Introduction to Data Mining

Lecture #22: Dimensionality Reduction-2

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In This Lecture

- Learn how to answer queries using SVD
- Learn the motivation and definition of CUR decomposition, an alternative method for SVD
- Compare CUR and SVD

Outline

- ➡ □ **SVD Case Studies**
- □ CUR Decomposition

Case study: How to query?

- Q: Find users that like ‘Matrix’
- A: Map query into a ‘concept space’ – how?

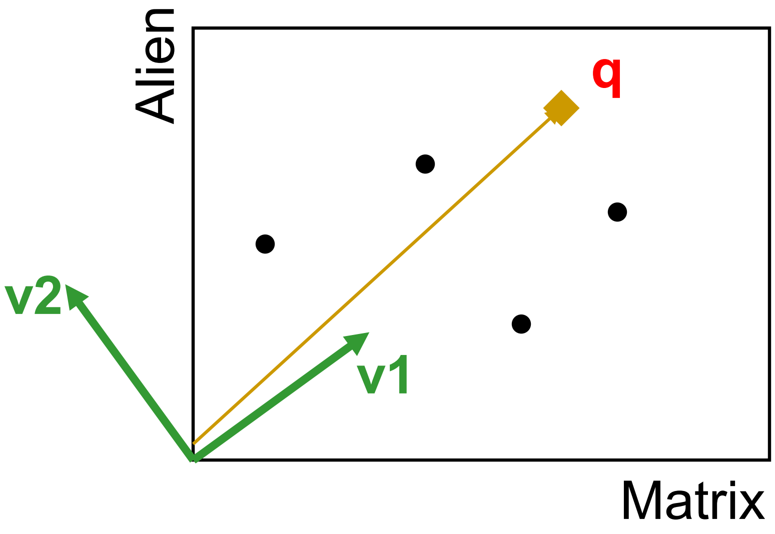
$$
\begin{array}{c|ccccc|}
 & \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \\
\hline
\uparrow & \mathbf{1} & \mathbf{1} & \mathbf{1} & 0 & 0 \\
\text{SciFi} & \mathbf{3} & \mathbf{3} & \mathbf{3} & 0 & 0 \\
\downarrow & \mathbf{4} & \mathbf{4} & \mathbf{4} & 0 & 0 \\
 & \mathbf{5} & \mathbf{5} & \mathbf{5} & 0 & 0 \\
\uparrow & 0 & \mathbf{2} & 0 & \mathbf{4} & \mathbf{4} \\
\text{Romnce} & 0 & 0 & 0 & \mathbf{5} & \mathbf{5} \\
\downarrow & 0 & \mathbf{1} & 0 & \mathbf{2} & \mathbf{2} \\
\hline
\end{array}
\sim
\begin{bmatrix}
\mathbf{0.13} & 0.02 \\
\mathbf{0.41} & 0.07 \\
\mathbf{0.55} & 0.09 \\
\mathbf{0.68} & 0.11 \\
0.15 & \mathbf{-0.59} \\
0.07 & \mathbf{-0.73} \\
0.07 & \mathbf{-0.29}
\end{bmatrix}
\times
\begin{bmatrix}
\mathbf{12.4} & 0 \\
0 & \mathbf{9.5}
\end{bmatrix}
\times
\begin{bmatrix}
\mathbf{0.56} & \mathbf{0.59} & \mathbf{0.56} & 0.09 & 0.09 \\
0.12 & -0.02 & 0.12 & \mathbf{-0.69} & \mathbf{-0.69}
\end{bmatrix}
$$

Case study: How to query?

- Q: Find users that like ‘Matrix’
- A: Map query into a ‘concept space’ – how?

$$\mathbf{q} = \begin{array}{ccccc} \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \\ \hline 5 & 0 & 0 & 0 & 0 \end{array}$$

Project into concept space:
Inner product with each ‘concept’ vector $\mathbf{v}_i$



