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Phase transition of traffic states with on-ramp

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Abstract

We study the phase transition on a highway in a modified anisotropic continuum model with an on-ramp, which is recently developed by Gupta and Katiyar (J. Phys. A: Math. Nucl. Gen. 38 (2005) 4069]. To investigate whether this model can describe several distinct traffic states that are identified from real-traffic data [Kerner and Rehborn, Phys. Rev. Lett. 79 (1997) 4030; Kerner, Phys. Rev. Lett. 81 (1998) 3797], we carry out numerical simulations with an open boundary condition. The observed transition between free flow and various types of congested flow such as localized clusters, stop-and-go traffic and different kinds of synchronized traffic flow is obtained by applying a triggering pulse through an on-ramp in our simulation.

We present the phase diagram for three representative values of the upstream boundary flux and for the whole range of the on-ramp flux. Several states like pinned localized cluster, triggered stop-and-go, recurring hump state, the oscillatory congested traffic and the homogeneous congested traffic are observed in phase transition from free flow to traffic-jam state. The phase diagram for our model near on-ramp is consistent with the results obtained by Lee et al. [Phys. Rev. E 59(5) (1999) 5101]. The results suggest that the modified model is able to describe all the three phases of traffic-flow theory developed by Kerner [Physica A 333 (2004) 379].

Keywords: Phase transition; Synchronized traffic flow; Anisotropic continuum model; On-ramp

1. Introduction

Traffic, a realization of an open one-dimensional many-body system, shows various complex behaviors. Recently, there has been much interest in the study of traffic flow near on-ramps of a highway system, which revealed a rich spectrum of phenomena and attracted research interests from physicists [1,2]. This is not only due to practical implications for optimizing freeway traffic but also because of the observed nonequilibrium phase transitions [2,3] and the various nonlinear dynamical phenomena like traffic jams [4], stop-and-go traffic [5] and synchronized traffic [2,6,7].

The transition between homogeneous free flow and the jammed state has been made by microscopic and macroscopic models without any inhomogenity in the system [4,8–10]. The stability analysis of the traffic flow model determines the two critical values ρ_{c1} and $\rho_{c2}(>\rho_{c1})$ of the density at which the homogeneous traffic

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flow becomes unstable. The traffic jam, however, can appear even below ρ_{c1}. If the density is greater then a different critical value $\rho_b(<\rho_{cl})$, an appearance of a localized perturbation, having finite amplitude, can lead to a self formation of local cluster of vehicles.

Measurements of traffic breakdown on various freeways by Kerner and Rehborn [2] indicate a first-order transition to synchronized traffic. Traffic data from several freeways in Germany [2,6] and Netherlands [11] indicate that synchronized traffic is the most common form of congested traffic. The synchronized traffic flow resembles the traffic jam in the sense that both states produce inhomogeneous density and flow profile.

The transition between free and synchronized flow is of hysteretic nature, i.e., the density immediately before the transition is higher and the average velocity lower than immediately after the inverse transition. The synchronized traffic flow is observed, in nearly all occasions, localized near ramp, and hysteretic phase transitions can be induced by the fluctuations of on-ramp flow. Helbing and Treiber [7] specifies that the transition is usually caused by a localized and short perturbation of traffic flow that starts downstream of the on-ramp and propagates upstream with a velocity of about $-15\,\mathrm{km/h}$. As soon as the perturbation passes the on-ramp, it triggers the breakdown, which spreads upstream in the course of time.

Lee et. al [12] proposed the recurring hump (RH) state as an origin of the (non-stationary type) synchronized traffic flow. They investigated the dynamic phase transition between RH state and free flow through numerical simulations of a hydrodynamic model proposed by Kerner and Konhauser [4]. They found that in RH state, velocity and density profiles oscillate around the ramp so the synchronized flow can be maintained for several hours.

