Los Operadores de Pauli

Expresión matricial para los operadores lineales de Pauli: $\mathbb{R}^2 \to \mathbb{R}^2$, definidos como

$$\begin{aligned} \sigma_z|+\rangle &= |+\rangle, & \sigma_z|-\rangle &= -|-\rangle \\ \sigma_x|+\rangle_x &= |+\rangle_x, & \sigma_x|-\rangle_x &= -|-\rangle_x, \\ \sigma_y|+\rangle_y &= |+\rangle_y, & \sigma_y|-\rangle_y &= -|-\rangle_y \end{aligned}$$

con la base canónica representada por: $|+\rangle \leftrightarrows \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|-\rangle \leftrightarrows \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Además, tenemos otros dos conjuntos de vectores base

$$\begin{aligned} |+\rangle_x &= \frac{1}{\sqrt{2}}[|+\rangle + |-\rangle], \quad & |-\rangle_x &= \frac{1}{\sqrt{2}}[|+\rangle - |-\rangle], \\ |+\rangle_y &= \frac{1}{\sqrt{2}}[|+\rangle + i|-\rangle], \quad & |-\rangle_y &= \frac{1}{\sqrt{2}}[|+\rangle - i|-\rangle], \end{aligned}$$

y sus formas asociadas $\langle x| \leftrightarrows (1,0) \quad \langle -| \leftrightarrows (1,0)$

Representaciones Matriciales

La representación matricial de $(\sigma_z^{(+)(-)})^i_j$

$$(\sigma_z^{(+)(-)})^i_j = \begin{pmatrix} \langle +|\sigma_z|+\rangle & \langle +|\sigma_z|-\rangle \\ \langle -|\sigma_z|+\rangle & \langle -|\sigma_z|-\rangle \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

La representación matriicial de $(\sigma_z^{(+x)(-x)})^i_j$

$$(\sigma_z^{(+x)(-x)})^i_j = \begin{pmatrix} {}_x\langle +|\sigma_z| + \rangle_x & {}_x\langle +|\sigma_z| - \rangle_x \\ {}_x\langle -|\sigma_z| + \rangle_x & {}_x\langle -|\sigma_z| - \rangle_x \end{pmatrix}$$

$$\begin{aligned} \sigma_z| + \rangle_x &= \sigma_z\left(\frac{1}{\sqrt{2}}[|+\rangle + |-\rangle]\right) = \frac{1}{\sqrt{2}}[\sigma_z|+\rangle + \sigma_z|-\rangle] = \frac{1}{\sqrt{2}}[|+\rangle - |-\rangle] = |-\rangle_x \\ \sigma_z|-\rangle_x &= \sigma_z\left(\frac{1}{\sqrt{2}}[|+\rangle - |-\rangle]\right) = \frac{1}{\sqrt{2}}[\sigma_z|+\rangle - \sigma_z|-\rangle] = \frac{1}{\sqrt{2}}[+\rangle + |-\rangle] = |+\rangle_x \end{aligned}$$

entonces,

$$(\sigma_z^{(+x)(-x)})^i_j = \begin{pmatrix} {}_x\langle +| - \rangle_x & {}_x\langle +| + \rangle_x \\ {}_x\langle -| - \rangle_x & {}_x\langle -| + \rangle_x \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

La representación matricial de $(\sigma_x^{(+y)(-y)})^i_j$

$$(\sigma_x^{(+y)(-y)})^i_j = \begin{pmatrix} {}_y\langle +|\sigma_z| + \rangle_y & {}_y\langle +|\sigma_z| - \rangle_y \\ {}_y\langle -|\sigma_z| + \rangle_y & {}_y\langle -|\sigma_z| - \rangle_y \end{pmatrix}$$

$$\sigma_z|+\rangle_y = \sigma_z\left(\frac{1}{\sqrt{2}}[|+\rangle + i|-\rangle]\right) = \frac{1}{\sqrt{2}}[\sigma_z|+\rangle + i\sigma_z|-\rangle] = \frac{1}{\sqrt{2}}[|+\rangle - i|-\rangle] = |-\rangle_y$$
$$\sigma_z|-\rangle_y = \sigma_z\left(\frac{1}{\sqrt{2}}[|+\rangle - i|-\rangle]\right) = \frac{1}{\sqrt{2}}[\sigma_z|+\rangle - i\sigma_z|-\rangle] = \frac{1}{\sqrt{2}}[|+\rangle + i|-\rangle] = |+\rangle_y$$

entonces,

$$(\sigma_x^{(+y)(-y)})^i_j = \begin{pmatrix} {}_y\langle +| - \rangle_y & {}_y\langle +| + \rangle_y \\ {}_y\langle -| - \rangle_y & {}_y\langle -| + \rangle_y \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Representación mantricial $(\sigma_y)^i_j$ en la base $\{\pm\}$

$$(\sigma_y^{(+)(-)})^i_j = \begin{pmatrix} \langle +|\sigma_y|+\rangle & \langle +|\sigma_y|-\rangle \\ \langle -|\sigma_y|+\rangle & \langle -|\sigma_y|-\rangle \end{pmatrix}$$

$$\sigma_y|+\rangle = \sigma_y\left(\frac{1}{\sqrt{2}}[|+\rangle_y + |-\rangle_y]\right) = -i|-\rangle$$
$$\sigma_y|-\rangle = \sigma_y\left(\frac{-i}{\sqrt{2}}[|+\rangle_y - |-\rangle_y]\right) = -i|+\rangle$$

$$(\sigma_y^{(+)(-)})^i_j = \begin{pmatrix} -i\langle +|-\rangle & -i\langle +|+\rangle \\ -i\langle -|-\rangle & -i\langle -|+\rangle \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$$