

刷人利器——深度优先搜索

一手微信主讲及令狐冲ffer都有课程版本 v7.0



什么时候使用 DFS?

在之前的课程中,我们知道了 Binary Tree 的问题大部分都是 DFS 今天的课程中,我们将更深入的讨论 DFS 的使用场景



独孤九剑——破索式

碰到让你找所有方案的题, 基本可以确定是 DFS 除了二叉树以外的 90% DFS 的题, 要么是排列, 要么是组合



找所有满足某个条件的方案

找到图中的所有满足条件的路径

路径=方案=图中节点的排列组合

很多题不像二叉树那样直接给你一个图(二叉树也是一个图)

点、边、路径是需要你自己去分析的

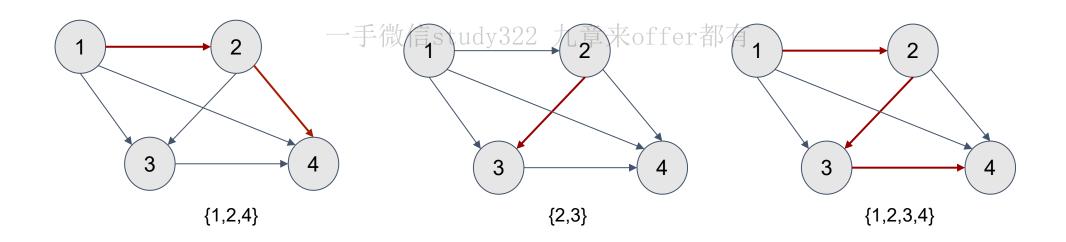
案例一: 找出一个集合的所有子集



点:集合中的元素

边:元素与元素之间用有向边连接,小的点指向大的点(为了避免选出 12 和 21 这种重复集合)

路径: = 子集 = 图中任意点出发到任意点结束的一条路径





找N个数组成的全排列 Permutation

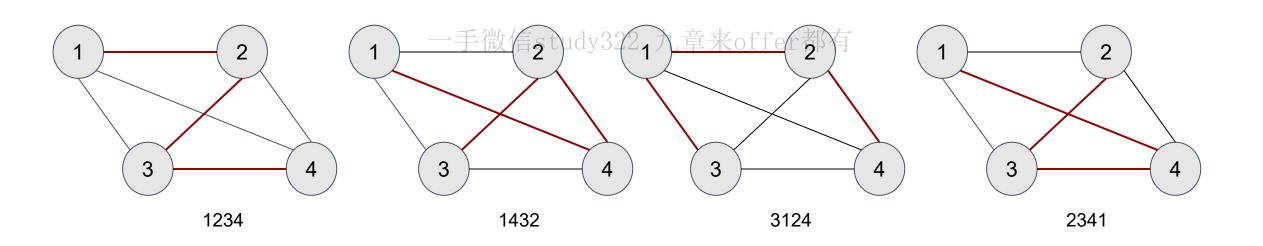
案例二: 求出 N 个数组成的全排列



点:每个数为一个点

边: 任意两两点之间都有连边, 且为无向边

路径: = 排列 = 从任意点出发到任意点结束经过每个点一次且仅一次的路径





BFS vs DFS

找所有方案的问题是否可以使用 BFS?

