Homework 3
Differential Equations

MATH 210-010 ⋄ Fall 2024

October 21, 2024

Due: Tuesday, October 22, 2024

Instructions: To complete a problem set, you must submit a zip file labeled `Yourlastname_HW#` to Dropbox no later than 11:59 PM on the due date above. For example, if I were to complete this assignment, my folder would be named `Emerick_HW3`. In this folder, a `py` file is to be submitted for each problem such that when the `py` file is executed, the output (as presented in Python) is the solution to the problem. Each `py` file must be saved as `Yourlastname_HW#_No#.py`. For example, if I were submitting the answer to Question Number 1 on Homework 3, the `py` file for that problem would be saved as `Emerick_HW3_No1.py`. Each `py` file should be well commented and be free of extraneous lines and commands. Also, each `py` file must output only what the problem asked to be outputted. Failure to abide by these simple homework submission guidelines may result in a deduction of points at my discretion.

Name:

Score:

For each problem below submit a separate `py` file with an initial comment that describes the objective of the `py` file. Always remember to begin your `py` file by importing appropriate libraries.

1.] Create a file that computes the numerical solution to the following initial value problem

$$\frac{dy}{dt} = ky\left(1 - \frac{y}{M}\right)\left(\frac{y}{m} - 1\right) \qquad y(0) = y_0,$$

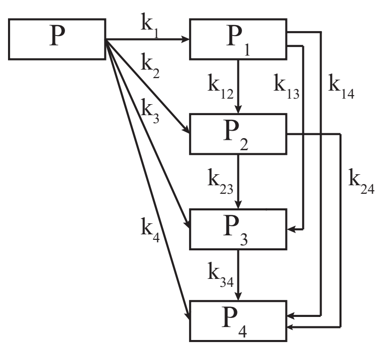
for parameter values $k > 0$, $m > 0$, and $M > 0$ and an initial value y_0. Your file should plot the solution to the equation with at least three solutions on the same graph, one with $y_0 < m$, $m < y_0 < M$, and one with $y_0 > M$.

2.] Consider the following system of that describes a predator-prey dynamic:

$$\begin{aligned}\frac{dx}{dt} &= Rx - 4x^2 - 3xy\\ \frac{dy}{dt} &= -2y + xy\end{aligned}$$

where $R > 0$ is a real-valued parameter. In this model, which population is the prey and which is the predator? Plot the vector field with the phase plane solution as well. Briefly justify your answer. Determine a value of R that makes both populations coexist.

3.] The following schematic represents a chemical process in which a parent peptide (denoted by P) breaks down into four smaller components (denoted by P_i for $i = 1, 2, 3, 4$). The larger components can also break down into smaller components, where P_4 is the smallest component. Here, assume each "bin" is a function of time and measures the concentration of each component in the solution.



Assume that the initial condition contains only the parent peptide, $P(0) = 1$, and all other components are not yet formed: $P_i(0) = 0$ for $i = 1, 2, 3, 4$. Set up and write down the linear differential equation model that simulates this process. Assume each reaction is constant and positive. Create a file that solves this model and outputs the solution when $k_1 = .09$, $k_2 = .8$, $k_3 = .07$, $k_4 = .6$, $k_{12} = .05$, $k_{13} = .5$, $k_{14} = .06$, $k_{23} = .7$, $k_{24} = .08$, and $k_{34} = .9$.