práctica 7

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1.

a.

D: producto defectuoso
N: producto no defectuoso
$P(D) = p, P(N) = 1 - p$
proceso estocástico $\{N_n : n \in \mathbb{N}\}$
$\Omega = \{w : w = (w_1, w_2, ...), w_i \in \{D, N\}\}$
$N_n(w) \sim Bin(n, p)$
$P(N_n = k) = \binom{n}{k} p^k (1-p)^{n-k}$

b.

-Cantidad de estados discreto, la cantidad de productos defectuosos($\mathbb{N}_0$)
-Instantes de observación discretos, cada vez que se fabrica un producto($\mathbb{N}_0$)
-Propiedad markoviana:

c.

$E = \mathbb{N}_0$

$$P = \begin{pmatrix} 1-p & p & 0 & 0 & 0 & 0 & ... \\ 0 & 1-p & p & 0 & 0 & 0 & ... \\ 0 & 0 & 1-p & p & 0 & 0 & ... \\ 0 & 0 & 0 & 1-p & p & 0 & ... \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix}$$

d.

Los estados son todos transitorios, porque para cada estado hay una probabilidad no nula de abandonarlo, y no regresar nunca porque cada vez se van agregando más productos defectuosos.

e.

$N_n \sim Bin(n, p)$ entonces:
$E(N_n) = np$
$V(N_n) = np(1-p)$

2.

a.

$$P(N_5 = 1, N_4 = 0, N_3 = 0, N_2 = 0, N_1 = 0) = p(0,0)p(0,0)p(0,0)p(0,0)p(0,1)p(N_0 = 0)$$
$$= 0.38^4 0.62$$

b.

$$P(N_5 = 2|N_4 = 1)P(N_4 = 1) = 0.62 * 0.136083 = 0.0843712$$

c.

$$P(N_{20} = 12|N_{12} = 9) = P(N_8 = 3) = P^8(0,3) = 0.0602776$$

d.

38 automóviles

4.

$E = \{S, N\}$

$$P = \begin{pmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{pmatrix}$$

5.

a.

$$P\{X_1 = b, X_2 = b, X_3 = b, X_4 = a, X_5 = c|X_0 = a\} = P(a,b)P(b,b)P(b,b)P(b,a)P(a,c)$$
$$= \frac{1}{3}\frac{3}{4}\frac{3}{4}\frac{1}{4}\frac{2}{3}$$
$$= \frac{1}{32}$$

b.

$$\begin{aligned} P\{X_1 = b, X_3 = a, X_4 = c, X_6 = b | X_0 = a\} &= P(a,b)P^2(b,a)P(a,c)P^2(c,b) \\ &= \frac{1}{3}0.1875\frac{2}{3}\frac{4}{3} \\ &\approx 0.05 \end{aligned}$$

c.

$$\begin{aligned} P\{X_2 = b, X_5 = b, X_6 = a\} &= P\{X_2 = b, X_5 = b, X_6 = a | X_0 = a\}P(X_0 = a) \\ &+ P\{X_2 = b, X_5 = b, X_6 = a | X_0 = b\}P(X_0 = b) \\ &+ P\{X_2 = b, X_5 = b, X_6 = a | X_0 = c\}P(X_0 = c) \\ &= P^2(a,b)P^3(b,b)P(b,a)P(X_0 = a) \\ &+ P^2(b,b)P^3(b,b)P(b,a)P(X_0 = b) \\ &+ P^2(c,b)P^3(b,b)P(b,a)P(X_0 = c) \\ &= 0.25 * 0.5468 * 0.25 * 0.4 \\ &+ 0.6458 * 0.5468 * 0.25 * 0.2 \\ &+ 0.1333 * 0.5468 * 0.25 * 0.4 \end{aligned}$$

6.

a.

$$E = \{0, 1, 2\}$$
$$P = \begin{pmatrix} 0 & 0.5 & 0.5 \\ 0.75 & 0 & 0.25 \\ 0 & 0 & 1 \end{pmatrix}$$

c.

$$(\pi_0 P^\infty) = \{0.43, 0.27, 0.3\}$$

8.

b.

$$E = \{a, b, c, d, e, f\}$$

F(i,j)	a	b	c	d	e	f
a	1	1	0	0	0	0
b	1	1	0	0	0	0
c	0	0	1	1	0	0
d	0	0	1	1	0	0
e	1	1	0	0	0	0
f	1	1	0	0	$\frac{1}{3}$	0

c.

R(i,j)	a	b	c	d	e	f
a	∞	∞	0	0		
b	∞	∞	0	0		
c	0	0	∞	∞	0	0
d	0	0	∞	∞	0	0
e			0	0		
f			0	0		