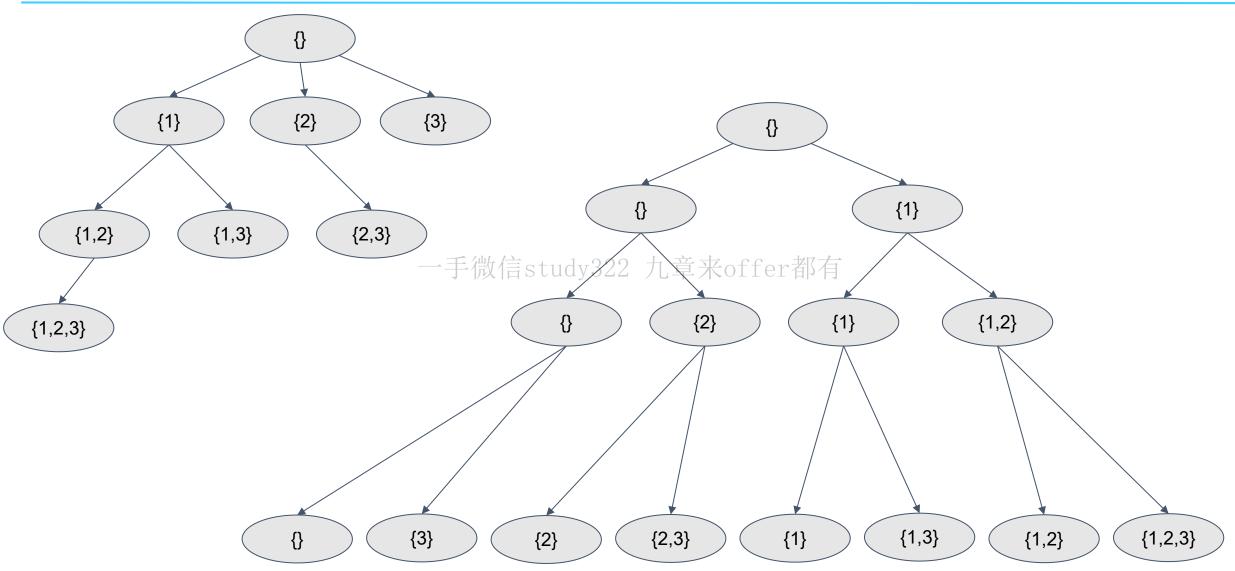


找路径→找点

一手微信改变"点"的定义ffer都有可以将找所有路径问题变为找所有点的问题

全子集问题的另外两种画图方法(找所有点)







BFS vs DFS

宽度优先搜索的空间复杂度取决于宽度深度优先搜索的空间复杂度取决于深度



什么是递归 Recursion?

函数 Function 自己调用自己

Recursion 是代码的实现方式,并不算是一种算法(也有一些书籍认为递归是算法,但是这样说并不准确)



递归在算法中干了啥

递归就是当多重循环层数不确定的时候 一个更优雅的实现多重循环的方式



```
if n == 1:
    for i in range(1, n + 1):
if n == 2:
    for i in range(1, n + 1):
        for j in range(1, n + 1):
            if i != j:
if n == 3:
    for i in range(1, n + 1):
        for j in range(1, n + 1):
            if i != j:
                for k in range(1, n + 1):
                    if k != i and k != j:
```

```
def recursion(self, n, visited, path):
    if len(path) == n:
        # do something
        return
    for i in range(1, n + 1):
       if i not in visited:
            path.append(i)
            visited.add(i)
            self.recursion(n, visited, path)
            path.pop()
            visited.remove(i)
```

递归干了啥



```
if (n == 1) {
   for (int i = 1; i <= n; i++) {
                                                     private void recursion(int n, Set<Integer> visited, List<Integer> path) {
if (n == 2) {
                                                         if (path.size() == n) {
   for (int i = 1; i <= n; i++) {
                                                             // do something
       for (int j = 1; j <= n; j++) {
                                                             return;
           if (i != j) {
                                              手微信study if (!visited.contains(i)) {
                                                                 path.add(i);
                                                                 visited.add(i);
if (n == 3) {
                                                                 recursion(n, visited, path);
   for (int i = 1; i <= n; i++) {
                                                                 path.remove(path.size() - 1);
       for (int j = 1; j <= n; j++) {
                                                                 visited.remove(i);
           if (i != j) {
               for (int k = 1; k <= n; k++) {
                  if (k != i && k != j) {
```

递归三要素



一般来说,如果面试官不特别要求的话,DFS都可以使用递归(Recursion)的方式来实现。 递归三要素是实现递归的重要步骤:

- 递归的定义
- 递归的拆解
- 递归的出口

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排列组合问题 Combination & Permutation



组合问题

问题模型: 求出所有满足条件的"组合"。

判断条件:组合中的元素是顺序无关的。

时间复杂度:与 2ⁿ 相关。

排列问题

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问题模型:求出所有满足条件的"排列"。

判断条件:组合中的元素是顺序"相关"的。

时间复杂度:与 n! 相关。



DFS时间复杂度通用计算公式

O(方案个数*构造每个方案的时间)

排列问题 = O(n! * n)

组合问题 = O(2ⁿ * n)



String Permutation II

www.lintcode.com/problem/string-permutation-ii www.jiuzhang.com/solutions/string-permutation-ii 字母换数字,换汤不换药



