Bayesian hypothesis testing
PSM 2

Bennett Kleinberg

5 March 2019

Probability, Statistics & Modeling II

Today

Bayesian statistics

- What is it? & How does it differ from NHST?
- What can it solve?
- Why should I care?
- How do I do it?

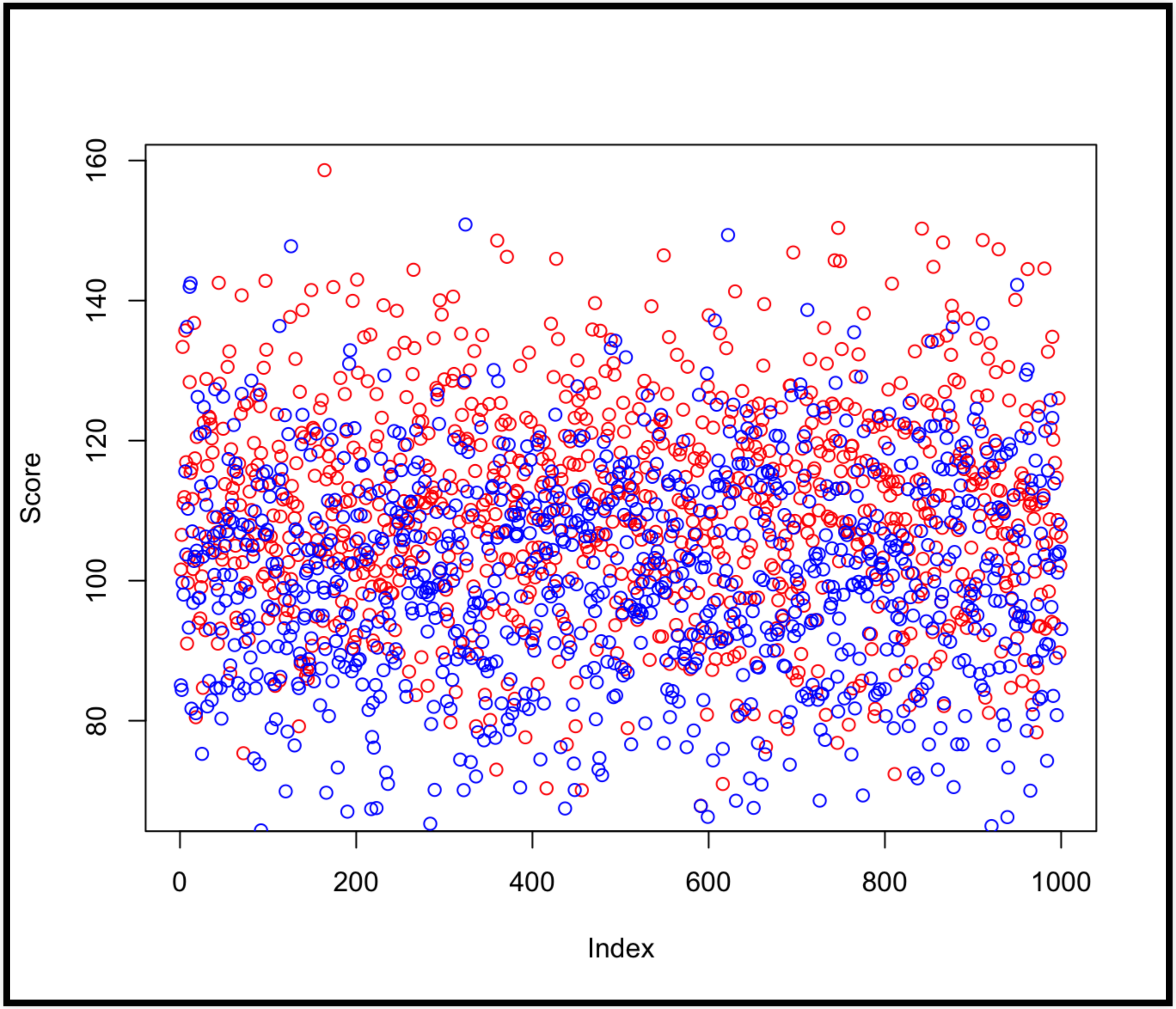
What is Bayesian Statistics?

What is it?

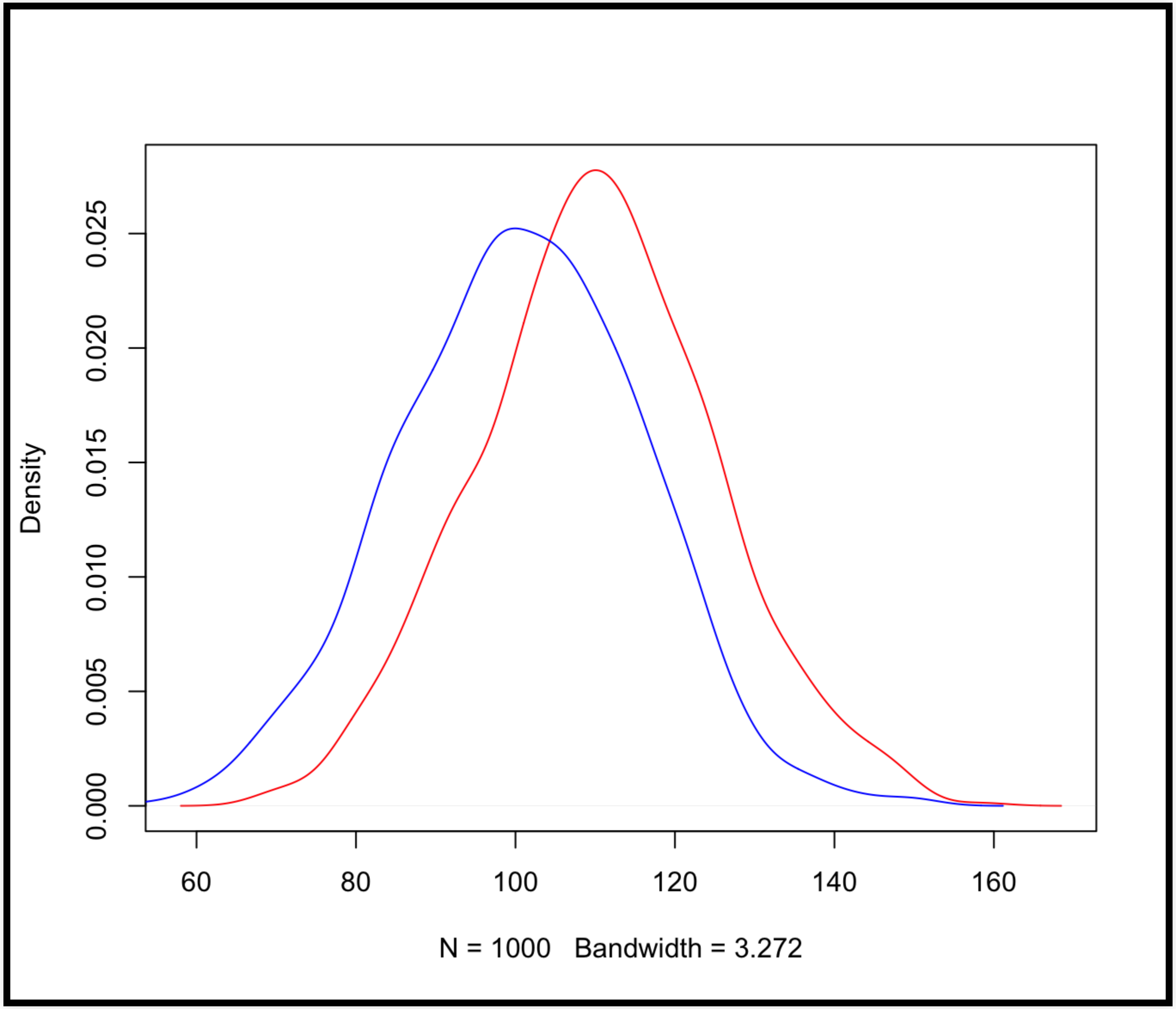
Rewind:

