, we can find the net potential at a point due to a group of point charges:

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \left(\frac{q_i}{r_i}\right)$$

(n point charges)

The previous sum is an algebric sum.

4.8- Potential due to an electric dipole:

If we apply the superposition principle we get:

$$V = \sum_{l=1}^{2} V_l = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right) = \frac{q}{4\pi\epsilon_0}\,\frac{r_{(-)} - r_{(+)}}{r_{(-)}.r_{(+)}}$$

Generally, for dipoles, we are interested by points relatively far from the dipole, so r>>d.

d: The distance between charges

$$r_{(-)} - r_{(+)} \sim d\cos\theta$$

$$r_{(-)}.r_{(+)} \sim r^2$$

By replacing the previous approximations, we find:

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\,cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\,cos\theta}{r^2}$$

(Electric dipole)

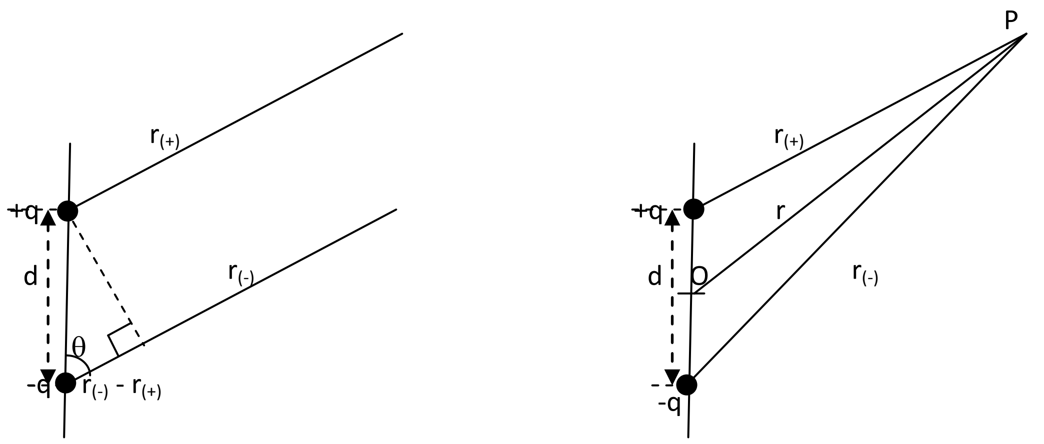


Figure 4.5: Electric dipole

<u>Induced dipole moment:</u>

Many molecules such as water, have permanent electric dipole moments. In other molecules, "nonpolar molecules", and isolated atoms, the negative and positive centers are superposed.

But when they are placed in an external field, a distorsion occurs in electron orbits and separated the centers of positive and negative charge.

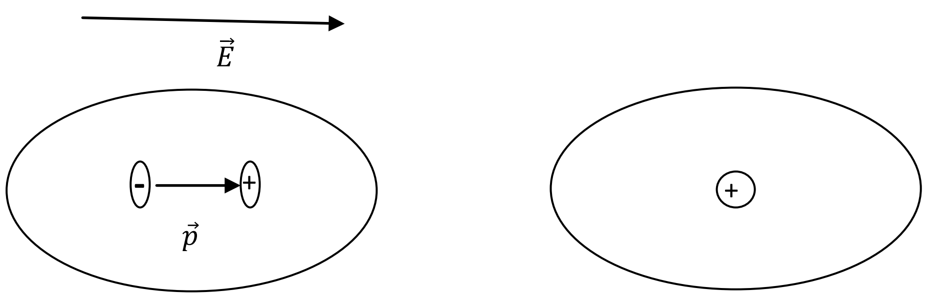


Figure 4.6: The electric field shifts the centers of the positive and negative charges, creating a dipole

4.9- Potential due to a continuous charge:

For a continuous charge→ a differential element of charge dq is chosen→ Determine the potential dV at P due to dq, and then integrate over the entire charge distribution

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r}$$

(positive or negative dq)

r : distance between P and dq

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\int\frac{dq}{r}$$

Line of charge:

