Online Appendix to Simulating Collusion:

Challenging Conventional Estimation Methods

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This online appendix contains summary statistics and result tables for the individual models we simulated. Linear regression, hazard rate (HR) estimation, Lasso cross-validation (CV) Regression (Tibshirani (1996)), and regressions corrected for Heckman Sample Selection (Heckman (1979)) are applied on data simulated based on Model I (Stigler (1964), II (Stigler (1964) and Harrington and Wei (2017), and III (Stigler (1964) and Bos et al (2018).

Appendix A Summary Statistics

Table A1 Summary Statistics Population (Detected and Undetected Cartels) of Model I

	Mean	Median	SD	Min	Max	Skew	N
Number of Firms n_f	4.74	5	1.52	2	8	-0.59	77'822
Detection Probability σ	0.24	0.25	0.08	0.10	0.35	-0.22	77'822
Start	442.35	427	308.34	1	1000	0.12	77'822
End	545.66	539	306.41	1	1000	0	77'822
Duration	104.31	35	159.45	1	1000	2.52	77'822
Ln(Duration+1)	3.30	3.58	1.91	0.69	6.91	-0.08	77'822
Detected	0.47	0	0.50	0	1	0.12	77'822
Times Caught	2.12	1	2.54	0	18	1.30	77'822
Repeat Offender	0.47	0	0.50	0	1	0.14	77'822

For Model I, this table presents the following summary statistics of the 77'822 simulated detected and undetected cartels within a total period of 1'000 time units: the exogenous industry and enforcement characteristics, the start and end date of the cartel, the duration, if it got detected, detected repeatedly, and how often it got detected.

Table A2 Summary Statistics Population (Detected and Undetected Cartels) of Model II

	Mean	Median	$^{\mathrm{SD}}$	Min	Max	Skew	N
Number of Firms n_f	5.19	6	1.38	2	8	-1.16	53'178
Detection Probability σ	0.23	0.25	0.09	0.10	0.35	-0.10	53'178
Start	422.15	401	314.10	1	1000	0.18	53'178
End	574.27	571	309.25	1	1000	-0.05	53'178
Duration	153.12	11	264.18	1	1000	2	53'178
Ln(Duration+1)	3.06	2.48	2.23	0.69	6.91	0.35	53'178
Detected	0.22	0	0.41	0	1	1.35	53'178
Times Caught	0.62	0	1.05	0	8	2.01	53'178
Repeat Offender	0.17	0	0.37	0	1	1.77	53'178

For Model II, this table presents the following summary statistics of the 53'178 simulated detected and undetected cartels within a total period of 1'000 time units: the exogenous industry and enforcement characteristics, the start and end date of the cartel, the duration, if it got detected, detected repeatedly, and how often it got detected.

 $\textbf{Table A3} \ \ \text{Summary Statistics Population (Detected and Undetected Cartels) of Model III}$

	Mean	Median	SD	Min	Max	Skew	N
Number of Firms n_f	3.62	4	1.28	2	7	0.16	809'858
Fines γ (% of Profit)	0.80	0.80	0.08	0.70	0.90	0.02	809'858
Leniency (% of Fine) θ	0.40	0.50	0.41	0	1	0.37	809'858
Detection Probability σ	0.22	0.20	0.08	0.10	0.35	0.08	809'858
Structured	0.48	0	0.50	0	1	0.07	809'858
Start	416.30	397	317.51	1	1000	0.19	809'858
End	524.04	516	315.19	1	1000	0.02	809'858
Duration	108.75	43	162.41	1	1000	2.60	809'858
Ln(Duration+1)	3.50	3.78	1.79	0.69	6.91	-0.20	809'858
Detected	0.53	1	0.50	0	1	-0.11	809'858
Times Caught	2.38	1	2.82	0	25	1.64	809'858
Repeat Offender	0.49	0	0.50	0	1	0.02	809'858

For Model III, this table presents the following summary statistics of the 809'858 simulated detected and undetected cartels within a total period of 1'000 time units: the exogenous industry and enforcement characteristics, the start and end date of the cartel, the duration, if it got detected, detected repeatedly, and how often it got detected.

Appendix B Linear Models Results

Table B4 Linear Regression on Cartel Duration for Model I, II and III

		Ln(Durat		
	mlrSample	mlrUndetect	mlrCartels	mlrHeck
Number of Firms n_f	-0.16^{***} (0.002)	-0.60^{***} (0.002)	-0.50*** (0.001)	-0.0003 (0.003)
Fines γ (% of Profit)	0.06*** (0.02)	0.03 (0.03)	0.03 (0.02)	0.07*** (0.02)
Leniency (% of Fine) θ	0.09*** (0.004)	1.58*** (0.01)	0.84*** (0.004)	$0.003 \\ (0.005)$
Detection Probability σ	-3.66*** (0.02)	-4.31*** (0.03)	-4.67^{***} (0.02)	-3.24^{***} (0.02)
Structured	-0.40^{***} (0.004)	-0.52^{***} (0.005)	-0.56*** (0.003)	-0.36^{***} (0.004)
Model II	0.53*** (0.01)	0.28*** (0.01)	0.23*** (0.01)	0.67*** (0.01)
Model III	-0.24^{***} (0.02)	-0.82^{***} (0.02)	-0.63^{***} (0.02)	-0.07^{***} (0.02)
Start	-0.001^{***} (0.0000)	-0.001*** (0.0000)	-0.002^{***} (0.0000)	-0.0002^{***} (0.0000)
Times Caught	-0.002^{**} (0.001)	-0.01^{***} (0.002)	0.05*** (0.001)	-0.05^{***} (0.001)
Repeat Offender	0.04*** (0.01)	1.26*** (0.01)	0.82*** (0.005)	-0.37^{***} (0.01)
IMR				$-0.71^{***} $ (0.01)
Constant	6.38*** (0.01)	6.98*** (0.02)	7.08*** (0.01)	6.06*** (0.01)
Observations R ² Adjusted R ²	$475'456 \\ 0.14 \\ 0.14$	465'402 0.43 0.43	940'858 0.42 0.42	475'456 0.15 0.15

Note: This table shows the estimation results of linear cross-sectional regressions to explain cartel duration (ln(duration+1)) at the industry level, for data simulated for all Models I, II, and III. Columns 2 - 5 estimate linear regression coefficients on the sample of detected cartels, the group of undetected cartels, the population of all cartels, and the sample corrected for Heckman Sample Selection, respectively. The estimated coefficients show standard errors in the sample, but do not test for the real population. Except of fines (γ) , all coefficients are significant.

