

CSCI 3230

Fundamentals of Artificial Intelligence

Chapter 8

FIRST ORDER LOGIC

Outline

- ▶ Syntax and Semantics
- ▶ Extensions and Notational Variations
- ▶ Using First-Order Logic (FOL)
- ▶ Logical Agents for the Wumpus World
- ▶ A Simple Reflex Agent
- ▶ Representing Change in the World (Model-Based)
- ▶ Deducing Hidden Properties of the World
- ▶ Toward a Goal-Based Agent
- ▶ Knowledge Engineering Process

First-order logic

First-order Predicate Logic

basic categories and their relations

- ▶ **First-order logic** makes a stronger set of ontological **(entities)** commitments. The world consists of **objects**, i.e., things with individual **identities** and **properties** that distinguish them from other objects.
- ▶ Among these objects, various **relations** hold. Some relations are **functions** – relations with only one “value” for a given “input”.
 - **Objects**: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries...**terms**
 - **Relations (Predicate)**: brother of, bigger than, inside, part of, has color, occurred after, owns...
 - **Properties (unary relations)**: red, round, bogus, prime, multistoried...
 - **Functions**: father of, best friend, third inning of, one more than...
 - **Facts**: “One plus two equals three”, “Squares neighboring the wumpus are smelly”...(atomic sentences)

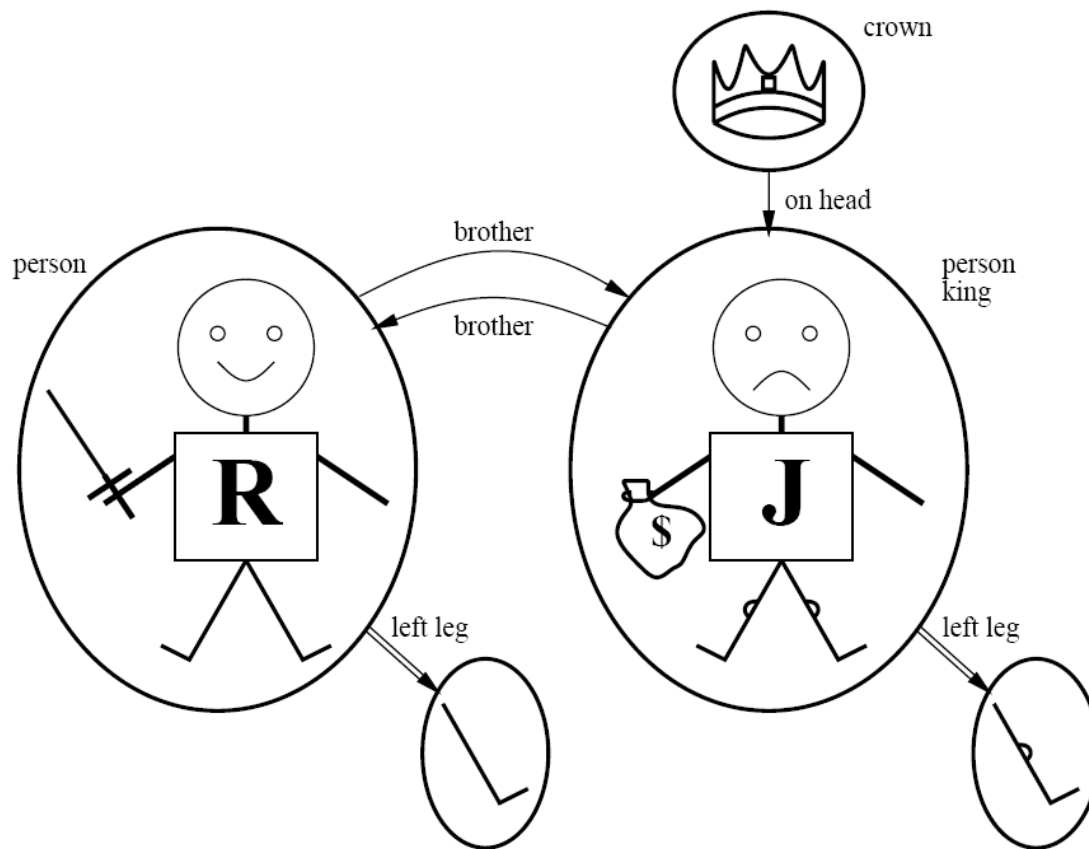
First-order logic

Objects?

Binary relations?

Unary relations
/properties?

Unary function?



predicates

Fig.8.0 A model containing **5** objects, **2** binary relations (brother, on head), **3** unary relations/properties (person, king, crown), and **1** unary function (left-leg).

First-order logic (FOL)

- ▶ FOL commits to the existence of **objects** and **relations**, it does **not** make an ontological commitment to such things as **categories**, **time**, and **events**. cf. OO classes
E.g. John fell in love with Mary 2 hours ago.
- ▶ Existence defined by **quantifiers**: $\forall \mid \exists$
- ▶ But FOL is **universal** in the sense that it can express ^{any algorithm} anything that can be programmed. cf. Turing Machine

Syntax and Semantics

- ▶ In propositional logic every expression is a **sentence**, which represents a fact.
- ▶ FOL has sentences, but also **terms**, which represent **objects**. **Constant symbols**, **variables**, and **function symbols** are used to build terms, and **quantifiers** and **predicate symbols** are used to build sentences.
 - **Constant symbols**: A, B, C, John...
 - An interpretation must specify which object in the world is referred to by each constant symbol.

