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Phase diagram of a continuum traffic flow model with a static bottleneck

Arvind Kumar Gupta · Isha Dhiman

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Abstract The phase transitions are investigated in a continuum speed-gradient model with a static bottleneck under open boundary conditions. The bottleneck situation has been studied using two different approaches—explicit and implicit. The phase diagrams showing different traffic states are presented. The effect of strength of bottleneck has been analyzed, and it is found that the strength parameter has no qualitative effect in the explicit case while has considerable effect in the implicit case. Furthermore, the results of both the approaches are compared, and the consistency between them is discussed.

Keywords Phase transition · Static bottleneck · Traffic flow model

1 Introduction

From the past few decades, the growing traffic congestion on roads has become a burden for most of the nations. Traffic congestion is characterized by slowing down of vehicles, lesser free space on roads and increased vehicular queueing which eventually leads to traffic jam. The aim of recent studies is to understand complex traffic phenomena arising in real traffic due to various reasons. Many efforts have been made in the past to investigate the effects of different traffic conditions such as road structure, driver's forecast effect, driving resistance and potential lane-changing using continuum as well as car-following modeling approach [3,9,15,28–30,32,34,35,39].

One of the commonly arising disrupting circumstances is the bottleneck situation. Many factors can be responsible for bottleneck situations like a road accident, construction on roads, narrowing down of roads, wrong timings of traffic lights and a slow moving vehicle (moving bottleneck). The speed of vehicles is significantly affected by the presence of bottleneck. In addition to this, the bottleneck results into some complex traffic phenomena such as stop-and-go traffic and long queue of vehicles.

The physical phenomenon of bottleneck has been given importance in the past [2,6,14,38]. The interruption caused by a bottleneck has been studied using macroscopic [34] as well as car-following modeling approach [33]. Due to the lack of explicit consideration of the bottleneck into model equations, almost all models were unable to effectively describe the dynamical properties of traffic in the presence of bottleneck. In this direction, Lattanzio et al. [17] proposed a continuum model by incorporating moving bottleneck explicitly. This model lacks to preserve conservation of vehicles and violates the anisotropic feature of traffic flow. Recently, Tang et al. improved the shortcomings of Lattanzio's model and examined its effects on various traffic flow phenomena such as shock and rarefaction waves, equilibrium flux in the case of static [36] as well

A. K. Gupta (✉) · I. Dhiman
Department of Mathematics, Indian Institute of Technology Ropar, Rupnagar 140001, Punjab, India
e-mail: akgupta@iitrpr.ac.in

as multi-static bottleneck [37]. The effect of bottleneck has also been investigated in multi-lane systems [4,21].

The phase diagram is an effective tool to investigate the dependency of traffic patterns on the model parameters. Since the study of phase transitions provides close insight about the traffic dynamics, this phenomenon has attracted the interest of many researchers. The phase transitions of the possible traffic patterns induced by an on-ramp on a freeway have been thoroughly studied using macroscopic [10,12,13,18,19] as well as microscopic approaches. The phenomenon of phase transitions due to bottleneck has also been studied in car-following models. Nagatani discussed the jamming transitions between oscillatory congested traffic (OCT) and homogeneous congested traffic (HCT) in a car-following model with a static bottleneck under periodic boundary conditions [22] and presented the phase diagram for noisy traffic state with open boundary conditions [23]. So far, the phase transitions in the presence of a single static bottleneck have not been studied for continuum models.

In this paper, we analyze the effects of a static bottleneck on the road qualitatively using two different approaches and classify the traffic patterns into different phases. In Sect. 2, we present explicit and implicit approaches to model the static bottleneck. The phase diagram and the different phase transitions for both the approaches are discussed in Sect. 3. In Sect. 4, we present a comparison between the proposed approaches. At last, we summarize our results.

2 Mathematical model with a static bottleneck

In literature, the simplest continuum traffic flow model (LWR model) for unidirectional one-lane road was developed independently by Lighthill and Whitham [20] and Richards [26] and can be expressed by the following conservation law:

$$\rho_t + (\rho V_e(\rho))_x = 0 \tag{1}$$

where ρ is the density and $V_e(\rho)$ is the equilibrium speed, which is given by steady-state strictly decreasing relationship between highway speed and density. The equilibrium speed–density relationship is fundamental as it gives direct connection to everyday driving experience, i.e., how a drivers speed is influenced by the presence of other vehicles in its vicinity and also takes care of traffic regulations and road conditions. This can be acquired either by fitting an analytic expression to real traffic data or by employing phenomenological assumptions about driver's behavior. Generally, the speed–density function should possess the following attributes.

