

RTDSP Week 10 Homework

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1 Design a lowpass IIR filter

1.1 specifications

- sampling frequency $f_s = 8kHz$
- passband cutoff frequency $\Omega_p = 2kHz$
- stopband cutoff frequency $\Omega_s = 2.5kHz$
- passband ripple $\delta_p = 1dB$
- stopband attenuation $1 - \delta_s = 60dB$

MatLab filter designer tool can be activated with command "filterDesigner".

1.2 Butterworth filter

Designer generated a 19th order filter, converted into 10 biquad sections.

Table 1: Butterworth filter SOS Matrix with Scale Values

stage	b_0	b_1	b_2	a_0	a_1	a_2	scale value
01	1	2	1	1	0.065667297825008810741032050373178208247	0.84752869182663104208330651090363971889	0.478298997412909987492213303994503803551
02	1	2	1	1	0.057082550418259582480917657676400267519	0.60599533914287069613635594578227028251	0.415770521083136668227808740994078107178
03	1	2	1	1	0.050723935585552173965773192776396172121	0.427101913138009270376471704366849735379	0.369456462180890343738326464517740532756
04	1	2	1	1	0.045963090723188854136438408204412553459	0.29315704602940945155964413970650639385	0.334780034188149611118490156513871625066
05	1	2	1	1	0.042392867755750046621088245046848896891	0.192709950031353438593129112632595933974	0.308775704446775822731297012069262564182
06	1	2	1	1	0.039743357588969567550929440358231659047	0.118166817992363476497175156509911175817	0.2894775438953325580785572128661442548
07	1	2	1	1	0.037833812965076887846471720422414364293	0.064442382880592730121094291462213732302	0.275569048961417373266868935388629324734
08	1	2	1	1	0.036544271569331258919799410023188102059	0.02816154284531288687354334331770893186	0.266176453603661045121953065972775220871
09	1	2	1	1	0.035798595676589113623311533274318207987	0.007182187027813756105554254816070169909	0.260745195676100716131173840039991773665
10	1	1	0	1	0.017777274867327930857330287039985705633	0	0.508888637433663859610533108934760093689

