```
> restart : with(plots) : assume(v > 0, L > 0) :
> f(x) := 0 : g(x) := piecewise(x < L/4, 4*v*x/L, x < L/2, (4*v/L)*(L/2-x), x < L, 0);
```

$$g := x \mapsto \begin{cases} \frac{4 \cdot v \cdot x}{L} & x < \frac{L}{4} \\ \frac{4 \cdot v \cdot \left(\frac{L}{2} - x\right)}{L} & x < \frac{L}{2} \\ 0 & x < L \end{cases} \tag{1}$$

```
> u := N→Sum(sin(n*Pi*x/L)*(a[n]*cos(n*Pi*c*t/L) + b[n]*sin(3·n*Pi*c*t/L)), n=1..N);
```

$$u := N \mapsto \sum_{n=1}^{N} \sin\left(\frac{n \cdot \pi \cdot x}{L}\right) \cdot \left(a_n \cdot \cos\left(\frac{n \cdot \pi \cdot c \cdot t}{L}\right) + b_n \cdot \sin\left(\frac{3 \cdot n \cdot \pi \cdot c \cdot t}{L}\right)\right) \tag{2}$$

```
> u := piecewise(0 ≤ x and x ≤ a, A1·exp(I·k1·x)·(1 − exp(−I·2·k1·x)), a ≤ x and x ≤ L, A2·exp(I·k2·x)·(1 − exp(I·2·k2·(a−x))));
```

$$u := \begin{cases} A1\, e^{I k1\, x} \left(1 - e^{-2 I k1\, x}\right) & 0 \le x \le a \\ A2\, e^{I k2\, x} \left(1 - e^{2 I k2\, (a - x)}\right) & a \le x \le L\sim \end{cases} \tag{3}$$

```
> #a:=0.5; L:=1;
> A1 := simplify(int(g(x)·exp(I·k1·x)·(1 − exp(−I·2·k1·x)), x=0..a));
Warning, unable to determine if (1/2)*L is between 0 and a; try to use assumptions or use the AllSolutions option
Warning, unable to determine if (1/4)*L is between 0 and a; try to use assumptions or use the AllSolutions option
```

$$A1 := \frac{-4\, I\, v\sim \left(\begin{cases} -2 \sin(k1\, a) + 2\, k1\, a \cos(k1\, a) & a \le \frac{L\sim}{4} \\ -4 \sin\left(\frac{L\sim\, k1}{4}\right) + (-2\, a + L\sim)\, k1 \cos(k1\, a) + 2 \sin(k1\, a) & a \le \frac{L\sim}{2} \\ 2 \sin\left(\frac{L\sim\, k1}{2}\right) - 4 \sin\left(\frac{L\sim\, k1}{4}\right) & \frac{L\sim}{2} < a \end{cases}\right)}{L\sim\, k1^2} \tag{4}$$

```
> A2 := simplify(int(g(x)·exp(I·k2·x)·(1 − exp(I·2·k2·(a−x))), x=a..L));
Warning, unable to determine if (1/2)*L is between a and L; try to use assumptions or use the AllSolutions option
Warning, unable to determine if (1/4)*L is between a and L; try to use assumptions or use the AllSolutions option
```