Syntax and Semantics

述語

- **Predicate symbols**: Brother, Bigger...
 - An interpretation specifies that a predicate symbol refers to a particular **relation** in the model. In a model, the relation is defined by the set of **tuples** of objects that satisfy it. A tuple is a collection of objects arranged in a fixed order. (Extensional definition)
E.g. { <John, Peter> ... <examples or brothers> }
- **Function symbols**: Cosine, FatherOf, LeftLegOf...
 - Some relations are functional – that is, any given object (John) is **related to exactly one** other object (LeftLeg) by the relation.
E.g. **LeftLegOf(John)**.

Syntax and Semantics

Sentence \rightarrow Atomic-Sentence

| Sentence Connective Sentence

| Quantifier Variable, ... Sentence

| \neg Sentence | (Sentence)

Atomic-Sentence \rightarrow Predicate(*Term*, ...) | *Term* = *Term* *// facts*

Term \rightarrow Function(*Term*, ...) | Constant | Variable *// objects*

Connective $\rightarrow \Rightarrow$ | \wedge | \vee | \Leftrightarrow

Quantifier $\rightarrow \forall$ | \exists

Constant $\rightarrow A$ | X_i | *John* | ...

Variable $\rightarrow a$ | x | s

Predicate \rightarrow Before | HasColor | Raining | ... *// relations:- unary(property) & binary*

Function \rightarrow Mother | LeftLegof | ... *// objects*

Fig 8.1 The syntax of first-order logic (with equality) in BNF (Backus–Naur Form)

Syntax and Semantics

► Terms

- A **term** is a logical expression that refers to an object. **Constant** symbols are therefore terms.
- The formal semantics of terms is straightforward. An interpretation specifies a functional relation referred to by the *function symbol*, and **objects(John)** referred to by the **terms** that are its arguments. E.g. John's left leg: **LeftLegOf(John)**

► Atomic sentences

- **Atomic sentences** that state facts. An atomic sentence is formed from a **predicate symbol** followed by a parenthesized list of terms. e.g.,

Brother (Richard, John)

states, under the interpretation given before, that Richard the Lionheart is the brother of King John.

Syntax and Semantics

Atomic sentences can have arguments that are complex terms:

Married(FatherOf(Richard), MotherOf(John))

states that Richard the Lionheart's father is married to King John's mother

(again, under a suitable interpretation)

- An atomic sentence is true if the relation(Teacher) referred to by the predicate symbol holds between the objects (Leung, CSCI 3230) referred to by the arguments.
E.g. Teacher (Leung, CSCI 3230)

Syntax and Semantics

► Complex sentences

- We can use **logical connectives** to construct more complex sentences, like propositional calculus.
 - $Brother(Richard, John) \wedge Brother(John, Richard)$ is true when John is the brother of Richard **and** Richard is the brother of John.
 - $Older(John, 30) \vee Younger(John, 30)$ is true when John is older than 30 **or** John is younger than 30.
 - $Older(John, 30) \Rightarrow \neg Younger(John, 30)$ states that **if** John is older than 30, **then** he is not younger than 30.
 - $\neg Brother(Robin, John)$ is true when Robin is **not** the brother of John.

Syntax and Semantics

► Quantifiers

- **Universal quantification** (\forall) (read: for all)

- “All cats are mammals”

$$\forall x \text{ Cat}(x) \Rightarrow \text{Mammal}(x)$$

- The preceding sentence is therefore equivalent to

$$\begin{aligned} &\text{Cat}(\text{Spot}) \Rightarrow \text{Mammal}(\text{Spot}) \wedge \\ &\text{Cat}(\text{Rebecca}) \Rightarrow \text{Mammal}(\text{Rebecca}) \wedge \\ &\text{Cat}(\text{Felix}) \Rightarrow \text{Mammal}(\text{Felix}) \wedge \\ &\text{Cat}(\text{Richard}) \Rightarrow \text{Mammal}(\text{Richard}) \wedge \\ &\text{Cat}(\text{John}) \Rightarrow \text{Mammal}(\text{John}) \wedge \end{aligned}$$

...

- Thus, it is true iff all these sentences are true, i.e., if P is true for all object x in the universe. Hence \forall is called a **universal** quantifier.

