Forecasting Volatility in NVDA Stock Returns With ARCH, GARCH, and GJR-GARCH Models

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I. **Introduction**

The rate of technological developments in recent years has caused technology stocks to skyrocket. With the recent developments in artificial intelligence, investors are more than ever drawn to technology stocks like NVDA. Nvidia is a world leader in artificial intelligence, pioneering and developing a range of AI services including chips, systems, and software. Nvidia has transformed the world's largest industries, and the surge of the company's stock has put it alongside household names like Amazon, Apple, Microsoft, and Google.

While stocks are inherently unpredictable, economists can use models to forecast volatility in stocks, or the conditional variance. Conditional variance is the variability of the stock at a certain time given past its variability. Figures 1 and 2 illustrate the log daily adjusted close price of the NVDA stock, and the first difference of the log which shows the percentage change.

Figure 1: Ln(Daily Adj Close Price)

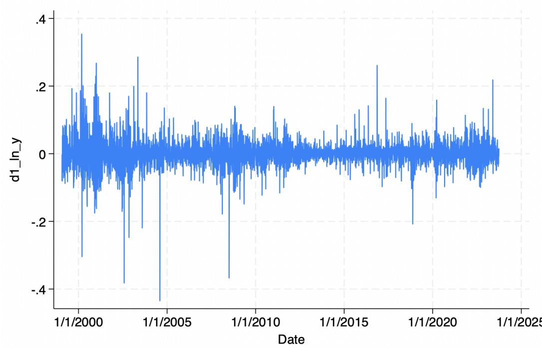


Figure 2: Growth Rate of Price (Returns)

The adjusted daily close price is a more accurate indicator of the overall stock value than the unadjusted daily close price, as the adjusted close includes the adjustments for stock splits and dividend distributions. The natural log shows the log of the daily close price, as seen in Figure 1, and the first difference shows the percentage change, or the growth rate, of the daily close price. The inspiration for modeling stock returns using the first difference of the natural log, comes from Indratno, Lestari, and Pasaribu (2016), who modeled the return of a stock (R_t), given the adjusted close price of a stock (X_t), as $R_t = \ln(X_t/X_{t-1}) = \ln(X_t) - \ln(X_{t-1})$. This is equivalent to taking the first difference of the natural log of the adjusted close price. This is shown in Figure 2, and the growth rate of the NVDA stock shows some heteroskedasticity in the returns. However, further tests will be done later on in this paper to confirm whether the returns are heteroskedastic.

Figures 3 and 4 show the histogram and QQ plot of the stock returns. The curved line below on the left represents a normal distribution, as well as the diagonal line below on the right. Both plots show that this data is not normally distributed. In comparing the histogram with the normal distribution, the shape of the data in the histogram does not follow the cuve of a normal distribution, and the points on the QQ plot deviate from the line representing normal distribution in the graph on the left. This indicates a high level of kurtosis, above the kurtosis of a normal distribution which is 3. The kurtosis of the NVDA returns will be examined further in Section III of this paper, where it will be computed numerically.

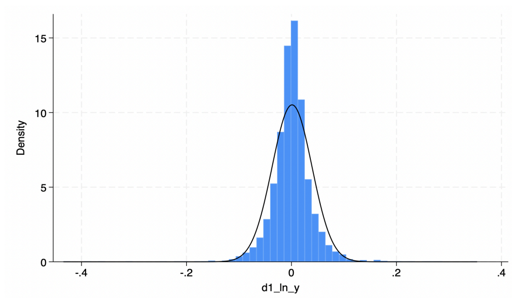


Figure 3: Histogram of Returns

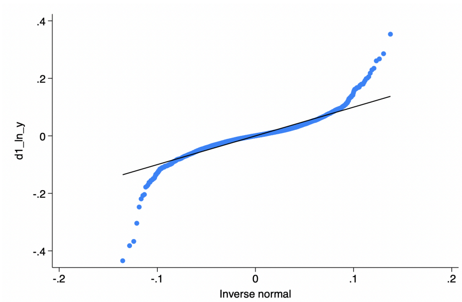


Figure 4: QQ Plot

II. Literature Review

Kanasro, Rohra, and Junejo (2009) examine the presence of volatility as well, in their case study of the Karachi Stock Exchange (KSE) of Pakistan. Through the use of the GARCH(1,1) model, they are able to confirm the presence of high volatility at the Kachari Stock Exchange throughout the study period. This implies random-walk behavior, implying that the market is very uncertain and risky for short-term and medium-term investors. This high volatility is a crucial piece of information that would deter a risk-averse investor, and high risk creates negative effects on the stock return and overall business of the market.

Lestari, Pasaribu, and Indranato (2016) aim to simulate and forecast the mean and variance of future returns of the Astra Agro Lestari Stock Exchange, conditional on past information. They analyze around 11 months of data from 2016, and state that analyzing volatility will show the conditional variance of the underlying stock return, and would improve the usefulness of stock price as an accurate signal for financial decisions. They use ARCH models and GARCH models to model the volatility of this stock, by first building an ARMA(1,1) model, using the residuals of the mean equation to test for ARCH effects, specifying the ARCH order and performing estimation, and refining the fitted ARCH model. They tested ARCH(1), ARCH(2), GARCH(1,1) and GARCH(1,2), and found that the GARCH(1,1) model had the lowest AIC and was most accurate in modeling stock returns.

Collin (2019) aims to find a model which best forecasts volatility of the Swedish stock market returns. Collin considers ARCH(1), GARCH(1,1), and EGARCH(1,1) models, and finds that EGARCH(1,1) generated the best forecasts. This aligned with the hypothesis of this paper, which indicated that models which account for features such as asymmetry would be superior in modeling volatility in stock returns. Additionally, she found that the ARCH(1) model slightly outperformed GARCH(1,1).