搜索去重的诀窍——选代表

错误方法: 把所有方案都放在 HashSet 里去重

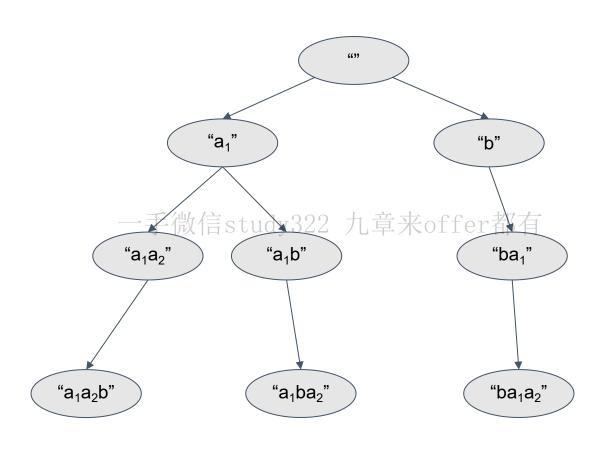
正确方法: 在每组重复方案中选出代表方案



aab

由于出现了两个a, 为了区分将两个a分别记做a1, a2 字符串变成了a1 a2 b 搜索过程中保证a2在a1的后面







```
public List<String> stringPermutation2(String str) {
                                                                                 // 递归的定义: 找到所有 permutation 开头的排列
                                                                                 private void dfs(char[] chars,
     char[] chars = str.toCharArray();
                                                                                              boolean[] visited,
    Arrays.sort(chars);
                                                                                              String permutation,
                                                                                              List<String> permutations) {
     boolean[] visited = new boolean[chars.length];
                                                                                    // 递归的出口: 当我找到一个完整的排列
    List<String> permutations = new ArrayList<>();
                                                                                    if(chars.length == permutation.length()){
                                                                                       permutations.add(permutation);
                                                                                       return;
    dfs(chars, visited, "", permutations);
     return permutations;
                                                                                    // 递归的拆解: 基于当前的前缀, 下一个字符放啥
                                                                                    for(int i = 0; i < chars.length; i++) {</pre>
                                                                                       if(visited[i]) {
                                                                                          continue;
                                             一手微信study322 九章来offe# 書車「不同位置的同样的字符,必须按照顺序用。
                                                                                       // => a' a" b => V
                                                                                       // \Rightarrow a'' a' b \Rightarrow x
                                                                                       // 不能跳过一个a选下一个a
                                                                                       if(i > 0 \&\& chars[i] == chars[i - 1] \&\& !visited[i - 1]) {
                                                                                          continue:
                                                                                       // make changes
                                                                                       visited[i] = true;
                                                                                       // 找到所有 permutation 开头的排列
                                                                                       // 找到所有 "a" 开头的
                                                                                       dfs(chars, visited, permutation + chars[i], permutations);
                                                                                       // backtracking
                                                                                       visited[i] = false;
```



```
def stringPermutation2(self, str):
    chars = sorted(list(str))
    visited = [False] * len(chars)
    permutations = []
    self.dfs(chars, visited, [], permutations)
    return permutations
```

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```
dfs(self, chars, visited, permutation, permutations):
if len(chars) == len(permutation):
    permutations.append(''.join(permutation))
for i in range(len(chars)):
    if visited[i]:
    if i > 0 and chars[i] == chars[i - 1] and not visited[i - 1]:
    visited[i] = True
    permutation.append(chars[i])
    self.dfs(chars, visited, permutation, permutations)
    permutation.pop()
    visited[i] = False
```



Combination Sum

http://www.lintcode.com/problem/combination-sum/

http://www.jiuzhang.com/solutions/combination-sum/

问:和 subsets 的区别有哪些?

与 Subsets 比较



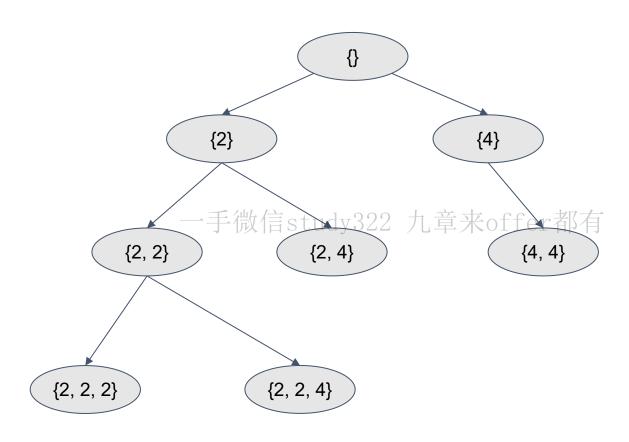
- Combination Sum 限制了组合中的数之和
 - 加入一个新的参数来限制
- Subsets 无重复元素, Combination Sum 有重复元素
 - 需要先去重
- Subsets 一个数只能选一次,Combination Sum 一个数可以选很多次
 - 搜索时从 index 开始而不是从 index + 1



candidates = [2, 4], target = 6

在搜索过程中保证选数字的有序性 只要选了一个4,之后就不会再选2 从target开始减,减成非正数就返回





回溯与硬拷贝



```
combinationSum(self, candidates, target):
candidates = sorted(list(set(candidates)))
results = []
self.dfs(candidates, target, 0, [], results)
return results
dfs(self, candidates, target, start, combination, results):
if target < 0:
    return
if target == 0:
    return results.append(list(combination))
for i in range(start, len(candidates)):
    combination.append(candidates[i])
    self.dfs(
        candidates,
        target - candidates[i].
        i.
        combination,
        results,
    combination.pop()
```

