# Q1 What is the true meaning of linearity?

Since linearity enables the computation to easily summing signals, this further makes lots of signal processing and mathematical techniques possible.

# Q2 Derive transfer function from the I/O equation? (FIR)

$$y(n) = b_0 x(n) + b_1 x(n-1) + \cdots + b_{L-1} x(n-L+1)$$
$$\therefore H(z) = \frac{Y(z)}{X(z)} = b_0 + b_1 z^{-1} + \cdots + b_{L-1} z^{-(L-1)} = \sum_{l=0}^{L-1} b_l z^{-l}$$

# Q3 Derive transfer function from the I/O equation? (IIR)

$$y(n) = \sum_{l=0}^{L-1} b_l x(n-l) - \sum_{m=1}^{M} a_m y(n-m)$$
$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{l=0}^{L-1} b_l z^{-l}}{1 + \sum_{m=1}^{M} a_m z^{-m}} = \frac{\sum_{l=0}^{L-1} b_l z^{-l}}{\sum_{m=0}^{M} a_m z^{-m}}$$

# Q4 Derive transfer function of MA filter from the I/O equation?

$$y(n) = \frac{1}{L}\left[x(n) + x(n-1) + \cdots + x(n-L+1)\right]$$
$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{L} \sum_{n=0}^{L-1} z^{-n} = \frac{1}{L} \frac{1-z^{-L}}{1-z^{-1}} = \frac{1-z^{-L}}{L \times (1-z^{-1})}$$

# Q5 Derive recursive I/O equation from the transfer function? (MA)

$$y(n) = y(n-1) + \frac{1}{L}\left[x(n) - x(n-L)\right]$$
$$\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1-z^{-L}}{L \times (1-z^{-1})}$$

https://forums.ni.com/t5/LabVIEW/Transfer-function-of-recursive-moving-average-filter/td-p/3172606

# Q6 Is an MA filter stable?

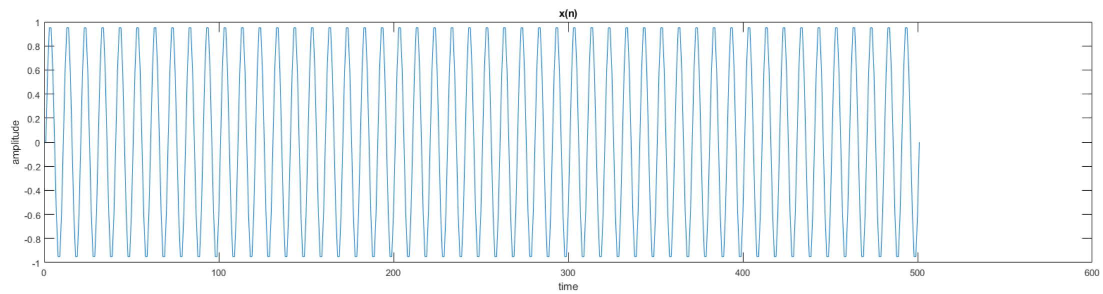
Since there are always $L-1$ poles at the origin, one pole at $z=1$, $L$ zeros spaced on the unit circle, there is no pole located outside the unit circle but one lies on the unit circle. Therefore MA filter is marginally stable.

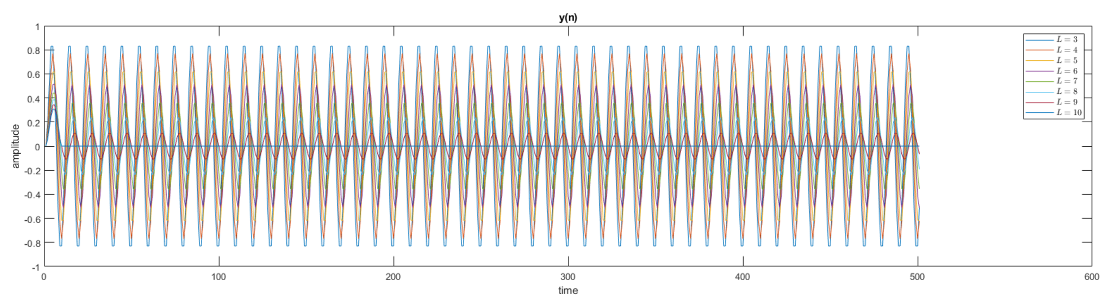
# Q7 Is an MA filter FIR or IIR?

MA filter is FIR filter.

