Jamitons: phantom traffic jams

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ABSTRACT Traffic on motorways can slow down for no apparent reason. Sudden changes in speed by one or two drivers can create a chain reaction that causes a traffic jam for the vehicles that are following. This kind of phantom traffic jam is called a 'jamiton' and the article discusses some of the ways in which traffic engineers produce mathematical models that seek to understand and, if possible, reduce this effect. Ways in which jamitons can be simulated in the classroom are also discussed.

You must have found yourself at some point in your life driving along a motorway at a decent speed when suddenly the cars ahead of you show their brake lights and what was a fast-moving stream of traffic becomes a slow-moving traffic jam. You sit in the car crawling along, sometimes stopping completely, and there is absolutely nothing you can do. As time goes by you think about what awful accident could have caused this terrible jam. Eventually, the cars around you start to move faster, the traffic begins to spread out and soon you are back to moving at your original speed. You see no evidence of an accident and wonder what could have caused this delay.

Jamitons

What you have just experienced is probably a phantom traffic jam – what is called a 'jamiton' by people who research traffic flow. It appears that sometimes, if the circumstances are right, traffic jams can have no apparent cause at all. Perhaps it would be more accurate to say that they can be caused by something really small such as someone braking suddenly, which causes everyone around them in the traffic stream to change their speed. It does not have to be very significant: helicopter traffic observers have seen free-flowing traffic seem to 'stutter' momentarily and then from that point moving in a direction opposite to that of the traffic flow they have watched a jam develop that can bring the stream of vehicles to a halt.

Traffic flow research

Traffic jams are undesirable for a variety of reasons. They waste the time of drivers and their passengers and can result in making people late for school, college or work, late for meetings and late for important deadlines. When traffic jams happen unpredictably, it becomes difficult to estimate journey times with any degree of certainty and drivers tend to set aside more time for a journey to allow for possible delays, taking time away from more useful activities. Vehicles sitting in traffic jams experience more wear and tear because of frequent accelerating and braking and they also produce more pollution. Because vehicles are so much closer to each other, there is an increased risk of collision. People sitting in traffic jams can become tense, frustrated and angry, increasing the possibility of dangerous 'road rage' behaviour and having a negative effect on their health. Of course, traffic jams make it particularly difficult for emergency vehicles to get to their destinations.

Figure 1 A traffic jam on the M25 motorway near London; image © Robin Webster and licensed for reuse under the Creative Commons Licence

For all these reasons there is pressure on road planners and, in England, the Highways Agency to reduce the incidence of traffic jams. Traffic jams that have no apparent cause – jamitons – are a particular focus of interest and have been the subject of research in the last three decades. There have been a number of attempts to construct mathematical models of traffic behaviour in order to identify the conditions under which jamitons can occur and, it is hoped, to be able to put in place measures that reduce the chances of them happening. The variable speed limits on some of our motorways are examples of measures to manage traffic flow and, while they do not stop jamitons from occurring, they can reduce their impact.

Mathematical models

Mathematical models are very much the staple diet of many, perhaps most, researchers in the sciences. A mathematical model that is sufficiently rich to give a good description of what happens in a particular scientific context is very useful. It is a lot cheaper to experiment with mathematics than it is to run expensive practical experiments, and a good model can make usable predictions. The best models are also explanatory and can add to our understanding of the situation.

A range of mathematical models for traffic flow have been used and the main types can be roughly classified into four broad categories. *Microscopic* models try to describe the individual vehicles and their interactions, usually using differential equations that try to capture individual driver behaviour in some way. *Mesoscopic* models use the same kind of mathematics as is used to model the behaviour of gas molecules with a mixture of statistics and mechanics. *Macroscopic* models treat the traffic flow as a kind of fluid and use partial differential equations to model behaviour. These have been particularly successful recently at modelling jamitons. The fourth broad category takes a completely different approach using *cellular automata* and computer simulations. We will look at these last two types in what follows.

Simulations with students

First, however, a physical demonstration of some kind might help bring the problem alive in a classroom context. A typical school classroom with somewhere in the region of 20–30 students provides an ideal resource for experimenting with jamitons.