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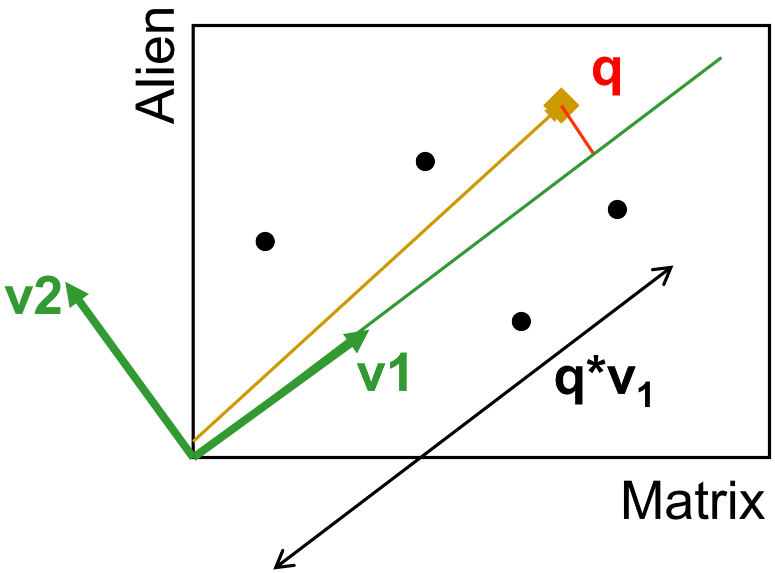


Case study: How to query?

Compactly, we have:

$$\mathbf{q}_{\text{concept}} = \mathbf{q}\ \mathbf{V}$$

$$\mathbf{q} = \begin{array}{ccccc} \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \\ \left[5 \right. & 0 & 0 & 0 & \left. 0 \right] \end{array} \mathbf{x} \underbrace{\begin{bmatrix} 0.56 & 0.12 \\ 0.59 & -0.02 \\ 0.56 & 0.12 \\ 0.09 & -0.69 \\ 0.09 & -0.69 \end{bmatrix}}_{\text{movie-to-concept similarities (V)}} \mathbf{=} \begin{bmatrix} \overset{\text{SciFi-concept}\ \downarrow}{2.8} & 0.6 \end{bmatrix}$$

Case study: How to query?

- **How would the user *d* that rated (‘Alien’, ‘Serenity’) be handled?**
  $d_{concept} = d\,V$

$$\mathbf{q} = \begin{array}{c} \begin{array}{ccccc} \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \end{array} \\ \begin{bmatrix} 0 & 4 & 5 & 0 & 0 \end{bmatrix} \end{array} \mathbf{x} \begin{bmatrix} 0.56 & 0.12 \\ 0.59 & -0.02 \\ 0.56 & 0.12 \\ 0.09 & -0.69 \\ 0.09 & -0.69 \end{bmatrix} = \begin{bmatrix} \overset{\text{SciFi-concept}}{\downarrow}\,5.2 & 0.4 \end{bmatrix}$$

movie-to-concept similarities (V)

Case study: How to query?

- **Observation:** User ***d*** that rated (‘*Alien*’, ‘*Serenity*’) will be **similar** to user **q** that rated (‘*Matrix*’), although ***d*** and **q** have **zero ratings in common**!

$$\begin{array}{rccccc} & \text{Matrix} & \text{Alien} & \text{Serenity} & \text{Casablanca} & \text{Amelie} \end{array}$$

$$\mathbf{d} = \begin{bmatrix} 0 & 4 & 5 & 0 & 0 \end{bmatrix} \dashrightarrow \begin{bmatrix} \overset{\substack{\text{SciFi-concept} \\ \downarrow}}{5.2} & 0.4 \end{bmatrix}$$

$$\mathbf{q} = \begin{bmatrix} 5 & 0 & 0 & 0 & 0 \end{bmatrix} \dashrightarrow \begin{bmatrix} 2.8 & 0.6 \end{bmatrix}$$

Zero ratings in common

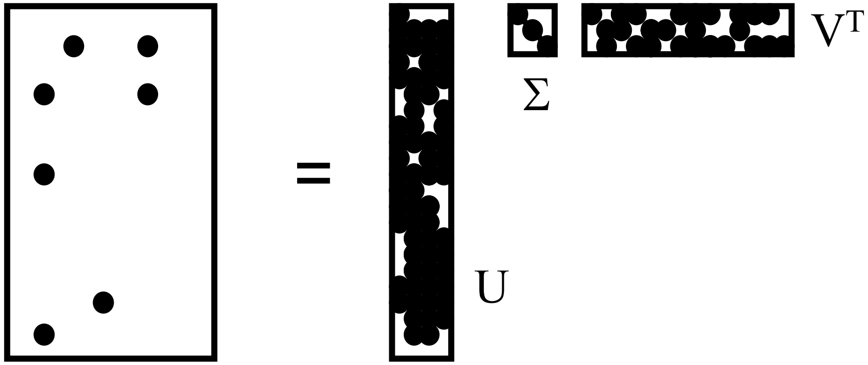
Similarity ≠ 0

SVD: Pros and Cons

- + **Optimal low-rank approximation** in terms of Frobenius norm (or Euclidean distance)
- - **Interpretability problem:**
  - ❑ A singular vector specifies a linear combination of all input columns or rows
- - **Lack of sparsity:**
  - ❑ Singular vectors are **dense!**

