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An Improved Macroscopic Model of Traffic Flow: Derivation and Links with the Lighthill-Whitham Model

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Abstract—We first recall a macroscopic model with two equations, that we have introduced in [1], and which completely resolves the severe inconsistencies of the class of Payne-Whitham models. In this short paper, we describe the effects of adding a relaxation term in the anticipation equation, and the main steps and mathematical difficulties to show rigorously the convergence to the Lighill-Whitham model when the relaxation time tends to 0.

Keywords—Traffic flow, Continuous models, Conservation laws, Relaxation limits.

1. INTRODUCTION

We are interested in continuum models of traffic flow, more precisely in "second-order" models, i.e., in models with two equations: the natural one (conservation of mass), and the phenomological one, which involves an "anticipation term", supposed to describe the response of (macroscopic) drivers to variations of the traffic density ahead. A well-known model with two equations is the Payne-Whitham system, which mimics the 2×2 system of ("isentropic") gas dynamics equations. As observed in the neat study of Daganzo [2], this model leads to severe inconsistencies, such as cars starting to move backward (!!), or some part of the information traveling faster than any car, etc....

In [1], we have introduced a new "second-order" continuum model of traffic flow, still based on a heuristic assumption, which completely suppresses these inconsistencies. This model and its heuristic derivation are briefly recalled in Section 2. For a kinetic derivation, we refer to [3]. In Section 3, we give some typical examples of solutions to the Riemann Problem for this model, in the case without source term. In Section 4, we add a relaxation term and recall the crucial role of the classical *subcharacteristic condition.* In Section 5, we announce and summarize a forthcoming detailed work on the relaxed model, and its *rigorous* convergence to the Lighthill-

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Whitham model [4] when the relaxation time τ vanishes. We also emphasize the role of *vacuum* in the modeling.

2. THE MODEL

Let us first briefly recall that the Payne-Whitham (PW) [5] type of models consists of two equations, the conservation of mass

$$\partial_t \rho + \partial_x(\rho v) = 0, \tag{2.1}$$

and the "momentum equation", which is in fact an "anticipation equation", written here in nonconservative form,

$$\partial_t v + v\,\partial_x v + \rho^- 1 p'(\rho)\,\partial_x \rho = \left(\tau^{-1}\right)\left(V(\rho) - v\right) + \nu\,\partial_x^2 v, \tag{2.2}$$

with nonnegative constants τ^{-1}, a relaxation time, and ν, a diffusion coefficient. In the above equation, $V(\rho)$ is an *equilibrium* velocity, decreasing with respect to ρ, e.g. (with suitable normalizations), $V(\rho) = 1 - \rho$.

As we already said, in the case where $\tau^{-1} = \nu = 0$, this PW model leads in particular to situations where cars can start going backward .... In this respect, adding some diffusion in the model still makes it worse, due to the infinite speed of propagation for the heat equation, which would imply immediately that some cars have moved backward. So, we definitely assume throughout the paper that

$$\nu = 0.$$

Let us first assume that we also have $\tau^{-1} = 0$; i.e., let us consider the purely hyperbolic case, without relaxation term.

In order to understand what goes wrong in the PW model, assume for instance that ahead of a driver travelling with speed v the density is increasing with respect to x, but that this denser group of drivers travels *faster* than him, so that the density ρ is decreasing with respect to $(x - v\,t)$. Clearly, the PW type of models predicts that this (macroscopic) driver would slow down, since ρ is increasing with respect to x! On the other hand, any reasonable driver would accelerate, since this denser traffic travels *faster* than him, so that *in his own framework* there are less and less cars immediately ahead of him; see Figure 1.

