Term Paper on Fundamentals of Automatic Controls, A.Y. 2023-2024

Azhar Moeez - 2066699,
Bellinato Tommaso - 2032597,
Lazzarini Matteo - 2066590

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1 First assignment

The initial request was to simulate the system and observe the free evolution by starting the magnet from a point higher than the equilibrium, in order to find the largest deviation from that value for which the magnet converges to equilibrium.

Let's analyze the considered parameters:

- z_0: the initial position of the system
- z_{eq}: position at which the system is in equilibrium
- c: variable that tracks the deviation from equilibrium

We initialize z_0 as:

$$z_0 = z_{eq} + c$$

The assignment was carried out by trial, to find the largest value of c for which the system converges to equilibrium. To do this, a `for` loop was defined that plots the position of the center of the magnet for each chosen value of c, observing the first value for which the simulation diverges.

Subsequently, only those two values were kept for a clearer view of the graph.

The obtained results allow us to understand that our system, without feedback control, converges to equilibrium and therefore remains stable if initialized with $z_{\text{eq}} \leq z_0 \leq 1.46$ m, while with $z_0 \geq 1.47$ m the system diverges.

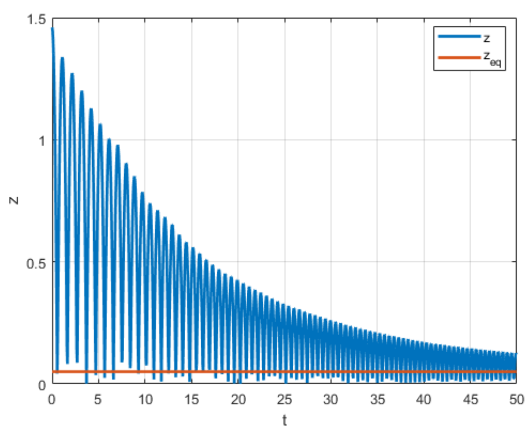

Figure 1: $z_0 = 1.46$ m

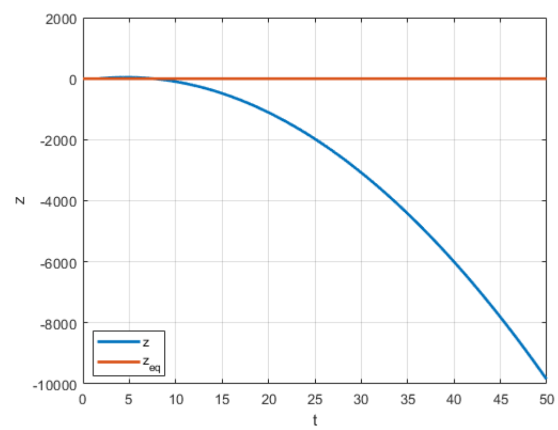

Figure 2: $z_0 = 1.47$ m

2 Second assignment

The second request was to try to find a purely proportional controller that would allow us to achieve a settling time < 1 sec when $z_{\text{ref}}(t) = z_0 + 0.02\ \text{step}(t)$ in closed loop, reporting the steps followed to create such a controller along with the simulation of the nonlinear SISO system in case of success; otherwise, explaining why it was not possible to create a controller that met all the specified requirements.

To study stability in closed loop, we draw the root locus and observe that the system is stable for $K > 0$.

We define an array containing plausible values of K arranged in ascending order and for each of them, we plot the corresponding trajectory of the nonlinear system.

We obtain the settling time of the **linearized system** corresponding to each value of K and extract the smallest one, observing that for the sample of K values (which covers a wide range), the settling time is **well** above the required maximum: the linearized system has the smallest settling time of about 6017 sec with $K_p = 5000$.