You need a space big enough for the class to stand in a large circle with one or two metres between them (one is enough; two would be ideal). Get the students to spread out evenly and first practise following each other slowly in a circle, keeping roughly equal distances apart. This will not be as easy as it seems! You might give them a specific minimum distance to maintain. You might also draw two chalk lines as a path for people to follow. This can be done with string and chalk and is a nice exercise for the class to engage in as a brief aside.

There are three key safety issues about this activity:

- reduce the chances of tripping by ensuring the surface is even and clear of obstacles;
- resist the temptation to have them move so fast they start to jog;
- have a clear rule that if anyone calls out '*Stop!*', everyone comes to an immediate halt.

Once you have your class able to maintain a steady circular movement, get them to speed up gradually but to stop short of jogging. It is quite likely that small jamitons will happen spontaneously – it only takes one person in the circle to pause or stumble a little and you will get a compression wave going round the circle in the opposite direction to the way people are walking. However, it is preferable to control the emergence of jamitons, so make sure that you can get regular, quite fast walking in a circle first, maintaining roughly the same distance between each other.

Once you can do this, you can begin to experiment with jamitons. Give the members of the circle a simple rule:

Keep walking as fast as you can but try to keep one arm's length from the person in front of you, slowing down and even stopping to avoid getting too close.

If your circle is quite tight, you may have to adapt this to something like:

Whatever you do, you must not make physical contact with the person in front of you.

Having established this rule, you can pick one person (preferably by some process that results in the members of the circle not knowing who that person is) and tell them that when they feel like it they can slow down a little bit and then catch up again. This should produce a compression wave of people bunching up that moves around the circle and eventually dissipates as people adjust their positions. For the first time, the result is likely to be quite chaotic with people breaking out of the circle. Practice will be needed to get it right (Figure 2).

It would be good if someone could film the activity. Even better would be to film it from above, say through an upstairs window overlooking the circle of walkers. Having once produced a jamiton, you can experiment with the length and nature of the pause by making the pause longer or asking people to catch up again quickly or slowly. You can pick more than one person to do the pause. You could set conditions on how people recover after being slowed down. For example, they could hang back until the person in front of them has reached a longer distance or they could move up as quickly as the rule about distances allows them. There are other rules you could try to apply such as the ones that we use with a cellular automaton simulation (see below).

Macroscopic models and the fundamental diagram

The first macroscopic models were developed in the 1930s. They focused on two measures of