Syntax and Semantics

- **Existential quantification (\exists)** (read: there exist(s))
 - We can make a statement about some object in the universe **without naming** it using an existential quantifier. To say, e.g., that Spot has a sister who is a cat:

$$\exists x \text{ Sister}(x, \text{Spot}) \wedge \text{Cat}(x)$$

- In general $\exists x$ is true for some object in the universe.

$$\begin{aligned} & (\text{Sister}(\text{Spot}, \text{Spot}) \wedge \text{Cat}(\text{Spot})) \vee \\ & (\text{Sister}(\text{Rebecca}, \text{Spot}) \wedge \text{Cat}(\text{Rebecca})) \vee \\ & (\text{Sister}(\text{Felix}, \text{Spot}) \wedge \text{Cat}(\text{Felix})) \vee \\ & (\text{Sister}(\text{Richard}, \text{Spot}) \wedge \text{Cat}(\text{Richard})) \vee \\ & (\text{Sister}(\text{John}, \text{Spot}) \wedge \text{Cat}(\text{John})) \vee \end{aligned}$$

...

(?? c.f. propositional logic??) The existentially quantified sentence is **true** just in case at least **one** of these disjuncts is true.

Syntax and Semantics

- Nested quantifiers

- Express more complex sentences using multiple quantifiers. E.g., “For all x and all y , if x is the parent of y then y is the child of x ” becomes

$$\forall x, y \text{ Parent}(x, y) \Rightarrow \text{Child}(y, x)$$

- $\forall x, y$ is equivalent to $\forall x \forall y$
All persons
- We can have mixtures. “Everybody loves somebody” means that for every person, there is someone that person loves:

$$\forall x \exists y \text{ Loves}(x, y)$$

- To say “There is someone who is loved by everyone” we write

$$\exists y \forall x \text{ Loves}(x, y)$$

$$? \exists x \forall y \text{ Loves}(x, y) ?$$

Syntax and Semantics

- ▶ The **order** of quantification is \therefore **important**.
- ▶ $\forall x (\exists y P(x, y))$, where $P(x, y)$ says that every object x in the relation P to some y .
- ▶ $\exists x (\forall y P(x, y))$ says that there is some object in the world that has a particular property, namely the property of being related by P to every object in the world.
- ▶ A minor difficulty – 2 quantifiers with the **same** variable name.
 - $\forall x [\text{Cat}(x) \vee (\exists x \text{ Brother}(\text{Richard}, x))]$ *Standardize apart; overloaded variable name;*
 - One interpretation: $\exists x \text{ Brother}(\text{Richard}, x)$ is a sentence about Richard (that he has a brother), not about x : So putting a $\forall x$ outside it has no effect.
- ▶ The term **well-formed formula** or **wff** is sometimes used for sentences that have all their variables properly introduced. E.g. $\forall x P(x, y)$, y is **not** properly introduced.

Syntax and Semantics



- Connections between \forall and \exists

- The 2 quantifiers are intimately connected with each other, through **negation**.
- When one says that everyone dislikes parsnips, one is also saying that there does not exist someone who likes them; and vice versa:

$$\forall x \neg \text{Likes}(x, \text{Parsnips}) \equiv \neg \exists x \text{ Likes}(x, \text{Parsnips})$$

- “Everyone like ice-cream” means that there is no one who does not like ice-cream:

$$\forall x \text{ Likes}(x, \text{Ice-cream}) \equiv \neg \exists x \neg \text{Likes}(x, \text{Ice-cream})$$

- De Morgan rules: thus, we do not really need both \forall and \exists , just as we do not need both \wedge and \vee . $\neg(\alpha \wedge \beta) \equiv (\neg \alpha \vee \neg \beta)$;
- $\neg \forall x R(x, y) \equiv \exists x \neg R(x, y)$??How to convert??
- Double negation + De Morgan: $\neg(\neg \forall x \text{ Likes}(x, \text{Ice-cream})) \equiv \neg \exists x \neg \text{Likes}(x, \text{Ice-cream})$

Syntax and Semantics

Equality

- FOL includes one more way to make **atomic sentences**, other than using a predicate and terms. We can use the **equality symbol** to make statements that 2 terms refer to the same object. E.g.,

Prefix function
Father (John) = Henry // infix

- Equality can be viewed as a **predicate symbol** with a predefined meaning, i.e., **identity relation**.

Syntax and Semantics

- The **equality** symbol can be used to describe the **properties** of a given **function**, as we did above for the Father symbol.

Father (John) = Henry

- It can also be used with **negation** to insist that two terms are **not the same object**. To say that Spot has at least 2 sisters:

$\exists x, y \text{ Sister(Spot, } x \text{) } \wedge \text{ Sister(Spot, } y \text{) } \wedge \neg(x = y)$

- Simply writing $\exists x, y \text{ Sister(Spot, } x \text{) } \wedge \text{ Sister(Spot, } y \text{)}$ would not assert the existence of 2 **distinct** sisters, because nothing says that x and y have to be different.

Extensions and Notational Variations

–Higher–order logic

- ▶ FOL gets its name from the fact that one can **quantify** over **objects** (the first–order entities that actually exist in the world) but **not** over **relations** or **functions** on those objects.
- ▶ **Higher–order logic allows** us to **quantify** over **relations** and **functions** as well as over **objects**. E.g., in higher–order logic 2 objects are equal iff all properties, p , applied to them are equivalent:

$$\forall x, y (x = y) \Leftrightarrow \forall p (p(x) \Leftrightarrow p(y))$$

- ▶ 2 functions, f & g , are equal iff they have the same value for all arguments:

$$\forall f, g (f = g) \Leftrightarrow \forall x (f(x) = g(x))$$

- ▶ Higher–order logics have strictly more expressive power. However, logicians have **little understanding** of how to reason effectively with sentences in higher–order logic, and the general problem is **undecidable**.