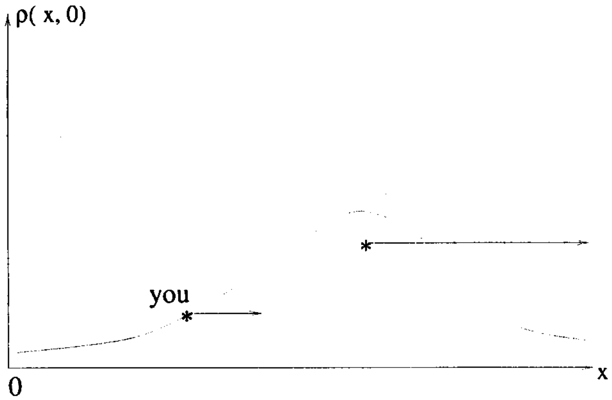


Figure 1. In such a case, would you brake, or accelerate?

With this simple remark in mind, the model that we have introduced in [1] is obvious: just replace in equation (2.2) the x derivative of the (pseudo) "pressure" by a *convective* derivative $\partial_t + v\,\partial_x$, to obtain

$$\begin{aligned} \partial_t \rho + \partial_x(\rho v) &= 0, \\ \partial_t(v + p(\rho)) + v\partial_x(v + p(\rho)) &= 0. \end{aligned} \tag{2.3}$$

In fact, by analogy with the PW model, the "pressure" here—which has never been a real pressure—is in fact a pseudopressure, whose derivative with respect to ρ is rather $(\rho)^{-1}p'(\rho)$, and is in fact homogeneous to a velocity; see Section 4. Of course we must prescribe initial data $U_0 := (\rho_0, v_0)$ whose components are two bounded nonnegative functions. Let us now briefly describe the main hyperbolic features of this model.

3. EXAMPLES OF SOLUTIONS TO THE RIEMANN PROBLEM

We refer to [1] for more details on this system. Here, let us just mention that (2.3) is (strictly) hyperbolic; i.e., it can be rewritten under the form

$$\partial_t U + A(U)\,\partial_x U = 0, \tag{3.1}$$

where the matrix

$$A(U) := \begin{pmatrix} v & \rho \\ 0 & v - \rho p'(\rho) \end{pmatrix} \tag{3.2}$$

has real eigenvalues

$$\lambda_1 = v - \rho\, p'(\rho) \le \lambda_2 = v, \tag{3.3}$$

distinct except at vacuum, i.e., for $\rho = 0$. We will see later that this loss of strict hyperbolicity at vacuum allows for the possibility of *instabilities* near vacuum; see also the remark in Section 5.

In fact, for a large class of pressure laws, the first eigenvalue turns out to be *genuinely nonlinear*, see [6–8] for basic references on nonlinear hyperbolic systems of conservation laws, whereas $\lambda_2 = v$ is *linearly degenerate*. Practically, that means that the simple waves associated to λ_1 will be either *shock waves* (braking) or *rarefaction waves* (acceleration), whereas the ones associated to λ_2 will be *contact discontinuities*, corresponding to situations where each car just follows the leading car, at the same speed.

Let us consider now the *Riemann invariants* (in the sense of Lax) of system (2.2), i.e., the functions w and z, respectively, associated to λ_1 and λ_2, which allow us to diagonalize the system. They are given here by

