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Rectangular shape management zone delineation using integer linear programming

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ABSTRACT

The spatial variability of soil properties is one of the main impairments to the productivity and crop quality in agriculture. Delineating the field into site-specific management zones is usually implemented to face within-field variability. Classical zoning methods, based on soil fertility variables, have a disadvantage: the zones have oval shapes which are not practical for the variable rate technology and machinery. In this work, we present a new zoning method that optimally delineates rectangular homogeneous management zones, using relative variance to guarantee the homogeneity. This zoning method, based on soil properties, relies on an integer linear programming model that is efficiently solved to optimality. Experimental results on real and generated instances validated the method and enabled a graphical visualization of the solution.

1. Introduction

One of the main aspects of precision agriculture is to provide farming management methods to respond to within-field variability. It relies on new technologies like satellite imagery, information technology, and geospatial tools to improve the decision-making process in agricultural production. As mentioned in Ortega and Santibáñez (2007), in contrast with "traditional" uniform field management, precision agriculture permits the application in a site-specific manner of agronomic practices such as fertilization, weed and pest control, as a function of the information compiled from collected field data. The impact of precision agriculture derives from the fact that most factors determining crop yield and quality are variable in space and time. To be more efficient, management decisions must be time- and site-specific and not rigidly programmed.

Within precision agriculture, an important area is the site-specific nutrient management since there is a need of delineating management zones within fields before planting the crops to improve the overall yield. More precisely, a management zone is a sub-region of a field that is relative homogeneous with respect to soil parameters, and for which a specific rate of inputs is needed (Roudier et al., 2008). Indeed, variable rate technology uses equipment to apply inputs at a precise location to achieve site-specific application rates of inputs to reduce input and labor costs, maximize productivity, and to reduce the impact wastage on the environment. Mainly, variable rate technology in agriculture includes fertilizer, lime, seeding, and pesticides.

As mentioned by Doerge (1999), the most meaningful factors to include in a management zone strategy are those with the most direct effects on crop yield: soil moisture relationships, soil pH, soil pathogen infestation, and extremes in soil nutrient levels (see also Cambardella et al. (1994), Ortega and Flores (1999)).

Trying to delineate management zones efficiently and accurately is a mayor challenge where decision support systems are needed (McBratney et al., 2005). In this study we used data of the soil properties to propose the Integer Linear Programming Management Zone delineation method (ILPMZ for short) based on a mathematical model that could be easily inserted in any decision support system. The main advantage of the zones that the ILPMZ zoning method computes, is that they have a rectangular shape which is an important characteristic for agriculture machinery. Moreover, rectangular parcels (or portions of them) allow easier adoption of variable rate technologies based on prescription maps than irregular parcels. Additionally, this zoning method could also be applied for drip irrigation designs.

There are several approaches in the literature for properly determining site-specific management zones. Most of them are based on clustering algorithms, i.e., they are classification-based approaches.

- Approaches based on information of the soil. For example Schepers et al. (2004) and Fraisse et al. (2001) use soil and relief information; Carr et al. (1991) base their zoning on topographic

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maps; while methods of Mulla (1991), Mortensen et al. (2003), and Bhatti et al. (1991) need soil sampling.

- Approaches based on yield maps, combining data from several seasons. We can cite Blackmore (2000), Diker et al. (2004) and Pedroso et al. (2010). Doerge (1999) pointed out that crop yield patterns from yield maps may not be stable enough across seasons to accurately define management zones without supplemental information.
- Integration of the two previous approaches as in Whelan et al. (2003), Franzen and Nanna (2003), Hornung et al. (2003, 2006). In Roudier et al. (2008) they use a watershed segmentation algorithm where the user can introduce morphologies of the desired zones.

The combination of the different layers of information can be performed by a cluster procedure using K-means or Fuzzy K-means methods (Ortega et al., 2002; Li et al., 2005; Jiang et al., 2011), or principal component analysis with a cluster method (Ortega and Flores, 1999). The Fuzzy K-means algorithm is widely used and the choice of the data layers processed by the clustering is an issue (Jaynes et al., 2005). A major drawback is the resulting fragmentation of the zones (Simbahan and Dobermann, 2006; Frogbrook and Oliver, 2007; Li et al., 2005). Moreover, this zones are oval shaped and disjoint due to the clustering methods.

Fig. 1 schemes the real word problematic we solve in this work: Some farmers become aware that precision agriculture leads to important saving (e.g., in fertilizers) by delineating management zones. For this, they invest in soil samples of their fields that are then analyzed in a laboratory (dots in the maps are the places where the soil samples where taken). The results of the properties of the soil samples can be visualized as thematic maps like the one in the left upper part of Fig. 1 (Organic Matter (OM) is used as soil property and MapInfo as visualization software). By using a clustering method (e.g., the one in MapInfo software) with the organic matter as soil chemical property we obtain the upper right map of the same figure. Each color of this map represents a management zone (green, orange, light blue, and dark blue). We can notice that the resulting zones are disjoint and are irregular shaped which difficulty the use of machinery and therefore the application of resources and inputs. Farmers then think about tracing a grid and delineate their own zones based on the clustering zones or on the thematic map which results in any of the solutions presented in the bottom of Fig. 1. The drawback of this approach is that there is an exponential number of different possible management zone delineations. In order to find the best delineation, farmers must try all of them (e.g., compute the costs of fertilizers) and this would literally take years to be completed. With the ILPMZ method we offer the farmer the best management zones delineation in minutes such that they are rectangular and the most homogeneous possible within each zone.