Standard errors are in parentheses. Significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

 ${\bf Table~B5~~ Hazard~Rate~on~Cartel~Duration~for~Model~I,~II~and~III}$

	HDG 1	Cartel		IIDII 1
	HRSample	HRUndetect	HRCartels	HRHeck
Number of Firms n_f	0.12***	0.60***	0.37***	-0.01***
,	(0.002)	(0.002)	(0.001)	(0.002)
Fines γ (% of Profit)	-0.10***	-0.05**	-0.05***	-0.11***
	(0.02)	(0.02)	(0.01)	(0.02)
Leniency (% of Fine) θ	-0.05***	-1.26***	-0.57***	0.02***
,	(0.004)	(0.01)	(0.003)	(0.004)
Detection Probability σ	3.34***	4.32***	4.02***	3.02***
·	(0.02)	(0.02)	(0.01)	(0.02)
Structured	0.36***	0.35***	0.42***	0.34***
	(0.003)	(0.004)	(0.002)	(0.003)
Model II	-0.48***	-0.34***	-0.37***	-0.58***
	(0.01)	(0.01)	(0.01)	(0.01)
Model III	0.22***	0.87***	0.51***	0.09***
	(0.02)	(0.02)	(0.01)	(0.02)
Start	0.001***	0.001***	0.001***	0.001***
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
Γimes Caught	0.01***	-0.34***	-0.05***	0.04***
	(0.001)	(0.003)	(0.001)	(0.001)
Repeat Offender	-0.03***	-0.26***	-0.38***	0.29***
_	(0.004)	(0.01)	(0.003)	(0.01)
MR				0.56***
				(0.01)
Observations	475'456	465'402	940'858	475'456
Log Likelihood	-2'806'284.00	-1'445'284.00	-4'442'193.00	-2'804'065.0

Note: This table shows the estimation results of Weibull Hazard Model to explain cartel death at the industry level, for data simulated for all Models I, II, and III. The estimated coefficients show the change of risk for cartel breakdown, if the covariate increases by 1 unit, keeping all others fixed. Columns 2 - 5 use HR estimation on the sample of detected cartels, the group of undetected cartels, the population of all cartels, and the sample corrected for Heckman Sample Selection, respectively. The estimated coefficients show standard errors in the sample, but do not test for the real population. All estimators are significant.

Standard errors are in parentheses. Significance at the 1%, 5%, and 10% level is indicated by ***, ***, and *, respectively.

 ${\bf Table~B6}~$ Sample Selection Bias Linear Regression - Models I, II and III

Coefficients	β^s	β^l	α_{IMR}	$bias_{IMR}$
Number of Firms n_f	-0.16	0	0.23	-0.16
Fines γ (% of Profit)	0.06	0.07	0.01	-0.01
Leniency (% of Fine) θ	0.09	0	-0.13	0.09
Detection Probability σ	-3.66	-3.24	0.61	-0.43
Start	-0.40	-0.36	0	0
Structured	0.53	0.67	0.05	-0.04
Model II	-0.24	-0.07	0.19	-0.14
Model III	0	0	0.24	-0.17
Times Caught	0	-0.05	-0.07	0.05
Repeat Offender	0.04	-0.37	-0.58	0.41
IMR		-0.71		
Constant	6.38	6.06	-0.45	0.32

This table shows, for the Linear Regression of the combined Models I, II, and III, the sample selection bias that we correct with the inverse Mill's ratio (IMR) following (Heckman (1979)). β^s is the estimated coefficients in the short model without IMR. β^l is the estimated coefficients in the corrected long model including IMR. α_{IMR} is the coefficient in the auxiliary regression between each variable and IMR. The last column shows the sample selection bias: $bias_{IMR} = \beta^l(IMR) * \alpha_{IMR}$.

Table B7 Sample Selection Bias Linear Regression - Model I

Coefficients	β^s	β^l	α_{IMR}	$bias_{IMR}$
Number of Firms n_f	-0.12	-0.02	0.15	-0.10
Start	0	0	0	0
Detection Probability σ	-2.93	-3.11	-0.27	0.18
Times Caught	0.04	-0.03	-0.10	0.07
Repeat Offender	0.21	-0.18	-0.57	0.39
IMR		-0.68		
Constant	5.79	5.87	0.11	-0.07

This table shows, for the Linear Regression of Model I, the sample selection bias that we correct with the IMR following (Heckman (1979)). β^s is the estimated coefficients in the short model without IMR. β^l is the estimated coefficients in the corrected long model including IMR. α_{IMR} is the coefficient in the auxiliary regression between each variable and IMR. The last column shows the sample selection bias: $bias_{IMR} = \beta^l(IMR) * \alpha_{IMR}$.

 ${\bf Table~B8~~Sample~Selection~Bias~Linear~Regression~-~Model~II}$

Coefficients	β^s	β^l	α_{IMR}	$bias_{IMR}$
Number of Firms n_f	-0.16	-0.09	0.14	-0.07
Start	0	0	0	0
Detection Probability σ	-1.43	-1.67	-0.46	0.25
Times Caught	0.07	-0.17	-0.44	0.24
Repeat Offender	0.27	0.18	-0.16	0.09
IMR		-0.54		
Constant	6.26	6.56	0.56	-0.30

This table shows, for the Linear Regression of Model II, the sample selection bias that we correct with the IMR following (Heckman (1979)). β^s is the estimated coefficients in the short model without IMR. β^l is the estimated coefficients in the corrected long model including IMR. α_{IMR} is the coefficient in the auxiliary regression between each variable and IMR. The last column shows the sample selection bias: $bias_{IMR} = \beta^l(IMR) * \alpha_{IMR}$.

