3.25

a.) N = 61063

1882^2= 270 mod 61063 and 270 = 2 \* 3^3 \* 5

1898^2 = 60750 mod 61063 and 60750 = 2 \* 3^5 \* 5^3

So,

1882^2 \* 1898^2 = (2 \* 3^3 \* 5) (2 \* 3^5 \* 5^3) = (2 \* 3^4 \*5^2)^2 = (4050)^2 mod 61063 🡨 4050 = b

And,

1882 \* 1898 = 3572036 = 30,382 mod 61063 🡨30,382 = a

GCD(N, a-b) = GCD(61063, (30,382 – 4,050)) = GCD(61063, 26332) = **227**

b.) N = 52907

339^2 = 480 mod 52907 = 2^5 \* 3 \* 5

763^2 = 192 mod 52907 = 2^6 \* 3

773^2 = 52907 mod 52907 = 2^6 \* 3^5

976^2 = 250 mod 52907 = 2 \* 5^3

So, 339^2 \*763^2\*773^2\*976^2 = (2^5 \* 3 \* 5)( 2^6 \* 3)( 2^6 \* 3^5)( 2 \* 5^3) = 2^18 \* 3^7 \*5^4 however this is not a power of 2 so we have to find a new combo. This combo is:

339^2\*773^2\*976^2 = (2^5 \* 3 \* 5)( 2^6 \* 3^5)( 2 \* 5^3) = 2^12 \* 3^6 \*5^4 = ( 2^6 \* 3^3 \* 5^2)^2 = 43200^2 🡨 43200 = b

And,

339 \* 773 \* 976 = 301024752 = 36829 mod 52907 🡨 36829 = a

GCD(N, b- a) = GCD(52907, (43200, 36829)) = GCD(52907, 6371) = **277**

c.) N = 198103

1189^2 = 27000 mod 198103 = 2^5 \* 3 \* 5

1605^2 = 27000 mod 198103 = 2 \* 7^3

2378^2 = 108000 mod 198103 = 2^5 \* 3^3 \* 5^3

2815^ 2 = 105 mod 198103 = 3 \* 5 \* 7

So,

1189^2 \* 1605^2 \* 2378^2 \*2815^ 2 = (2^5 \* 3 \* 5)( 2 \* 7^3)( 2^5 \* 3^3 \* 5^3)( 3 \* 5 \* 7)

= 2^3 \* 3^2 \* 5^2 \* 7^2 however this is not a power of two so another combination so I find a new combination which is:

1605^2 \* 2378^2 \*2815^ 2 = ( 2 \* 7^3)( 2^5 \* 3^3 \* 5^3)( 3 \* 5 \* 7) = (2^6 \* 3^4 \* 5^4 \* 7^4 ) = (2^3 \*3^2 \* 5^2 \* 7^2)^2 = (88200)^2 mod 198103 🡨 b

So,

1605 \* 2378 \* 2815 mod 198103 = 64248 🡨 a

GCD ( N, b – a) = GCD (198103 , (88200 – 64248)) = GCD (198103, 23952) = 499

d.) N = 2525891

1591^2 = 5390 mod 2525891 = 2 \* 5 \* 7^2 \* 11

3182^2 = 21560 mod 2525891 = 2^3 \* 5 \* 7^2 \* 11

4773^2 = 48150 mod 2525891 = 2 \* 3^2 \* 5 \* 7^2 \* 11

5275^2 = 40824 mod 2525891 = 2^3 \* 3^6 \* 7

5401^2 = 13860000 mod 2525891 = 2^4 \* 3^2 \* 5^3 \* 7 \* 11

1591^2 \* 3182^2 \*4773 ^ 2 \* 5275^2 \* 5401^2 = ( 2 \* 5 \* 7^2 \* 11)( =2^3 \* 5 \* 7^2 \* 11)( 2 \* 3^2 \* 5 \* 7^2 \* 11)( 2^3 \* 3^6 \* 7)( 2^4 \* 3^2 \* 5^3 \* 7 \* 11) = (2^6 \* 3^4 ^ 5^3 \* 7^4 \* 11^2)^2 = (18825760800)^2

So,

1591 \* 3182 \*4773 \* 5275 \* 5401 = 739064

GCD (N, b- a) = GCD(2525891, 18825686936) = 1