Su, Huang, and Lin (2011) use the GJR-GARCH, GARCH-M, and GJR-GARCH-M models to forecast volatility of FuBon and Cathay financial holdings.

The authors find that while the GJR-GARCH works well, the GARCH-M model works better. Therefore, the authors adopt the asymmetric GARCH model, GJR-GARCH-M(1,1) to represent the leverage effect. For our paper we will focus on the GJR-GARCH model.

Nugroho et. al (2019) uses GARCH, GARCH-M, GJR-GARCH, and log-GARCH models for returns volatility of financial data such as the DJIA and S&P 500 from January 2000 to December 2017. The authors comment that the GJR-GARCH(1,1) model captures an asymmetric behavior by allowing the current conditional variance to have a differing response to the past positive and negative returns. This is important to check for stock returns, so we will estimate the GJR-GARCH model for the NVDA stock returns in this paper as well to determine whether there are asymmetric leverage effects in this data set.

The above papers have found varying results for which model they concluded was best in forecasting volatility of stock returns. However, some of these papers only use the AIC to evaluate the models, while there are other criteria that can be used to find the order of ARCH and GARCH models, such as BIC which is known to be better at identifying a model. The approach in this paper will consider both of these information criteria in selecting a model. Furthermore, these papers do not test for structural break or unit root, which will both be considered throughout the approach of this paper in order to improve upon past research conducted in the area of modeling stock returns.

III. Data Set Information

The focus of this paper is on the variable of the adjusted daily closing price of the NVDA stock, which is a numeric variable in the dataset. To obtain the data required to analyze this topic, data was collected from Yahoo Finance, an open source data provider for financial data. The dataset is up to date as of September 29, 2023. It includes 6,213 observations and dates back to January 22, 1999, not including weekends since trading does not occur on weekends. While this paper focuses on the adjusted daily closing price and the date, the dataset also includes information about the daily open price, daily highest price, daily lowest price, daily close price, and volume of shares. This paper focuses on adjusted daily closing price as opposed to daily closing price because it is a better indicator of the stock value as it accounts for stock splits and dividend distributions. Figure 5 displays the summary statistics for the variables.

Stats	Price	Return
Mean	**37.69747**	**.0011353**
SD	**78.21098**	**.0379241**
Skewness	**3.024967**	**-.1503008**
Kurtosis	**13.004**	**15.32674**

Figure 5: Summary Statistics

Figure 5 confirms the results of the QQ plot in Figure 4. In the plot, there are several points curving away from the line representing normal distribution, which indicates high levels of kurtosis. In a normal distribution, the kurtosis is 3, and as seen from the summary statistics in

Figure 5, the kurtosis of this data is 15.327. This confirms that the data is not normally distributed since the kurtosis is high.

IV. Model Selection

To model this data, the ARCH (Auto Regressive Conditional Heteroskedastic) Model and GARCH (Generalized Auto Regressive Conditional Heteroskedastic) are both used. These will allow for the modeling of the conditional variance of the return of the stock. This is useful for the topic of this paper, since the conditional variance of the stock return depends on time. The models can then be used to forecast the conditional variance of the stock, which is how financial experts evaluate stocks and risk since stocks are otherwise unpredictable. ARCH and GARCH models are most appropriate in dealing with data with high kurtosis, including financial returns which often have thicker tails than normality implies. The NVDA stock returns exhibit these characteristics, so the ARCH and GARCH models will be applied. Additionally, the GJR-GARCH model will be used as well to check for asymmetric leverage effects in the series.

V. Results

In conducting the initial unit root tests on the dataset, DF-GLS is used to determine the optimal lag number. When conducting DF-GLS on price, the optimal lag using Ng-Perron and minimum MAIC is found to be 32, while the optimal lag using the minimum SIC is found to be 11. When conducting DF-GLS on the returns, the optimal lag using Ng-Perron and minimum MAIC is found to be 33, while using the minimum SIC produces an optimal lag of 26. These tests all conclude a non-rejection of the null hypothesis of a unit root, since the test statistic is not significant. These results are displayed in Figure 6 below.

Price	Number of Lags	Test Statistic	Critical Values
MIN SIC	11	0.297	-3.480 (1%), -2.839 (5%), -2.551 (10%)
MIN MAIC	32	-0.071	-3.480 (1%), -2.834 (5%), -2.547 (10%)
Ng-Perron	32	-0.071	-3.480 (1%), -2.834 (5%), -2.547 (10%)
Return	Number of Lags	Test Statistic	Critical Values
MIN SIC	26	-2.447	-3.480 (1%), -2.836 (5%), -2.548(10%)
MIN MAIC	33	-2.214	-3.480 (1%), -2.834 (5%), -2.547 (10%)
Ng-Perron	33	-2.214	-3.480 (1%), -2.834 (5%), -2.547 (10%)

Figure 6: DF-GLS Results

Next, the Augmented Dickey Fuller (ADF) test for unit roots is conducted using these optimal lag numbers, as seen in Figure 7 below. The ADF test finds that for prices, a lag of 32 produces a test statistic of 0.858, which is greater than the 10% critical value of -3.12. Furthermore, a lag of 11 produces a test statistic of 1.171, which is also greater than the 10% critical value of -3.12. In both cases, using either of the optimal lag numbers results in the conclusion that the null hypothesis of a unit root can not be rejected, which indicates presence of a unit root. Next, the ADF test is conducted on returns. Using a lag of 33 produces a test statistic of -13.283 and a lag of 26 produces a test staitstic of -14.098. Both test statistics are less than the 1% critical value of -3.96. As a result the null hypothesis of a unit root is rejected, indicating that there is no unit root in the return series.