 ${\bf Table~B9~~Sample~Selection~Bias~Linear~Regression~-~Model~III}$

Coefficients	β^s	β^l	α_{IMR}	$bias_{IMR}$
Number of Firms n_f	-0.18	-0.01	0.25	-0.17
Fines γ (% of Profit)	0.06	0.07	0.01	-0.01
Leniency (% of Fine) θ	0.10	0.01	-0.13	0.09
Start	0	0	0	0
Structured	-0.40	-0.36	0.06	-0.04
Detection Probability σ	-3.85	-3.31	0.79	-0.54
Times Caught	-0.01	-0.05	-0.06	0.04
Repeat Offender	0.01	-0.37	-0.56	0.38
IMR		-0.68		
Constant	6.25	6.02	-0.33	0.23

This table shows, for the Linear Regression of Model II, the sample selection bias that we correct with the IMR following (Heckman (1979)). β^s is the estimated coefficients in the short model without IMR. β^l is the estimated coefficients in the corrected long model including IMR. α_{IMR} is the coefficient in the auxiliary regression between each variable and IMR. The last column shows the sample selection bias: $bias_{IMR} = \beta^l(IMR) * \alpha_{IMR}$.

Table B10 Linear Regression and HR for Cartel Duration on Model I - ICC on Stigler - Detection independent of Collusion

	elameSlm	$\frac{\operatorname{Ln}(\operatorname{Duration} + 1)}{\operatorname{mirIndetect}}$	tion+1) $mlrCartels$	HrHeck	HRSample	Cartel Death	Death HRCartels	ды НВН
	ordinac iiii	200000000000000000000000000000000000000		11001111111	ordinaconi	200000000000000000000000000000000000000		100000000000000000000000000000000000000
N Firms n_f	-0.12^{***}	-0.57***	-0.39***	-0.02***	0.09	0.79***	0.30	0.01*
•	(0.005)	(0.01)	(0.004)	(0.01)	(0.004)	(0.01)	(0.003)	(0.01)
Start	-0.001^{***}	-0.001^{***}	-0.002***	-0.0002^{***}	0.001***	0.001***	0.001***	0.001***
	(0.000)	(0.0000)	(0.0000)	(0.0001)	(0.0000)	(0.0000)	(0.0000)	(0.0001)
Detection Prob. σ	-2.93***	-1.60***	-2.61***	-3.11***	2.78***	1.42***	2.73***	2.94***
	(0.08)	(0.08)	(0.00)	(0.08)	(0.01)	(0.06)	(0.05)	(0.01)
Times Caught	0.04	-0.01	0.13***	-0.03***	-0.03***	-0.56***	-0.13***	0.02***
	(0.004)	(0.01)	(0.003)	(0.01)	(0.004)	(0.02)	(0.003)	(0.005)
Repeat Offender	0.21	2.06***	1.20***	-0.18***	-0.22***	-0.19***	-0.59***	0.08***
	(0.02)	(0.03)	(0.02)	(0.03)	(0.02)	(0.05)	(0.01)	(0.02)
IMR				***89.0-				0.53***
				(0.03)				(0.03)
Constant	5.79***	6.13***	5.81***	5.87**				
	(0.03)	(0.05)	(0.03)	(0.03)				
Observations	36'615	41,207	7,822	36,615	36'615	41,207	7'822	36'615
\mathbb{R}^2	0.10	0.47	0.50	0.11				
Adjusted \mathbb{R}^2	0.10	0.47	0.50	0.11				
Log Likelihood					$-218^{\circ}572.00$	-114'465.60	-350'666.30	-218'383.30

Note: This table shows the estimation results of linear cross-sectional regressions to explain cartel duration (ln(duration+1)) and the estimation results of a Weibull Hazard Model to explain cartel death, both at the industry level, for data simulated for Model I. Columns 2 - 5 estimate linear regression coefficients, while columns 6 - 9 estimate HR coefficients, both on the sample of detected cartels, the group of undetected cartels, the population of all cartels, and the sample corrected for Heckman Sample Selection, respectively. The estimated coefficients show standard errors in the sample, but do not test for the real population. The estimated HR coefficients show the change of risk for cartel breakdown if the covariate increases by 1 unit, keeping all others fixed. Standard errors are in parentheses. Significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

Table B11 Linear Regression and HR for Cartel Duration on Model II - ICC on Stigler - Detection depends on number of Firms

		$\operatorname{Ln}(\operatorname{Duration} + 1)$	ion+1)			Cartel Death	Death	
	mlrSample	mlrUndetect	mlrCartels	mlrHeck	HRSample	HRUndetect	HRCartels	HRHeck
N Firms n_f	-0.16***	***26.0—	-0.72***	***60.0-	0.11**	0.96	0.56***	0.07***
•	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Start	-0.002***	-0.002***	-0.002***	-0.001***	0.002^{***}	0.001***	0.001^{***}	0.002***
	(0.0001)	(0.0000)	(0.0000)	(0.0001)	(0.0001)	(0.0000)	(0.000)	(0.0001)
Detection Prob. σ	-1.43***	-0.86***	-1.17***	-1.67***	1.20***	0.67***	1.12***	1.35***
	(0.14)	(0.00)	(0.08)	(0.14)	(0.12)	(0.06)	(0.06)	(0.12)
Times Caught	0.07	0.75	0.68***	-0.17***	-0.04***	-1.52***	-0.49***	0.10***
	(0.02)	(0.02)	(0.01)	(0.03)	(0.02)	(0.03)	(0.01)	(0.03)
Repeat Offender	0.27***	-0.54***	-0.31***	0.18***	-0.28***	1.26	0.34***	-0.21***
	(0.04)	(0.00)	(0.03)	(0.04)	(0.03)	(0.08)	(0.03)	(0.03)
IMR				-0.54***				0.33***
				(0.00)				(0.02)
Constant	6.26***	8.82***	2.60***	6.56				
	(0.05)	(0.05)	(0.04)	(0.06)				
Observations	11,733	41,445	53,178	11,733	11,733	41,445	53,178	11,733
\mathbb{R}^2	0.17	0.51	0.56	0.17				
Adjusted \mathbb{R}^2	0.16	0.51	0.56	0.17				
Log Likelihood					-77'014.56	-120'764.40	-209'666.50	-76'995.30

