

Homework 1

1.12

a) Take $a = 4$ and $b = 2$

- 1) Set $u = 1$, $g = a = 4$, $x = 0$, and $y = b = 2$
- 2) Y is not zero
- 3) $4 = 2 * 2 + 0$
- 4) $S = 1 - 2 * 0 = 1$
- 5) $U = 0$ and $g = 2$
- 6) $X = 1$ and $y = 0$
- 7) Y is zero, go $v = (2 - 4 * 0) / 2 = 1$

Therefore, $au + bv = \gcd(a, b)$ is true because $4 * 0 + 1 * 2 = 2$

b) Implement with Java

c)

- i) $(1258 * 13) + (527 * -31) = 17$; $g = 17$, $u = 13$, $v = -31$
- ii) $(1056 * 8) + (228 * -37) = 12$; $g = 12$, $u = 8$, $v = -37$
- iii) $(167181 * -4430) + (163961 * 4517) = 7$; $g = 7$, $u = -4430$, $v = 4517$
- iv) $(239847 * 59789) + (3892394 * -970295) = 1$; $g = 1$, $u = 59789$, $v = -970295$

d) If $b = 0$, then a is the gcd and $u = 1$ and $v = 0$.

e)

- i) $(1258 * -18) + (527 * 43) = 17$; $g = 17$, $u = -18$, $v = 43$
- ii) $(1056 * 51) + (228 * -11) = 12$; $g = 12$, $u = 51$, $v = -11$
- iii) $(167181 * 4517) + (163961 * -4430) = 7$; $g = 7$, $u = 4517$, $v = -4430$
- iv) $(239847 * 59789) + (3892394 * -970295) = 1$; $g = 1$, $u = 59789$, $v = -970295$

1.23

a) $X = 31$

b) $X = 5764$

c) $X = 221$

d) $X = a + my$

$$a + mk \equiv b \pmod{n}$$

$$a + mk - b = nj$$

$$mk - nj = b - a$$

since $\gcd(m, n) = 1$, then $mu + nv = 1$

$$\text{then, } mu(b - a) + nv(b - a) = b - a$$

$$x = a + mu(b - a) = a + (1 - nv)(b - a) = b + nv(b - a)$$

This shows that $x \equiv a \pmod{m}$ and $x \equiv v \pmod{n}$

1.25

a.) $17^{183} \pmod{256}$

183 in binary = 10110111

X	17^{2^0}	17^{2^1}	17^{2^2}	17^{2^4}	17^{2^5}	17^{2^7}
X(mod 256)	17	33	65	1	1	1

$17 \cdot 33 \cdot 65 \cdot 1 \cdot 1 \cdot 1 \pmod{256} = \mathbf{113}$

b.) $2^{477} \pmod{1000}$

477 in binary = 111011101

X	2^{2^0}	2^{2^2}	2^{2^3}	2^{2^4}	2^{2^6}	2^{2^7}	2^{2^8}
X(mod 256)	2	16	256	536	616	456	936

$2 \cdot 16 \cdot 256 \cdot 536 \cdot 616 \cdot 456 \cdot 936 \pmod{1000} = \mathbf{272}$

c.) $11^{507} \pmod{1237}$

507 in binary = 111111011

X	11^{2^0}	11^{2^1}	11^{2^3}	11^{2^4}	11^{2^5}	11^{2^6}	11^{2^7}	11^{2^8}
X(mod 256)	11	121	388	867	830	1128	748	380

$11 \cdot 121 \cdot 388 \cdot 867 \cdot 830 \cdot 1128 \cdot 748 \cdot 380 \pmod{1237} = \mathbf{322}$