Verification : Homework 5

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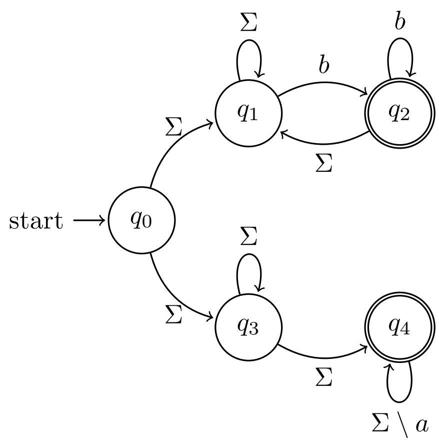
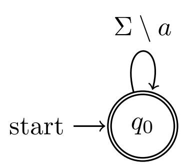
Exercise 1

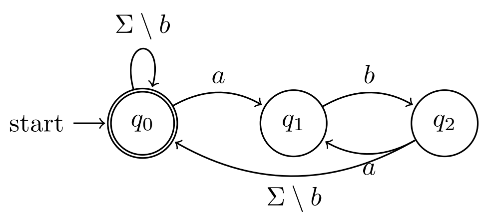
a)



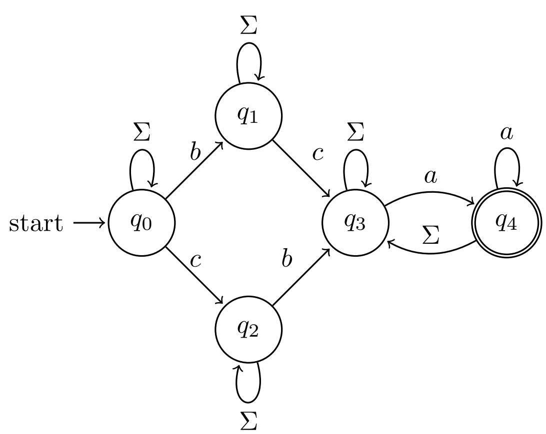
b)



c)



d)



Exercise 2

Let L_i be recognized by a BA $\mathcal{A}_i = (Q_i, \Sigma, I_i, T_i, F_i)$
Let L_i' be recognized by a DBA $\mathcal{A}_i' = (Q_i', \Sigma, q_{init_i}, T_i', F_i)$

a)

$L_1 \cap L_2$ is recognized by $\mathcal{A} = (Q_1 \times Q_2 \times \{1,2\}, \Sigma, I_1 \times I_2 \times \{1\}, T, F_1 \times Q_2 \times \{1\})$, where T is defined as follows :

$$T = \bigcup_{i \in \{1,2\}} \{((q_1, q_2, i), a, (q_1', q_2', i)) | (q_1, a, q_1') \in T_1,\ (q_2, a, q_2') \in T_2,\ q_i \notin F_i\} \cup$$
$$\{((q_1, q_2, 1), a, (q_1', q_2', 2)) | (q_1, a, q_1') \in T_1,\ (q_2, a, q_2') \in T_2,\ q_1 \in F_1\} \cup$$
$$\{((q_1, q_2, 2), a, (q_1', q_2', 1)) | (q_1, a, q_1') \in T_1,\ (q_2, a, q_2') \in T_2,\ q_2 \in F_2\}$$

Intuitively, this automaton runs two copies of the automata running in parallel and switches from copy i to the other only when it reaches a state in F_i. An accepting word would then necessarily switch infinetely many times from a copy to another, that is, reaching infinitely many times both F_1 and F_2

b)

We may notice that the previous construction is a deterministic Büchi automaton if $\mathcal{A}_1$ and $\mathcal{A}_2$ are. This observation is sufficient to prove that $L_1' \cap L_2'$ is recognized by a DBA.

c)

Let q be a fresh state.
$L_1 \cup Ł_2$ is recognized by $\mathcal{A} = (\{q\} \sqcup Q_1 \sqcup Q_2, \Sigma, \{q\}, T, F_1 \sqcup F_2)$,
where $T = T_1 \sqcup T_2 \sqcup \{(q, a, q_1) | \exists q_{s_1} \in I_1 \text{ s.t } (q_{s_1}, a, q_1) \in T_1\} \sqcup$
$\{(q, a, q_2) | \exists q_{s_2} \in I_2 \text{ s.t } (q_{s_2}, a, q_2) \in T_2\}$
Intuitively, this automaton chooses non-deterministically which automaton will actually be executed.
An example of this construction may be find in Exercise 1, item a).

d)

$L_1' \cap L_2'$ is recognized by $\mathcal{A}' = (Q_1' \times Q_2', \Sigma, (q_{init_1}, q_{init_2}), T_1' \times T_2', F_1' \times Q_2' \cup Q_1' \times F_2')$,
The construction is similar to the one used for the intersection but uses only one copy and accepts if and only if one of the two sets of targets is visited infinitely many times (by the pigeonhole principle).