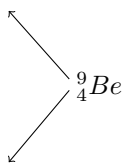


# 1 Wave-particle duality

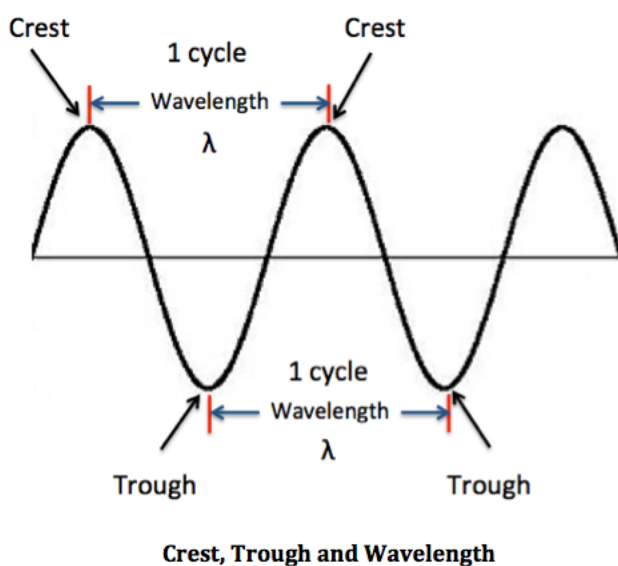
The mass is the protons plus the neutrons

The atomic number is the number of electrons, which equals the number of protons



## 1.1 Characteristics of a Wave

- Wave Length ( $\lambda$ ) — Distance between two consecutive points of equal vibrational phase.



$$1\mu\text{m} = 10^{-6}\text{m}$$

$$1\text{nm} = 10^{-9}\text{m}$$

$$1\text{\AA} = 10^{-10}\text{m}$$

$$1\text{pm} = 10^{-12}\text{m}$$

- Frequency ( $\nu$ ) — Number of vibrations or complete cycles per unit of time.

$$1\text{MHz} = 10^6\text{Hz}$$

$$1\text{GHz} = 10^9\text{Hz}$$

$$1\text{THz} = 10^{13}\text{Hz}$$

- Period ( $T$ ) — Duration of a cycle  $V = \frac{1}{T}$
- Wave Number — Number of complete cycles per unit of time

$$\bar{\nu} = \frac{1}{\lambda}$$

TODO: Is the name of this right?

- Propagation Speed — Speed with which the wave propagates in a given direction.

$$\nu = \frac{c}{\lambda}$$

$$\lambda = cT$$

TODO: Add graphics for the wave, showing with amplitude as the Y axis, time as the X axis, and pointing to the nodes, peaks, and minimums or whatever their name is in english.

Note that the frequency, and the period are, unlike the wavelength, independent of the medium through which the wave travels.

## 1.2 Energy of an Electromagnetic Wave

TODO: Align these sideways

$$E = h\nu$$

$$E = nh\nu$$

Where  $E$  is the energy of a photon or electron,  $h$  is Planck's constant ( $h = 6.62607 \times 10^{-34} \text{ J s}$ )

## 1.3 Electromagnetic Spectrum

TODO: Add graphics here. It shows the different radiations from microwave to gamma rays, there's also an image showing the combinations of RGB lights, and it also mentions the largest and smallest (red, violet respectively) frequencies that the human eye can see.

## 1.4 Interference

Interference is a phenomenon associated with the wave-like behavior of electromagnetic radiation. The interference between two waves travelling through the same region in space yields another wave, which is the algebraic sum of the two interfering waves.

TODO: Graphs for constructive and destructive interference.

## 1.5 Stationary Waves

REVIEW NEEDED

The interference between two waves travelling at the same speed, but in opposing directions creates a *stationary wave* — a wave in which, throughout time, the nodes and peaks (TODO: Peaks isn't the word I want, I mean the part which is either a peak or valley) stay at the same point in space

TODO: Graph for "wave in a box" stationary wave.