$$w = v + p(\rho), \qquad z = v, \tag{3.4}$$

and they satisfy, at least for smooth solutions,

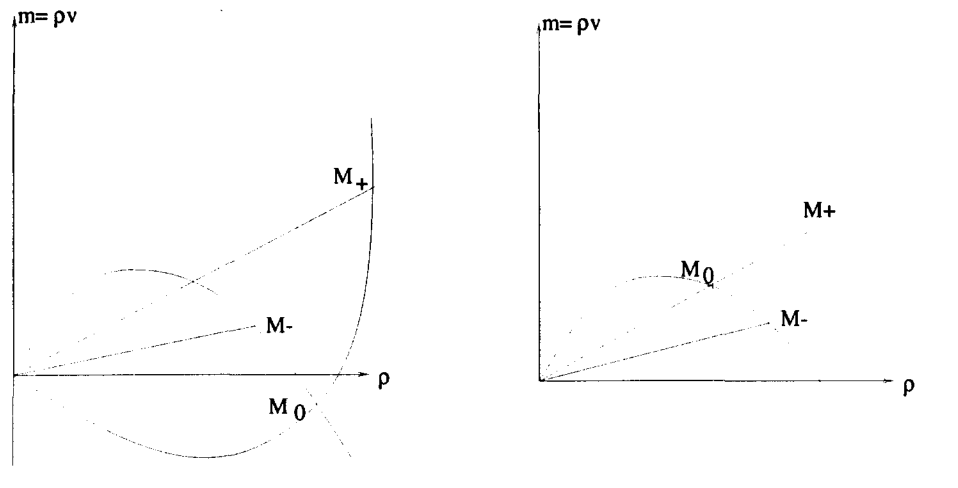
$$\begin{aligned} \partial_t w + \lambda_2\, \partial_x w &= \partial_t(v + p(\rho)) + v\, \partial_x(v + p(\rho)) = 0, \\ \partial_t z + \lambda_1\, \partial_x z &= \partial_t v + (v - \rho p'(\rho))\, \partial_x v = 0. \end{aligned} \tag{3.5}$$

Classically, z is constant across a wave of the second family (a contact discontinuity) separating two constant states U_l and U_r, and the first Riemann invariant w is constant across a 1-wave in the case of a rarefaction wave, which is classical. Here this is also true across a shock, which is much more exceptional. For such systems, sometimes called *à la Temple* [9], shock curves and rarefaction curves are therefore coinciding.

The solution of the Riemann problem is now easy to construct: given the data $U_\pm = (\rho_\pm, v_\pm)$ in $\mathbb{R}^2$, draw the curves

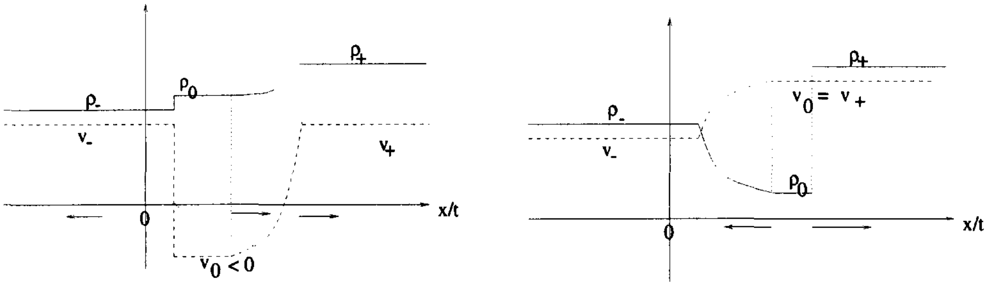
$$w = w_- := w(U_-) \qquad \text{and} \qquad z = v = z_+ := z(U_+),$$

consider their (unique) intersection point U_0, and check the inequalities between the different propagation speeds, to decide which type of 1-simple wave (1-shock wave is $v_- > v_+$, 1-rarefaction wave if $v_- < v_+$, possibly with an intermediate vacuum state if $v_- < w_- < v_+$), always followed by a 2-contact discontinuity, allows to connect U_- to U_0 and U_0 to U_+. The solution just consists of the juxtaposition of all these pieces of solutions.



