Simulate Collusion: Application Examples for Cartel Simulation based on stochastic interest rates

knitr::opts\_chunk$set(echo = TRUE, warning = FALSE, tidy=TRUE, tidy.opts = list(width.cutoff=80))

rm(list = ls())

Set Seed for Reproducibility

set.seed(123)

Read in Data. Baseline Model 1 with default parameters.

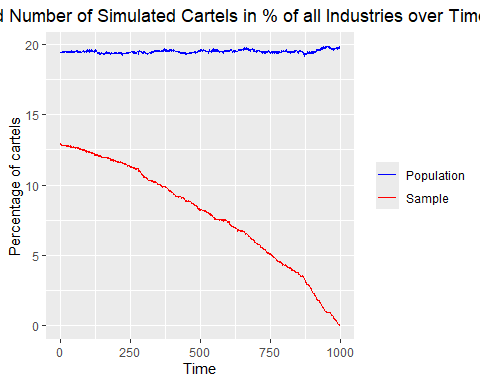
sim\_list <- sim\_col\_r()  
# sim\_list <- sim\_col\_r(model = 1, periods = 1000, n\_industries = 300, r\_min =  
# 0.001, r\_max = 0.3, n\_firms\_min = 2, n\_firms\_max = 100, alpha = 500) #  
# Bellert, Günster (2025)  
cartels\_duration <- get\_durations(sim\_list$cartels\_detected, sim\_list$cartels\_undetected,  
 sim\_list$interest\_r, sim\_list$deltas, sim\_list$parms, model = 1)  
cartels\_duration <- data.frame(cartels\_duration)

Add nonlinear interdependencies between variables

cartels\_duration <- add\_nonlinears(cartels\_duration, model = 1)

Plot cartel time series

c\_det <- rowSums(sim\_list$cartels\_detected)/(dim(sim\_list$cartels\_detected)[2] \*  
 dim(sim\_list$cartels\_detected)[3]) # number of detected cartels / number of industries  
c\_pop <- rowSums(sim\_list$cartels\_population)/(dim(sim\_list$cartels\_population)[2] \*  
 dim(sim\_list$cartels\_population)[3]) # number of cartels / number of industries  
sim\_cartels <- ts(data = cbind(c\_pop, c\_det))  
colnames(sim\_cartels) <- c("Population", "Sample")  
pallete <- c("blue", "red")  
  
autoplot(sim\_cartels \* 100) + xlab("Time") + ylab("Percentage of cartels") + ggtitle("Aggregated Number of Simulated Cartels in % of all Industries over Time") +  
 scale\_colour\_manual(values = pallete) + theme(legend.title = element\_blank(),  
 plot.title = element\_text(hjust = 0.5))

 Summary Statistics

describe(cartels\_duration)

## vars n mean sd median trimmed mad min max range  
## parm\_id 1 2708 17.85 8.68 19.00 18.02 8.90 1.00 39.00 38.00  
## industry 2 2708 14.54 8.67 14.00 14.36 10.38 1.00 30.00 29.00  
## cartel 3 2708 5.52 6.46 3.00 4.09 2.97 1.00 37.00 36.00  
## detected 4 2708 0.32 0.47 0.00 0.28 0.00 0.00 1.00 1.00  
## nTc 5 2708 0.88 1.14 0.00 0.67 0.00 0.00 7.00 7.00  
## rep\_off 6 2708 0.24 0.43 0.00 0.18 0.00 0.00 1.00 1.00  
## start 7 2708 400.24 340.77 370.00 382.43 523.36 1.00 1000.00 999.00  
## end 8 2708 645.52 324.72 713.00 672.60 425.51 1.00 1000.00 999.00  
## duration 9 2708 246.27 321.86 77.00 185.91 112.68 1.00 1000.00 999.00  
## lduration 10 2708 3.86 2.30 4.36 3.88 3.05 0.69 6.91 6.22  
## n\_firms 11 2708 4.33 1.42 5.00 4.37 1.48 2.00 8.00 6.00  
## sigma 12 2708 0.24 0.09 0.25 0.25 0.07 0.10 0.35 0.25  
## in\_cartel 13 2708 1.00 0.00 1.00 1.00 0.00 1.00 1.00 0.00  
## mean\_r 14 2708 0.12 0.09 0.14 0.12 0.10 -0.14 0.40 0.54  
## var\_r 15 2708 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00  
## mean\_delta 16 2708 0.82 0.02 0.81 0.81 0.02 0.76 0.88 0.12  
## nfirms\_sigma 17 2708 1.05 0.52 1.00 1.02 0.59 0.20 2.45 2.25  
## skew kurtosis se  
## parm\_id -0.20 -0.84 0.17  
## industry 0.13 -1.18 0.17  
## cartel 2.03 3.91 0.12  
## detected 0.77 -1.42 0.01  
## nTc 1.44 2.07 0.02  
## rep\_off 1.21 -0.53 0.01  
## start 0.21 -1.43 6.55  
## end -0.45 -1.18 6.24  
## duration 1.23 0.21 6.18  
## lduration -0.19 -1.52 0.04  
## n\_firms -0.23 -0.94 0.03  
## sigma -0.30 -1.17 0.00  
## in\_cartel NaN NaN 0.00  
## mean\_r -0.11 -0.74 0.00  
## var\_r 4.32 25.61 0.00  
## mean\_delta 0.29 -0.70 0.00  
## nfirms\_sigma 0.52 -0.53 0.01