To the best of our knowledge, this is the first approach that directly offers rectangular shape zones which is an important characteristic for variable rate technologies since it facilitates the work and operation of machinery. Indeed, broadcast seeders (used for spreading lime or fertilizer), manure spreader or sprayers are usually towed behind a tractor. If a management zone is rectangular then it is easier for the farmer to indicate its limits to the tractor driver. Therefore, agricultural inputs are spread exactly in the management zone that requires them.

The ILPMZ method delineates the most homogeneous rectangular management zones from a field with respect to the properties of the soil. It consists of three main stages:

(a) *Instance generation*. In this stage, data from grid soil sampling of a given field is processed: for each soil sample we have its coordinates, and a set of soil properties (pH, organic matter, phosphorus, nitrogen, etc.). Then, a thematic map of the field is created with respect to the wished soil property (or properties) and all the quarters (or potential zones) are computed together with their variances.

(b) *Mathematical model*. With the input of stage a), we propose an Integer Linear Programming (ILP) that is then solved with a branch and bound algorithm. The aim of the ILP is to find a set of the most homogeneous quarters that minimizes the sum of their variance and that covers the whole field.
A main contribution is the insertion of the relative variance into the model to guarantee the homogeneity of the managements zones.

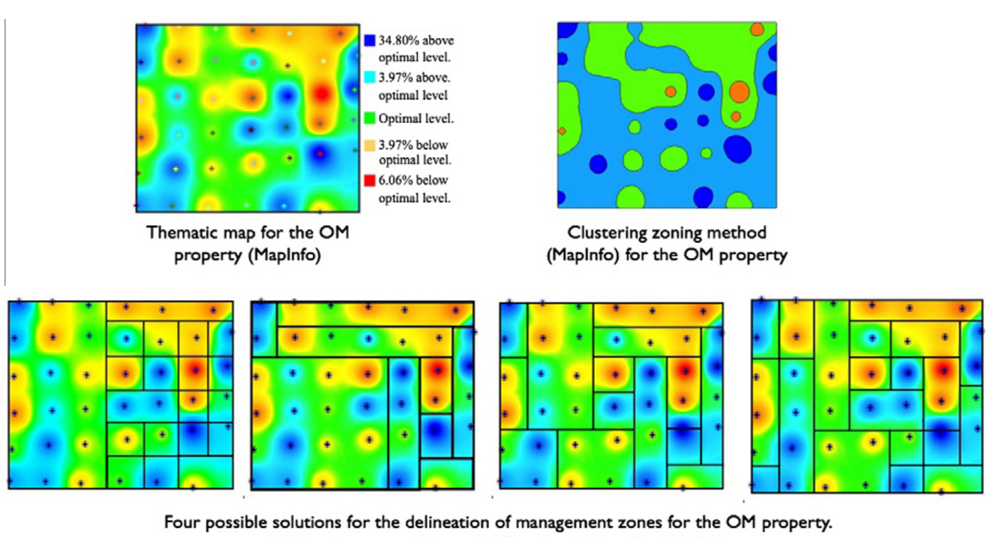

Fig. 1. Left upper map: thematic map for the organic matter property (OM) obtained with MapInfo. Right upper map: Clustering zoning method from MapInfo for the OM property. Each color of this map represents a management zone (green, orange, light blue, and dark blue). Bottom maps: four (out of millions) different solutions of management zones obtained by making a grid based on the smallest size of a zone. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

(c) *Visualization*. This module translates the solution given by the mathematical model into a graphical view of the most homogeneous rectangular zones for the field.

The main novelty of procedure ILPMZ is that step (a) is not only an instance generator, it also computes the different potential management zones of the parcels. This way step (b) becomes tractable since the mathematical model can be exactly solved in seconds.

The rest of this paper is as follows. In Section 2 we introduce our ILPMZ delineation method. The instance generation phase is described in Section 2.1, the Mathematical model is introduced in Section 2.2 while the visualization phase is described in Section 2.3. In Section 3 we experimentally test our methodology, prove its efficiency, and accuracy. Finally, Section 4 concludes the work.

2. Materials and methods

The ILPMZ method delineates rectangular management zones from a field with respect to some properties of the soil. As mentioned, it consists of three main stages: Instance generation, Mathematical model, and Visualization. Each one of these parts of the ILPMZ methodology is explained with more details in this section.

2.1. Instance generation

The objective of this stage is to generate an instance for the Mathematical Model stage from the soil samples that have been taken from the field. These soil samples are approximately equidistant in the field since they were generated from a systematic grid sampling with the help of the Software SMS Mobile and a GPS receiver model AgLeader 1500 with e-diff. Sample positions were collected in geographic coordinates (lon, lat) using the WGS84 datum. Coordinates were then converted to UTM zone 19 S and from them to Cartesian coordinates, in meters.