The existence of boundary conditions, beyond causing the phenomenon to be stationary, has another important consequence. The wave functions that describe it [the wave] cannot be ordinary. Its wavelength depends on imposed physical constraints (in this case a distance between points in space) and differ from each other through an integer.

$$\lambda = \frac{2L}{n} \quad n = 1, 2, 3, \dots$$

TODO: I forgot how to properly align these, so I did “ ” for the time being.

With this we can see that the solutions are discrete, as opposed to being continuous, and the factor distinguishing them is the integer  $n$ , which we call a quantum number.

TODO: Add graph showing waves with different  $n$ 's

## 1.6 Electromagnetic Radiation

The propagation of electromagnetic radiation is a wave-like movement, and with it we associate a period  $T$ , a frequency  $\nu$ , and a wavelength  $\lambda$ . While in some cases the electromagnetic radiation will present typical wave-like behavior, such as diffraction, on others it behaves in a particle-like manner. One example of the particle-like nature of radiation is the well known “photoelectric effect” experiment.

TODO: Diagram of the photoelectric setup.

From this experiment we could verify that:

1. The emission of electrons on off the plate depends on the frequency of the incident radiation, and not on the intensity of the incident beam.
2. The number of emitted electrons is proportional to the intensity of the beam
3. Emission is immediate, even for low-intensity beams.
4. Electrons bind (connect) to the metal with an energy  $W$ , which has to the incident beam must provide to release the electron. The kinetic energy of said electron is such that:

$$E_c = \frac{1}{2} m v^2 = E_{\text{photon}} - W$$
$$E_c = h\nu - W$$

TODO: Properly align these.

Below a certain *critical frequency*  $\nu_0$ , there will be no photoemission, because the energy  $h\nu$  is insufficient to “beat” the attraction potential  $W$ . We then have

$$h\nu_0 = W$$

TODO: Add the graph that is at the beginning of page 5.

The intensity of the incident beam will be a measurement of the number of *quanta* transmitted by the radiation, given that the higher the intensity the more photons are emitted per unit of time. The interpretation of the photoelectric effect suggests that light is a particle, and that the energy for each transmitted particle is  $h\nu$ . Einstein named these particles *photons*.

In other experiments, such as the dark-body emission, the particle-like description of light is the only one compatible with the results. We can now see that the wave-like and particle-like aspects of light are intimately associated, and that the best description of radiation is a *dual* one.

## 1.7 de-Broglie Hypothesis

TODO: Insert all the formulas at the end of page 5. Take care of formatting them in a sensible way.

de-Broglie, with the dualistic properties of radiation in mind, proposed that this type of behavior, verified for electromagnetic radiation, would be valid for all the other entities which had, up until that point, been identified as particles. This idea would be described mathematically by the relation:

$$\lambda = \frac{h}{p}$$

## 1.8 Electrons

- Particle-like Behavior
  - The determination of the mass of the electron attributed to it a particle-like characteristic.
- Wave-like Behavior
  - Electrons suffer diffraction when interfering with crystals.

## 1.9 Heisenberg's Uncertainty Principle

A direct consequence of the dualistic behavior is the impossibility of measuring with precision the position and the momentum of a particle. Subsequently we must, at the quantum level, abandon the concept of trajectory since, to describe it we must know at any given time the position and momentum of a given particle.

## 1.10 Schrödinger's Equation

$$\begin{aligned} E &= E_p + E_k \implies E\psi = (E_p + E_k)\psi \\ \implies -\frac{h^2}{8\pi^2m} \frac{d^2\psi(x)}{dx^2} + E_p\psi(x) &= E_{\psi(x)} \end{aligned}$$

TODO: Add the following as a “subtitle” to the equations, ‘Time independent Schrödinger equation on 1 dimension, describing any system that exhibits particle-wave behavior’

Where  $\psi$  is the wave function.

## 1.11 Physical Interpretation of Schrödinger's solutions

The wave function  $\psi(x)$  does not have any physical meaning at first, but its square  $\psi(x)^2$  is proportional to the density of particles at the point  $x$ . If  $\psi(x)$  represents a single particle, its square will be