	Number of Lags	Test Statistic	Critical Values
Price	11	1.171	-3.960 (1%), -3.410 (5%), -3.120 (10%)
	32	0.858	-3.960 (1%), -3.410 (5%), -3.120 (10%)
Return	26	-14.098 ***	-3.960 (1%), -3.410 (5%), -3.120 (10%)
	33	-13.283 ***	-3.960 (1%), -3.410 (5%), -3.120 (10%)

Figure 7: ADF Results

To confirm the results of the ADF test, the KPSS test is also used. For both prices and returns, the 10%, 5%, and 1% critical values are 0.119, 0.146, and 0.216 respectively. When testing on prices for up to 60 lags, all test statistics are greater than the critical values. Using the bandwidth formula (integral(4*(T/100)^2/9)) with T being the number of samples to determine the best lag, it is determined that the optimal lag is 4. As seen in Figure 8 below, at 4 lags, the null hypothesis of a trend stationary series is rejected for price since the test statistic is greater than the critical values, implying that the price series has a unit root. On the other hand, for the returns, the null hypothesis of a stationary series is not rejected, implying that there is no unit root in the returns. This aligns with the results from the ADF test. As a result, it has been shown that the price series has a unit root, and the return series does not. However, these tests do not take into account structural breaks. As a result, the result of the unit root test may be biased.

	Number of Lags	Test Statistic	Critical Values
Price	4	16.7***	0.216 (1%), 0.176 (5%), 0.146 (10%)
Return	4	0.08	0.216 (1%), 0.176 (5%), 0.146 (10%)

Figure 8: KPSS Results

Next, in order to test for structural breaks, the Yabu-Perron and Kim-Perron process is used to find a significant break. This test is an improvement from the zandrews test, which is biased for AO-B and AO-C models with breaks in the trend. As seen in Figure 9 below of the Yabu-Perron results, when conducting the test using the AO-B model allowing for a break in the trend, the break is found to be at time 3860, or at date May 27, 2014. However, this break is not found to be significant so we do not conduct Kim-Perron tests on it. On the other hand, using the AO-C model allowing for a break in both the trend and intercept, the break is found to be at time 4358, or at date May 17, 2016. This break is found to be significant so we conduct the Kim-Perron unit root test including this break, and find test statistics significant at the 5% significance level, as shown in Figure 10 below of the Kim-Perron results. The critical values for Model 3 are found in the 1989 Perron paper. Therefore, we can reject the null hypothesis of a unit root when using the AO-C model with a break at time 4358.

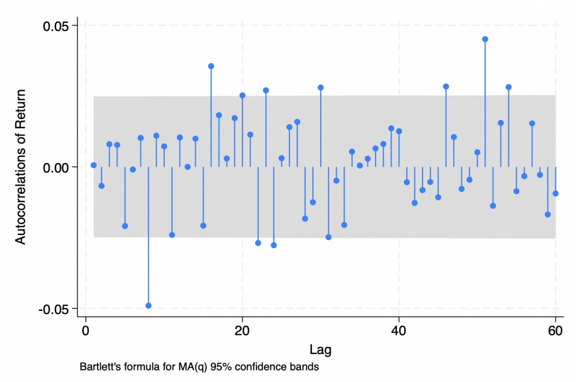
Model	Criteria	Break	EXP Test Statistic	Critical Values
2	1	3860	-0.212	2.61 (1%), 1.28 (5%), 0.74 (10%)
2	2	3860	-0.189	2.61 (1%), 1.28 (5%), 0.74 (10%)
3	1	4358	39.834 ***	4.57 (1%), 2.79 (5%), 2.15 (10%)
3	2	4358	39.205 ***	4.57 (1%), 2.79 (5%), 2.15 (10%)

Figure 9: Yabu-Perron Results

Model	Criteria	khat	Test statistic	Lambda	Critical Values
3	1	8	-4.587**	0.7	-4.75 (1%), -4.18 (5%), -3.86(10%)
3	2	0	-4.656**	0.7	-4.75 (1%), -4.18 (5%), -3.86(10%)

Figure 10: Kim-Perron Results

Next, the series is examined for ARCH effects. Since the plot of the returns exhibits signs of showing heteroskedasticity, it will be helpful to also examine the autocorrelation and partial autocorrelation functions of the NVDA stock returns. As seen from Figures 11 and 12 below, the autocorrelation and partial autocorrelation functions both exhibit spikes at lag 8. These spikes may be significant and could be an indicator of ARCH effects up to lag 8, so this must be tested statistically for any significant ARCH effects in the data.