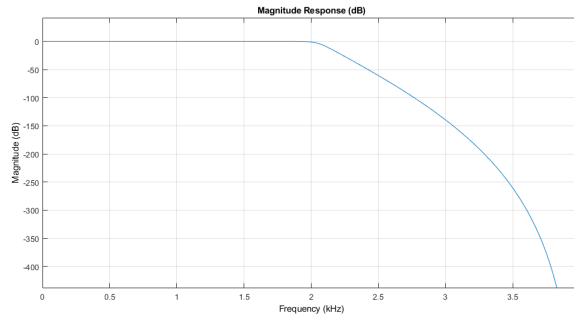


Figure 1: Butterworth Magnitude Response

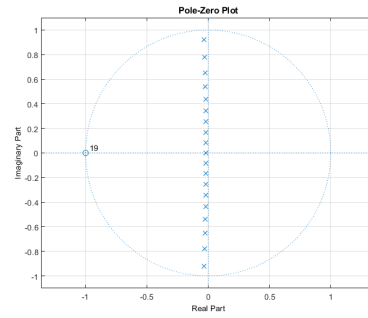


Figure 2: Butterworth Pole-Zero Plot

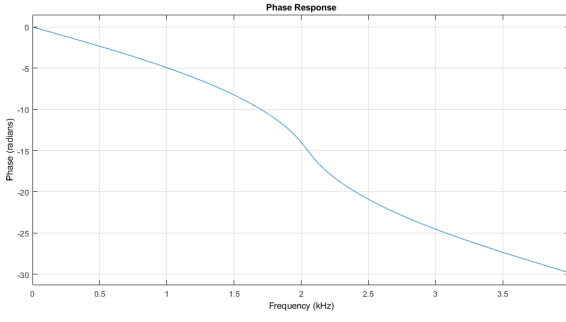


Figure 3: Butterworth Phase Response

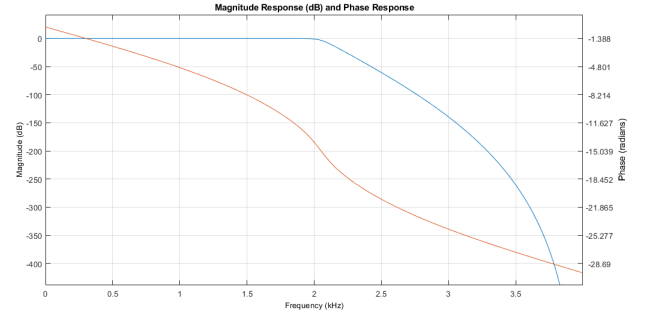


Figure 4: Butterworth Magnitude and Phase Response

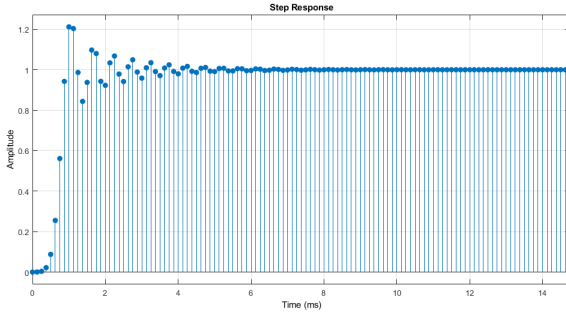


Figure 5: Butterworth Step Response

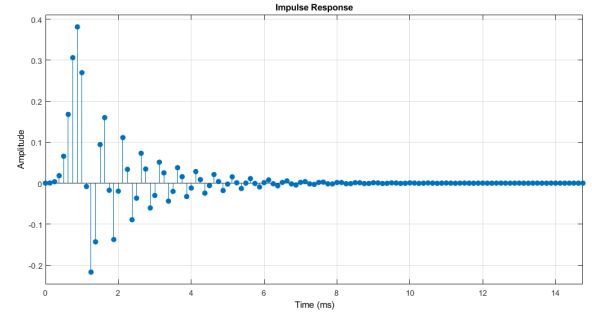


Figure 6: Butterworth Impulse Response

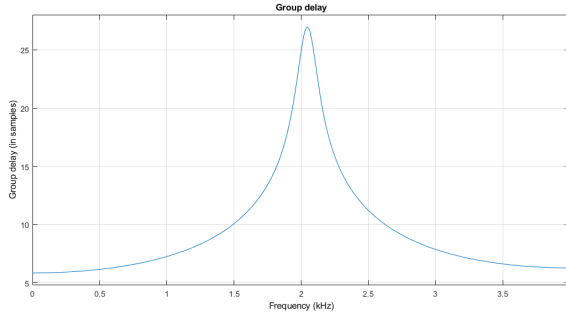


Figure 7: Butterworth Group Delay

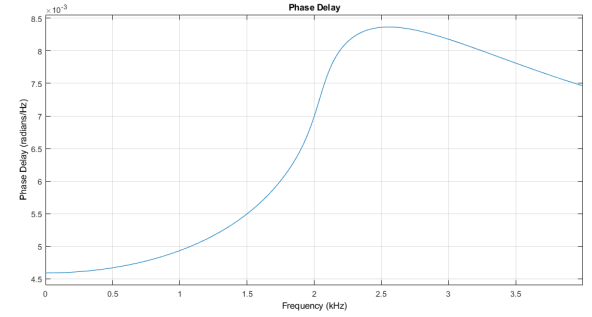


Figure 8: Butterworth Phase Delay

1.3 Chebyshev Type II filter

Designer generated a 9th order filter, converted into 5 biquad sections.

Table 2: Chebyshev Type II filter SOS Matrix with Scale Values

stage	b_0	b_1	b_2	a_0	a_1	a_2	scale value
01	1	0.791346445526044828255862739752046763897	1	1	0.160856768910089042989852714526932686567	0.785752066013098215968568638345459476113	0.697372709877414731494127408950589597225
02	1	0.99659806905118930586695569218136370182	1	1	0.222259373763130513301788937496894504875	0.465219024690403326616205958998762071133	0.56313137750497155309403751743957400322
03	1	1.37704458940432705738032836961792781949	1	1	0.32362720427270869061331819994800724089	0.240540047123330352762948791678354609758	0.463176369155356815188184782527969218791
04	1	1.80146393162399642839943680883152410388	1	1	0.412862663685468311847159839089727029204	0.099336539665886458561061544969561509788	0.397793910596263067258604451126302592456
05	1	1	0	1	0.224123106519700715510978739075653720647	0	0.612061553259850343877701561723370105028

