# Model-Based RL I

## Basic Components of MBRL

Kurtland Chua

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- How do I used a learned model to solve the RL problem?

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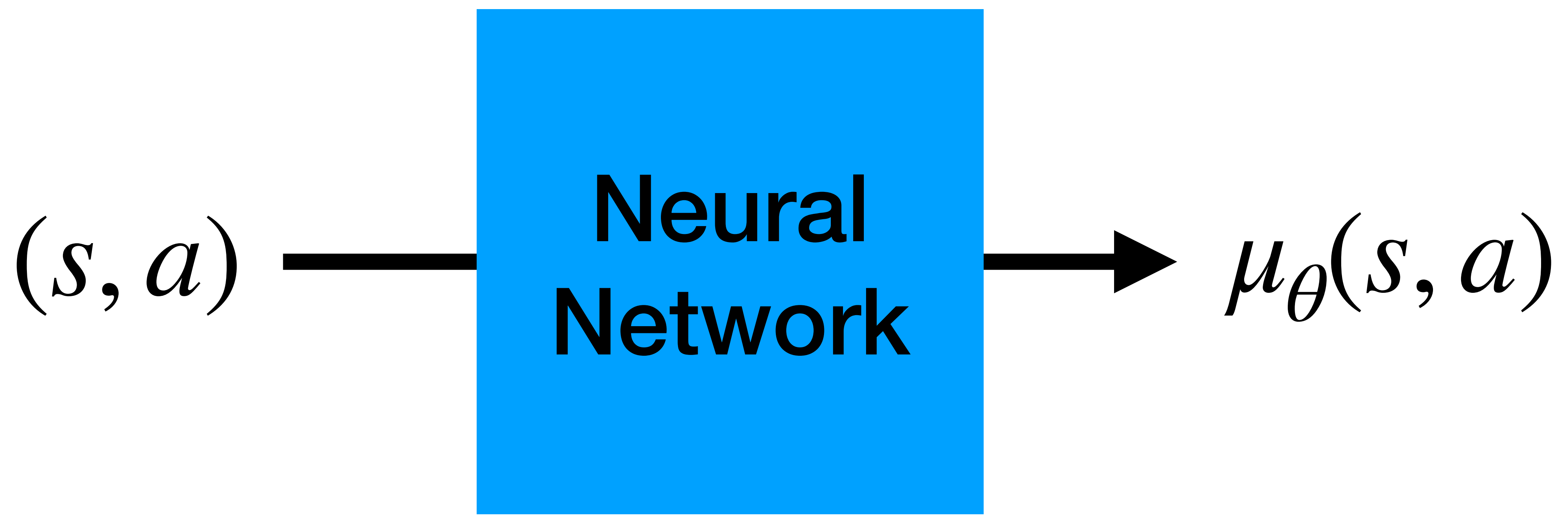

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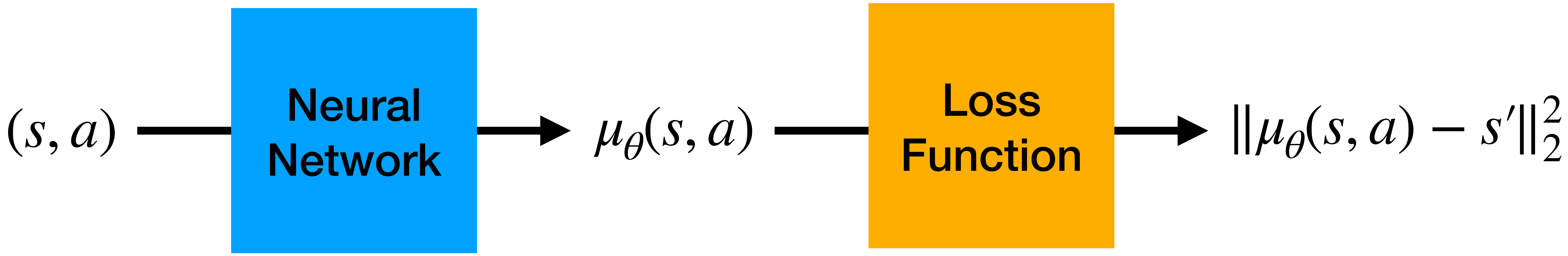

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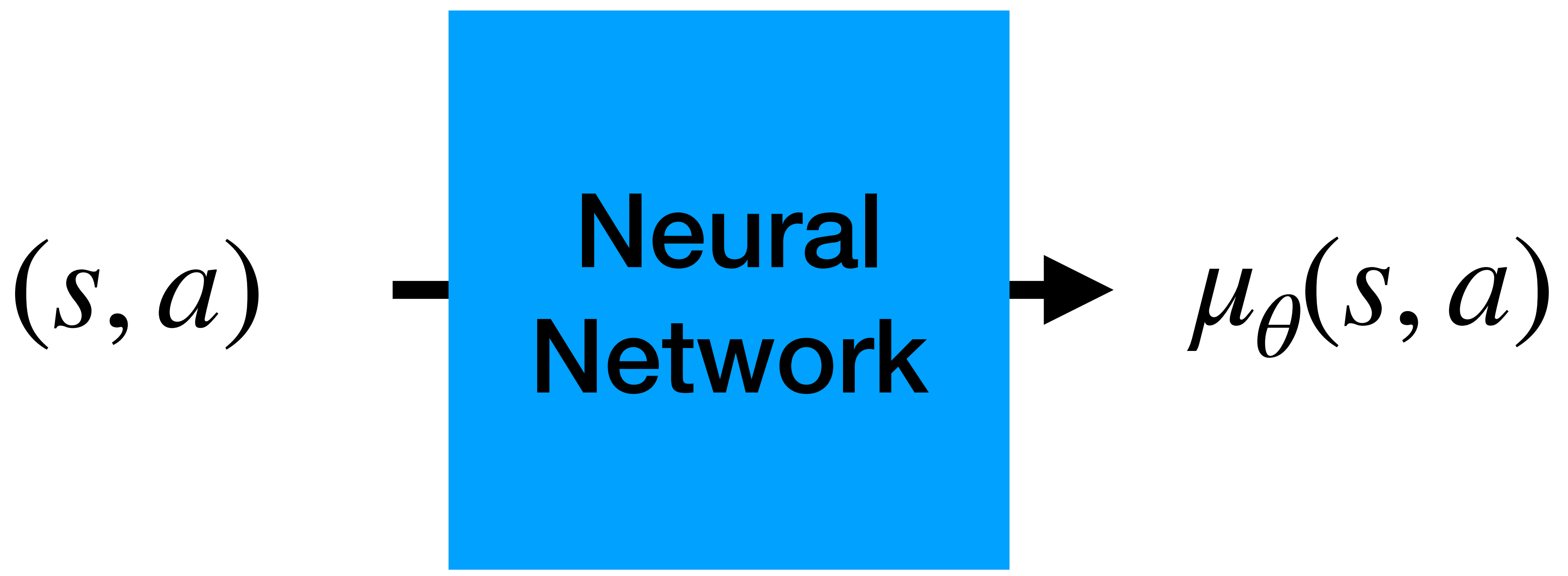

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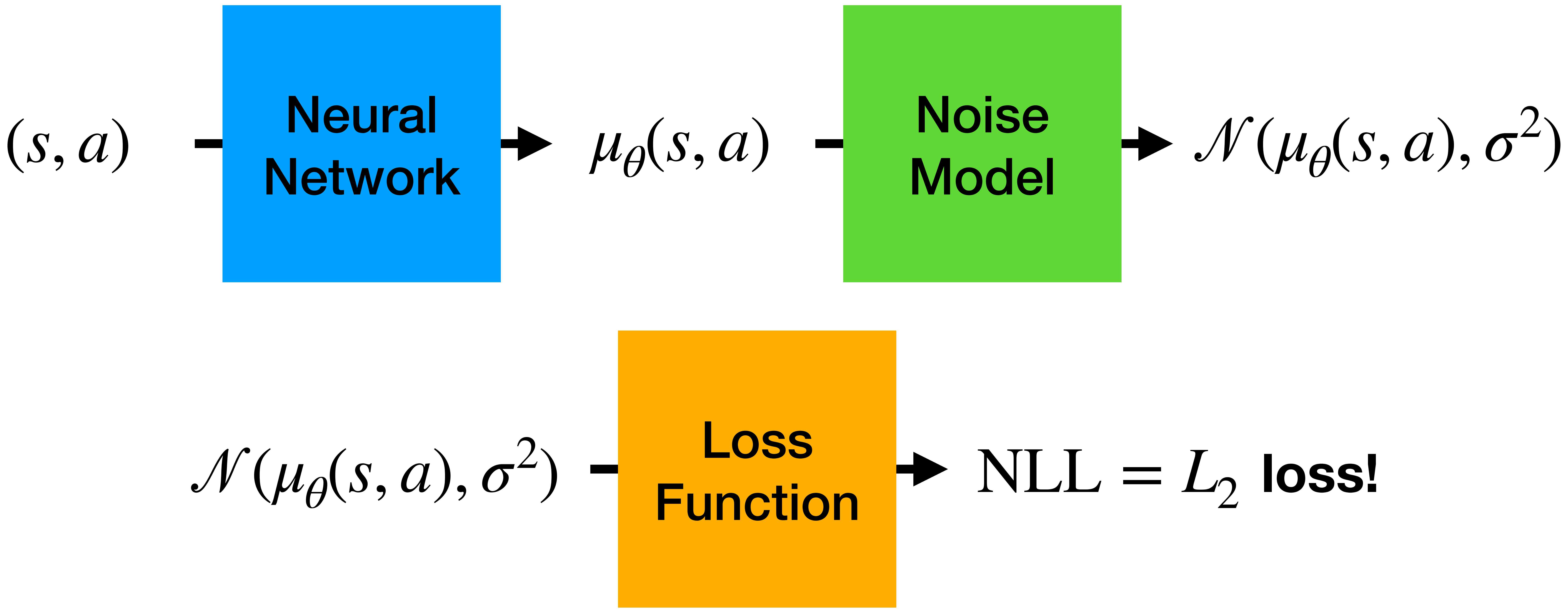

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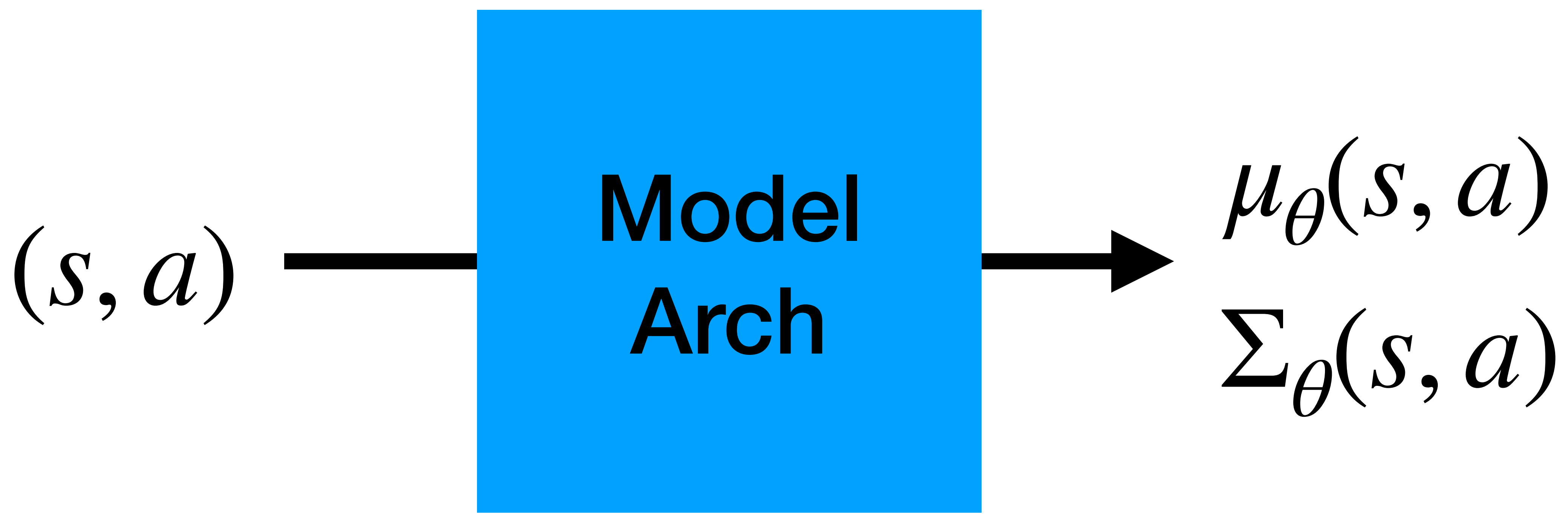

