Developing a BDD-Based CTL Model Checker

CS 6840 Formal System Design

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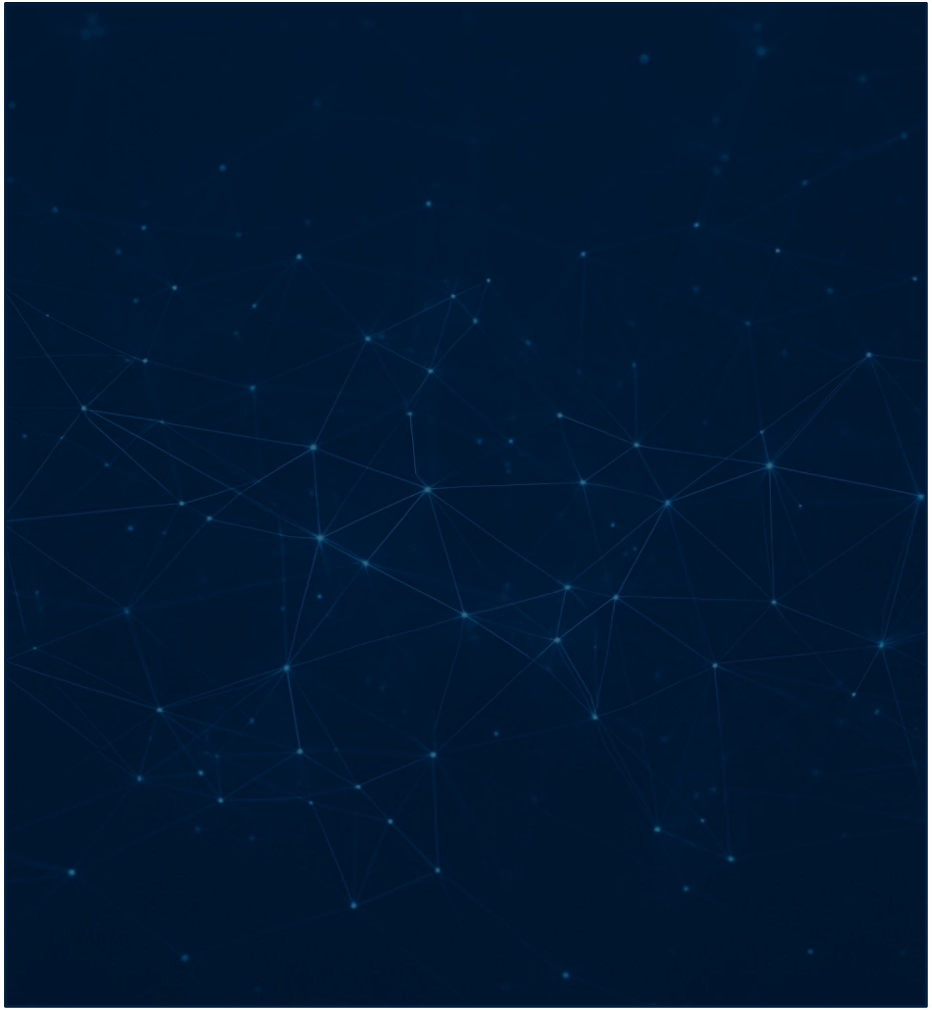
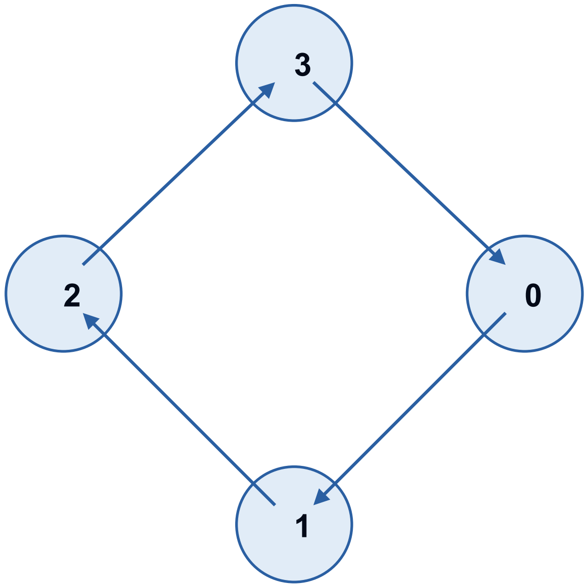
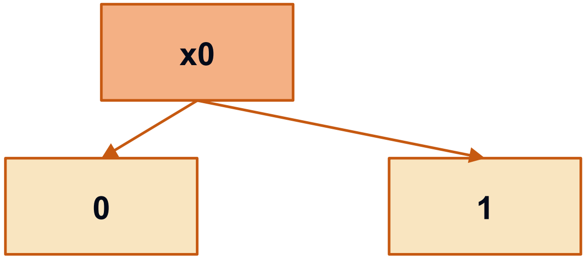