Frobenius norm:

$$\|X\|_F = \sqrt{\Sigma_{ij} X_{ij}^2}$$



Outline

☑ SVD Case Studies

➡ □ **CUR Decomposition**

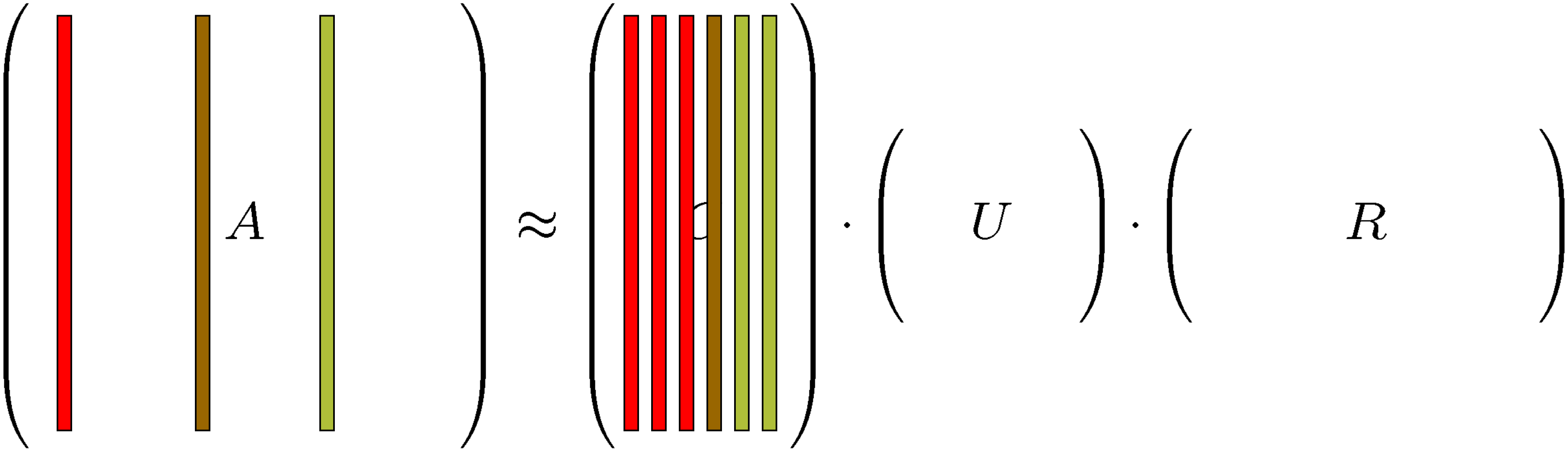
CUR Decompositior

Frobenius norm: $\|X\|_F = \sqrt{\Sigma_{ij} X_{ij}^2}$

- Goal: Express A as a product of matrices C,U,R

  Make $\|A-C\cdot U\cdot R\|_F$ small

- "Constraints" on C and R:



A C U R

CUR Decompositior

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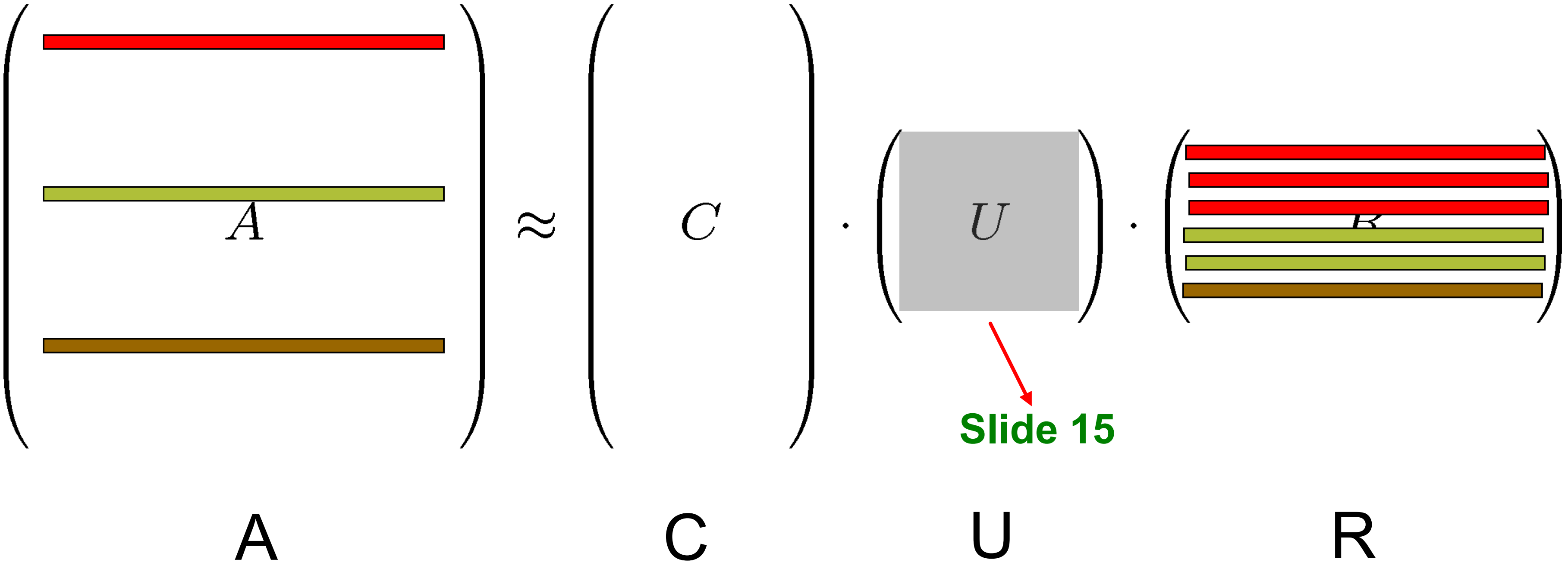


CUR: How it Works

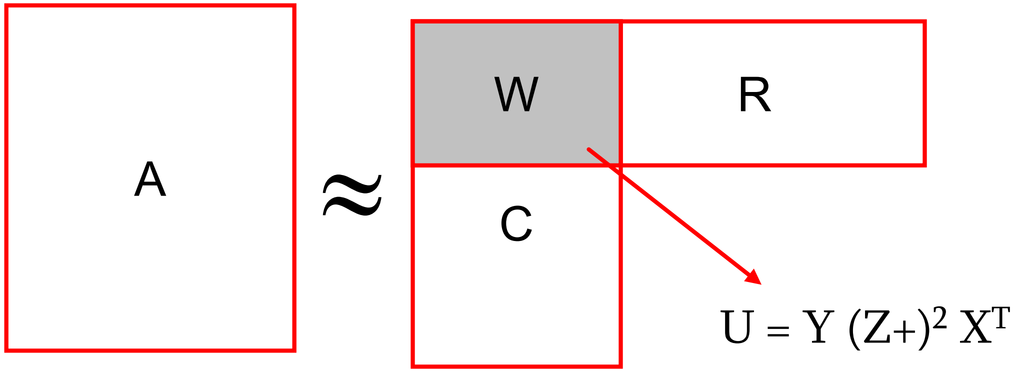
- **Sampling columns (similarly for rows):**

```
**Input**: matrix $\mathbf{A} \in \mathbb{R}^{m \times n}$, sample size $c$
**Output**: $\mathbf{C}_d \in \mathbb{R}^{m \times c}$
    for $x = 1 : n$        [column distribution]
        $P(x) = \sum_i \mathbf{A}(i,x)^2 / \sum_{i,j} \mathbf{A}(i,j)^2$
    for $i = 1 : c$        [sample columns]
        Pick $j \in 1 : n$ based on distribution $P(x)$
        Compute $\mathbf{C}_d(:,i) = \mathbf{A}(:,j) / \sqrt{cP(j)}$
```

Note this is a randomized algorithm; same column can be sampled more than once

Computing U

- Let **W** be the "intersection" of sampled columns **C** and rows **R**
  - Let SVD of **W** = **X Z Y**^{T}
- **Then: U = Y (Z$^+$)^2 X^T**
  - Z^+: **reciprocals of non-zero singular values:** $Z^+_{ii} = 1/ Z_{ii}$
  - Z^+ is called "**pseudoinverse**" of Z



