# Notes For *Deterministic Nonperiodic Flow*

## Introduction

- Some patterns of change over time are regular, others are not.
- There are many situations where fluids flow in a regular/periodic manner.
- There are even more situations where things are very irregular and unpredictable, most importantly, weather.
- Very little is known about the dynamics of these systems
- Plan to analyze what causes flow to be nonperiodic/unpredictable using a model of fluid flow with differential equations
- It is nearly impossible to symbolically solve such chaotic equations, the best people can do is an approximations that is dependent on measured conditions.
- Hydrodynamical system studied has both heat being injected and removed, making it dissipative and nonconservative, meaning that it is constantly seeking but unable to reach a state of rest.
- The level of forcing will be controlled by linear, non-chaotic equations.
- These linear forcing equations will give rise to complex, nonperiodic behavior of fluid convection.

## Phase space

- Imagine a mathematical system where multiple dependent variables are a function of time.
- The value of each dependent variable can be assigned to an axis, meaning that change over time in the situation can be shown as a line on a 3d plot.
- The differential equations controlling the dependent variables will never return values beyond a certain level because they represent a system with a limited amount of energy.
- Therefore, the path on the phase space (plot) will stay inside a set region.
- This behavior is represented by conservation of energy

## The instability of nonperiodic flow

- Will show that nonperiodic flow is unstable
- introduced some notation: P(t) represents the trajectory of the state of a system being modelled
- Set up system of equations so that it is bounded and continuous
- Since P(t) is confined to a limited range and continuous on infinite domain, it must come close to repeating itself.
- Trajectories either repeat themselves often or not at all.
- Stable trajectories continue to remain close together if they start close together, unstable trajectories diverge from each other.
- Stable trajectories are uniformly stable if they can stay close together after being perturbed from any location.

- Any trajectory that perfectly repeats itself is periodic because it must continue where it left off.
- Quasi-periodic trajectories are bound to repeat themselves given enough time, nonperiodic trajectories may not repeat themselves even over infinite time.
- Any trajectory that is stable will eventually become quasi-periodic
- Any nonperiodic trajectory must be unstable.
- This means that if a nonperiodic system is perturbed extremely lightly, it is likely to be heavily affected over time.

## Numerical integration of nonconservative systems

- Gives a scheme for how he will approximate a solution through numerical integration.
- Uses computer to solve these approximations to great accuracy with a program that iterates on previous approximations.

## The convection equations of Saltzman

- The Saltzman equations are differential equations that simulate fluid flow in the most simple way possible.
- These equations are derived from applying other equations to an idealized situation.
- Equation's dependent variables represent the strength of convection and the intensity of the vertical temperature difference

## Applications of linear theory

- Perturb convection equations to examine how stable they are
- Solved equations with perturbation with numerical integration procedure

## Numerical integration of the convection equations

- Solved Saltzman equations in computer with values chosen so that it would be "supercritical"
- Calculated many many iterations
- Simulated fluid is unable to find a stable convective cycle because it is sloshing around too quickly.
- Sloshes start small, grow rapidly, and then reverse direction unpredictably
- Represented trajectory of dependent variables on line plots and 3d graph projection
- Graphs never repeat themselves at any point
- Scatter plot of values does not capture unpredictability, trajectory is necessary
- Merely knowing one previous point does not allow researchers to predict successive states of the trajectory reliably.

- Prediction approach does work if the system is periodic.
- Trajectory can come arbitrarily close to repeating itself, but if it is unstable, it never actually will.
- Trajectory is infinitely long but fills a finite volume

## Conclusion

- Simulations of and real measurements of many fluid dynamics situations are chaotic even though the inputs are perfectly regular.
- A simple set of differential equations that simulate convection are chaotic if the amount of heat input is high enough.
- The only possible solution to this set of equations in this situation is nonperiodic and can only be found with an approximation.
- All of the solutions to this equation are unstable and most of them are chaotic.
- Long-range weather prediction is impossible because small measurement inaccuracies will eventually result in wildly different simulation output.
- Has the earth's weather ever actually repeated itself? No!
- Still open as to what the longest range for weather forecasts is, that is dependent on simulation advancedness.