

# Programming Paradigms 2025

## Session 6: Higher-order functions

### Problems for solving and discussing

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#### Please read this

If you finish working on a problem and there is more time, *please have a look at the lettered problems at the end of the problem set*. We will cover all of the numbered problems in class anyway.

#### Problems that we will definitely talk about

1. (10 minutes)

Define a function `dbs` over lists using higher-order functions. The list should take a list of pairs of numbers and return the list of tuples in which the second element in the first tuple is two times the first element. As an example `dbs [(1,4) ,(3,6) ,(9,2) ,(4,8) ]` should return the list `[(3,6),(4,8)]`.

2. (15 minutes)

The `within` function takes a list of numbers and a pair of numbers, returns a list of numbers which are in the input list and within the range (inclusive) given by the input pair.

The elements in the output list appear to be in the same order they appeared in the input list. If the input pair is `(n1,n2)`, assume that `n1` is the lower bound of the range and `n2` is the upper bound of the range.

As an example, `within [1,3,4,5,2] (1,3)` should give us `[1,3,2]` and `within [1,3,4,5,2] (3,1)` should give us `[]`.

Define `within` using the higher-order functions in Chapter 7.

3. (10 minutes)

Implement the `sumrows` function. The function takes a list of number lists returns a one-dimensional list of numbers with each number equal to the sum of the corresponding row in the input list. If a list is empty, its sum is `0`.

As an example, `sumrows [[1,2], [3,4]]` should give us `[3, 7]`, and `sumrows [[],[],[1]]` should give us `[0,0,1]`.

Define `sumrows` using the higher-order functions in Chapter 7.

4. (15 minutes)

The base of the natural exponential function  $e = 2.718\dots$  can be written as the limit of the infinite series

$$\sum_{k=0}^{\infty} \frac{1}{k!}$$

The function `approx` should give us the approximation of  $e$  that we find by adding the first  $n$  terms of this infinite series. That is,

$$\text{approx } n = \sum_{k=0}^n \frac{1}{k!}$$

Define `approx` using the higher-order functions in Chapter 7; the factorial function  $k!$  is defined by

`fact k = product [1..k]`

### 5. (10 minutes)

Here is a function. What does it do? And why? Use the explanation at the very end of Section 7.3 in the book to help you answer the "why"-question.

```
fingo :: [a] -> [a] -> [a]

fingo xs ys = foldr (:) xs ys
```

## More problems to solve at your own pace

In your solutions, remember the learning goals of this session! **You must use the higher-order functions from Chapter 7 – not recursion or list comprehension!!**

- a) The partition function generalizes the `isolate` function from Session 5. The partition function takes a predicate `p` and a list `xs` and returns the pair of lists of elements which do and do not satisfy the predicate, respectively.
  - a) Define partition using `filter`.
  - b) Define partition using `foldr`.
- b) How can we implement the `filter` function using `foldr`?
- c) Use `foldr` to define a function `remove` which takes two strings as its arguments and removes every letter from the second list that occurs in the first list. For example, `remove "first" "second"` should give us "econd". First find out what the type of the function should be.
- d) The function `map` can be applied to any function, so we can write `map map`. What is the type of `map map`? Figure this out *without* asking the Haskell interpreter – try to justify your answer and only then ask the interpreter.
- e) The function `min2` takes a list of numbers and returns the second-smallest number of the input list. If a list contains duplicates, the second-smallest number and the smallest number can be identical; then the function should return that number. Assume that every input lists contains at least two numbers. As an example, `min2 [2110, 4820, 3110, 4120]` should give us `3110` and `min2 [2110, 4820, 2110, 4120]` should give us `2110`. Define the `min2` function using the higher-order functions in Chapter 7.

## Bibliography

- [1] Wikipedia. Continued fractions. [https://en.wikipedia.org/wiki/Continued\\_fraction](https://en.wikipedia.org/wiki/Continued_fraction).