Extensions and Notational Variations

–Functional and predicate expressions using the λ operator

- ▶ Useful to be able to construct **complex** predicates and functions from **simpler** components (e.g. $P \wedge Q$), or complex terms from simpler ones (e.g. $x^2 + y^3$).
- ▶ The **operator λ** (the Greek letter lambda) is used for this purpose. The function that takes the difference of the squares of its first and second arguments:

$$\lambda x, y \ x^2 - y^2$$

- ▶ This **λ -expression** can then be applied to arguments to yield a logical term in the same way that an ordinary, **named function** can:

$$(\lambda x, y \ x^2 - y^2)(25, 24) = 25^2 - 24^2 = 49$$

$$f(x, y)$$

Extensions and Notational Variations

–Functional and predicate expressions using the λ operator

- ▶ E.g., the **two-place predicate** "are of differing gender and of the same address" can be written

$$\lambda x, y \text{ Gender}(x) \neq \text{Gender}(y) \wedge \text{Address}(x) = \text{Address}(y)$$

the application of a predicate **λ -expression** to an appropriate number of arguments yields a logical sentence.

- ▶ The use of **λ** in this way does **not increase the formal expressive power** of FOL, because any sentence that includes a λ -expression can be rewritten by "plugging in" its arguments to yield a standard term or sentence.

Extensions and Notational Variations

–The uniqueness quantifier $\exists!$

- ▶ Some authors use the notation

$$\exists!x \text{ King}(x)$$

to mean "there exists a **unique** object x satisfying $\text{King}(x)$ " or more informally, "there's exactly one King."

–not a new quantifier, $\exists!$, but a convenient **abbreviation** for the longer sentence

$$\exists x \text{ King}(x) \wedge \forall y \text{ King}(y) \Rightarrow x = y$$

- ▶ A more complex example is "Every country has exactly one ruler":

$$\forall c \text{ Country}(c) \Rightarrow \exists!r \text{ Ruler}(r, c)$$

Extensions and Notational Variations

–The uniqueness operator ι

- ▶ More convenient to have a term representing the **unique object** directly. The notation $\iota x P(x)$ is commonly used.

(The symbol ι is the Greek letter iota).

To say that "the **unique** ruler of Freedonia is dead" or equivalently "the r that is the ruler of Freedonia is dead", we would write

$\text{Dead}(\iota r \text{ Ruler}(r, \text{Freedonia}))$

- ▶ This is just an **abbreviation** for the following sentence:

$\exists! r \text{ Ruler}(r, \text{Freedonia}) \wedge \forall s \text{ Ruler}(s, \text{Freedonia}) \Rightarrow \text{Dead}(s)$

-Notational variations

Some variations: (see table)

Prolog has 3 main differences:

- ▶ It uses **uppercase** letters for **variables** and **lowercase** for **constants**.
- ▶ Prolog also **reverses the order of implications**, writing $Q:-P$ instead of $P \Rightarrow Q$.
- ▶ A **comma** is used both to separate arguments and for conjunction, and a **period** marks the end of a sentence:

Cat (X) :- furry(X), meows(X), has(X, claws).

Because **Lisp** does **not distinguish** between **uppercase** and **lowercase** symbols, **variables** are usually distinguished by an initial **?** or **\$** character, e.g.:

(forall ?x (implies (and (furry ?x) (meows ?x) (has ?x claws))
(cat ?x)))

Using First-Order Logic

–The kinship domain

In knowledge representation, a **domain** is a section of the world about which we wish to express some knowledge.

The kinship domain

includes **facts** such as "Elizabeth is the mother of Charles" and "Charles is the father of William." and **rules** such as "If x is the mother of y and y is a parent of z, then x is a grandmother of z."

The objects in our domain are people whose **properties** include gender, related by **relations** such as parenthood, brotherhood, marriage, and so on. We \therefore have 2 **unary predicates**, Male and Female.

Using First-Order Logic

-The kinship domain

Most of the kinship relations will be binary predicates: *Parent, Sibling, Brother, Sister, Child, Daughter, Son, Spouse, Wife, Husband, Grandparent, Grandchild, Cousin, Aunt, Uncle, Nephew(male), Niece(f)*. *How to define Mother from above terms??*

We will use **functions** for Mother and Father, because every person has exactly one of each of these (at least according to nature's design).