1. $V_e(\rho = 0) = v_{max}$ (free velocity).
2. $V_e(\rho = \rho_m) = 0$; ρ_m is maximum or jam density.
3. $\frac{dV_e(\rho)}{d\rho} < 0$ and $\frac{d^2(\rho V_e(\rho))}{d\rho^2} < 0$.

Although LWR model exhibits a wide range of phenomena such as traffic sound waves, shock and rarefaction waves, it has some limitations due to the assumption of no deviation of speed from equilibrium value. So, the LWR model cannot explain non-equilibrium situations such as stop-and-go traffic. To conquer this drawback, later, several higher order models were developed [1,7,8,11,24,25,27]. These models use two partial differential equations (mass conservation and momentum conservation) in analogy with one-dimensional fluid flow. A general class of these models can be represented by

$$\rho_t + (\rho v)_x = 0 \tag{2}$$

$$v_t + v v_x = -\left(\frac{1}{\rho}\right) P_x + \frac{1}{\tau}\big(V_e(\rho) - v\big) \tag{3}$$

where $P = P(\rho, v)$ denotes the traffic pressure term and τ is relaxation time. As a specific case, if $P = P(\rho)$ i.e. pressure is a function of density only, the models are classified as density-gradient (DG) models [24]. On the other hand, if $P = P(v)$ i.e. pressure is a function of velocity only, then the models are known as speed-gradient (SG) models [15].

However, Daganzo [5] pointed out an important limitation of many second-order models which was physically unacceptable. In these models, the future conditions of traffic were affected by traffic conditions behind the flow which violated the anisotropic behavior of vehicles. To overcome this problem, Jiang et al. [15] proposed a continuum SG model as

$$\rho_t + (\rho v)_x = 0 \tag{4}$$

$$v_t + v v_x = \frac{1}{\tau}\big(V_e(\rho) - v\big) + c_0 v_x \tag{5}$$

where c_0 is the propagation speed of a small perturbation. This model is anisotropic and can describe some complex traffic phenomena [15].

The common occurrence of bottleneck on highways adds to the importance of studying bottleneck situation. We present two different approaches to investigate the impacts of bottleneck on the uniform flow. In order to examine phase transitions due to bottleneck, we analyze the SG model (Eqs. (4), (5)). The phase diagram as well as different phase transitions obtained using both the approaches is presented in Sect. 3.

2.1 Explicit approach

Due to the importance of bottleneck in traffic flow, substantial work has been done on studying the bottleneck situations [2,6,14,17,36–38] but these models are not suited to study directly the impact of bottleneck as they did not consider this situation explicitly. To describe this effect, recently, Lattanzio et al. [17] proposed a model with a moving bottleneck given by:

$$\begin{aligned} &\rho_t + f(x, y(t), \rho)_x = 0\,; \qquad \rho(0,x) = \rho_0(x) \\ &\dot{y}(t) = w(\rho(t,y)) \qquad\qquad y(0) = y_0 \end{aligned} \tag{6}$$

where $y = y(t)$ and $w(\rho)$ are bottleneck's position and its speed, respectively. The flux function f is defined as follows:

$$f(x, y(t), \rho) = \rho v(\rho)\phi(x - y(t)) \tag{7}$$

where $\phi(x - y(t))$ is the cutoff function representing capacity drop due to the presence of bottleneck. Equation (7) shows that the flux will be reduced near the bottleneck. So, Eqs. (6) and (7) can be used to examine the effects of moving bottleneck on dynamic properties of traffic flow. However, Tang et al. [36] pointed out that not only this model violates the conservation law of traffic flow, but also fails to satisfy anisotropic property of traffic flow. Tang et al. [36] proposed a modified SG model for the static bottleneck by incorporating the friction effect due to the flux ($\tilde{q}$) produced by the static bottleneck explicitly as follows:

$$\rho_t + (\rho v)_x = 0 \tag{8}$$

$$v_t + v v_x = \frac{1}{\tau}\big(V_e(\rho) - v\big) + c_0 v_x + F \tag{9}$$

Here, the friction term F signifies the explicit consideration of bottleneck. It was assumed that the friction produced by a static bottleneck is not qualitatively different from the friction produced by an on-ramp [31], and so, we have:

$$F = -\mu s(x,t)\rho v \tag{10}$$

where μ is friction coefficient and $s(x,t) = \tilde{q}/L_0$, L_0 is the length of bottleneck and $\tilde{q}$ is given by

$$\tilde{q} = \rho v(\rho)(1 - \phi(x - y)) \tag{11}$$

where y is the location of static bottleneck. In real-world traffic, the terms $s(x,t)$ and friction effect F depend upon many factors like structure of bottleneck and strength of bottleneck. This dependency can make these terms complex to model.