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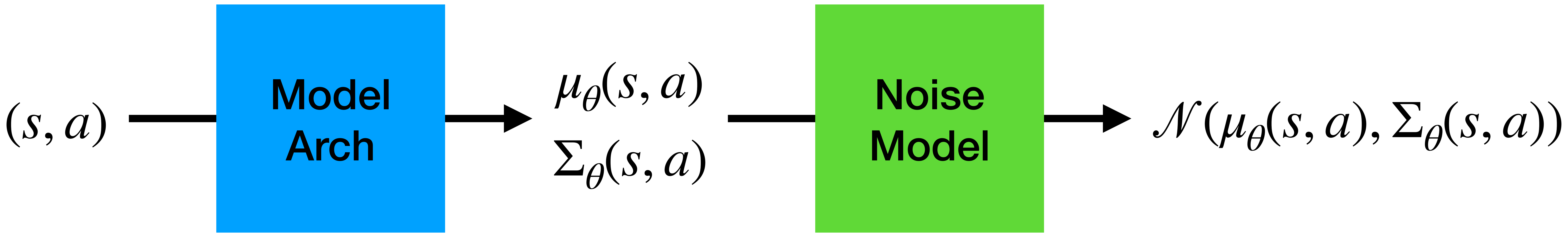

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    - Yes! **But not the one we necessarily care about!**

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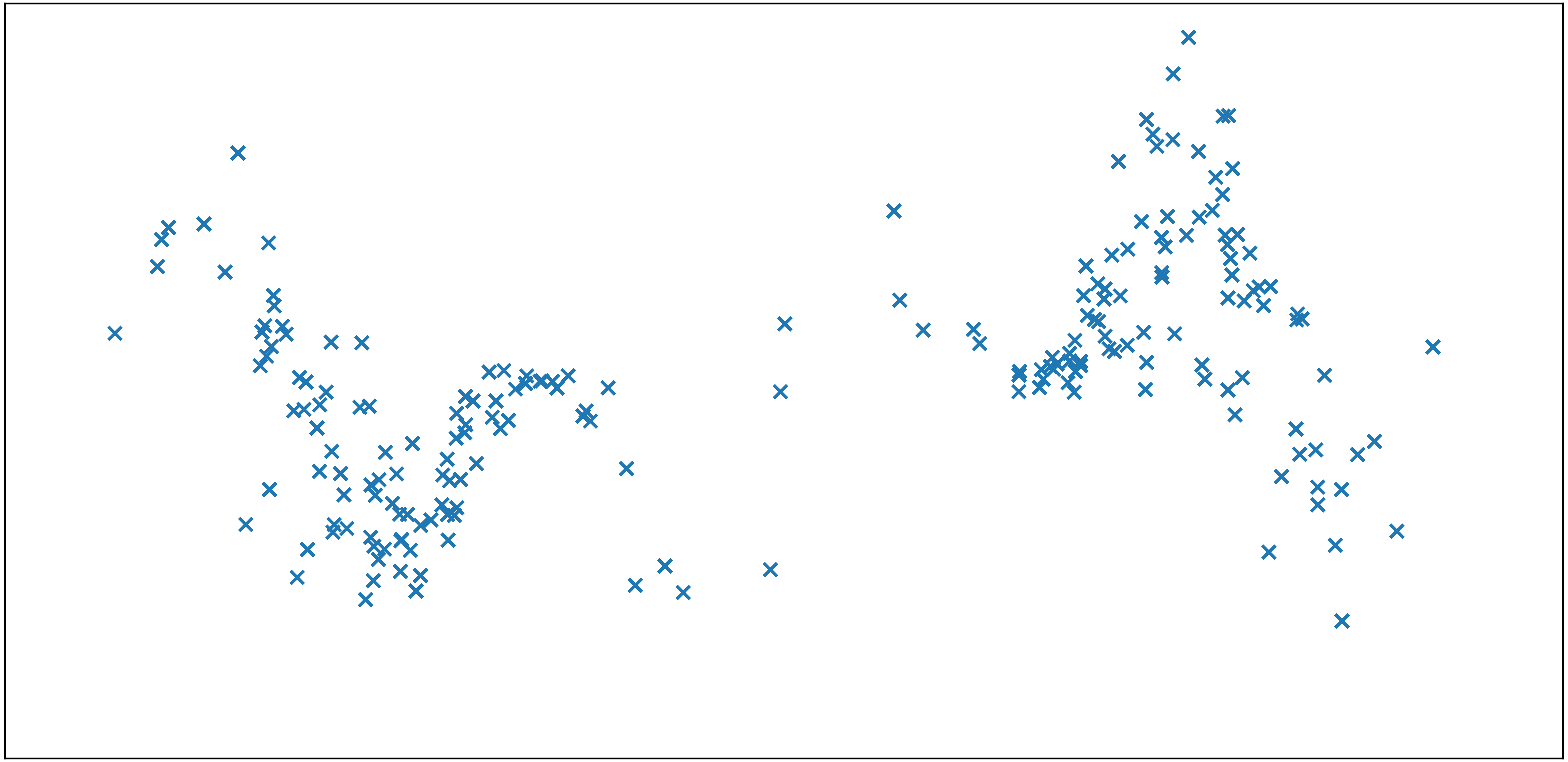

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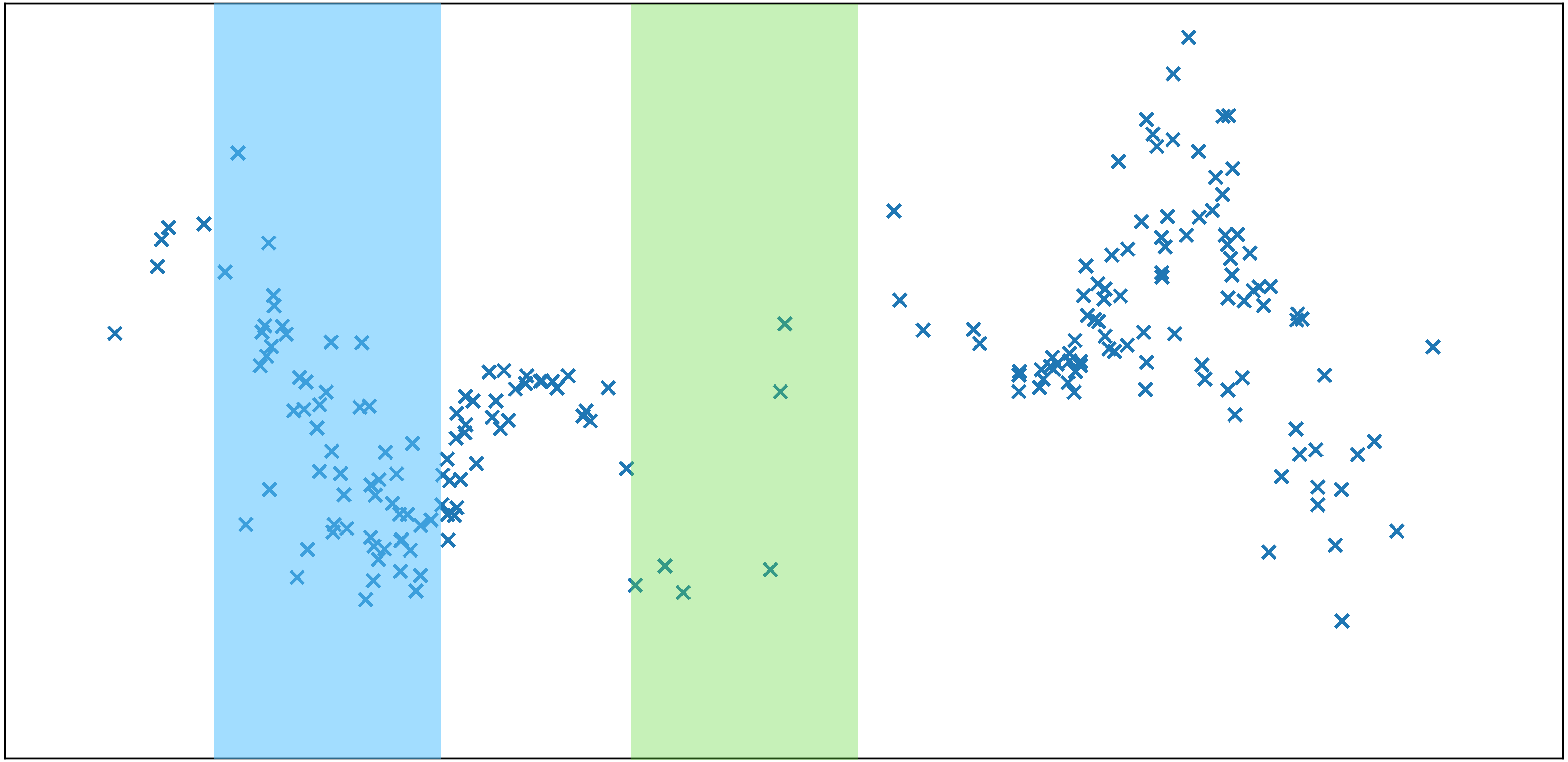