An example of the soil sample data needed by our methodology can be visualized in Table 1. This table presents the data of 40 soil samples of an agriculture field close to Santiago, Chile (we use this field along the work and call it Real Field instance). This field has 256 m width and 305.6 m long (around 7.82 ha). The samples are approximately spaced by 50 m one from each other so four soil samples are needed to cover an ha. Then, the samples are labeled (first and fourth column of the table) and their positions are translated into a Cartesian map (coordinates (x, y) in the table). Finally, the information about each soil property is presented (pH, organic matter rate (OM), amount of phosphorus (P), and sum of bases (SB). Phosphorus is the most limiting factor in Chilean soils while SB and OM are good indicators of overall soil fertility. Organic matter was determined by the wet oxidation method, extractable P by the Olsen method, while SB corresponds to the sum of bases determined by the CH3COONH4 method of INIA (2006).

The visualization of this data is called a thematic map. In Fig. 2 two thematic maps are presented from the data of Table 1. The left map is the thematic map for the OM property and the right one is for the P property. Here we used MapInfo with the default grid and the inverse distance weighting interpolator. It can be noted that any interpolation method such as kriging, nearest neighbor or other could have been used, since data is spatially dependent.

These thematic maps reveal the diversity of the soil in the field. It is easy to conclude that applying the same amount of inputs (seed, fertilizers, pesticides, water, etc.) throughout the field, would result in few zones at optimum level. With only these thematic maps, it is an extremely difficult task to delineate the most homogeneous managements zones of the field. Moreover, the management zones should be rectangular to be a realistic solution for a farmer. It is known that fields with rectangular shape are better in terms of machinery efficiency. Additionally, for drip irrigation design, rectangular fields are usually used. This leads to the ILPMZ management zone delineation problem which belongs to the NP-hard class. In this article, a new integer linear programming is proposed in order to obtain the best solution among the combinatorial number of zoning patterns of the field.

With the information of the samples, we proceed to generate all the potential zones (or quarters) of the field. Unlike a cutting stock problem, here we are not generating all possible patterns of a field. Instead, we are forming the potential rectangles that could be a zone. This search is the key point of ILPMZ and it can be done in $\mathcal{O}(WidthF \times LengthF)$ where *WidthF* is the number of samples in the width of the field while *LengthF* is the number of samples in its length.

An illustration is given in Fig. 3. The left hand side of this figure shows a thematic map of a small field with nine samples. On the right side, all potential zones are marked. For this example, we have a total of 36 rectangular quarters (generated by Algorithm 1 presented below). Usually, the soil samples are almost equidistant,

Table 1
Coordinates and values of the soil properties for each one of the 40 soil samples taken from a 7.82 ha field close to Santiago, Chile. OM is in %, P in mg kg^{-1} and SB in $Cmol(+)$ kg^{-1}.

Sample	Coordinates (x,y)	Soil properties				Sample	Coordinates (x,y)	Soil properties			
		pH	OM	P	SB			pH	OM	P	SB
1	0.00, 9.14	5.2	11.8	8.0	5.89	21	297.68, 166.36	5.6	10.4	4.0	8.26
2	48.97, 8.46	5.5	12.8	4.0	7.97	22	253.87, 160.20	5.4	18.7	11.0	8.88
3	97.52, 5.57	5.2	14.9	10.0	7.63	23	206.99, 157.26	5.6	10.5	11.0	6.03
4	150.52, 9.42	5.4	14.0	7.0	11.44	24	158.29, 155.16	5.5	16.8	3.0	9.48
5	201.07, 8.25	5.5	11.2	4.0	6.36	25	105.27, 153.53	5.4	14.8	5.0	7.85
6	250.24, 0.00	5.4	14.7	4.0	9.31	26	56.47, 156.87	5.5	12.6	5.0	5.38
7	298.57, 84.00	5.6	12.5	6.0	10.03	27	6.15, 151.48	5.4	15.1	7.0	6.50
8	249.94, 78.89	5.6	9.6	4.0	7.99	28	6.33, 204.03	5.4	11.7	5.0	5.88
9	208.71, 73.33	5.5	14.3	6.0	8.20	29	58.83, 205.57	5.5	16.0	4.0	8.09
10	160.73, 66.20	5.5	15.0	6.0	9.23	30	108.59, 207.64	5.4	13.8	4.0	8.18
11	102.69, 59.51	5.4	14.5	5.0	6.64	31	159.65, 203.22	5.6	12.6	3.0	7.95
12	53.66, 58.30	5.4	11.1	6.0	6.00	32	206.04, 199.18	5.4	14.4	6.0	7.50
13	2.81, 52.71	5.3	14.1	5.0	5.67	33	255.23, 205.16	5.4	15.4	5.0	8.23
14	6.93, 101.13	5.3	16.3	6.0	5.51	34	303.14, 212.73	5.7	11.2	5.0	9.51
15	58.25, 105.04	5.4	12.7	7.0	6.36	35	278.06, 242.75	5.2	16.6	22.0	7.30
16	104.05, 107.24	5.4	14.2	6.0	7.80	36	208.60, 243.31	5.5	15.6	8.0	9.21
17	156.53, 111.44	5.5	11.4	5.0	6.72	37	158.68, 247.47	5.5	16.1	5.0	9.51
18	204.49, 114.91	5.5	11.5	8.0	6.11	38	108.00, 249.65	5.4	13.9	6.0	6.90
19	250.37, 119.77	5.4	16.7	6.0	8.75	39	58.16, 253.69	5.5	15.4	5.0	9.69
20	296.17, 124.74	5.5	13.5	5.0	7.81	40	12.72, 254.37	5.4	10.7	4.0	7.71