```
combinationSum(self, candidates, target):
candidates = sorted(list(set(candidates)))
results = \Pi
self.dfs(candidates, target, ∅, [], results)
return results
dfs(self, candidates, target, start, combination, results):
if target < 0:
    return
if target == 0:
    return results.append(combination)
for i in range(start, len(candidates)):
    self.dfs(
        candidates.
        target - candidates[i],
        combination + [candidates[i]],
        results,
```



```
public List<List<Integer>> combinationSum(int[] candidates, int target) {
   List<List<Integer>> results = new ArrayList<>();
   // 集合为空
   if (candidates.length == 0) {
       return results;
   // 排序和夫重
   candidates = removeDuplicates(candidates);
   // dfs
   dfs(candidates, target, 0, new ArrayList<Integer>(), results);
   return results;
                                                             private void dfs(int[] candidates, int target, int start, List<Integer>
                                                             ____combination, List<List<Integer>>> results) {
                                                       study32/1 到达远界OI
private int[] removeDuplicates(int[] candidates)
   //排序
                                                                 if (target == 0) {
   Arrays.sort(candidates);
                                                                     results.add(new ArrayList<Integer>(combination));
   //去重
                                                                     return;
   int index = 0;
                                                                 // 递归的拆解: 挑一个数放入current
   for (int i = 0; i < candidates.length; i++) {</pre>
       if (candidates[i] != candidates[index]) {
                                                                 for (int i = start; i < candidates.length; i++) {</pre>
                                                                    // 剪枝
           candidates[++index] = candidates[i];
                                                                     if (target < candidates[i]) {</pre>
                                                                         break;
   int[] candidatesNew = new int[index + 1];
   for (int i = 0; i < index + 1; i++) {
                                                                     combination.add(candidates[i]);
                                                                     dfs(candidates, target - candidates[i], i, combination, results);
       candidatesNew[i] = candidates[i];
                                                                     combination.remove(combination.size() - 1);
   return candidatesNew;
```

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k Sum II

http://www.lintcode.com/problem/k-sum-ii/

http://www.jiuzhang.com/solution/k-sum-ii/

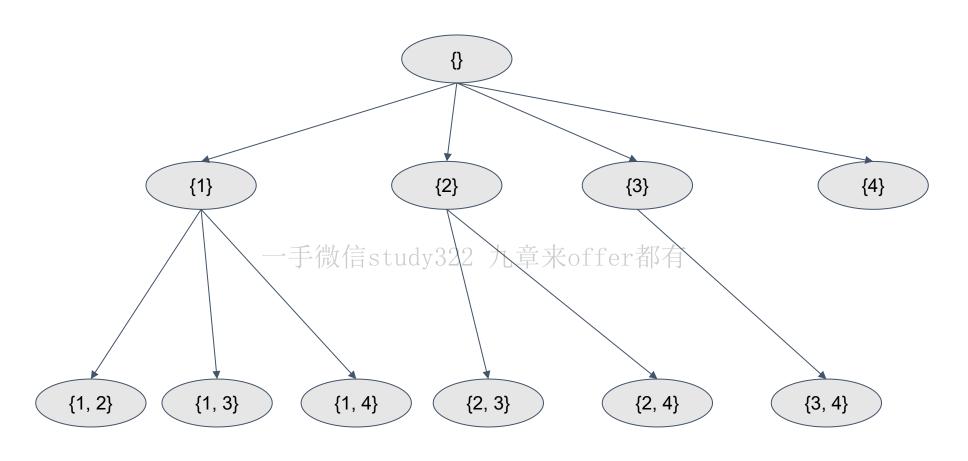
找出所有 k 个数之和 = target 的组合



[1,2,3,4], k = 2, target = 5

一手微保证选数的有序性^{er都有} 选了后面的数之后就不能选择前面的数







```
def kSumII(self, A, k, target):
    A = sorted(A)
    subsets = []
    self.dfs(A, 0, k, target, [], subsets)
    return subsets

def dfs(self, A, index, k, target, subset, subsets):
    if k == 0 and target == 0:
        subsets.append(list(subset))
        return

if k == 0 or target <= 0:
    return

for i in range(index, len(A)):
    subset.append(A[i])
    self.dfs(A, i + 1, k - 1, target - A[i], subset, subsets)
    subset.pop()</pre>
```