A gas-kinetic approach is used by Helbing and Treiber [7] to describe the hysteretic phase transitions between free flow and synchronized flow. They proposed homogeneous congested traffic (HCT) state as an explanation for the (stationary type) synchronized traffic flow. In the subsequent study, Helbing et al. [13] investigated the phase diagram of the different kinds of congested traffic and identified new dynamic phases such as the standing localized cluster (SLC), the triggered stop and go (TSG) and the oscillating congested traffic (OCT) states. Analytical conditions for the existence of these states are discussed and it is found that the phase diagram is universal for a class of traffic-flow models. Lee et al. [14,15] study the phase diagram of the continuum traffic-flow model with an open boundary condition. For several representative values of the upstream boundary flux q_{up}, many traffic states are found for the whole range of the on-ramp flux q_{rmp}. Two non-trivial analytical solutions for SLC and HCT states are also discussed.

In this paper, we study the phase diagram of traffic flow in the presence of an on-ramp using a recently developed continuum model. This new model overcomes the problem of negative flows and negative speeds (i.e., wrong-way travel) that exists in almost every higher-order continuum models and therefore can describe the traffic-flow dynamics more realistically. Aw and Rascle [16] also introduced a model to avoid negative speeds by replacing the space derivative of the study with a convective derivative. Klar and Wegener [17] proposed a derivation of the Aw and Rascle's model.

In the next section, we presented a modified continuum model of traffic flow with an on-ramp. In Section 3, we investigate the possible traffic states for given values of the upstream flux and the input flux from on-ramp. Finally in Section 4, we summarize our results.

2. Mathematical model with an on-ramp

Paralleled with experiments, many physical models have been proposed [18,19]. At present, traffic problems have been investigated by many models: car-following models [8,20,21], the cellular automaton models [9], continuum models [4,16,22–27] and so on. Continuum models differ from car-following models with regard to simulations. With continuum models one has to deal with two coupled partial differential equations instead of a few hundred or even thousands of ordinary differential equations in the car-following case. In contrast to microscopic traffic models, which delineate the position and velocity of all interacting vehicles, macroscopic traffic models restrict to the description of the collective vehicle dynamics in terms of the freeway location and time.

In this work, we adopt the one-lane continuum traffic-flow model, which was first introduced in Ref. [22] and modified to include the effect of an on-ramp.

$$\rho_t + (\rho v)_x = q_{in}(t)\phi(x), \tag{1}$$

$$v_t + vv_x = a[\overline{V}(\rho) - v] + a\overline{V}'(\rho)\left[\frac{\rho_x}{2\rho} + \frac{\rho_{xx}}{6\rho^2} - \frac{\rho_x^2}{2\rho^3}\right] - 2\beta c(\rho)v_x, \tag{2}$$

where $\overline{V}'(\rho) = \frac{d\overline{V}(\rho)}{d\rho}$, $c^2(\rho) = -\frac{a\overline{V}'(\rho)}{2}$ and $\phi(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{x^2}{2\sigma^2}}$.

These two equations are analogous to the continuity equation and the Navier–Stokes equation, respectively. Here $\rho(x,t)$ is the local vehicle density, $v(x,t)$ is the local velocity and traffic flow is understood as $q(x,t) = \rho(x,t)v(x,t)$. $\overline{V}(\rho)$ is the safe velocity that is achieved in the steady-state homogeneous traffic flow. The intrinsic properties on the main highway is prescribed by $\overline{V}(\rho)$ and two parameters a and $c(\rho)$. a is the driver's sensitivity, which is taken as constant in our model while $c(\rho)$ represents the anticipation parameter, which will vary with $\rho(x,t)$. In all previously developed models the anticipation is taken as constant but in real situations anticipation parameters depends on the traffic density [4,24–26]. The source term in the continuity equation represents the external flux through an on-ramp. The spatial distribution of the external flux $\phi(x)$ is localized near on-ramp i.e., $(x = 0)$, and normalized so that $q_{in}(t)$ denotes the total incoming flux at time t.