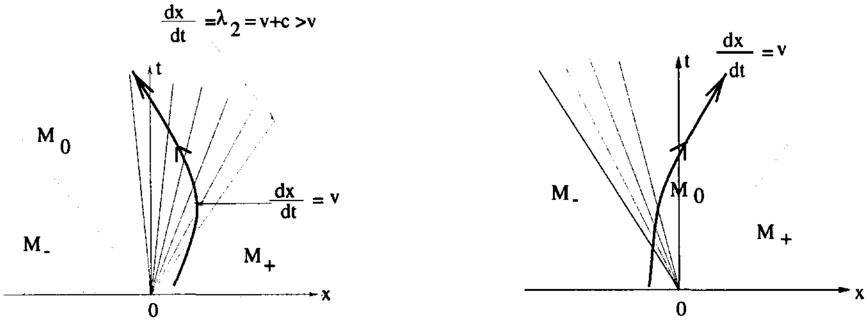
(a) (b)

Figure 2. Example of Riemann problems for the PW model (a), and for our model (b). Note that in (a) the cars behind brake immediately and instantaneously reach negative speeds (!), although cars ahead travel *faster*!



(a) (b)

Figure 3. Note that negative velocities appear with the PW model (a), not with our model (b).



(a) (b)

Figure 4. The trajectories $\frac{dx}{dt} = v$ cross the 2- (respectively, 1-) rarefaction wave from the right to the left with PW model (a) from the left to the right with our model (b): some part of the information travels *faster* than cars ahead, and modifies their trajectories (!) in PW model (a), not in our model (b).

As a striking example, let us refer to one of the examples described in [1]; see Figures 2–4.

In this example, and in *all* the other ones that we have constructed, the model satisfies *all* the natural qualitative requirements that one can impose: the velocities—and of course the densities—remain nonnegative. They stay in a bounded (convex) invariant region, and therefore, they remain bounded from above. Due to the usual inequalities on the eigenvalues of the solution, *no* wave travels faster than the speed of the vehicles. Finally, the model *always* predicts the natural waves: shocks when braking, contacts when following, rarefactions, possibly with appearance of vacuum, when accelerating, instabilities near vacuum, especially with a few slow drivers ahead, discontinuities at the end of a queue followed by vacuum, etc....

The only (slight) drawback of this model is the following: assume that the cars in front are much faster than the cars behind, so that there is a rarefaction wave between U_- and the intermediate state U_0, with U_0 at vacuum. In this case, the maximal speed reached by the cars in this rarefaction turns out to be

$$v_{\max}(U_-) := v_- + p(\rho_-),$$

and therefore, *depends* on the initial state U_-. Although this is a much milder inconsistency, this is clearly unrealistic: this speed should be—after possibly some adaptation time—the maximal allowed possible speed v_m (at vacuum), which is involved in the definition of the bounded invariant region. This is one of the reasons to add a relaxation term in the model.

4. THE MODEL WITH A RELAXATION TERM

In this section, we take into account the above observation, and therefore, we add a relaxation term in the anticipation equation, to obtain

$$\begin{aligned}
&\partial_t \rho + \partial_x(\rho v) = 0, \\
&\partial_t((v + p(\rho)) + v\partial_x(v + p(\rho)) = \tau^{-1}\left(V(\rho) - v\right),
\end{aligned} \tag{4.1}$$

where τ is a relaxation time, and $V(\rho)$ an *equilibrium* velocity.

We describe below some important mathematical and practical issues in this model, and we announce new results, whose detailed statements and proofs will be given in a forthcoming paper [10], as well as in the Ph.D. Thesis of Aw [11]. First, for mathematical purposes, it is useful to multiply the first equation by $w = v + p(\rho)$, the second by ρ, and to add these two equations, in order to obtain the system under conservative form

$$\begin{aligned}
&\partial_t \rho + \partial_x(\rho v) = 0, \\
&\partial_t(\rho(v + p(\rho))) + \partial_x(\rho v(v + p(\rho))) = \tau^{-1}\rho(V(\rho) - v).
\end{aligned} \tag{4.2}$$

Indeed, we recall, see again [6–8], that as in the case without source-term on the right-hand side, one selects the so-called *entropy admissible* weak solutions (possibly discontinuous) by considering additional conservation laws of this system, i.e., the *entropy-entropy flux* pairs of functions with scalar values

$$(\eta(U),\, q(U)) := (\eta(\rho, v),\, q(\rho, v)).$$

We recall that an *entropy* weak solution is required to satisfy for any *convex* entropy $\eta(U)$ the inequality

$$\partial_t \eta + \partial_x q \le \tau^{-1}\partial_v \eta\,(V(\rho) - v), \tag{4.3}$$

and the equality whenever U is smooth. We recall that the equality is obtained by multiplying the first equation in (4.1) by $\frac{\partial \eta}{\partial \rho}$, the second one by $\frac{\partial \eta}{\partial v}$, and adding these two equations.