```
public List<List<Integer>> kSumII(int[] A, int k, int target) {
   Arrays.sort(A);
   List<List<Integer>> subsets = new ArrayList();
    dfs(A, 0, k, target, new ArrayList<Integer>(), subsets);
   return subsets;
}
private void dfs(int[] A, int index, int k, int target,
                List (Integer) subset,
if (k == 0 && target == 0) {
       subsets.add(new ArrayList<Integer>(subset));
       return;
   if (k == 0 | target <= 0) {
       return;
    for (int i = index; i < A.length; i++) {</pre>
       subset.add(A[i]);
       dfs(A, i + 1, k - 1, target - A[i], subset, subsets);
       subset.remove(subset.size() - 1);
```



休息一会儿

一手微信。 Take a Break



Letter Combinations of Phone Number

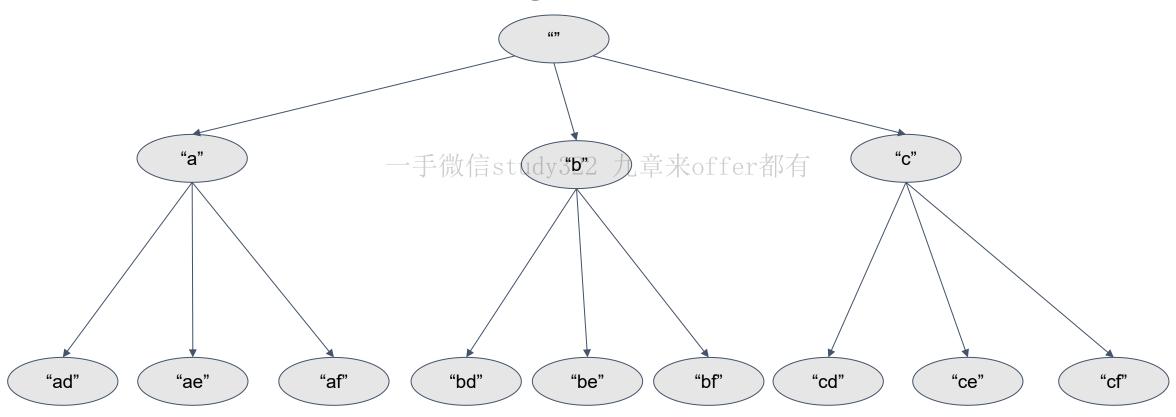
http://www.lintcode.com/problem/letter-combinations-of-a-phone-number/

http://www.jiuzhang.com/solution/letter-combinations-of-a-phone-number/

什么是点?什么是边?什么是路径?



digits=23





```
KEYBOARD = {
    '2': 'abc',
    '3': 'def',
    '4': 'ghi',
    '5': 'jkl',
    '6': 'mno',
    '7': 'pqrs',
    '8': 'tuv',
    '9': 'wxyz',
class Solution:
    @param digits: A digital string
   @return: all posible letter combinations
    def letterCombinations(self, digits):
        if not digits:
            return
       results = []
       self.dfs(digits, 0, [], results)
        return results
    def dfs(self, digits, index, chars, results):
        if index == len(digits):
            results.append(''.join(chars))
            return
        for letter in KEYBOARD[digits[index]]:
            chars.append(letter)
            self.dfs(digits, index + 1, chars, results)
            chars.pop()
```