To study the effect of an on-ramp, we perform extensive numerical simulations over a system of 32.2-km-long highway with an on-ramp right in the middle (at $x = 0$). Open boundary conditions are used. The values of the density and velocity at the upstream boundary are kept fixed at ρ_{up} and $\overline{V}(\rho_{up})$, respectively, while the values at the downstream boundary are obtained by linear extrapolation from the neighborhood. The numerical method developed in Ref. [22] is used to carry out simulations. In this paper, the following parameters are adopted for our model:

$$\beta = 4.0, \quad \sigma = 200\,\text{m}, \quad u_f = 120\,\text{km/h}, \quad \rho_m = 0.2\,\text{veh/m}, \quad T = \frac{1}{a} = 25\,\text{s},$$

$$\overline{V}(\rho) = u_f\left[\left(1 + \exp\left(\frac{\frac{\rho}{\rho_m} - 0.25}{0.06}\right)\right)^{-1} - 3.72\times 10^{-6}\right]. \tag{3}$$

The critical values for our model corresponding to the above parameters are $\rho_{c_1} = 0.034\,\text{veh/m}$, $\rho_{c_2} = 0.096\,\text{veh/m}$, $q_{c_1} = \rho_{c_1}V(\rho_{c_1}) = 3230\,\text{veh/h}$, $q_{c_2} = \rho_{c_2}V(\rho_{c_2}) = 244\,\text{veh/h}$, which can easily be found out by the linear stability analysis [22]. The maximum flow is given by $q_{max} = \max_\rho\{\rho V(\rho)\} = 3350\,\text{veh/h}$. In the previous studies of homogeneous highways without ramps, numerical works also find that the homogeneous flow is stable to large perturbations only when the density and flow are less than another critical value of $\rho_b = 0.03\,\text{veh/m}$ and $q_b = \rho_b V(\rho_b) = 3030\,\text{veh/h}$, respectively. When the traffic density is less than ρ_b, homogeneous flow is the only stable traffic state. If the density is above ρ_b, traffic jams begin to emerge and well-formed clusters moving with constant velocities are observed. So ρ_b plays an important role in finding out the transition between the homogeneous flow and traffic congestion.

3. Phase diagram of the traffic states

Due to the linear instability above ρ_{c_1}, nontrivial effects of the on-ramp are more likely to appear below ρ_{c_1}. The transition between the homogeneous flow and congestion can be triggered by fluctuations of the on-ramp flow. In order to study the criterion of perturbations to induce such transitions, we apply a triggering pulse to a steady state, by changing the values of $q_{in}(t)$ for a short period as

$$q_{in}(t) = \begin{cases} q_{rmp} + \Delta q, & t_0 < t < t_0 + \Delta t, \\ q_{rmp}, & \text{otherwise}, \end{cases} \tag{4}$$

where Δq and Δt are the extra flow and duration of the extra on-ramp flow, respectively. The study of Lee et al. [14] motivated us to choose three values of q_{up}, $2400\,(\ll q_b)\,\text{veh/h}$, $2900\,(< q_b)\,\text{veh/h}$ and

3140 ($>q_b$) veh/h. On the other hand, q_{rmp} is varied continuously up to $q_{rmp}^{max} \equiv q_{max} - q_{up}$. In order to show the phase diagram of the traffic states, we investigated all three values of q_{up} and it is observed that the traffic flow can remain in the free-flow state if the on-ramp flow is less than the critical input flux through the onramp. The critical on-ramp flux q_{rmp}^c is given by

$$q_{rmp}^c = q_c - q_{up}. \tag{5}$$

The on-ramp flow ($0 < q_{rmp} < q_{rmp}^c$) provides a small transition layer near the ramp, which transform the upstream flow into downstream flow. The safe velocity is achieved in both the upstream and downstream end and flow is simply q_{up} and $q_{up} + q_{rmp}$ in upstream and downstream, respectively.

For $q_{up} > q_b$, traffic jam can be created from the free flow by applying a finite-amplitude perturbation. When $q_{rmp} > q_{rmp}^c$, the downstream flux is larger than the instability limit q_c and the free flow becomes linearly unstable with respect to infinitesimal amplitude perturbations.