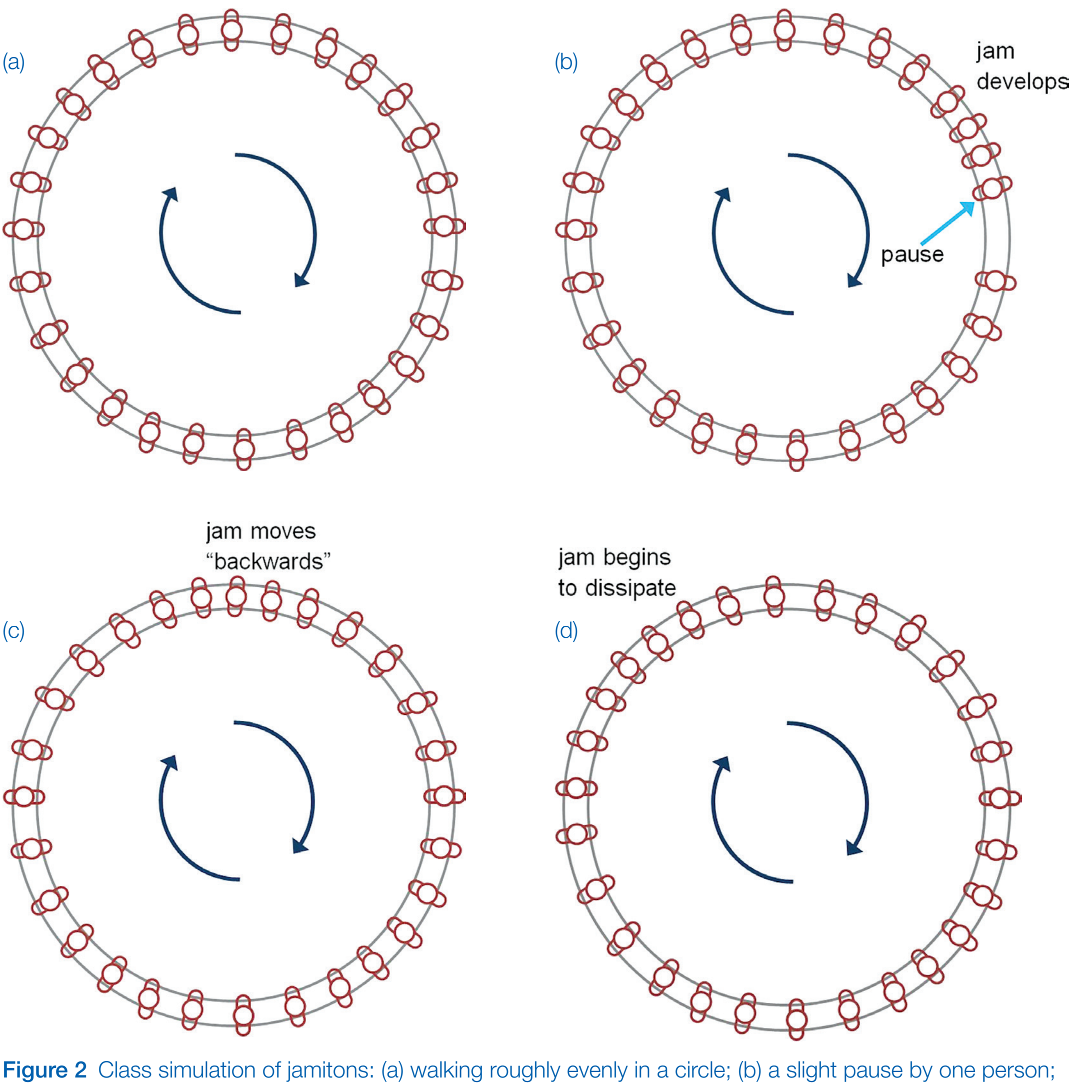

Figure 2 Class simulation of jamitons: (a) walking roughly evenly in a circle; (b) a slight pause by one person; (c) the jam starts to move backwards; (d) the jam begins to dissipate

traffic flow that are relatively easy to obtain and current models still focus on these measures:

- the traffic *density* – the number of vehicles in a given distance at a particular point in time, usually given the symbol ρ;
- the traffic *flowrate* – the number of vehicles passing a particular point on the road in a given period of time, usually given the symbol Q.

What are typical values for these measures? The usual assumption made is that a typical car is about 5 m long (the research was originally done in the USA where cars tend to be longer than in the UK, where the average car length is more like 4 m) and that the closest that cars come to each other is about one-half of a length so that the average space occupied by a car is about 7.5 m. This suggests that at maximum density there would be $100/7.5 = 13.3$, or just over 13 cars in every 100 m. The units of this would be 'cars per 100 metres'. It is more natural, using the SI approach, to use a unit such as 'cars per metre', which, for maximum density, would be $1/7.5 = 0.133$ cars per metre. If we use UK figures, the average space occupied by a car is more like 6 m and the maximum density is $1/6 = 0.167$ cars per metre (Figure 3 and 4). Notice that we are keeping it simple and avoiding the problem of including lorries and buses.

One way around this difference in values is to use a *relative* measure of density. If the maximum density for the particular place where we are doing our modelling is ρ_{max} then we can define the relative density to be $r = \rho/\rho_{max}$. This has no units and varies between zero (no cars on the road) and 1 (when the cars are at maximum density).

The flowrate is particularly easy to measure, since it is just a matter of counting how many cars pass a specific point in a given time. Its units are 'cars per minute' or 'cars per second'. Suppose that cars are travelling on a fast motorway at an average speed of 70 m.p.h. A mile is 1609.34 m, so 70 m.p.h. is $(1609.34 \times 70)/(60 \times 60) = 31.3\ \mathrm{m\,s^{-1}}$.

In the UK, the Government recommends a 2 s gap between cars in traffic. In 2 s a car travels about 62.6 m at 70 m.p.h. so let's assume that drivers are being reasonably sensible and keeping the total distance from the back of any one car in the traffic stream to the back of the car in front of it to 62.6 m. This means that it takes exactly 2 s to cover this distance and that a car passes a particular spot on the road every 2 s. This corresponds to a flowrate of 0.5 cars per second. In reality, drivers are not that sensible and flowrates of 1 or more are often observed, even at high speeds.