Although Tang et al. [36] examined the effect of bottleneck on many traffic phenomena such as equilibrium flux and perturbation, it is worth to mention that the important phenomena of phase transitions produced due to a static bottleneck have not been studied so far. In this direction, we examine the continuum model given by Eqs. (8), (9), (10) and (11) under open boundary conditions to identify various traffic patterns in the presence of a static bottleneck.

2.2 Implicit approach

As pointed out by Lighthill and Whitham [20], the maximum value of flow rate reduces inside the region of bottleneck. The flow–density curve reduces in its vertical scale. The jam density (ρ_m) also reduces inside the bottleneck, and this reduction in jam density can be ignored if we consider the region of bottleneck only on a small portion of a long highway. Due to the capacity dropping, each vehicle will experience a reduction in its velocity on passing through the bottleneck which further leads to reduction in the magnitude of free velocity. Hence, it is well justified to investigate the effect of bottleneck in relation to free velocity (v_f). More the capacity dropping due to the bottleneck, more will be the reduction in free velocity. Thus, decreasing values of v_f inside bottleneck can be thought of as increasing the bottleneck strength on road. On the similar lines, Nagatani [23] also analyzed the phase transitions due to bottleneck using a car-following model by adding a noise in the traffic flow. It will be interesting to see whether this approach is able to generate different traffic patterns in the presence of a static bottleneck for a continuum model.

We consider a one-dimensional road of length L with a static bottleneck which is positioned on a small region between L_1 and L_2 $(0 < L_1 < L_2 < L)$. For $x \in [0, L_1) \cup (L_2, L]$, i.e. the region outside the bottleneck, the traffic dynamics are governed by the speed-gradient continuum model given by Eqs. (4) and (5), while for $x \in [L_1, L_2]$, the flow is governed by the modified SG model (Eqs. (4) and (5)) in which velocity-dynamics equation is being replaced with the following equation

$$v_t + v v_x = \frac{1}{\tau}\big(V_{eb}(\rho) - v\big) + c_0 v_x \tag{12}$$

where $V_{eb}(\rho) < V_e(\rho)$ is the modified equilibrium speed inside the bottleneck. The main idea behind this approach is that the vehicle will try to gain equilibrium speed even inside the bottleneck region, which is obviously lesser than the equilibrium speed outside the bottleneck due to the capacity dropping inside the bottleneck.

3 Phase diagram

In this section, we investigate the phase diagram of the traffic flow with a static bottleneck. Numerical simulation is conducted on a road of length $L = 32.2$ km under open boundary conditions. Since we wish to study the qualitative behavior of various traffic patterns, we adopt the well-recognized numerical scheme [8,10,15,31,37] for SG model in both the approaches given by Eqs. 13, 14 and 15. Applying finite-difference scheme to model given by Eqs. 8 and 9, we get

$$\rho_i^{j+1} = \rho_i^j + \frac{\Delta t}{\Delta x}\left(\rho_{i-1}^j v_i^j - \rho_i^j v_{i+1}^j\right) \tag{13}$$

(a) If traffic is heavy i.e. $v_i^j < c_0$,

$$v_i^{j+1} = v_i^j + \frac{\Delta t}{\Delta x}\left(c_0 - v_i^j\right)\left(v_{i+1}^j - v_i^j\right) + \frac{\Delta t}{\tau}\left(V_e - v_i^j\right) + g_i^j \tag{14}$$