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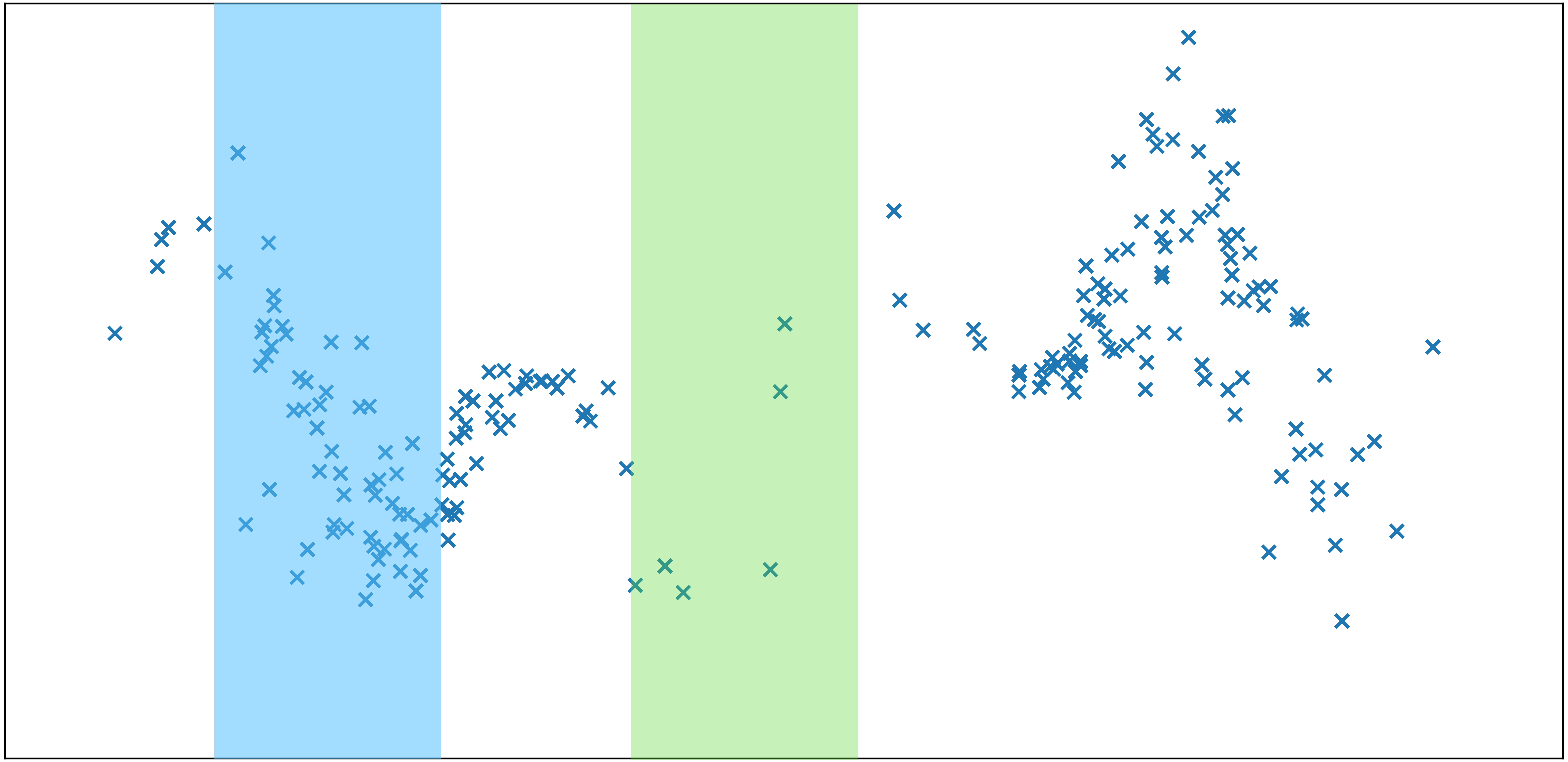

Where do we need more data?

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- Irreducible Error: Inherent environment noise.

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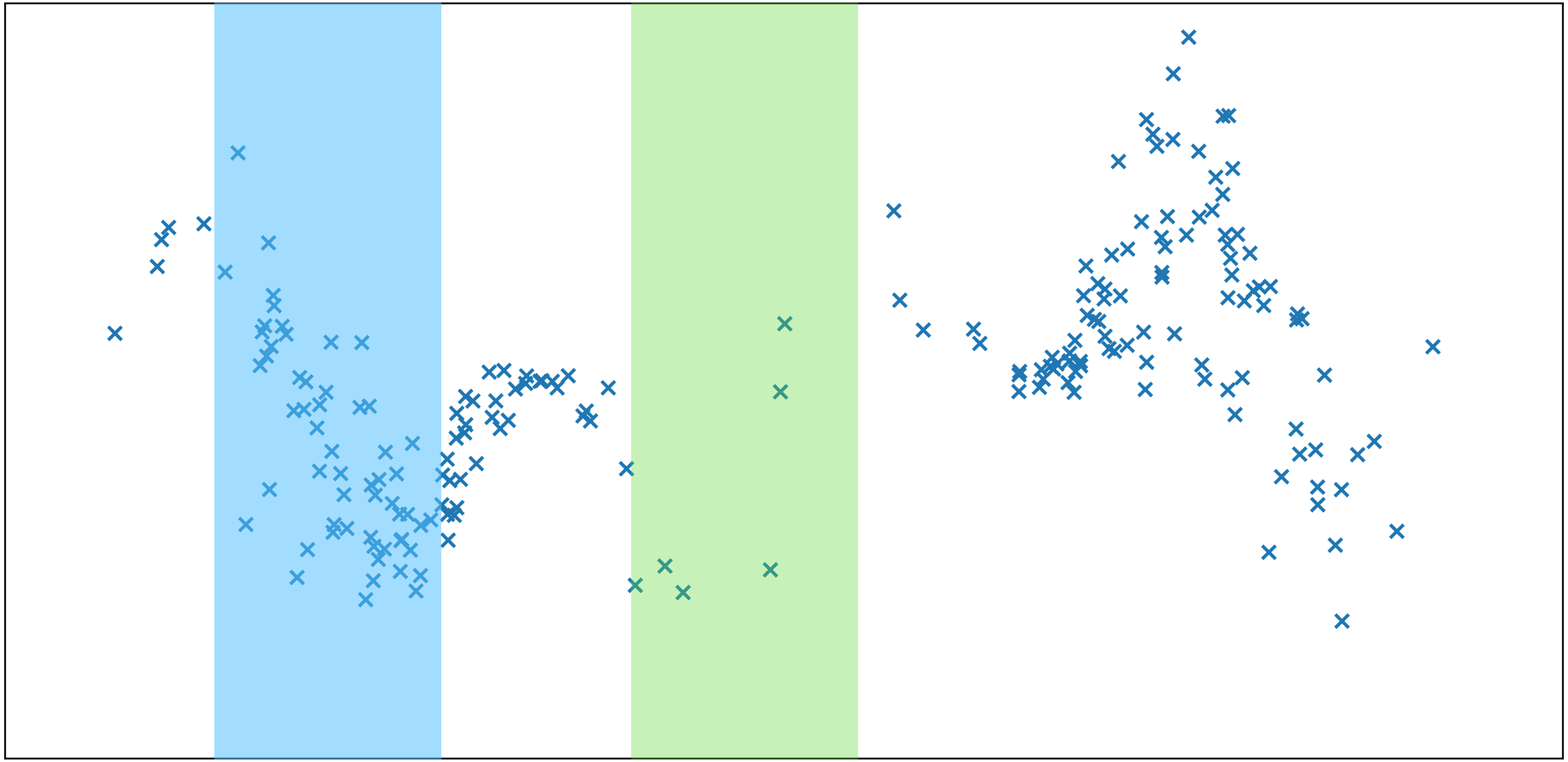

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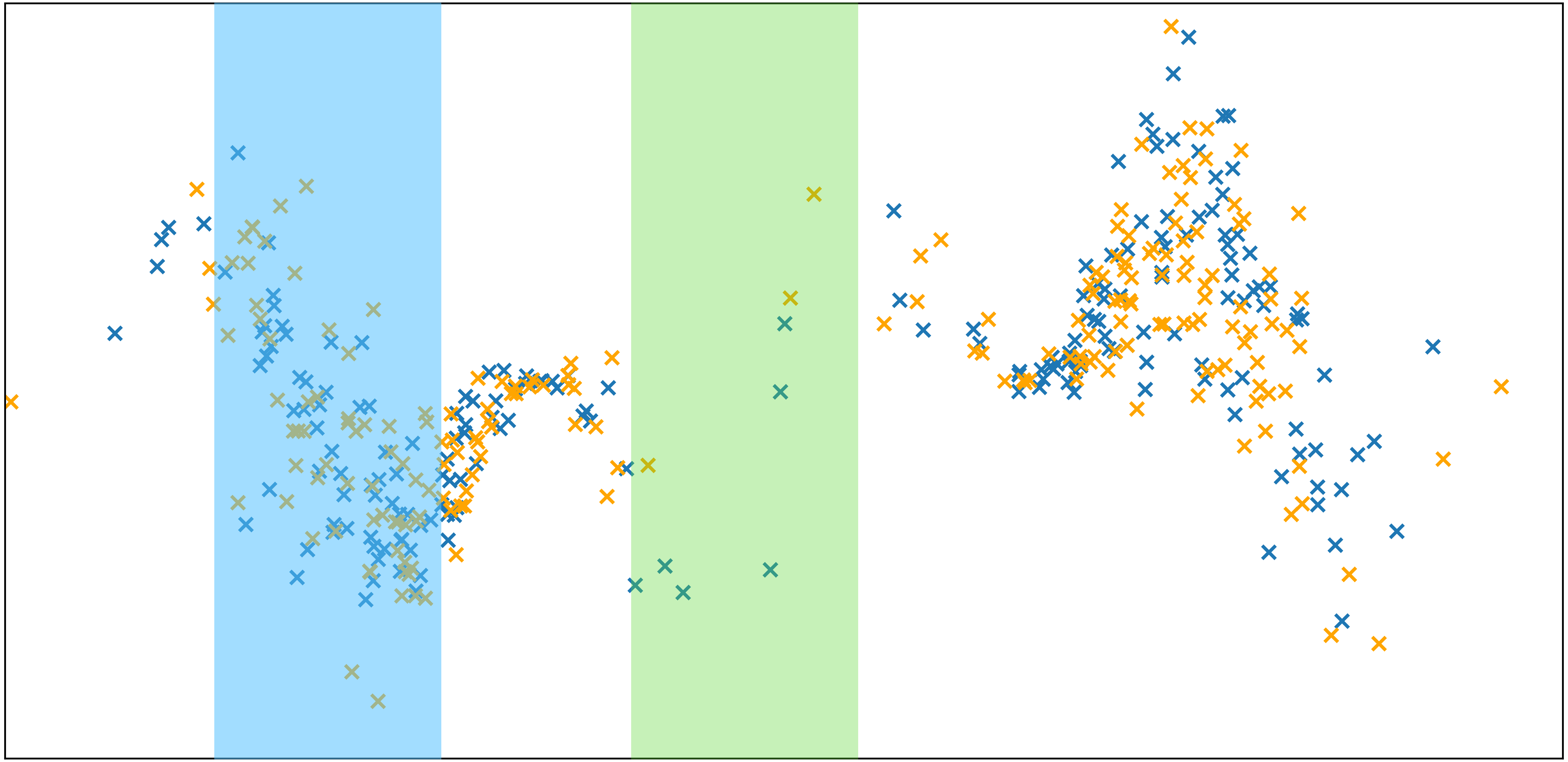

