Mini-Project 1 – Optimization with Calculus

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Abstract

In this project we will find the maximum area which is a 2-variable function of Lumber and height that will maximize the size of the deck. In this project we will introduce the problem of maximization, work through the work of solving, deriving, and creating a maximum function. Next we will show and review the results of the function and graph followed by a discussion of what the results show. Finally, we will conclude with a summary of other options on how to further our research on maximization and minimization of a deck. We will make connections between different variables and find more variables such as cost that we can add to the problem.

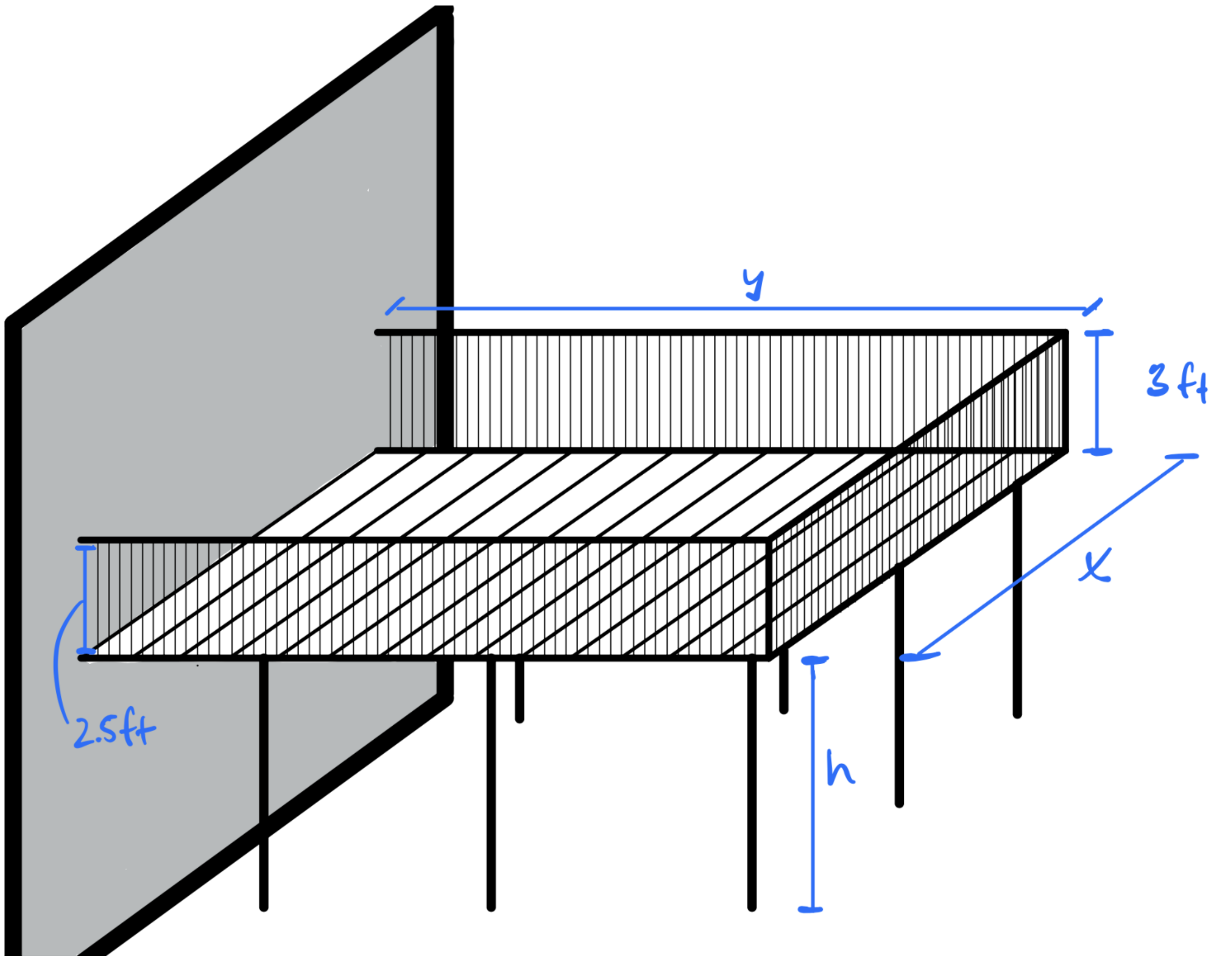

Figure 1: Drawing of deck

1 Introduction

We will maximize the size of the deck given a certain amount of lumber and height of the deck. We will take into account the frame, pillars, spindles, and boards of the deck. There will be 2 big spindles for extra support of the railing that will be 3 feet. These will be placed on the opposite side of the boarder (house). The floor boards will be flush against one another and will each be 6 inches wide. The deck will have a spindle every 4 inches and a pillar every 5 feet. When we calculate the number of pillars we must round up to the nearest whole number. Other maximization problems show that we should end up with a square deck in order to use the lumber in the most efficient way. We will analyze the graph and see how the results reflect this observation.

2 Problem Statement

In this problem we will look at the maximization of a deck with a certain amount of lumber and a chosen height off the ground that must be grater than 2.5 feet. The chosen height will be the same as the height of the pillars that hold the deck up. For this problem we will assume the use of one type of lumber for all parts. The parts will insist of pillars, spindles, 2 big spindles, and a railing above and below the spindles that will span the perimeter of the deck. As previously stated, the maximization should yield a square deck. Other important numbers to keep the deck up to code include: the big spindles being 3 feet tall, the spindles being 4 inches apart, and having a pillar every 5 feet in order to support the deck properly. 1

3 Methodology

In order to solve for the maximum area of the deck I found the function L or the lumber in terms of the variables x, y, and h. I found the length, width, height, and amount needed for each part of the deck. Once I had this function I simplified and solved for y. This gives A as a function of x, h, and L where h and L are constants. This allows for the derivative of A to be taken in terms of x. Since A was a fraction with only constants in the denominator the derivative of the numerator was sufficient. Then x was solved for and plugged into y. One this was done we had A in terms of just L and h which was the goal.