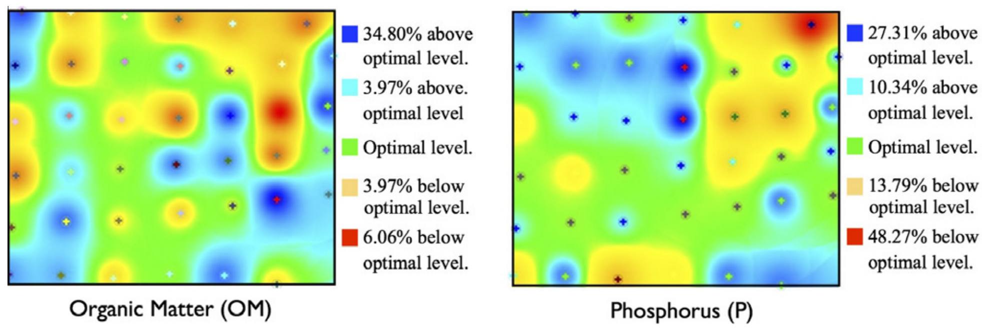

Fig. 2. Thematic maps of organic matter and phosphorus generated with the information of Table 1.

e.g., four soil samples (two width for two long) are needed to cover an ha and the minimum area covered is of a quarter of ha. The number of soil samples to cover an ha may change according to each farmer's requirements.

The total number of potential zones $|Q|$ can be computed by the following formula:

$$|Q| = \left(\sum_{i=1}^{WidthF} i\right)\left(\sum_{j=1}^{LengthF} j\right).$$

Indeed, this determination of all potential zones is not hard since there are only a polynomial number of them. This manner, some of the computations are *done* outside the mathematical model of step b), making it more tractable to solve.

Algorithm 1. Quarters generation of a field.

```
INPUT: WidthF, LengthF, MinWidthQ, MinLengthQ, soil samples
for j = MinWidthQ To WidthF do
  for l = 0 To (WidthF − 1) do
    if (j + l) ⩽ WidthF then
      for i = MinLengthQ To LengthF do
        for k = 0 To (LengthF − 1) do
          if (k + i) ⩽ LengthF then
            creation of a new quarter
          end if
        end for
      end for
    end if
  end for
end for
```

The determination of all possible quarters is implemented by Algorithm 1. The input of this algorithm is the soil samples data (as in Table 1), the number of samples in the width of the field (*WidthF*), the number of samples in the length of the field (*LengthF*), the minimum quantity of samples the width of a quarter (zone) must contain (*MinWidthQ*), and the minimum quantity of samples the length of a quarter must contain (*MinLengthQ*). The algorithm starts creating the smallest quarters width wise. Then it checks if there is still some width to cover. After, it checks the length.

With this algorithm we can create the correspondence matrix $C = \{c_{ij}\}$. If $c_{ij} = 1$, then quarter i covers sample point j, $c_{ij} = 0$ otherwise. Once all the potential quarters are enumerated, we also compute for each one of them its variance, i.e., we compute the variance of a particular soil property for the set of the samples included in a potential quarter. The correspondence matrix of the field of Fig. 3 appears in Table 2. The rows are the quarters, the columns are the sample points except for the last column that

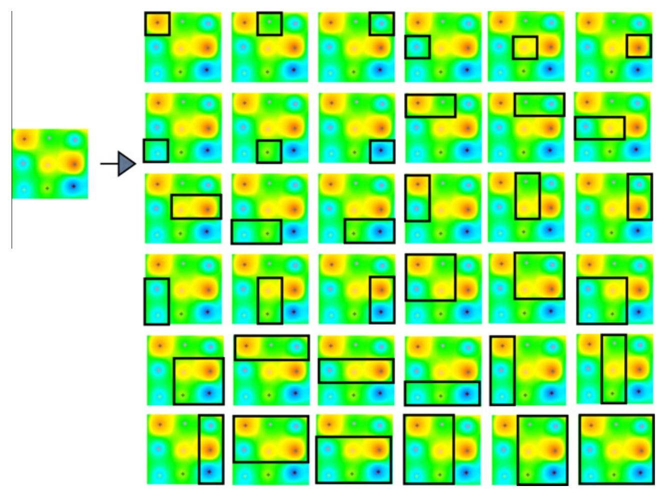

Fig. 3. The 36 quarters of a field.

corresponds to the variance of the quarter. Notice quarter 1 only covers soil sample 1 and therefore, its variance is 0. Quarter 6 covers soil samples 1, 2, and 3, while quarter 36 covers all soil samples (i.e., there is only one zone that is equal to the field).

Most of the fields are not initially rectangular (as it is in the example of Fig. 3). In this case, the ILPMZ method inserts dummy soil samples to fill a rectangle where the field can be contained. This dummy samples are like the real ones: equidistant one from each other. Nevertheless, their data about the properties is very high with respect to the real samples. This manner, the mathematical model make this dummy soil samples to be alone or with other

Table 2
Correspondence matrix C for the field of example of Fig. 3 with the variance of each quarter i.