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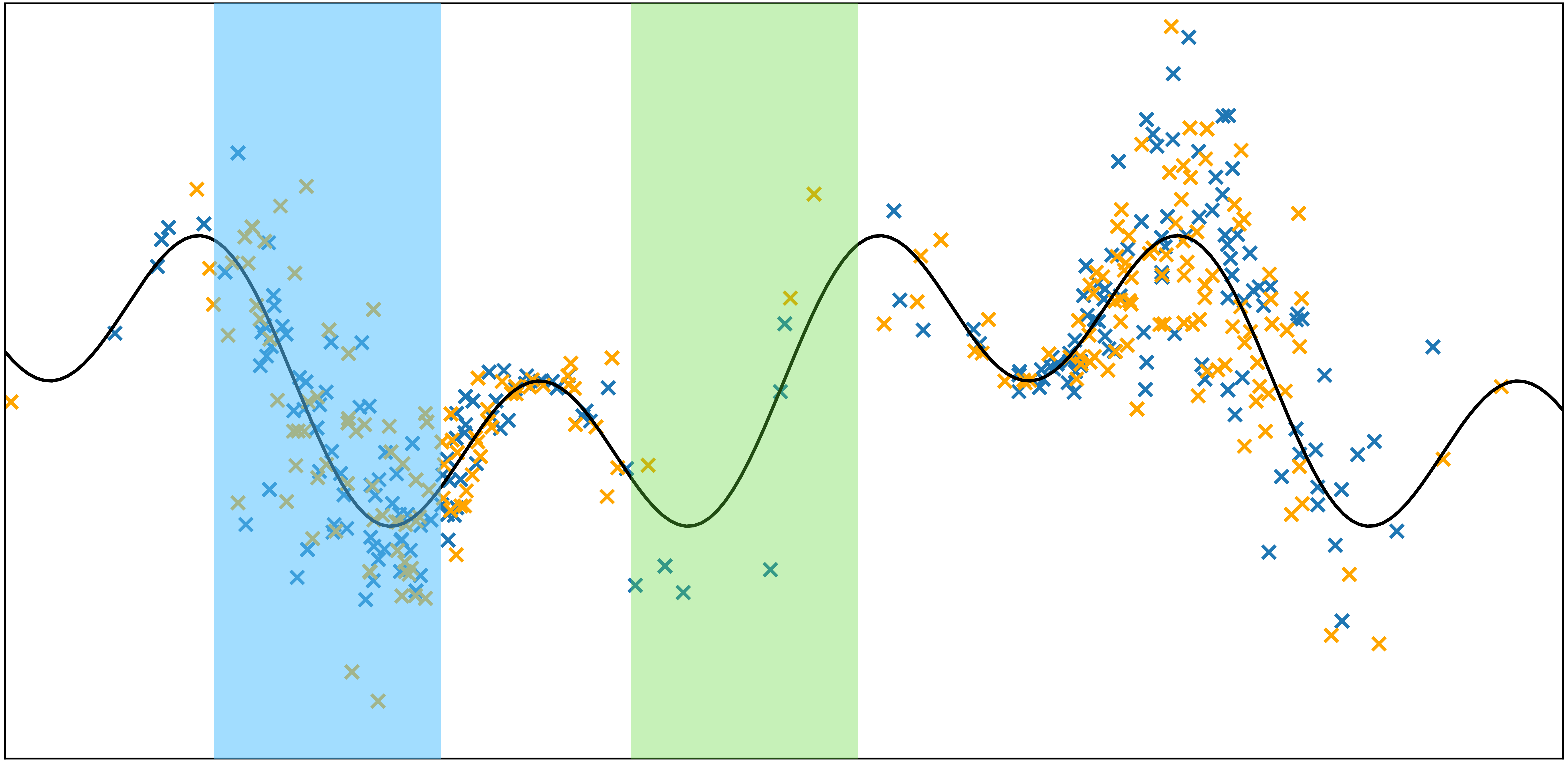

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**Key Takeaway: Train an ensemble of models to allow the agent to quantify "addressable" uncertainty when it needs to.**

# Part III: Trajectory Prediction

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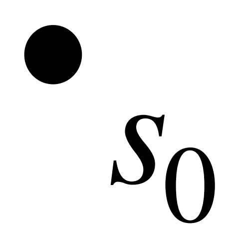

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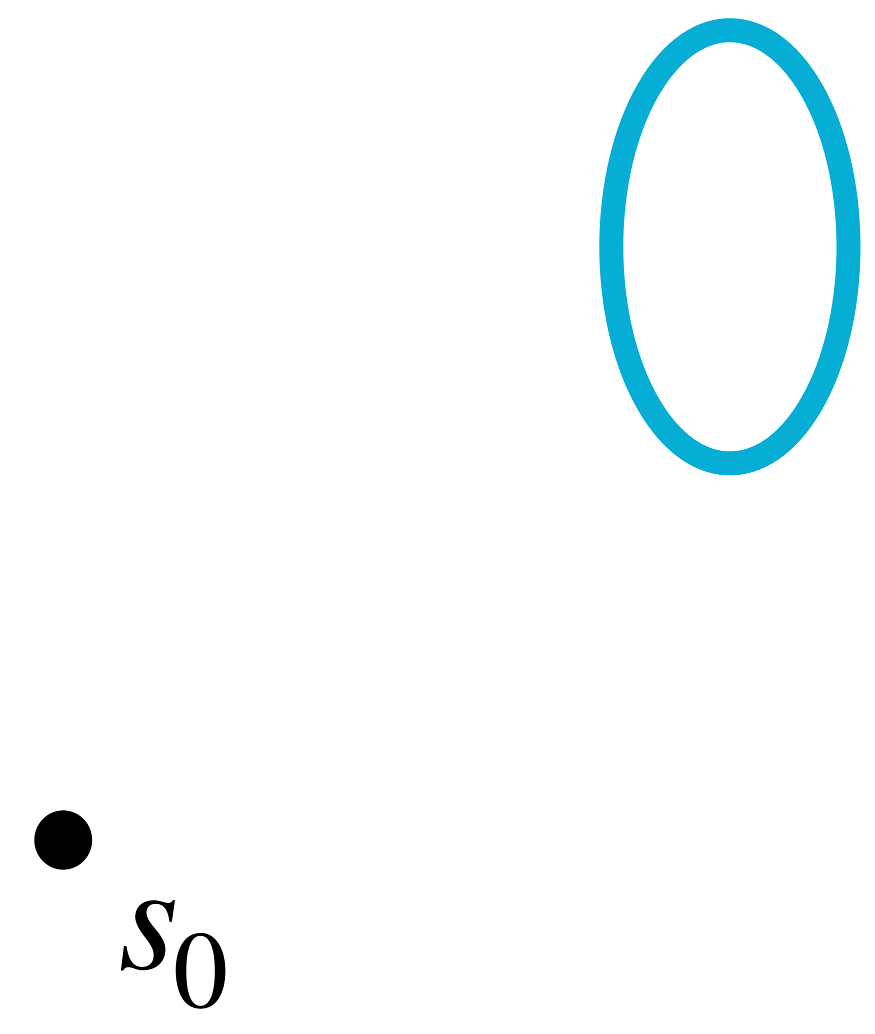

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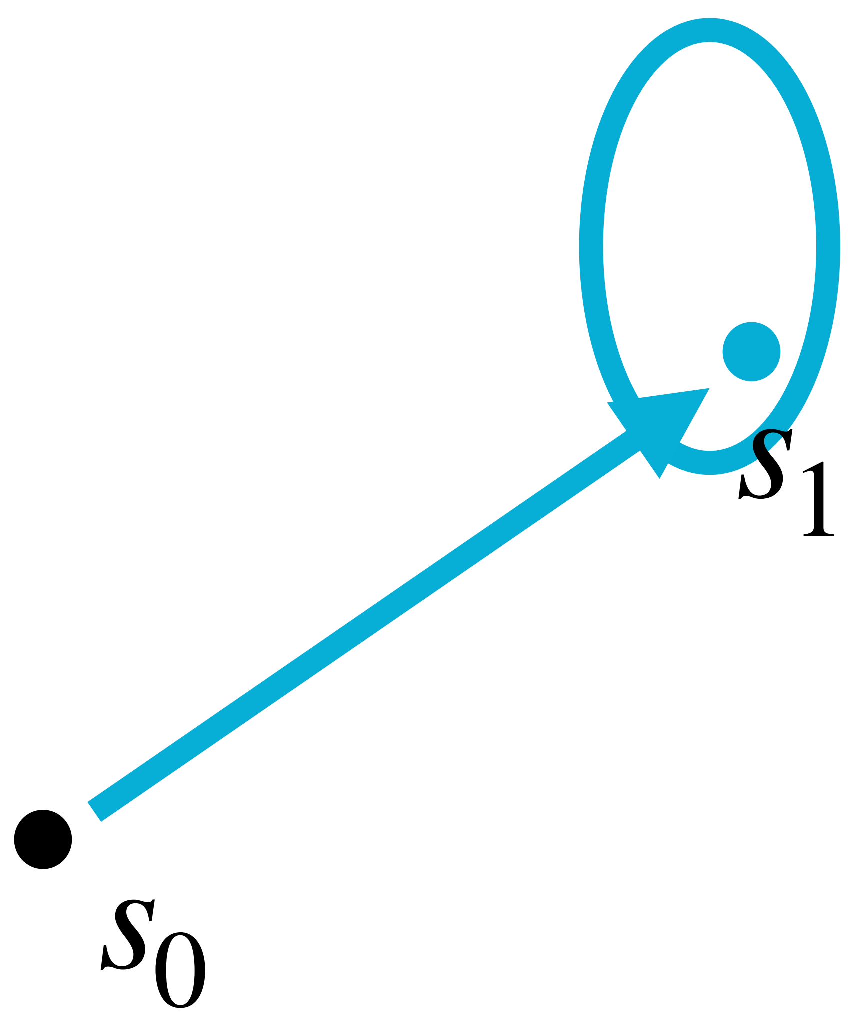

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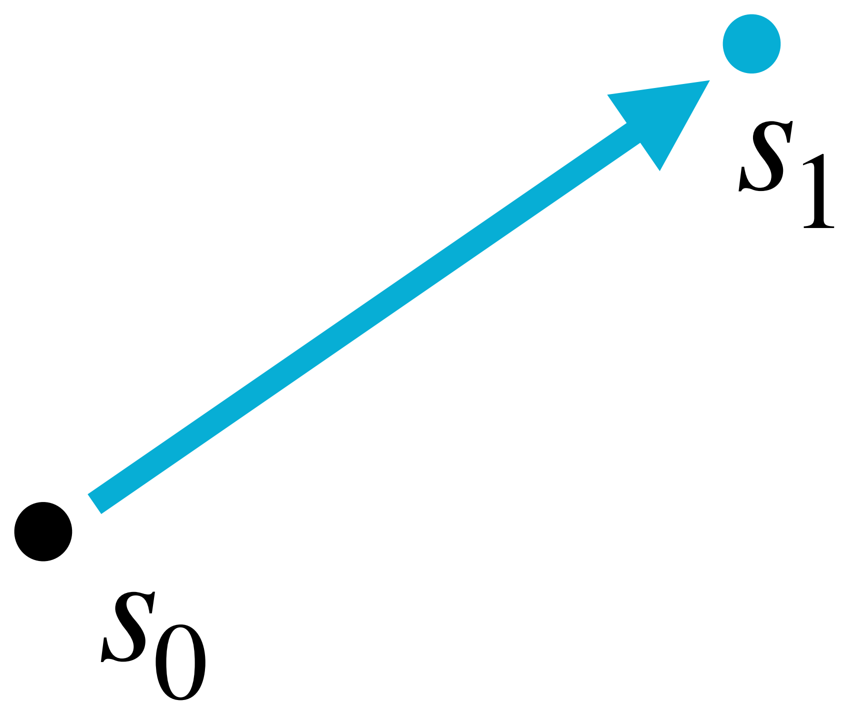