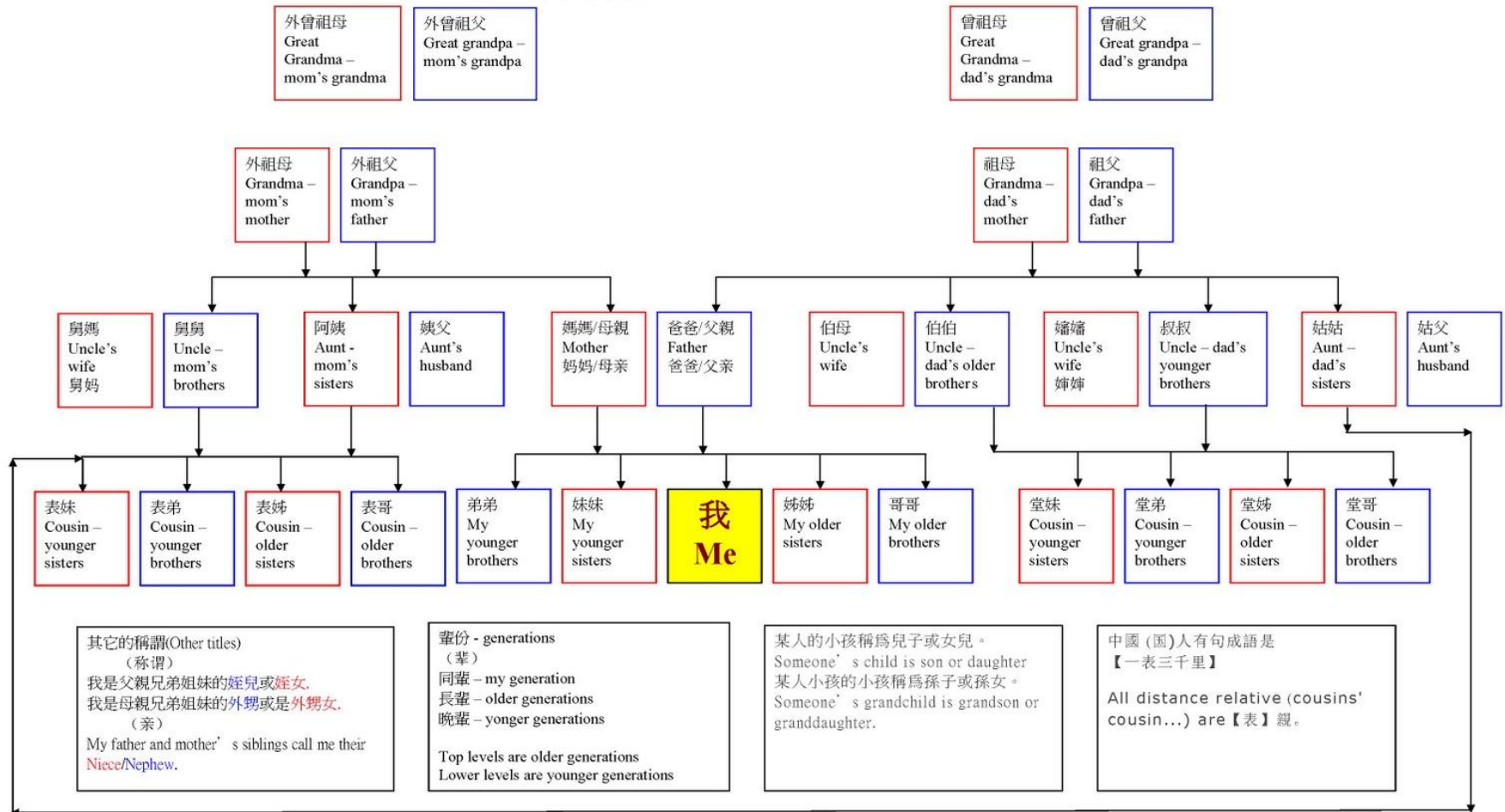
E.g.. One's Mother is one's female parent:

$$\forall m, c \text{ Mother}^{\text{function}}(c) = m \Leftrightarrow \text{Female}^{\text{property}}(m) \wedge \text{Parent}^{\text{relation}}(m, c)$$

One's husband is one's male spouse:

$$\forall w, h \text{ Husband}(h, w) \Leftrightarrow \text{Male}(h) \wedge \text{Spouse}(h, w)$$

中英文親戚稱謂表/亲戚称谓表/Relatives



Using First-Order Logic

–The kinship domain

Male and **female** are disjoint categories:

$$\forall x \text{ Male}(x) \Leftrightarrow \neg \text{Female}(x)$$

Parent and **child** are inverse relations:

$$\forall p, c \text{ Parent}(p, c) \Leftrightarrow \text{Child}(c, p)$$

A **grandparent** is a parent of one's parent:

$$\forall g, c \text{ Grandparent}(g, c) \Leftrightarrow \exists p \text{ Parent}(g, p) \wedge \text{Parent}(p, c) \text{ ?}\exists p\text{?}$$

Sibling (parent)??