As to the modeling, there is now a second heuristic choice to be made: the choice of function V. A mathematically crucial assumption is the so-called *subcharacteristic condition*, see, e.g., [12–15], and in the context of traffic flow [16,17]. This condition turns out to be here

$$-p'(\rho) \le V'(\rho) \le 0. \tag{4.4}$$

We recall that the two functions p and V have the dimensions of velocities. A particular choice of equilibrium velocity could be, for instance,

$$V(\rho) = \alpha\,(p(\rho_m) - p(\rho)), \qquad 0 \le \alpha \le 1, \tag{4.5}$$

where ρ_m, the maximal allowed density in the model, corresponds to the $v = 0$ and to the invariant bounded region, which in the plane of characteristic coordinates $w = v + p(\rho)$, $z = v$ is the triangle

$$R := \{(w, z);\, 0 \le z = v \le w = v + p(\rho) \le w_m := v_m = p(\rho_m)\}\,.$$

In Figure 5 below, we show this region R as well as the equilibrium curves $v = V(\rho)$ in the two cases:

- the *strictly subcharacteristic case*, in which the inequality in (4.5) is strict,
- and the *subcharacteristic case*, in which the inequality in (4.5) is large.

In fact, we have considered the extreme case, we could call it the *characteristic case*, where

$$V'(\rho) = -p'(\rho) \text{ or } = 0,$$

which corresponds to choosing $\alpha = 1$ in (4.5), and more precisely, see Figure 5 below,

$$V(\rho) = \max(p(\rho_m) - p(\rho),\, 0). \tag{4.6}$$

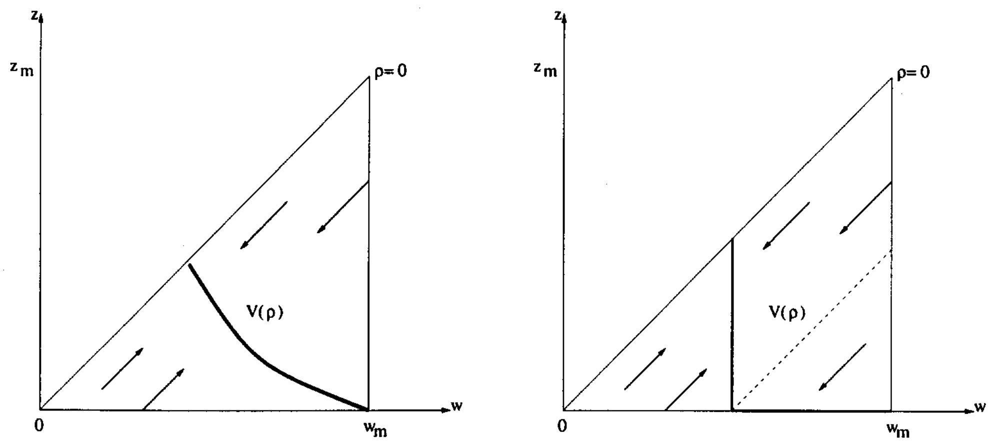


Figure 5. Invariant region R and equilibrium curve $v = V(\rho)$ in the subcharacteristic case (left) and in the characteristic case (right).