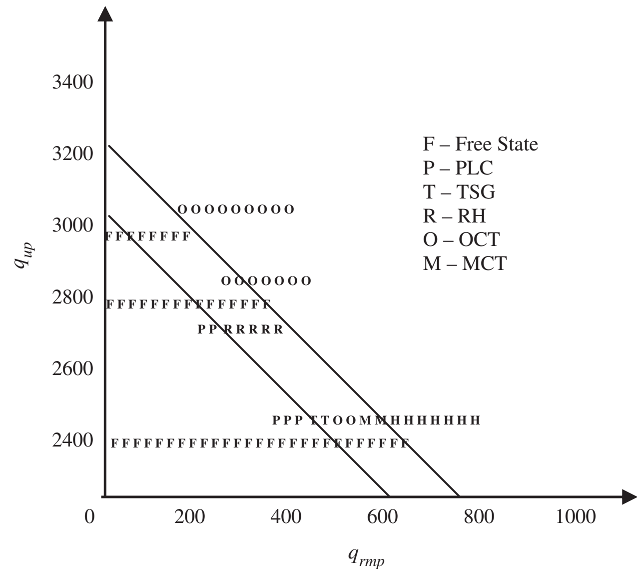

Fig. 1. Phase diagram of traffic states for three representative values of $q_{up} = 2400$ ($\ll q_b$), 2900 ($<q_b$) and 3140 ($>q_b$) veh/h. The dashed line indicate the theoretical phase boundaries, i.e., $q_{rmp} + q_{up} = q_c$ and $q_{rmp} + q_{up} = q_b$. Some symbols are shifted vertically for illustrative purpose.

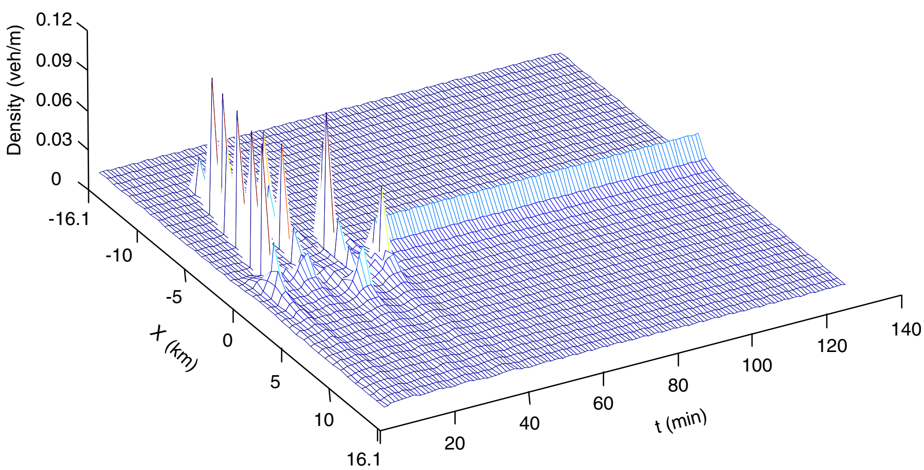

Fig. 2. The spatio-temporal evolution of the density of PLC state for $q_{up} = 2900$ veh/h and $q_{rmp} = 189$ veh/h. The on-ramp is at $x = 0$ km.

Fig. 1 represents the phase diagram for three representative values of q_{up}. As it is already discussed, free flow can exist until $q_{rmp} \geqslant q_{rmp}^c$. For both the $q_{up} = 2400$ and 2900 veh/h, a nontrivial time-independent traffic state appears. This state can be generated in our simulation from the free flow by applying a triggering pulse (4). In this state, the density and velocity are homogeneous far from the on-ramp in both the upstream and downstream directions. Similar type of state is also predicted in Refs. [13–15,27] and named as pinned localized cluster/standing localized cluster (PLC/SLC). Fig. 2 describes the formation of PLC. When we gave a small perturbation in q_{rmp}, the perturbation moves in the downstream direction with increasing amplitude. After some time it moves backward and localized near the on-ramp with a definite shape. For $q_{up} = 2900$ veh/h, the PLC state will occur only when the value of q_{rmp} is greater than q_{rmp}^b, while for $q_{up} = 2400$ veh/h, the PLC state appears much before q_{rmp}^b.