CUR: Provably good approx. to SVD

- **If we carefully choose # of columns and rows,**

SVD error

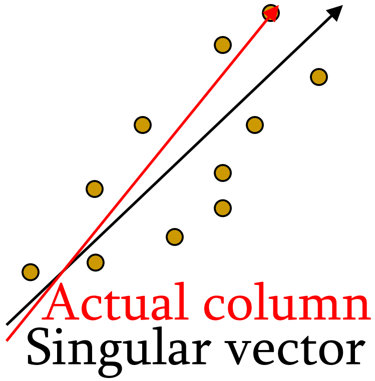
$$\|A - CUR\|_F \le (2+\epsilon)\,\|A - A_k\|_F$$

CUR error

- with probability 98%

In practice:
Pick 4*k* cols/rows
for a “rank-k” approximation

CUR: Pros & Cons



+ Easy interpretation

- Since the basis vectors are actual columns and rows

+ Sparse basis

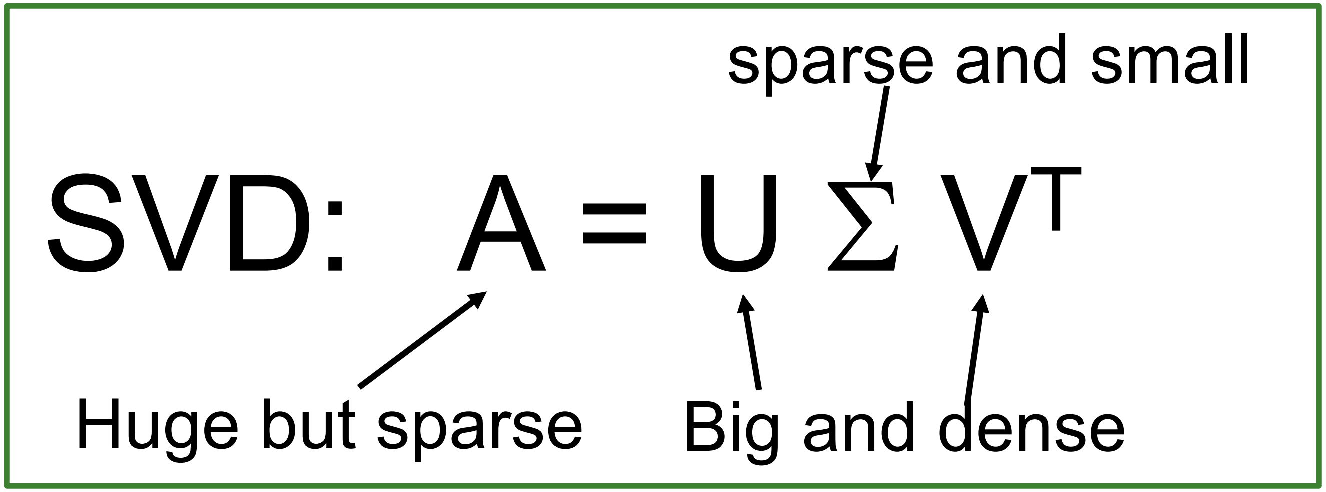
- Since the basis vectors are actual columns and rows

– Not the ‘optimal’ solution for minimizing error

– Duplicate columns and rows

- Columns of large norms will be sampled many times
  - CMD method (Sun et al., 2007) solves the problem