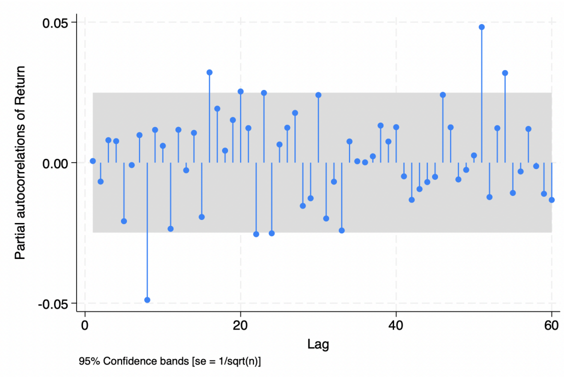


Figure 11: Autocorrelation of Returns

Figure 12: Partial Autocorrelation of Returns

To statistically test for ARCH effects in the returns of the NVDA stock, the Engle LM (Lagrange Multiplier) method is used to test for autocorrelation in the squared residuals of the mean equation. As seen from the results in Table 1 in the Appendix, since the p-value is statistically significant at all lag levels, the test shows that there is evidence of autocorrelation in the squared residuals at all lag levels. This implies autocorrelation in the variance of NVDA stock returns. To confirm the Engle LM test, the test will also be conducted by hand. To do this, a regression is estimated, the residuals of the mean equation are extracted and squared, then an autogregressive model is estimated on the squared residuals. Finally, statistical testing is done on the coefficients on the AR model to determine whether they are jointly significant. As seen from the regression in Table 2 in the Appendix, the coefficients on the lagged squared residuals have statistically significant p-values. In testing if the coefficients on the lagged squared residuals are jointly significant, Engle used the chi-square version of the test, where the chi-square test statistic is equal to (R^2)N. As seen from Table 3 in the Appendix, the p-value on this test is much less than 0.05. Therefore the null hypothesis of “no ARCH" is rejected for the series, and the series exhibits ARCH effects. The Ljung Box test is also used on the residuals of the mean equation to confirm these results. This test investigates whether a variable is white noise, as white noise variables can not be correlated. The results of the Ljung-Box test, as shown in Table 4 in the Appendix, indicate that there is volatility clustering and significant autocorrelation since the p-value is 0. There is evidence of ARCH effects of length at least equal to 1, therefore an ARCH or GARCH model would be most useful to model the series.

To find the optimal lag length to include in the ARCH model, the AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion) are used. As seen from Figure 13 below, the AIC finds the optimal lag length to be 10, while the BIC finds the optimal lag length to be 8. The lag length chosen here is 8, as this is what was indicated from the autocorrelation and partial autocorrelation functions, and because the BIC is known to find a more parsimonious model.

Lags	AIC	BIC
1	-23588.18	-23567.97
2	-23880.61	-23853.68
3	-24150.33	-24116.66
4	-24268.44	-24228.04
5	-24493.25	-24446.11
6	-24573.30	-24519.43
7	-24615.37	-24554.76
8	-24703.68	**-24636.33**
9	-24703.82	-24629.75
10	**-24710.02**	-24629.21

Figure 13: AIC and BIC Lag Selection

An ARCH(q) model consists of two equations:

1) A mean equation $Y_t = B + E_t$
2) A variance equation $E_t = (a_0 + a_1E^2_{t-1} + a_2E^2_{t-2} + \dots + a_pE^2_{t-q})^{1/2}u_t$, where:

- Conditional variance = $V(Y_t \mid \Omega_{t-1}) = a_0 + a_1E^2_{t-1} + a_2E^2_{t-2} + \dots + a_pE^2_{t-q}$
- Unconditional variance = $V(Y) = a_0 / (1 - a_1 - a_2 - a_3 - a_4 - a_5 - a_6 - a_7 - a_8)$.

Therefore, as seen from Figure 13 and Table 5 in the Appendix, the chosen model using the lowest BIC of ARCH(8) has the equations:

1) $Y_t = 0.0019 + E_t$
2) $E_t = (0.0003 + 0.0901E^2_{t-1} + 0.0649E^2_{t-2} + 0.1714E^2_{t-3} + 0.0933E^2_{t-4} + 0.1793E^2_{t-5} + 0.1334E^2_{t-6} + 0.0799E^2_{t-7} + 0.1512E^2_{t-8})^{1/2}u_t$

Where the conditional variance is:

- $V(Y_t \mid \Omega_{t-1}) = 0.0003 + 0.0901E^2_{t-1} + 0.0649E^2_{t-2} + 0.1714E^2_{t-3} + 0.0933E^2_{t-4} + 0.1793E^2_{t-5} + 0.1334E^2_{t-6} + 0.0799E^2_{t-7} + 0.1512E^2_{t-8})$

And the unconditional variance is:

- $V(Y) = 0.0003 / (1- 0.0901- 0.0649- 0.1714- 0.0933- 0.1793- 0.1334- 0.0799- 0.1512)$

For variance to be stationary, all coefficients a_i must be $-1 < a_i < 1$. And the sum of the coefficients must be less than one. This ARCH(8) model is stationary because each coefficient from the estimated ARCH model is less than one, and because the sum of coefficients = 0.9633 < 1 so the variance of the estimated model is stationary. According to the significance test in Table 6 in the Appendix, the sum of the coefficients is statistically significant from 1 since the p-value of this test is less than 0.05. The variance equation in an ARCH(q) process is simply an AR(q) process on the variance rather than the mean equation. While this model is stationary, we must also test for any remaining volatility clustering in the residuals. As seen from Table 7 in the Appendix, the Ljung-Box Q^2 test on the residuals of the ARCH(8) mean equation indicates that there is no significant volatility clustering left over in the residuals because the p-value is greater than 0.05. In other words, this ARCH(8) model is able to capture the vast majority of the conditional heteroskedasticity.

Next we will look at the GARCH (Generalized Autoregressive Conditional Heteroskedasticity models). While the ARCH model is able to model financial data, the generalized model is known to be an improvement from the ARCH models because it mimics an infinite order ARCH model in the same way that an invertible MA process is equivalent to an infinite order AR process. In addition, the GARCH model does not require as many lags in the variance equation as the ARCH model. GARCH(p,q) models have the following variance equations:

$$Y_t = B + E_t$$

$$E_t = \sigma_t u_t \text{ where } u_t \sim N(0,1).$$

$$\sigma_t^2 = [a_0 + a_1 E^2_{t-1} + a_2 E^2_{t-2} + \ldots + a_p E^2_{t-p}] + [y_1 \sigma^2_{t-1} + y_2 \sigma^2_{t-2} + \ldots + y_q \sigma^2_{t-q}]$$

We will try a few GARCH(p,q) models. In practice, it is common to use values of p and q where p <=2 and q<=2. The fit of these GARCH models to the data will be evaluated using the AIC and BIC information criterion. According to Figure 14 below, the AIC and BIC both choose the optimal p to be p=1, and the optimal q to be q=1. These values of p and q display the smallest values for AIC and BIC. As a result, we will select the GARCH(1,1) model to estimate the NVDA stock returns with.