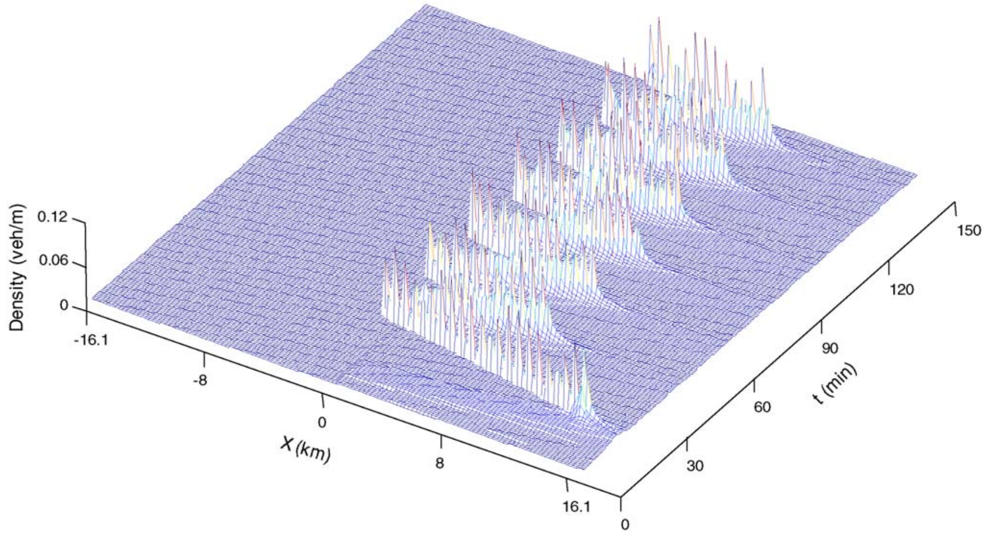

Fig. 3. The evolution of RH state for $q_{up} = 2900$ veh/h and $q_{rmp} = 280$ veh/h. The periodic oscillations are localized near the on-ramp in downstream direction.

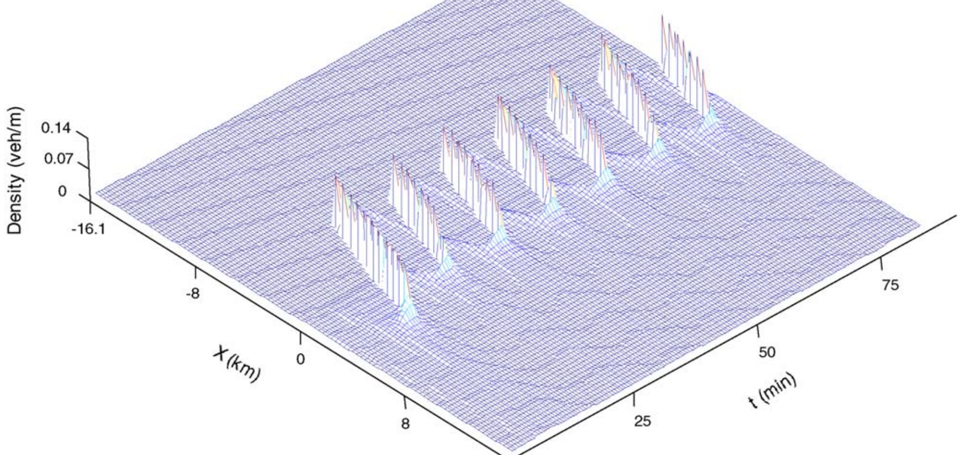

Fig. 4. The spatio-temporal dynamics of TSG state for $q_{up} = 2400$ veh/h and $q_{rmp} = 690$ veh/h.

