

An Ant looking for food

Bella Shao

2023-06-19

Introduction

An ant leaves its ant hill to look for food. The ant walks at speed of 10cm per 1 second and it moves randomly towards north, south, east and west with equal probability.

Question 1

The food is located 20cm away from the ant hill. In this case, the ant walks towards one of the four directions randomly at each step, and it starts exactly 20cm away from the food. This process is a random walk with $|10|\text{cm}$ per second. Each step randomly moves to one of the four directions with equal probability.

```
Walk_Q1 <- function(t){ # t is the second/step

  walks <- 0 #starts from 20cm outside the center
  #walks <- runif(n = 1, min = 0.01, max = 10.01) #start on the line of (10cm,0cm) and (0cm,10cm)

  for (s in 1:t){

    walks <- c(walks, walks[length(walks)] + sample(c(10,-10, 10,-10), size = 1, replace = TRUE))
  }

  return(walks)
}

# run this process many times
t = 100
outcomes_Q1 <- replicate(n = 500, Walk_Q1(t))

#the first occurrence at 20cm or -20cm means the ant successfully foraged the food

simu_timeQ1 <- apply(outcomes_Q1, 2, function(v){which(abs(v) == 20)[1]})

mean(simu_timeQ1) # the average time (seconds) an ant spent to forage food outside 20cm away from ant h

## [1] 4.876
```

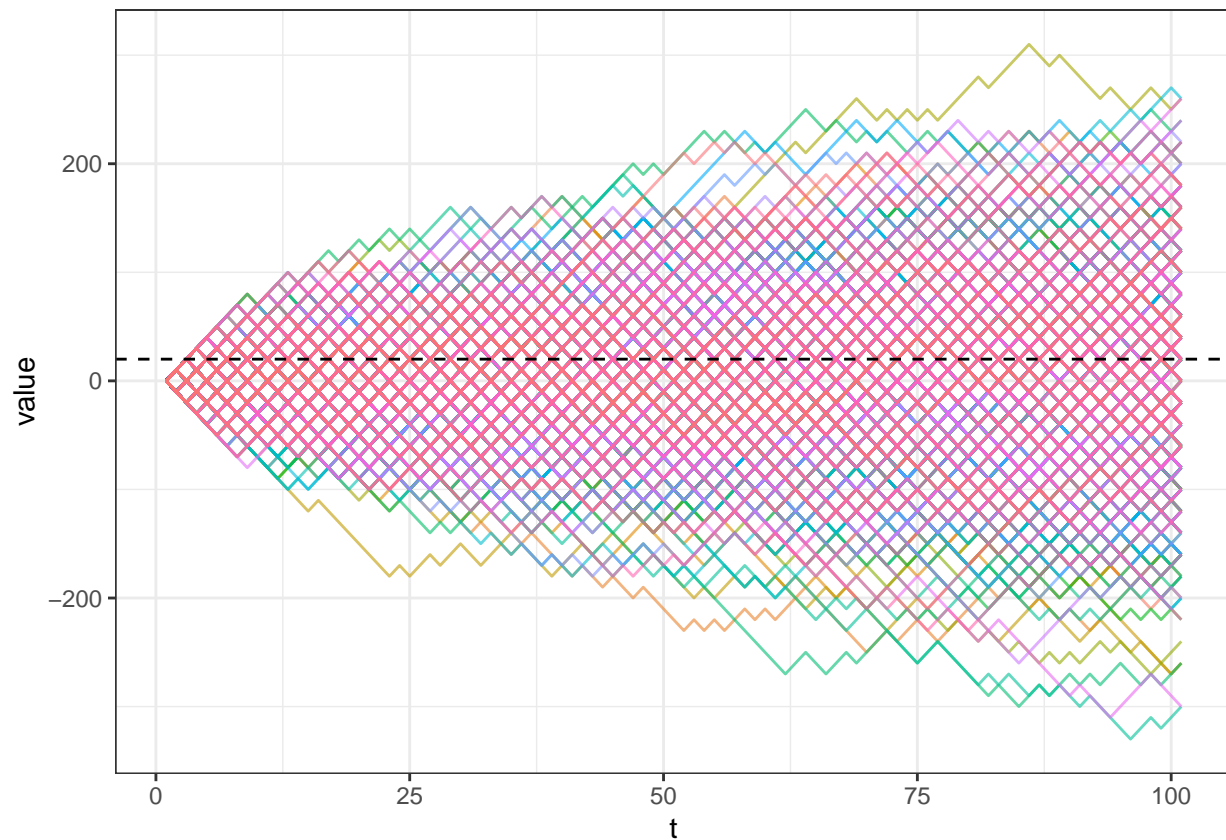
```
sd(simu_timeQ1)
```

```
## [1] 2.524554
```

```
dt1 <- data.table(t = 1:(t+1),  
                  outcomes_Q1)
```

```
dt1 <- melt(dt1, id.vars = "t")
```

```
ggplot(dt1, aes(x = t, y = value, col = variable)) +  
  geom_line(alpha = 0.6) +  
  theme_bw() + theme(legend.position = "none") +  
  geom_hline(yintercept = 20, linetype = "dashed")
```