(b) If traffic is light i.e. $v_i^j \geq c_0$,

$$v_i^{j+1} = v_i^j + \frac{\Delta t}{\Delta x}\left(c_0 - v_i^j\right)\left(v_i^j - v_{i-1}^j\right) + \frac{\Delta t}{\tau}\left(V_e - v_i^j\right) + g_i^j \tag{15}$$

where $g_i^j = -\frac{\Delta t \mu (1-\phi)(\rho_i^j v_i^j)^2}{L_0}$ and $g_i^j = 0$ for explicit and implicit approaches, respectively. Here, i and j denote the road section and time, respectively. The above difference scheme is suitable for the traffic flow as it maintains the physical properties of the traffic flow even under extreme conditions. The location of bottleneck is assumed to be in the middle of the road. The parameters are set as: $v_f = 30$ m/s, $\rho_m = 0.2$ veh/m, $\tau = 10$ s, $\Delta t = 1$ s, $\Delta x = 100$ m and $c_0 = 10$ m/s.

3.1 Phase diagram with explicit approach

Now, we wish to explore the phase transitions of traffic states qualitatively by considering the effect of bottleneck explicitly. For simplicity, we assume that the cutoff function $\phi(x, t)$ is not depending upon the density. We define the structure of bottleneck given by cutoff function as follows [36]:

$$\phi(\xi) = \begin{cases} 1 & ; \ \xi > 0 \quad or \quad \xi \leq -2\delta \\ \beta & ; \ -\delta \leq \xi \leq 0 \\ (\beta - 1)\xi/\delta + 2\beta - 1 & ; \ -2\delta \leq \xi < -\delta \end{cases}$$

where $\delta > 0$, $0 < \beta < 1$ are parameters representing the strength and scope that the bottleneck affects at its upstream, respectively. Under the assumption that even in the presence of bottleneck, drivers will try to attain same equilibrium speed, we adopt the following equilibrium speed–density relation [16]:

$$V_e(\rho) = v_f\left[\left(1 + exp\left(\frac{\frac{\rho}{\rho_m} - 0.25}{0.06}\right)\right)^{-1} - 3.72 \times 10^{-6}\right] \tag{16}$$

Here, v_f and ρ_m are free velocity and jam density, respectively. The other parameters required in this approach are set as

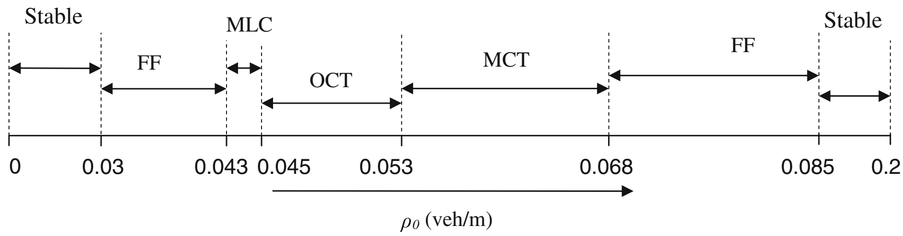
$\beta = 0.8, \quad \delta = 100$ m, $\quad \mu = 0.1 \quad$ and $\quad L_0 = 100$ m

In order to qualitatively characterize the distinct patterns of traffic flow, we look for various phases inside the region of instability. It is expected that on a long highway, the presence of static bottleneck on a small portion will not affect the region of instability. The stability criterion for SG model without bottleneck has been found as [15]

$$\rho_0 V_e'(\rho) < -c_0 \tag{17}$$

With the aforementioned parameters used in this approach, the instability region comes out to be $0.03 \leq \rho_0 \leq 0.085$.