A **sibling** is another child of one's parents:

$$\forall x, y \text{ Sibling}(x, y) \Leftrightarrow x \neq y \wedge \exists p \text{ Parent}(p, x) \wedge \text{Parent}(p, y)$$

Using First-Order Logic

–Axioms, definitions and theorems

- ▶ **Mathematicians** write **axioms** to capture the **basic facts** about a domain, **define other concepts in terms** of these **basic facts**, and then use the **axioms** and **definitions** to prove **theorems**. But the sentences that are in the **knowledge base** initially are sometimes called "**axioms**", or "**definitions**".
- ▶ Important question: how do we know when we have written down **enough axioms** to fully specify a domain? In many domains, there is **no clearly** identifiable basic set.
- ▶ The converse problem also arises: do we have **too many** sentences? E.g., **do we need** the following sentence, specifying that siblinghood is a symmetric relation?
 - $\forall x, y \text{ Sibling}(x, y) \Leftrightarrow \text{Sibling}(y, x)$

Using First-Order Logic

–Axioms, definitions and theorems

- ▶ Answer is NO. From $\text{Sibling}(\text{John}, \text{Richard})$, we can **infer** that

- ▶ $\exists p \text{ Parent}(p, \text{John}) \wedge \text{Parent}(p, \text{Richard})$.

And from that we can **infer?** $\text{Sibling}(\text{Richard}, \text{John})$. (last axiom, p27)

$\forall x, y \text{ Sibling}(x, y) \Leftrightarrow x \neq y \wedge \exists p \text{ Parent}(p, x) \wedge \text{Parent}(p, y)$

- ▶ In mathematics, an **independent axiom** is one that **cannot be derived** from all the other axioms. Mathematicians strive to produce a **minimal set** of axioms that are **all independent**.
- ▶ In **AI**, common to include **redundant** axioms, not because of what can be proved, but because they make proof **more efficient**.
- ▶ An **axiom** of the form $\forall x, y P(x, y) \equiv \dots$ is often called a **definition** of P , because it defines exactly for **what** objects P does and does not hold. Possible to have **several** definitions for the same predicate; e.g., a triangle could be defined both as a **polygon** with **3 sides** and as a polygon with **3 angles**.

Using First-Order Logic

-The domain of sets

Set is a predicate that is true only of sets. The following eight axioms provide this:

1. The only **sets** are the empty set and those made by adjoining something to a set.

$$\forall s \text{ Set}(s) \Leftrightarrow (s = \text{EmptySet})$$

$$\forall (\exists x, s_2 \text{ Set}(s_2) \wedge s = \text{Adjoin}(x, s_2))$$

There does not exist

2. The **empty set** has no elements adjoined into it. (In other words, there is no way to decompose EmptySet in to a smaller set and an element.)

$$\neg \exists x, s \text{ Adjoin}(x, s) = \text{EmptySet}$$

3. **Adjoining** an element already in the set has no effect:

$$\forall x, s \text{ Member}(x, s) \Leftrightarrow s = \text{Adjoin}(x, s)$$

Using First-Order Logic

-The domain of sets

4. The only **members** of a set are the elements that were adjoined into it. We express this **recursively**, saying that x is a member of s iff s is equal to some set s_2 adjoined with some element y , where either y is the same as x or x is a member of s_2 .

$$\forall x, s \text{ Member}(x, s) \Leftrightarrow \exists y, s_2 (s = \text{Adjoin}(y, s_2) \wedge (\overset{\text{newly added}}{x=y} \vee \overset{x \text{ already existed}}{\text{Member}(x, s_2)}))$$

5. A set is **subset** of another iff all of the first set's members are members of the second set.

$$\forall s_1, s_2 \text{ Subset}(s_1, s_2) \Leftrightarrow (\forall x \text{ Member}(x, s_1) \Rightarrow \text{Member}(x, s_2))$$

6. Two sets are **equal** iff each is a subset of the other.

$$\forall s_1, s_2 (s_1 = s_2) \Leftrightarrow (\text{Subset}(s_1, s_2) \wedge \text{Subset}(s_2, s_1))$$

Using First-Order Logic

-The domain of sets

7. An object is a member of the **intersection** of two sets if and only if it is a member of each of the sets.

$$\forall x, s_1, s_2 \text{ Member}(x, \text{Intersection}(s_1, s_2)) \Leftrightarrow \\ \text{Member}(x, s_1) \wedge \text{Member}(x, s_2)$$

8. An object is a member of the **union** of two sets if and only if it is a member of either set.

$$\forall x, s_1, s_2 \text{ Member}(x, \text{Union}(s_1, s_2)) \Leftrightarrow \\ \text{Member}(x, s_1) \vee \text{Member}(x, s_2)$$

The domain of lists is very similar to the domain of sets. The difference is that lists are ordered, and the same element can appear more than once in a list.

Using First-Order Logic

–Asking questions and getting answers

- ▶ To add the kinship sentences to a knowledge base KB, e.g.

$$\text{TELL}(\text{KB}, (\forall m, c \text{ Mother}(c) = m \\ \Leftrightarrow \text{Female}(m) \wedge \text{Parent}(m, c)))$$

- ▶ Now if we tell it

$$\text{TELL}(\text{KB}, \text{Female}(\text{Maxi}) \wedge \text{Parent}(\text{Maxi}, \text{Spot}) \\ \wedge \text{Parent}(\text{Spot}, \text{Boots}))$$

then we can

- ▶ $\text{ASK}(\text{KB}, \text{Grandparent}(\text{Maxi}, \text{Boots}))$

and receive an **affirmative** answer.