```
public List<String> letterCombinations(String digits) {
    List<String> combinations = new ArrayList();
   if (digits == null | digits.length() == 0) {
        return combinations;
                                               public static String[] KEYBOARD = {
    dfs(digits, 0, "", combinations);
                                                    11 11
   return combinations;
                                                   "abc",
                                                    "ghi",
private void dfs(String digits,
                                                    "jk1",
                 int index,
                                                    "mno",
                 String combination
                List(String combinations) {
                                                    "pgrs",
    if (index == digits.length()) {
                                                    "tuv",
        combinations.add(combination);
                                                   "WXYZ",
        return;
                                               };
   int digit = digits.charAt(index) - '0';
   for (int i = 0; i < KEYBOARD[digit].length(); i++) {</pre>
        dfs (
            digits,
            index + 1,
            combination + KEYBOARD[digit].charAt(i),
            combinations
        );
```



Follow up

如果有一个词典(Dictionary) 要求组成的单词都是词典里的 如何优化?

面试评分标准



Strong Hire: 两问的 DFS 都能写出来,第二问使用 Trie 或者 Hash 都可以,无需提示

Hire / Weak Hire: 写完第一问的 DFS, 第二问给出正确思路和方法, 但是没写完, 需要部分提示

No Hire: 第一问没写完,或者 bug 很多

Strong No: 没思路不会做

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矩阵上的 DFS

http://www.lintcode.com/problem/word-search-ii/

http://www.jiuzhang.com/solution/word-search-ii/

矩阵 (Matrix) 也是图

给出一个由小写字母组成的矩阵和一个字典。找出所有同时在字典和矩阵中出现的单词。



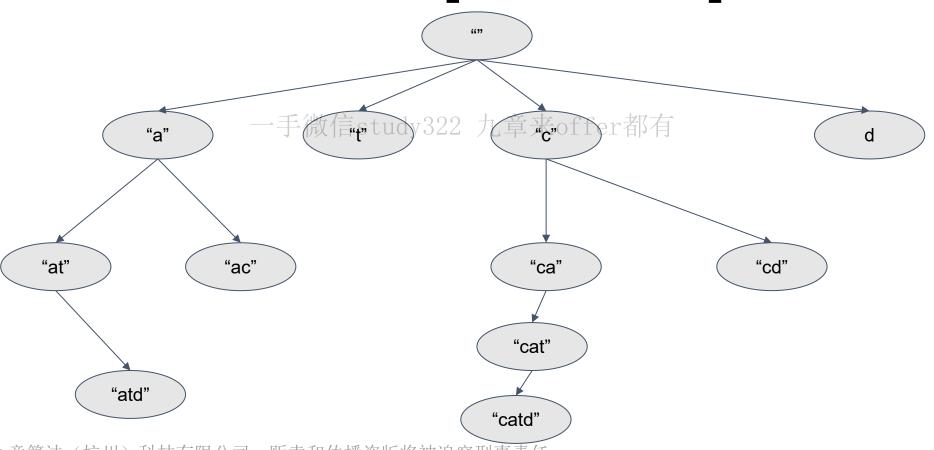
使用哪种搜索策略

A: for 词典里的每个单词 { check 单词是否在矩阵里 }

B: for 矩阵的每个单词 { check 单词是否在词典里}



board = ["at","cd"] words = ["cat","at"]



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代码



```
public List<String> wordSearchII(char[][] board, List<String> words) {
   if (board == null || board.length == 0) {
       return new ArrayList<>();
                                                                                             public static int[] DX = {0, 1, -1, 0};
                                                                                             public static int[] DY = {1, 0, 0, -1};
   if (board[0] == null || board[0].length == 0) {
                                                                                             private boolean inside(char[][] board, int x, int y) {
       return new ArrayList<>();
                                                                                                return x >= 0 && x < board.length && y >= 0 && y < board[0].length;
   boolean[][] visited = new boolean[board.length][board[0].length];
                                                                                             private void dfs(char[][] board,
   Map<String, Boolean> prefixIsWord = getPrefixSet(words);
                                                                                                             boolean[][] visited,
   Set<String> resultSet = new HashSet<>();
                                                                                                             int x,
                                                                                                             int y,
   for (int i = 0; i < board.length; i++) {</pre>
                                                                                                             String word,
        for (int j = 0; j < board[i].length; j++) {
                                                                                                            Map<String, Boolean> prefixIsWord,
           visited[i][j] = true;
                                                                                                            Set<String> resultSet) {
           dfs(board, visited, i, j, String.valueOf(board[i][j]), prefixIsWord, resultSet);
                                                                                                if (!prefixIsWord.containsKey(word)) {
           visited[i][j] = false;
                                                        if (prefixIsWord.get(word)) {
                                                                                                    resultSet.add(word);
   return new ArrayList<String>(resultSet);
                                                                                                for (int i = 0; i < 4; i++) {
private Map<String, Boolean> getPrefixSet(List<String> words) {
                                                                                                    int adjX = x + DX[i];
   Map<String, Boolean> prefixIsWord = new HashMap<>();
                                                                                                    int adjY = y + DY[i];
   for (String word : words) {
        for (int i = 0; i < word.length() - 1; i++) {
                                                                                                    if (!inside(board, adjX, adjY) || visited[adjX][adjY]) {
           String prefix = word.substring(0, i + 1);
                                                                                                        continue;
           if (!prefixIsWord.containsKey(prefix)) {
                prefixIsWord.put(prefix, false);
                                                                                                    visited[adjX][adjY] = true;
                                                                                                    dfs(board, visited, adjX, adjY, word + board[adjX][adjY], prefixIsWord, resultSet);
                                                                                                    visited[adjX][adjY] = false;
        prefixIsWord.put(word, true);
   return prefixIsWord;
```

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```
DX = [0, 1, -1, 0]
DY = [1, 0, 0, -1]
class Solution:
    def wordSearchII(self, board, words):
        if board is None or len(board) == 0:
            return []
        if board[0] is None or len(board[0]) == 0:
            return []
        visited = [[0] * len(board[0]) for _ in range(len(board))]
        prefix_is_word = self.get_prefix_set(words)
        result set = set()
        for i in range(len(board)):
            for j in range(len(board[i])):
                visited[i][j] = True
                self.dfs(board, visited, i, j, board[i][j], prefix_is_word, result_set)
                visited[i][j] = False
        return list(result_set)
        get prefix set(self, words):
        prefix_is_word = {}
        for word in words:
            for i in range(len(word)):
                prefix = word[:i + 1]
                if prefix not in prefix_is_word:
                    prefix is word[prefix] = False
                prefix is word[word] = True
        return prefix_is_word
```

```
def inside(self, board, x, y):
    return x >= 0 and x < len(board) and y >= 0 and y < len(board[0])

def dfs(self, board, visited, x, y, word, prefix_is_word, result_set):
    if word not in prefix_is_word:
        return

if prefix_is_word[word]:
    result_set.add(word)

for i in range(4):
    adjX = x + DX[i]
    adjY = y + DY[i]
    if not self.inside(board, adjX, adjY) or visited[adjX][adjY]:
        continue

    visited[adjX][adjY] = True
    self.dfs(board, visited, adjX, adjY, word + board[adjX][adjY], prefix_is_word,
        result_set)
    visited[adjX][adjY] = False</pre>
```