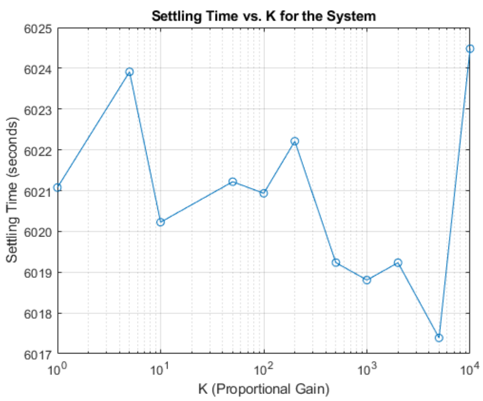

Figure 3: Settling time corresponding to the array of K

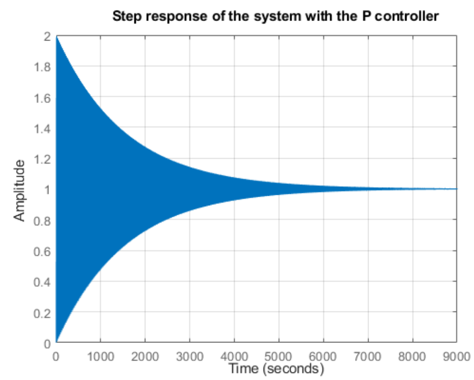

Figure 4: Simulation of the linearized system with a proportional controller having $K_p = 5000$

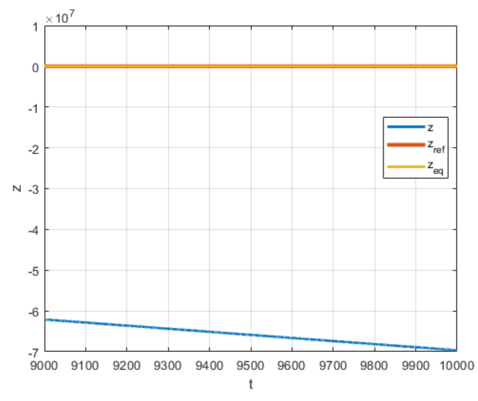

Figure 5: Simulation of the position of the center of the magnet with $K_p = 5000$

We also observe that the settling time found does **NOT** correspond to the one for which the simulated system converges to $z_{\text{ref}} = 0.07$ m: the **nonlinear SISO model** is barely stable and requires an appropriate controller to increase its "stability basin".

It is concluded that it is not possible to create a purely proportional controller that meets the specified requirements for the reasons mentioned above.

3 Third assignment

The third request was to develop a PID controller so that the system would converge to the reference of the previous assignment using the MATLAB tools we deemed most useful, reporting the steps followed for the development of such a controller and the simulation of the nonlinear SISO system.

First, we derived the transfer function of the system, then used an integrated MATLAB tool called `pidTuner`, which allows setting the PID controller parameters (K_p, K_i, K_d) so that the overshoot is not excessive and the stability margin is adequate. It is also possible to visualize the system's rise and settling times.

Once the PID controller parameters were obtained, we implemented them in our system, verifying through the Bode Diagram that the phase margin was adequate, as it can be seen from the graph, it is always above 90 deg.

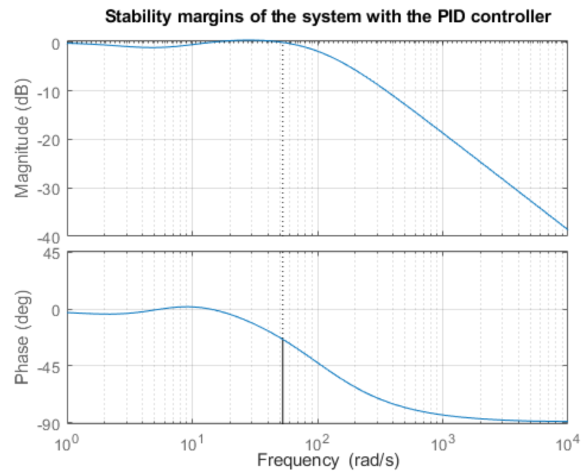

Figure 6: Stability margins

Finally, we plotted the closed-loop simulation with the PID controller and the nonlinear SISO system, achieving good results in terms of rise time, overshoot, settling time, and correct convergence to $z_{\text{ref}} = 0.07$ m.