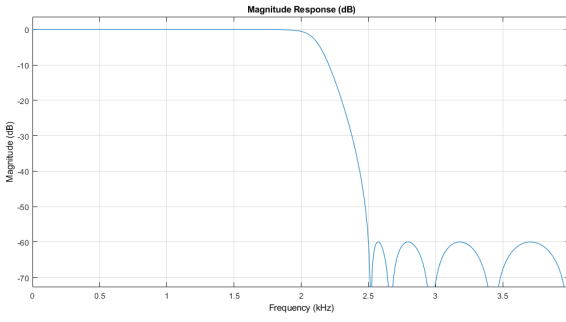


Figure 9: Chebyshev II Magnitude Response

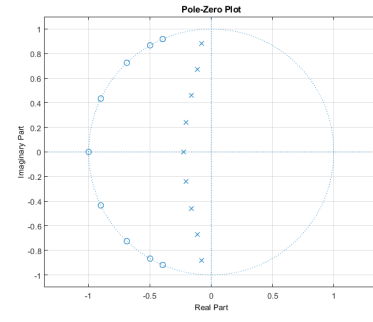


Figure 10: Chebyshev II Pole-Zero Plot

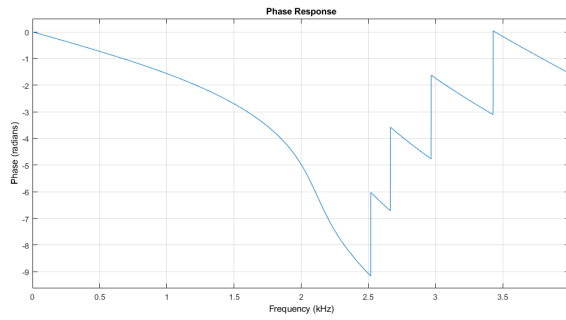


Figure 11: Chebyshev II Phase Response

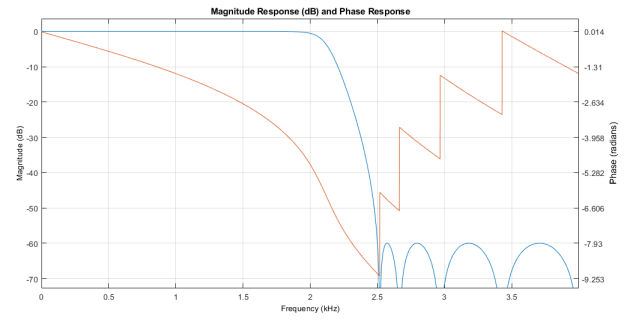


Figure 12: Chebyshev II Magnitude and Phase Response

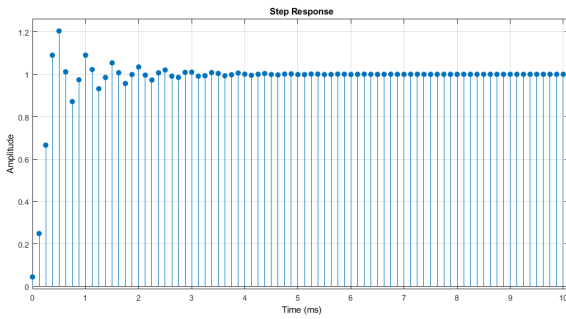


Figure 13: Chebyshev II Step Response

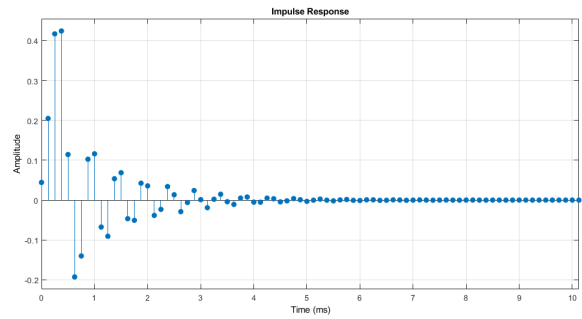


Figure 14: Chebyshev II Impulse Response

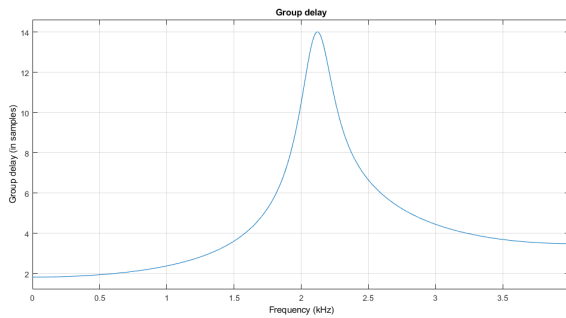


Figure 15: Chebyshev II Group Delay

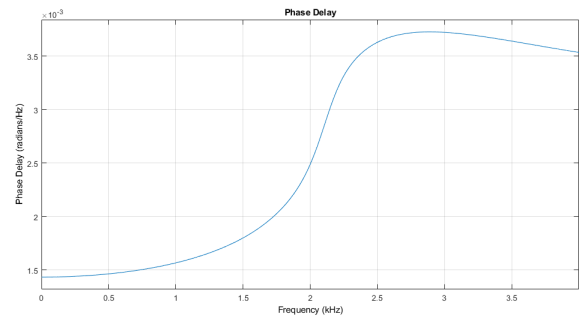


Figure 16: Chebyshev II Phase Delay

1.4 Elliptic filter

Designer generated a 6th order filter, converted into 3 biquad sections.

Table 3: Elliptic filter SOS Matrix with Scale Values

stage	b_0	b_1	b_2	a_0	a_1	a_2	scale value
01	1	1.002643979498020865293028691667132079601	1	1	-0.339055849383870733237955619188142009079	0.698272792228797345437385502009419724345	0.959420058471034953306855186383472755551
02	1	1.798927466167703048682824373827315866947	1	1	-0.949266688284802317987498554430203512311	0.370331600304015129410117879160679876804	2.556431085694567784116770781110972166061
03	1	0.615981808299446154109091366990469396114	1	1	-0.003780167335060244816064001227573498909	0.921067486511090161016568345075938850641	0.013362342077117634470129203805299766827

