Mini-Project 1 – Optimization with Calculus

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September 16, 2024

Abstract

An optimization is the systematic process of maximizing or minimizing an objective function while considering a set of constraints. The present work investigates two- and three-dimensional objects and aims to optimize objective functions associated with perimeter, area, surface area, and volume. Each problem is augmented with an extra parameter that considers the borders surrounding the object. Each instance yields an analytical solution derived using techniques established in single-variable calculus. In the context of two dimensions, the maximum area is directly proportional to the square of the enclosed perimeter. The square root of the provided area determines the minimum perimeter obtained. The optimal volume in the three-dimensional scenario is determined by the surface area.

1 Introduction

This paper presents a generalization of a calculus optimization problem. An area characterised by a two-dimensional rectangle and a two-dimensional semi-circle. The semi-circle is positioned atop the rectangle. The objective of this task is to minimize the perimeter while considering a limitation associated with the area. Each individual problem is formulated in a conventional optimization style and methods derived from calculus are employed to determine the ideal dimensions and related optimized objective function. We elucidate the solution by considering the constraint parameter and analyze the variation of the optimized objective function value with the input parameters.

Following the establishment of the solution to the fundamental models, we proceed to expand the problem to encompass several borders. In this particular scenario, there exist four rectangles and four semi-circles. Hence, there exist boundaries on the exterior and for each individual wall situated within. This can be subsequently divided into section S. We develop and ascertain a resolution to the initial challenges depending on the input parameter of the number of parts.

The present work is structured in the following manner: Within Section 1, we present the two-dimensional optimization problems together with a detailed explanation of the section parameter. The formulation of the answer to the optimization problem utilizing fundamental ways developed in calculus is presented in Section 2. In Section 3, a comprehensive analysis of the outcomes obtained from each model is provided, along by visual representations that illustrate the relationship between each optimized value and the input variables and parameters. This paper includes Sections 4 and 5 to concisely outline our results and propose potential directions for future study.

2 Problem Statement

The aim of this study is to investigate the relationship between the perimeter and the size of a greenhouse structure including four rectangular walls and four semi-circles divided into sections referred to just as S. By adjusting the number of sections S while keeping the area constant, optimal perimeter minimization is attained. The contribution of each section to the overall layout has significant consequences for material use and design efficiency.

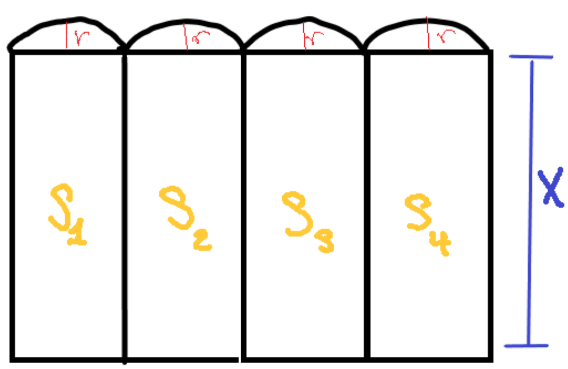

Figure 1: Greenhouse with four rectangles each with their corresponding semi-circles and divided into sections.

3 Methodology

The variable S denotes the number of section, which can range from 1 to any whole number. Each section can effect the overall perimeter. A denoted the total area, as 2000 square feet.

3.1 Variables and Structure

There are four rectangular walls and four semi-circular ends in the greenhouse divided into S sections

- S which can range from 1 to any real whole number is the number of sections that influence the total perimeter.
- A is the greenhouse's total area

3.2 Math Formulas

To show the relationship between the area A, radius r, and height x of the greenhouse, us the area formula

$$A = 2Srx + \frac{S}{2}\pi r^2 \tag{1}$$

Use the area formula to find the values of x and r.

To calculate the perimeter P, use the formula

$$P = (S+1)x + (2+\pi)Sr \tag{2}$$

For each value of S, determine the matching perimeter P by choosing a real number for A

4 Results

A relationship between perimeter and area was established by calculating the perimeter P using an area of 2000 square feet. In order to investigate the impact on the perimeter, the number of sections S was varied within the range of 1 to 4.The results show that the perimeter increases proportionally with the area for every value of S, which was expected.

4.1 Ideal Setup:

The graph demonstrates that reducing the number of sections S leads to a proportionally smaller perimeter for a given area. At an area of 1000 square feet, the perimeter was notably smaller for S= 1 than for S= 4 .

4.2 Graphical Analysis:

The graphed data (Figure 2) clearly show a rising trend: as area A grows, the difference in perimeter between the various S values similarly grows. These findings indicate that the selection of S becomes even more crucial in regulating the perimeter for bigger greenhouses.

5 Discussion

The findings indicate that the quantity of sections S has a crucial role in affecting the operational efficiency of the greenhouse's perimeter. Designs that exhibit a decreased quantity of sections (lower S) consistently result in a smaller perimeter for a given available space.

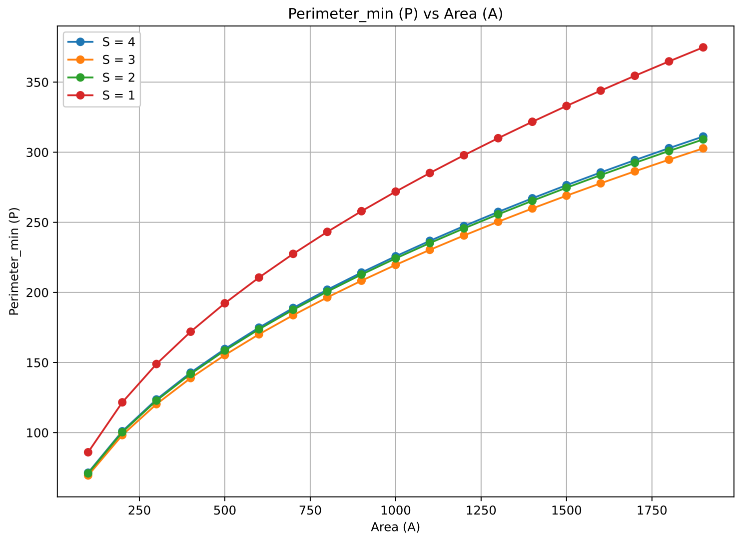

Figure 2: Graph of Minimum Perimeter vs. Area

5.1 Implications for greenhouse design:

Minimizing the perimeter is crucial for reducing construction material expenses from a pragmatic perspective, as a smaller perimeter requires less material for walls and other structural reinforcements. Greenhouse builders with resource constraints or a limited budget can realize significant cost reductions by choosing a design with a reduced number of sections.

In greenhouses with greater dimensions, the selection of parameter S significantly affects the operational efficiency. While decreasing the number of sections reduces the perimeter, It could result in compromises in other features of the greenhouse performance, like ventilation, structural stability, or cosmetic appeal.

6 Conclusion

The present study examined the correlation between the perimeter and size of a greenhouse characterized by four rectangular walls and four semi-circular ends. The objective of the project was to reduce the perimeter for optimal material utilization by manipulating the number of sections S and computing the perimeter for different areas. Study findings indicate that decreasing the quantity of structures leads to a reduction in the perimeter, thereby enhancing the efficiency of the design, particularly for larger greenhouses.

6.1 Optimization of Height and Volume:

Incorporating height and volume optimization into the analysis, alongside the perimeter, has the potential to enhance design efficiency and cost-effectiveness by optimizing space use.

6.2 Alternative Geometric Forms:

Conduct experiments using various geometries, such as elliptical or parabolic ends, to ascertain if alternative designs offer a more optimal equilibrium between visual attractiveness, structural effectiveness, and material utilization.