- ▶ The sentences added using TELL – called **assertions**
- ▶ The questions asked using ASK – called **queries** or **goals** (different to an agent's desired states).
- ▶ Thus, a query with existential variables is asking "Is there an x such that ...," and we solve it by providing such an x. The standard form for an answer of this sort is a **substitution** or **binding list** – a set of variable/term pairs.

Logical Agents for the Wumpus World

We will consider 3 agent architectures:

1. **reflex** agents that merely classify their percepts and act accordingly;
2. **model-based agents** that construct an internal representation of the world and use it to act; and
3. **goal-based agents** that form goals and try to achieve them. (Goal-based agents are usually also model-based agents.)

```
function KB-Agent (percept) returns an action  
  static: KB, a knowledge base  
           t, a counter, initially 0, indicating time  
  Tell(KB, Make-Percept-Sentence(percept, t))  
  action  $\leftarrow$  Ask(KB, Make-Action-Query(t))  
  Tell(KB, Make-Action-Sentence(action, t))  
  t  $\leftarrow t + 1$   
  return action
```

Fig 8.2 A generic knowledge-based agent

A Simple Reflex Agent

- ▶ The simplest possible kind of agent has rules **directly** connecting **percepts to actions**. These rules resemble **reflexes** or instincts. E.g., if the agent sees a **glitter**, it should do a **grab** in order to pick up the gold.

$\forall s, b, u, c, t \text{ Percept}([s, b, \text{Glitter}, u, c], t) \Rightarrow \text{Action}(\text{Grab}, t)$

- ▶ The connection between percept and action can be mediated by rules for perception, which **abstract the immediate perceptual input into more useful** forms:

$\forall b, g, u, c, t \text{ Percept}([\text{stench}, b, g, u, c], t) \Rightarrow \text{Stench}(t)$ ^{concepts}

$\forall s, g, u, c, t \text{ Percept}([s, \text{Breeze}, g, u, c], t) \Rightarrow \text{Breeze}(t)$

$\forall s, b, u, c, t \text{ Percept}([s, b, \text{Glitter}, u, c], t) \Rightarrow \text{AtGold}(t)$

...

Then a connection can be made from these **predicates** to action choices:

$\forall t \text{ AtGold}(t) \Rightarrow \text{Action}(\text{Grab}, t)$

A Simple Reflex Agent

Limitations of simple reflex agents

- ▶ They will have a hard time in the wumpus world. A pure reflex agent **cannot** know for sure when to climb, because **neither** having the gold nor being in the start square is part of the **percept**; they are things the agent knows by forming a representation (model) of the world. (state)
- ▶ They also cannot avoid infinite **loops**. Randomization provides some relief, but risking many fruitless actions.

Representing Change in the World

(+Situation)
location only

(Model Based Agents)

The easiest way to **deal with change** is simply to **change the knowledge base**; to erase the sentence that says the agent is at [1,1], and replace it with says at [1,2]. But all knowledge about the **past is lost**, and it prohibits **speculation** about different possible futures.

Situation Calculus

It conceives of the world as consisting of **a sequence of situations**, each of which is a "**snapshot**" of the state of the world.

Situations are generated from previous situations by actions, as shown in the following figure.

Representing Change in the World (+Situation)