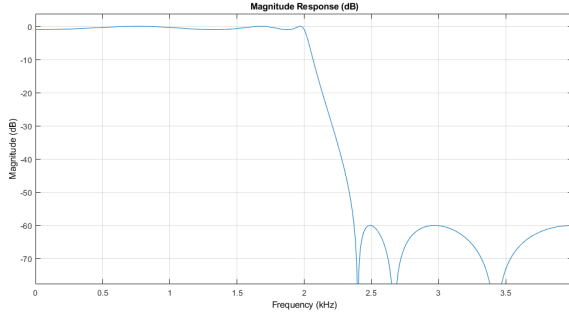


Figure 17: Elliptic Magnitude Response

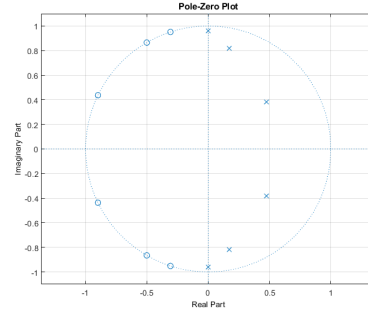


Figure 18: Elliptic Pole-Zero Plot

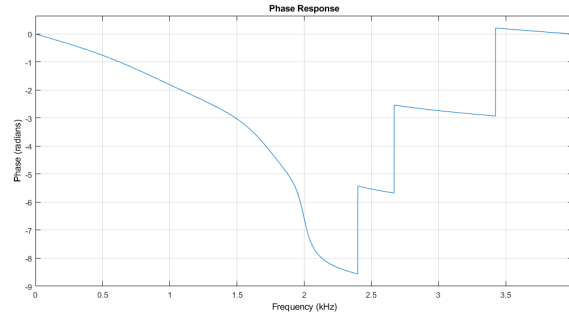


Figure 19: Elliptic Phase Response

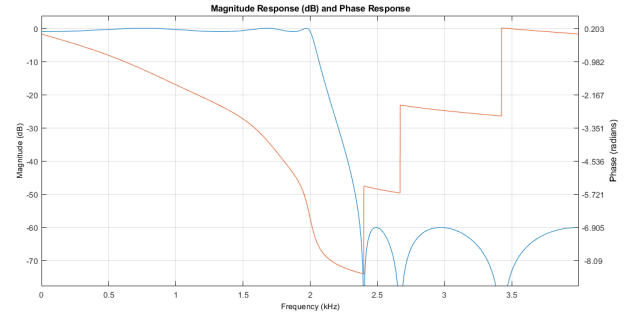


Figure 20: Elliptic Magnitude and Phase Response

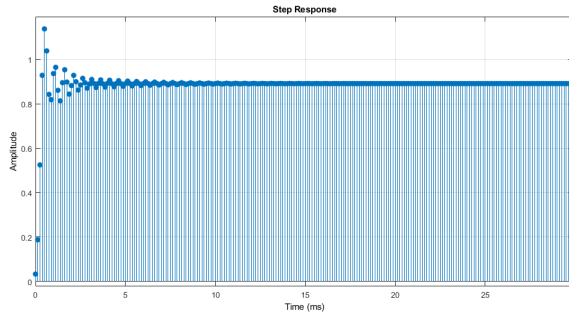


Figure 21: Elliptic Step Response

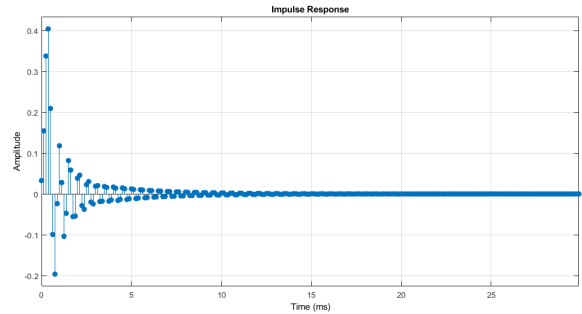


Figure 22: Elliptic Impulse Response

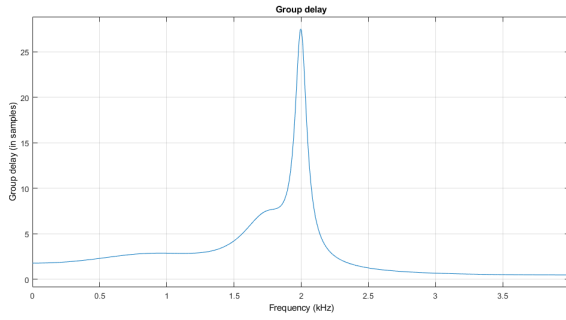


Figure 23: Elliptic Group Delay

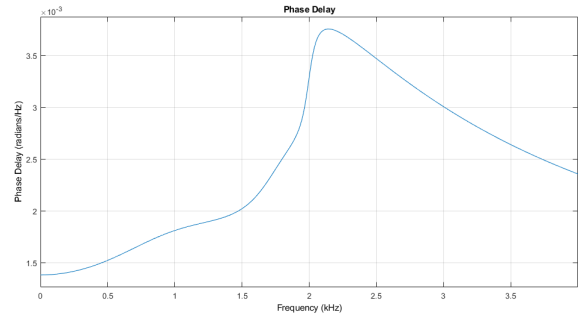


Figure 24: Elliptic Phase Delay

1.5 Bessel filter

Since Filter Designer in MatLab doesn't have the option to design Bessel filter, I used my own code to try to design a Bessel filter that