ARCH(p)	GARCH(q)	AIC	BIC
1	0	-23588.18	-23567.97
1	1	**-24768.51**	**-24741.57**
1	2	-24650.41	-24623.47
2	0	-23589.66	-23569.46
2	1	-24733.23	-24706.30
2	2	-24590.61	-24563.68

Figure 14: Model Selection Using Information Criterion

As seen in Table 9 in the Appendix of our estimated GARCH(1,1) model on the NVDA stock returns, this GARCH(1,1) model has the following equations:

$$Y_t = 0.002 + E_t$$

$$E_t = \sigma_t u_t$$

$$\sigma_t^2 = 0.00001 + 0.0615E^2_{t-1} + 0.9341\sigma^2_{t-1}$$

After estimating the model, some post-estimation tests are conducted. In order for the model to be stationary, the coefficients must all be less than 1. This is confirmed from the estimated model in Table 8 of the Appendix. Additionally, the sum of the coefficients must sum to less than 1. This is also confirmed by Table 8 because the sum of coefficients: = 0.0614679 + 0.9341273 = 0.9955 < 1. Finally, we must test whether this is significantly different from 1 in order to confirm that the model is stationary. As seen from Table 9 in the Appendix, the sum is confirmed to be significantly different from 1 because the p-value of the test below is less than 0.05 so the estimated GARCH(1,1) model is stationary. Next, the Ljung-Box Q^2 test is used on the estimated GARCH(1,1) model to test for any volatility clustering left over in the residuals. If there is still volatility clustering leftover, a better model should be estimated. As seen from Table 10 in the Appendix, the Ljung-Box Q^2 test on the GARCH(1,1) indicates that there is no significant volatility clustering left over in the residuals because the p-value is greater than 0.05. In other words, this GARCH(1,1) model is able to capture the majority of conditional heteroskedasticity.

Lastly, this paper will examine the series for leverage effect using the asymmetrical GJR-GARCH model, which can capture asymmetric responses to unexpected events. A leverage effect occurs when volatility rises when returns fall, and declines when returns rise. An asymmetric response occurs when the volatility rises quickly and unexpectedly, and does not decrease as fast as it increases. The GJR-GARCH model decomposes the effect of E_{t-1}^2 from the GARCH(1,1) equation into the sum of two different effects via a dummy-variable interaction:

$$\sigma_t^2 = a_0 + a_1E_{t-1}^2 + a_2D_{t-1}E_{t-1}^2 + y_1\sigma_{t-1}^2$$

$$D_{t-1} = \{\ 1, \text{if } E >= 0; \quad 0, \text{otherwise.}\ \}$$

Thus, it can be seen from Table 11 in the Appendix that the equation for our estimated GJR-GARCH(1,1) is as follows:

$$\sigma_t^2 = 0.00001 + 0.0993E_{t-1}^2 - 0.0615D_{t-1}E_{t-1}^2 + 0.9273\sigma_{t-1}^2$$

The Sign Bias test is conducted to determine if there is a leverage effect. Since the a_1 coefficient 0.0993 is significant, as seen in Table 11, this shows that there is a leverage effect present in the series, and that the sign of the current period shock is helpful in predicting the conditional volatility of the returns.

To check if this model is stationary, the coefficients must all be less than 1. This is confirmed from the estimated model in Table 11. Additionally, the sum of the coefficients must sum to less than 1. This is also confirmed by Table 11 because the sum of coefficients = 0.995 < 1. Finally, we must test whether this is significantly different from 1 in order to confirm that the model is stationary. As seen from Table 12 in the Appendix, the sum is confirmed to be significantly different from 1 because the p-value of the test below is less than 0.05 so the estimated GJR-GARCH(1,1) model is stationary. Next, the Ljung-Box Q^2 test is used on the model to test for any volatility clustering left over in the residuals. As seen from Table 13 in the Appendix, the Ljung-Box Q^2 test on the GJR-GARCH(1,1) indicates that there is no significant volatility clustering left over in the residuals because the p-value is greater than 0.05. In other words, this model is able to capture the vast majority of the conditional heteroskedasticity.

After estimating the models, we will compare the three models we have estimated and evaluate them using the AIC and BIC information criterion to determine which is the best fitting model for the NVDA stock returns. As seen in Figure 15 below, which summarizes the results of the models we estimated, all coefficients on all three models are significant at the 1% significance level. In selecting the best model, Figure 16 shows that the min AIC and BIC both select the GJR-GARCH(1,1) model as the best fitting model. Furthermore, in examining the correlation table below in Figure 17, it can be seen that the predicted conditional variances from these models are highly correlated.

	(1) Return	(2) Return	(3) Return
Return			
_cons	**0.00192*** **	**0.00200*** **	**0.00168*** **
ARCH			
L.arch	**0.0901*** **	**0.0615*** **	**0.0993*** **
L2.arch	**0.0649*** **		
L3.arch	**0.171*** **		
L4.arch	**0.0933*** **		
L5.arch	**0.179*** **		
L6.arch	**0.133*** **		
L7.arch	**0.0799*** **		
L8.arch	**0.151*** **		
L.garch		**0.934*** **	**0.927*** **
L.tarch			**-0.0615*** **
_cons	**0.000279*** **	**0.0000119*** **	**0.0000133*** **
N	**6212**	**6212**	**6212**

* p<0.05, ** p<0.01, *** p<0.001

Figure 15: Results of the Three Estimated Models

Akaike's information criterion and Bayesian information criterion

Model	N	ll(null)	ll(model)	df	AIC	BIC
ARCH	6,212	.	12361.84	10	-24703.68	-24636.33
GARCH	6,212	.	12388.25	4	-24768.51	-24741.57
GJR_GARCH	6,212	.	12419.13	5	-24828.26	-24794.59

Note: BIC uses N = number of observations. See [R] IC note.