The information that led us to a flowrate of 0.5 also tells us what the traffic density is on that stretch of motorway. Since there is one car every 62.6 m, the density is $1/62.6 = 0.016$ cars per metre and, using a UK maximum density of 0.167 cars per metre, this gives a relative density of $0.016/0.167 = 0.096$. Notice that if we divide the

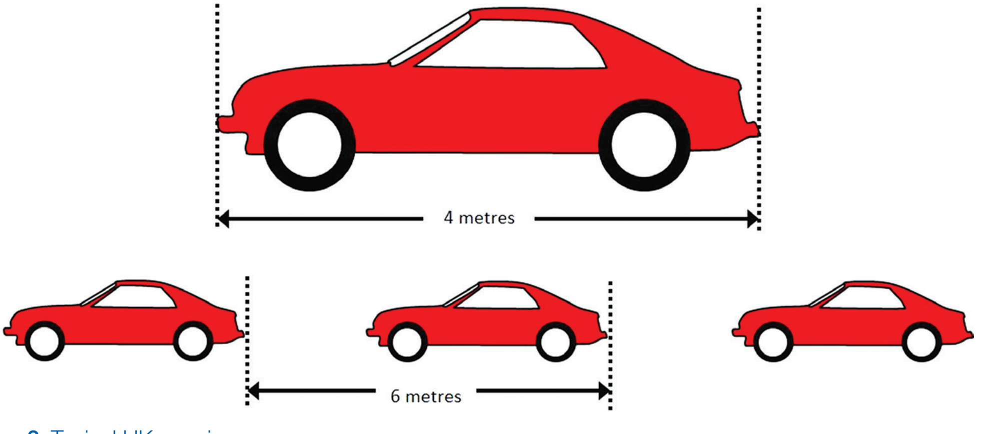

Figure 3 Typical UK car size

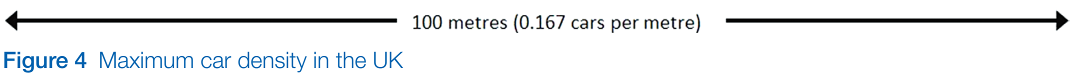

Figure 4 Maximum car density in the UK

flowrate by the density we get 0.5/0.016=31.3, which is the average speed of the traffic in $m\,s^{-1}$.

Early traffic models in the 1930s assumed a simple relationship between traffic flowrate and traffic density. Using relative density $r=\rho/\rho_{max}$, this was expressed as:

$$Q=4Q_{max}\,r(1-r)$$

where Q_{max} is the maximum value of the traffic flowrate.

The resulting graph of traffic flowrate is an upside-down U-shaped quadratic. It crosses the r axis at 0 and 1. When the density is 0, there are no cars on the road and the flowrate is thus also 0. When the density reaches the maximum, cars are as close together as they can be and they have to slow right down. In this simple model, it is assumed that they come to a halt, so the flowrate will again be 0. (In practice, drivers can be rash enough to still be moving at this density!) The largest value of the flowrate is half-way along the range of densities at $r=½$. If we substitute $r=½$ into the equation above, we get:

$$Q=4Q_{max}\,½(1-½)=4Q_{max}\,¼=Q_{max}$$

and that is why we need a factor of 4 so that the maximum value of Q comes out to be Q_{max} and not $Q_{max}/4$.

The graph in Figure 5 shows the simple theoretical relationship between Q and r, whereas Figure 6 shows some actual 2003 data from the Minnesota Department of Transportation. While the shapes of the graphs are vaguely similar, there are also many important differences. The real data are much more spread out and the highest traffic flowrate occurs at a relative density of less than ½. Notice that at the maximum density of 1 there are points on the graph that correspond to a positive flowrate, so that even at that density there are cars still moving.

Graphs such as these are called the 'fundamental diagram' of traffic flow and much mathematical effort has been expended in trying to get a model that produces the kind of diagram that is close to that of actual data, such as in Figure 6.