For $q_{up} = 2900\,\text{veh/h}$, an increase in the value of q_{rmp} gives rise to a different state, called as RH state (recurring hump state). Fig. 3 shows the phase transition from the PLC state to the RH state. It is clear from the figure that in this state (RH state) a cluster does not remain stationary but moves back and forth in a localized region near the on-ramp. Due to the oscillatory pattern in the congested region a different name, oscillating pinned localized cluster (OPLC) is also proposed [28]. In Ref. [12], this state is proposed as the origin of the synchronized flow.

The RH state presents only for the intermediate value of the $q_{up} = 2900\,\text{veh/h}$. This state does not occur for small values of q_{up}, but a different type of state called TSG state is proposed in the phase diagram. Fig. 4, describes the phase transition from the PLC state to the TSG state. In TSG state, when passing the ramp, an initial localized perturbation causes a secondary perturbation traveling in downstream direction [13]. Since the amplitude of this secondary perturbation is always small, regardless of the amplitude of the primary perturbation, it grows its amplitude and the triggered perturbation changes its direction and propagation speed and finally induces another small perturbation when passing the ramp.

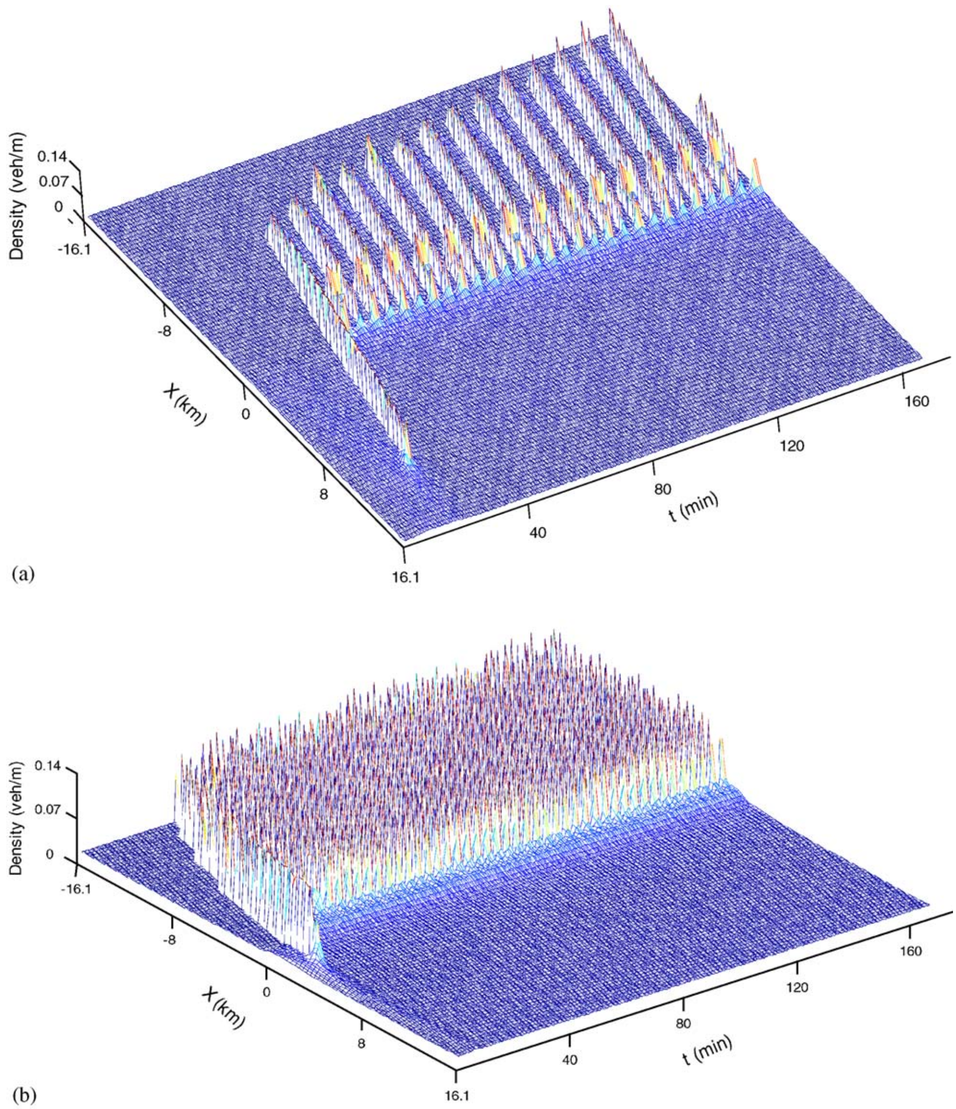

Fig. 5. The spatio-temporal evolution of OCT state for $q_{up} = 2400\,\text{veh/h}$ and a. $q_{rmp} = 700\,\text{veh/h}$ and b. $q_{rmp} = 730\,\text{veh/h}$. The large value of q_{rmp} generates a large number of clusters.

The OCT state is achieved when we further increase the value of q_{rmp} and the value of q_{up} is small. The phase transition from TSG state to OCT state can easily be seen in Figs. 5a and b. In this state, clusters are self-generated near the on-ramp, repeatedly, and move in the upstream direction. This state is compared with traffic jam state by Lee et al. [14].