Figure 16: AIC and BIC Model Evaluation

	ARCH_hat	GARCH_~t	GJR_GA~t
ARCH_hat	1.0000		
GARCH_hat	0.8236	1.0000	
GJR_GARCH_~t	0.8227	0.9657	1.0000

Figure 17: Correlation of the Predicted Conditional Variance

Finally, we want to predict the future conditional variance using our selected GJR-GARCH(1,1) model. As seen in Figure 18 below, predicting an additional 500 rows of data shows that the conditional variance of the NVDA stock returns converges to 0. In other words, the conditional variance is shown to converge to the unconditional variance, which is as expected for stock returns.

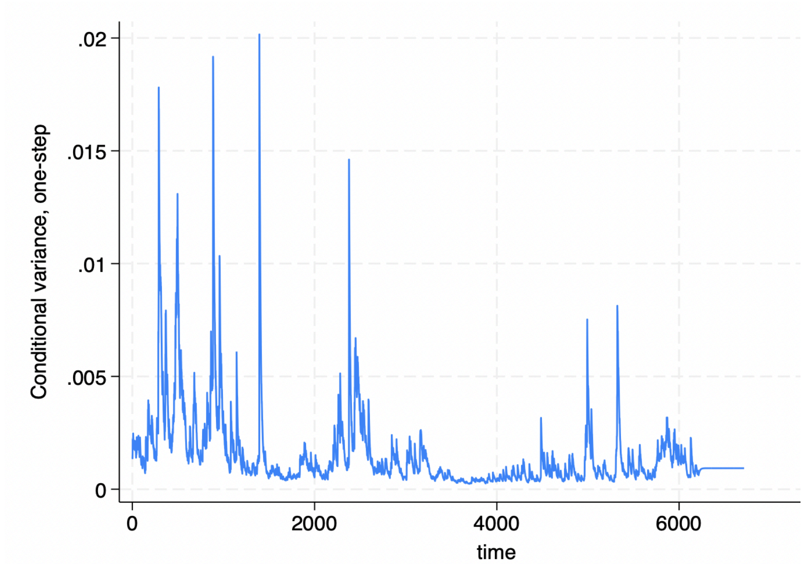


Figure 18: Predicting Future Variance

VI. Conclusion

In conclusion, when exploring the volatility of the adjusted daily closing price of the NVDA stock we first took the log, then the first difference of the log to get the stock returns, or percentage change of the prices. The graphs of the data indicate heteroskedasticity visually, and the large kurtosis value confirms a fat tailed distribution.

To conduct the unit root test, we used the DF-GLS test to determine the optimal lag, then tested the ADF test using the optimal lag to find that the price series has a unit root, while the return series (the differenced log of prices) does not have a unit root. This was confirmed with the KPSS test as well. However, since these tests do not account for structural breaks in the data, the Yabu-Perron and Kim-Perron process was also used to find a significant break at time 4358 and rejected the null hypothesis of a unit root to confirm prior tests that may have been biased without accounting for breaks in the data.

To test for ARCH effects, the Engle-LM test and Ljung-Box tests are used and find that ARCH effects are present in the data. An optimal lag of 8 is determined, making the best ARCH model an ARCH(8). To improve the model, we use the GARCH model and test for max p and q of 2, finding the optimal GARCH(p,q) to be a GARCH(1,1) model. Lastly, to further improve this, we estimate a GJR-GARCH(1,1) model to account for asymmetric leverage effects and find that this is the best fit model for the NVDA stock returns. Using this model to predict the next 500 rows of data, it is shown that the conditional variance of the NVDA behaves as expected and converges to unconditional variance.

VII. References

Collin, Emil. "Forecasting Volatility on Swedish Stock Returns." *Umea Universitet*, 2019, https://www.diva-portal.org/smash/get/diva2:1330964/FULLTEXT01.pdf.

Lestari, Karunia & Pasaribu, Udjianna & Indratno, Sapto. (2016). Modelling Stock Return Volatility using ARCH and GARCH Models. https://www.researchgate.net/publication/343392639_Modelling_Stock_Return_Volatility_using_ARCH_and_GARCH_Models

Nugroho, D B, et al. "Empirical performance of GARCH, GARCH-M, Gjr-GARCH and log-GARCH models for returns volatility." *Journal of Physics: Conference Series*, vol. 1307, no. 1, 2019, p. 012003, https://doi.org/10.1088/1742-6596/1307/1/012003.

Kanasro, Hakim Ali, et al. "Measurement of Stock Market Volatility through ARCH and GARCH Models: A Case Study of Karachi Stock Exchange." *Australian Journal of Basic and Applied Sciences*, 2009, pp. 3123–3127, https://www.researchgate.net/profile/Dr-Hakim-Kanasro/publication/228423218_Measurement_of_Stock_Market_Volatility_through_ARCH_and_GARCH_Models_A_Case_Study_of_Karachi_Stock_Exchange/links/5abb4804aca2722b0b7bdfe8/Measurement-of-Stock-Market-Volatility-through-ARCH-and-GARCH-Models-A-Case-Study-of-Karachi-Stock-Exchange.pdf.

Su, Y. C., et al. "GJR-GARCH model in value-at-risk of Financial Holdings." *Applied Financial Economics*, vol. 21, no. 24, 2011, pp. 1819–1829, https://doi.org/10.1080/09603107.2011.595677.