Null hypothesis significance testing

Hypothesis testing the old way



Hypothesis testing the old way



Hypothesis testing the old way

NULL hypothesis testing

- $H_0 : M_A \approx M_B$
  - there is no difference in the means between Group A and Group B
- $H_A : M_A \neq M_B$
  - there is a difference in the means between Group A and Group B

Directed hypotheses:

- $H_A : M_A > M_B$
- $H_A : M_A < M_B$

Hypothesis testing the old way

NULL hypothesis testing

Purpose:

- test whether the data allow us to reject H_0
- remember: rejecting $H_0 \neq$ accepting H_A
- remember: not rejecting $H_0 \neq M_A == M_B$
- obsession with the *p*-value

In fact: all we can ever say is whether H_0 was rejected or not!

There are more problems

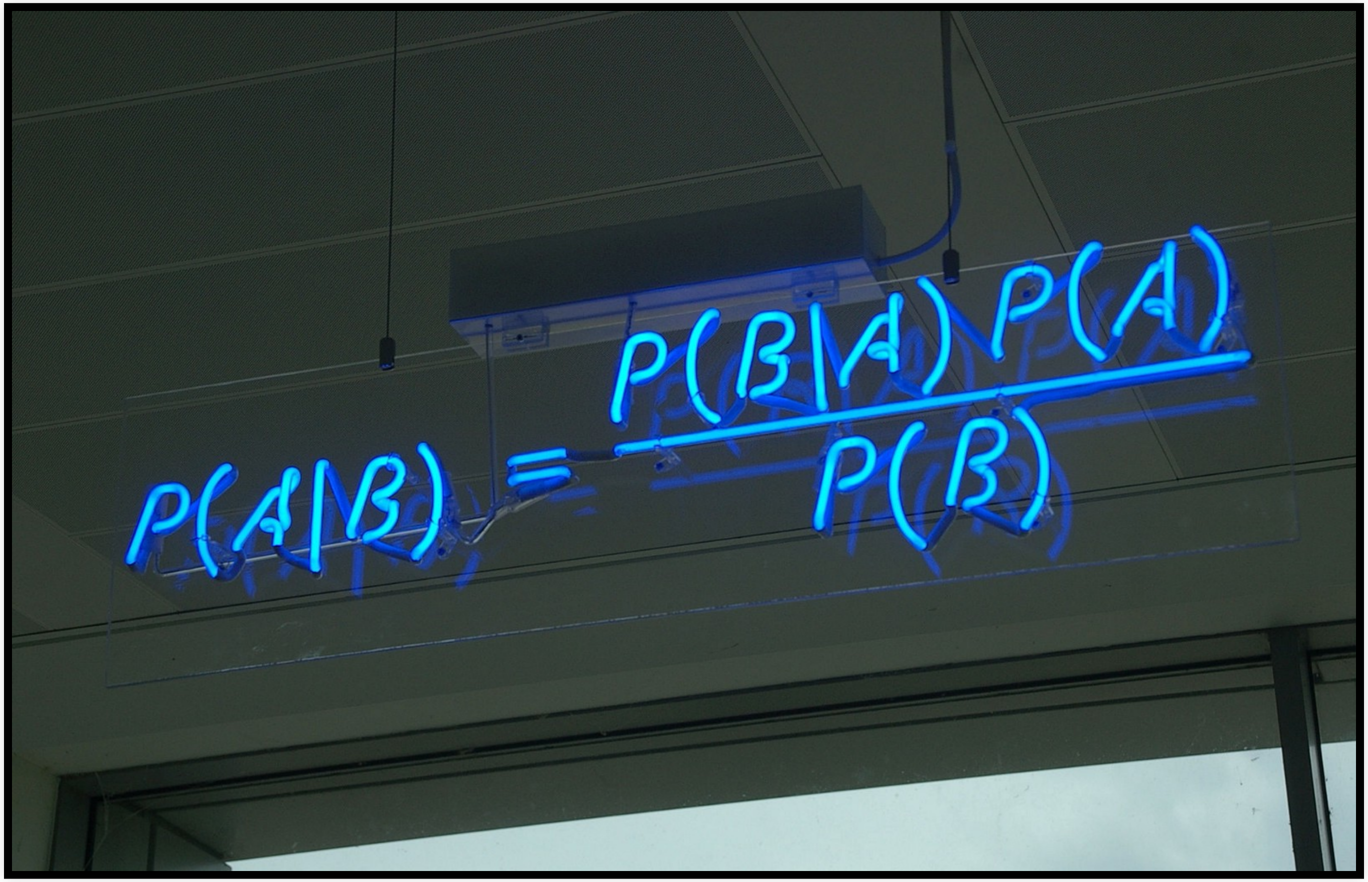
- we’re bad at interpreting NHST results
- strong assumptions about the data
- no stopping rule (increase *n* and everything becomes significant)

Hypothesis testing in general

- core of inference testing
- core of scientific endeavour
  - think of the ‘reproducibility crisis’
  - we want to avoid fishing expeditions

So: we deseperately need hypotheses, but NHST is weak

Enter



What is it?

