práctica 4

martín rossi

1.
recorrido: $R_x = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$
probabilidad puntual:

$$p(X = i) = \frac{1}{36}, i \in \{2, 12\}$$

$$p(X = i) = \frac{1}{18}, i \in \{3, 11\}$$

$$p(X = i) = \frac{1}{12}, i \in \{4, 10\}$$

$$p(X = i) = \frac{1}{9}, i \in \{5, 9\}$$

$$p(X = i) = \frac{5}{36}, i \in \{6, 8\}$$

$$p(X = 7) = \frac{1}{6}$$

probabilidad acumulada:

$$F(t) = \begin{cases} 0 & t < 2 \\ \frac{1}{36} & 3 \leq t < 4 \\ \frac{1}{12} & 4 \leq t < 5 \\ \frac{1}{6} & 5 \leq t < 6 \\ \frac{5}{18} & 6 \leq t < 7 \\ \frac{5}{12} & 7 \leq t < 8 \\ \frac{13}{18} & 8 \leq t < 9 \\ \frac{5}{6} & 9 \leq t < 10 \\ \frac{11}{12} & 10 \leq t < 11 \\ \frac{35}{36} & 11 \leq t < 12 \\ 1 & 12 \leq t \end{cases}$$

$$E(X) = \sum_{i=2}^{12} i * p(X = i) = 7$$

$$Var(X) = E(X^2) - E(X)^2 = \frac{329}{6} - 49 = \frac{35}{6}$$

$$SD(X) = \sqrt{Var(X)} = 2.4152$$

2.

a.

$$p(T=3) = \frac{1}{3}, p(T=4) = \frac{1}{6}, p(T=5) = \frac{1}{6}, p(T=6) = \frac{1}{3}$$

b.

$$P(3 < T \leq 5) = P(T=4) + P(T=5) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

c.

$$E(T) = \sum_{i=3}^{6} i * p(T=i) = 3 * \frac{1}{3} + 4 * \frac{1}{6} + 5 * \frac{1}{6} + 6 * \frac{1}{3} = \frac{9}{2}$$

$$Var(T) = E(T^2) - E(T)^2 = \frac{131}{6} - \frac{81}{4} = \frac{19}{12}$$

$$SD(T) = \sqrt{Var(T)} = 1.2583$$

3.

a.

$$P(X=1) = \frac{1}{8}, P(X=2) = \frac{1}{4}, P(X=3) = \frac{3}{8}, P(X=4) = \frac{1}{4}$$

b.

$$P(1 \leq X \leq 3) = \frac{3}{4}, P(X < 3) = \frac{3}{8}, P(X > 1.4) = \frac{7}{8}$$

4.

a.

$$Y \sim Bin(3, 0.05)$$

b.

$$\begin{aligned} P(Y > 1) &= P(Y=2) + P(Y=3) \\ &= \binom{3}{2} * 0.05^2 * 0.95 + \binom{3}{3} * 0.05^3 \\ &= 0.0071 + 0.0001 \\ &= 0.0072 \end{aligned}$$

5.
a.

$$R_z = \mathbb{N}$$

b.

$$P(Z = 5) = q^4 * p = 0.95^4 * 0.05 = 0.0407$$

6.
con reposicion:

$$X \sim Bin(3, \frac{1}{5}) \qquad F(x) = P(X \leq x) = \begin{cases} 0 & x < 0 \\ 0.512 & x = 0 \\ 0.896 & x = 1 \\ 0.992 & x = 2 \\ 1 & x \geq 3 \end{cases}$$

sin reposicion:

$$P(X = k) = \frac{\binom{4}{k}\binom{16}{3-k}}{1140}, k = 0, 1, 2, 3 \qquad F(x) = P(X \leq x) = \begin{cases} 0 & x < 0 \\ 0.4912 & x = 0 \\ 0.9123 & x = 1 \\ 0.9965 & x = 2 \\ 1 & x \geq 3 \end{cases}$$

7.

X : cantidad de piezas defectuosas

$$p = \frac{3}{25}$$

$$P(X = 1) = \frac{\binom{3}{1}\binom{22}{4}}{\binom{25}{5}} = 0.4130$$

$$100 * P(X = 1) = 100 * 0.4130 = 41.3$$

8.
a.

$$P(X = k) = (1 - p)^{(k-1)} * p$$

b.

$$P(X = 5) = (1 - p)^4 * p$$

c.

$$\frac{\delta}{\delta p}[(1-p)^4 * p] = -(5*p-1)(1-p)^3$$

$$\begin{aligned}-(5*p-1)(1-p)^3 = 0 &\implies \\ 5*p-1=0 &\implies \\ p = 0.2\end{aligned}$$

en $p = 0.2$ alcanza el maximo

9.

Y : numero de llantas sacadas hasta que 5 esten bien (pascal)

$p = 0.8$

a.

$$P(Y=8) = \binom{7}{4} * 0.8^5 * 0.2^3 = 0.09175$$

b.

$$\begin{aligned}E(Y) &= \sum_{i=5}^{\infty} i * P(Y=i) \\ &= \sum_{i=5}^{\infty} i * \binom{i-1}{4} * 0.8^5 * 0.2^{(i-5)} \\ &= 0.32768 * \sum_{i=0}^{\infty} (i+5) * \binom{i+4}{4} * 0.2^i \\ &= 6.25\end{aligned}$$

10.

a.

$$P(X=0) = \frac{e^{-3} * 3^0}{0!} = 0.0498$$

b.

$$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.42319$$

c.

$$P(X \geq 1) = 1 - P(X=0) = 1 - 0.0498 = 0.9502$$

tomando X : numero de moleculas en λcm^3

$$P(X \geq 1) = 1 - P(X = 0) = 1 - e^{\lambda} > 0.99 \implies e^{\lambda} < 0.01 \implies \lambda < ln(0.01)$$

11.

X : numero de grietas en 4 metros

en una distribucion poisson λ es el promedio o esperanza

$$\begin{aligned} P(X \geq 1) = 1 - P(X = 0) &< 0.5 \\ P(X = 0) &> 0.5 \\ \frac{e^{\lambda} * 3^0}{0!} &> 0.5 \\ \lambda &> ln(0.5) \end{aligned}$$

$$E(\frac{X}{4}) = \frac{E(X)}{4} > \frac{ln(0.5)}{4}$$

12.

a.

como la distribucion esta dada por hora con $\lambda = 30$, por dos minutos seria $\lambda = 1$

$$P(X = 0) = e^{-1} = 0.3679$$

b.

Y : cantidad de intervalos sin clientes en 15 experimentos $\sim Bin(15, 0.3679)$

$$P(Y \leq 5) = \sum_{i=0}^{5} P(Y = i) = 0.5058$$

13.

X : cantidad de defectos en un rollo de 100 metros

$$0.4 = P(X \geq 1) = 1 - P(X = 0) = 1 - \frac{e^{-\lambda} * \lambda^0}{0!} = 1 - e^{-\lambda}$$

$$0.6 = e^{-\lambda} \implies \lambda = -ln(0.6) = 0.5108$$

a.

Y : cantidad de defectos en 50 metros $\sim Po(\frac{\lambda}{2})$

$$P(Y \leq 2) = P(Y = 0) + P(Y = 1) + P(Y = 2) = 0.9977$$

b.

$P(X \geq 1) = 0.4$

Z : cantidad de rollos con al menos un defecto en 10 rollos $\sim Bin(10, 0.4)$

$P(Z \leq 1) = P(Z = 0) + P(Z = 1) = 0.04636$

c.

$$E(Z) = \sum_{i=0}^{10} i * P(Z = i) = 4$$