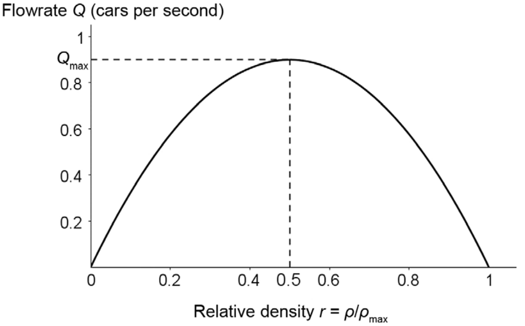

Figure 5 A 1930s quadratic model of the fundamental diagram

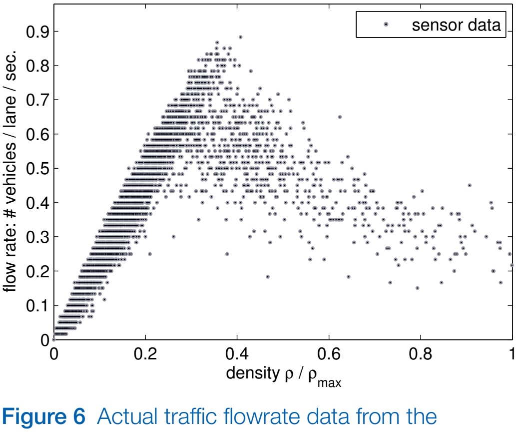

Figure 6 Actual traffic flowrate data from the Minnesota Department of Transportation in 2003; reproduced with permission from Seibold *et al*. (2013)

Two macroscopic models

There are two macroscopic models that stand out in their ability to provide fundamental diagrams that approach the kind of diagram that real traffic flow produces. The first of these is the *Payne–Whitham model*. It still uses the basic quadratic relationship between traffic flow and density but it provides a simple model of the way in which jamitons cause the spreading of points around the basic diagram. In this model, a jamiton happens at a range of traffic densities, which are called the 'sonic' density for each jamiton. It contributes a line of data sets to the fundamental diagram (Figure 7).

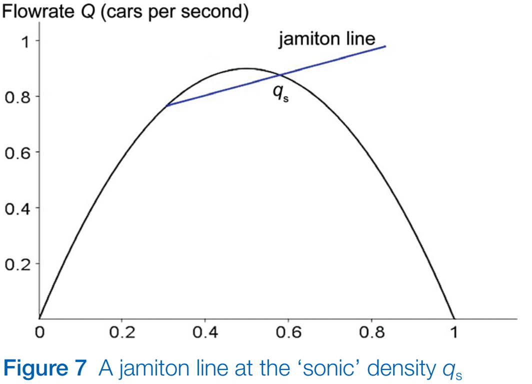

Figure 7 A jamiton line at the 'sonic' density q_s

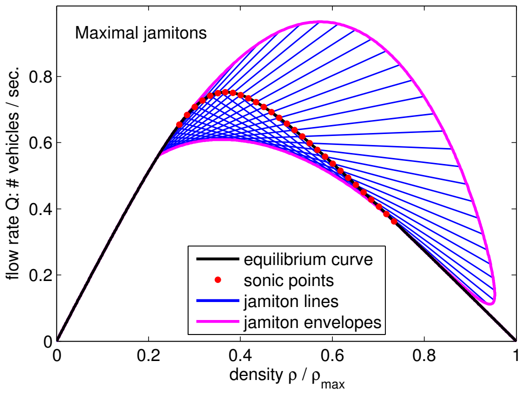

Figure 8 One of the solutions predicted by the Payne-Whitham model; reproduced with permission from Seibold *et al*. (2013)

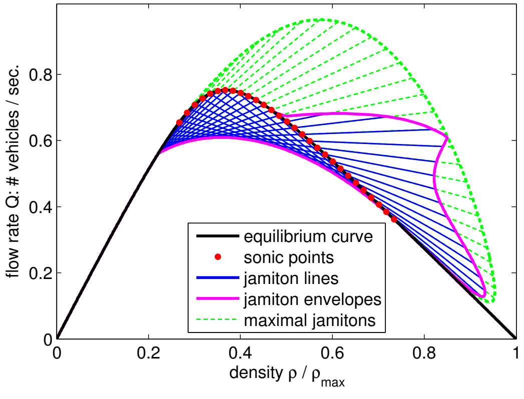

Figure 9 One of the solutions predicted by the Inhomogeneous Aw–Rascle–Zhang model; reproduced with permission from Seibold *et al*. (2013)