The role of the subcharacteristic condition is crucial: indeed, let the relaxation time τ go to 0, and perform a (formal) Chapman-Enskog expansion, i.e., assume that the solution

$$U = (\rho,\, v) := U^\tau = (\rho^\tau,\, v^\tau)$$

is uniformly bounded when τ goes to 0, *as well as its derivatives*—which is obviously wrong near discontinuities!—and make the following Ansatz:

$$v = V(\rho) + \tau V_1(\rho) + \cdots\,.$$

From the second equation (4.1), we see that at leading order

$$V_1 = -(\partial_t + v\,\partial_x)(v + p(\rho)) \sim -\,(V'(\rho) + p'(\rho))\,(\partial_t\,\rho + V(\rho)\partial_x\,\rho)\,.$$

Using now the first equation to approximate $\partial_t \rho$, we obtain

$$V_1(\rho) \sim (V'(\rho) + p'(\rho))\, V'(\rho)\, \partial_x \rho.$$

Differentiating this approximation with respect to x implies in turn in the first equation

$$\partial_t\, \rho + \partial_x(\rho\, V(\rho)) = -\tau\, \partial_x(V_1(\rho)) \sim \tau\, \partial_x\left(-\left(V'(\rho) + p'(\rho)\right) V'(\rho)\, \partial_x \rho\right).$$

Therefore, at least formally, the nonlinear diffusion coefficient

$$D(\rho) := -\left(V'(\rho) + p'(\rho)\right) V'(\rho)$$

must be nonnegative, or (better) slightly positive, to stabilize the system, which is the justification of subcharacteristic condition (4.5), strict or large.

We should also note that this formal diffusion has the great advantage, in the strict case, to regularize the solution without having the huge drawback of a real diffusion term mentioned in Section 2: the speed of propagation in (4.1) remains *bounded* and does *not* produce cars going backward .... Moreover, this condition is also essential for the stability of the numerical schemes that we are mentioning in the next section, and for the convergence when τ goes to 0; see [18–21].

5. CONVERGENCE OF NUMERICAL SCHEMES AND RELAXATION TO THE LIGHTHILL-WHITHAM CASE WHEN τ GOES TO 0

We just describe here the main steps, some of the difficulties, and we make some comments, referring again to forthcoming papers [10,11] for more details. We proceed in three steps.

- First, for a fixed relaxation time τ, construct a numerical approximation, using a splitting: first approximate the system of conservation laws either by the Glimm or by the Godunov scheme, using in either case the resolution of the Riemann problem in [1]. Then approximate the (stiff) ODE like, e.g., in [18,20,22].
- Then construct an entropy weak solution, globally defined in time, as the limit of this numerical approximation when the mesh size $h := (\Delta x, \Delta t)$ tends to 0.
- Then let τ tend to 0, and show that the solution tends to the solution of the Lighthill-Whitham model [4,12]

$$\partial_t \rho + \partial_x(\rho V(\rho)) = 0, \qquad \rho(x, 0) = \rho_0(x). \tag{5.1}$$

Here, the purpose of the numerical scheme(s) is *not* to produce numerical results, although the algorithm is fairly simple, but to construct a globally defined weak solution and then give the *principle* of a rigorous proof of convergence to the Lighthill-Whitham model when $\tau \to 0$.

STEP 1. First, for a fixed τ and a fixed mesh, we construct a numerical approximation to the initial value problem (4.2), written in conservative form, with (nonnegative) initial data $\rho_0(x)$ and $y_0(x)$: We recall that R is the invariant triangle described in Figure 5, so that the density and the velocity remain bounded and nonnegative (!). Moreover, all the straight lines parallel to the first bisector correspond to lines of constant density. In particular, the first bisector itself is the vacuum line $\rho = 0$, and the directions of the vector-field

$$\frac{d\rho}{dt} = 0, \qquad \frac{dv}{dt} = \tau^{-1}\left(V(\rho) - v\right) \tag{5.2}$$

are therefore parallel to this bisector as indicated in Figure 5.

Now, let us describe two possible schemes.