of a Weibull Hazard Model to explain cartel death, both at the industry level, for data simulated for Model II. Columns 2 - 5 estimate linear regression coefficients, while columns 6 - 9 estimate HR coefficients, both on the sample of detected cartels, the group of undetected cartels, the population of all cartels, and the sample corrected for Heckman Sample Selection, respectively. The estimated coefficients show standard errors in the sample, but do not test for the real population. The estimated HR coefficients show the change of risk for cartel breakdown if the covariate increases by 1 unit, Note: This table shows the estimation results of linear cross-sectional regressions to explain cartel duration (ln(duration+1)) and the estimation results keeping all others fixed.

Standard errors are in parentheses. Significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

 Table B12
 Linear Regression and HR for Cartel Duration on Model III - ICC on Harrington et al.

	mlrSample	Ln(Duration+1 mlrUndetect mlrC	tion+1) mlrCartels	mlrHeck	HRSample	Cartel Death HRUndetect HR	Death HRCartels	HRHeck
N Firms n_f	-0.18^{***}	-0.56***	-0.49*** (0.002)	-0.01** (0.003)	0.13***	0.58***	0.37***	-0.001 (0.003)
Fines γ (% of Profit)	(50.0) ***90.0	0.02	0.02	0.07***	-0.10***	(2005) -0.05**	-0.05***	-0.11^{**}
Leniency (% Fine) θ	(0.02) 0.10^{***} (0.004)	$(0.05) \\ 1.59*** \\ (0.01)$	(0.02) $0.87***$ (0.004)	$\begin{pmatrix} 0.02 \\ 0.01 \\ (0.005) \end{pmatrix}$	(0.02) -0.05^{***} (0.004)	$^{(0.02)}_{-1.28**}$	(0.01) $-0.59***$ (0.003)	$\begin{array}{c} (0.02) \\ 0.02^{***} \\ (0.004) \end{array}$
Start	-0.001^{***} (0.0000)	-0.001^{***} (0.000)	-0.002^{***} (0.0000)	-0.0002^{***} (0.0000)	0.001^{***} (0.0000)	0.001^{***} (0.0000)	0.001^{***} (0.0000)	0.001^{***} (0.000)
Structured	-0.40^{***} (0.004)	-0.52^{***} (0.005)	-0.56^{***}	-0.36^{***} (0.004)	0.37*** (0.003)	0.38***	0.44^{***} (0.002)	0.34***
Detection Prob. σ	-3.85*** (0.02)	-4.83^{***}	-5.16^{***}	_3.31*** (0.03)	3.51^{***} (0.02)	5.02^{***}	4.43^{***}	3.10***
Times Caught	-0.01***	-0.004** (0.002)	0.04***	-0.05***	0.01^{***}	-0.31^{***}	-0.05*** (0.001)	0.04***
Repeat Offender	0.01**	1.16**	0.76***	_0.37***	$\begin{array}{c} (0.001) \\ -0.01 \\ (0.005) \end{array}$	-0.26^{***}	-0.35^{***}	0.29***
IMR	(10:0)	(10.0)	(600.0)	-0.68** -0.01)	(0000)	(10:0)	(*00.0)	0.54^{***} 0.01
Constant	6.25***	6.04*** (0.03)	6.53***	6.02***				
Observations R^2 Adjusted R^2	427'108 0.14 0.14	382,750 0.43 0.43	809'858 0.41 0.41	$427108 \\ 0.14 \\ 0.14$	427'108	382,750	809,858	427'108
Log Likelihood					-2'509'783.00	-1'200'409.00	-3.867.860.00	-2.507.956.00

Note: This table shows the estimation results of linear cross-sectional regressions to explain cartel duration (ln(duration+1)) and the estimation results of a Weibull Hazard Model to explain cartel death, both at the industry level, for data simulated for Models IIIa and IIIb. Columns 2 - 5 estimate linear regression coefficients, while columns 6 - 9 estimate HR coefficients, both on the sample of detected cartels, the group of undetected cartels, the population of all cartels, and the sample corrected for Heckman Sample Selection, respectively. The estimated coefficients show standard errors in the sample, but do not test for the real population. The estimated HR coefficients show the change of risk for cartel breakdown if the covariate increases by 1 unit, keeping all others fixed.

Standard errors are in parentheses. Significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

Appendix C Lasso Results

 ${\bf Table~C13~~Sample~Selection~Bias~Lasso~Regression~-~Models~I}$

Coefficients	β^s	β^l	α_{IMR}	$bias_{IMR}$
Number of Firms n_f	0.26	0.16	-0.22	0.11
n_f^3	-0.01	0	0.01	0
Detection Probability σ	-4.01	-4.25	-0.50	0.24
σ^3	3.93	5.97	4.30	-2.04
$n_f \sigma$	0.12	0.04	-0.17	0.08
Start	0	0	0	0
Times Caught	0.02	-0.02	-0.09	0.04
Repeat Offender	0.18	-0.05	-0.48	0.23
IMR		-0.47		
Constant	5.09	5.54	0.95	-0.45

This table shows, for the Lasso CV Linear Regression of Model I, the sample selection bias that we correct with the IMR following (Heckman (1979)). β^s is the estimated coefficients in the short model without IMR. β^l is the estimated coefficients in the corrected long model including IMR. α_{IMR} is the coefficient in the auxiliary regression between each variable and IMR. The last column shows the sample selection bias: $bias_{IMR} = \beta^l(IMR) * \alpha_{IMR}$.