Fig. 1 Phase diagram with explicit approach



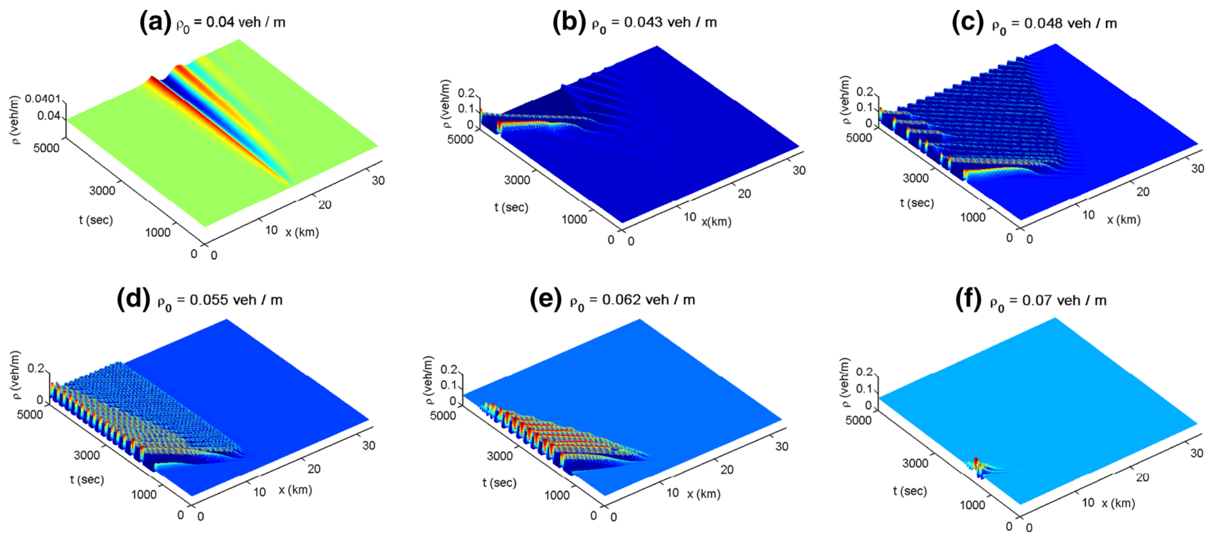


Fig. 2 Spatiotemporal evolution of different traffic states **a** Perturbation localized at bottleneck **b** MLC **c** OCT **d** MCT **e** Intermediate phase between MCT and free flow **f** Free-flow state

Various traffic patterns are obtained for different values of initial homogeneous density and are represented in the form of phase diagram (Fig. 1). The spatiotemporal evolution of different phases (Fig. 2) depicts that the presence of bottleneck on the road affects the traffic flow significantly.

It is found that for smaller values of initial density, even inside the region of instability, i.e. $0.03 < \rho_0 \leq 0.039$ veh/m, the bottleneck has no significant effect on the uniformity of traffic flow. In this region, the perturbation induced by bottleneck is localized on the position of bottleneck itself with considerably small amplitude. The density in both upstream and downstream directions remains homogeneous, and a small transition layer exists at the position of bottleneck. Such state is called as free-flow state. It is to be noted that although we are inside the region of instability, we are not getting disruptions in traffic flow even in the presence of bottleneck. This can be understood as follows. Since the density is low, vehicles will find sufficient space even inside the bottleneck and move with the desired velocity. Tang et al. [36] also pointed that the static bottleneck has negative effect on the small perturbation when initial density is relatively low as driver's expected speed is relatively high. As soon as ρ_0 becomes 0.04 veh/m, the disturbance due to bottleneck starts to increase its amplitude slowly but remains localized at bottleneck. After sufficiently long time, few more small amplitude perturbations are generated in the downstream direction (Fig. 2a). This phase stays up to $\rho_0 = 0.043$ veh/m and for $\rho_0 = 0.043$ veh/m the effect of bottleneck emerges after a very long time (approximately 50 minutes) where two localized perturbations move independently in backward direction and grow their amplitude with time (Fig. 2b). This state can be referred as moving localized clusters (MLC). Further increase in ρ_0 affects the downstream where the primary perturbation emerges and moves backward. This state continues till $\rho_0 = 0.045$ veh/m and can be regarded as an intermediate state between two distinct traffic patterns MLC and oscillatory congested traffic (OCT), the latter of which start emerging at $\rho_0 = 0.045$ veh/m. Here, two backward moving low-amplitude primary perturbations combine to generate a secondary perturbation of higher amplitude (Fig. 2c).

Further increase in value of ρ_0 leads to increase in perturbations in the downstream. This state persists up to $\rho_0 = 0.052$ veh/m until we identify a new traffic phase known as MCT (Mixed Congested Traffic). In this phase, the congested region consists of two types of subregions (Fig. 2d). The region just upstream to the bottleneck is homogenous having density higher than the uniform density and the other subregion consists of backward moving clusters. In this phase, one important observation is that there exist a specific value of initial homogeneous density (ρ_b) such that for $\rho_0 \leq \rho_b$, the region of impact of bottleneck is widely spread in both upstream and downstream direction and for $\rho_0 \geq \rho_b$, the affected region is restricted up to upstream of the bottleneck. In our case, the value of ρ_b is 0.055 veh/m. Further discussion about ρ_b is given in Sect. 4. The traffic continues to remain in MCT state till homogeneous density becomes $\rho_0 = 0.058$ veh/m, when two kinds of subregions merge into single region comprising of uniformly spaced clusters (Fig. 2e). Further increase in ρ_0 leads to decay of clusters with time, and the region of impact due to the bottleneck induced perturbation starts shrinking. The traffic flow is again found to be in stable state for larger values of homogeneous density when $\rho_0 \geq 0.07$ veh/m (Fig. 2f). This value is much lesser than the up critical density ($\rho_{c2} = 0.085$ veh/m). Similar effect has been observed by Tang et al. [36] which supports our results. Hence, the static bottleneck has positive effect on the stability of traffic flow when initial density is relatively high as driver's expected speed is high.