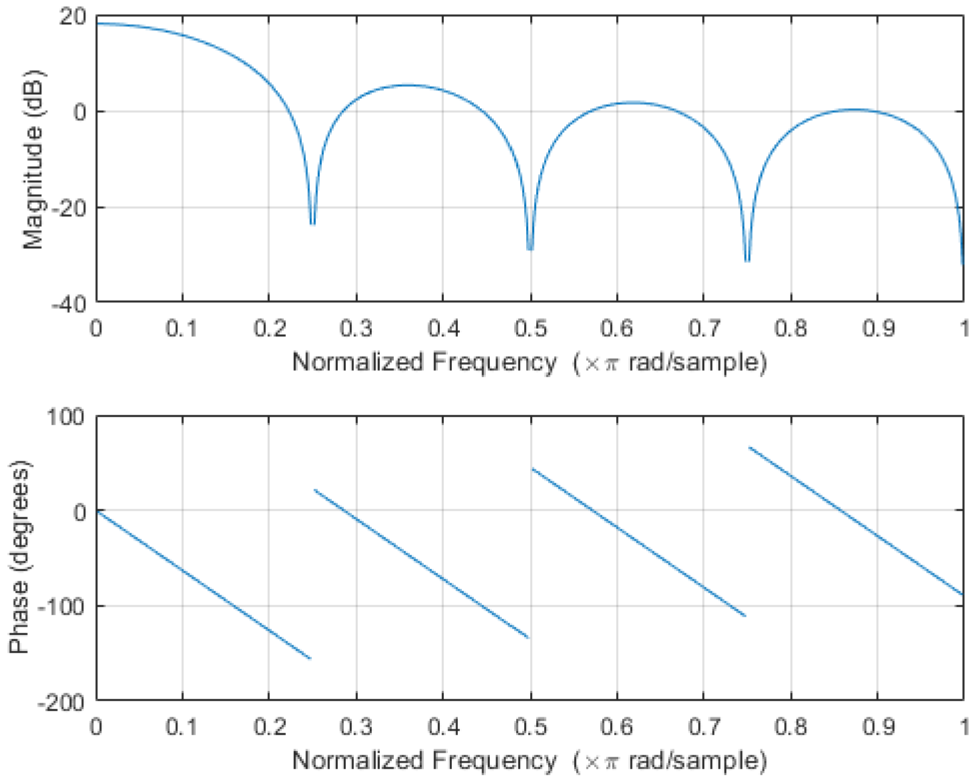
# Q8 When the input to the MA filter is a sinewave, explain it's output waveform in both time and frequency domain.

In time domain, the output of MA filter decreases its amplitude as the $L$ of MA filter increases.
Code in code1.m




In frequency domain of $L = 8$ MA filter, magnitude have bumps which creates lowest points on $n \times 0.25\pi$. It is because those lowest points are where zeros are located on unit circle.




## Q9 We generate 100 samples of sinewave with $A = 1$, $f = 1000Hz$, and sampling rate of $10kHz$.

### (a) What is the digital frequency $\omega$ and $F$?

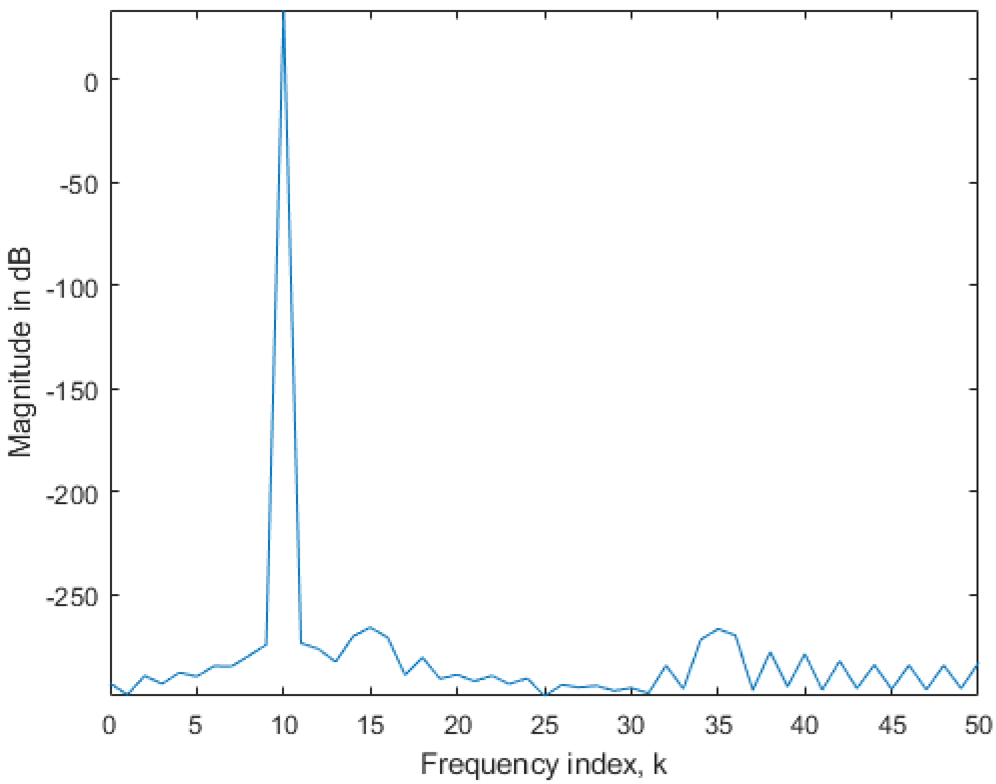
$$\Delta_\omega = \frac{2\pi}{N} = \frac{2\pi}{100} \approx 0.0628$$
$$\Delta_f = \frac{f_s}{N} = \frac{1e4}{100} = 100$$