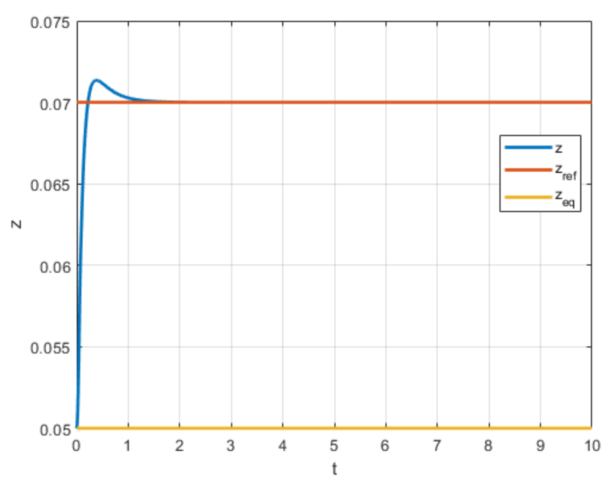

Figure 7: Simulation of the position of the center of the magnet

4 Fourth assignment

The fourth request was to choose a type of controller of our choice and push the center of the magnet as high as possible with an appropriate z_{ref} and initializing the system at equilibrium.

For convenience we used the controller from the previous assignment, *re-tuning* it to achieve the desired result via the `pidTuner` tool and further manual adjustments.

We noticed that increasing the PID parameters or z_{ref} beyond a certain value does not further affect the maximum height the magnet can reach, which we determined to be $z = 0.11079$ m. This is because the nonlinear system imposes a maximum limit on the current input that can be generated by the controller.

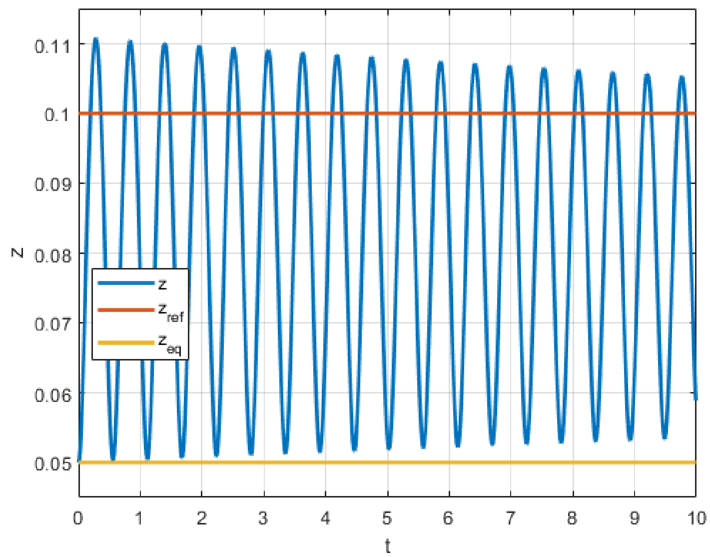

Figure 8: Simulation of the position of the center of the magnet

5 Fifth assignment

The fifth request was to use a controller of our choice and verify that the tracking error of the trajectory of our magnet with respect to the two given references was less than $error_{\text{max}} = 0.002$ m.

Here too, we initially used the PID controller developed in the third assignment. In this case, the re-tuning process was not performed because the results obtained were consistent with the requirements; therefore, we decided to keep the same controller with the same parameters.

5.1 1st trajectory

Let's take the following trajectory as a reference:

$$z_{\text{ref}}(t) = z_0 + 0.01 \sin(2\pi t)$$

The results obtained by plotting the graph with respect to the first reference are very good: it can be seen that the maximum error is well within the range of $\pm error_{\text{max}}$.