SVD vs. CUR

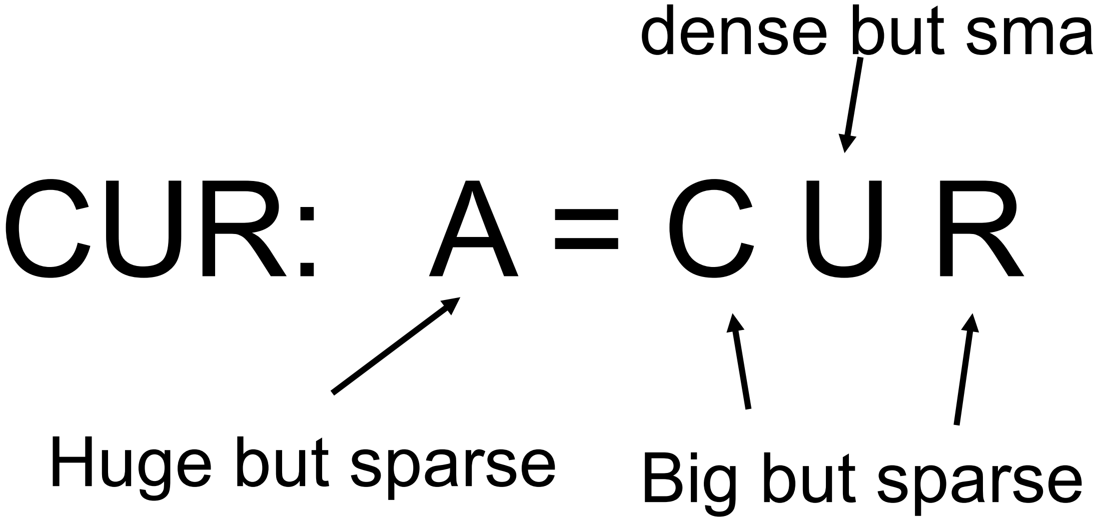


sparse and small

SVD: $A = U \Sigma V^T$

Huge but sparse

Big and dense



dense but small

CUR: $A = C U R$

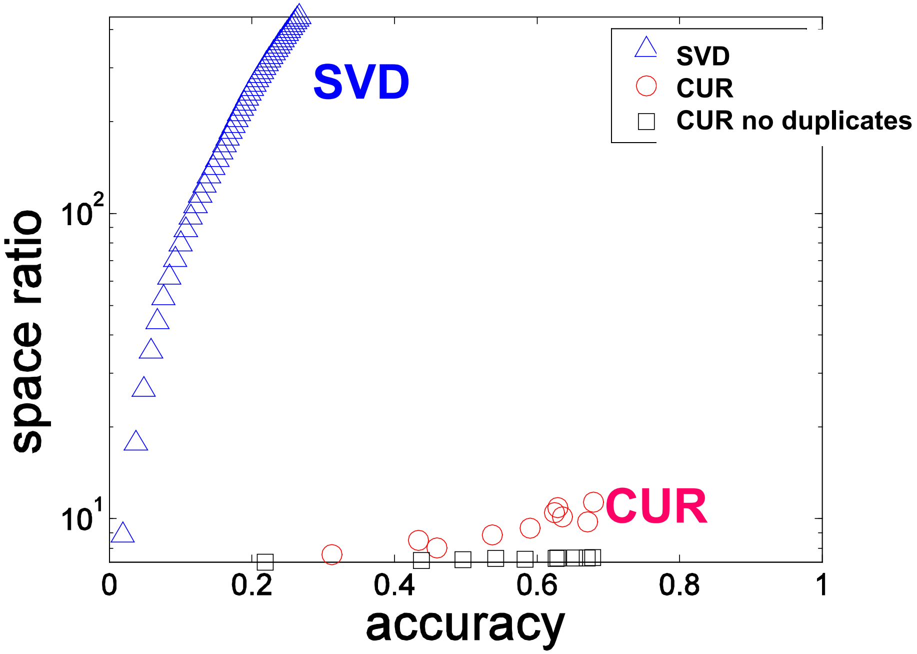
Huge but sparse

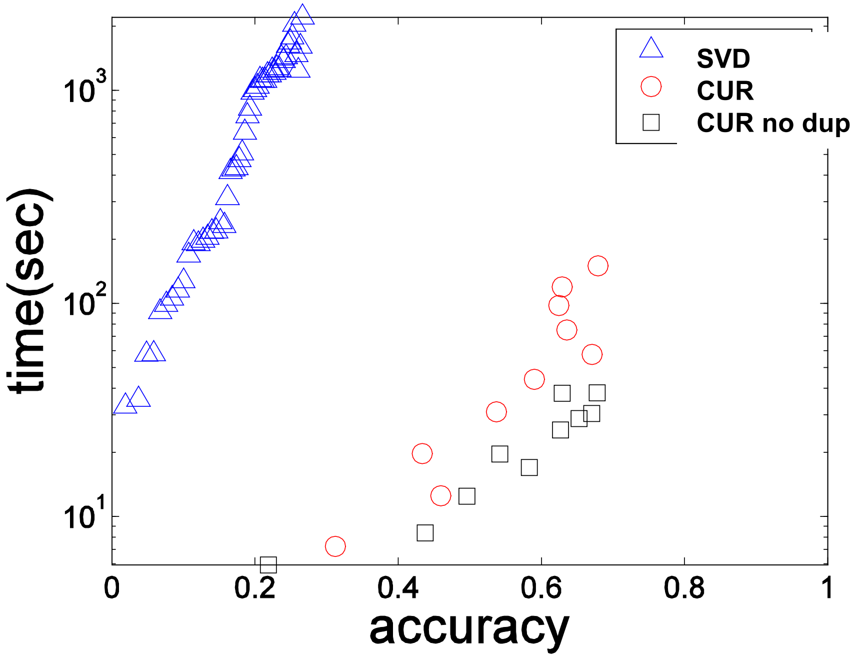
Big but sparse

SVD vs. CUR: Simple Experiment

- **DBLP bibliographic data**
  - Author-to-conference big sparse matrix
  - A_{ij}: Number of papers published by author *i* at conference *j*
  - 428K authors (rows), 3659 conferences (columns)
    - **Very sparse**
- **Want to reduce dimensionality**
  - How much time does it take?
  - What is the reconstruction error?
  - How much space do we need?

Results: DBLP- big sparse matrix





- **Accuracy:**
  - 1 – relative sum squared errors
- **Space ratio:**
  - #output matrix entries / #input matrix entries
- **CPU time**

SVD는 hyper parameter를 정해줘야 할 필요가 없고 error bound가 확실하므로 쓰기 훨씬 편하다.

Sun, Faloutsos: *Less is More: Compact Matrix Decomposition for Large Sparse Graphs*, SDM '07.

Further Reading: CUR

- Drineas et al., *Fast Monte Carlo Algorithms for Matrices III: Computing a Compressed Approximate Matrix Decomposition*, SIAM Journal on Computing, 2006.

- J. Sun, Y. Xie, H. Zhang, C. Faloutsos: *Less is More: Compact Matrix Decomposition for Large Sparse Graphs*, SDM 2007

- *Intra- and interpopulation genotype reconstruction from tagging SNPs*, P. Paschou, M. W. Mahoney, A. Javed, J. R. Kidd, A. J. Pakstis, S. Gu, K. K. Kidd, and P. Drineas, Genome Research, 17(1), 96-107 (2007)

- *Tensor-CUR Decompositions For Tensor-Based Data*, M. W. Mahoney, M. Maggioni, and P. Drineas, Proc. 12-th Annual SIGKDD, 327-336 (2006)

Conclusion

POPULAR SCIENCE
THE FUTURE NOW
THE CONTROL CENTERS
Using Data to Feed the World, Solve Cold Cases, Battle Malware, Predict Our Fate
OFFICER ALGORITHM
Can a Crime Be Prevented Before It Begins?
