Phase 1 (part 1): Critical Points and Relative Extrema

1.] Find all the critical points of the following functions on the given intervals:

(a) $f(x) = 3x^4 + 8x^3 - 18x^2$ on $[-1, 2]$

$f'(x) = 12x^3 + 24x^2 - 36x = 0$

$12x(x^2 + 2x - 3) = 0$

$12x(x+3)(x-1) = 0$

$x = 0,\ x = -3,\ x = 1$

Critical Points:

$c_1 = 0,\ c_2 = 1$

(b) $g(x) = x^{1/3}(x-2)^2$ on $(-\infty, \infty)$

$g'(x) = \frac{1}{3}x^{-2/3}(x-2)^2 + x^{1/3}2(x-2)$

$g'(x) = \dfrac{(x-2)^2}{3x^{2/3}} + 2x^{1/3}(x-2)$

$g'(x) = \dfrac{(x-2)^2 + 6x(x-2)}{3x^{2/3}}$

$g'(x) = \dfrac{(x-2)(x-2+6x)}{3x^{2/3}}$

$g'(x) = \dfrac{(x-2)(7x-2)}{3x^{2/3}}$

$f'(x)$ is undefined at $x = 0$

$f'(x) = 0$ at $x = 2,\ 2/7$

Critical Points:

$c_1 = 0,\ c_2 = 2/7,\ c_3 = 2$

2.] Determine and classify all relative extrema of the the following functions:

(a) $f(x) = \frac{2}{3}x^3 + 4x^2 - 10x + \frac{5}{3}$

$f'(x) = 2x^2 + 8x - 10$

$f'(x) = 2(x^2 + 4x - 5)$

$f'(x) = 2(x+5)(x-1)$

Critical Points: $c_1 = -5, c_2 = 1$

$f''(x) = 4x + 8$

$f''(-5) = 4(-5) + 8 = -12 < 0$

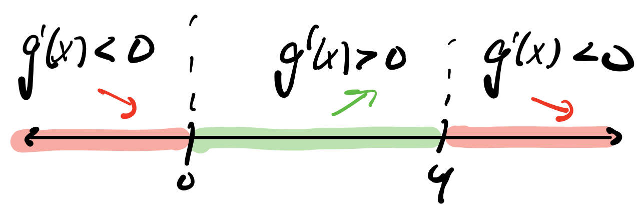
$f''(1) = 4(1) + 8 = 12 > 0$

Relative Max at $x = -5$
Relative Min at $x = 1$

(b) $g(x) = x^2 e^{-x/2}$

$g'(x) = 2xe^{-x/2} - x^2 e^{-x/2} \cdot \frac{1}{2}$

$g'(x) = \frac{1}{2} x e^{-x/2}(4 - x)$



Critical Points: $c_1 = 0, c_2 = 4$

Relative Min at $x = 0$
Relative Max at $x = 4$