Outline

- Motivation & Problem
- CTL & Temporal Operators
- Implementation & Design
- Benchmarks & Validation
- Lessons, Future Work & Conclusion

Motivation & Problem





- **CTL checks temporal properties on transition systems**
- Explicit enumeration enumerates all reachable states, leading to state-space explosion
- BDDs encode state sets symbolically, compacting many states into a succinct graph
  but require careful variable ordering to remain efficient

[1][2]

CTL & Temporal Operators

Existential

EX ϕ
Exists next: some successor satisfies ϕ

EF ϕ
Exists eventually: some path eventually reaches ϕ

EG ϕ
Exists globally: some path stays in ϕ forever

E[ϕ U ψ]
Exists until: along some path ϕ holds until ψ holds

Universal

AX ϕ
For all next: every successor satisfies ϕ

AF ϕ
For all eventually: along all paths ϕ eventually holds

AG ϕ
For all globally: along all paths ϕ holds globally

A[ϕ U ψ]
For all until: on every path ϕ holds until ψ holds

EF p: a path exists where p eventually holds

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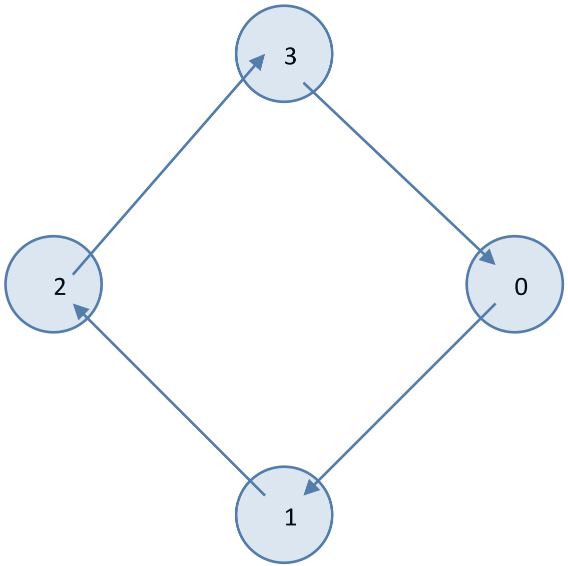
Transition System Representation



Ring topology example

Current Bits	Next Bits
s0	s0_next
s1	s1_next

Labeling

Each state has a set of propositions (e.g., p, q) that hold in that state.

Transition relation

Encoded as a single BDD over current and next bits.

