

Q.2.

(i) [22, 24, 24, 28, 28, 28, 28, 26, 26, 26, 21, 19, 20, 29, 22, 24, 24, 24, 23, 24, 20, 16, 10, 10, 8, 11, 6, 9, 9, 12, 15, 19]

(2) 160 bits because $2^5 = 32$ so we need 5 bits per signal and total $32 \times 5 = 160$ bits

(3) Differences: 5.75, -0.5, 0, 1, 0, 0, -0.75, 0.25, 0, 0, -1.25, -0.5, 0.25, 0, 0.5, 0.5, 0, 0, -0.25, 0.25, -1, -1, -1.5, 0, -0.5, 0.75, -1.25, 0.75, 0, 0.75, 0.75, 1.

max diff = 1, min diff: -1.5

$2^{3.58} = 12$, we have uniformly distributed levels between $[-1.5, 1]$ and 1 more level for the first signal so we need 4 bits per signal, and total $32 \times 4 = 128$ bits

(4) Compression ratio is $5/4$

(5) Huffman coding for the differences, so we have following chart. totally 103 bits for these signals

Q.3

Arithmetic Compression:

- ① Totally 8 types of different outcomes.
They are AAA: 0.512; AAB: 0.128; ABA: 0.128
ABB: 0.032; BAA: 0.128; BAB: 0.032; BBA: 0.032
BBB: 0.008

- ② Following arrangement:

0	
AAA	000
—	—
AAA 100	0.512 = .1000011
—	—
AAB 101	0.64 = .10100011
—	—
ABA 110	0.768 = .11000100
—	—
ABB 1100	0.8 = .11001100
—	—
BAA 1101	0.928 = .11101101
—	—
BAB 1110	0.96 = .11110101
—	—
BBA 11110	0.992 = .11111101
—	—
BBB 11111	1

- ③ Avg code length is $2+3+3+4+4+4+5+5/8 = 3.375$

- ④ Bits for message "ABA(3) BBA(5) ABB(4)
AAA(2) BBB(5)" = $3+5+4+2+5 = 19$ bits

- ⑤ By stopping the shrink when the interval bounds get too close can improve code length.

Q4

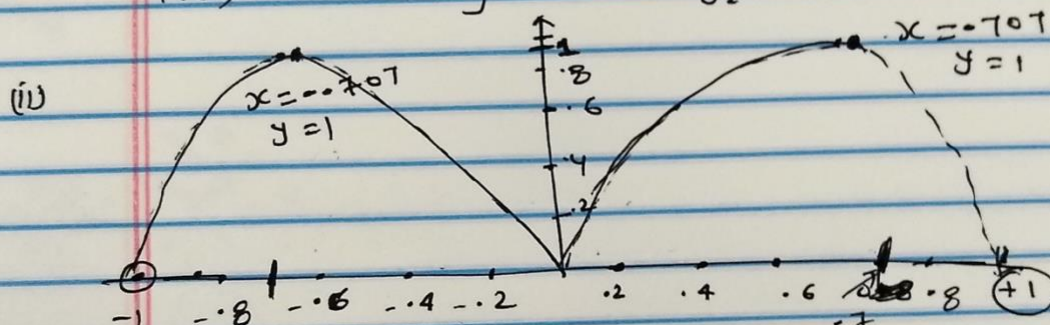
Entropy:

(i)

$$P(X) = x^2; \quad P(Y) = (1-x^2)$$

So

$$\begin{aligned} H(x) &= -\sum p_i \log p_i \\ &= -x^2 \log_2(x^2) - (1-x^2) \log_2(1-x^2) \\ &= -x^2 \log_2 x^2 - 1 + x^2 \log_2(1-x^2) \\ H(x) &= -2x^2 \log_2 x + x^2 \log_2(1-x^2) - 1 \end{aligned}$$



(iii) Minimum entropy can occur in the case where only one symbol is present in the entire gamut.

$$\begin{aligned} P(X) &= 1 & \text{or} & & P(Y) &= 1 \\ x^2 &= 1 & & & 1-x^2 &= 1 \\ x &= \pm 1 & & & \text{or} & & x = 0 \end{aligned}$$

So entropy will be minimum at $x = \pm 1, 0$

(iv) Entropy will be maximum when both probabilities are equal $P(X) = P(Y)$

$$\begin{aligned} 1-x^2 &= x^2 \\ x^2 &= \frac{1}{2} \Rightarrow x = \pm \frac{1}{\sqrt{2}} \\ x &= \pm 0.707 \end{aligned}$$