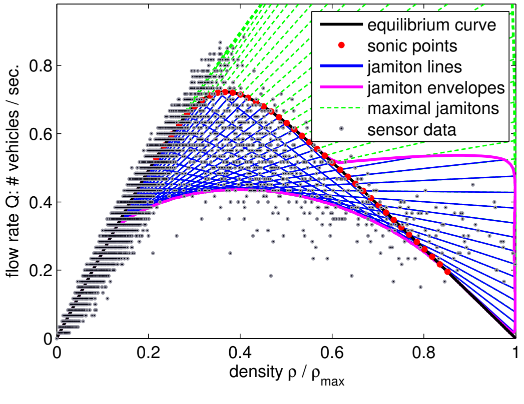

Figure 10 The Inhomogeneous Aw–Rascle–Zhang model superimposed over the Minnesota data; reproduced with permission from Seibold *et al*. (2013)

This jamiton line shows how the traffic 'bunches' and 'spreads' around the jamiton. The slope of the line gives us the speed at which the jamiton moves backwards through the traffic flow.

These jamiton lines can be combined into a fundamental diagram showing the range of different possible jamitons predicted by the model. Figure 8 shows one set of solutions predicted by the Payne–Whitham model. Compared with the actual data in Figure 6, this is beginning to give something that vaguely approaches the kind of thing that happens in practice.

The second of the two macroscopic models is the *Inhomogeneous Aw–Rascle–Zhang model*. While this model differs only in quite small details from the Payne-Whitham model, it results in a different relationship between traffic flowrate and density and produces fundamental diagrams such as the one in Figure 9.

This model has been matched to the Minnesota data in Figure 6. Figure 10 shows the fundamental diagram for the model and for the Minnesota data on the same graph.

While there is still a lot more work to be done, the results obtained from the Aw–Rascle–Zhang model are very encouraging and help us understand the way in which jamitons tend to be formed.

An alternative modelling approach

A completely different way of attempting to model traffic flow behaviour is to use a simple simulation method that is rich enough to mimic actual traffic behaviour. A recent approach has been to use what is called a 'cellular automaton'. The idea of a cellular automaton goes back to John Horton Conway's 'Game of Life' (see *Website*).

For the purposes of simulating traffic flow, our cellular automaton consists of a long strip of boxes or 'cells'. They represent a typical car length with safety space (7.5 m in the USA, 6 m in the UK). Each cell is either blank, meaning there is no car there, or contains a non-negative whole number that represents a car moving at that speed. Figure 11 shows the left-hand end of such a strip.

In this example there are three cells visible with cars moving at speed 2 and four with cars moving at speed 1. The empty cells are spaces between cars.

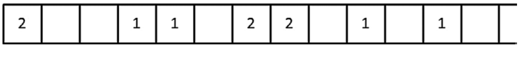

Figure 11 Cellular automaton traffic simulation – starting strip (left-hand end)

There are several different ways of using a cellular automaton to model traffic flow but they all work in the same kind of way. A set of rules is applied in strict sequence to each cell and once these have been applied the new strip represents the traffic flow after a fixed length of time – typically fractions of a second. A very simple set of rules was used in 1992 by Nagel and Schreckenberg that is still rich enough to provide a model that does appear to mimic traffic behaviour.

In their model, there is a low maximum speed, which we can take as just 2, so cars move at speeds 0, 1 and 2 only. There is also a probability level p used in their randomisation rule (Rule 3), which we can set at, say, $p = ⅓$. Nagel and Schreckenberg use four rules, as described below (Figure 12).

- **Rule 1: acceleration**
This rule mimics drivers' tendency to want to go at the highest allowable speed. All cars that have not already reached the maxim speed of 2 accelerate by one unit (so 0 becomes 1 and 1 becomes 2). In our example, all cars are now moving at the top speed.

- **Rule 2: safety distance**
This rule mimics drivers' need to keep a reasonable distance from the car in front. If a car has d empty cells in front of it and its speed after applying Rule 1 is larger than d, then reduce its speed to d. In our example, only one car remains at speed 2, four go to speed 1 and two have stopped.

- **Rule 3: randomisation**
This rule mimics the observed tendency that drivers have to brake too hard occasionally. If the speed is greater than 0 then with probability p the speed is reduced by 1. In our example, we can imagine that we threw a fair six-sided die and if it landed with a 5 or a 6 on top then we applied braking. Let us suppose this happened to the first and third cars moving at speed 1.