Rooted in two ideas of probability:

Frequentist vs Bayesian

Laymen’s explanation

I have misplaced my phone somewhere in the home. I can use the phone locator on the base of the instrument to locate the phone and when I press the phone locator the phone starts beeping.

Problem: Which area of my home should I search?

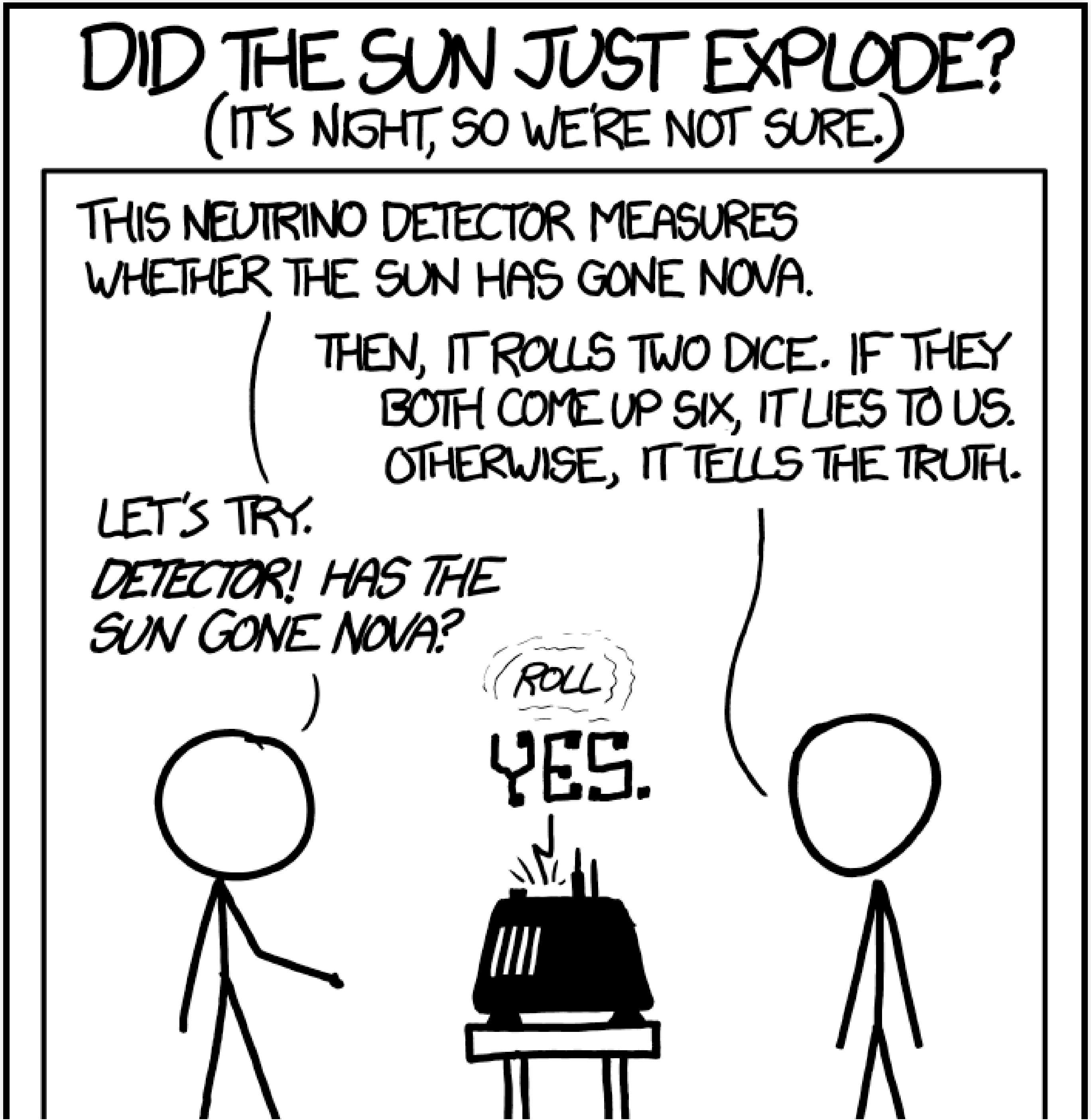
From this SO post

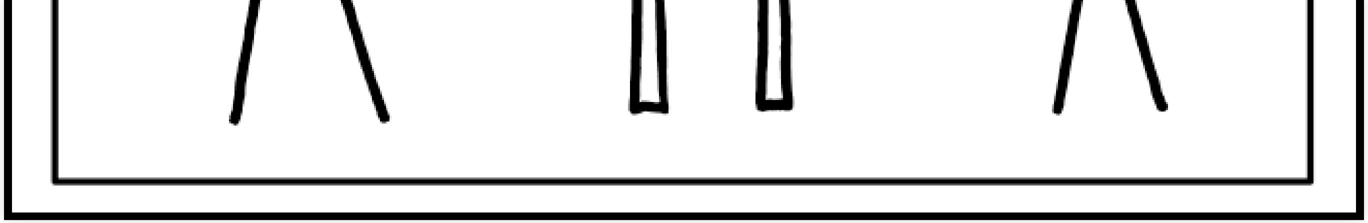
Frequentist Reasoning

I can hear the phone beeping. I also have a mental model which helps me identify the area from which the sound is coming. Therefore, upon hearing the beep, I infer the area of my home I must search to locate the phone.