		Sample point j									σ_i^2
		1	2	3	4	5	6	7	8	9	
Potential quarter i	1	1	0	0	0	0	0	0	0	0	0
	2	0	1	0	0	0	0	0	0	0	0
	3	0	0	1	0	0	0	0	0	0	0
	4	1	1	0	0	0	0	0	0	0	0.21
	5	0	1	1	0	0	0	0	0	0	0.57
	6	1	1	1	0	0	0	0	0	0	0.57
	7	0	0	0	1	0	0	0	0	0	0.00
	8	0	0	0	0	1	0	0	0	0	0.00
	9	0	0	0	0	0	1	0	0	0	0.00
	10	0	0	0	1	1	0	0	0	0	2.40
	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
	36	1	1	1	1	1	1	1	1	1	2.07

dummy samples which facilitates their elimination in the visualization stage of the ILPMZ method.

Next stage of the ILPMZ algorithm is the Mathematical Model. It requires the correspondence matrix of the potential quarters together with their variances.

2.2. Mathematical model

Once we have the data from the samples transformed into a correspondence matrix, we proceed to run an integer linear programming model. This model minimizes the sum of the variance of the potential quarters such that they cover the whole field and that comply with a given relative variance that guarantees the homogeneity of the management zones. This manner, the field will have delineated homogeneous rectangular management zones.

Let I be the set of potential quarters and J the set of soil samples of the field. Each quarter i has n_i soil sample points. The total number of soil samples points is N. Farmers do not wish to have tiny management zones because of their machinery, so let LS be the maximum number of zones in the field while LI is the minimum one. Now we can state the decision variables of our model:

$$x_i = \begin{cases} 1 & \text{if quarter } i \text{ is chosen}, \\ 0 & \text{otherwise}. \end{cases}$$

Our proposed integer linear programming is named ILP and is as follows.

$$\min \sum_{i \in I} \sigma_i x_i \tag{1}$$

$$\textit{subject to} \sum_{i \in I} c_{ij} x_i = 1 \quad \forall j \in J \tag{2}$$

$$\sum_{i \in I} x_i \leqslant LS \tag{3}$$

$$\sum_{i \in I} x_i \geqslant LI \tag{4}$$

$$x_i \in \{0, 1\} \quad \forall i \in I$$

Objective function (1) minimizes the sum of the variance of each chosen zone (or potential management zone). Restriction (2) ensures that every point sample j is covered by only one zone, i.e., the whole filed is partitioned into non-overlapping zones. Constraints (3) and (4) limit the number of zones in which the field will be partitioned.

To guarantee a homogeneous zoning delineation we propose to introduce to the ILP model the relative variance since it has been proved to be a high quality criterion to measure the efficiency of a zoning method Ortega and Santibáñez (2007). Suppose a set of quarters Q satisfy restrictions (2)–(4), i.e., quarters in Q cover all the field and satisfy the minimum and maximum number of allowed zones, then the relative variance of Q is as follows:

$$RV(Q) = 1 - \frac{\sum_{i \in Q} \sigma^2_{w_i}}{\sigma^2_T},$$

where σ^2_T is the total variance of all the field and sum of the $\sigma^2_{w_i}$ for $i \in Q$ is the variance within defined as follows:

$$\sum_{i \in Q} \sigma^2_{w_i} = \frac{\sum_{i \in Q} (n_i - 1)\sigma^2_i}{N - |Q|}. \tag{5}$$

Numerator in (5) considers the number of samples n_i in quarter i (minus one degree of freedom) as a weight and the denominator takes into account the number of selected quarters (total number N minus the number of quarters $|Q|$). Therefore, we introduce the following restriction to ILP:

$$\left(1 - \frac{\sum_{i \in I} (n_i - 1)\sigma^2_i x_i}{\sigma^2_T [N - \sum_{i \in I} x_i]}\right) \geqslant \alpha. \tag{6}$$

Restriction (6) implies that the relative variance of all the chosen quarters is at least α which is a parameter that has to be greater than 0.5 (value given by the experts) to guarantee an homogeneous behavior of the zoning method. This restriction can be easily put into a linear form:

$$(1 - \alpha)\sigma^2_T \left[N - \sum_{i \in I} x_i\right] \geqslant \sum_{i \in I} (n_i - k)\sigma^2_i x_i. \tag{7}$$

Model ILP enhanced with restriction (7) delineates a field into rectangular and most homogeneous management zones. ILP is an NP-hard problem[1] that can be solved with a branch and bound method[2] for a field with 30×30 soil samples (see Section 3). This efficiency is mainly due to the elegance of ILP: few restrictions but sufficiently close to the convex hull of the solution space. Therefore, there is no need of extra valid inequalities and even less, there is no need of an heuristic algorithm which would provide solutions without optimality guarantee.

To the best of our knowledge, there is not a field in Chile with more than 30×30 soil samples. Although, there could be larger instances where a different solution methodology would be required since ILP is NP-hard. The key point of determining the potential zones at the instance generation stage makes the ILP mathematical model simpler but still NP-hard.

2.3. Visualization

The solution of ILP, as it is given by standard optimization software is not friendly for a farmer. A visualization stage must translate the solution of ILP into a map that is useful for the farmer. This map must indicate the characteristics of each one of the final zones so that the farmer knows, for example, where to fertilize. Notice that the dummy soil samples that we inserted in the Instance Generation stage must be deleted at this point since they will be naturally discarded by the method (see more detail in Section 3).