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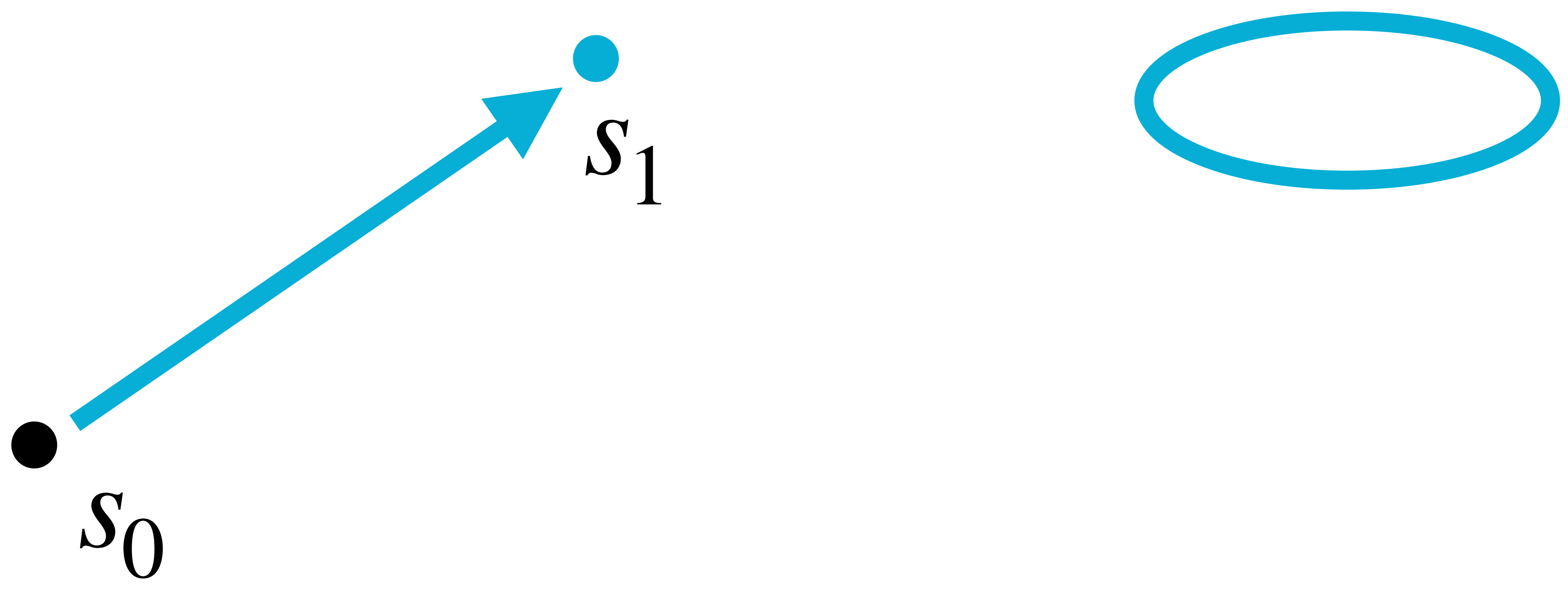

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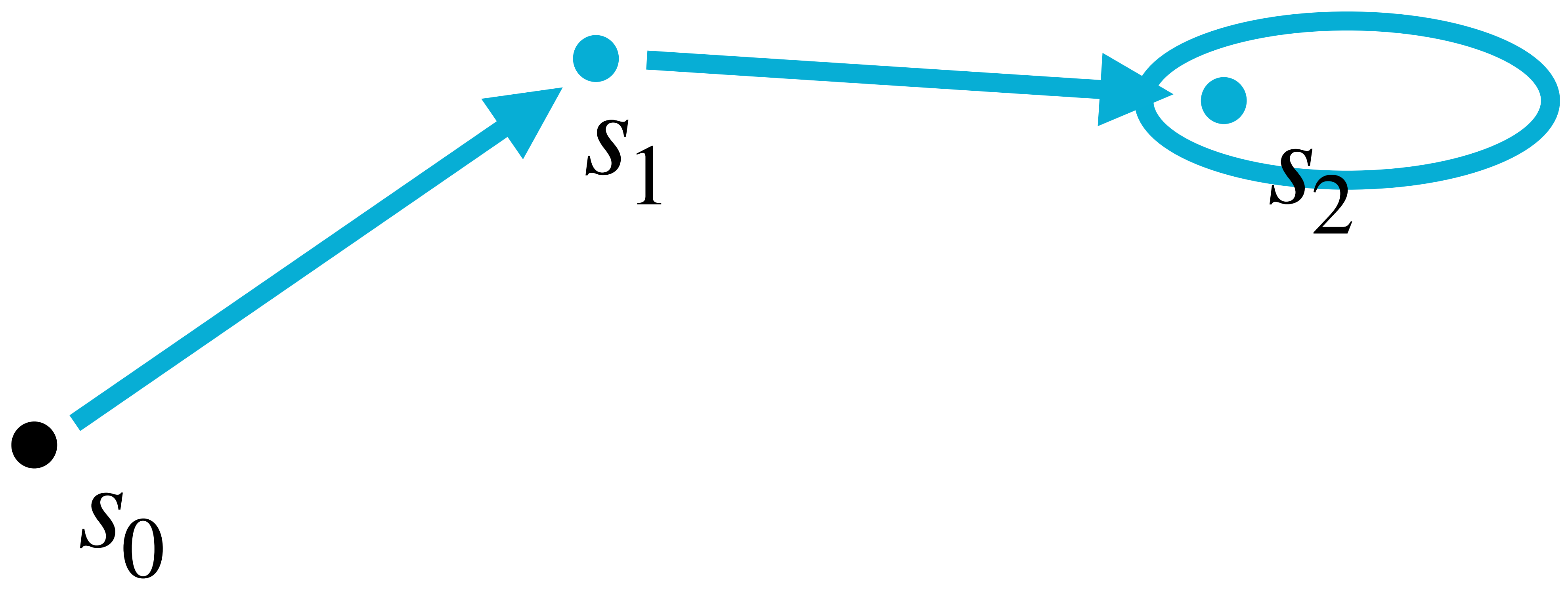

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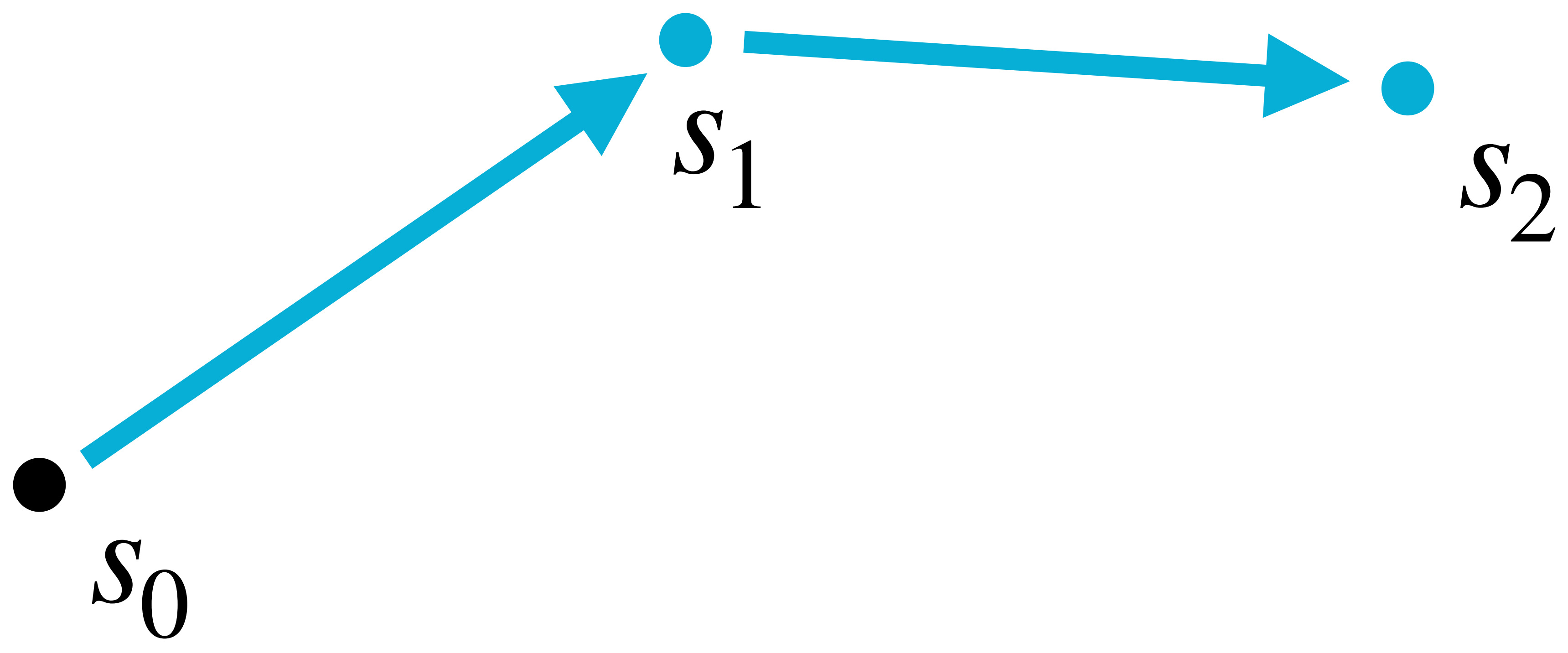

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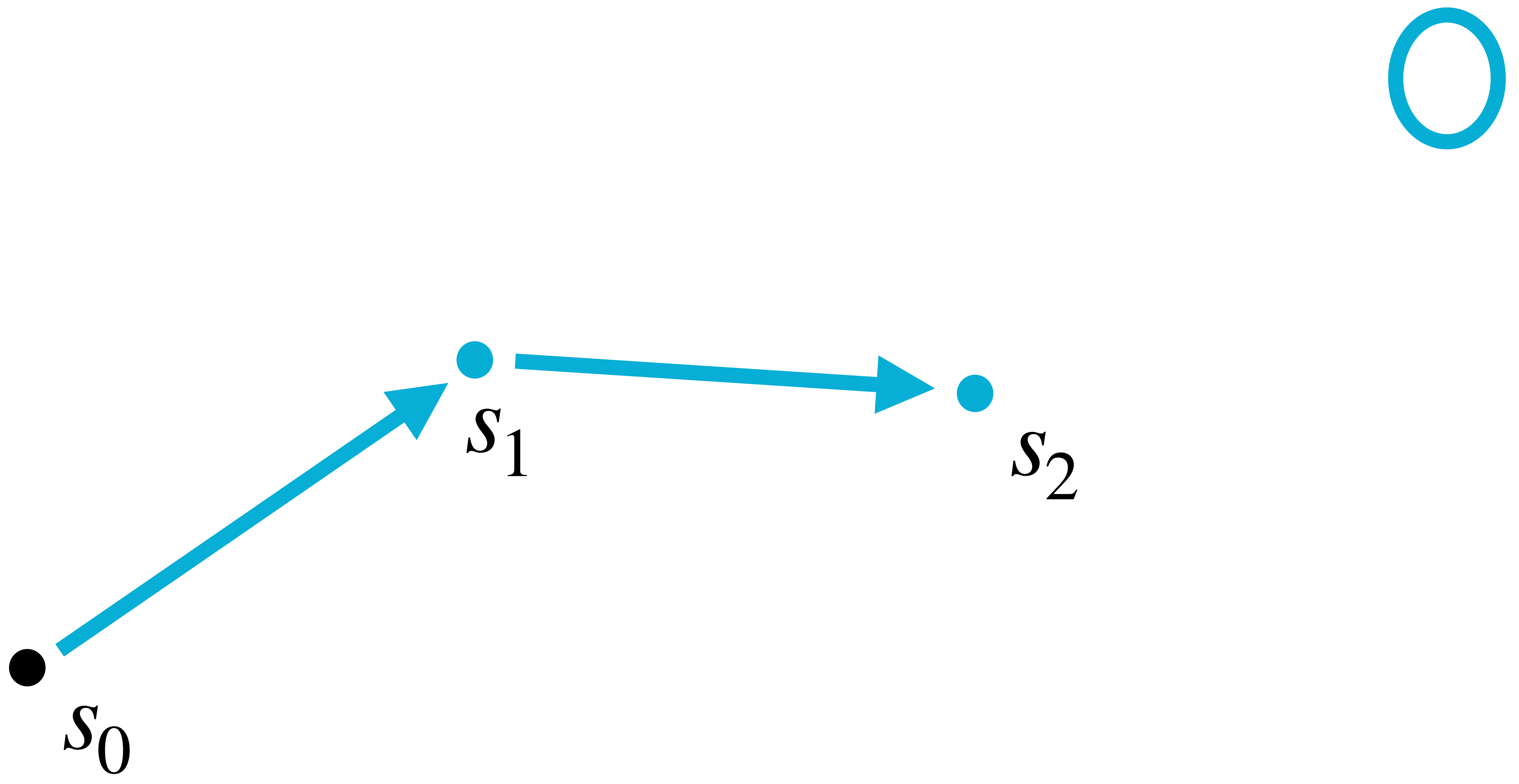