(i) First, replace (x,t) by oblic coordinates $(x',t) := (x - ct, t), c \geq v_m$, so that in the new coordinates all the speed of propagation, as well as the new velocities $v' := v - c \leq 0$, are nonpositive.

(ii) Define mesh sizes $\Delta x' = \Delta x$ in space and Δt in time, satisfying the CFL condition

$$\operatorname{Sup}\{|\lambda_k(U)|;\ k = 1,2,\ U \in R\} \times \frac{\Delta t}{\Delta x} \leq 1.$$

Now define an approximation of the solution $U_j^n := (\rho_j^n,\ v_j^n)$, in conservative variables

$$Y_j^n := (\rho_j^n,\ y_j^n) := \left(\rho_j^n,\ \rho_j^n\ \left(v_j^n + p\left(\rho_j^n\right)\right)\right)$$

on the cell $I_j = (x'_{j-1/2}, x'_{j+1/2})$ at time $t_n = n\,\Delta t$ as follows.

(iii) Define Y_j^n for $n = 0$ by

$$Y_j^0 := \left(\int_{I_j} \rho_0(x)\,dx,\ \int_{I_j} \rho_0(x)(v_0(x) + p(\rho_0(x)))\,dx\right).$$

(iv) At time t_n, knowing Y_j^n—or equivalently U_j^n whenever $\rho_j^n > 0$—use the following splitting to define Y_j^{n+1}.

- First use either the Glimm or the Godunov scheme to define the approximation $Y_j^{n+1/2}$ at time t_{n+1} of the solution to system (4.2), rewritten in variables $(x',\ t)$. We recall the principle of these two schemes: starting at time t_n with piecewise constant values, consider the solution to equation (5.1) at the new time step t_{n+1}, and replace it either by its *average value* on each cell, in the case of Godunov, see, e.g., [23], or by an equidistributed random sampling on each cell; see [7,24].

In either case, between times t_n and $t_n + 1$, thanks to the CFL condition and to the finite speed of propagation, the solution in each cell is the solution to the Riemann problem, and, e.g., the flux at point $x'_{j+1/2}$ is $F(W(0; Y_j^n, Y_{j+1}^n)) = F(Y_{j+1}^n) = (\rho_{j+1}^n, y_{j+1}^n)$, where $F(Y) = (\rho(v-c), y(v-c))$ and $W(x'/t; Y_-, Y_+)$ is the value at point (x',t) of the centered solution to the Riemann problem with data Y_- and Y_+. We recall that all the propagation speeds are nonpositive in the (x',t) variables.

Next, approximate the ODE system

$$\partial_t Y = \left(0,\ \tau{-1}\,\rho_j^n\ \left(V\left(\rho_j^n\right) - v_j^n\right)\right),$$

e.g., by

$$\left(Y_j^{n+1}\right) = \exp\left(-\frac{\Delta t}{\tau}\right) Y_j^{n+1/2} + \left(1 - \exp\left(-\frac{\Delta t}{\tau}\right)\right) \left(0,\ V\left(\rho_j^{n+1/2}\right) - v_j^{n+1/2}\right).$$

STEPS 2 AND 3. Now we first let the mesh size tend to 0, with a fixed ratio $r = \Delta t/\Delta x'$, and with a fixed relaxation time τ, to construct a globally defined entropy weak solution to (4.1). Then we let τ go to 0.

In order to justify rigorously the convergence, there are three main difficulties.

- In the third step, there is the hard problem of extending the (convex) entropies for the limit equation into (convex) entropies for system (4.1), which must be minimal at equilibrium, i.e., for $v = V(\rho)$; see, e.g., [14,15,18–20]. We do not comment further on this problem here, again referring to [10,11] for the proof.
- The second ingredient, crucial in Steps 2 and 3, is to use the existence of the bounded invariant triangle R, and mostly the subcharacteristic condition (4.4) to show that the two steps of the algorithm preserve the L^∞, uniformly with respect to the mesh size and also with respect to τ.
- The third problem—harder—is to show that no spurious oscillation develops. For this, it is natural to study the BV stability, uniformly with respect to the mesh size and also with respect to τ.

