Mini-Project 3: Gravitational Motion Simulations

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Abstract

As part of this project, I'm looking into how gravity affects things by modeling problems with one or two bodies in both two and three dimensions. I use numerical methods to look at how trajectories behave, see if they are stable, and see what part initial conditions play. The first part is about how a single mass moves when it is pulled by gravity, and the second part is about how two things that are also pulled by gravity interact with each other. These models show how gravity works and help us understand orbital paths, stability, and how the center of mass moves.

1 Introduction

A lot about basic physics can be learned from studying gravitational motion, especially when it comes to how orbits work. This project takes a look at two events:

- A **one-body problem** is a way to model how one body moves when gravity pulls on it.
- A **two-body problem** in which two masses interact in three dimensions, with the center of mass of the system being integrated.

Differential equations based on Newtonian mechanics are used to solve each situation, and numerical solutions are used to show how the solutions work.

2 Problem Statement

The main goal is to visualize and understand the gravitational interactions between celestial bodies by:

1. Following the path of a single mass that is affected by a big mass that stays in one place.
2. Figure out how the pull of gravity between two things works in three dimensions.

The goals include modeling the paths, looking at how different starting speeds and distances affect them, and checking how stable the paths are over time.

3 Methodology

I used the `solve_ivp` function from `SciPy` in Python to solve the differential equations, and `matplotlib` was used to make the animation.

3.1 One-Body Problem

When I thought about the one-body problem, I found the position (x, y) and speed (v_x, v_y) of a mass m_1 moving past a fixed mass m_2. x and y are the coordinates of the object, and G is the gravity constant.

$$
\begin{aligned}
\frac{dx_1}{dt} &= v_{x1} \\
\frac{dy_1}{dt} &= v_{y1} \\
\frac{dz_1}{dt} &= v_{z1} \\
\frac{dv_x}{dt} &= -\frac{G \cdot m_2 \cdot (x_1 - x_2)}{(x^2 + y^2)^{3/2}} \\
\frac{dv_y}{dt} &= -\frac{G \cdot m_2 \cdot (y_1 - y_2)}{(x^2 + y^2)^{3/2}} \\
\frac{dv_z}{dt} &= -\frac{G \cdot m_2 \cdot (z_1 - z_2)}{(x^2 + y^2)^{3/2}}
\end{aligned}
$$

3.2 Two-Body Problem in 3D

To solve the two-body problem, I put it in a three dimension scenario and took into account the pull of gravity between masses 1 and 2. The equations now read:

$$
\begin{aligned}
\frac{dx_2}{dt} &= v_{x2} \\
\frac{dy_2}{dt} &= v_{y2} \\
\frac{dz_2}{dt} &= v_{z2} \\
\frac{d^2x_2}{dt^2} &= -\frac{G \cdot m_1 \cdot (x_2 - x_1)}{r^3} \\
\frac{d^2y_2}{dt^2} &= \frac{G \cdot m_1 \cdot (y_2 - y_1)}{r^3} \\
\frac{d^2z_2}{dt^2} &= \frac{G \cdot m_1 \cdot (z_2 - z_1)}{r^3}
\end{aligned}
$$

where $r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. Similar equations apply to the y and z components.

4 Results

4.1 One-Body Simulation

The smaller mass moves around the larger mass in the one-body simulation. Its path is affected by its starting speed and distance. Figure 1 displays the path of m_1 and displays stable orbits for certain starting conditions.

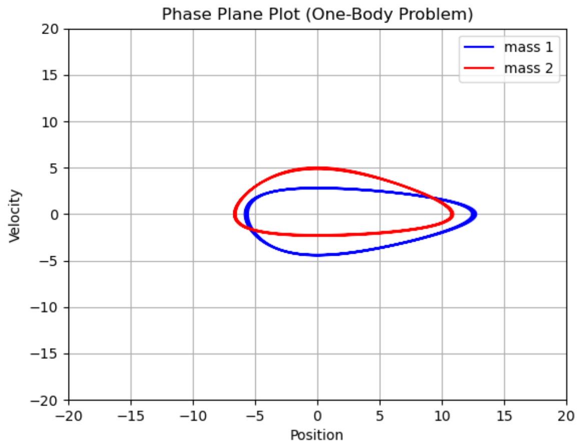

Figure 1: One-body problem: Path of mass m_1 around stationary m_2.

4.2 Two-Body Simulation in 3D

It shows a dynamic relationship between two bodies whose masses move around a common center of mass. Figure 2 shows their paths and the path of the center of two masses. The paths of m_1 and m_2 and the center of mass are shown in Figure 2.

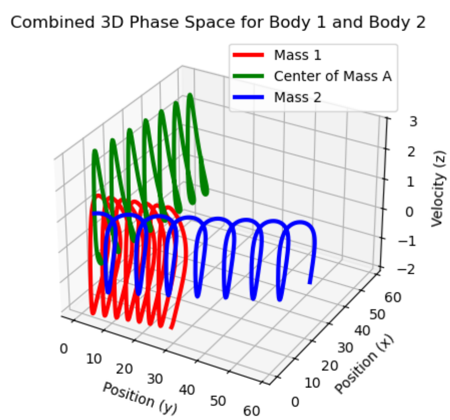

Figure 2: Two-body problem: Paths of m_1, m_2, and the center of mass.

5 Discussion

The simulations highlight:

- **One-Body Problem:** Trajectories stay stable under certain starting conditions, with the starting speed and distance having a big effect on path stability. .
- **Two-Body Problem:** Both masses circle a common center of mass, showing how gravity works. The center of mass moves closer to the middle when the masses are alike.

It is shown here how the starting conditions affect the stability of orbits and the balance of gravity around the center of mass in two-body systems.

6 Conclusion

In this project, gravitational motion was simulated in both one-body and two-body scenarios. This gave us a look at orbital paths and let us study the effects of gravity and stability. In the future, it might be possible to model other solar systems by looking into interactions between many bodies or changing the strength of gravity.