VIII. Tables

```
. quietly reg Return

. estat archlm, lags(1/10)
LM test for autoregressive conditional heteroskedasticity (ARCH)
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |         72.443               1                   0.0000
       2     |        151.382               2                   0.0000
       3     |        184.093               3                   0.0000
       4     |        212.660               4                   0.0000
       5     |        278.900               5                   0.0000
       6     |        289.317               6                   0.0000
       7     |        290.870               7                   0.0000
       8     |        293.846               8                   0.0000
       9     |        294.283               9                   0.0000
      10     |        301.229              10                   0.0000
---------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

Table 1: Engle LM test for ARCH effects

```
. predict e, resid
(1 missing value generated)

. gen e2 = e^2
(1 missing value generated)

. reg e2 L(1/10).e2

      Source |       SS           df       MS      Number of obs   =     6,202
-------------+----------------------------------   F(10, 6191)     =     31.60
       Model |  .008932043        10  .000893204   Prob > F        =    0.0000
    Residual |  .174969749     6,191  .000028262   R-squared       =    0.0486
-------------+----------------------------------   Adj R-squared   =    0.0470
       Total |  .183901792     6,201  .000029657   Root MSE        =    .00532

------------------------------------------------------------------------------
          e2 | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
          e2 |
         L1. |   .0690254   .0127017     5.43   0.000     .0441256    .0939252
         L2. |   .0861708   .0127317     6.77   0.000     .0612123    .1111293
         L3. |    .050492   .0127764     3.95   0.000     .0254458    .0755381
         L4. |   .0518506   .0127906     4.05   0.000     .0267765    .0769247
         L5. |    .095306   .0127991     7.45   0.000     .0702152    .1203967
         L6. |   .0360877   .0127991     2.82   0.005      .010997    .0611783
         L7. |   .0133327   .0127902     1.04   0.297    -.0117405    .0384058
         L8. |   .0188037    .012775     1.47   0.141    -.0062398    .0438473
         L9. |   .0069982   .0127297     0.55   0.583    -.0179564    .0319528
        L10. |   .0343297   .0126974     2.70   0.007     .0094383    .0592211
             |
       _cons |   .0007711   .0000789     9.78   0.000     .0006165    .0009257
------------------------------------------------------------------------------
```

Table 2: Engle LM Test by Hand for ARCH effects

```
. test L1.e2 L2.e2 L3.e2 L4.e2 L5.e2 L6.e2 L7.e2 L8.e2 L9.e2 L10.e2

 ( 1)  L.e2 = 0
 ( 2)  L2.e2 = 0
 ( 3)  L3.e2 = 0
 ( 4)  L4.e2 = 0
 ( 5)  L5.e2 = 0
 ( 6)  L6.e2 = 0
 ( 7)  L7.e2 = 0
 ( 8)  L8.e2 = 0
 ( 9)  L9.e2 = 0
 (10)  L10.e2 = 0

       F( 10,  6191) =   31.60
            Prob > F =    0.0000

. scalar teststat = e(r2)*e(N)

. display e(r2)*e(N)
301.22888

. display chi2tail(10,teststat)
8.548e-59
```

Table 3: Engle Chi-Square Test for ARCH effects

```
Portmanteau test for white noise
---------------------------------------
 Portmanteau (Q) statistic =  1175.8284
 Prob > chi2(40)           =     0.0000
```

Table 4: Ljung-Box Test for White Noise

ARCH family regression

Sample: 2 thru 6213 — Number of obs = 6212; Wald chi2(.) = .; Prob > chi2 = .

Log likelihood = 12361.84

Return	Coefficient	OPG std. err.	z	P>|z|	[95% conf.	interval]
Return						
_cons	.0019212	.0003064	6.27	0.000	.0013206	.0025217
ARCH						
arch						
L1.	.090117	.0097487	9.24	0.000	.0710099	.1092241
L2.	.0648575	.0113096	5.73	0.000	.0426912	.0870238
L3.	.1713533	.0098528	17.39	0.000	.152042	.1906645
L4.	.0932916	.0073856	12.63	0.000	.0788161	.1077672
L5.	.1792744	.0068307	26.25	0.000	.1658866	.1926623
L6.	.1333546	.0086711	15.38	0.000	.1163596	.1503497
L7.	.0798582	.0091957	8.68	0.000	.0618351	.0978813
L8.	.1511738	.0097583	15.49	0.000	.1320479	.1702996
_cons	.0002789	.0000105	26.57	0.000	.0002584	.0002995

Table 5: Estimated ARCH(8) Model

```
[ARCH]L.arch + [ARCH]L2.arch + [ARCH]L3.arch + [ARCH]L4.arch + [ARCH]L5.arch + [ARCH]L6.arch + [ARCH]L7.arch + [ARCH]L8.arch = 1

    chi2(  1) =    4.21
  Prob > chi2 =    0.0402
```

Table 6: ARCH(8) Stationary Test

```
. wntestq w2

Portmanteau test for white noise
---------------------------------------
 Portmanteau (Q) statistic =    24.3729
 Prob > chi2(40)           =     0.9755
```

Table 7: ARCH(8) Ljung-Box Q^2 Test

```
. arch Return, arch(1) garch(1) nolog

ARCH family regression

Sample: 2 thru 6213                             Number of obs     =       6212
                                                Wald chi2(.)      =          .
Log likelihood = 12388.25                       Prob > chi2       =          .

------------------------------------------------------------------------------
             |                 OPG
      Return | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
Return       |
       _cons |    .002003   .0003652     5.49   0.000     .0012873    .0027188
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .0614679   .0020349    30.21   0.000     .0574797    .0654562
             |
       garch |
         L1. |   .9341273   .0020886   447.26   0.000     .9300337    .9382208
             |
       _cons |   .0000119   7.68e-07    15.54   0.000     .0000104    .0000134
------------------------------------------------------------------------------
```

Table 8: Estimated GARCH(1,1)

```
 [ARCH]L.arch + [ARCH]L.garch = 1

           chi2(  1) =   21.65
         Prob > chi2 =    0.0000
```

Table 9: GARCH(1,1) Stationary Test

```
Portmanteau test for white noise
---------------------------------------
 Portmanteau (Q) statistic =    10.8810
 Prob > chi2(40)           =     1.0000
```

Table 10: GARCH(1,1) Ljung-Box Q^2 Test

```
ARCH family regression

Sample: 2 thru 6213                             Number of obs     =      6212
                                                Wald chi2(.)      =         .
Log likelihood = 12419.13                       Prob > chi2       =         .