3.2 Phase diagram with implicit approach

To observe the phase transitions between different traffic states in implicit approach, we use the numerical scheme discussed in this section. The modified equilibrium speed for the region inside the bottleneck is given by:

$$V_{eb}(\rho) = v_b\left[\left(1+exp\left(\frac{\frac{\rho}{\rho_m}-0.25}{0.06}\right)\right)^{-1} - 3.72\times 10^{-6}\right] \tag{18}$$

where v_b is the reduced free velocity. For the sake of comparison between two approaches, we have used the similar equilibrium speed–density relation here as given by Eq. 16. The only difference in the equilibrium speed–density relation inside and outside the bottleneck is the free-flow speed. On reaching the entrance of bottleneck region L_1, the vehicle's free velocity v_f reduces to v_b, which remains constant throughout the bottleneck region. As soon as vehicle comes out of the bottleneck region, they will try to attain the same free velocity v_f. The parameters used here are as follows: $v_b = 20$ m/s, $L_1 = 16.0$ km and $L_2 = 16.2$ km.

Figure 3 shows spatiotemporal evolution of different traffic states for different values of initial homogeneous density (ρ_0). For $\rho_0 \leq 0.027$ veh/m, no disruptions in uniform traffic flow are found except with a small rise in the density at the position of bottleneck. This transition layer is completely localized and does not affect upstream as well as downstream regions. At $\rho_0 = 0.028$ veh/m, we observe a drastic change in traffic pattern consisting of two subregions in the upstream direction. A small subregion of homogeneous high density (HCT) is formed just upstream to the bottleneck while the other subregion comprises of high-density uniformly spaced clusters moving backwards (OCT). Such traffic state can be called as OCT-HCT (Fig. 3a). The HCT region remains fixed in its width as time increases while the OCT region grows in upstream direction with time. Additionally, one finds a homogeneous low-density region widely spread in downstream to the bottleneck. This can be understood as when a vehicle comes out of bottleneck, its velocity will increase while the flow rate at this position still equals to the dropped capacity inside the bottleneck. Due to this, the vehicle will observe a decrease in the density which leads to formation of a low-density region in downstream. This observation is in accordance with the findings of LWR model [20]. On increasing ρ_0 further up to 0.031 veh/m, the OCT subregion expands in upstream direction. It is to be noted that this phase exists when ρ_0 is in the stable region of SG model without bottleneck. For further discussion, we refer to Sect. 4. As soon as $\rho_0 = 0.031$ veh/m, we identify a new traffic phase. In this phase, the HCT subregion in the upstream has not changed while we observe that the OCT region further gets divided into two subregions which exhibits MCT pattern as found in explicit case (Sect. 3.1). We call this phase as MCT-HCT (Fig. 3b). The low-density homogeneous region in the downstream shrinks in size

