APM466 Assignment 1

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Fundamental Questions - 25 points

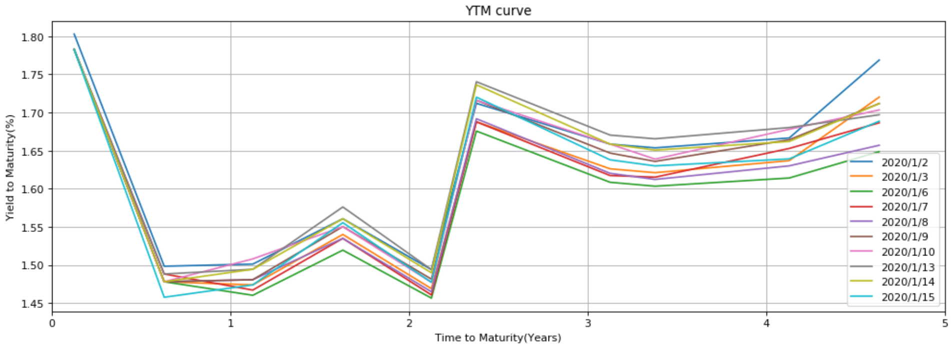
1.
   (a) It is a tool for governments to raise money for their projects and daily operation, and bonds can also be a tool for government to accomplish fiscal and monetary policies.
   (b) Yield curve is an indicator of non-risk equity's liability, government regard it as a indicator of interest rate, and also use it as pricing tool of other equity in the market.
   (c) In order to decrease money supply, government has to borrow more money from public. Therefore, government will sell bonds and attract people by low bond price and high yield.

2. In order to make sure the difference on the maturity date for consecutive bonds in 6 months, I dominantly choose the bonds mature in March and September. Besides that, the bond issued in the 20th century will also be neglected since their liability remained at a low level. However, there is no bond matured in September in 2022 and 2023, I use bonds matured in June instead.
   CAN 1.50 Mar 1 2020; CAN 0.75 Sept 1 2020; CAN 0.75 Mar 1 2021; CAN 0.75 Sept 1 2021; CAN 0.50 Mar 1 2022; CAN 2.75 June 1 2022; CAN 1.75 Mar 1 2023; CAN 1.50 June 1 2023; CAN 2.25 Mar 1 2024; CAN 1.50 Sept 1 2024.

3. The PCA evaluate how data dispersed, it can also be illustrated by covariance matrix. Eigenvectors of vocariance matrix illustrate how direction of our data dispersed, and corresponding eigenvalues evaluate the maginitude on its diretions. Therefore, eigenvalues and eigenvectors of covariance matrix of stochastic process can indicate the dispersion of events in those stochastic processes.

Empirical Questions - 75 points

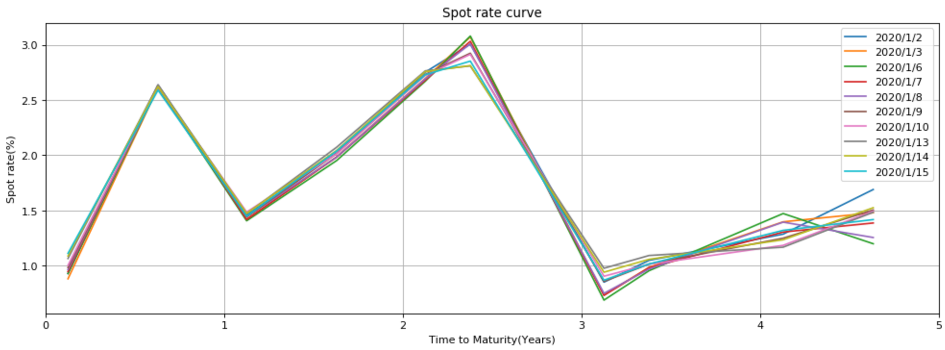
4.
   (a) I the cash flow of each period to calculate compounded yield to maturity. The equation I used is:
   $$bondprice = \sum_{t=0}^{T} \frac{Coupon}{(1+\frac{ytm}{2})^t} + \frac{Par}{(1+\frac{ytm}{2})^T} \tag{1}$$
   (b) Step 1: for bonds mature less than 6 months, they will be calculated by equation below:
   $$r(T) = -\frac{log(P/N)}{T} \tag{2}$$
   where P is the price of bond, N means par value, and T is the time to maturity.
   Step 2: for those bonds mature longer than 6 months, I will use the bond mature at least half



year earlier, regards them as zero coupon bond and calculate their spot rate. The equation is written down below:

$$P = C * e^{-r(t_1)*t_1} + (C + Par) * e^{-r(t_2)*t_2} \tag{3}$$

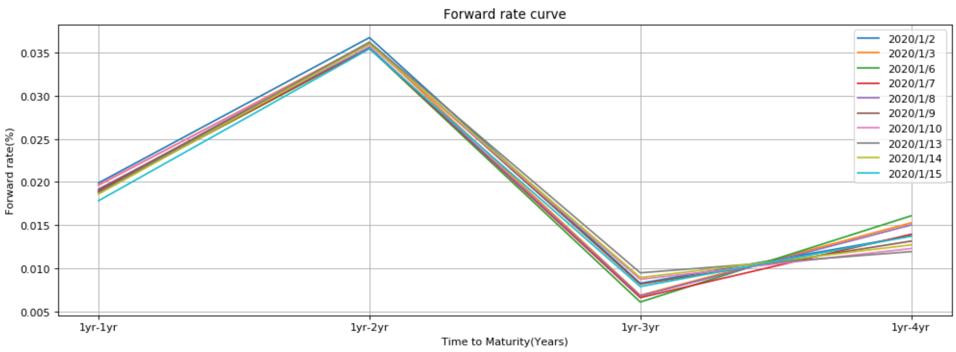
C represent coupon payment, r(t) represent spot rate in corresponding period, t is corresponding time to maturity. The result is down below:



(c) Step1: Select bonds mature in March only.
Step2: For t years forward rate on year 1, the forward rate is calculated by:

$$f_{1,t} = \sqrt[t]{\frac{(1 + S_{t+1})^{t+1}}{1 + S_1}} - 1 \tag{4}$$

where S_t is the spot rate of corresponding period. The result is illustrated on figure below:



5. The co-variance matrix for log-return of yield to maturity is:
$$\begin{bmatrix} 1.41211756e-05 & 2.29587465e-05 & 2.19344462e-05 & 2.58436690e-05 & 2.27131442e-05 \\ 2.29587465e-05 & 1.37167854e-04 & 4.58351253e-05 & 8.97514632e-05 & 9.19142817e-05 \\ 2.19344462e-05 & 4.58351253e-05 & 8.68599429e-05 & 9.95369531e-05 & 1.00152938e-04 \\ 2.58436690e-05 & 8.97514632e-05 & 9.95369531e-05 & 1.36922962e-04 & 1.55566982e-04 \\ 2.27131442e-05 & 9.19142817e-05 & 1.00152938e-04 & 1.55566982e-04 & 2.56975884e-04 \end{bmatrix}$$

   The co-variance matrix for log-return of forward rate is:
$$\begin{bmatrix} 7.65412776e-04 & 6.38817797e-05 & 1.25964191e-03 & -8.11960642e-04 \\ 6.38817797e-05 & 1.31393259e-04 & 1.31060685e-03 & -7.54621172e-04 \\ 1.25964191e-03 & 1.31060685e-03 & 1.50330331e-02 & -9.97435073e-03 \\ -8.11960642e-04 & -7.54621172e-04 & -9.97435073e-03 & 9.04476414e-03 \end{bmatrix}$$

6. The eigenvalues for log-return of yield to maturity are:
   $4.79963792e-04, 8.93808349e-05, 5.28863637e-05, 7.71071456e-06, 2.10611275e-06$
   The eigenvectors for log-return of yield to maturity are:
   $[0.09665464, -0.10762347, -0.18467199, -0.92622775, -0.29508216]^T$
   $[0.3710123, -0.87219665, 0.23850662, 0.02561942, 0.20995502]^T$
   $[0.3525261, 0.10872532, -0.69218193, -0.03514344, 0.61931678]^T$
   $[0.5152873, -0.03783732, -0.37443962, 0.35113049, -0.68523696]^T$
   $[0.68059688, 0.46307378, 0.53822858, -0.13006967, 0.1254689]^T$
   The first eigenvalue: $4.79963792e-04$ counts around 76% of sum of eigenvalues.

   The eigenvalues for log-return of forward rate are:
   $2.26558462e-02, 1.65320593e-03, 6.57411462e-04, 8.13970615e-06$
   The eigenvectors for log-return of forward rate are:
   $[0.0681955, 0.09517055, 0.99054103, 0.07155703]^T$
   $[0.06657819, 0.10476356, -0.08606083, 0.98852692]^T$
   $[0.79921884, 0.57923255, -0.10171303, -0.12406998]^T$
   $[-0.59343573, 0.80278068, -0.03280945, -0.04796621]^T$
   The largest eigenvalue : $2.26558462e-02$ counts around 90% of sum of eigenvalues.

   The largest eigenvector of vocariance matrix always represent the largest variance of of the data, and its magnitude is the corresponding eigenvalue.

References and GitHub Link to Code

1. For general intuition of bonds:$https://www.investopedia.com/terms/g/government-bond.asp$

2. Code for ytm calculation:
   $https://github.com/jamesmawm/Mastering-Python-for-Finance-source-codes/blob/master/B03898_05_{Co}$

3. General intuition of the PCA:
   $https://towardsdatascience.com/a-one-stop-shop-for-principal-component-analysis-5582fb7e0a9c$

4. Code for covariance matrix and its eigenvalue and eigenvector:
   $https://github.com/shin2suka/Calculating-Curves/blob/master/a1_466.py$