

The intermediate Jacobian of the cubic threefold

Master's Thesis draft

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Extended Abstract

We begin by reviewing the history and modern theory of Abelian varieties, emphasizing their significance in complex geometry as invariants of more complicated algebraic varieties.

As a motivating example, we discuss the intermediate Jacobian of a smooth cubic threefold. Clemens and Griffiths showed that this principally polarized Abelian fivefold is naturally realized as the Albanese variety of the Fano surface of lines on the threefold. The interplay between the geometry of the Fano surface and the Hodge structure of the threefold yields strong birational information: in particular, the intermediate Jacobian furnishes a transcendental obstruction to rationality, leading to the classical result that the cubic threefold is not rational.

We then place this case study in a broader context by interpreting the intermediate Jacobian of the cubic threefold through the framework of Prym varieties. In this viewpoint, the resulting principally polarized Abelian variety can be described as the Prym variety of an étale double cover of a curve, and the Abel–Jacobi embedding of the Fano surface corresponds to the Abel–Prym map.

Time permitting, we conclude with a brief overview of related transcendental methods in algebraic geometry and their applications to questions of rationality and classification.