References

Bos I, Davies SW, Harrington JE, et al (2018) Does Enforcement Deter Cartels? A Tale of Two Tails. International Journal of Industrial Organization 59:372–405

Harrington JE, Wei Y (2017) What Can the Duration of Discovered Cartels Tell Us About the Duration of All Cartels? The Economic Journal 127(604):1977–2005

Heckman JJ (1979) Sample Selection Bias as a Specification Error. Econometrica: Journal of the econometric society pp 153–161

Stigler GJ (1964) A Theory of Oligopoly. Journal of Political Economy 72(1):44–61

Tibshirani R (1996) Regression Shrinkage and Selection via the Lasso. Journal of the Royal Statistical Society Series B (Methodological) 58(1):267–288

 ${\bf Table~C14~~Sample~Selection~Bias~Lasso~Regression~-~Models~II}$

Coefficients	β^s	β^l	α_{IMR}	$bias_{IMR}$
Number of Firms n_f	0.33	0.27	-0.20	0.05
n_f^3	-0.01	-0.01	0.01	0
Detection Probability σ	-2.34	-2.64	-1.09	0.30
σ^3	3.19	4.81	5.96	-1.62
$n_f \sigma$	0.11	0.08	-0.13	0.04
Start	0	0	0	0
Times Caught	0.04	-0.08	-0.42	0.11
Repeat Offender	0.21	0.17	-0.15	0.04
IMR		-0.27		
Constant	5.32	5.71	1.44	-0.39

This table shows, for the Lasso CV Linear Regression of Model II, the sample selection bias that we correct with the IMR following (Heckman (1979)). β^s is the estimated coefficients in the short model without IMR. β^l is the estimated coefficients in the corrected long model including IMR. α_{IMR} is the coefficient in the auxiliary regression between each variable and IMR. The last column shows the sample selection bias: $bias_{IMR} = \beta^l(IMR) * \alpha_{IMR}$.

 ${\bf Table~C15~~Sample~Selection~Bias~Lasso~Regression~-~Models~III}$

Coefficients	β^s	β^l	α_{IMR}	$bias_{IMR}$
Number of Firms n_f	0.40	0.19	-0.46	0.21
n_f^3	-0.01	-0.01	0.01	-0.01
Detection Probability σ	-5.44	-9.09	-8.03	3.65
σ^2	11.03	22.02	24.20	-10.99
σ^3	-6.74	-20.94	-31.27	14.20
$n_f \sigma$	-0.83	-0.36	1.02	-0.46
$n_f \sigma \gamma^3$	0.03	0.03	0.01	0
Leniency (% of Fine) θ	-0.22	-0.14	0.17	-0.08
$n_f \theta$	0.11	0.06	-0.11	0.05
Structured	-0.39	-0.37	0.04	-0.02
Start	0	0	0	0
Times Caught	-0.01	-0.04	-0.07	0.03
Repeat Offender	-0.04	-0.26	-0.48	0.22
IMR		-0.45		
Constant	5.41	6.18	1.68	-0.76

This table shows, for the Lasso CV Linear Regression of Model III, the sample selection bias that we correct with the IMR following (Heckman (1979)). β^s is the estimated coefficients in the short model without IMR. β^l is the estimated coefficients in the corrected long model including IMR. α_{IMR} is the coefficient in the auxiliary regression between each variable and IMR. The last column shows the sample selection bias: $bias_{IMR} = \beta^l(IMR) * \alpha_{IMR}$.

 ${\bf Table~C16~}$ Sample Selection Bias Lasso Regression - Models I, II and III

Coefficients	β^s	β^l	α_{IMR}	$bias_{IMR}$
Number of Firms n_f	0.16	0.02	-0.23	0.14
n_f^3	-0.01	0	0.01	-0.01
Detection Probability σ	-8.44	-11.34	-4.81	2.90
σ^2	14.02	26.38	20.47	-12.36
σ^3	-8.44	-25.83	-28.81	17.40
$n_f \sigma$	-0.07	0.09	0.28	-0.17
Fines γ (% of Profit)	1.77	1.66	-0.17	0.10
γ^3	-1.43	-1.34	0.14	-0.09
Leniency (% of Fine) θ	0.03	-0.05	-0.14	0.08
$n_f heta$	0.02	0.02	0	0
Structured	-0.37	-0.36	0.02	-0.01
Model II	0.54	0.65	0.18	-0.11
Model III	-0.67	-0.44	0.37	-0.23
Start	0	0	0	0
Times Caught	0	-0.04	-0.07	0.04
Repeat Offender	0.02	-0.30	-0.53	0.32
IMR		-0.60		
Constant	6.25	6.75	0.82	-0.49

This table shows, for the Lasso CV Linear Regression of the combined Models I, II, and III, the sample selection bias that we correct with the IMR following (Heckman (1979)). β^s is the estimated coefficients in the short model without IMR. β^l is the estimated coefficients in the corrected long model including IMR. α_{IMR} is the coefficient in the auxiliary regression between each variable and IMR. The last column shows the sample selection bias: $bias_{IMR} = \beta^l(IMR) * \alpha_{IMR}$.