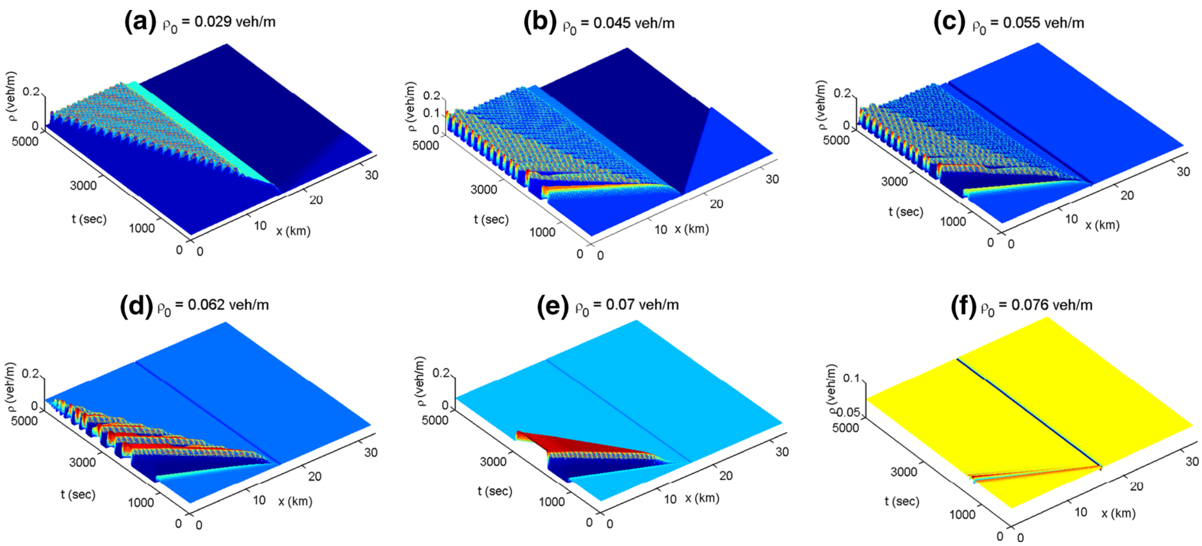


Fig. 3 Spatiotemporal evolution of different traffic states with $v_b = 20$ m/s **a** MLC **b** MCT-HCT **c** MCT-HCT with no bottleneck effect in downstream **d** Intermediate phase between MCT-HCT and HCT **e** HCT **f** Free-flow state

while the traffic pattern in upstream does not change on further increasing initial homogeneous density. At another critical density $\rho_d = 0.055$ veh/m (Fig. 3c), the low-density region in downstream completely disappears leaving only a small fall in the density at the position of bottleneck and the bottleneck effect is now restricted to upstream only (Fig. 4). It is noteworthy that ρ_d is same as ρ_b found in explicit case (Sect. 3.1) which shows consistency in the results of both the approaches. At this point, the density in the HCT region in upstream matches with the density in downstream which forms a homogeneous region. We observe the change in the traffic pattern from this point onwards. On further increasing ρ_0, two secondary perturbations combine to produce a comparatively higher-amplitude perturbation and moves backwards (Fig. 3d). This process continues more effectively as ρ_0 increases and these perturbations move far away from the upstream to the bottleneck with time. The HCT phase is identified at $\rho_0 = 0.068$ veh/m, which dies out completely as initial density becomes $\rho_0 = 0.075$ veh/m. Figure 3e, f shows HCT and stable uniform flow, respectively.

Now, we examine the effect of reduced free velocity on various traffic patterns due to bottleneck. Based on the simulation results, the phase diagram in (ρ_0, v_b) space is obtained and shown in Fig. 5. Smaller values of v_b signify high strength of bottleneck, and $v_b = 30$ m/s represents the uniform road without bottleneck.