- **Rule 4: driving**
This rule actually makes the cars move forward if they can. Each car moves forward the number of cells that corresponds to its speed and it retains its speed. So, cars at speed 0 stay where they are, those at speed 1 move forward one cell and those at speed 2 move forward two cells. Notice that there will always be an empty space for the car to move into because of Rule 2. In our example, one car moves forward two spaces and two move forward one space.

2			2	2		2	2		2		2	

2			0	1		0	1		1		1	

2			0	0		0	1		0		1	

		2	0	0		0		1	0			1

Figure 12 Cellular automaton: after applying Rules 1–4 (top to bottom, respectively)

Figure 13 shows the initial state as well as the state after applying all four rules. This can be thought of as two 'generations' of the simulation. Even in this very simple model, we can see a small jamiton beginning to develop.

2			1	1		2	2		1		1	
		2	0	0		0		1	0			1

Figure 13 Two generations of the cellular automaton traffic simulation, before and after the four rules are applied

Of course, it is quite hard work applying this model by hand but it is particularly well suited to computer simulation. There are free software applications on the internet that show this model being simulated. For example, there is a Java applet at www.thp.uni-koeln.de/~as/Mypage/simulation.html that lets you change some of the parameters of the model and gives a running image of the simulated traffic flow. Figure 14 is an example of the kind of output it produces. The results in this figure show a set of jamitons moving in the opposite direction to the traffic flow. The simulation uses a large circular road with a fixed number of cars moving round it.

The value of this model is that it can be used to simulate specific aspects of driver behaviour and, by running the model with different parameters, the impact of that behaviour can be seen quickly and evaluated. One example of making such a change is a variant of the original Nagel and Schreckenberg model in which different probabilities are used in the randomisation rule (Rule 3) for cars that were initially stationary (smaller probability) and those that were moving. This models the lesser likelihood of a driver who has just started moving to brake suddenly for no apparent reason.

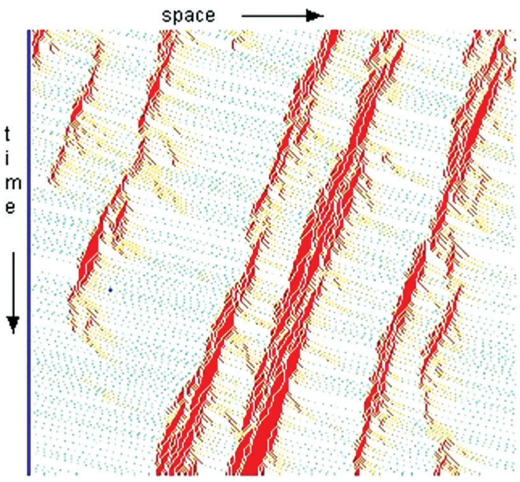

Figure 14 Sample output from the applet at www.thp.uni-koeln.de/~as/Mypage/simulation.html

Modelling with show-me boards

Despite the slow process of evolution of doing the modelling by hand it is, nevertheless, worth using as a demonstration with students. It can also be a lot of fun if it is done in an organised way. One approach is to have students stand in a line or a large circle, each with a show-me board, pen, dice and a small board-eraser. Each show-me board should have a strip of, say, five cells drawn on it (Figure 15). The idea is that the boards are held up next to each other to form a long cellular automaton. Every four or five boards there needs to be a student with a camera (a mobile phone camera will do) facing the board so that a picture can be taken at each generation. Quite a bit of organisation is needed to get the photographs taken and then to stitch them together into a running record of the simulation. Indeed, how to go about organising the simulation is a good project for students in itself.

One consideration is how to change the probability for the randomisation rule. Throwing dice, perhaps combined with a coin, is a particularly good way of doing this and can stimulate a discussion of how to get particular probability values.

Another consideration for this active form of modelling is whether to form a circle or a line. With a circle there is a fixed number of cars and as they come off the end of the line they join on at the beginning again. If a line is used, then some kind of mechanism needs to be agreed for adding new cars on at the beginning as the original cars move along the strip. This could be as simple as 'add a car going at speed 2 to every third cell from the left'.

Once collected, the data can be entered into a spreadsheet and, using the conditional cell formatting facility, a colour code could be used to give a clearer impression of the movement of cars along the 'road'. Depending on the spreadsheet skills of the group, the experiment could be followed up with a challenge to programme the spreadsheet to produce the simulation.

