Verification : TD4

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Exercice 1

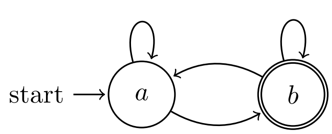
Let $\mathcal{A} = (Q, \Sigma, I, T, (F_i)_{i\in\mathbb{N}})$.
We build $(\mathcal{A}_i)_{i\in\mathbb{N}}$ such that $\mathcal{A}_i = (Q, \Sigma, I, T, F_i)$.
Then we merge them by $T'(f_i, \alpha) = T(f_i, \alpha)_{i+1}$

Exercice 2

• $Rat(\Sigma^\omega) \subseteq Rec(\Sigma^\omega)$
Let $(X_i)_{i\leq m} \in Rat(\Sigma^*)$ and $(Y_i)_{i\leq m} \in Rat(\Sigma^+)$ and (A_{X_i}, A_{Y_i}) their automata.
Then $\sum\limits_{i\leq m}(A_{X_i}.A_{Y_i}^\omega)$ recognize $\bigcup\limits_{i\leq m}(X_i.Y_i^\omega)$.
• $Rec(\Sigma^\omega) \subseteq Rat(\Sigma^\omega)$
$X_f = L(\mathcal{A}_f)$ vu comme un automate fini. $Y = \bigcup L(\mathcal{A}_{ff})$, où les $\mathcal{A}_{ff}$ sont des copies de $\mathcal{A}$ dont l'état initial est f.
$\mathcal{A}_{ff}$ admet un état f' copie de f pour éviter d'accepter le mot vide.

Exercice 3

a)



b)

Let $\mathcal{A}$ be such an automata and $f_0 \in F$ reachable from q_0 by a word u_0. Then, because $\mathcal{A}$ is deterministic and $u_0ba^\omega \in L$, there exists $u_1 = bv_1$ such that $T^*(q_0, u_0u_1) \in F$.
By induction, we may then build an infinite word $\underset{i\in\mathbb{N}}{\cdot} u_i$ containing an infinite number of b and recognized by $\mathcal{A}$. In particular, $L(\mathcal{A}) \neq L$

b)

L may be rewritten $\bigcup X.Y^*$ with the same construction as above. In that case $L' = \bigcup X.Y^\omega$
Then w has infinitely many prefixes if and only if $w \in Ł'$, that is $L' = \overrightarrow{L}$