如何使用 hashset 来做前缀查询?



Follow up

https://www.lintcode.com/problem/word-search-iii/

https://www.lintcode.com/problem/boggle-game/ (Airbnb)

在矩阵上同时能够找出最多多少个不重叠的单词?

本题不作为面试要求



找所有的最短路径

http://www.lintcode.com/problem/word-ladder-ii/

http://www.jiuzhang.com/solutions/word-ladder-ii/



算法1:直接 DFS

通过打擂台的方式记录下所有最短路径 缺点是什么?



算法2:直接 BFS

一手微信队列里存什多来offer都有

A: 单词节点

B: 单词路径



算法2: 直接 BFS

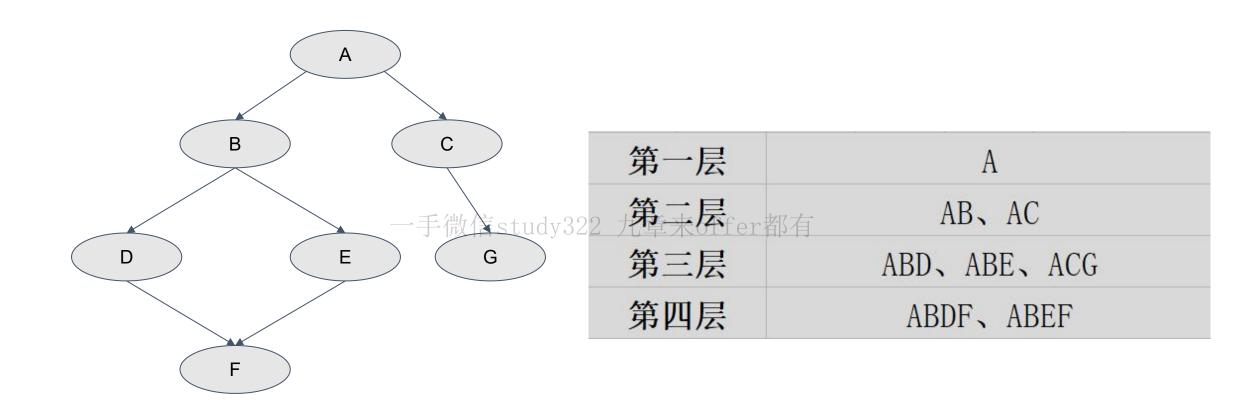
一手微信队列里存什多来offer都有

A: 单词节点

B: 单词路径

如果使用 BFS 找所有的路径,那么队列里放的就是路径







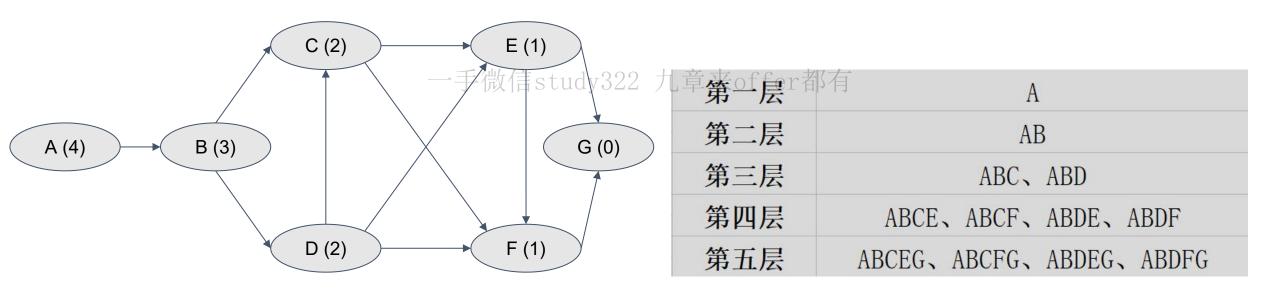
最优算法: BFS + DFS

BFS: 求出所有点到终点的距离

DFS: 沿着离终点越来越近的路线找到所有路径



start:A end:G





```
public List<List<String>> findLadders(String start, String end, Set<String> dict) {
   dict.add(start);
                                                                                private Map<String, Integer> bfs(String end, Map<String, Set<String>> indexes) {
   dict.add(end);
                                                                                    Map<String, Integer> distance = new HashMap<String, Integer>();
   Map<String, Set<String>> indexes = buildIndexes(dict);
                                                                                    distance.put(end, 0);
                                                                                    Queue < String > queue = new ArrayDeque < String > ();
   Map<String, Integer> distance = bfs(end, indexes);
                                                                                    queue.offer(end);
                                                                                    while (!queue.isEmpty()) {
   List<List<String>> results = new ArrayList<List<String>>();
                                                                                        String word = queue.poll();
   List<String> path = new ArrayList<String>();
                                                                                        for (String nextWord : getNextWords(word, indexes)) {
   path.add(start);
                                                                                            if (!distance.containsKey(nextWord)) {
   dfs(start, end, distance, indexes, path, results);
                                                                                                 distance.put(nextWord, distance.get(word) + 1);
                                                    一手微信study322 九章来ofqueue 可fer(nextWord);
   return results;
private Map<String, Set<String>> buildIndexes(Set<String> dict) {
                                                                                    return distance;
   Map<String, Set<String>> indexes = new HashMap<String, Set<String>>();
   for (String word : dict) {
       for (int i = 0; i < word.length(); i++) {</pre>
                                                                                private List<String> getNextWords(String word, Map<String, Set<String>> indexes) {
           String key = word.substring(0, i) + "%" + word.substring(i + 1);
                                                                                    List<String> words = new ArrayList<String>();
           if (indexes.containsKey(key)) {