Correlations

df\_cor <- Filter(function(x) sd(x) != 0, cartels\_duration)  
df\_cor <- select(df\_cor, -c(industry, cartel))  
cor(df\_cor)

## parm\_id detected nTc rep\_off start  
## parm\_id 1.0000000 -0.40003904 -0.43463667 -0.31035459 0.31001437  
## detected -0.4000390 1.00000000 0.51595326 0.36350774 -0.41945505  
## nTc -0.4346367 0.51595326 1.00000000 0.85175975 0.07907463  
## rep\_off -0.3103546 0.36350774 0.85175975 1.00000000 0.17379382  
## start 0.3100144 -0.41945505 0.07907463 0.17379382 1.00000000  
## end -0.1838240 -0.30337208 0.18400454 0.20529736 0.53307104  
## duration -0.5136867 0.13803060 0.10191962 0.02311749 -0.52094361  
## lduration -0.6312934 0.41138255 0.43561024 0.29653834 -0.45052721  
## n\_firms 0.9806076 -0.43763128 -0.49004536 -0.36400202 0.29965026  
## sigma 0.1865477 0.15271088 0.23893317 0.24120670 0.07964984  
## mean\_r -0.3739227 0.06544833 0.07250863 0.07290290 -0.04677827  
## var\_r -0.3391750 0.09799320 0.06707937 0.01295915 -0.33498134  
## mean\_delta 0.3767310 -0.06399273 -0.06742465 -0.06749011 0.05262354  
## nfirms\_sigma 0.7858059 -0.19781191 -0.18341256 -0.10282428 0.24746171  
## end duration lduration n\_firms sigma  
## parm\_id -0.183824027 -0.51368667 -0.63129340 0.980607611 0.186547726  
## detected -0.303372079 0.13803060 0.41138255 -0.437631281 0.152710884  
## nTc 0.184004536 0.10191962 0.43561024 -0.490045364 0.238933168  
## rep\_off 0.205297363 0.02311749 0.29653834 -0.364002016 0.241206701  
## start 0.533071039 -0.52094361 -0.45052721 0.299650255 0.079649844  
## end 1.000000000 0.44449828 0.33831628 -0.172823720 -0.071565878  
## duration 0.444498284 1.00000000 0.81832058 -0.491615547 -0.156531533  
## lduration 0.338316279 0.81832058 1.00000000 -0.637135154 -0.027111883  
## n\_firms -0.172823720 -0.49161555 -0.63713515 1.000000000 -0.009611012  
## sigma -0.071565878 -0.15653153 -0.02711188 -0.009611012 1.000000000  
## mean\_r 0.008801452 0.05840632 0.08079891 -0.377181218 -0.017069398  
## var\_r 0.297613541 0.65492150 0.52956836 -0.329221714 -0.080195301  
## mean\_delta -0.001557775 -0.05728696 -0.07793729 0.379774785 0.018396639  
## nfirms\_sigma -0.145000305 -0.40829003 -0.42720078 0.658611326 0.707820047  
## mean\_r var\_r mean\_delta nfirms\_sigma  
## parm\_id -0.373922696 -0.33917498 0.376730980 0.7858059  
## detected 0.065448333 0.09799320 -0.063992732 -0.1978119  
## nTc 0.072508629 0.06707937 -0.067424650 -0.1834126  
## rep\_off 0.072902899 0.01295915 -0.067490106 -0.1028243  
## start -0.046778271 -0.33498134 0.052623541 0.2474617  
## end 0.008801452 0.29761354 -0.001557775 -0.1450003  
## duration 0.058406324 0.65492150 -0.057286957 -0.4082900  
## lduration 0.080798909 0.52956836 -0.077937291 -0.4272008  
## n\_firms -0.377181218 -0.32922171 0.379774785 0.6586113  
## sigma -0.017069398 -0.08019530 0.018396639 0.7078200  
## mean\_r 1.000000000 0.03235240 -0.998496734 -0.2655065  
## var\_r 0.032352398 1.00000000 -0.029186897 -0.2588364  
## mean\_delta -0.998496734 -0.02918690 1.000000000 0.2684771  
## nfirms\_sigma -0.265506457 -0.25883638 0.268477132 1.0000000