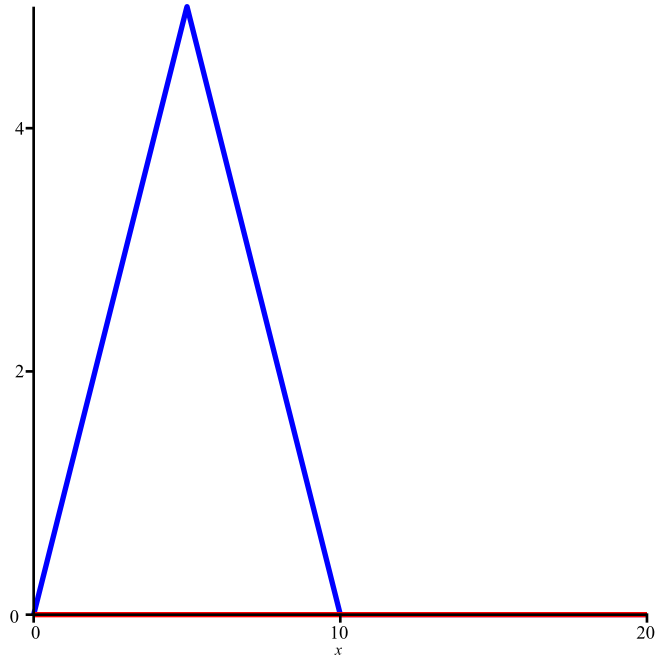
$$A2 := -\frac{2\,v\sim\left(\int_a^{L\sim} -\left(\begin{cases} 2\,x & x<\frac{L\sim}{4} \\ L\sim-2\,x & x<\frac{L\sim}{2} \\ 0 & x<L\sim \end{cases}\right)\left(e^{I\,k2\,x}-e^{I\,k2\,(2\,a-x)}\right)\,dx\right)}{L\sim} \quad (5)$$

> $sol := u;$

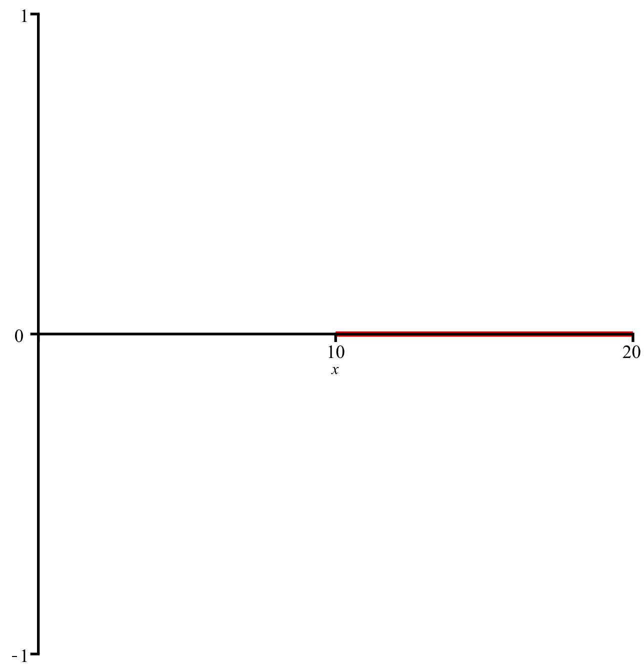
$$sol := \begin{cases} \dfrac{-4\,I\,v\sim\left(\begin{cases} -2\sin(k1\,a)+2\,k1\,a\cos(k1\,a) & a\le\frac{L\sim}{4} \\ -4\sin\left(\frac{L\sim\,k1}{4}\right)+(-2\,a+L\sim)\,k1\cos(k1\,a)+2\sin(k1\,a) & a\le\frac{L\sim}{2} \\ 2\sin\left(\frac{L\sim\,k1}{2}\right)-4\sin\left(\frac{L\sim\,k1}{4}\right) & \frac{L\sim}{2}<a \end{cases}\right)e^{I\,k1\,x}\left(1-e^{\cdots}\right.}{L\sim\,k1^2} \\ -\dfrac{2\,v\sim\left(\int_a^{L\sim} -\left(\begin{cases} 2\,x & x<\frac{L\sim}{4} \\ L\sim-2\,x & x<\frac{L\sim}{2} \\ 0 & x<L\sim \end{cases}\right)\left(e^{I\,k2\,x}-e^{I\,k2\,(2\,a-x)}\right)\,dx\right)e^{I\,k2\,x}\left(1-e^{2\,I\,k2\,(a-x)}\right)}{L\sim} \end{cases}$$

> $sol2 := subs(\{L=20, v=5, a=10\}, sol) : vel := diff(sol2, t) :$

> $plot([subs(\{L=20, v=5, a=10\}, g(x)), eval(vel, t=0)], x=0..20, color=[blue, red],$
> $thickness=2, tickmarks=[4, 4]);$



> $animate(sol2, x=0..20, t=0..50, frames=50, thickness=2, tickmarks=[4, 3]);$



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