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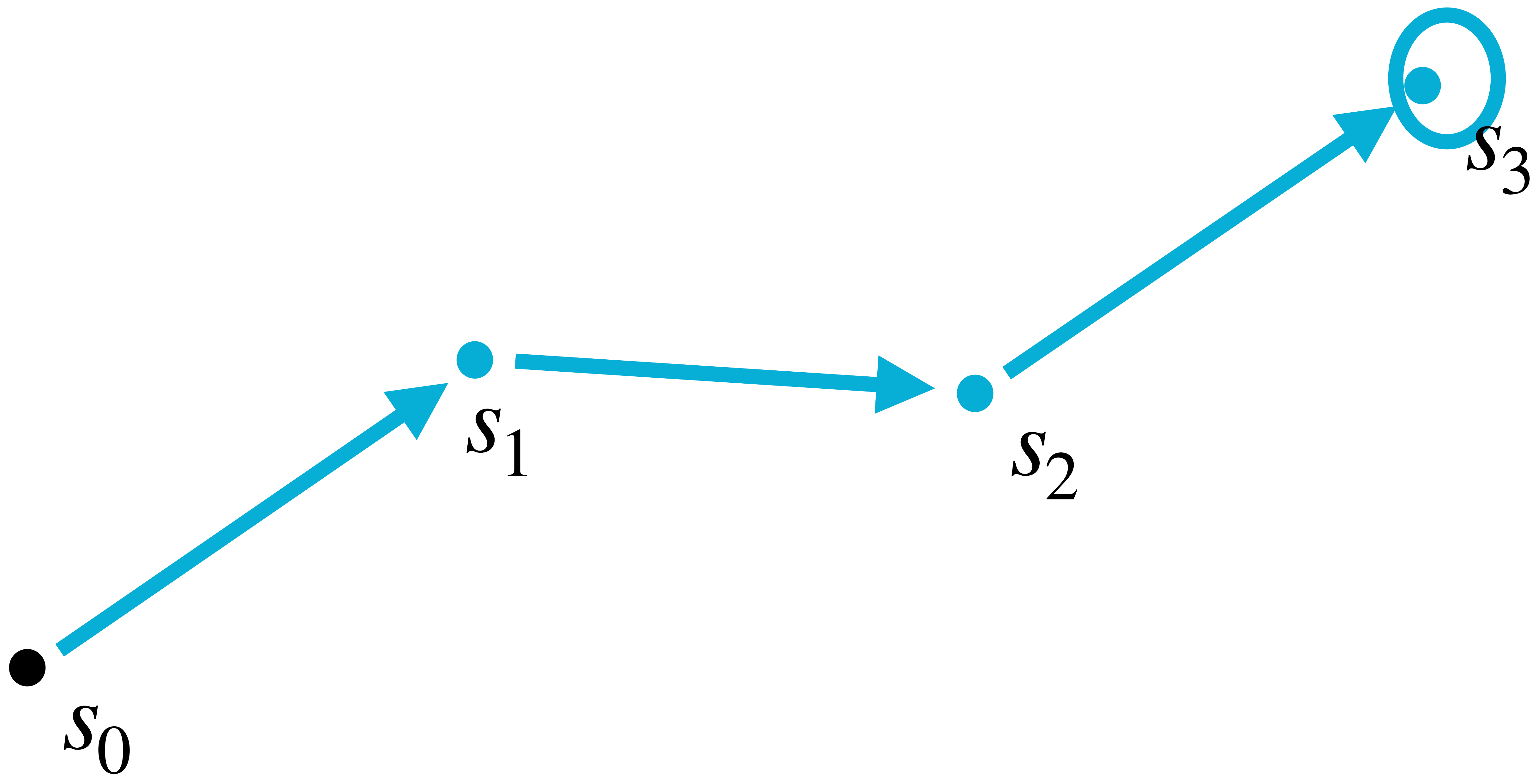

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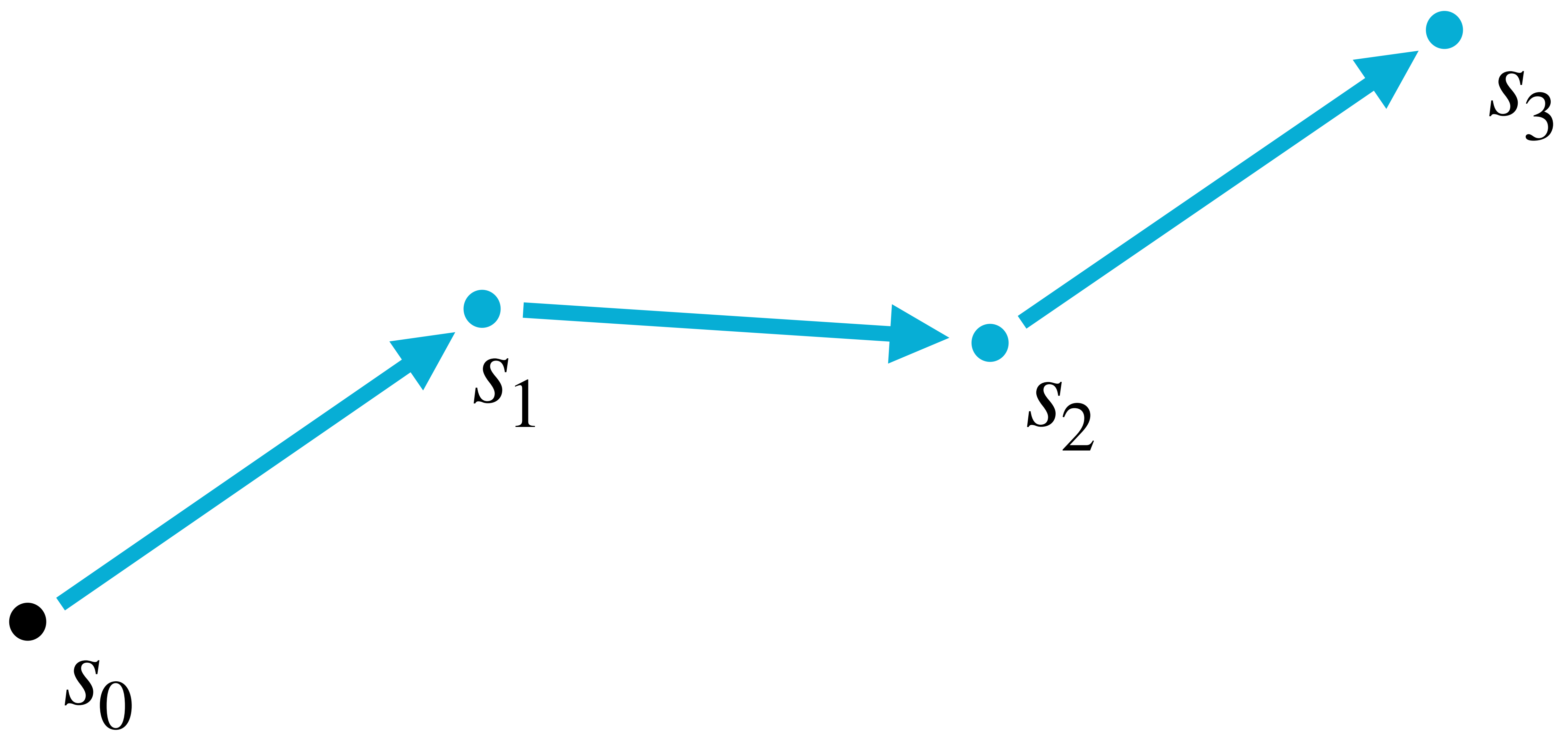

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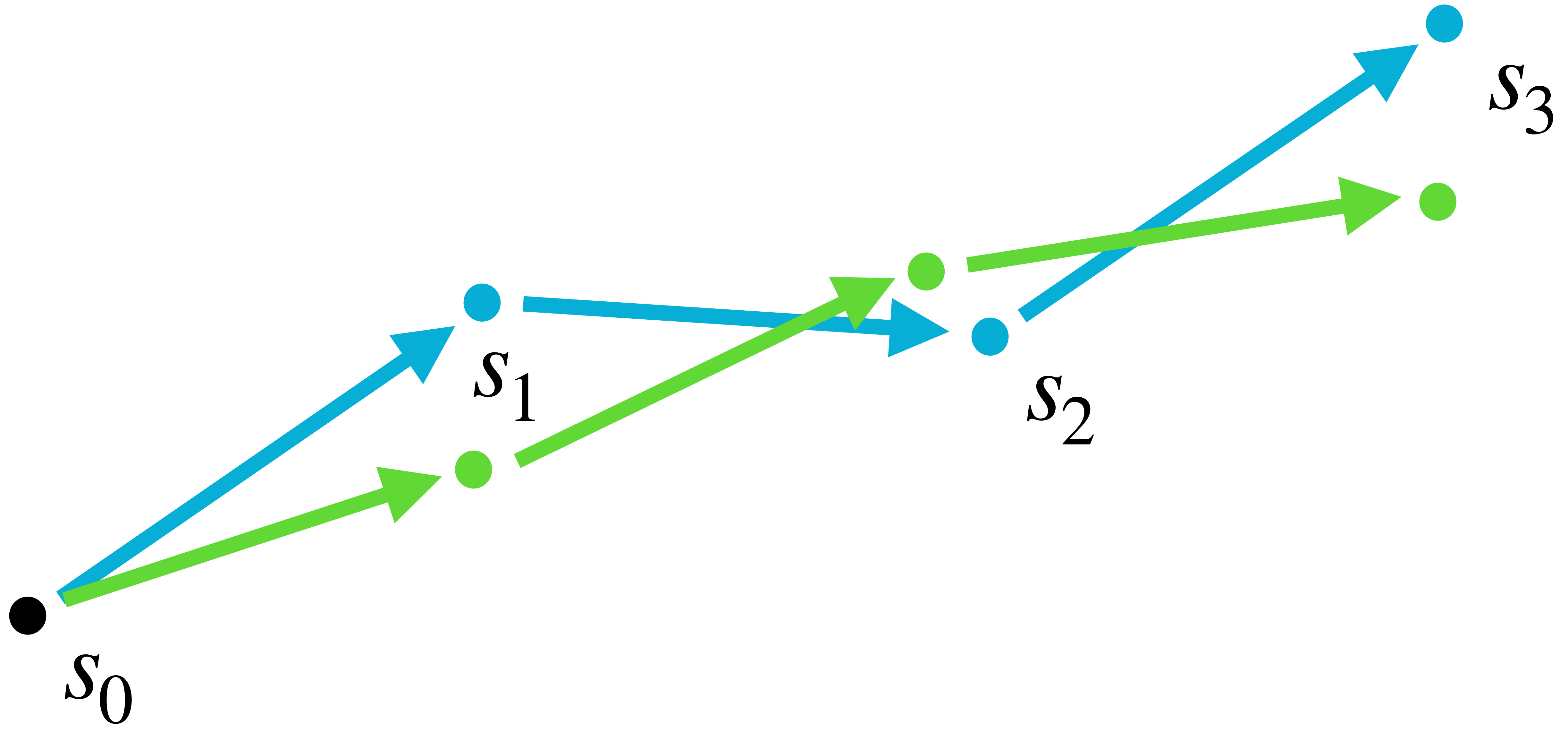