Here, due to (4.4), the second step—the resolution of the ODE—preserves the total variation of the solution, see [18], but there are big differences between the Godunov scheme and the Glimm scheme.

(a) In each cell, the updated value U_j^{n+1} picked by the Glimm scheme is an *intermediate* value in the solution to the Riemann problem, and therefore, for a very special Temple system like (4.2), preserves the *total variation* of the approximate solution, at least *away from vacuum*, where the velocity v and the other Riemann invariant $w = v + p(\rho)$ are not defined; see the remark below. Now define

$$d(U_-, U_+) := |w_- - w_+| + |v_- - v_+|,$$

and define at each time t_n the (Glimm) functional

$$\mathcal{F}(U^n) := \sum_{j=-\infty}^{+\infty} d\left(U_j^n, U_{j+1}^n\right).$$

In the case of the Glimm scheme, the conclusion is then that $\mathcal{F}(U^n)$ decreases with respect to time. Therefore, if $\mathcal{F}(U^0) < +\infty$, then the total variation of w and v are uniformly controlled, which is the key point to pass to the limit in Steps 2 and 3.

Unfortunately, the above statement is true provided that *we stay away from vacuum*, and *a priori* this assumption is only guaranteed if $m := \inf \rho_0(x) > 0$ and $\mathcal{F}(U^0) \leq h(m)$, where h is some known function depending on function p. The mathematical result is interesting, but this is a very severe restriction, since we want to be able to handle the case of the vacuum.

REMARK. There is some philosophical issue here: for mathematical purposes, we could define the velocity v and $w = v + p(\rho)$ at vacuum, even if this is physically meaningless! Unfortunately, that would require a fixing; otherwise when solving the Riemann problem, we would have to interrupt an acceleration wave in order to avoid that some cars would go faster than *nonexisting cars* in front of them! When $\tau = +\infty$, i.e., when there is no source-term in (4.1), such a fixing can be introduced without increasing the total variation of the solution, but this definitely fails when there is a source-term, even when condition (4.4) holds.

(b) In the case of the Godunov scheme, one could modify the distance d at vacuum: set $D(U_-, U-+) = d(U_-, U-+)$ when both densities $\rho_\pm$ are positive, $D = 0$ when both states are at vacuum, and $D(U_-, U_+) := 2w_m$ is the maximal possible distance between two points in R when one *and only one* of the states $U_\pm$ is at vacuum. Finally, define at each time t_n the modified (Glimm) functional

$$\mathcal{F}(U^n) := \sum_{j=-\infty}^{+\infty} D\left(U_j^n, U_{j+1}^n\right).$$

Note that this new functional has the great advantage of not involving any definition of v and w at vacuum. Moreover, we can exploit the fact that, contrarily to the Glimm scheme, the Godunov scheme *reduces* at each time step the number of cells which are at vacuum, since it uses *averages* of the solution to the Riemann problem. Unfortunately, for this very reason—v is a nonlinear function of the conservative variables—there are situations where this scheme does *not* preserve the total variation of the solution, unless we introduce a Godunov scheme with variable cells, which describes the trajectories and exploits the underlying Lagrangian nature of the system, and which has (almost) all the good properties we need, including the control of the above-modified Glimm functional. We refer to [10,11] for more details.

A last remark concerns the convergence to the Lighthill-Whitham (LW) model: in the *characteristic* case, the limit equation is not exactly the usual LW model. Indeed, see Figure 5, the limit flux is either $\rho V(\rho)$ or 0, depending on the value of ρ. The resulting function $f(\rho)$ is nonconcave, and this is already enough to allow more complex (composite) waves, even with such a simplified model.

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