It is interesting to notice that the spacing between the clusters depends on the on-ramp flux. Smaller value of on-ramp flux gives rise to an OCT state having larger spacing and vice versa (Figs. 5a and b). For sufficiently small q_{rmp}, one can find out the homogeneous part between the neighboring clusters. Helbing et al. [13] define this state as TSG state and find out a shape boundary between OCT and TSG states by using a gas-kinetic-based model. But Lee et al. [14,15] did not get any sharp phase transition between OCT and TSG states in their simulation.

For the smallest q_{up}, further increase in the value of q_{rmp} leads to state mixed congested traffic (MCT), which is depicted in Fig. 6. It illustrates the transition of OCT state to MCT state, where a part of congested region near the on-ramp is homogeneous and the rest of the region is filled with clusters. This state is not seen by Helbing et al. [13]. In Ref. [14] a direct transition from OCT state to HCT state is proposed, while in our simulation MCT state is an intermediate state between OCT and HCT states. This will validate the results found by Lee et al. [14]. It is clear from the Fig. 7 that the congested region is almost homogeneous and expands backwards monotonically in this state. This state is similar to traffic shock caused by a large on-ramp flux, which works as a bottleneck for such type of traffic jams. Lee et al. [14,15] relate this state to the stationary synchronized flow.

Therefore, the above result shows a good agreement with the results found by Lee et al. [12,14,15], Helbing and Treiber [7] and Helbing et al. [13].

4. Conclusion

In the literature of macroscopic traffic-flow theory, the model plays an important role in explaining certain aspects of the system. Recent experimental observations of real-traffic patterns have discovered new phases of traffic flow such as synchronized flow and revealed a large number of features that are consistent with the diverse nonlinear dynamical phenomena observed in real-traffic flow.

In this paper, we investigated an anisotropic continuum traffic flow model, recently proposed by Gupta and Katiyar [22] with a source term representing the on-ramp flux on a highway. Our model is isotropic and also

