2. (a) Assume the following random variables:

- L : Lightning
- S : Storm
- C : Camp Fire
- F : Forest Fire

$$P(F) = \sum_{\text{Possible values for L,C}} P(F|L,C) \times P(L,C)$$

$$= 0.5P(L)P(C) + 0.4P(L)P(\neg C) + 0.1P(\neg L)P(C) + 0.01P(\neg L)P(\neg C)$$

$P(C)$ is given; we now have to compute $P(L)$.

$$P(L) = 0.5P(S) + 0.05P(\neg S) = 0.05 + 0.045 = 0.095$$

$$P(\neg L) = 0.5P(S) + 0.95P(\neg S) = 0.05 + 0.855 = 0.905 = 1 - P(L)$$

With those calculations (as well as a sanity check), we can revisit the original question.

$$P(F) = 0.5 \times 0.095 \times 0.75 + 0.4 \times 0.095 \times 0.25 + 0.1 \times 0.905 \times 0.75 + 0.01 \times 0.905 \times 0.25$$

$$= 0.035625 + 0.009500 + 0.067875 + 0.0022625 = 0.1152625$$

(b) Thunder has no influence on the probability of a camp fire (though I doubt the truth of that in the real world), but it does change the probability of lightning–

$$P(L|T) = \frac{P(T|L)P(L)}{P(T)} = \frac{P(T|L)P(L)}{P(T|L)P(L) + P(T|\neg L)P(\neg L)}$$

$$P(T|L)P(L) = 0.95 \times 0.095 = 0.09025$$

$$P(T|\neg L)P(\neg L) = 0.2 \times 0.905 = 0.181$$

$$P(L|T) = \frac{P(T|L)P(L)}{P(T|L)P(L) + P(T|\neg L)P(\neg L)} = \frac{0.09025}{0.09025 + 0.181} = \frac{0.09025}{0.27125} \approx 0.332719$$

We can now use the same calculations as above, but assuming the probability of lightning is $P(L|T)$.

$$P(F) = 0.5P(L)P(C) + 0.4P(L)P(\neg C) + 0.1P(\neg L)P(C) + 0.01P(\neg L)P(\neg C)$$

$$\approx 0.5 \times 0.332719 \times 0.75 + 0.4 \times 0.332719 \times 0.25 + 0.1 \times 0.667281 \times 0.75 + 0.01 \times 0.667281 \times 0.25$$

$$= 0.124770 + 0.033272 + 0.050046 + 0.001668 = 0.207956$$

(c) To compute the probability of a storm given a forest fire, we first have to compute the probability of lightning given the same. To do so using what we know, we need to make use of a few rules:

$$P(L|F) = \frac{P(F|L)P(L)}{P(F)}$$

$$P(F|L) = \frac{P(F,L)}{P(L)}$$

$$P(F,L) = P(F,L,C) + P(F,L,\neg C) = P(F|L,C)P(L|C)P(C) + P(F|L,\neg C)P(L|\neg C)P(\neg C)$$

$$= 0.5 \times 0.095 \times 0.75 + 0.4 \times 0.095 \times 0.25 = 0.035625 + 0.0095 = 0.045125$$

$$P(F|L) = \frac{P(F,L)}{P(L)} = \frac{0.045125}{0.095} = 0.475$$

$$P(L|F) = \frac{P(F|L)P(L)}{P(F)} = \frac{0.475 \times 0.095}{0.1152625} \approx 0.391498$$

Now we can calculate $P(S)$ conditionally, using $P(L|F)$ in place of $P(L)$. Or something. Turns out my calculations all use circular logic and don't make much sense.

(d)

$$P(L|\neg S) = 0.05; P(\neg L|\neg S) = 0.95$$

$$P(T|\neg S) = P(T|\neg S, L)P(L|\neg S) + P(T|\neg S, \neg L)P(\neg L|\neg S) =$$

$$= P(T|\neg S, L) \times 0.05 + P(T|\neg S, \neg L) \times 0.95$$

What even?

(e)

$$P(C, F) = P(F|C)P(C)$$

$$= \Big(P(F|C, L)P(L) + P(F|C, \neg L)P(\neg L)\Big) \times P(C)$$

$$= (0.5 \times 0.095 + 0.1 \times 0.905) \times 0.75$$

$$= (0.0475 + 0.0905) \times 0.75 = 0.138 \times 0.75 = 0.1035$$