Table C17 Lasso CV Regression and HR for Cartel Duration on Model I - ICC on Stigler - Detection independent of Collusion

	LasSample	LasUndetec LasCi	$ ext{tion}+1)$ LasCartels	LasHeck	HRLasSample	Cartel Death HRLasUnd HRL	Death HRLasCartels	HRLasHeck
Start	-0.001***	-0.001^{***}	-0.002***	-0.0004^{***}	0.001***	0.001***	0.001***	0.001***
N Firms n_f	-1.37***	4.87***	2.50***	0.16***	1.04***	12.61***	-1.49***	-0.14^{***}
$n_{ m f}^2$	(0.14) $0.43***$	(0.17) $-1.29***$	(0.09) -0.64^{***}	(0.02)	(0.12) $-0.33***$	(1.42) $-1.56***$	(0.07) $0.38***$	(0.02)
n_{s}^{3}	(0.04) $-0.04***$	(0.04) $0.09***$	(0.02) $0.04***$	-0.005***	(0.03) $0.03***$	(0.25) $0.07***$	(0.02) $-0.02***$	0.004***
Detection Prob. σ	(0.003) $-5.04**$	(0.003) $-17.14***$	(0.002) $-12.64***$	(0.0004) $-4.25***$	(0.002) $4.97**$	(0.01) $18.48***$	(0.001) $12.14***$	(0.0004) 3.72***
	(2.38)	(2.25)	(1.71)	(0.36)	(2.05)	(2.12)	(1.38)	(0.31)
<i>d</i> -	4.74 (10.82)	(10.53)	(7.93)		-6.61 (9.34)	-15.51 (9.02)	-15.50° (6.41)	
σ^3	-2.64	-34.76^{**}	-17.19	5.97	5.93	22.24^*	18.06^{*}	-4.94***
	(15.46)	(15.51)	(11.53)	(1.49)	(13.35)	(13.29)	(9.30)	(1.29)
$n_f\sigma$	0.12**	1.69***	1.39***	0.04	0.06	-2.31 ***	-1.12^{***}	0.005
Times Caught	0.02^{***}	0.03***	0.03	(0.00) -0.02^{***}	-0.02^{***}	(0.10) $-0.49***$	$(0.03) \\ -0.12^{***}$	0.03
)	(0.004)	(0.01)	(0.003)	(0.01)	(0.004)	(0.02)	(0.003)	(0.004)
Repeat Offender	0.17***	1.75***	0.99***	*0.05	-0.19***	-0.23***	-0.51**	-0.03
IMR	(0.02)	(0.03)	(0.02)	(0.02) -0.47^{***}	(0.02)	(cn:n)	(0.01)	0.33^{***}
Constant	*****	***90 1	2 17***	(0.03) 7.7.4**				(0.03)
	(0.24)	(0.28)	(0.17)	(0.09)				
Observations R. ²	36'615 0.12	41,207 0.49	7'822 0.53	36'615 0.13	36'615	41,207	7,822	36'615
$ m Adjusted~R^2$ $ m Log~Likelihood$	0.12	0.49	0.53	0.13	-218'190.40	-113'185.60	-348'310.50	-218'178.10

Note: This table shows the estimation results of Lasso CV linear cross-sectional regressions to explain cartel duration (ln(duration+1)) and the estimation results of a Lasso CV Weibull Hazard Model to explain cartel death, both at the industry level, for data simulated for Model I. Columns 2 - 5 estimate linear regression coefficients, while columns 6 - 9 estimate HR coefficients, both on the sample of detected cartels, the group of undetected cartels, the population of all cartels, and the sample corrected for Heckman Sample Selection, respectively. The estimated coefficients show standard errors in the sample, but do not test for the real population. The estimated HR coefficients show the change of risk for cartel breakdown if the covariate increases by 1 unit, keeping all others fixed.

Standard errors are in parentheses. Significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

Table C18 Lasso CV Regression and HR for Cartel Duration on Model II - ICC on Stigler - Detection depends on number of Firms

	LasSample	Ln(Duration+1) LasUndetec LasC	ion+1) LasCartels	LasHeck	HRLasSample	Cartel Death HRLasUnd HRI	Death HRLasCartels	HRLasHeck
Start	-0.002***	-0.002^{***}	-0.002***	-0.001***	0.002***	0.001***	0.001***	0.002***
N Firms n_f	(0.0001) -1.98**	(0.0000) 8.53***	(0.0000) 5.88**	(0.0001) $0.27***$	$(0.0001) \\ 1.37^{***}$	(0.000)	(0.0000) -4.30***	(0.0001) -0.26***
$n_{\tilde{\tau}}^{2}$	(0.24) 0.61^{***}	(0.18) $-2.16***$	(0.13) $-1.48***$	(0.04)	(0.20) $-0.43***$	(1.48) $-0.55**$	(0.11) $1.08***$	(0.03)
r 20 1	(0.06)	(0.04) $0.15***$	(0.03) 0.10^{***}	-0.01	(0.05) $0.04***$	(0.26) -0.0004	(0.03) $-0.07***$	0.01**
J i	(0.01)	(0.003)	(0.002)	(0.001)	(0.004)	(0.01)	(0.002)	(0.001)
Detection Prob. σ	-2.36^{***} (0.61)	-5.81^{***} (0.51)	-7.30^{***} (0.40)	-2.64^{***} (0.61)	4.17 (3.61)	5.79*** (2.11)	8.33*** (1.68)	1.84^{***} (0.55)
σ^2					-11.18 (16.44)	0.25	$\frac{(-1.5)}{-5.39}$	
σ^3	3.12	-2.29	-0.19	4.81*	12.46	$\frac{(5.91)}{1.97}$	7.29	-4.34*
	(2.52)	(1.64)	(1.46)	(2.55)	(23.52)	(13.27)	(11.46)	(2.29)
$n_f \sigma$	0.13	0.97	1.19***	0.08	0.01	-0.94***	-1.11**	0.04
	(0.10)	(0.08)	(0.02)	(0.10)	(0.0)	(0.16)	(0.02)	(0.09)
Times Caught	0.02	0.62^{***}	0.54***	-0.08**	(0.09)	-1.30^{***}	-0.41^{***}	0.03
Repeat Offender	0.20^{***}	-0.44^{***}	-0.23^{***}	0.17^{***}	-0.22^{***}	1.07***	0.26^{***}	-0.21^{***}
•	(0.03)	(0.05)	(0.03)	(0.04)	(0.03)	(0.08)	(0.03)	(0.03)
$_{ m IMR}$				-0.27^{***} (0.06)				0.12^{**} (0.06)
Constant	7.99***	-2.81^{***}	-0.25	5.71***				
	(0.30)	(0.26)	(0.19)	(0.16)				
Observations P2	11,733	41,445	53,178	11,733	11,733	41,445	53,178	11,733
$ m Adjusted~R^2$	0.20	0.54	0.59	0.20				
Log Likelihood					-76'836.46	$-118^{\circ}987.30$	-207'949.80	-76'867.61