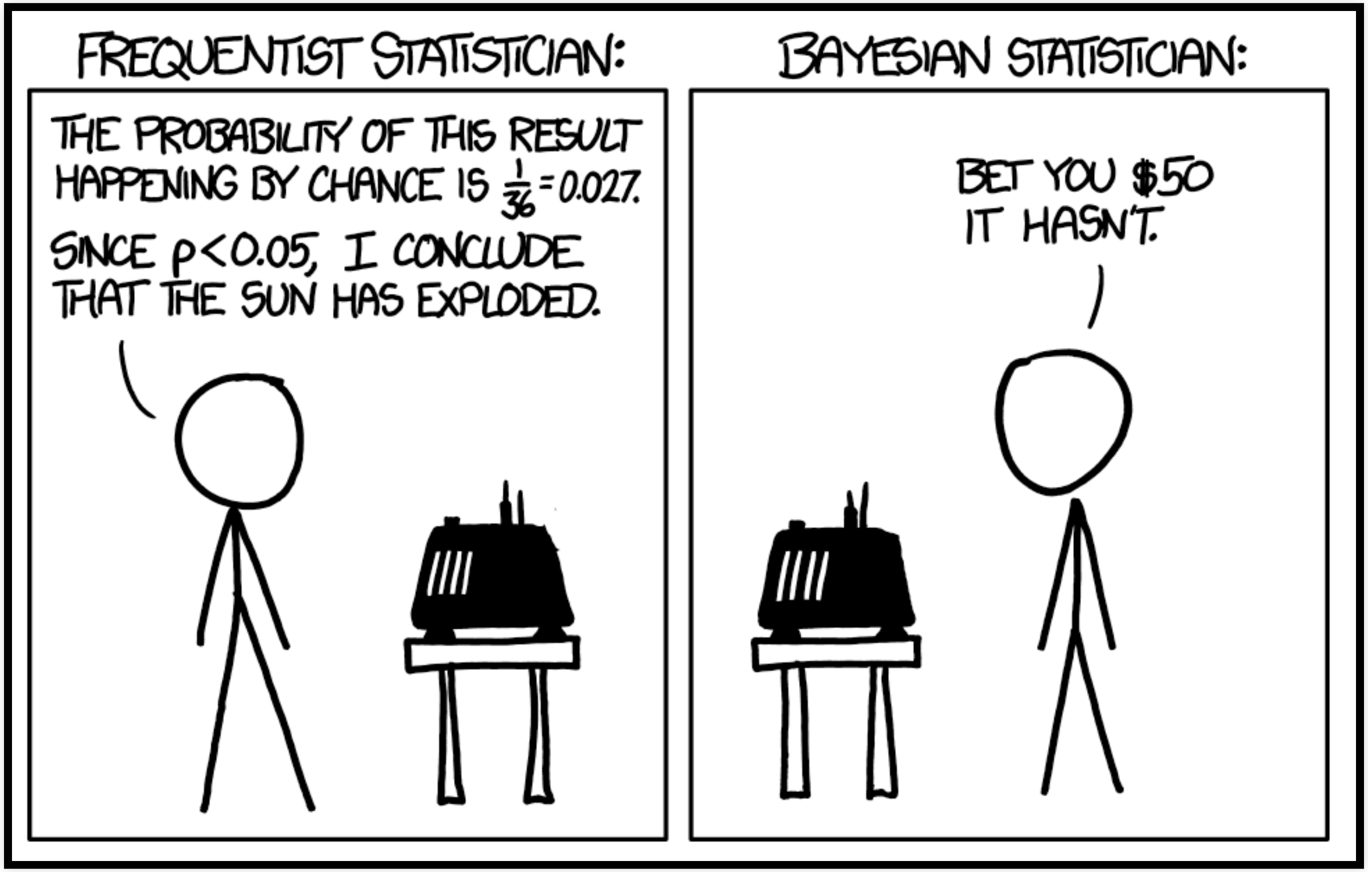
Bayesian Reasoning

I can hear the phone beeping. Now, apart from a mental model which helps me identify the area from which the sound is coming from, I also know the locations where I have misplaced the phone in the past. So, I combine my inferences using the beeps and my prior information about the locations I have misplaced the phone in the past to identify an area I must search to locate the phone.


DID THE SUN JUST EXPLODE?
(IT'S NIGHT, SO WE'RE NOT SURE.)
THIS NEUTRINO DETECTOR MEASURES WHETHER THE SUN HAS GONE NOVA.
THEN, IT ROLLS TWO DICE. IF THEY BOTH COME UP SIX, IT LIES TO US. OTHERWISE, IT TELLS THE TRUTH.
LET'S TRY.
DETECTOR! HAS THE SUN GONE NOVA?
ROLL
YES.



Img source


FREQUENTIST STATISTICIAN:
BAYESIAN STATISTICIAN:
THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS 1/36 = 0.027. SINCE p < 0.05, I CONCLUDE THAT THE SUN HAS EXPLODED.
BET YOU $50 IT HASN'T.

What is it?

Remember?

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)}$$

$$P(terrorist|alarm) = \frac{P(alarm|terrorist) * P(terrorist)}{P(alarm)}$$

Translated to hypothesis testing

- $P(H)$: prob. that hypothesis *H* **prior** to the data
- $P(D)$: marginal prob. of the data (same for all hyp.)
- $P(D|H)$: compatibility of the data with the hyp. (**likelihood**)

We want to know:

$P(H|D)$: prob. of the hyp. given the data (**posterior**)

$$P(H|D) = \frac{P(D|H) * P(H)}{P(D)}$$

$$posterior = \frac{likelihood * prior}{marginal}$$

Formally

Since: $P(D)$ does not involve the hypothesis, …

$$P(H|D) \propto P(D|H) * P(H)$$

Conceptually

$$posterior \propto likelihood * prior$$

- posterior: what we know after having seen the data (i.e. what we learned from the data)
- prior: our *prior* beliefs
- likelhood: observation

Think for a second

- this means that evidence can/must convince
- if you know that the sun is unlikely to have exploded, the evidence must be very, very strong to convince you otherwise

Bayesian inference: about updating beliefs with the data.

Bayesian hypothesis tests

If for any *H*:

$$P(H|D) \propto P(D|H) * P(H)$$

… then maybe we can compare the evidence $P(H_0|D)$ with the evidence $P(H_A|D)$?

Bayes factors

Important: no special status for H_0!

Suppose we have not seen the data, then:

$$odds_{0A} = \frac{P(H_0)}{P(H_A)}$$

or:

$$odds_{prior} = \frac{prior_{H_0}}{prior_{H_A}}$$

Bayes factors

What we want for two hypotheses H_0 and H_A is:

- $P(D|H_A)$: compatibility of the data with H_A
- … versus …
- $P(D|H_0)$: compatibility of the data with H_0

$$\frac{P(H_A|D)}{P(H_0|D)} = \frac{P(D|H_A)}{P(D|H_0)} * \frac{P(H_A)}{P(H_0)}$$

How much more likely the data are under H_A compared to H_0.

