Around the world traffic congestion causes a drain on the economy due to time and fuel wasted and collisions associated with traffic congestion. In 2011 congestion in America alone caused a loss of \$121 billion due to 5.5 billion hours of extra travel time and 2.9 billion gallons of wasted fuel. Although congestion levels in America are much higher now than several decades ago, they have dropped below the peak in 2005, but will increase when the economy improves[15]. As personal vehicle ownership increases globally, traffic congestion continues to be a persistent problem. It is no surprise then that research in the physics of traffic is developed with the ultimate goal of mitigating road traffic. Traffic control strategies such as ramp metering and variable speed limits are in place today, but a good model of traffic dynamics is needed for proper coordination of control strategies.

The topic of traffic models is vast and here focus is set on freeway sections. In particular no ramps or intersection are taken. The 1950's saw the development of the Lighthill-Whitham-Richards (LWR) model [9, 14], which became the seminal model for traffic flow, still studied today. This model is a conservation law for vehicles, based on fluid dynamics. Let ρ the lineic vehicular density (veh/m) and q the traffic flux (veh/s), dynamics follow the equation $\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial x} = 0$. The simplicity of the model enables many developments. Simple yet powerful numerical discretization schemes such as Godunov ([8, 12]) or cell transmission models ([6, 7]). These are used in everyday's transportation research (e.g. Berkeley Advanced Traffic Simulation system).

Yet as a first-order model it has inherent shortcomings. Most of these have been pointed out in [5]. The LWR model fails to describe realistically what happens when a vehicle crosses a shock. Although the Rankine-Hugoniot condition can help guarantee macroscopic mass conservation, at the microscopic the speed of travelling particle would be discontinuous. It is also well known that this first order model fails to predict light traffic dynamics accurately. Considering all drivers as similar helps build an elegant model but does not account for plattons disolving because users' desired speeds vary from one person to another. Stop and go behavior otherwise named traffic oscillations cannot appear in the model with expanding amplitudes although this phenomenon is observed in practice. This phenomenon has attracted more an more attention in transportation research. Jamitons, traffic jam that appears without the presence of a bottleneck have been reproduced in small experiments ([16]). Empirical studies have focused on detecting and quantifying this effect ([21]) on actual freeways in day-to-day traffic. Car-following behaviors ([10]) and lane-changing ([4, 1]) are often seen as the cause of these oscillations. Several approaches have been developped so as to model these observations.

This article will focus on macroscopic second order models as opposed to microscopic and mesoscopic approaches. Payne and Whitham have developped in parallel an identical system of equations that aimed at a finer traffic modelling ([13, 17]). Their main aim was describing accurately what happens inside a traffic shock. To that end, they followed the same approach as in fluid dynamics and used a higher order model that would capture momentum related features. The Payne-Whitham model (PW) therefore consists of a mass conservation equation identical to that of LWR seconded by a momentum equation $\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} + \frac{p'(\rho)}{\rho}\frac{\partial \rho}{\partial x} = \frac{V(\rho)-v}{\tau} + \nu\frac{\partial^2 v}{\partial x^2}$ where the pressure function$p$ is strictly increasing and the speed equilibrium V is decreasing. As Daganzo pointed later in his review of second order models ([5]), this approach was flawed both in the derivation of its equations and its predictions. The formulation of PW relies on the assumption that spacing and speed are slow varying which would yield neglibile 2^{nd} and 3^{rd} derivatives for these quantities. This is contradictory with the observations of Newell ([11]) where his car following model predicts sharp and quick changes in these quantities. In terms of physics, modelling traffic by a gas-like model was highly unrealistic as fuilds feature an anysotropic propagation of the information. A given gas particle will be influenced by both those behind and in front of it. Things are very different with traffic where cars will mostly react to downstream conditions. This is incompatible with the slope a characteristic line being greater than cars' speeds in the PW model. Also, at the upstream end of a jam with low density, traffic would flow in reverse if $\nu \neq 0$ which is completely irrealistic. Finally, gas particles are physically identical when drivers have different habits and desired speeds. The heterogenoues composition of traffic is not taken into account in the PW model, hence another flaw.

After this requiem, second order traffic models where not forgotten though. Zhang improved the mathematical structure of his first model ([19]) which was highly similar to that of PW in its momentum equation $\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}+\rho\left(V_*^{'}(\rho)\right)^2\frac{\partial \rho}{\partial x} = \frac{V(\rho)-v}{\tau}$ where $\rho V_*^{'}(\rho)$ represents the information propagation speed in traffic. This improvement was conducted indepently of Aw and Rascle and eventually gave birth to an identical model (ARZ) ([2, 18]). The second order equation in that model, $\frac{\partial(v+p(\rho))}{\partial t} + v\frac{\partial(v+p(\rho))}{\partial x} = \frac{\rho(V(\rho)-v)}{\tau}$,does not present any wrong way travels. Indeed in corresponds to a setup of the PW model where $\nu = 0$. Also, it does not feature any gas-like behavior that contradicts the elementary physics of traffic. The key element for the model to present that feature was formulating the evolution of the pseudo-pressure term $v + p(\rho)$ in a convective manner. This indeed changed the information propagation from a gas-like anysotropic behavior to a quantity that would be carried by the particles in the system at speed v. This model has since been thoroughly studied. Talk about furthur developments (take where it is cited). Talk about Godunov scheme.

Want a model that is suitable for all traffic regimes (need for control).

Talk about behavioral models old ([11]) and recent ([3]) and data driven approaches ([20]).

Similarities with Saint-Venant. Cite Bayen's work with Saint-Venant.

The objective of this article is two-fold. First we aim at developping strategies that enforce the readability and easiness of use of the ARZ model. Solutions to these non-linear equations are practically hard to derive in an efficient manner. Linearizing the system about an equilibrium in therefore a sound strategy to open the way to usual control schemes with multiple inputs and outputs. In particular the techniques that have extensively developed by X. Litrico in [] can readily be applied to that framework. The other objective is assessing the fit quality of the model by practically comparing its output with actual data collected as part of the NGSIM project. In particular, the second order linearized model will be compared to its first order counterparts.

This article presents several new contributions

Oscillations without wavelets.

Objectives
Contribution
Organization

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