A summary of the ILPMZ method is presented in Fig. 4. Initially we only have the soil samples from the field we want to delineate into rectangular homogeneous management zones. If the field is not rectangular, then we must add dummy samples that will be then deleted. Then, the method proceeds with the Instance Generation stage where the thematic maps of the different soil properties are presented, the potential quarters are generated together with their variance, and finally, the correspondence matrix is computed. This information is the input of the mathematical model ILP which is solved by a branch and bound method. If there is not a feasible solution then one must adjust either the value of LS (maximum number of zones in the field) or reduce parameter α which is equal to relax the homogenization of the zones.

Indeed, a strict bound on LS or in the shape of the quarters may lead to the unfeasibility of ILP. Then, the method must be flexible in one of these parameters with respect to the farmers wishes. More details are presented in Section 3.

3. Experimental results

In this section we validate the ILPMZ method in eleven different fields:

[1] A knapsack problem or a general assignment problem can be easily reduced to ILP.

[2] A branch and bound is an enumeration algorithm. Therefore, when it ends it gives the optimal solution (more details in Wolsey (1998)).

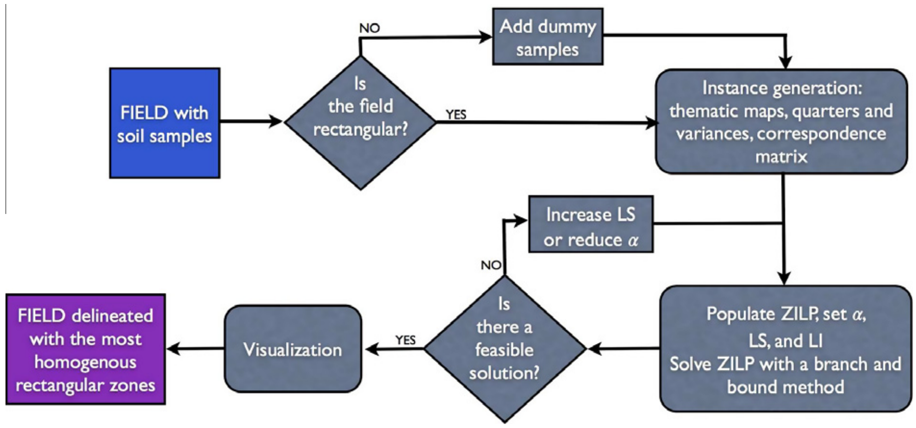

Fig. 4. Summary of the ILPMZ method.

- Real field: vineyard close to Santiago, Chile, with 256 m width and 305.6 m long (around 7.82 ha). Tables 1 and 2 of Section 2.1 describe this instance.
- Large fields: 10 fictitious fields based on generated data (realistic parameters) with at most 900 soil samples (30×30 samples). This fields are to test the efficiency of the ILPMZ method.

Two codes need to be executed for the ILPMZ method on a standard personal PC[3]:

- Quarters generation: Algorithm 1 creates all the possible quarters that could be a management zone. This algorithm was coded in Visual Basic for Applications for Excel.
- B& B: The branch and bound algorithm to solve mixed integer linear programs was implemented by GAMS/CPLEX 12.2 using default options, except for the optimal criterion fixed at 0.

Table 3 present the experimental results, for OM, P, pH and, SB soil properties of an vineyard field close to Santiago, Chile, that has a total of 42 soil samples (six soil samples width by seven length) with a total number of quarters $|Q| = 588$ (generated by Algorithm 1 of Section 2.1). For this field the minimum shape of a management zone, $w \times l = 1 \times 1$ means the minimum zone is w soil samples width for l soil samples long (one can translate soil samples into distance easily). The homogenization parameter α is set to 0.5. First column of Table 3 is the maximum number of management zones the farmer wishes for this field. Second column presents the solution of the objective function of ILP for the total variance where"–" means that no feasible solution could be found (parameters α must be reduced). Last column corresponds to the resulting number of management zones used to partition the field.

Table 3 shows that the ILPMZ with the homogeneity parameter $\alpha = 0.5$ leads to high quality solutions. The maximum number of management zones *LS* plays an important role since in most of the solutions this bound is reached. The smaller the management zones, the most homogeneous they are going to be. Nevertheless, a farmer does not wish to have tiny management zones. When *LS* is too restrictive, the ILP may not find feasible solutions that satisfy $\alpha = 0.5$. Therefore, one could relax this parameter by reducing it until the ILP finds a solution as presented in Fig. 4.

The relationship between variance and quarters for the OM and P soil properties with α equal to 0.5 is showed in Fig. 5. As expected, the number of quarters increases when the value of the variance decreases. Hence, the maximum number of zones given by the farmer to partition the field is an important aspect to consider since the value of the variance within the field depends on it.

Table 3
ILPMZ method applied to a 7.82 ha field close to Santiago, Chile, for the MO, SB, pH, and P soil properties, with the homogenization parameter $\alpha = 0.5$, and total size of quarters $|Q| = 588$.

OM			SB		
LS	σ^2	Zones	*LS*	σ^2	Zones
42	0.00	42	42	0.00	42
20	3.12	20	20	0.93	20
15	4.73	15	15	1.34	13
10	9.43	10	10	1.64	10
9	11.52	9	7	3.59	7
8	–	–	6	–	–
pH			P		
42	0.000	42	42	0.00	42
20	0.002	19	20	1.25	18
15	0.004	12	15	1.95	15
10	0.006	10	10	3.46	10
5	0.014	5	5	4.24	5
4	0.021	4	3	6.21	3
3	–	–	2	–	–

Fig. 6 is the visualization of the ILPMZ applied to the real instance related to the SB soil property. The left side has a minimum size of the management zones of $w \times l = 1 \times 1$ while the right side is set to $w \times l = 2 \times 1$. As mentioned, the result of the zoning drastically vary depending on the minimum size of the quarter and the homogeneity parameter α.