satisfies the requirements. My code is available at https://github.com/belongtothenight/RTDSP_Code/blob/main/src/w10_bessel.m.

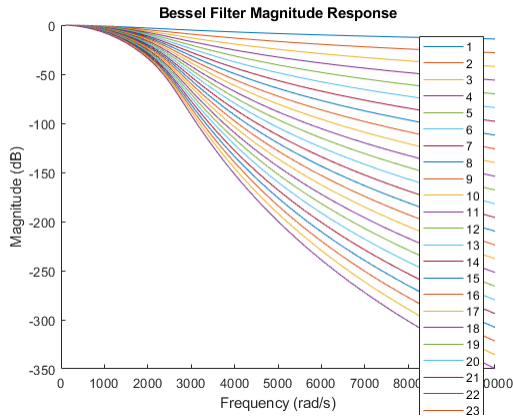


Figure 25: Bessel filter Magnitude Response with different orders

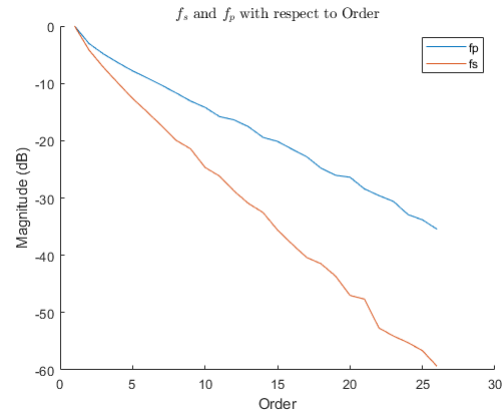


Figure 26: f_s and f_p with different orders

From figure 25, we can see that as the filter order increases, the magnitude response decrease faster. From figure 26, we can see that as the filter order increases, the f_s and f_p both decrease.

However, since MatLab limits the maximum order of Bessel filter to 25, I can't design a Bessel filter that satisfies the requirements. Also, the magnitude response at f_p drops way too fast as the order increases, so I don't think Bessel filter is a good choice for this problem.