Called the Bayes Factor BF_{A0}

The evidence in the data favors one hypothesis, relative to another, exactly to the degree that the hypothesis predicts the observed data better than the other.

What is a Bayes factor? (Morey, 2014)

Stepwise example

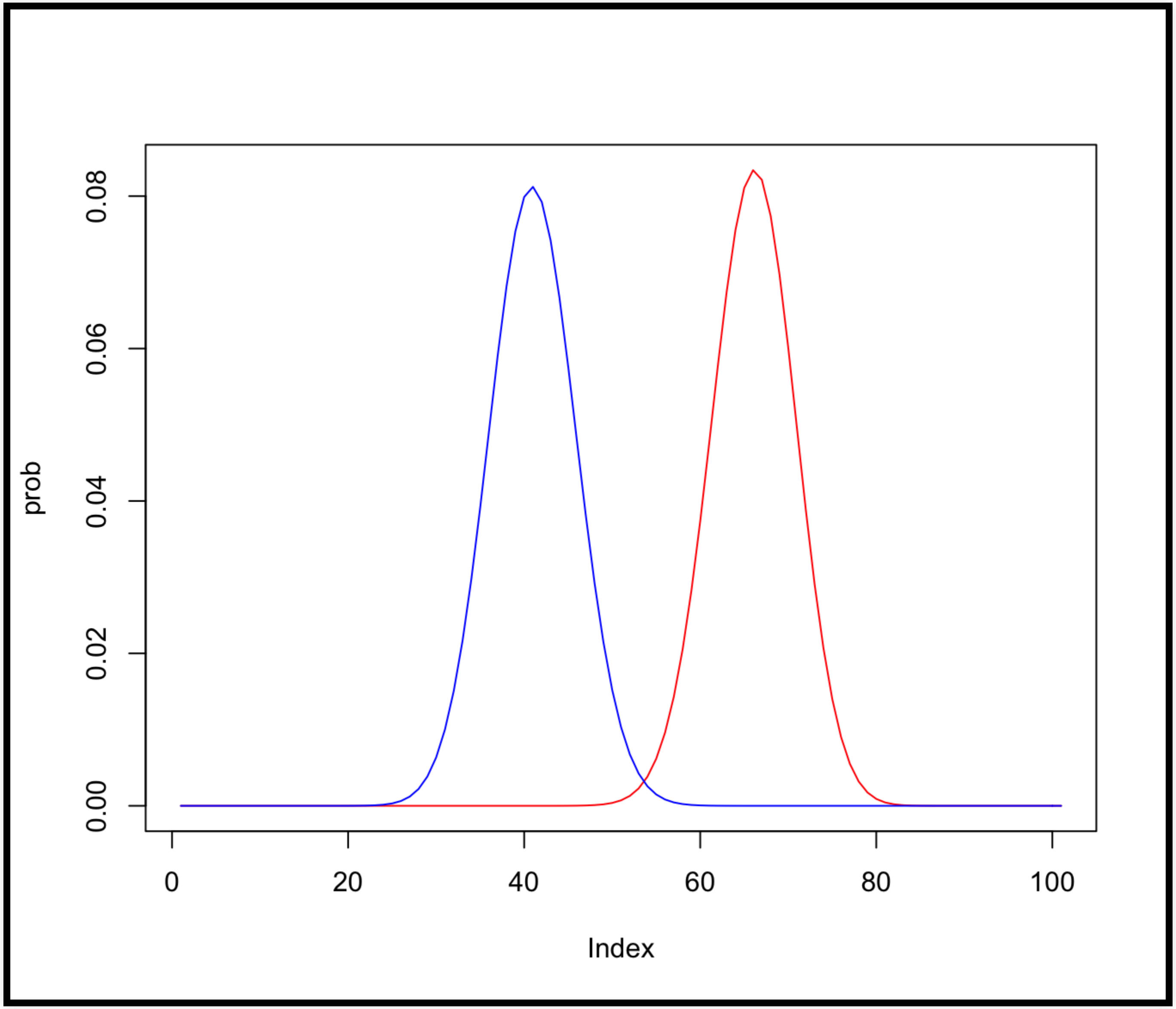
Suppose we have two lines of thought re. successful re-integration after prison:

- Optimists
- Skeptics

Optimists say that 65% of offenders can be re-integrated in society; skeptics say it's 40%.

Data: 100 offenders and their outcome (successful vs fail)

- $H_{optimists} = 0.65$
- $H_{skeptics} = 0.40$



0.00
0.02
0.04
0.06
0.08
prob
0
20
40
60
80
100
Index

Now the data come in

- 100 ex-prisoners
- 58 successfully re-integrated in society
- 42 not
- $58/100 = 0.58$