------------------------------------------------------------------------------
             |                 OPG
      Return | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
Return       |
       _cons |   .0016767   .0003821     4.39   0.000     .0009277    .0024257
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .0992924    .003424    29.00   0.000     .0925815    .1060034
             |
       tarch |
         L1. |  -.0614546   .0032504   -18.91   0.000    -.0678252    -.055084
             |
       garch |
         L1. |   .9272999   .0022226   417.21   0.000     .9229437    .9316562
             |
       _cons |   .0000133   9.09e-07    14.62   0.000     .0000115    .0000151
------------------------------------------------------------------------------
```

Table 11: GJR-GARCH(1,1) Results

```
 ( 1)  [ARCH]L.arch + [ARCH]L.tarch + [ARCH]L.garch = 1

           chi2(  1) =  343.79
         Prob > chi2 =    0.0000
```

Table 12: GJR-GARCH(1,1) Stationary Test

```
Portmanteau test for white noise
---------------------------------------
 Portmanteau (Q) statistic =    12.1365
 Prob > chi2(40)           =     1.0000
```

Table 13: GJR-GARCH(1,1) Ljung-Box Test

IV. Do File:

```
drop _all
clear all
import excel "/Users/belladavies/Desktop/SENIOR YEAR/CAS EC 405/Paper/NVDA.xlsx",
sheet("NVDA") firstrow
gen time = _n
tsset time

rename AdjClose Price
gen Ln_Price = ln(Price)
gen Return = d.Ln_Price

* Initial Graphs and Statistics
tsline Price
tsline Ln_Price
tsline Return
histogram Return, bin(60) normal
qnorm Return

tabstat Price Return, statistics(mean, sd, skewness, kurtosis)

* GLS to find # Lags
dfgls Price
dfgls Return

* Augmented Dickey Fuller Test
dfuller Price, lags(11) trend regress
dfuller Price, lags(32) trend regress

dfuller Return, lags(26) trend regress
dfuller Return, lags(33) trend regress

* KPSS Test
kpss Price
kpss Return

* ACF & PACF
ac Return, lags(60)
pac Return, lags(60)
```

```
* Engle LM Test for ARCH Effects
quietly reg Return
estat archlm, lags(1/10)

* Engle LM Test "by hand"
quietly reg Return
predict e, resid
gen e2 = e^2
reg e2 L(1/10).e2

test L1.e2 L2.e2 L3.e2 L4.e2 L5.e2 L6.e2 L7.e2 L8.e2 L9.e2 L10.e2
scalar teststat = e(r2)*e(N)
display e(r2)*e(N)
display chi2tail(10,teststat)

* Ljung Box Test
wntestq e2

* Lag-selection using AIC and BIC
forvalues lags=1/10{
        display "Lags = " `lags'
        quietly arch Return, arch(1/`lags')
        estat ic
}

* Estimating ARCH(8) model
arch Return, arch(1/8) nolog
estimates store ARCH


* Ljung-Box Q^2 test on the ARCH(8) squared residuals
predict w, resid
predict ARCH_hat, variance
gen w2 = (w^2)/ARCH_hat
wntestq w2

* Testing ARCH(8) for stationary
display [ARCH]L1.arch + [ARCH]L2.arch + [ARCH]L3.arch + [ARCH]L4.arch +
[ARCH]L5.arch + [ARCH]L6.arch + [ARCH]L7.arch + [ARCH]L8.arch
```

```
test [ARCH]L1.arch + [ARCH]L2.arch + [ARCH]L3.arch + [ARCH]L4.arch + [ARCH]L5.arch
+ [ARCH]L6.arch + [ARCH]L7.arch + [ARCH]L8.arch = 1

* Trying GARCH models and selecting a model using AIC and BIC
quietly arch Return, arch(1) nolog
estat ic
quietly arch Return, arch(1) garch(1) nolog
estat ic
quietly arch Return, arch(1) garch(2) nolog
estat ic
quietly arch Return, arch(2) nolog
estat ic
quietly arch Return, arch(2) garch(1) nolog
estat ic
quietly arch Return, arch(2) garch(2) nolog
estat ic

* Estimating GARCH(1,1)
arch Return, arch(1) garch(1) nolog
estimates store GARCH

* Testing GARCH(1,1) for stationary
display [ARCH]L1.arch + [ARCH]L1.garch
test [ARCH]L1.arch + [ARCH]L1.garch = 1

* Ljung-Box Q^2 test on the GARCH(1,1) squared residuals
predict w, resid
predict GARCH_hat, variance
gen w2 = (w^2)/GARCH_hat
wntestq w2

* Estimating GJR-Garch(1,1)
arch Return, arch(1) garch(1) tarch(1) nolog
estimates store GJR_GARCH

* Testing GJR-GARCH(1,1) for stationary

display [ARCH]L1.arch + [ARCH]L1.tarch + [ARCH]L1.garch
test [ARCH]L1.arch + [ARCH]L1.tarch + [ARCH]L1.garch = 1
```

```
* Ljung-Box Q^2 test on the GJR-GARCH(1,1) squared residuals
predict s, resid
predict GJR_GARCH_hat, variance
gen s2 = (s^2)/GJR_GARCH_hat
wntestq s2

* Summarizing the results of the 3 models
esttab ARCH GARCH GJR_GARCH, not
estimates stats ARCH GARCH GJR_GARCH
correlate ARCH_hat GARCH_hat GJR_GARCH_hat

* Predicting future conditional variance with GJR-GARCH
arch Return, arch(1) garch(1) tarch(1) nolog
tsappend, add(500)
predict varhat, variance
tsline varhat
```