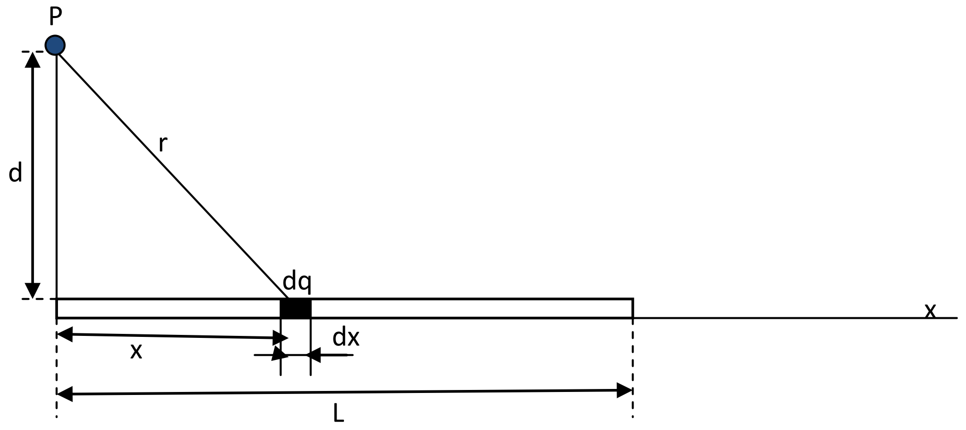


Figure 4.7: A thin, uniformly charged rod produces an electric potential V at P

$$dq = \lambda\, dx$$

$$r = \sqrt{x^2 + d^2}$$

The elemental potential dV:

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, dx}{\sqrt{x^2 + d^2}}$$

The rod charge is positive, V =0 at infinity

$$V = \int dV = \int_0^L \frac{1}{4\pi\varepsilon_0}\frac{\lambda}{(x^2 + d^2)^{1/2}}\, dx = \frac{\lambda}{4\pi\varepsilon_0}\int_0^L \frac{dx}{(x^2 + d^2)^{1/2}}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[Ln\left(x + (x^2 + d^2)^{1/2}\right)\right]_0^L = \frac{\lambda}{4\pi\varepsilon_0}\left[Ln\left(L + (L^2 + d^2)^{1/2}\right) - Ln\, d\right]$$

$$V = \frac{\lambda}{4\pi\varepsilon_0}\left[Ln\left(\frac{L + (L^2 + d^2)^{1/2}}{d}\right)\right]$$

Charged disk:

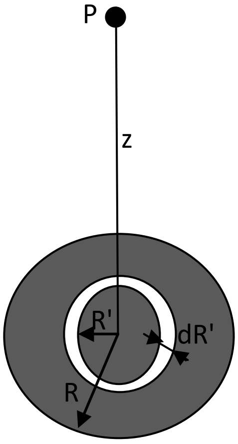


Figure 4.8: A plastic disk of radius R, charged on its top surface with surface charge density σ

$$dq = \sigma\,(2\pi R')\,dR'$$

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma\,(2\pi R')\,dR'}{\sqrt{R'^2 + z^2}}$$

$$V = \int_0^R dV = \frac{\sigma}{2\varepsilon_0}\int_0^R \frac{R'dR'}{\sqrt{R'^2 + z^2}} = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + z^2} - z\right)$$

$$V == \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R^2 + z^2} - z\right)$$

(Charged disk)

4.10- Calculating the field from the potential:

- We want to deduce the electric field from potential
- We know that the electric field lines are perpendicular to equipotential lines

Suppose that a positive test charge q moves through a displacement $\overrightarrow{ds}$ from one equipotential surface to the next surface

$$dW = -q_0\,dV = \vec{F}.\overrightarrow{ds} = q_0\,\vec{E}.\overrightarrow{ds} = q_0\,E\,cos\theta\,ds$$
$$-q_0\,dV = q_0\,E\,cos\theta\,ds$$
$$E\,cos\theta = -\frac{\partial V}{\partial s}$$

Since E cosθ is the component of $\vec{E}$ in the direction of $\overrightarrow{ds}$

$$E_s = -\frac{\partial V}{\partial s}$$

The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes in that direction

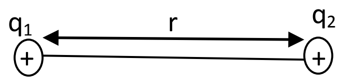
$$E_x = -\frac{\partial V}{\partial x} \quad ; \quad E_y = -\frac{\partial V}{\partial y} \quad ; \quad E_z = -\frac{\partial V}{\partial z}$$

$$\vec{E} = -\nabla V = -\frac{\partial V}{\partial x}\vec{\imath} - \frac{\partial V}{\partial y}\vec{\jmath} - \frac{\partial V}{\partial z}\vec{k}$$

4.11- Electric potential energy of a system of point charges:

The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance

Example:



To find the electric potential energy→ We must built mentally the system→ starting with both charges infinitely far away and at rest→ We bring q_1 from infinity, no work, charge alone → We bring q_2, we must do work, q_1 exerts a force on q_2 during the move

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}$$

$$W = q_2 V = q_2 . \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$