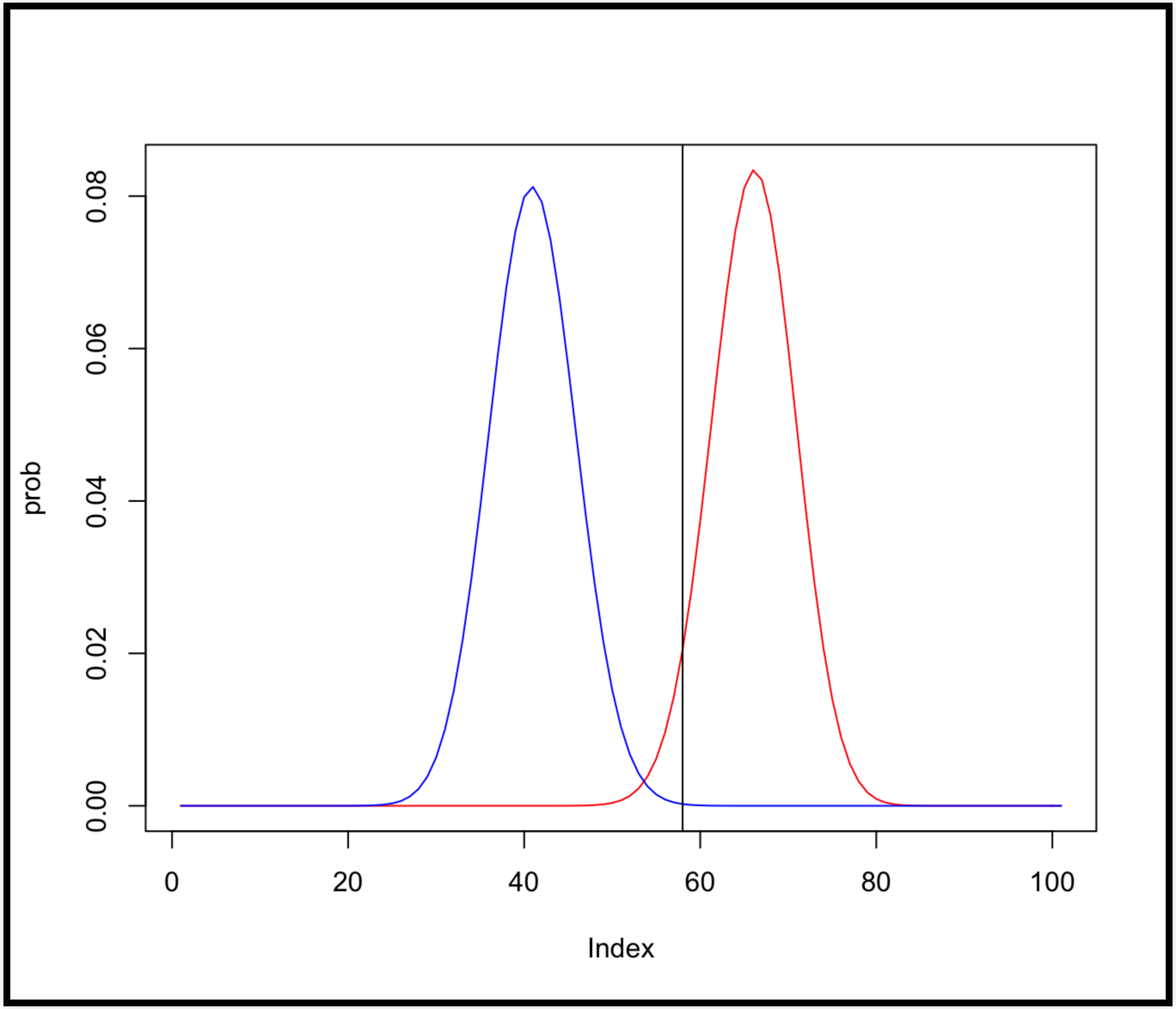
Closer to the optmists, but how much?

Relative weight of evidence

How much does the evidence change our beliefs?

Plausibility of the hypotheses $H_{opt.} = 0.65$ and $H_{skept.} = 0.40$ changes according to Bayes' rule!

Probability of observations



Probability of observations

- 58 successes:
- for $H_{opt.} = 0.65$ = 0.0284
- for $H_{skept.} = 0.40$ = 0.0001

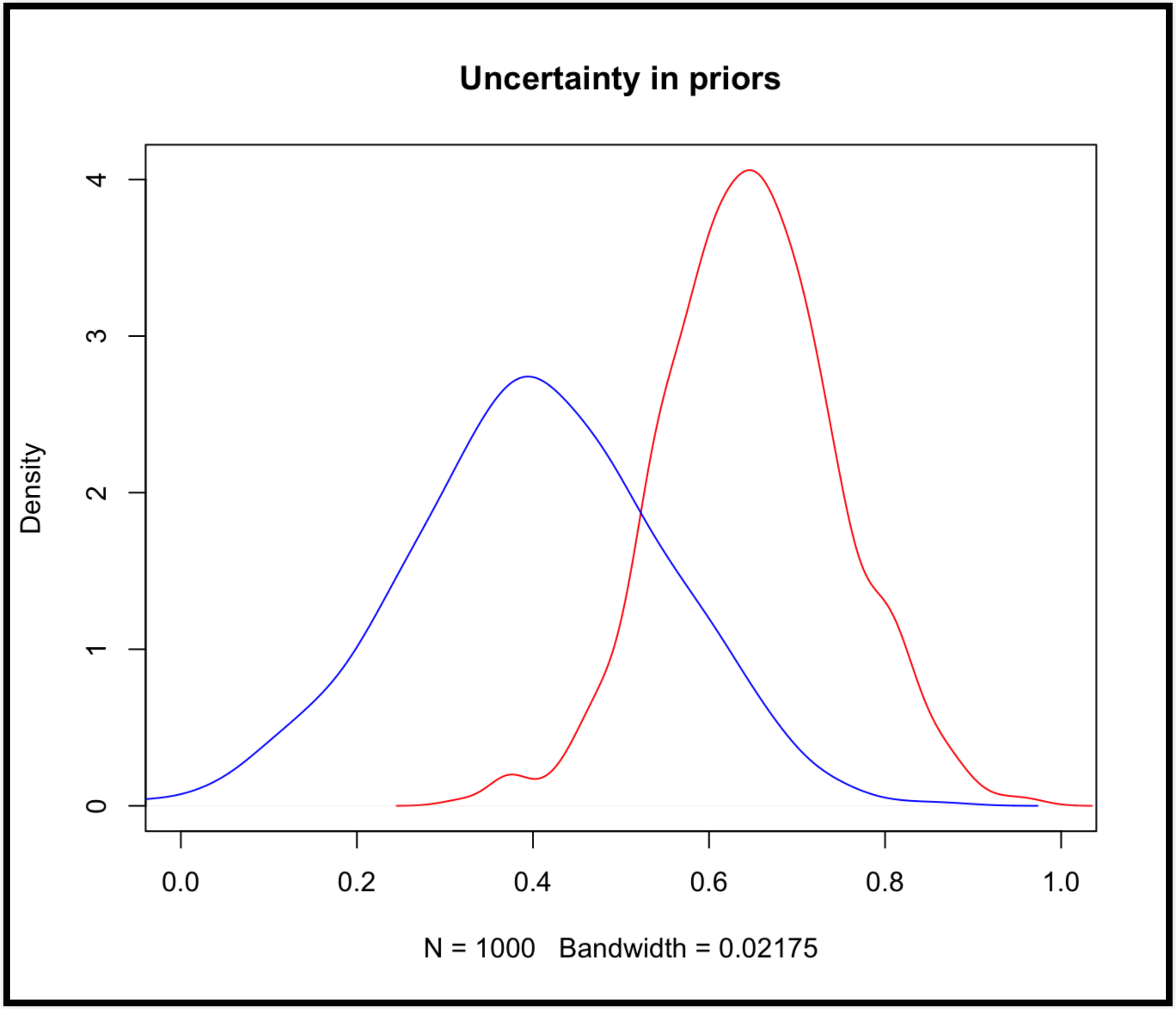
So: $\frac{H_{opt.}}{H_{skept.}} = \frac{0.0284}{0.0001} = 250.03$

Bayes factor

$$BF = \frac{P(D|H_{opt.})}{P(D|H_{skept.})} = 250.03$$

The data are 250 times more likely under $H_{opt.}$ than under $H_{skept.}$

But what about uncertain priors?