NEW WAYS OF SEEING
A Gallery of Extraordinary Infographics
PLUS
Juan Enriquez Reprograms Life
James Gleick Unsplits the Bit
AND Lawrence Weschler Questions the Cloud
SPECIAL ISSUE
DATA IS POWER
HOW INFORMATION IS DRIVING THE FUTURE

Data contains value and knowledge

What did we learn?

- **We learned to mine different types of data:**
  - Data is high dimensional
  - Data is a graph
  - Data is infinite/never-ending

- **We learned to use different models of computation:**
  - MapReduce
  - Streams and online algorithms
  - Single machine in-memory

What did we learn?

- **We learned to solve real-world problems:**
  - Recommender systems
  - Market basket analysis
  - Spam detection
  - Duplicate document detection
  - Ranking in graphs
  - Community detection
  - Increasing the revenue of search engines
  - Trending topics in social network news feeds

What did we learn?

- **We learned various "tools":**
  - Linear algebra (SVD, Rec. Sys., Communities)
  - Pruning technique (frequent itemsets)
  - Hashing (LSH, Bloom filters)
  - Link Analysis (PageRank, HITS, Random Walk)
  - Graph Algorithms (betweenness, BFS)
  - Clustering (k-means, CURE)
  - Sampling (reservoir sampling)
  - Greedy Algorithm (Balance)

Final Remark

- This course covered extensive materials; thanks for all the hard works (quizzes, homeworks, exams, questions, etc.)
- The knowledge you learned from this course would be helpful in many ways
  - E.g., which subjects to study more; understanding and evaluating other people’s work
- Takeaway points
  - Summarize each topic using few sentences so that you can really use them (e.g., Bloom Filter)
    - 1) What is the problem to solve? 2) What is the main idea? 3) Result?
  - Learn the way how each problem is solved
    - To apply the similar techniques to solve other problems in the future

Thank You!