2. (a) Assume the following random variables:

- L : Lightning
- S : Storm
- C : Camp Fire
- F : Forest Fire

$$P(F) = \sum_{\text{Possible values for L,C}} P(F|L, C) \times P(L, C)$$

$$= 0.5P(L)P(C) + 0.4P(L)P(\neg C) + 0.1P(\neg L)P(C) + 0.01P(\neg L)P(\neg C)$$

$P(C)$ is given; we now have to compute $P(L)$.

$$P(L) = 0.5P(S) + 0.05P(\neg S) = 0.05 + 0.045 = 0.095$$

$$P(\neg L) = 0.5P(S) + 0.95P(\neg S) = 0.05 + 0.855 = 0.905 = 1 - P(L)$$

With those calculations (as well as a sanity check), we can revisit the original question.

$$P(F) = 0.5 \times 0.095 \times 0.75 + 0.4 \times 0.095 \times 0.25 + 0.1 \times 0.905 \times 0.75 + 0.01 \times 0.905 \times 0.25$$

$$= 0.035625 + 0.009500 + 0.067875 + 0.0022625 = 0.1152625$$

(b) Thunder has no influence on the probability of a camp fire (though I doubt the truth of that in the real world), but it does change the probability of lightning–

$$P(L|T) = \frac{P(T|L)P(L)}{P(T)} = \frac{P(T|L)P(L)}{P(T|L)P(L) + P(T|\neg L)P(\neg L)}$$

$$P(T|L)P(L) = 0.95 \times 0.095 = 0.09025$$

$$P(T|\neg L)P(\neg L) = 0.2 \times 0.905 = 0.181$$

$$P(L|T) = \frac{P(T|L)P(L)}{P(T|L)P(L) + P(T|\neg L)P(\neg L)} = \frac{0.09025}{0.09025 + 0.181} = \frac{0.09025}{0.27125} \approx 0.332719$$

(c)

(d)