In Tables 4 and 5 are showed the results obtained by ILPMZ when varying the minimum size of a zone ($w \times l$) in the Real Instance (first column). Second column is related to parameter α. It is first set to 0.5, but if no solution can be found (ILP is unfeasible) then it is reduced by 0.1. In the table we show the first α for which a feasible solution could be found. Third column *LS* is the maximum number of zones desired by the farmer. Once we have an α that makes ILP to be feasible, then we decrease *LS*. Fourth column represent the total number of potential zones $|Q|$ (taking into account $w \times l$). Column σ^2 is the objective function of the ILP. Last column is the number of management zones delineated by ILPMZ. Each one of the tests took no more than 0.63 s in a personal PC. From Tables 4 and 5 we remark that if the minimal size of a zone drastically affects the delineation of the zones. Indeed, if $w \times l$ is 3×3 for the Real Instance, then it would be difficult for ILPMZ to find homogeneous zones for this field. The option is either to reduce this minimum size of the zones or reduce the wished homogeneity within the zones. Experts suggest that best delineations of

[3] PC Intel Core 2 Duo of 2.0 GHz, and 4 GB of RAM.

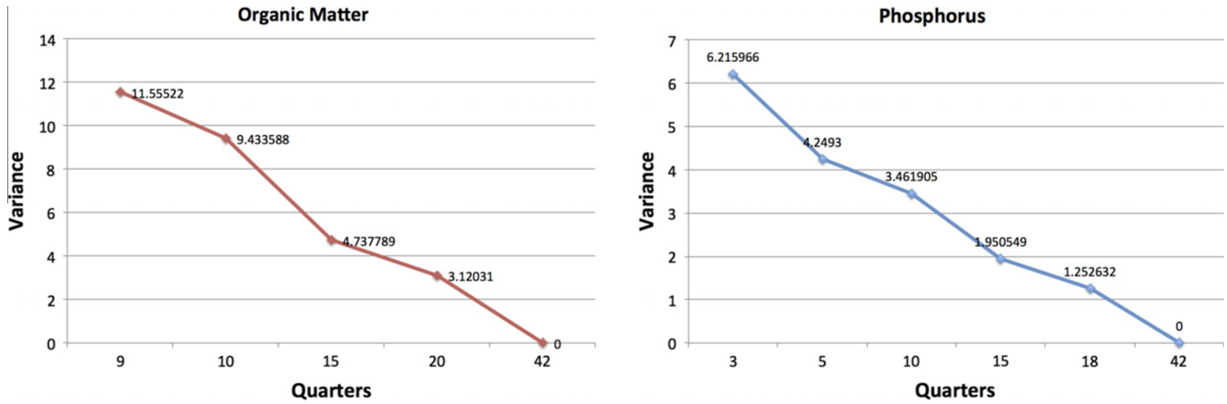

Fig. 5. Relationship between Variance and Quarters applied to a 7.82 ha field close to Santiago, Chile, for the OM and P soil properties for $\alpha = 0.5$.

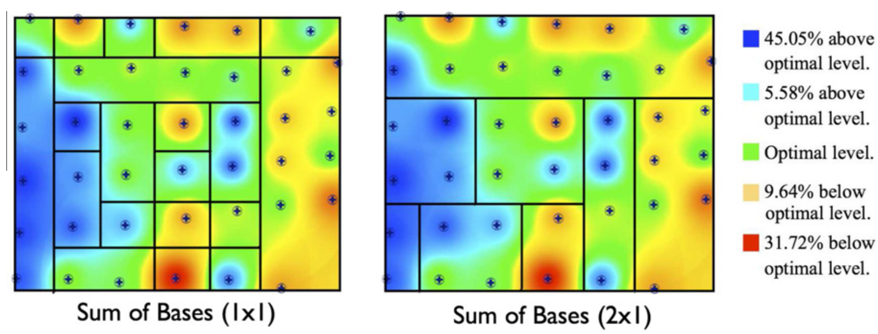

Fig. 6. ILPMZ applied to a real field close to Santiago, Chile, related to the SB soil property with minimum size of the management zones $w \times l = 1 \times 1$ and $w \times l = 2 \times 1$, respectively.

Table 4
ILPMZ method applied to the Real Instance for the OM and SB soil properties.