Figure 16 is a snapshot from an *Excel* spreadsheet simulation of a cellular automaton with traffic moving from left to right and time moving downwards (as in Figure 14). Each successive simulation follows the one above it. The red cells are stationary cars, light green are cars moving at speed 1 and dark green are cars moving at speed 2. The white cells are empty. To make the visual impact clearer, the row and column headings are not shown, and distance and time labels have not been added. The figure illustrates how the positions of bunching of traffic move as time passes.

Conclusion

Two very different kinds of mathematical approaches to modelling the flow of traffic have been explored with the intention of understanding phantom traffic jams or 'jamitons'. In each case, a description has been given of how the approach to modelling traffic flow can be demonstrated physically with a typical school class. Understanding traffic flow in general and jamitons in particular continues to be a real and open problem and, so far, what the models appear to

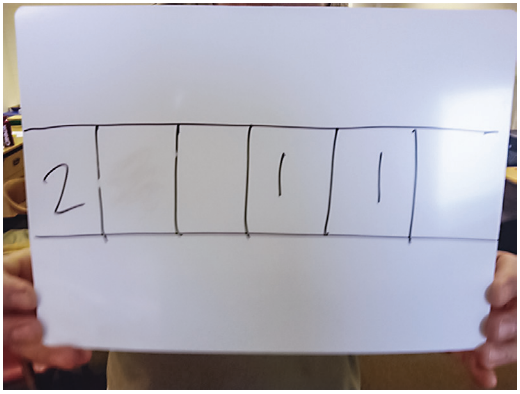

Figure 15 A show-me board element in the long strip

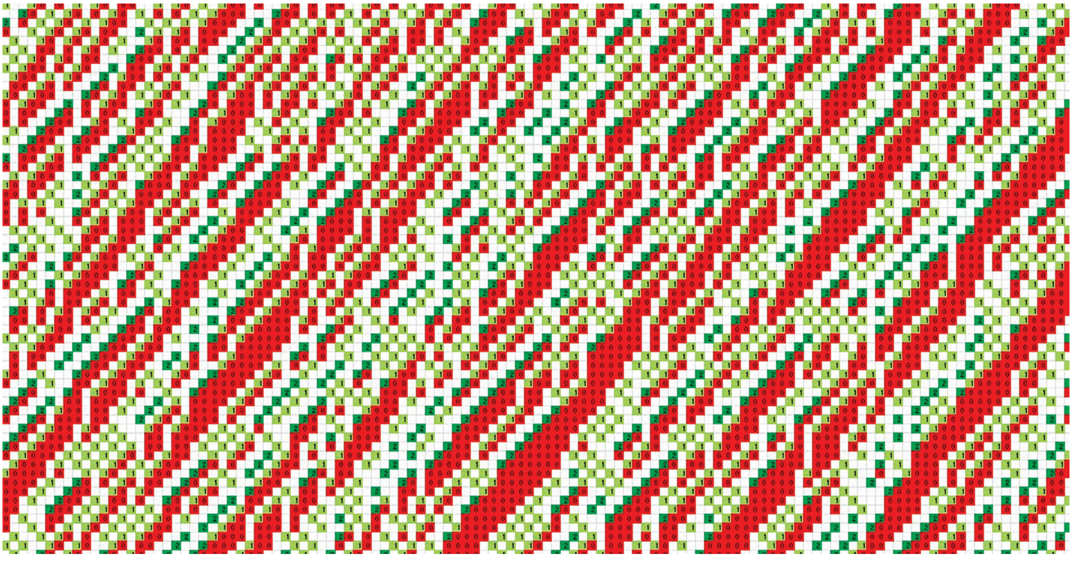

Figure 16 A selection of cells from a spreadsheet simulation with colour-coded cells (cars moving left-to-right; red indicates stationary cars)

show is that jamitons are an endemic feature of any large moving body of traffic in which the drivers have less-than-perfect skills. Perhaps the final solution will have to be the science fiction idea of computer-controlled traffic flow in which there are no sudden changes of speed to generate jamitons!

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Website

John Conway Talks About the Game of Life: http://thinkorthwim.com/2007/05/27/john-conway-talks-about-the-game-of-life/.

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