Mean comparison tests for sumstats

t.test(duration ~ detected, data = cartels\_duration, var.equal = FALSE)

##   
## Welch Two Sample t-test  
##   
## data: duration by detected  
## t = -8.0336, df = 2219.6, p-value = 1.521e-15  
## alternative hypothesis: true difference in means between group 0 and group 1 is not equal to 0  
## 95 percent confidence interval:  
## -118.33914 -71.90085  
## sample estimates:  
## mean in group 0 mean in group 1   
## 215.7122 310.8322

t.test(n\_firms ~ detected, data = cartels\_duration, var.equal = FALSE)

##   
## Welch Two Sample t-test  
##   
## data: n\_firms by detected  
## t = 26.608, df = 1938.1, p-value < 2.2e-16  
## alternative hypothesis: true difference in means between group 0 and group 1 is not equal to 0  
## 95 percent confidence interval:  
## 1.233705 1.430043  
## sample estimates:  
## mean in group 0 mean in group 1   
## 4.760609 3.428736

t.test(sigma ~ detected, data = cartels\_duration, var.equal = FALSE)

##   
## Welch Two Sample t-test  
##   
## data: sigma by detected  
## t = -8.3003, df = 1851.6, p-value < 2.2e-16  
## alternative hypothesis: true difference in means between group 0 and group 1 is not equal to 0  
## 95 percent confidence interval:  
## -0.03438520 -0.02124136  
## sample estimates:  
## mean in group 0 mean in group 1   
## 0.2336235 0.2614368

Linear Regression

reg <- lm(lduration ~ start + n\_firms + sigma + mean\_r + var\_r, data = cartels\_duration)  
summary(reg)

##   
## Call:  
## lm(formula = lduration ~ start + n\_firms + sigma + mean\_r + var\_r,   
## data = cartels\_duration)  
##   
## Residuals:  
## Min 1Q Median 3Q Max   
## -5.6052 -1.2476 -0.0461 1.1861 3.6685   
##   
## Coefficients:  
## Estimate Std. Error t value Pr(>|t|)   
## (Intercept) 8.188e+00 1.597e-01 51.272 <2e-16 \*\*\*  
## start -1.348e-03 9.248e-05 -14.580 <2e-16 \*\*\*  
## n\_firms -8.636e-01 2.398e-02 -36.014 <2e-16 \*\*\*  
## sigma 1.270e-01 3.422e-01 0.371 0.711   
## mean\_r -3.428e+00 3.367e-01 -10.181 <2e-16 \*\*\*  
## var\_r 1.980e+03 9.420e+01 21.022 <2e-16 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Residual standard error: 1.504 on 2702 degrees of freedom  
## Multiple R-squared: 0.5732, Adjusted R-squared: 0.5724   
## F-statistic: 725.7 on 5 and 2702 DF, p-value: < 2.2e-16