OM						SB					
$w \times l$	α	LS	$\|Q\|$	σ^2	Zones	$w \times l$	α	LS	$\|Q\|$	σ^2	Zones
1 × 2	0.1	42	441	17.35	6	1 × 2	0.4	42	441	7.24	7
1 × 2	0.1	20	441	17.35	6	1 × 2	0.4	20	441	7.24	7
1 × 2	0.1	5	441	22.18	5	1 × 2	0.4	7	441	7.24	7
1 × 2	0.1	4	441	–	–	1 × 2	0.4	6	441	–	–
2 × 1	0.4	42	420	19.50	11	2 × 1	0.5	42	420	7.75	8
2 × 1	0.4	20	420	19.50	11	2 × 1	0.5	20	420	7.75	8
2 × 1	0.4	9	420	21.49	9	2 × 1	0.5	7	420	7.76	7
2 × 1	0.4	8	420	–	–	2 × 1	0.5	6	420	–	–
2 × 2	0.0	42	315	4.61	1	2 × 2	0.3	42	315	5.09	4
2 × 2	0.0	20	315	4.61	1	2 × 2	0.3	20	315	5.09	4
2 × 2	0.0	10	315	4.61	1	2 × 2	0.3	4	315	5.09	4
2 × 2	0.0	5	315	4.61	1	2 × 2	0.3	3	315	–	–
3 × 3	0.0	42	150	4.61	1	3 × 3	0.2	42	150	3.17	2
3 × 3	0.0	20	150	4.61	1	3 × 3	0.2	20	150	3.17	2
3 × 3	0.0	10	150	4.61	1	3 × 3	0.2	2	150	3.17	2
3 × 3	0.0	5	150	4.61	1	3 × 3	0.2	1	150	–	–

management zones are achieved with a homogeneity parameter α greater than 0.5. Therefore, it would be better to reduce the minimum size of the wished zones than reducing parameter α. Notice that the number of zones obtained by ILPMZ (last column) is not always equal to LS which is the maximum number of zones that a farmer would want.

Table 6 presents the solution for the Large instance set. The main purpose is to test the ILPMZ method on intances that have different sizes. The columns of this table are as follows: number of the instance (first column), number of soil samples in the width (*WidthF*) and length (*LengthF*) of the field, total number of soil samples (fourth column), total number of quarters generated (fifth column), total time in minutes for generating all the quarters ("Q time" column), loading time of the model in seconds ("Loading time" column), and B& B time in seconds (last column). For all instances the B& B is totally executed until the optimal solution is found. We can say that the ILP model is very efficient since for the large instances it does not take more than 4 min. The main issue is the loading time of the model which contains all the information about the quarters of the field. Nevertheless, since this ILPMZ is only needed

Table 5
ILPMZ method applied to the Real Instance for the pH and P soil properties.

pH						P					
$w \times l$	α	*LS*	$\lvert Q\rvert$	σ^2	Zones	$w \times l$	α	*LS*	$\lvert Q\rvert$	σ^2	Zones
1 × 2	0.4	42	441	0.01	4	1 × 2	0.5	42	441	6.21	3
1 × 2	0.4	20	441	0.01	4	1 × 2	0.5	20	441	6.21	3
1 × 2	0.4	4	441	0.01	4	1 × 2	0.5	3	441	6.21	3
1 × 2	0.4	3	441	–	–	1 × 2	0.5	2	441	–	–
2 × 1	0.4	42	420	0.05	9	2 × 1	0.3	42	420	102.6	3
2 × 1	0.4	20	420	0.05	9	2 × 1	0.3	20	420	102.6	3
2 × 1	0.4	8	420	0.06	8	2 × 1	0.3	3	420	102.6	3
2 × 1	0.4	7	420	–	–	2 × 1	0.3	2	420	–	–
2 × 2	0.2	42	315	0.07	5	2 × 2	0.3	42	315	102.6	3
2 × 2	0.2	20	315	0.07	5	2 × 2	0.3	20	315	102.6	3
2 × 2	0.2	5	315	0.07	5	2 × 2	0.3	3	315	102.6	3
2 × 2	0.2	4	315	–	4	2 × 2	0.3	2	315	–	–
3 × 3	0.1	42	150	0.02	2	3 × 3	0.2	42	150	53.07	3
3 × 3	0.1	20	150	0.02	2	3 × 3	0.2	20	150	53.07	3
3 × 3	0.1	2	150	0.02	2	3 × 3	0.2	3	150	53.07	3
3 × 3	0.1	1	150	–	–	3 × 3	0.2	2	150	–	–

Table 6
ILPMZ applied to the set of large instances.

Instance	*WidthF*	*LengthF*	# Samples	$\lvert Q\rvert$	*Q* time	Loading time	B& B time
1	6	7	42	588	0.03	0.63	0.33
2	10	10	100	3025	0.40	1.26	0.46
3	15	10	150	6600	1.33	4.71	2.67
4	15	15	225	14400	4.25	8.65	3.40
5	15	20	300	25200	18.30	31.15	19.95
6	20	20	400	44100	22.78	48.11	22.10
7	20	25	500	68250	54.40	92.51	41.26
8	25	25	625	105625	84.88	227.35	121.27
9	25	30	750	151125	174.05	415.82	223.95
10	30	30	900	216225	250.96	1852.54	193.66

at the beginning of the planning period, the total time is reasonable. Notice that instance 9 is smaller in size than 10 but it is harder to solve by the branch and bound (although, the magnitude order remains the same). This behavior is frequent for integer linear programmings since an instance is not only harder because of its size but also because of the parameters of the variables and resources.

4. Conclusions

Dividing the field into site-specific management zones is an interesting manner to face within-field variability. Classical zoning methods, based on soil properties, have a disadvantage: the zones have oval shapes which are not practical for the fertilization machinery which is most of the times towed by a tractor.

In this work, we present a new zoning method that optimally delineates rectangular homogeneous management zones, using relative variance to guarantee the homogeneity within the zones. Our ILPMZ method based on an integer linear programming model can be efficiently solved even for a large fields.

Experimental results show that the ILPMZ method is efficient and practical so it could be embedded in any decision system.

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