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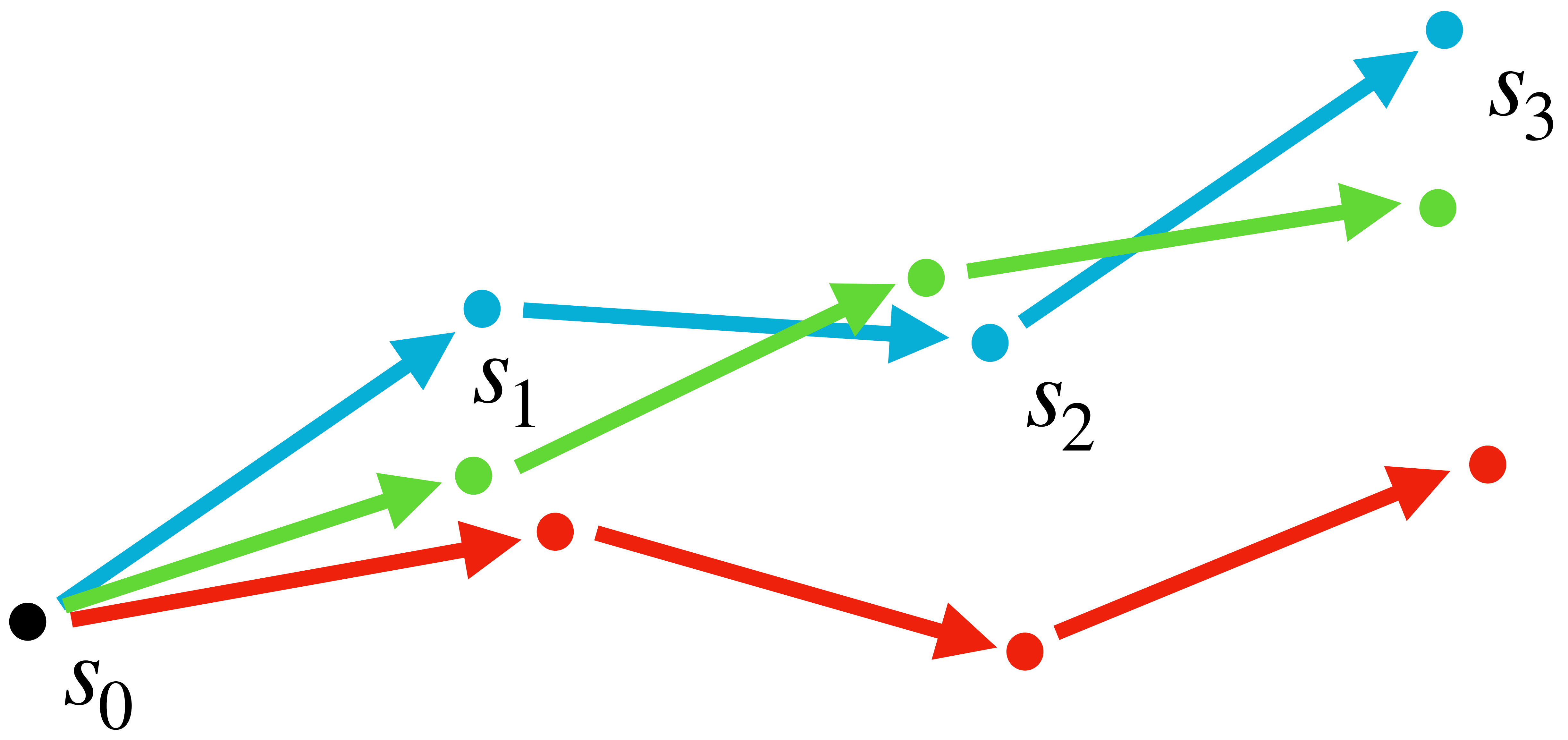

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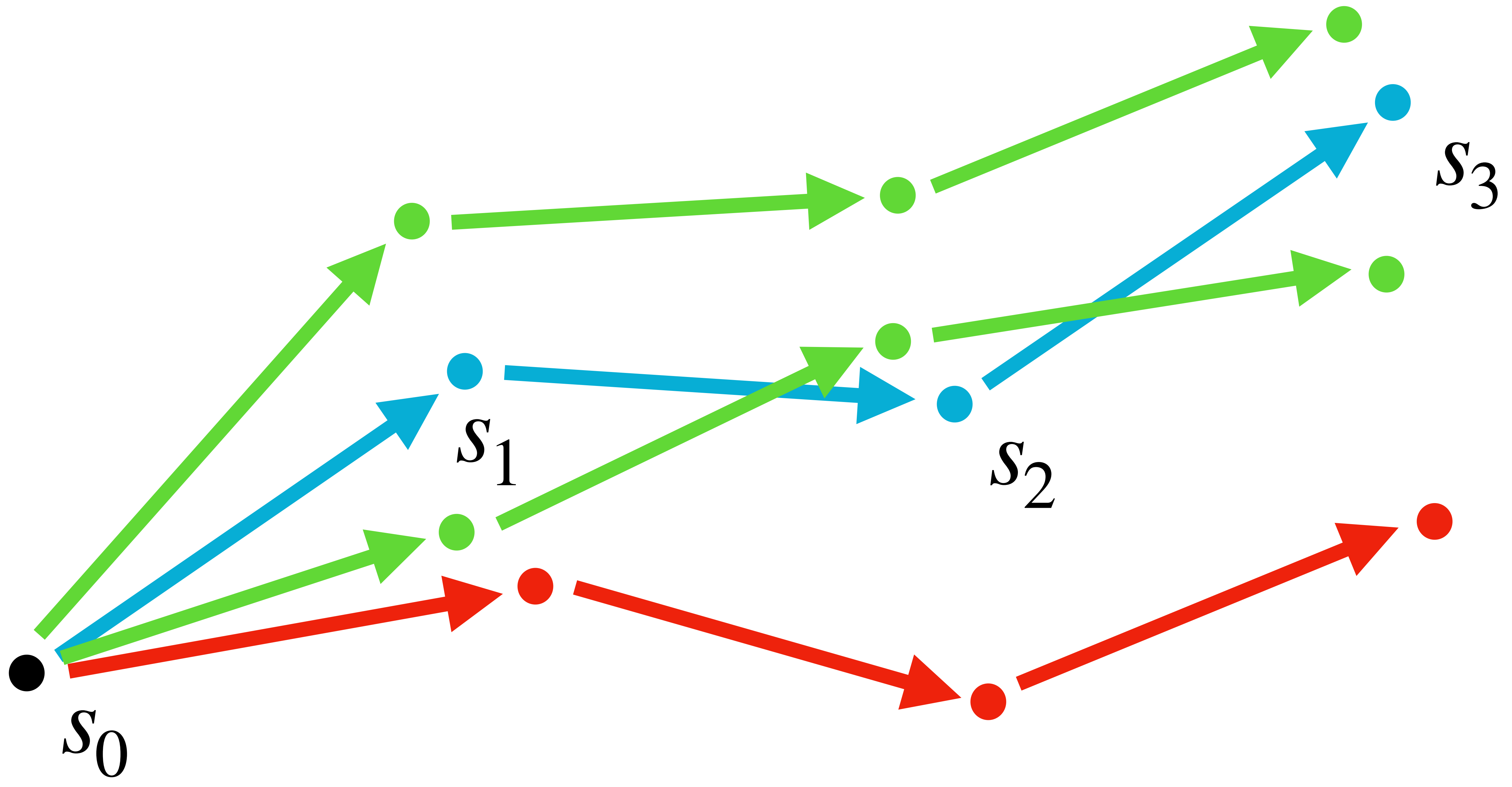

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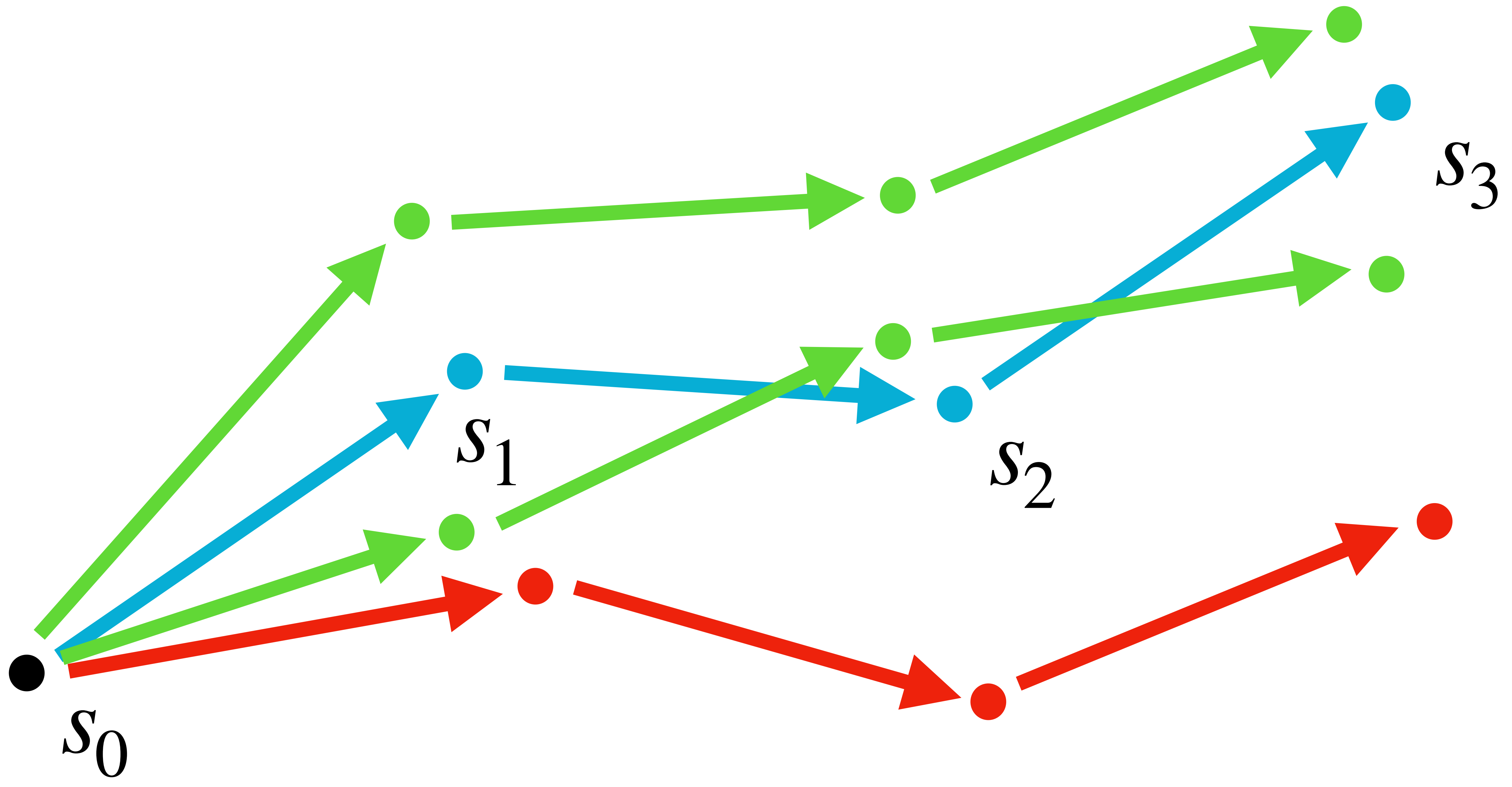


Known as ***particle-based sampling***

# Part IV: From Prediction to Control

# Simple Idea: Open-Loop Control

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- Sample a bunch of action sequences, check which one does the best, and use that sequence for the entire episode!