                                                                                    for (int i = 0; i < word.length(); i++) {</pre>
               indexes.get(key).add(word);
                                                                                        String key = word.substring(0, i) + "%" + word.substring(i + 1);
           } else {
                                                                                        for (String w : indexes.get(key)) {
               indexes.put(key, new HashSet<>());
                                                                                            words.add(w);
               indexes.get(key).add(word);
                                                                                    return words;
   return indexes;
```



```
private void dfs(String curt,
                String target,
                Map<String, Integer> distance,
                Map<String, Set<String>> indexes,
                List<String> path,
                List<List<String>> results) {
   if (curt.equals(target)) {
       results.add(new ArrayList<String>(path));
       return;
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   for (String word : getNextWords(curt, indexes)) {
       if (distance.get(word) != distance.get(curt) - 1) {
           continue;
       path.add(word);
       dfs(word, target, distance, indexes, path, results);
       path.remove(path.size() - 1);
```



```
from collections import deque
class Solution:
   def findLadders(self, start, end, dict):
       dict.add(start)
       dict.add(end)
       indexes = self.build indexes(dict)
       distance = self.bfs(end, indexes)
       results = []
       self.dfs(start, end, distance, indexes, [start], results)
       return results
   def build indexes(self, dict):
       indexes = {}
       for word in dict:
           for i in range(len(word)):
               key = word[:i] + '%' + word[i + 1:]
               if key in indexes:
                   indexes[key].add(word)
               else:
                   indexes[key] = set([word])
       return indexes
```

```
bfs(self, end, indexes):
    distance = {end: 0}
    queue = deque([end])
    while queue:
        word = queue.popleft()
        for next word in self.get next words(word, indexes):
            if next word not in distance:
                distance[next word] = distance[word] + 1
                queue.append(next word)
    return distance
def get_next_words(self, word, indexes):
   words = []
    for i in range(len(word)):
        key = word[:i] + '%' + word[i + 1:]
        for w in indexes.get(key, []):
            words.append(w)
    return words
def dfs(self, curt, target, distance, indexes, path, results):
    if curt == target:
        results.append(list(path))
    for word in self.get next words(curt, indexes):
        if distance[word] != distance[curt] - 1:
        path.append(word)
        self.dfs(word, target, distance, indexes, path, results)
        path.pop()
```

后续学习



- 在第 30 章的互动课中学习非递归的方式实现排列组合类 DFS
 - 如何求下一个排列
 - 通用的非递归算法
- 在第 31 章的互动课中学习两道经典的 DFS 题
 - N 皇后问题及程序结构的艺术
 - 数独及搜索顺序的优化算法

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