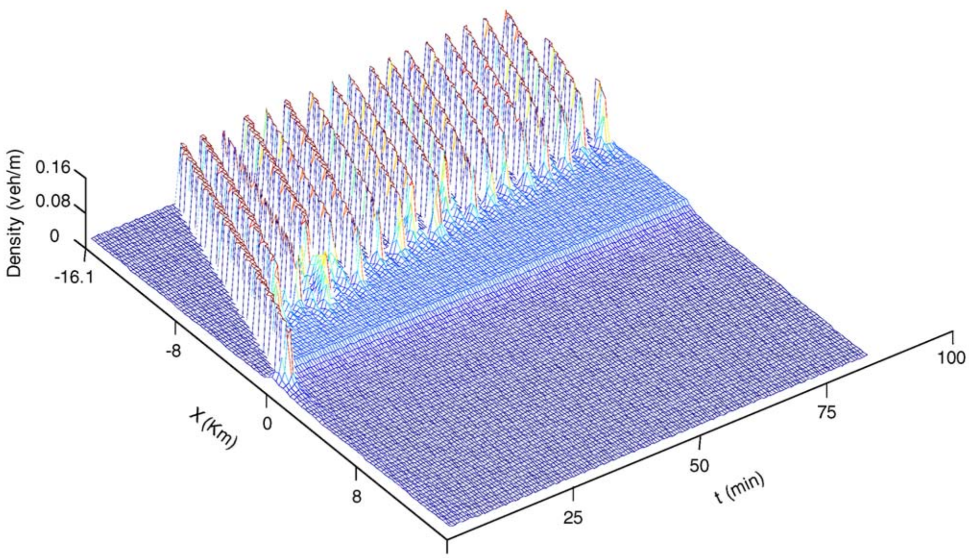

Fig. 6. The spatio-temporal evolution of MCT state for $q_{up} = 2400\,\text{veh/h}$ and $q_{rmp} = 800\,\text{veh/h}$. The congested region contains both the homogeneous and non-homogeneous part.

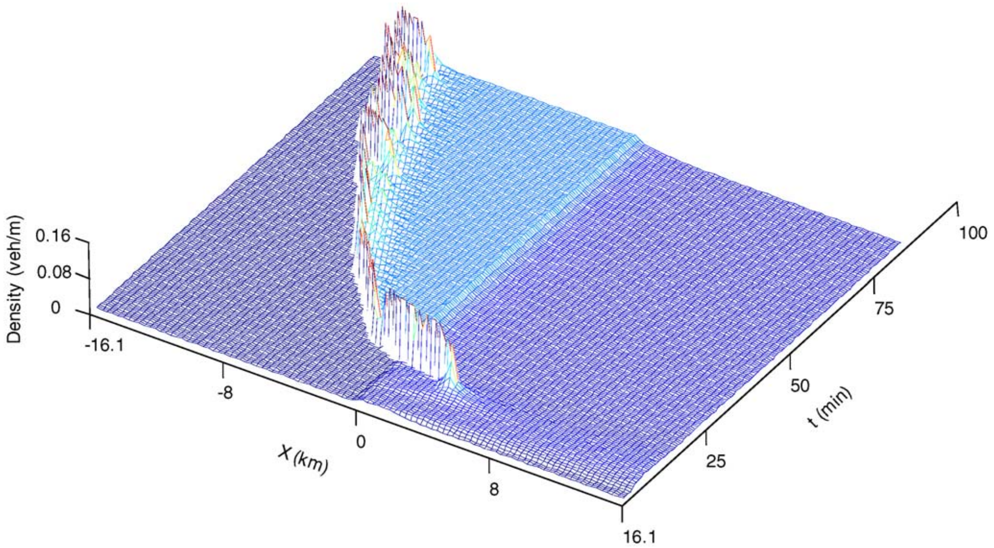

Fig. 7. The spatio-temporal evolution of HCT state for $q_{up} = 2400\,\text{veh/h}$ and $q_{rmp} = 930\,\text{veh/h}$. The congested region is almost homogeneous and expanded with time.

overcomes the problem of negative flows and negative speeds (i.e., wrong-way travel). In order to show that the emodel can describe phase transition, we carry out numerical simulations and find out that the highway with a source term displays a variety of traffic-flow states, which are not earlier studied by the authors [22] in a homogeneous system. For numerical simulation, a simple finite-difference scheme developed in Ref. [22] is adopted with the open boundary condition. To identify the different states in the phase transition on the highway, we varied the on-ramp flux while keeping upstream boundary flux as a constant. By applying a triggering pulse for a short period, various states are identified. The phase diagrams of three representative values of the upstream boundary flux are discussed and are given in Fig. 1. Our findings show good agreement with those presented in Refs. [12–15].

In order to validate the model further investigations like comparison with the empirical data should be carried out. Without having the deficiencies presented in the literature of higher-order models, our model provides an accurate description of traffic-flow situations and is able to explain the complex features of traffic-flow theory. Due to the advance of intelligent transportation system, further research will focus on experimental validation of this general model and control on congestion through on-ramp.

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