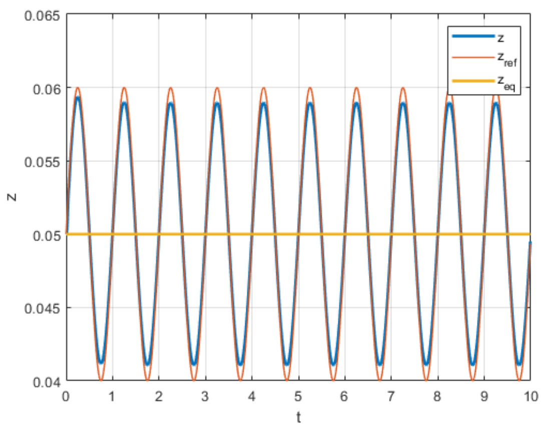

Figure 9: Simulation of the position of the center of the magnet

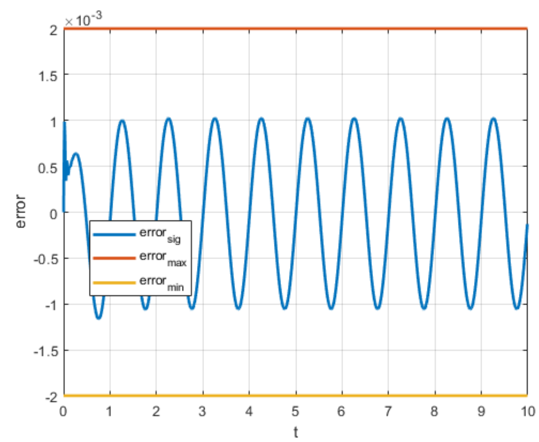

Figure 10: Tracking error

5.2 2nd trajectory

Now let's take the following trajectory as a reference:

$$z_{\text{ref}}(t) = z_0 + 0.01\text{squarewave}(2\pi t)$$

For the second reference, as can be seen from the error graph, in each step there is a very large peak that rises and falls very quickly until it returns to acceptable values. This behavior is a consequence of the type of reference used.

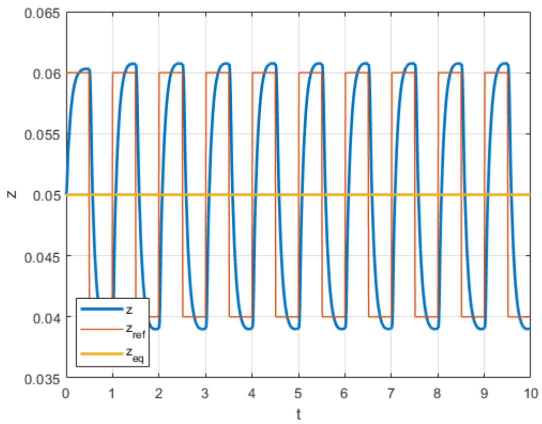

Figure 11: Simulation of the position of the center of the magnet

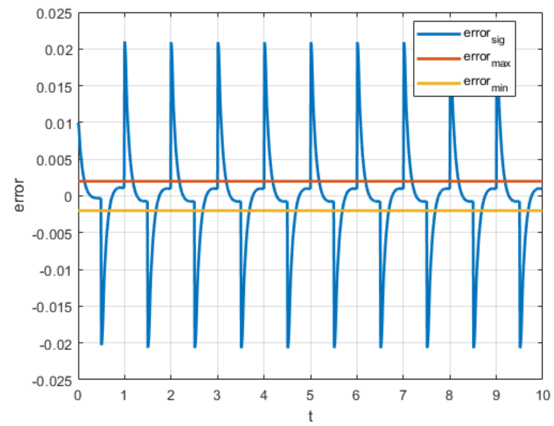

Figure 12: Tracking error

Regardless of the type of controller chosen, we would need one that allows the system to have an infinitesimal rise time, practically equal to 0, to ensure that the magnet follows the trajectory within an acceptable value.

If the magnet starts at $t = 0$ with $z = 0.05$, it still takes some time, albeit little, to reach the required reference, both during the ascent and descent phases; the time taken to reach the square wave reference is precisely what causes the error peaks: it can be noted, in fact, that once the reference is reached, the system returns to an acceptable value range.

In conclusion, we believe it is not possible to create a controller that allows an error within the range of $\pm error_{\max}$ for the second reference: it would require practically infinite modes to follow the reference as specified.