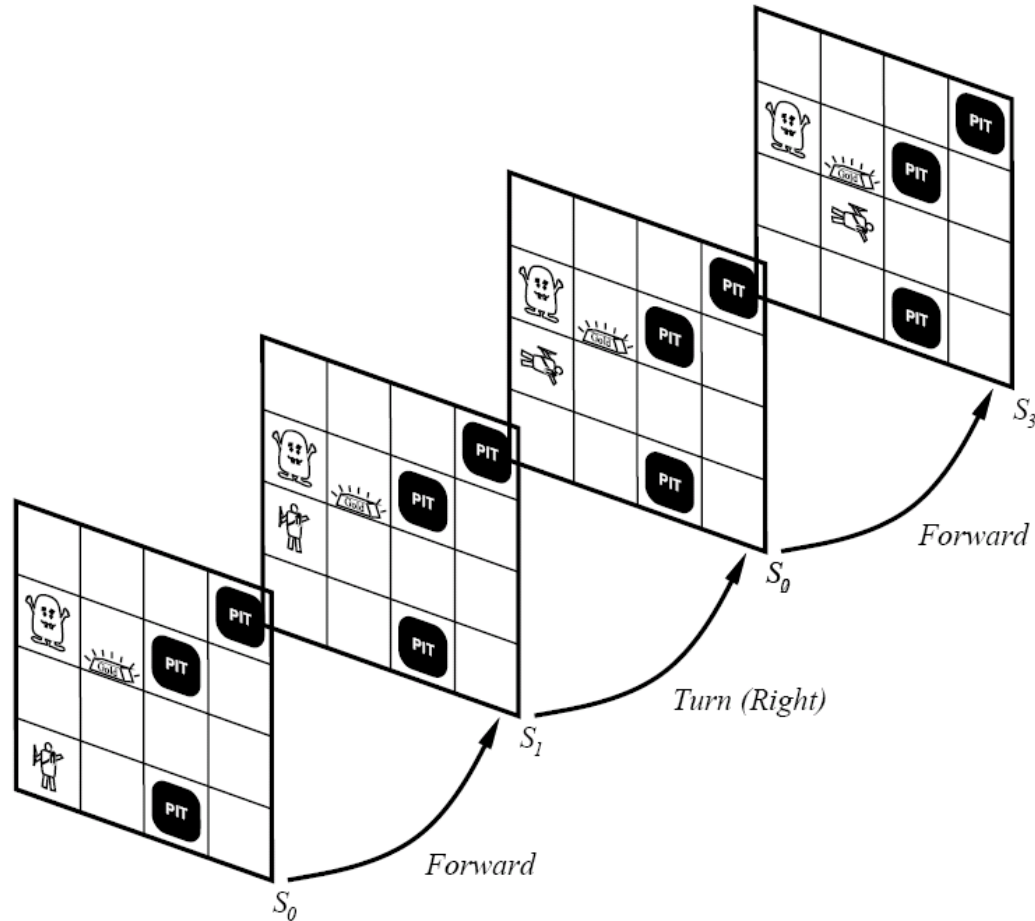


Fig 8.3, In situation calculus, the world is a sequence of situations linked by actions.

Deducing Hidden Properties of the World

(Situation: location + properties)

Once the agent knows where it is, it can associate properties, **situations (s)**, with the **places (l)**.

$$\forall l, s \text{ At}(\text{Agent}, l, s) \wedge \text{Breeze}(s) \Rightarrow \text{Breezy}(l)$$
$$\forall l, s \text{ At}(\text{Agent}, l, s) \wedge \text{Stench}(s) \Rightarrow \text{Smelly}(l)$$

It is useful to know if a place is breezy or smelly because we know that wumpus and the pits cannot move about.

Deducing Hidden Properties of the World

(Situation: location + properties)

2 main kinds of synchronic rules: (same world state(time))

- ▶ *Causal rules:* Causal rules reflect the assumed **direction of causality** in the world: some hidden property of the world **causes** certain percepts to be generated. E.g., we might have rules stating that squares adjacent to wumpuses are smelly and squares adjacent to pits are breezy:

$$\forall I_1, I_2, s \text{ At(Wumpus, } I_1, s) \wedge \text{Adjacent}(I_1, I_2) \Rightarrow \text{Smelly}(I_2)$$

$$\forall I_1, I_2, s \text{ At(Pit, } I_1, s) \wedge \text{Adjacent}(I_1, I_2) \Rightarrow \text{Breezy}(I_2)$$

?percepts

- Systems that **reason with causal rules** are called **model-based reasoning** systems, which help to understand the reasoning chain explicitly

Deducing Hidden Properties of the World

(Situation: location + properties)

Diagnostic rules: Diagnostic rules infer the presence of hidden properties **directly from the percept-derived information**.

$$\forall l, s \text{ At}(\text{Agent}, l, s) \wedge \text{Breeze}(s) \Rightarrow \text{Breezy}(l)$$

For deducing the presence of wumpuses, a **diagnostic rule** can only draw **weak conclusion**.

$$\forall l_1, s \text{ Breezy}(l_1) \Rightarrow \exists l_2 \text{ At}(\text{Pit}, l_2, s) \wedge \text{Adjacent}(l_1, l_2)$$

Symptoms (percepts) \Rightarrow diagnosis

- ▶ Bi-conditional sentence can be diagnostic and casual rules:

$$\forall s \text{ Breezy}(s) \Leftrightarrow \exists r \text{ Pit}(r) \wedge \text{Adjacent}(r, s)$$

Toward a Goal-Based Agent

- Once the **gold is found**, the aim now is to return to the start square as quickly as possible. We would like to infer that the agent now has the **goal** of being at **location [1,1]**:

$$\forall s \text{ Holding (Gold, } s) \Rightarrow \text{GoalLocation([1,1] ,} s \text{)}$$

- 3 ways to find the action sequence:
 - ▶ **Inference**: Write axioms that allow us to ASK the KB for a **sequence of actions** to achieve the goal safely.
 - ▶ **Search**: Use best-first search (Chap.4) to find a path to the goal.
 - ▶ **Planning**: Use special-purpose reasoning systems designed **to reason about action**.

Knowledge Engineering Process

1. Identify the task (problem specification/ definition)
2. Assemble the relevant knowledge – knowledge acquisition (system analysis)
3. Decide on a vocabulary of predicates, function and constants (design)
4. Encode general knowledge about the domain (coding)
5. Encode a description of the specific problem instance – precepts/ inputs to handle (coding sub-programs, I/O)
6. Pose queries to the inference procedure to get answer (testing & evaluation)
7. Debug the knowledge base (testing & evaluation)

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