práctica 5

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1.
a.

$$\int_{-1}^{0} kx^2 = 1$$
$$\left.\frac{kx^3}{3}\right|_{-1}^{0} = 1$$
$$\frac{k}{3} = 1$$
$$k = 3$$

b.

$$E(X) = \int_{-1}^{0} x3x^2 = \int_{-1}^{0} 3x^3 = -0.75$$

$$E(X^2) = \int_{-1}^{0} x^2 3x^2 = \int_{-1}^{0} 3x^4 = 0.6$$
$$Var(X) = E(X^2) - E(X)^2 = 0.6 - (-0.75)^2 = 0.0375$$

c.

$$F(x) = \int_{-1}^{x} 3x^2 = x^3 + 1$$

d.

$$F(x) = x^3 + 1 = 0.5 \implies x^3 = -0.5 \implies x = -0.7937$$

4. Es la probabilidad de que la funcion de densidad de A sea mayor a 1/3

$$\int_{\frac{1}{3}}^{1} 4(1-x)^3 dx = \frac{16}{81}$$

5.

$$f(x) = \begin{cases} \frac{1}{b-a} = \frac{1}{3} & 1 \leq x \leq 4 \\ 0 & sino \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{3} & 1 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

$$P(X > 2) = 1 - F(2) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(2 < X < 3) = F(3) - F(2) = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$P(X < 1.5) = F(1.5) = \frac{1}{6}$$

6.

$$f(x) = \begin{cases} \frac{1}{5} & 20 \leq x \leq 25 \\ 0 & sino \end{cases}$$

$$F(x) = \begin{cases} 0 & x < 20 \\ \frac{x-20}{5} & 20 \leq x \leq 25 \\ 1 & x > 25 \end{cases}$$

a.
$P(X \leq 23) = F(23) = \frac{3}{5}$
b.
$E(X) = \frac{b+a}{2} = \frac{45}{4}$
7.

$$f(x) = \begin{cases} 0.01 * e^{-0.01x} & x > 0 \\ 0 & x < 0 \end{cases}$$

$$F(x) = \begin{cases} 1 - e^{-0.01x} & x > 0 \\ 0 & x < 0 \end{cases}$$

$$E(X) = \frac{1}{\alpha} = 100$$

a.
$P(X < E(X)) = P(X < 100) = F(100) = 1 - e^{-0.01*100} = 1 - e^{-1}$
b.
$F(X) = 0.5 \implies 1 - e^{-0.01x} = 0.5 \implies e^{-0.01x} = 0.5 \implies -0.01x = ln(0.5)$
$x = 69.3147$

c.
$P(X > 200) = 1 - F(200) = 1 - (1 - e^{-0.01*200}) = e^{-2}$
se define
Y : cantidad de componentes entre 3 que duran mas de 200 horas, $Y \sim Bin(3, e^{-2})$

$$\begin{aligned}
P(Y \geq 2) &= P(Y = 2) + P(Y = 3) \\
&= \binom{3}{2}(e^{-2})^2(1 - e^{-2}) + (e^{-2})^3 \\
&= 3e^{-4}(1 - e^{-2}) + e^{-6} \\
&= 0.0476
\end{aligned}$$

8.
X_t : numero de accidentes en t dias $\sim Po(\frac{2t}{7})$
Y : dias que pasan entre dos accidentes $\sim Exp(\frac{2}{7})$
a.

$$P(Y > 3) = 1 - F_Y(3) = 1 - (1 - e^{-\frac{2*3}{7}}) = e^{-\frac{6}{7}}$$

b. Z : cantidad de dias hasta el tercer accidente $\sim Erl(\frac{2}{7}, 3)$

$$P(Z > 12) = 1 - F_z(12) = 1 - \frac{\gamma(3, 12 * \frac{2}{7})}{2!} = 1 - \frac{\gamma(3, \frac{24}{7})}{2!}$$

10.

$$\begin{aligned}
P(-a < Z < a) &= 2 * P(0 < z < a) && \text{(Z simetrica alrededor de 0)} \\
&= 2 * (F(a) - F(0)) \\
&= 2F(a) - 2F(0) \\
&= 2F(a) - 2 * 0.5 && (\mu = 0 \text{ es la mediana por Z normal}) \\
&= 2F(a) - 1 \\
&= 2P(Z < a) - 1
\end{aligned}$$

11.

$$Z = \frac{X - 500}{50} \sim N(0,1) \implies X = 50Z + 500$$

a.

$$P(X > 580) = P(50Z + 500 > 580) = P(Z > \frac{8}{5}) = 1 - F_Z(\frac{8}{5}) = 1 - 0.9452 = 0.0548$$

b.

$$P(X < 450) = P(50Z + 500 < 450) = P(Z < -1) = F_Z(-1) = 0.1587$$

12.

$$Z = \frac{X - 0.13}{0.005} \sim N(0,1) \implies X = 0.005X + 0.13$$

$$\begin{aligned} P(0.12 < X < 0.14) &= P(\frac{0.12 - 0.13}{0.005} < Z < \frac{0.14 - 0.13}{0.005}) \\ &= 2F_Z(2) - 1 && \text{(teorema ejercicio 10)} \\ &= 2 * 0.9772 - 1 \\ &= 0.9545 \end{aligned}$$

Y : cantidad de 4 que cumplen con las especificaciones $\sim Bin(4, 0.9545)$

$P(Y = 4) = 0.9545^4 = 0.8301$

13.
a.

$$P_{D_1}(D_1 > 45) = P_Z(Z > \frac{5}{36}) = 1 - F_Z(\frac{5}{36}) = 0.4443$$

$$P_{D_2}(D_2 > 45) = P_Z(Z > \frac{0}{9}) = 1 - F_Z(0) = 1 - 0.5 = 0.5$$

Conviene mas el segundo producto
b.

$$P_{D_1}(D_1 > 48) = P_Z(Z > \frac{8}{36}) = 1 - F_Z(\frac{8}{36}) = 1 - 0.5871 = 0.4129$$

$$P_{D_2}(D_2 > 48) = P_Z(Z > \frac{3}{9}) = 1 - F_Z(\frac{3}{9}) = 1 - 0.6293 = 0.3707$$

Conviene mas el primero