INTRODUCTION TO ARTIFICIAL INTELLIGENCE

BI-ZUM / BIE-ZUM

QUANTUM ANNEALING

Prof. RNDr. **PAVEL SURYNEK**, Ph.D.

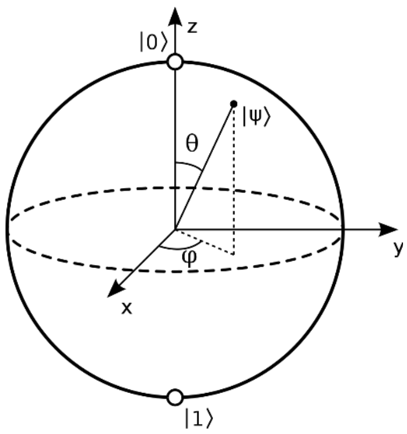
KAM FIT ČVUT

pavel.surynek@fit.cvut.cz

http://users.fit.cvut.cz/~surynek

http://courses.fit.cvut.cz/bi-zum

QUANTUM COMPUTERS

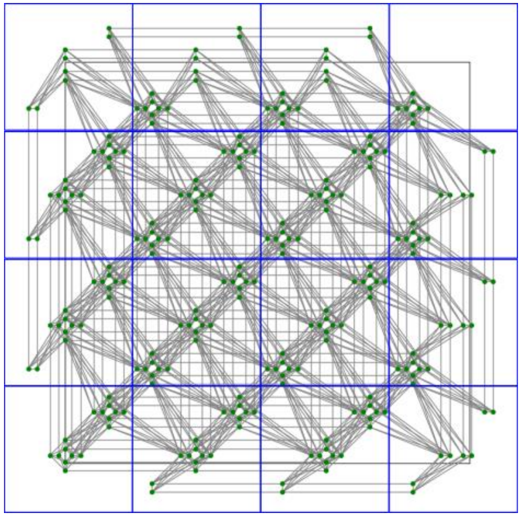


- Basic concept: **qubit**
  - superposition of two basic states |0⟩ and |1⟩
  - qubit state |ψ⟩ = α|0⟩ + β|1⟩, where are α and β complex numbers
  - probability of |0⟩ after measurement is |α|, probability of |1⟩ is |β|

- **Major paradigms**
  - gate based
    - quantum gates – produce qubits from input qubits
  - adiabatic quantum computing
    - quantum annealing
  - and more…

Operator	Gate(s)		Matrix
Pauli-X (X)	X		$\begin{bmatrix}0 & 1\\1 & 0\end{bmatrix}$
Pauli-Y (Y)	Y		$\begin{bmatrix}0 & -i\\i & 0\end{bmatrix}$
Pauli-Z (Z)	Z		$\begin{bmatrix}1 & 0\\0 & -1\end{bmatrix}$
Hadamard (H)	H		$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}$
Phase (S, P)	S		$\begin{bmatrix}1 & 0\\0 & i\end{bmatrix}$
$\pi/8$ (T)	T		$\begin{bmatrix}1 & 0\\0 & e^{i\pi/4}\end{bmatrix}$
Controlled Not (CNOT, CX)			$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{bmatrix}$
Controlled Z (CZ)	Z		$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{bmatrix}$
SWAP			$\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$
Toffoli (CCNOT, CCX, TOFF)			$\begin{bmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{bmatrix}$

ADIABATIC QUANTUM COMPUTERS



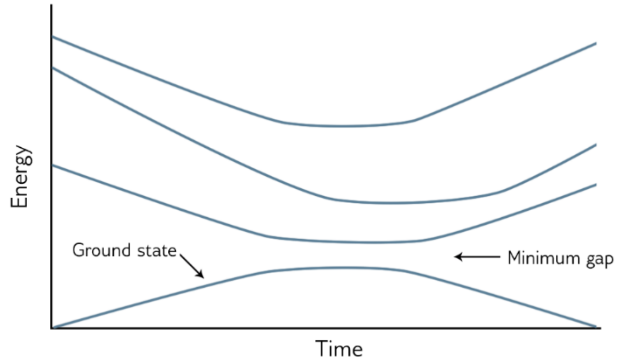
- **Limited number of qubits** (a few 1000s)
  - many more than gate-based model (a few 100s)
- **Limited connectivity**
  - hardware does not implement a fully connected graph
  - chained qubits (use extra qubits)
    - 5000 qubits and current topology can implement a fully connected graph with 177 qubits
- **But simple: one sample** – annealing, readout, cooling

