

1. (a) $0.12 + 0.24 + 0.08 + 0.16 = 0.6$

(b) $\frac{0.12+0.08}{P(\neg Y)} = \frac{0.2}{0.6} = \frac{1}{3}$

(c) $\frac{0.06}{P(\neg X, Y)} = \frac{0.06}{0.1} = 0.6$

(d) X, Y independent $\implies P(X|Y) = P(X|\neg Y), P(Y|X) = P(Y|\neg X)$

$$P(X|Y) = \frac{0.3}{0.4} = \frac{3}{4}$$

$$P(X|\neg Y) = \frac{0.2}{0.6} = \frac{1}{3} \neq \frac{3}{4} \implies X, Y \text{ not independent.}$$

(e) Y, Z independent $\implies P(Y|Z) = P(Y|\neg Z), P(Z|Y) = P(Z|\neg Y)$

$$P(Y|Z) = \frac{0.24}{0.6} = \frac{2}{5}$$

$$P(Y|\neg Z) = \frac{0.16}{0.4} = \frac{2}{5}$$

$$P(Z|Y) = \frac{0.24}{0.4} = \frac{3}{5}$$

$$P(Z|\neg Y) = \frac{0.36}{0.6} = \frac{3}{5}$$

Since the conditional probabilities of Y and Z are the same, the same must hold for $P(\neg Y|Z)$, etc. (as $P(Y) + P(\neg Y) = 1 = P(Z) + P(\neg Z)$). These equalities imply independence between Y and Z .