Uncertainty in priors
Density
0
1
2
3
4
0.0
0.2
0.4
0.6
0.8
1.0
N = 1000 Bandwidth = 0.02175

Prior beliefs as distributions

- rather than specific point estimates, we use distributions
- $H_{optimists}$ becomes a distribution (here normal distr.)
- $H_{skeptics}$ becomes a distribution (here normal distr.)

Bayesian estimation can handle this.

What can it solve?

What can it solve?

- all hypothesis testing questions!
- those with uncertainty
- aaaaand ….

What can it solve?

It can solve the sh** H_0 problem!!!!!

Now we can quantify relative evidence:

$$BF_{01} = \frac{P(H_0|D)}{P(H_1|D)}$$

Relative evidence of H_0 over H_1

Why should I care?

Why should I care?

Original Articles

Why Isn't Everyone a Bayesian?

B. Efron

Pages 1-5 | Received 01 Jul 1985, Published online: 27 Feb 2012

Efron, 1985

Why should I care?

- Bayesian framework widely considered superior
- Bayesian logic fits with “science” better than NHST
- tools problem is overcome
- will become standard in the future

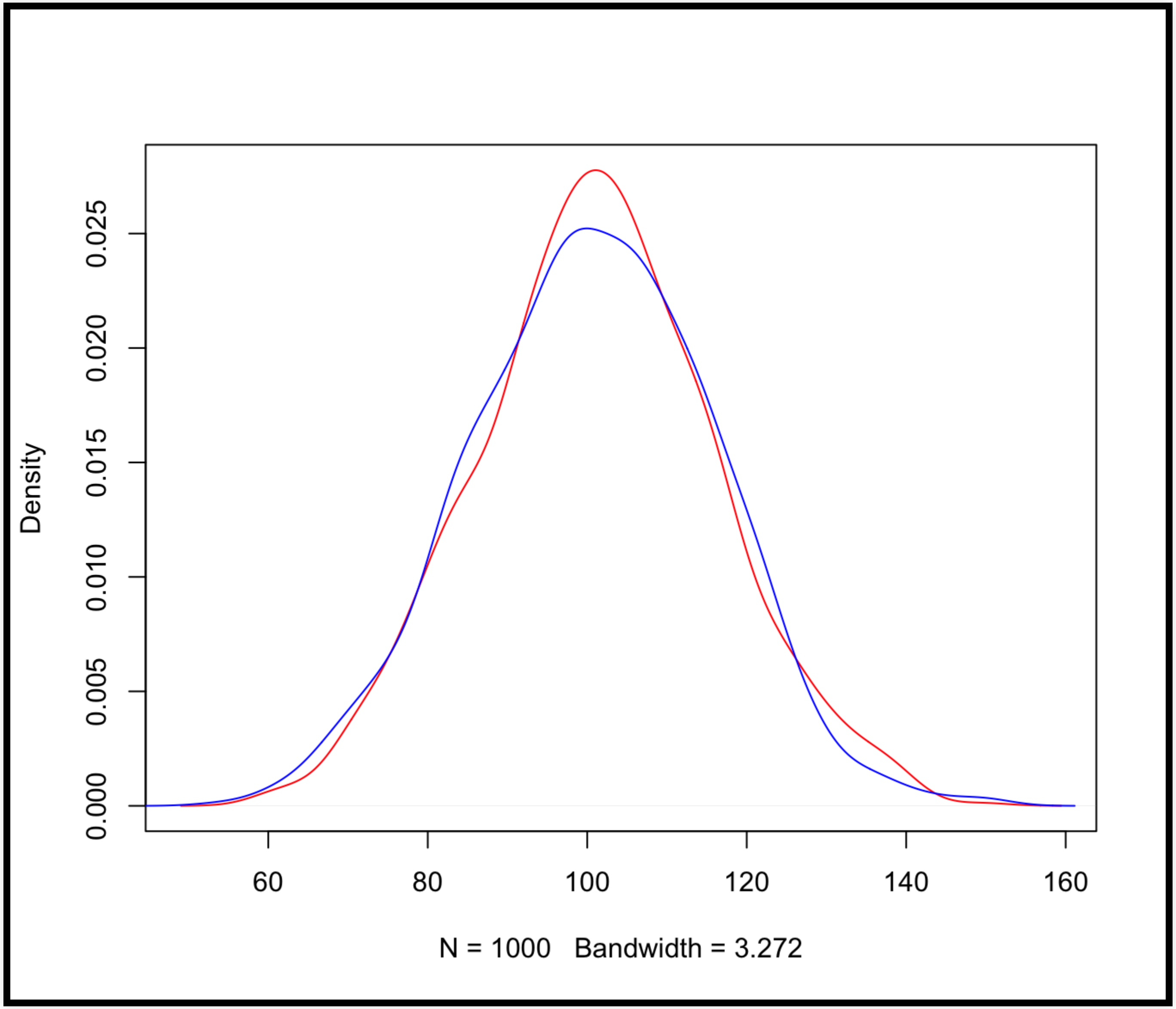
How do I do it?

How do I do it?

Two approaches in PSM2:

- the `BayesFactor` R package
- JASP

How do I do it?


