时间复杂度低于 O(n) 的算法

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除了二分法以外还有如下几种

快速幂算法 O(logN)

辗转相除法 O(logN)

分解质因数 O(√N)

分块检索法 O(√N)

快速幂算法

求 a^n % b

其中 a, b, n 都是 int 范围 (2^{31} - 1)

递归的做法 —— 最不容易写错


```java
class Solution {
    /*
     * @param a, b, n: 32bit integers
     * @return: An integer
     */
    public int fastPower(int a, int b, int n) {
        if (n == 1) {
            return a % b;
        }
        if (n == 0) {
            return 1 % b;
        }

        long product = fastPower(a, b, n / 2);
        product = (product * product) % b;
        if (n % 2 == 1) {
            product = (product * a) % b;
        }
        return (int) product;
    }
};
```

```python
class Solution:
    """
    @param a: A 32bit integer
    @param b: A 32bit integer
    @param n: A 32bit integer
    @return: An integer
    """
    def fastPower(self, a, b, n):
        if n == 0:
            return 1 % b
        if n == 1:
            return a % b

        # a^n = (a^n/2) ^ 2
        power = self.fastPower(a, b, n // 2)
        power = (power * power) % b

        # 如果 n 是奇数，还需要多乘以一个 a，因为 n // 2 是整除
        if n % 2 == 1:
            power = (power * a) % b

        return power
```

二进制的做法 —— 非递归，比较巧妙


```java
public int fastPower(int a, int b, int n) {
    long ans = 1, tmp = a;

    while (n != 0) {
        if (n % 2 == 1) {
            ans = (ans * tmp) % b;
        }
        tmp = (tmp * tmp) % b;
        n = n / 2;
    }

    return (int) ans % b;
}
```

```python
def fastPower(self, a, b, n):
    ans = 1
    while n > 0:
        if n % 2 == 1:
            ans = (ans * a) % b
        a = a * a % b
        n = n // 2
    return ans % b
```


$$a^{(1010)2} = a^{(1000)2} * a^{(10)2}$$

比如 n=5,可以看做 $a^{(101)2}\%b$ （5的二进制是101）

拆开也就是 $a^{(100)2} * a^{1}\%b$

因此相当于我们把 n 做二进制转换，碰到 1 的时候，称一下对应的 a 的幂次

而 a 的幂次我们只需要知道 a^1, $a^{(10)2}$, $a^{(100)2}$ ... 也就是 a^1, a^2, a^4 ...

因此不断的把 a = a * a 就可以了

中间计算的时候，随时可以 % b 避免 overflow 其不影响结果，这是 % 运算的特性。

分块检索算法

将长度为 N 的区间分成 √N 的大小的小区间

总共 √N 个小区间，每个小区间统计局部的数据

因此在这些区间中进行增删查改的效率是 O(√N)

统计每个数前面比他小的数

https://www.lintcode.com/problem/count-of-smaller-number-before-itself/

https://www.jiuzhang.com/solution/count-of-smaller-number-before-itself/

[1, 2, 7, 8, 5] 每个数前面比他小的数分别为 [0, 1, 2, 3, 2]

Python 代码


```python
class BlockArray:
    def __init__(self, max_value):
        self.blocks = [
            Block()
            for _ in range(max_value // 100 + 1)
        ]

    def count_smaller(self, value):
        count = 0
        block_index = value // 100
        for i in range(block_index):
            count += self.blocks[i].total

        counter = self.blocks[block_index].counter
        for val in counter:
            if val < value:
                count += counter[val]
        return count

    def insert(self, value):
        block_index = value // 100
        block = self.blocks[block_index]
        block.total += 1
        block.counter[value] = block.counter.get(value, 0) + 1
```

```python
class Block:
    def __init__(self):
        self.total = 0
        self.counter = {}
```

```python
class Solution:
    """
    @param A: an integer array
    @return: A list of integers includes the inde
    """
    def countOfSmallerNumberII(self, A):
        if not A:
            return []

        block_array = BlockArray(10000)
        results = []
        for a in A:
            count = block_array.count_smaller(a)
            results.append(count)
            block_array.insert(a)
        return results
```

Java 代码


```java
class BlockArray {
    public Block[] blocks;
    public int blockSize;

    public BlockArray(int capacity) {
        blockSize = (int) Math.sqrt(capacity);
        int blockCount = capacity / blockSize + 1;
        blocks = new Block[blockCount];
        for (int i = 0; i < blockCount; i++) {
            blocks[i] = new Block(blockSize);
        }
    }

    public int countSmaller(int value) {
        int index = value / blockSize;
        int count = 0;
        for (int i = 0; i < index; i++) {
            count += blocks[i].total;
        }

        for (int i = 0; i + index * blockSize < value; i++) {
            count += blocks[index].counter[i];
        }

        return count;
    }

    public void insert(int value) {
        int index = value / blockSize;
        blocks[index].total++;
        blocks[index].counter[value - index * blockSize]++;
    }
}
```

```java
class Block {
    public int total;
    public int[] counter;
    public Block(int blockSize) {
        this.total = 0;
        this.counter = new int[blockSize];
    }
}
```

```java
public class Solution {
    /**
     * @param A: an integer array
     * @return: A list of integers includes the index of t
     */
    public List<Integer> countOfSmallerNumberII(int[] A) {
        List<Integer> results = new ArrayList<>();
        if (A == null || A.length == 0) {
            return results;
        }

        BlockArray blockArray = new BlockArray(10000);
        for (int i = 0; i < A.length; i++) {
            results.add(blockArray.countSmaller(A[i]));
            blockArray.insert(A[i]);
        }

        return results;
    }
}
```