Question 2

The food is still at the same location, but the ant can only start moving at the coordinates of $(10cm, 0)$ and $(0, 10cm)$. In this case, the starting point for an ant is uniformly distributed within the range of 0 and 10.

```
Walk_Q2 <- function(t2){ # t is the second/step
```

```

walks <- runif(n = 1, min = 0, max = 10) #start on the line of (10cm,0cm) and (0cm,10cm)

for (s in 1:t2){

  walks <- c(walks, walks[length(walks)] + sample(c(10,-10, 10,-10), size = 1, replace = TRUE))
}

return(walks)

}

# run this process many times
t2 = 100
outcomes_Q2 <- replicate(n = 500, Walk_Q2(t2))

#the first occurrence at 20cm or -20cm means the ant successfully foraged the food

simu_timeQ2 <- apply(outcomes_Q2, 2, function(v){which(abs(v) == 20)[1]})

mean(simu_timeQ2) # the average time (seconds) an ant spent to forage food outside 20cm away from ant h

## [1] NA

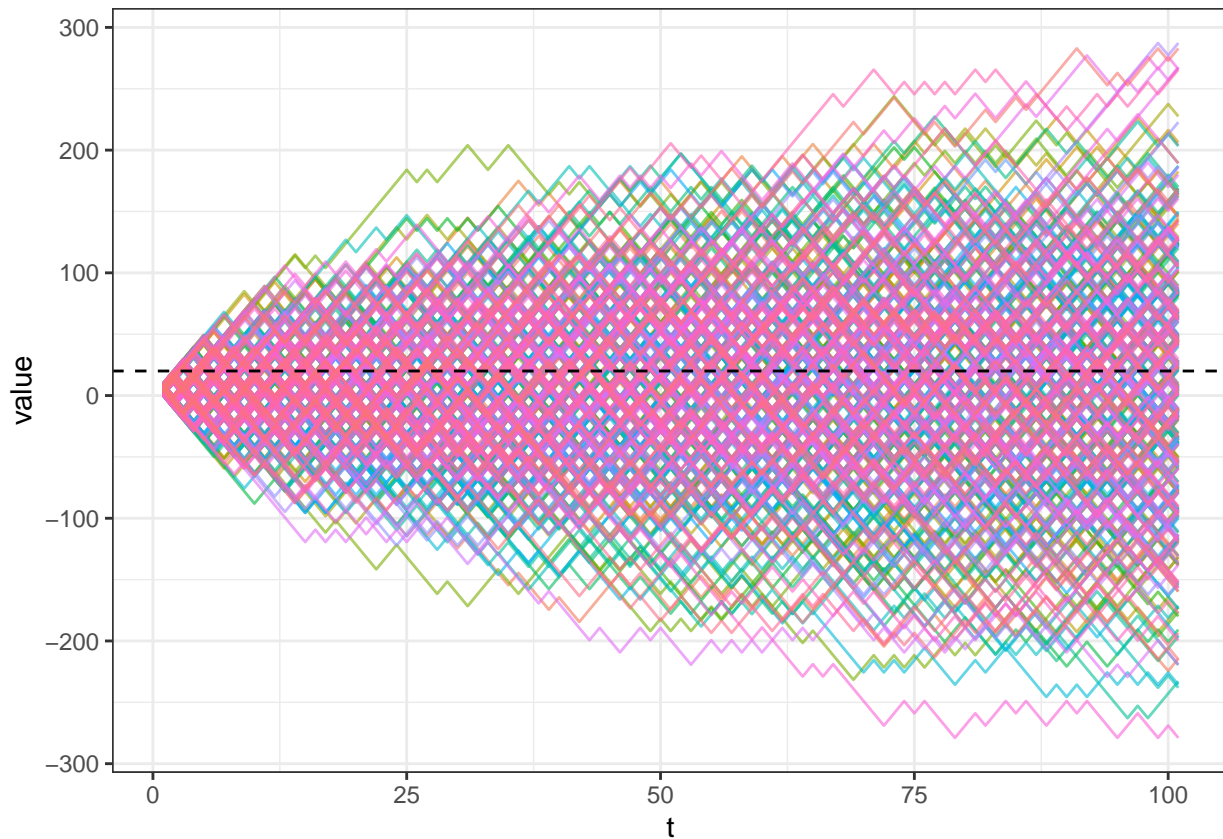
# The average time for foraging the food is "NA", which means non converging. Thus, the ant cannot reach the food

dt2 <- data.table(t = 1:(t2 + 1), outcomes_Q2)

dt2 <- melt(dt2, id.vars = "t")

ggplot(dt2, aes(x = t, y = value, col = variable)) +
  geom_line(alpha = 0.6) +
  theme_bw() + theme(legend.position = "none") +
  geom_hline(yintercept = 20, linetype = "dashed")

```



Question 3

The ant hill is located at $x = 0\text{cm}$, $y = 0\text{cm}$, and the food is located at a closed boundary $(\frac{x-2.5\text{cm}}{30\text{cm}})^2 + (\frac{y-2.5\text{cm}}{40})^2 < 1$.

```
Walk_Q3 <- function(t3){

  x_cor <- 0
  y_cor <- 0

  for (i in 1:t3)
  {
    #if walk along x coordinate
    x_cor <- c(x_cor, x_cor[length(x_cor)] + sample(c(10,-10), size = 1, replace = TRUE))

    #if walk along y coordinate
    y_cor <- c(y_cor, y_cor[length(y_cor)] + sample(c(10,-10), size = 1, replace = TRUE))
  }

  boundary = ((x_cor - 2.5)/30)^2 + ((y_cor - 2.5)/40)^2 < 1

  return(boundary)
}
```

```
# run this process many times
t3 = 100
outcomes_Q3 <- replicate(n = 1000, Walk_Q3(t3))

#the first occurrence at 20cm or -20cm means the ant successfully foraged the food

simu_timeQ3 <- apply(outcomes_Q3, 2, function(v){which(v == FALSE)[1]}) # the first occurrence of FALSE
mean(simu_timeQ3)
```

```
## [1] 9.118
```

```
sd(simu_timeQ3)
```

```
## [1] 5.868604
```