Simplified Model for Simulation

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- **Notations**
  - i denotes each salesperson and t denotes each month.
  - $P_{it} \in \{1,2\}$: salesperson i's position at time t.
- **State Variables**
  - $N_{it} = (N_{it}^1, N_{it}^2)'$: the numbers of level-p salesforce in the salesperson i's team.
    * Each salesman can add only level-1 salesmen in her team. Promotion and demotion in a team are modeled as a random process.
      · Level-1 salesperson: $(N_{i,t+1}^1, N_{i,t+1}^2)' = (\epsilon_{it}^{1Stay} + (N_{it}^2 - \epsilon_{it}^{2Stay}), \epsilon_{it}^{2Stay} + (N_{it}^1 - \epsilon_{it}^{1Stay}))'$ where $\epsilon_{it}^{1Stay} \sim B(N_{it}^1, p^{1stay})$ and $\epsilon_{it}^{2Stay} \sim B(N_{it}^2, p^{2stay})$ when $Q_t = 3$ and $\epsilon_{it}^{2Stay} = N_{it}^2$ otherwise.
      · Level-2 salesperson: $(N_{i,t+1}^1, N_{i,t+1}^2)' = (\epsilon_{it}^{1Stay} + n_{it} + (N_{it}^2 - \epsilon_{it}^{2Stay}), \epsilon_{it}^{2Stay} + (N_{it}^1 - \epsilon_{it}^{1Stay}))'$
  - Q_t: month type within a quarter. $Q_t = 1$ if t is the start of the quarter and $Q_t = Q_{t-1} + 1$ otherwise.
  - $s_{i,t-1}$: last month sales amount.
  - $S_{it} = s_{it-2} + s_{i,t-1}$: salesperson i 's accumulated sales for the last two months.
  - $s_{T(i),t-1}$: last month team sales amount.
  - $S_{T(i)t} = s_{T(i),t-2} + s_{T(i),t-1}$: salesperson i 's whole team's accumulated sales for the last two months.
- **Actions**
  - There are two different kinds of efforts: e^a and e^b
  - e_{it}^a :efforts to sell insurance contracts. This effort induces higher sales.
    * $s_{it} = s(e_{it}^a, \epsilon_{it}^a)$ where ϵ_{it}^a is a random shock which affects the sales s_{it}.
    * e_{it}^a is a discrete action variable.
  - e_{it}^b: effort to acquire new level-1 salesperson for her team, which leads to new addition of team members, n_{it}
    * $n_{it} = n(e_{it}^b, \epsilon_{it}^b)$ where ϵ_{it}^b denotes a random shock which affects the new addition of team members.
    * e_{it}^b is a discrete action variable.
- **Timing of the Model**
  - At the start of each month, a salesman observes current states and decides her effort levels, e_{it}^a and e_{it}^b.
  - The sales and addition of team members are realized during the month.
  - At the end of each month, the compensations are given and promotions happen based on her accumulated performance for the last three month.

- At the end of each quarter month (3, 6, 9, and 12), demotions occur based on her accumulated performance for the last three month.

- **Assumptions**
  - (A1) I assume that a sales manager has no control over his whole team member's sales performance. This implies that once a salesperson is added to a team, his sales performance is stochastic. Using this assumption, a team's sales can be denoted as $s_{T(i)t} = s_T(N_{it}, \epsilon_{it}^T)$ where ϵ_{it}^T is a random shock which affects the team's sales level.
  - (A2) I assume a monotonic relationship between effort and outcomes (i.e. sales and number of added salespeople).

- **Utility functions & Other functions**
  - Utility of a salesperson: $u(s_{it}, N_{it}) = 0.3 * s(e_{it}^a, \epsilon_{it}^a) + 0.1 * 0.3 * s_T(N_{it}, \epsilon_{it}^T) + I(Q_t = 3) * b_q - \rho(e_{it}^a, e_{it}^b)$
  - Individual sales: $s(e_{it}^a, \epsilon_{it}^a) = \alpha * e_{it}^a + \epsilon_{it}^a$
  - Team sales
    - Level-1 salesperson: $s_T(N_{it}^1, \epsilon_{it}^{T1}) = \tau_1 * N_{it}^1 * \epsilon_{it}^{T1}$
    - Level-2 salesperson: $s_T(N_{it}^1, N_{it}^2, \epsilon_{it}^{T1}, \epsilon_{it}^{T2}) = \tau_1 * N_{it}^1 * \epsilon_{it}^{T1} + \tau_2 * N_{it}^2 * \epsilon_{it}^{T2}$
  - Quarterly commission: $b_q = \delta * 0.3 * (S_{it} + s_{it})$ where $\delta = I(4000 \leq 0.3 * (S_{it} + s_{it}) < 6000) * 0.22 + I(6000 \leq 0.3 * (S_{it} + s_{it}) < 12000) * 0.25 + I(12000 \leq 0.3 * (S_{it} + s_{it}) < 24000) * 0.3$
    $+ I(24000 \leq 0.3 * (S_{it} + s_{it}) < 48000) * 0.35 + I(48000 \leq 0.3 * (S_{it} + s_{it})) * 0.45$
  - Cost of efforts: $\rho(e_{it}^a, e_{it}^b) = \theta_1 * (e_{it}^a)^2 + \theta_2 * (e_{it}^b)^2$
  - Adding new salespeople: $n_{it}^* = \beta * e_{it}^b + \epsilon_{it}^b$: Ordered Probit
    - $n_{it} = \begin{cases} 0 & if\ n_{it}^* \leq 0, \\ 1 & if\ 0 < n_{it}^* \leq \mu_1, \\ 2 & if\ \mu_1 \leq n_{it}^* \leq \mu_2 \\ 3 & if\ \mu_2 < n_{it}^* \end{cases}$