4 Results

As seen in 2 More lumber yields a higher max area for the deck. The other main takeaway from this graph is a higher deck leads to less area. Putting more lumber into the pillars to build up the deck will lead to less lumber to building a larger deck. With this in mind, the maximum area for the deck will come when the deck is 2.5 feet off the ground. This is the minimum amount in order for the deck to meet standards.

5 Discussion

Looking at this graph shows us a direct correlation between area and height. As the height of the deck increases the area decreases. This is because the height is taking lumber away from the deck. We can see that their must be some height in which there wouldn't be enough leftover lumber in order to build a deck. We can also see that more lumber yields a deck with more area. We can assume that this correlation keeps going as lumber available exceeds 500 cubic feet. Thinking about cost is another interesting maximization. We can see more lumber leads to more area. Since it looks like lumber increases at an exponential rate, buying more lumber should be increase the area for less cost per cubic foot. Trying to maximize cost would make the height 2.5 feet.

6 Conclusion

As discussed in the discussion, it would be cool to find the cheapest part of the deck and see how maximizing that part can effect the area while minimizing costs. I would've also liked to find how much lumber we would need when given a certain height and length of each side of the deck. This problem could've involved minimization of lumber or the minimization of costs. You could then compare how the cheapest deck and the most efficient deck dimensions compare to each other. These two would likely be the same because using the least amount of materials will likely be the cheapest. I was only able to find a deck with a house on one border, but another problem would be extending the house to cover two borders of the deck. This would use less lumber while getting the same or even more area.

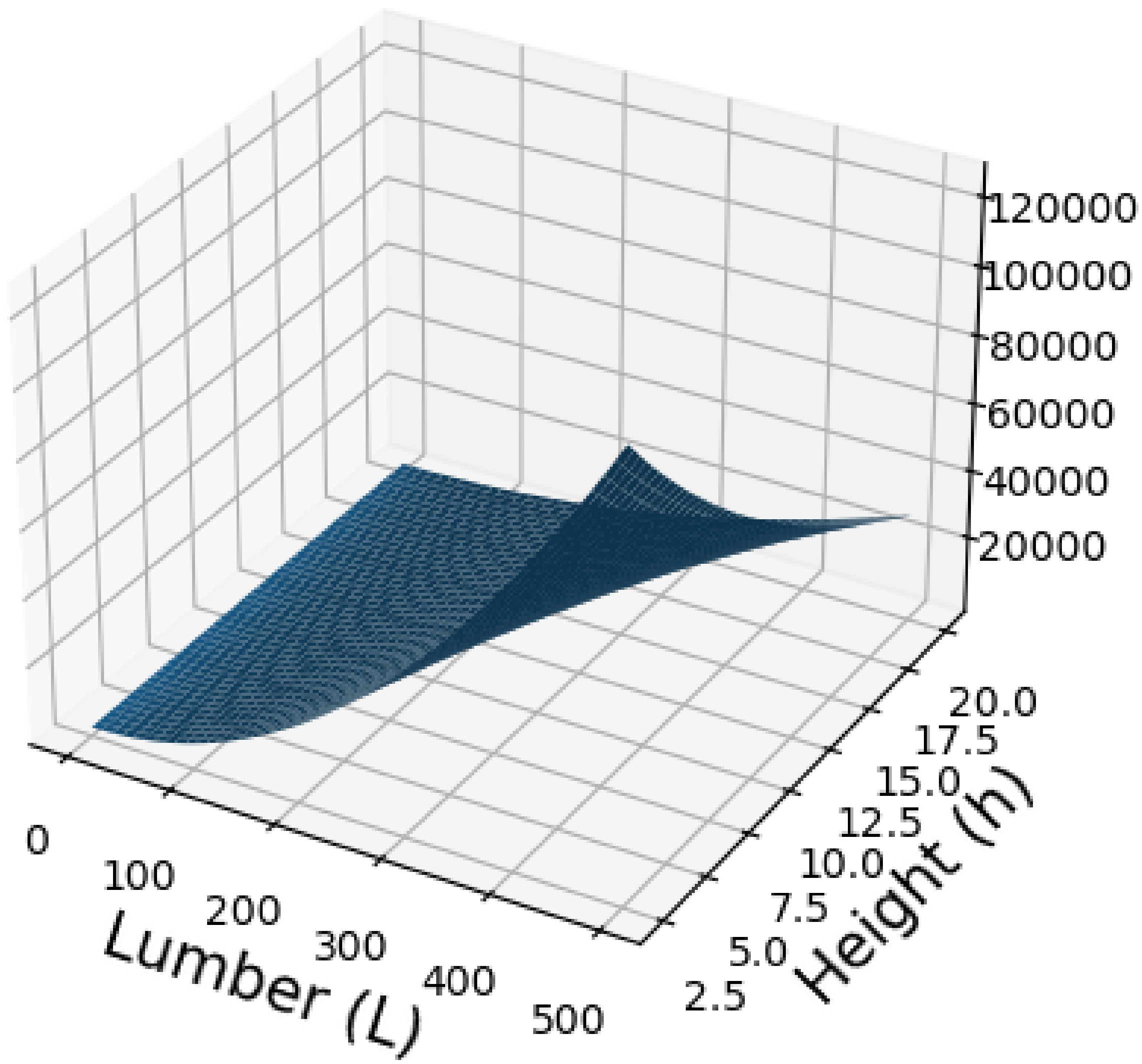

Figure 2: Graph of Max function of Lumber (L)