Density
0.000
0.005
0.010
0.015
0.020
0.025
60
80
100
120
140
160
N = 1000 Bandwidth = 3.272

Is there a difference?

```r
tapply(mydata$score, mydata$group, mean)
```

```
##        A        B
## 101.2419 100.6370
```

- $H_0 : M_A \approx M_B$
- $H_1 : M_A \neq M_B$

Old school NHST

```
t.test(score ~ group
       , data = mydata
       , var.eq=TRUE)
```

```
##
##  Two Sample t-test
##
## data:  score by group
## t = 0.90114, df = 1998, p-value = 0.3676
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -0.7115953  1.9214737
## sample estimates:
## mean in group A mean in group B
##        101.2419        100.6370
```

Effect size

Cohen's *d*

```
d = 0.90*(sqrt(1/1000 + 1/1000))
d
```

```
## [1] 0.04024922
```

BayesFactor R

```r
library(BayesFactor)
ttestBF(formula = score ~ group
       , data = mydata)
```

```
## Bayes factor analysis
## --------------
## [1] Alt., r=0.707 : 0.07520633 ±0%
##
## Against denominator:
##   Null, mu1-mu2 = 0
## ---
## Bayes factor type: BFindepSample, JZS
```

Reference

BayesFactor R

$BF_{10} = 0.075,$ so:

$$BF_{01} = 1/0.075 = 13.33$$

–> Evidence quantified for both hypotheses!

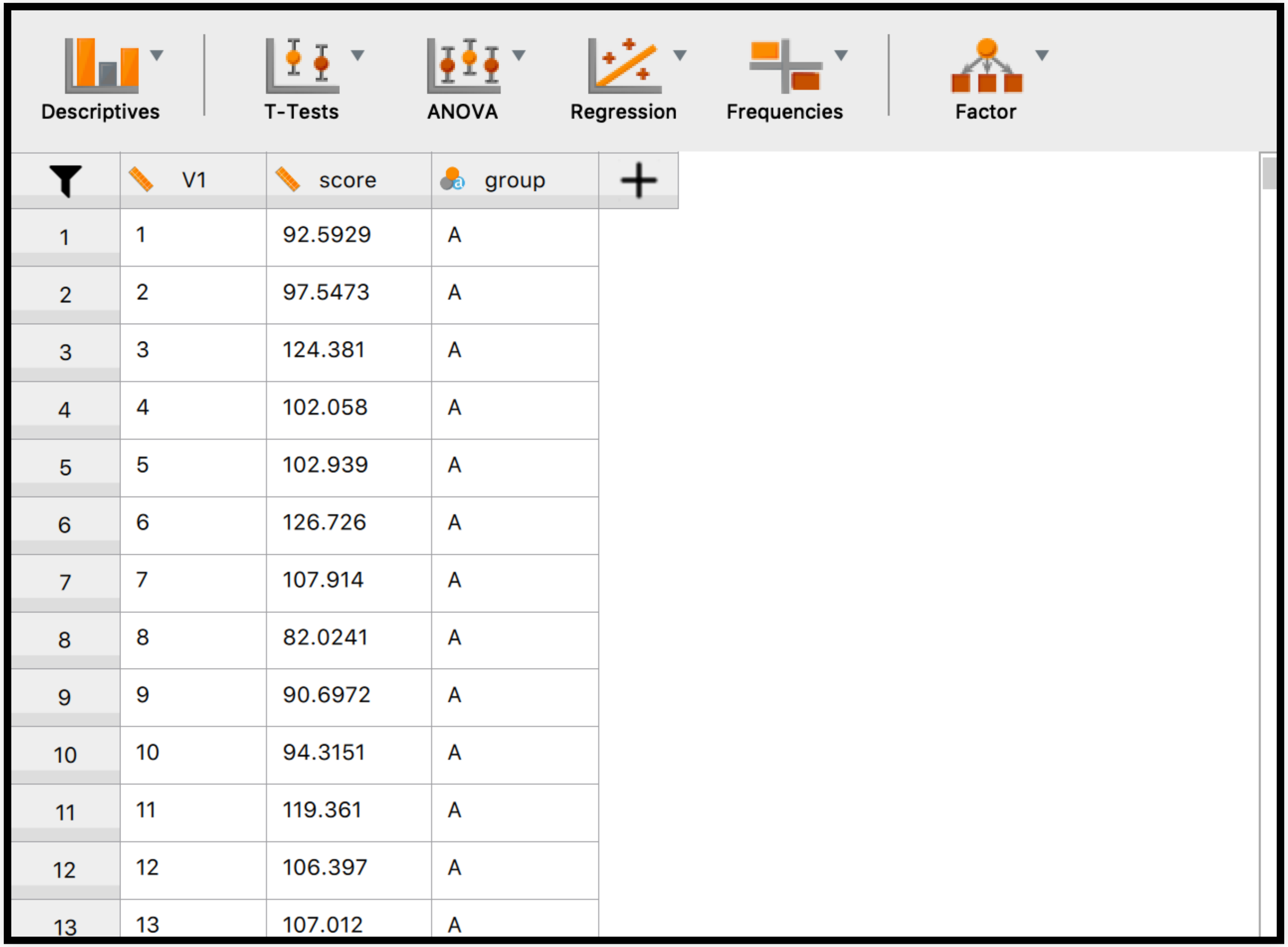
How do I do it?

JASP

JASP

A Fresh Way to
Do Statistics

JASP



Descriptives
T-Tests
ANOVA
Regression
Frequencies
Factor

	V1	score	group
1	1	92.5929	A
2	2	97.5473	A
3	3	124.381	A
4	4	102.058	A
5	5	102.939	A
6	6	126.726	A
7	7	107.914	A
8	8	82.0241	A
9	9	90.6972	A
10	10	94.3151	A
11	11	119.361	A
12	12	106.397	A
13	13	107.012	A

JASP

Independent Samples T-Test

Independent Samples T-Test

	t	df	p	Cohen's d
score	0.901	1997	0.368	0.040

Note. Welch's t-test.

JASP

Bayesian Independent Samples T-Test ▼

Bayesian Independent Samples T-Test

	BF_{01}	error %
score	13.29	1.078e −5

Psychonomic Bulletin & Review
February 2018, Volume 25, Issue 1, pp 219–234 | Cite as

How to become a Bayesian in eight easy steps: An annotated reading list

Authors   Authors and affiliations

Alexander Etz, Quentin F. Gronau, Fabian Dablander, Peter A. Edelsbrunner, Beth Baribault

Etz et al. 2018

Tutorial

- Bayesian hypothesis testing in practice
- use R and JASP

Next week

- Q&A session
- final questions before exam

END