# Hazard Rate Estimation

periods <- dim(sim\_list$cartels\_population)[1]  
data\_pop <- cartels\_duration  
data\_sample <- data\_pop[data\_pop$detected == 1, ]  
data\_undetect <- data\_pop[data\_pop$detected == 0, ]  
  
data\_pop$dead <- ifelse(data\_pop$end < periods, 1, 0)  
data\_sample$dead <- ifelse(data\_sample$end < periods, 1, 0)  
data\_undetect$dead <- ifelse(data\_undetect$end < periods, 1, 0)  
  
hazC <- phreg(Surv(duration, dead) ~ start + n\_firms + sigma + mean\_r + var\_r, data = data\_pop,  
 dist = "weibull")  
hazS <- phreg(Surv(duration, detected) ~ start + n\_firms + sigma + mean\_r + var\_r,  
 data = data\_sample, dist = "weibull")  
hazU <- phreg(Surv(duration, dead) ~ start + n\_firms + sigma + mean\_r + var\_r, data = data\_undetect,  
 dist = "weibull")  
  
stargazer(hazS, hazU, hazC, title = "HR Regression for Cartel Duration on Model I",  
 type = "text", column.labels = c("HRSample", "HRUndetect", "HRCartels"), df = FALSE,  
 digits = 3)

##   
## HR Regression for Cartel Duration on Model I  
## =========================================================  
## Dependent variable:   
## ------------------------------------------  
## duration duration   
## HRSample HRUndetect HRCartels   
## (1) (2) (3)   
## ---------------------------------------------------------  
## start 0.002\*\*\* -0.0004\*\*\* 0.00004   
## (0.0001) (0.0001) (0.0001)   
##   
## n\_firms 0.030 2.425\*\*\* 0.578\*\*\*   
## (0.031) (0.183) (0.024)   
##   
## sigma 1.110\*\* -0.795\*\* 0.357   
## (0.440) (0.346) (0.266)   
##   
## mean\_r -0.012 18.298\*\*\* 2.774\*\*\*   
## (0.373) (1.438) (0.280)   
##   
## var\_r -1,829.672\*\*\* -38,029.590\*\*\* -4,862.372\*\*\*  
## (185.539) (2,337.278) (227.926)   
##   
## log(scale) 5.904\*\*\* 21.292\*\*\* 8.799\*\*\*   
## (0.136) (1.582) (0.265)   
##   
## log(shape) 0.294\*\*\* -0.275\*\*\* -0.522\*\*\*   
## (0.028) (0.022) (0.017)   
##   
## ---------------------------------------------------------  
## Observations 870 1,838 2,708   
## Log Likelihood -5,686.590 -3,655.444 -10,784.150   
## =========================================================  
## Note: \*p<0.1; \*\*p<0.05; \*\*\*p<0.01

# Plot for ICC and Discount Factor of 5 Example Industries

set.seed(123)  
n\_firms <- 4  
gamma <- 0.8  
theta <- 1  
sigma\_all <- 0.25  
sigma\_t <- 1 - (1-sigma\_all)^(1/200)  
struc <- 1  
  
r\_min <- 0.001  
r\_max <- 0.3  
  
sim\_list\_M1 <- sim\_col\_r(model = 1,n\_firms\_min = 4, n\_firms\_max = 4, n\_industries = 5, sigma = 0.25, gamma = 0.8, theta = 1, struc = 1)  
sim\_list\_M3\_1 <- sim\_col\_r(model = 3,n\_firms\_min = 4, n\_firms\_max = 4, n\_industries = 5, sigma = 0.25, gamma = 0.8, theta = 1, struc = 1)  
sim\_list\_M3\_0 <- sim\_col\_r(model = 3,n\_firms\_min = 4, n\_firms\_max = 4, n\_industries = 5, sigma = 0.25, gamma = 0.8, theta = 0, struc = 1)  
x <- sim\_list\_M3\_1$ICC  
x <- data.frame(x)  
df <- cbind(sim\_list\_M1$ICC[,1,1], sim\_list\_M3\_1$ICC[, 1, 1], sim\_list\_M3\_0$ICC[, 1, 1], sim\_list\_M3\_1$deltas[,,1])  
df <- data.frame(df)  
sim <- ts(df)  
colnames(sim) <- c("ICC MI, II", "ICC MIII (no leniency)", "ICC MIII (full leniency)", paste("Industry", 1:ncol(sim\_list\_M3\_0$deltas[,,1])))  
pallete <- c('blue4', 'blue', 'red', 'lightcoral', 'aquamarine3', 'cyan4', 'brown', 'orange')  
y\_axis <- c(0.725, 0.875) # for singular picture  
autoplot(sim) +  
 ylim(y\_axis) +  
 xlab("Time") +  
 ylab("") +  
 scale\_colour\_manual(values=pallete) +  
 theme(legend.position = "none",  
 plot.caption = element\_text(hjust = 0.5, size = 10),# move caption to the middle  
 axis.title = element\_text(size = 10)  
 )