[4]

Symbolic CTL Model Checker

- **Parse formulas with Lark grammar**
- Represent boolean connectives as BDD operations (∧, ∨, ¬)
- Use predecessor function for EX/AX
- Compute least/greatest fixpoints for EF/AF and EG/AG
- Evaluate until operators using iterative fixpoints

```
def eval(node):
    kind = node[0]
    if kind == 'atom': return ap_bdd(node[1])
    if kind == 'not': return ~eval(node[1])
    if kind == 'and': return eval(node[1]) &
eval(node[2])
    if kind == 'or': return eval(node[1]) |
eval(node[2])
    if kind == 'ex': return pre(eval(node[1]))
    if kind == 'ax': return ~pre(~eval(node[1]))
    if kind == 'ef': return least_fix(lambda Y:
eval(node[1]) | pre(Y))
    if kind == 'af': return least_fix(lambda Y:
eval(node[1]) | ~pre(~Y))
    if kind == 'eg': return greatest_fix(lambda
Y: eval(node[1]) & pre(Y))
    if kind == 'ag': return greatest_fix(lambda
Y: eval(node[1]) & ~pre(~Y))
    if kind == 'eu': return least_fix(lambda Y:
eval(ψ) | (eval(φ) & pre(Y)))
```

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Explicit CTL Model Checker

- **ExplicitTransitionSystem stores adjacency sets**
- Predecessor function computes all predecessors via set comprehension
- Boolean connectives operate on Python sets
- Least/greatest fixpoints iterate until the set of states stabilizes
- Returns states satisfying a formula; initial states ⊆ result implies satisfaction

```
def eval(node):
    kind = node[0]
    if kind == 'atom': return {s for s in S if
ap in L(s)}
    if kind == 'not': return S - eval(node[1])
    if kind == 'and': return eval(node[1]) &
eval(node[2])
    if kind == 'or': return eval(node[1]) |
eval(node[2])
    if kind == 'ex': return pre(eval(node[1]))
    if kind == 'ax': return S - pre(S -
eval(node[1]))
    if kind == 'ef': return least_fix(lambda Y:
eval(node[1]) | pre(Y))
    if kind == 'af': return least_fix(lambda Y:
eval(node[1]) | (S - pre(S - Y)))
```

[6]

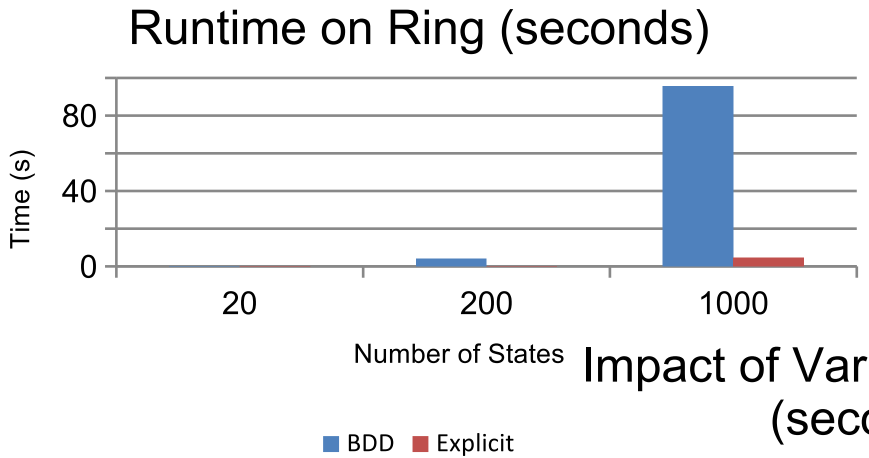
Example Usage & Validation

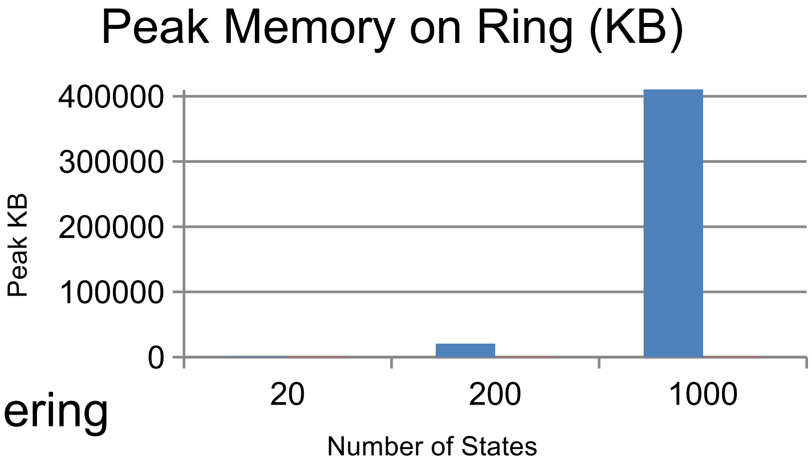
A minimal script builds small systems and queries both backends.
The results agree across all formulas, serving as a quick sanity check.