estimation results of a Lasso CV Weibull Hazard Model to explain cartel death, both at the industry level, for data simulated for Model II. Columns 2 - 5 estimate linear regression coefficients, while columns 6 - 9 estimate HR coefficients, both on the sample of detected cartels, the group of undetected cartels, the population of all cartels, and the sample corrected for Heckman Sample Selection, respectively. The estimated coefficients show standard errors in the sample, but do not test for the real population. The estimated HR coefficients show the change of risk for cartel breakdown if the Note: This table shows the estimation results of Lasso CV linear cross-sectional regressions to explain cartel duration (ln(duration+1)) and the covariate increases by 1 unit, keeping all others fixed.

Standard errors are in parentheses. Significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

Table C19 Cartel Duration with CV Lasso on Model III - ICC on Harrington et al.

HRLasHeck	0.001*** (0.000) -0.14*** (0.01)	0.005 ***	-0.05*** (0.01) 0.08***	(0.01) $-0.03***$ (0.004) $0.35***$	(0.003) 7.12^{***} (0.57) -14.55^{***}	(2.59) $11.95***$ (3.79) $0.29***$	(0.03) (0.001) (0.001) (0.01) (0.01) (0.01)	427'108
Death HRLasCartels	0.001*** (0.0000) -0.73*** (0.03) (0.22***	(0.01) -0.01*** -0.06***	(10.0) ***02.0	$egin{array}{l} (0.01) \\ -0.31** \\ (0.002) \\ 0.42*** \end{array}$	(0.002) $9.44***$ (0.09)	-21.25*** (0.31) -0.30***	(0.004) (0.001) (0.004)	809'858 -3'841'901.00
Cartel Death HRLasUnd HRL	0.001*** (0.0000) 1.96*** (0.06) 0.08***	(0.01) (0.001)	-0.04*** (0.01) -3.78***	$egin{pmatrix} (0.03) \\ 0.52^{***} \\ (0.01) \\ 0.36^{***} \end{pmatrix}$	(0.004) 27.77*** (0.68) 47.40***	(3.06) $-122.98***$ (4.58) $-5.49***$	(0.04) (0.03) (0.01)	382750 -1'183'409.00
HRLasSample	0.001*** (0.0000) -0.33*** (0.01)	0.01*** (0.0002) -0.54**	$\begin{array}{c} (0.20) \\ 0.23* \\ (0.14) \\ 0.14*** \end{array}$	(0.01) $-0.07***$ (0.004) $0.37***$	(0.11)	-7.71*** (0.39) 0.72***	(0.00) (0.001) (0.001) (0.005)	427'108
LasHeck	-0.0004*** (0.0000) 0.19*** (0.01)	-0.01^{***} (0.0003)	0.03^{***} (0.01) -0.14^{***}	(0.01) $0.06**$ (0.005) $-0.37***$	(0.004) -9.09*** (0.66) 22.02***	$\begin{array}{c} (3.04) \\ -20.94^{***} \\ (4.44) \\ -0.36^{***} \end{array}$	$\begin{array}{c} (0.03) \\ -0.04^{***} \\ (0.001) \\ -0.26^{***} \\ (0.01) \\ -0.45^{***} \\ (0.01) \end{array}$	6.18*** (0.06) 427'108 0.15 0.15
tion+1) LasCartels	-0.002*** (0.0000) 0.58*** (0.04) -0.22***	(0.01) (0.01) (0.001) $(0.04**)$	***09.0-	$egin{pmatrix} (0.01) & & & & & & & \\ 0.42^{***} & & & & & \\ (0.003) & & & & & & \\ & -0.54^{***} & & & & & \end{bmatrix}$	(0.003) $-11.16**$ (0.10)	21.48** (0.39) 0.56***	(0.005) (0.001) (0.005)	6.05 *** (0.05) 809'858 0.45 0.45
LasUndetec LasCa	-0.001*** (0.000) -0.40*** (0.06) -0.13***	(0.02) 0.01^{***} (0.001)	0.02* (0.01) $1.06***$	$egin{pmatrix} (0.02) \\ 0.12^{***} \\ (0.005) \\ -0.49^{***} \end{pmatrix}$	(0.004) -8.71*** (0.78) -60.58***	$egin{array}{c} (3.67) \\ 121.65*** \\ (5.49) \\ 2.69*** \\ (0.03) \end{array}$	0.05 *** (0.002) 1.00 *** (0.01)	7.19*** (0.09) 382'750 0.45
LasSample	-0.001^{***} (0.000) 0.41^{***} (0.01)	-0.01^{***} (0.0002) 1.40^{***}	(0.31) -0.70^{***} (0.16) -0.22^{***}	$egin{array}{l} (0.01) \\ 0.11** \\ (0.004) \\ -0.39*** \end{array}$	(0.004) $-3.09***$ (0.12)	9.32*** (0.46) -0.83***	$\begin{array}{c} (0.03) \\ -0.01*** \\ (0.001) \\ -0.04*** \\ (0.01) \end{array}$	4.52*** (0.16) 427'108 0.15 0.15
	$\begin{array}{c} \text{Start} \\ \text{N Firms } n_f \\ \\ n_f^2 \end{array}$	n_f^3 Fines $\gamma~(\%~{\rm of~Profit})$	γ^3 Leniency (% Fine) θ	$n_f heta$ Structured	Detection Prob. σ σ^2	σ^3 $n_f \sigma$	Times Caught Repeat Offender IMR	Constant Observations R ² Adjusted R ² Log Likelihood

Note: This table shows the estimation results of Lasso CV linear cross-sectional regressions to explain cartel duration (ln(duration+1)) and the estimation results of a Lasso CV Weibull Hazard Model to explain cartel death, both at the industry level, for data simulated for Models IIIa and IIIb combined. Columns 2 - 5 estimate linear regression coefficients, while columns 6 - 9 estimate HR coefficients, both on the sample of detected cartels, the population of all cartels, and the sample corrected for Heckman Sample Selection, respectively. The estimated coefficients show standard errors in the sample, but do not test for the real population. The estimated HR coefficients show the change of risk for cartel breakdown if the covariate increases by 1 unit, keeping all others fixed.