### (b) If we compute DFT with $N = 100$, what value of $k$ corresponding to $1000Hz$?

$k = 0.1$

### (c) Sketch the amplitude spectrum $|X(k)|$.

Code in code2.m




# Q10

## (a) Why we see a line at $k = 10$?

Since the sampling frequency is 10000, and N is 100, which means
$k = \frac{f}{f_s} \times N = \frac{1000}{10000} \times 100 = 10$

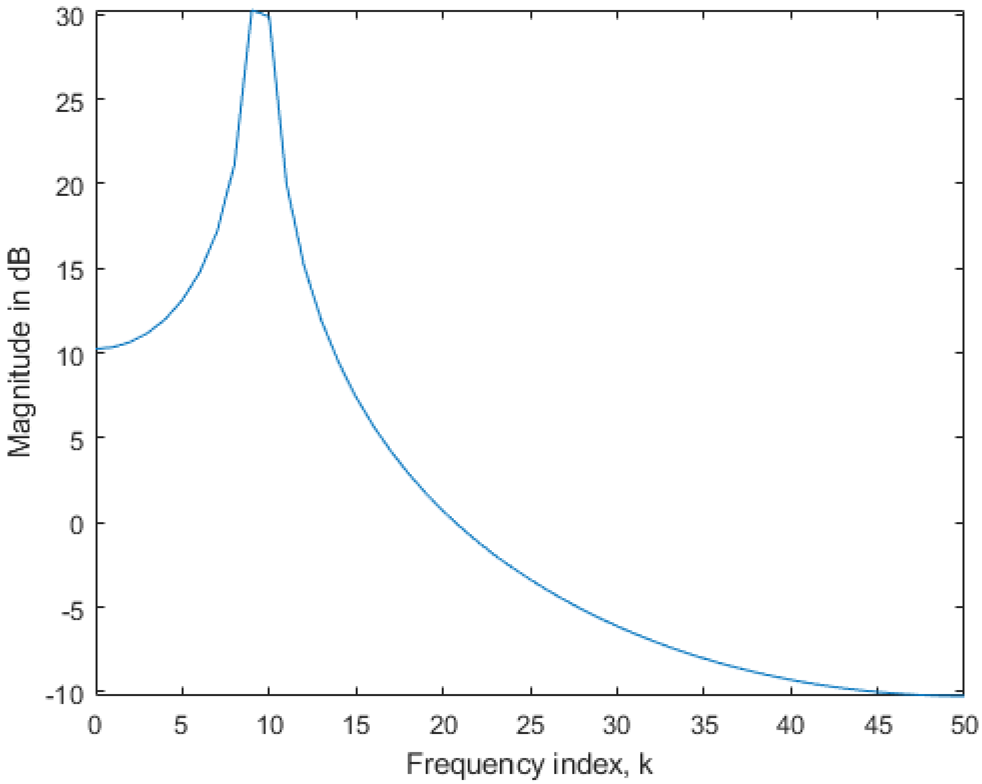
## (b) If we see a line spectrum at $k = 15$, what is the sinewave frequency?

Since $k = 1.5 \times k'$, $f_s = 1.5 \times f_s' = 15000$.

## (c) If $f$ is $950Hz$, sketch the amplitude spectrum, what happened?

When $f = 950Hz$, the center frequency started shifting left.
The reason behind this wider range of frequency is becuase we use 100 samples to compute FFT, which results in $\frac{f_s}{samples} = \frac{10000}{100} = 100Hz$ frequency resolution. This results in this wider frequency range. If we use $f_s = 850$ would results in roughly the same plot.

Code in code3.m

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