Benchmarks for Online Stochastic Operations Research Problems

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Abstract

- Benchmarks to compare reinforcement learning algorithms with state of the art for solving online stochastic operations research problems
- 3 1 Introduction
- 4 Operations Research is an awesome field. Many online stochastic problems exist in practice but
- solutions make simplistic assumption. Reinforcement learning is a good fit to these problems and
- 6 there have been some initial results. However, no standard set of problems or algorithms to compare
- 7 against exist. We prepare these benchmarks to compare different algorithms and push the community
- 8 towards developing better ones.

9 2 Related Work

10 3 Bin Packing

- In the classic bin packing problem, we fit given items of varying sizes into as few fixed size bins as
- possible. In the online stochastic version of this problem, each item arrives one at a time and its size
- varies as per an unknown distribution. We formulate the problem as a Markov Decision Process and
- compare reinforcement learning algorithms against a well known baseline called Sum of Squares that
- asymptotically converges to a good solution regardless of the item size distribution.

16 3.1 Related Work

Online stochastic bin packing [?]

18 3.2 Problem Formulation

- 19 Items can be of different types $j \in \{1, ..., J\}$. The size of type j is s_j and the probability that an item
- is of type j is p_j . Without loss of generality, we assume item types are enumerated in the increasing
- order of their size: $s_1 < s_2 < ... < s_J$. Items arrive one at a time and are packed into bins of size
- 22 B, where $s_J < B < \infty$. We assume the item sizes s_j and bin size B are integers. We assume the
- number of bins are unlimited and denote the sum of item sizes in a bin as level h. After n items have
- been packed, we denote the number of bins at level h as $N_h(n)$, where h = 1, ..., B.

- 25 Our objective is to reduce the number of non-empty bins. We can achieve this by minimizing the
- total waste (i.e. empty space) in the partially filled bins. Hence our objective is to minimize waste at
- 27 any point in time:

$$W(n) \triangleq \sum_{h=1}^{B-1} N_h(n)(B-h)$$
 (1)

- 28 3.3 Baseline Algorithm
- 29 3.4 Reinforcement Learning Algorithm
- 30 3.5 Results
- 31 4 Newsvendor
- 32 5 Vehicle Routing
- 33 5.1 Literature Review
- 34 5.2 Problem Formulation
- 35 We consider a version of VRP that is of a food delivery driver (e.g. using Amazon Restaurants).
- 36 Orders arrive at the driver's phone app over time in a dynamic manner over time. Each order has a
- 37 reward (e.g. delivery fee and tip) associated with it and it is assigned to a specific restaurant in the
- city. "City" here means the whole Euclidean space in which the VRP problem lives. Orders arrive
- 39 according to a Poisson process and the rate depends on the region of the city that the order is created
- in. Also, the reward of an order comes from a Gamma distribution, again, with parameters specific to
- 41 the region. The driver has to accept an order and pick the items up from the restaurant the order is
- mapped to. The order needs to be delivered within a time window since its creation, which is imposed
- 43 as a hard constraint. If the driver does not accept a specific order, it remains open for a while and
- 44 then disappears according to a probability distribution, meaning that either it has expired or accepted
- by some other driver. There is a capacity on the number of orders a driver can carry in the vehicle,
- 46 however there is no limit on the number of orders that are accepted (but not delivered) by the driver
- 47 at the same time. Finally, there is a cost associated with travel The driver's goal is to maximize the
- 48 average reward over an infinite horizon.
- 49 This problem is known as stochastic and dynamic capacitated vehicle routing problem with pick up
- 50 and delivery, time windows and service guarantee. (SDPDPTW with service guarantee).

5.3 Baseline Solution

- 52 The problem for a given set of orders can be expressed and solved using the following Mixed Integer
- 53 Programming formulation.
- 54 **Sets**
 - V: Current vehicle location, $V = \{0\}$
 - P: Pick up nodes (copies of the restaurant nodes, associated with the orders that are not in transit)
 - D: Delivery nodes representing the orders that are not in transit
 - A: Nodes representing the orders that are accepted by the driver; $A \subset D$
 - T: Delivery nodes representing the orders that are in transit
 - R: Nodes representing the restaurants, used for final return)
 - N : Set of all nodes in the graph, $N = V \cup P \cup D \cup T \cup R$
 - E: Set of all edges, $E = \{(i, j), \forall i, j \in N\}$

Decision variables

 x_{ij} : Binary variable, 1 if the vehicle uses the arc from node i to j, 0 otherwise; $i, j \in N$

 y_i : Binary variable, 1 if the order i is accepted, 0 otherwise; $i \in P$

 Q_i : Auxiliary variable to track the capacity usage as of node $i; i \in N$

 B_i : Auxiliary variable to track the time as of node i; $i \in N$

Parameters

n: Number of orders available to pick up, n = |P|

 C_{ij} : Symmetric Manhattan distance (in miles) matrix between node i and $j, (i, j) \in E$

 q_i : Supply (demand) at node $i, q_0 = |T|; q_i = 1, \forall i \in P; q_i = -1, \forall i \in D \cup T; q_i = 0 \in R$

 $m: Travel\ cost\ per\ mile$

 r_i : Revenue for order associated with pick up node $i, i \in P$

U: Vehicle capacity

M: A very big number

t: Time to travel one mile

d: A constant positive service time per stop

Model

$$\underset{x,y,Q,B}{\text{maximize}} \quad \sum_{i} r_{i} y_{i} - m \sum_{(i,j) \in E} C_{ij} x_{ij} \tag{2a}$$

subject to

$$\sum_{i \in N} x_{ij} = y_i \qquad \forall i \in P, \tag{2b}$$

$$\sum_{j \in N} x_{ij} = y_i \qquad \forall i \in P, \qquad (2b)$$

$$\sum_{j \in N} x_{ij} - \sum_{j \in N} x_{i+n,j} = 0 \qquad \forall i \in P, \qquad (2c)$$

$$y_i = 1 \qquad \forall i \in A, \tag{2d}$$

$$y_i = 1 \qquad \forall i \in A,$$
 (2d)
$$\sum_{j \in N} x_{ij} = 1 \qquad \forall i \in T,$$
 (2e)

$$\sum_{j \in N} x_{0j} = 1,\tag{2f}$$

$$\sum_{j \in N \setminus R} x_{ji} = 1 \qquad \forall i \in R, \tag{2g}$$

$$\sum_{j \in N \setminus R} x_{ji} = 1 \qquad \forall i \in R,$$

$$\sum_{j \in N \setminus R} x_{ji} - \sum_{j \in N} x_{ij} = 0 \qquad \forall i \in P \cup D \cup T,$$
(2g)

$$Q_i + q_j - M(1 - x_{ij}) \leqslant Q_j \qquad \forall i, j \in N, \tag{2i}$$

$$\max(0, q_i) \leqslant Q_i \qquad \forall i \in N, \tag{2j}$$

$$\min\left(U, U + q_i\right) \geqslant Q_i \qquad \forall i \in N, \tag{2k}$$

$$B_i + d + C_{ij}t - M(1 - x_{ij}) \leqslant B_i \qquad \forall i, j \in N, \tag{21}$$

$$B_i + C_{i,j+n}t \leqslant B_{i+n} \qquad \forall i \in P, \tag{2m}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i, j \in N, \tag{2n}$$

$$y_i \in \{0, 1\} \qquad \forall i \in P \tag{20}$$

Conclusion

Amazing! Awesome results! Accept!

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61 Do not include acknowledgments in the anonymized submission, only in the final paper.