Standard errors are in parentheses. Significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively.

Table C20 Cartel Duration with CV Lasso on Model I, II and III

	LasSample	Ln(Duration+1) LasUndetec LasCa	$ ext{tion}+1)$ LasCartels	LasHeck	HRLasSample	Cartel HRLasUnd	Cartel Death Jnd HRLasCartels	${ m HRLasHeck}$
N Firms n_f	0.26** (0.05) $-0.03**$	0.45** (0.05) $-0.30**$	**	0.02** (0.01)	-0.28** (0.04)	1.15** (0.05)	***************************************	-0.01 (0.01)
,	(0.01)	(0.01)	(0.001)		(0.01)	(0.01)	(0.001)	
n_f^3	-0.01***	0.02***	-0.001***	-0.003***	0.003**	-0.002***	0.003***	0.002***
7,2	(0.001)	(0.001)	(0.0002)	(0.0002) $1.66***$	(0.001)	(0.001)	(0.0001)	$(0.0001) \\ -0.64*$
- č	***&*	6U U	***&U U	(0.38)	**************************************	****	****	(0.33)
	(0.01)	(0.01)	(0.01)	(0.32)	(0.01)	(0.01)	(0.01)	(0.27)
Leniency (% Fine) θ	0.03***	0.66***	-0.55*** (0.01)	-0.05*** (0.01)	-0.05	-2.69***	0.44***	0.03**
$\theta_f n$	0.02^{***}	0.21***	0.40	0.02***	-0.001	0.29	-0.28	-0.002
) Detection Prob	(0.004) $-543***$	(0.005) $-18.35***$	(0.003) $-13.46***$	(0.004) $-11.34***$	(0.004) $_{4.77***}$	(0.01) $23.45***$	(0.002)	(0.003)
	(0.11)	(0.14)	(0.08)	(0.62)	(0.09)	(0.15)	(0.07)	(0.53)
σ^{2}				26.38***				-18.72^{***}
σ^3	11.91***	24.02***	23.64***	-25.83***	-10.07***	-29.63	-22.95***	(5.70) $16.65***$
	(0.42)	(0.50)	(0.35)	(4.21)	(0.36)	(0.43)	(0.27)	(3.60)
$n_f \sigma$	-0.09***	2.19***	1.26***	0.09***	0.10***	-2.97***	-0.93***	-0.07***
Structured	(0.02) $-0.37***$	$(0.02) \\ -0.51***$	$(0.01) \\ -0.51***$	(0.02) $-0.36***$	0.35***	(0.02) 0.35***	$(0.01) \\ 0.40***$	(0.02) $0.34***$
	(0.004)	(0.005)	(0.003)	(0.004)	(0.003)	(0.004)	(0.002)	(0.003)
Model II	0.54***	0.30***	0.36***	0.65	-0.48***	-0.39***	-0.40***	-0.57***
Model III	(0.01) $-0.30***$	$(0.01) -0.90^{***}$	$(0.01) -0.90^{***}$	(0.01) -0.44^{***}	$(0.01) \\ 0.24^{***}$	$\substack{(0.01)\\0.62^{***}}$	$(0.01) \\ 0.73***$	$\substack{(0.01)\\0.21^{***}}$
	(0.01)	(0.01)	(0.01)	(0.08)	(0.01)	(0.01)	(0.01)	(0.07)
Start	-0.001**	-0.001^{***}	-0.002^{***}	-0.0003***	0.001***	0.001***	0.001***	0.001***
Times Caught	(0.0000) -0.001	0.05***	0.07***	(0.0000) -0.04***	0.002***	(0.0000) -0.35***	(0.000) -0.07***	0.04***
)	(0.001)	(0.002)	(0.001)	(0.001)	(0.001)	(0.003)	(0.001)	(0.001)
Repeat Offender	0.02***	1.06***	0.69***	-0.30	-0.02***	-0.21***	-0.32***	0.24
9	(0.01)	(0.01)	(0.004)	(0.01)	(0.004)	(0.01)	(0.003)	(0.01)
IMK				0.60 (0.01)				0.48****
Constant	5,95***	7.57***	7.83***	(0.01) 6,75***				(0.01)
	(0.06)	(0.07)	(0.02)	(0.05)				
Observations	475,456	465,402	940,858	475,456	475'456	465,402	940'858	475'456
\mathbb{R}^2	0.15	0.45	0.46	0.15				
Aajustea r. ⁻ Log Likelihood	0.13	0.45	0.40	0.13	-2'804'453.00	-1'432'521.00	-4'409'298.00	-2'803'121.00

cartels, the group of undetected cartels, the population of all cartels, and the sample corrected for Heckman Sample Selection, respectively. The estimated coefficients show standard errors in the sample, but do not test for the real population. The estimated HR coefficients show the change of risk for cartel breakdown if the covariate increases by 1 unit, keeping all others fixed.

Standard errors are in parentheses. Significance at the 1%, 5%, and 10% level is indicated by ***, **, and *, respectively. Note: This table shows the estimation results of Lasso CV linear cross-sectional regressions to explain cartel duration (ln(duration+1)) and the estimation results of a Lasso CV Weibull Hazard Model to explain cartel death, both at the industry level, for data simulated for Models T, II, IIIa, and IIIb combined. Columns 2 - 5 estimate linear regression coefficients, while columns 6 - 9 estimate HR coefficients, both on the sample of detected