

$$a = x^5 + 3x^4 - 5x^3 - 3x^2 + 2x + 2$$

$$b = x^5 + x^4 - 2x^3 + 4x^2 + x + 5$$

a)  $F_2[x]$   $a = x^5 + x^4 + x^3 + x^2$   $b = x^5 + x^4 + x + 1$

$$x^5 + x^4 + x^3 + x^2 = (x^5 + x^4 + x + 1)(1) + (x^3 + x^2 + x + 1)$$

$$x^5 + x^4 + x + 1 = (x^3 + x^2 + x + 1)(x^2 + 1) + (0)$$

$$\gcd(x^5 + x^4 + x^3 + x^2, x^5 + x^4 + x + 1) = \underline{x^3 + x^2 + x + 1}$$

b)  $F_3[x]$   $a = x^5 + x^3 + 2x + 2$   $b = x^5 + x^4 + x^3 + x^2 + x + 2$

$$x^5 + x^3 + 2x + 2 = (x^5 + x^4 + x^3 + x^2 + x + 2)(1) + (2x^4 + 2x^2 + x)$$

$$x^5 + x^4 + x^3 + x^2 + x + 2 = (2x^4 + 2x^2 + x)(2x + 2) + (x^2 + 2x + 2)$$

$$2x^4 + 2x^2 + x = (x^2 + 2x + 2)(2x^2 + 2x) + (0)$$

$$\gcd(x^5 + x^3 + 2x + 2, x^5 + x^4 + x^3 + x^2 + x + 2) = \underline{x^2 + 2x + 2}$$

c)  $F_5[x]$   $a = x^5 + 3x^4 + 2x^2 + 2x + 2$   $b = x^5 + x^4 + 3x^3 + 4x^2 + x$

$$x^5 + 3x^4 + 2x^2 + 2x + 2 = (x^5 + x^4 + 3x^3 + 4x^2 + x)(1) + (2x^4 + 2x^3 + 3x^2 + x + 2)$$

$$x^5 + x^4 + 3x^3 + 4x^2 + x = (2x^4 + 2x^3 + 3x^2 + x + 2)(3x + 1) + (4x^3 + x^2)$$

$$2x^4 + 2x^3 + 3x^2 + x + 2 = (4x^3 + x^2)(3x + 1) + (2x^2 + x + 2)$$

$$4x^3 + x^2 = (2x^2 + x + 2)(2x + 2) + (4x + 1)$$

$$2x^2 + x + 2 = (4x + 1)(3x + 2) + (0)$$

$$\gcd(x^5 + 3x^4 + 2x^2 + 2x + 2, x^5 + x^4 + 3x^3 + 4x^2 + x) = \underline{x + 1}$$

d)  $F_7[x]$   $a = x^5 + 3x^4 + 2x^3 + 4x^2 + 2x + 2$   $b = x^5 + x^4 + 5x^3 + 4x^2 + x + 5$

$$x^5 + 3x^4 + 2x^3 + 4x^2 + 2x + 2 = (x^5 + x^4 + 5x^3 + 4x^2 + x + 5)(1) + (2x^4 + 4x^3 + x + 4)$$

$$x^5 + x^4 + 5x^3 + 4x^2 + x + 5 = (2x^4 + 4x^3 + x + 4)(4x + 3) + (3x)$$

$$2x^4 + 4x^3 + x + 4 = (3x)(3x^3 + 6x^2 + 5) + (4)$$

$$3x = (4)(6x + 1) + 0$$

$$\underline{\gcd = 1}$$