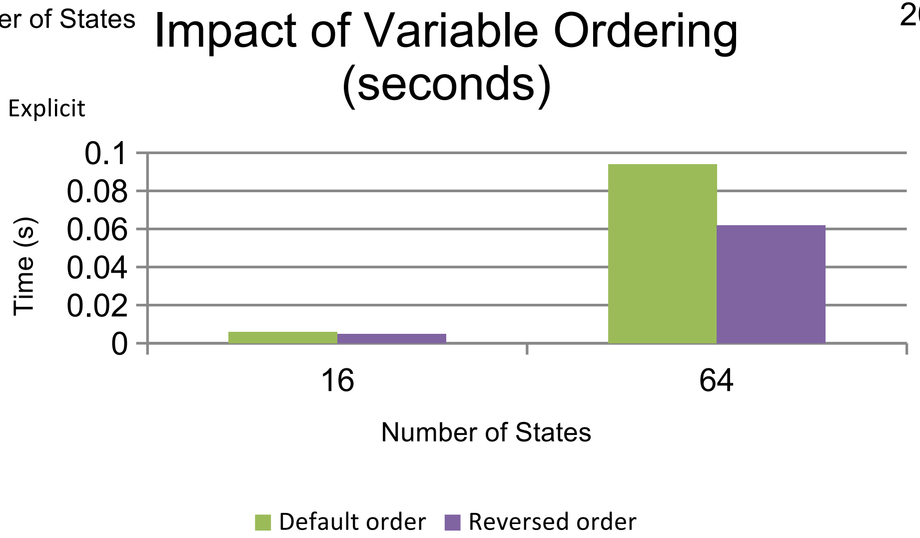
Formula	BDD	Explicit
EF p	True	True
AG p	False	False
AF p	True	True
EG q	False	False
EX p	True	True
AX p	True	True
E[q U p]	True	True
A[q U p]	False	False

[7]

Benchmark Results







Explicit checker handles larger rings significantly faster & uses far less memory.

Reversing BDD variable order yields noticeable speedups on chains.

[8][9]

Testing & Verification

16 unit tests ensure correctness across both backends.
Each test builds a small system and checks whether a CTL formula holds starting from the initial state.

Test Name	Formula	Result
test_ef_p	EF p	True
test_ag_p_false	AG p	False
test_af_p_true	AF p	True
test_eg_q_false	EG q	False
test_eu_q_until_p_true	E[q U p]	True
test_au_q_until_p_false	A[q U p]	False
test_ex_p_true	EX p	True
test_ax_q_false	AX q	False

Lessons Learned & Future Work

Variable ordering dramatically affects BDD sizes and performance
Symbolic methods excel when state spaces are large but regular
Python’s dd library and Lark parser streamline prototyping

Add fairness constraints and support additional logics
Experiment with dynamic variable reordering strategies
Scale to larger industrial benchmarks and compare with other tools

[11][12]

Conclusion

We delivered a complete CTL model checker with both symbolic and explicit backends.
Our benchmarks reveal that while symbolic methods can compress large state spaces, explicit enumeration often outperforms them on small examples.
Rigorous testing and documentation ensure correctness and reproducibility.
This project lays a foundation for further exploration in formal verification and model checking.

Thank you!

[13][14]