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**Algorithm 2** Open-Loop Control

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**Require:** Number of sequences $m$, action sequence proposal distribution $\mu$, dynamics model $\hat{p}$.

1: Sample i.i.d. action sequences $\left\{(a_0^{(i)}, a_1^{(i)}, \ldots, a_{T-1}^{(i)})\right\}_{i=1,\ldots,m}$ from $\mu$.

2: **for** $i = 1, \ldots, m$ **do**

3: Sample from the model $\hat{p}$ to perform Monte Carlo estimation of

$$R_i = \mathbb{E}\left[\sum_{t=0}^{T-1} r(s_t, a_t^{(i)})\right].$$

4: $i^* \leftarrow \operatorname{argmax}_i R_i$.

5: Apply action sequence $(a_0^{(i^*)}, a_1^{(i^*)}, \ldots, a_{T-1}^{(i^*)})$ to the environment.

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"Open loop": not adapting response to observed environment state.

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  2. Using a very accurate model.
     - If action sequence is not well-modeled, may want to replan actions based on new states seen.

# Closing the loop: Model Predictive Control

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**Require:** Number of sequences $m$, action sequence proposal distribution $\mu$, dynamics model $\hat{p}$.

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2: Sample i.i.d. action sequences $\left\{(a_t^{(i)}, a_{t+1}^{(i)}, \dots, a_{t+T-1}^{(i)})\right\}_{i=1,\dots,m}$ from $\mu$.

3: **for** $i = 1, \dots, m$ **do**

4: Sample from the model $\hat{p}$ to perform Monte Carlo estimation of

$$R_i = \mathbb{E}\left[\sum_{s=0}^{T-1} r(s_{t+s}, a_{t+s}^{(i)}) \;\middle|\; s_t\right].$$

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- How to specify closed-loop behaviors? Policies!

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---

**Algorithm 4** Model-Predictive Control with Policies

---

**Require:** Number of policy candidates $m$, policy proposal distribution $\mu$, dynamics model $\hat{p}$.

1: **for** every timestep $t$ **do**
2: Sample $m$ policy candidates $\pi_{\theta_1}, \ldots, \pi_{\theta_m}$ from $\mu$.
3: **for** $i = 1, \ldots, m$ **do**
4: Sample from the model $\hat{p}$ to perform Monte Carlo estimation of

$$R_i = \mathbb{E}\left[\sum_{s=0}^{T-1} r(s_{t+s}, a_{t+s}) \;\middle|\; s_t\right], \quad a_{t+s} \sim \pi_{\theta_i}(s_{t+s}).$$

5: Apply the action from the best policy.

---

# Summary of Control Policies

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| Control Policy | Improvement |
| --- | --- |
| | |
| | |
| | |

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| --- | --- |
| Open-loop control | |
| | |
| | |

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| **Control Policy** | **Improvement** |
|---|---|
| Open-loop control | |
| (Closed-loop) Model Predictive Control | |
| | |

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| MPC with (Adaptive) Policies | |

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| Control Policy | Improvement |
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| Open-loop control | |
| (Closed-loop) Model Predictive Control | Replanning at every step to take into account current state. |
| MPC with (Adaptive) Policies | Fixes mismatch between evaluated policies (open-loop) and the overall policy (closed-loop). |

# Part V: Putting Everything Together Suddenly (PETS)

# Probabilistic Ensembles with Trajectory Sampling (PETS)

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**Deep Reinforcement Learning in a Handful of Trials using Probabilistic Dynamics Models**

# Probabilistic Ensembles with Trajectory Sampling (PETS)

## Deep Reinforcement Learning in a Handful of Trials using Probabilistic Dynamics Models

**Kurtland Chua** **Roberto Calandra** **Rowan McAllister** **Sergey Levine**
Berkeley Artificial Intelligence Research
University of California, Berkeley
{kchua, roberto.calandra, rmcallister, svlevine}@berkeley.edu

# PETS components

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| Component | PETS choice |
| --- | --- |
| | |
| | |
| | |

# PETS components

| Component | PETS choice |
|---|---|
| Model | |
| | |
| | |

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| **Component** | **PETS choice** |
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| Extras/Useful Tricks | |

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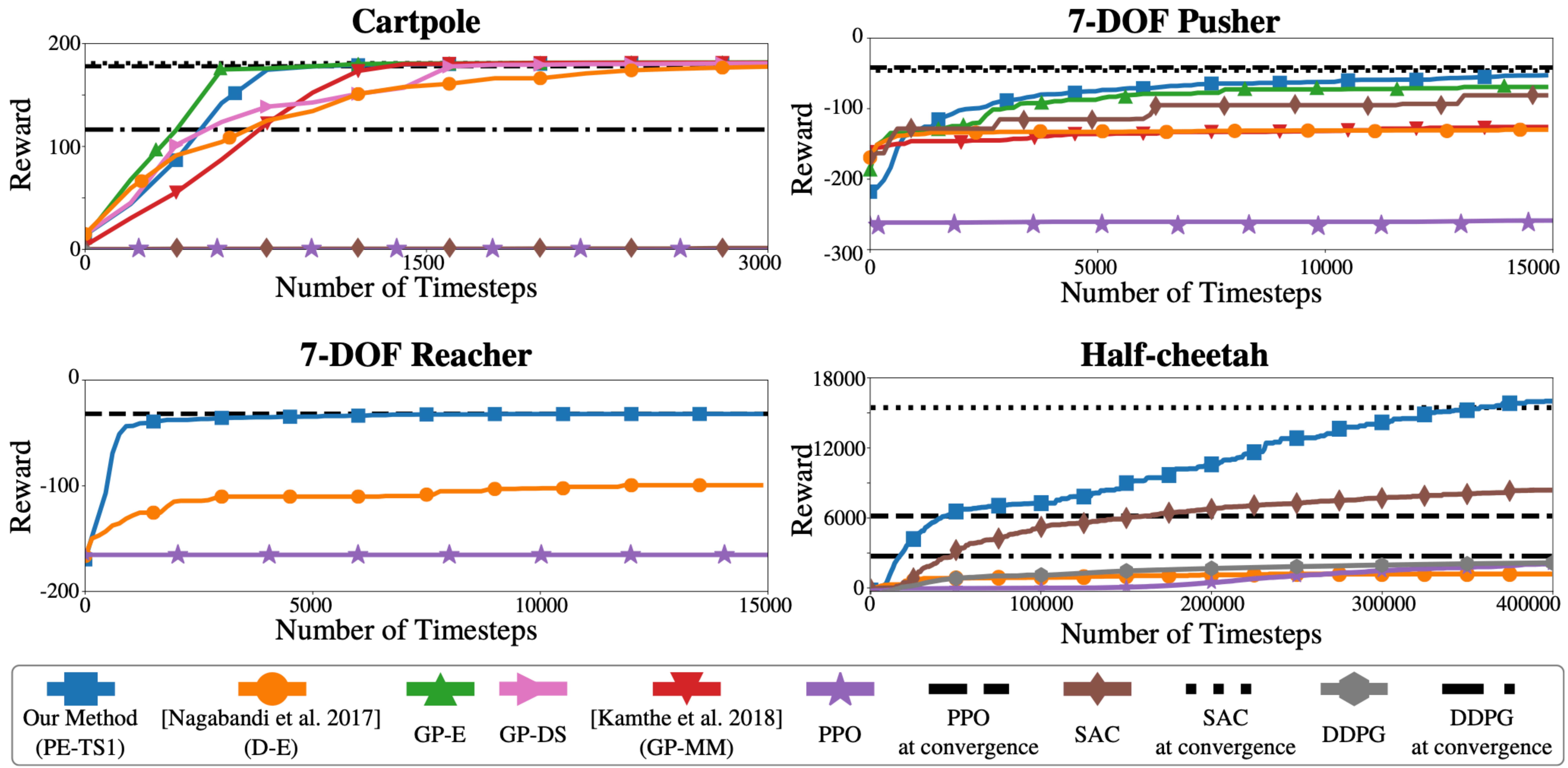
| Model | Ensemble of neural networks<br>Heteroskedastic noise model<br>MLE |
| Controller | Basic MPC |
| Extras/Useful Tricks | Cross-entropy method-based optimization;<br>Model predicts state change;<br>Warm-start MPC |

# Some experimental results



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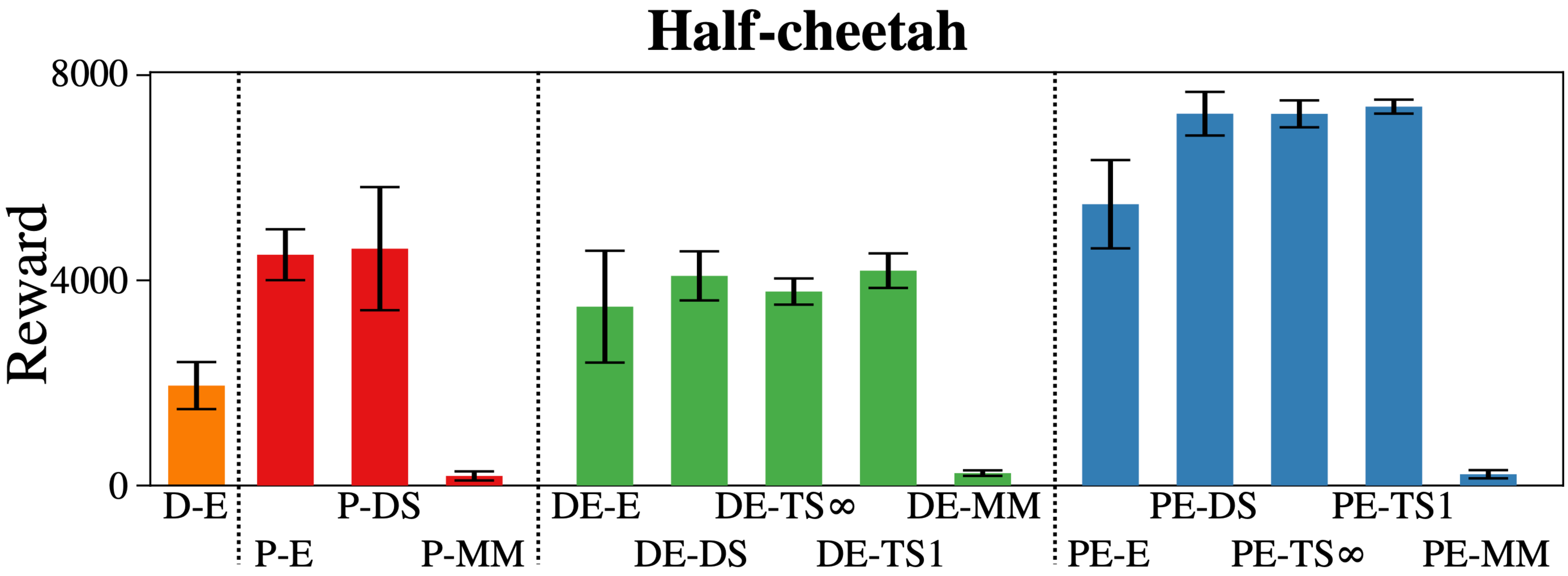

# PETS on HalfCheetah

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You may not like it, but this is what peak performance looks like