- **State transitions**
  - $N_{i,t+1}$
    - Level-1 salesperson: $(N_{i,t+1}^1, N_{i,t+1}^2)' = (\epsilon_{it}^{1Stay} + (N_{it}^2 - \epsilon_{it}^{2Stay}), \epsilon_{it}^{2Stay} + (N_{it}^1 - \epsilon_{it}^{1Stay}))'$ where $\epsilon_{it}^{1Stay} \sim B(N_{it}^1, p^{1stay})$ and $\epsilon_{it}^{2Stay} \sim B(N_{it}^2, p^{2stay})$ when $Q_t = 3$ and $\epsilon_{it}^{2Stay} = N_{it}^2$ otherwise.
    - Level-2 salesperson: $(N_{i,t+1}^1, N_{i,t+1}^2)' = (\epsilon_{it}^{1Stay} + n_{it} + (N_{it}^2 - \epsilon_{it}^{2Stay}), \epsilon_{it}^{2Stay} + (N_{it}^1 - \epsilon_{it}^{1Stay}))'$
  - $s_{it} = s(e_{it}^a, \epsilon_{it}^a)$
  - $S_{i,t+1} = s_{i,t-1} + s(e_{it}^a, \epsilon_{it}^a)$
  - $s_{T(i)t} = s_T(N_{it}, \epsilon_t^T)$
  - $S_{T(i),t+1} = s_{T(i),t-1} + s_T(N_{it}, \epsilon_t^T)$

- **Salesperson's problem**
  - **Salesperson at $P_{it} = 1$**

$$
\begin{aligned}
&V_1(N_{it}^1, N_{it}^2, S_{it}, s_{i,t-1}, S_{T(i)t}, s_{T(i),t-1}, Q_t) = \\
&max_{e_{it}^a} \int_{\epsilon_{it}^1} \Big(u(s(e_{it}^a, \epsilon_{it}^a), N_{it}^1) + \\
&\beta \left(\begin{array}{c} I(\epsilon_{it}^a \in E^{Promo}(e_{it}^a)) * V_2(N_{i,t+1}^1, N_{i,t+1}^2, S_{i,t+1}, s_{it}, S_{T(i),t+1}, s_{T(i)t}, Q_{t+1}) \\ +I(\epsilon_{it}^a \in E^{Stay}(e_{it}^a)) * V_1(N_{i,t+1}^1, N_{i,t+1}^2, S_{i,t+1}, s_{it}, S_{T(i),t+1}, s_{T(i)t}, Q_{t+1}) \end{array} \right) \Big) dF(\epsilon_{it}^1) \\
&\text{where } \epsilon_{it}^1 = (\epsilon_{it}^a, \epsilon_{it}^{T1}, \epsilon_{it}^{1Stay}, \epsilon_{it}^{2Stay})
\end{aligned}
$$

$$E^{Promo}(e_{it}^a) = \{\epsilon_{it}^a : 0.3 * (S_{it} + s(e_{it}^a, \epsilon_{it}^a)) \geq 2,000\} \text{ and } E^{Stay}(e_{it}^a) = R - E^{Promo}(e_{it}^a)$$

- **Salesperson at** $P_{it} = 2$

$$
\begin{aligned}
&V_2(N_{it}^1, N_{it}^2, S_{it}, s_{i,t-1}, S_{T(i)t}, s_{T(i),t-1}, Q_t) = \\
&max_{e_{it}^a, e_{it}^b} \int_{\epsilon_{it}^2} \Big(u(s(e_{it}^a, \epsilon_{it}^a), N_{it}) + \\
&\beta \left(\begin{array}{c} I(\epsilon_{it}^a \in E^{Stay}(e_{it}^a)) * V_2(N_{it+1}^1, N_{it+1}^2, S_{i,t+1}, s_{it}, S_{T(i),t+1}, s_{T(i)t}, Q_{t+1}) \\ +I(Q_t = 3 \;\&\; \epsilon_{it}^a \in E^{Demo}(e_{it}^a)) * V_1(N_{i,t+1}^1, N_{i,t+1}^2, S_{i,t+1}, s_{it}, S_{T(i),t+1}, s_{T(i)t}, Q_{t+1}) \end{array} \right) \Big) dF(\epsilon_{it}^2) \\
&\text{where } \epsilon_{it}^2 = (\epsilon_{it}^a, \epsilon_{it}^b, \epsilon_{it}^{T1}, \epsilon_{it}^{T2}, \epsilon_{it}^{1Stay}, \epsilon_{it}^{2Stay})
\end{aligned}
$$

$$E^{Demo}(e_{it}^a) = \{\epsilon_{it}^a : 0.3 * (S_{it} + s(e_{it}^a, \epsilon_{it}^a)) < 600\}, \text{ and } E^{Stay} = R^3 - E^{Demo}$$