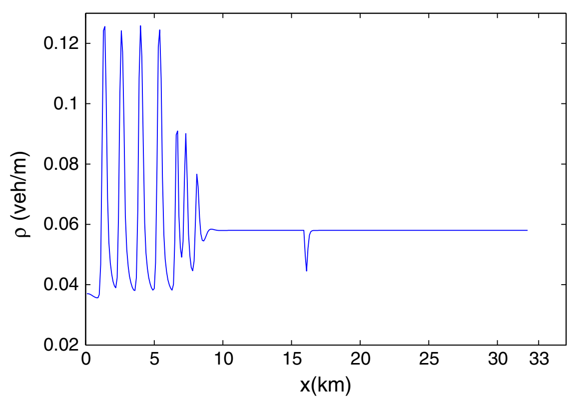


Fig. 4 Density profile corresponding to Fig. 3c at $t = 4{,}000$ s

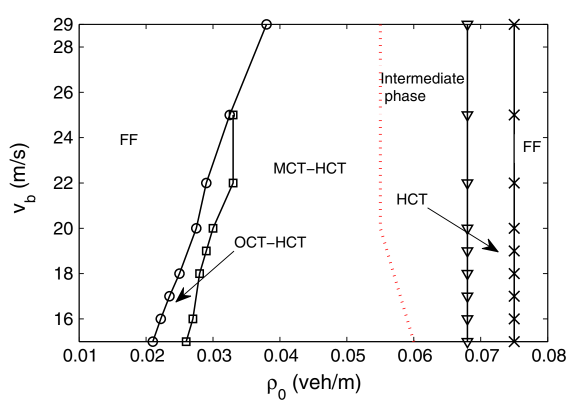


Fig. 5 Phase diagram with implicit approach

The dotted line in the diagram shows the critical value ρ_d after which the effect of bottleneck vanishes in the downstream to the bottleneck. The number of different traffic phases reduces with increase in reduced free velocity. As a specific case, the phase OCT-HCT disappears for $v_b \geq 25$ m/s. The value of critical density ρ_d becomes equal to the value of ρ_b for low strength of bottleneck. Also, in the range $25 \leq v_b \leq 30$ m/s, the congested traffic patterns occur within the region of instability which shows the consistency of our results with those obtained in [15].

4 Comparison between both the approaches

The phase diagram with implicit approach (Fig. 5) represents the phase transitions from one traffic state to another on varying the magnitude of reduced free velocity. The number of phases decreases with decrease in bottleneck strength. Hence, it is reasonable to conclude that strength of bottleneck affects the traffic states considerably.

In the explicit case, we also examined the effect of the strength parameter β and scope of bottleneck δ on traffic patterns and found that the traffic patterns do not change, which shows that these parameters do not play significant role. The effect of strength of bottleneck is unnoticeable in explicit approach which seems unrealistic. In our opinion, further modifications are required to explicitly model the bottleneck which can describe the complex traffic phenomena more realistically.

The cluster effect in explicit approach is found to emerge strictly within the region of instability, while in implicit approach, we found large amplitude perturbations even inside the stability region ($\rho_0 < \rho_{c1}$) for lower values of v_b. This can be explained as follows. Since the stability criterion given by Eq. (17) is obtained for SG model without bottleneck [15], it is quite natural that the stability region will deviate from the results obtained in [15] when the bottleneck strength is high and vice-versa.

In explicit case, the perturbations induced by bottleneck also affect the downstream direction $0.043 \leq \rho_0 \leq \rho_b$ (Fig. 2), while in implicit approach, the homogeneous low-density region exists in the downstream for $0.028 \leq \rho_0 \leq \rho_d$ (Fig. 3). This difference persists for $v_b \leq 25$ m/s, after which we find consistency in the downstream pattern of both the approaches

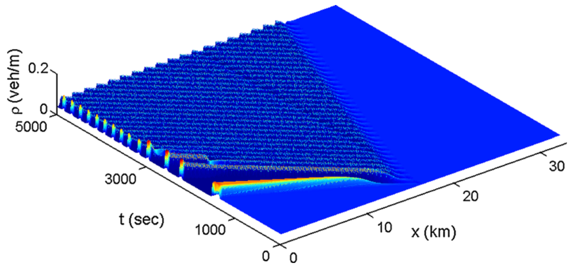


Fig. 6 Traffic pattern for $v_b = 29$ m/s and $\rho_0 = 0.05$ veh/m

as shown in Fig. 6. Additionally, we have found that $\rho_d = \rho_c$ for $v_b \geq 20$ m/s. Note that the value of ρ_b is the minima of the stability curve obtained from Eq. 17. From these observations, we can infer that as v_b approaches to v_f, the bottleneck effect decreases and one finds consistency between the results of both the approaches.

Figures 2 and 3 show that HCT phase is not observed for any value of initial homogeneous density in the explicit approach whereas in the implicit approach, HCT phase exists as an intermediate phase between MCT-HCT and FF.

5 Conclusion

In this paper, we presented two different approaches to investigate the static bottleneck in a continuum traffic flow model. In the explicit case, we adopted the approach proposed by Tang et al. [36] to investigate the phase transitions between different states while in the implicit approach, the bottleneck effect is modeled in terms of reduced free velocity of vehicles inside the bottleneck. Different traffic states like OCT, MCT, OCT-HCT, MCT-HCT and HCT are observed, and the phase transitions from one state to another are discussed. For both the approaches, the phase diagrams are presented and simulation results are compared. It is found that the results in implicit approach are more sensitive to the bottleneck strength parameter as compared to the explicit approach. The proposed modeling approaches might be helpful to understand the consequences of bottleneck more convincingly. The future work will focus on the study of the effect of bottleneck on multi-lane highways [4,21].

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