ADIABATIC QUNTUM COMPUTATION

- **Adiabatic computation**

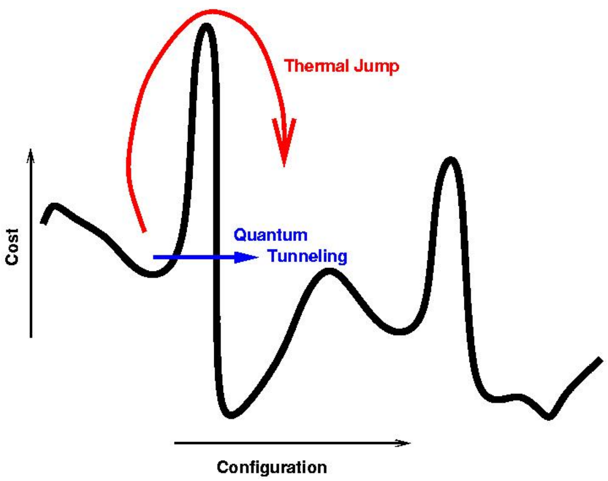
$$\mathcal{H}_{ising} = \underbrace{-\frac{A(s)}{2}\left(\sum_i \hat{\sigma}_x^{(i)}\right)}_{\text{Initial Hamiltonian}} + \underbrace{\frac{B(s)}{2}\left(\sum_i h_i \hat{\sigma}_z^{(i)} + \sum_{i>j} J_{i,j}\hat{\sigma}_z^{(i)}\hat{\sigma}_z^{(j)}\right)}_{\text{Final Hamiltonian}}$$

- By varying A(s) and B(s) over time
  - 0 ↗ 1, 1 ↘ 0
- The system moves from initial Hamiltonian and stays in the ground state (minimal energy)



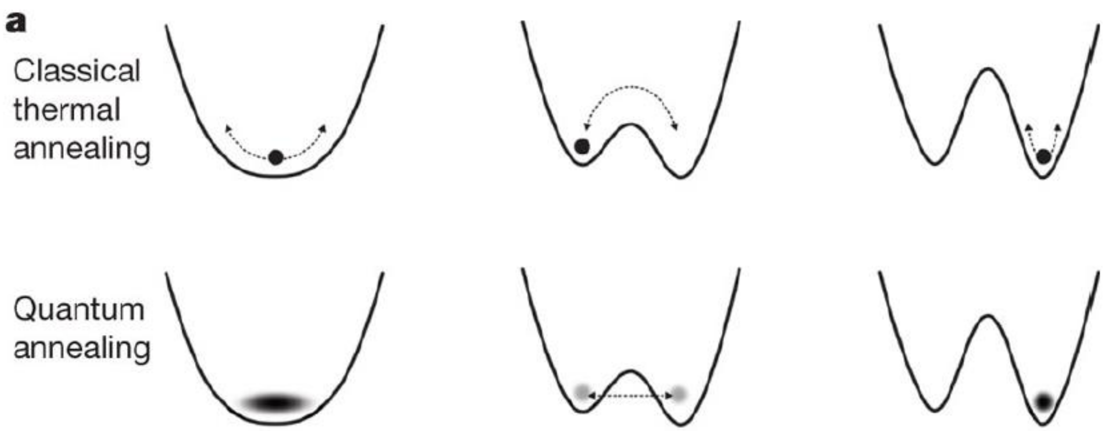
QUANTUM ANNEALING

- **Adiabatic computation** can be interpreted as quadratic optimization problem
  - n Boolean variables $x_1, x_2, \ldots, x_n$
    - decision variables (we want to determine their values)
  - minimize
    $\sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} a_i' x_i^2 + \sum_{i,j} a_{ij} x_i x_j$
  - $a_i, a_{i'}, a_{ij}$ are real-valued coefficients
- **Equivalent to:**
  - minimize $\sum_{i \leq j} q_{ij} x_i x_j$ (QUBO)
    - using matrices min $x^T Q x$
  - can be solved by classical simulated annealing but adiabatic computation can utilize quantum tunneling effect
  - $q_{ij} \neq 0$ requires hardware connection between qubits i and j



QUANTUM EFFECTS

- Temperature corresponds to the strength of tunneling effect
  - any height can be crossed
  - but the tunnel can be only of limited length depending on the temperature
  - the temperature can change the landscape
- If a certain cooling scheme is followed, then the process can reach the global optimum thanks to tunneling effect



PROBLEM COMPILATION TO **QUBO**

- **QUBO:** Quadratic Unconstrained Binary Optimization
  - minimize $\sum_{i \le j} q_{ij} x_i x_j$
- How to express constraints in an unconstrained formalism?
- Use **penalties**.
  - Examples:
    - (pseudo-Boolean) constraint $x+y \le 1$, penalty: xy
    - constraint $x = y$, penalty: $x+y - 2xy$
    - implication $x \Rightarrow y$, equivalent to $x \le y$, penalty: $x - xy$
  - Penalties are compositional – addition, penalty coefficients
- More complex compilation, **one-hot encoding**
  - n Boolean variables $x_1, x_2, \ldots, x_n$, exactly one has value 1
  - (pseudo-Boolean) constraint $\sum_{i=1}^{n} x_i = 1 \Leftrightarrow \sum_{i=1}^{n} x_i - 1 = 0$, penalty: $(\sum_{i=1}^{n} x_i - 1)^2 \Leftrightarrow -\sum_{i=1}^{n} x_i + 2 \sum_{i<j} x_i x_j$