

# Package ‘rcbmm’

January 30, 2021

**Type** Package

**Title** Regularized Copula-Based Mixture Model: Estimation and Model Selection

**Version** 0.1.0

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**Description** Model-based clustering through regularized copula-based mixture models with arbitrary marginal distributions. Estimation via the expectation-conditional-maximization algorithm. Shrinkage driven methods used to adaptively select optimal correlation structure regulated by a tuning parameter. Model selection based on average silhouette width and BIC.

**Encoding** UTF-8

**LazyData** true

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**RoxygenNote** 7.1.1

**Suggests** testthat

**Imports** cluster,  
utils,  
parallel,  
stats,  
copula,  
mvtnorm,  
mclust,  
fitdistrplus,  
Rcpp

**LinkingTo** Rcpp

## R topics documented:

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|            |                               |
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| angles2rho | <i>Matrix conversion tool</i> |
|------------|-------------------------------|

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## Description

A tool for converting a matrix of angles to a correlation matrix using Choleksy factor

## Usage

```
angles2rho(theta)
```

## Arguments

theta            A  $(p - 1)$  times  $(p - 1)$  matrix of angles.

## Value

A  $p$  times  $p$  correlation matrix.

## See Also

[rho2angles](#)

## Examples

```
theta <- matrix(rep(0.5, 4), 2, 2)
angles2rho(theta)
```

---

|           |   |
|-----------|---|
| cm_step_1 | <i>Conditional-maximization step 1 (M-step 2)</i> |
|-----------|---|

---

## Description

Implements the conditional-maximization step 1 of the ECM algorithm for regularized copula-based mixture model.

**Usage**

```
cm_step_1(
  x,
  K,
  z,
  mvdc,
  margins,
  trace = TRUE,
  restrictions = TRUE,
  variance_tolerance = 1e-05
)
```

**Arguments**

|                    |   |
|--------------------|---|
| x                  | A numeric matrix or data frame of observations. Rows correspond to observations and columns correspond to variables.  |
| K                  | The number of mixture components.   |
| z                  | A numeric matrix representing the current value of the posterior probabilities of membership of the observations after the expectation step of the ECM algorithm. Columns are associated with a mixture component and rows are associated with observations. The current value of the probabilities should be computed via <a href="#">e.step</a> . |
| mvdc               | A list of objects of class mvdc. Each element of the list corresponds to a mixture component and contains the previous estimates for the component distribution's marginal and copula parameters.   |
| margins            | A character vector specifying the marginal distributions of the components in the mixture. The vector must have a length equal to the number of columns in x. Each element must be equal to "norm", "beta" or "gamma".  |
| trace              | A logical value indicating if an update regarding the step's progress should be displayed.  |
| restrictions       | A logical value indicating if the variance of each Beta marginal should be restricted   |
| variance_tolerance | The lower bound for the variance of each Beta marginal, if restrictions = TRUE.   |

**Value**

A list of objects of class mvdc. Each element of the list corresponds to a mixture component and contains the updated estimates for the component distribution's marginal parameters and the same estimates as were passed for the estimates of the copula parameter.

**See Also**

[cm.step.2.ecm](#)

cm\_step\_2

*Conditional-maximization step 2 (M-step 2)***Description**

Implements the conditional-maximization step 2 of the ECM algorithm for regularized copula-based mixture model.

**Usage**

```
cm_step_2(x, K, z, mvdc, margins, lambda, trace = TRUE)
```

**Arguments**

|         |   |
|---------|---|
| x       | A numeric matrix or data frame of observations. Rows correspond to observations and columns correspond to variables.  |
| K       | The number of mixture components.   |
| z       | A numeric matrix representing the current value of the posterior probabilities of membership of the observations after the expectation step of the last iteration of the ECM algorithm. Columns are associated with a mixture component and rows are associated with observations.  |
| mvdc    | A list of objects of class mvdc. Each element of the list corresponds to a mixture component and contains the current estimates for the component distribution's marginal and previous estimates for copula parameters. The estimates for the marginal parameters should have been updated by <a href="#">cm.step.1</a> . |
| margins | A character vector specifying the marginal distributions of the components in the mixture. The vector must have a length equal to the number of columns in x. Each element must be equal to "norm", "beta" or "gamma".  |
| lambda  | A numeric value indicating the value of the tuning parameter for regularization.  |
| trace   | A logical value indicating if an update regarding the step's progress should be displayed.  |

**Value**

A list with the following elements

- mvdc A list of objects of class mvdc. Each element of the list corresponds to a mixture component and contains the updated estimates for the component distribution's copula parameter and the same estimates as were passed for the estimates of the marginal parameters.
- penalty The shrinkage penalty to apply to the log-likelihood of the model, resulting from the estimates of the copula parameters and the value of lambda.

**See Also**

[cm.step.1](#) [ecm](#)

ecm

*Model estimation through ECM***Description**

Implements the expectation-conditional-maximization algorithm for a regularized copula-based mixture model given initial parameter values, starting with the expectation step.

**Usage**

```
ecm(
  x,
  K,
  lambda,
  start = NULL,
  margins,
  trace = TRUE,
  maxit = 1000,
  epsilon = 1e-06,
  dist_mat = NULL
)
```

**Arguments**

|          |  |
|----------|--|
| x        | A numeric matrix or data frame of observations. Rows correspond to observations and columns correspond to variables.   |
| K        | An integer specifying the number of components for which a regularized copula-based mixture model should be fitted.  |
| lambda   | A numeric value indicating the value of the tuning parameter for regularization.   |
| start    | A list providing the starting values for ECM. The list is produced by <a href="#">fit.rcbmm</a> .  |
| margins  | A character vector specifying the marginal distributions of the components in the mixture. The vector must have a length equal to the number of columns in x. Each element must be equal to "norm", "beta" or "gamma". |
| trace    | A logical value indicating if an update regarding the algorithm's progress should be displayed after each iteration.   |
| maxit    | A numeric value indicating the maximal number of ECM iterations.   |
| epsilon  | A numeric value specifying the tolerance associated with determining when convergence of the ECM algorithm has been achieved.  |
| dist_mat | An object of type dist for calculating silhouette values.  |

**Value**

- K The number of mixture components.
- lambda The value of the tuning parameter.
- z A numeric matrix representing the posterior probabilities of membership of the observations after the expectation step of the last iteration of the ECM algorithm. Columns are associated with a mixture component and rows are associated with observations.

- **clusters**A classification vector indicating the associated cluster of each observation. The classification corresponds to **z**.
- **loglik**A numeric vector displaying the penalized log-likelihood after each iteration of ECM.
- **param\_number**The number of independent parameters associated with the model.
- **BIC**The BIC value of the model. Computed using the unpenalized log-likelihood after the last iteration of the ECM algorithm.
- **mixing\_probs**The mixing proportions associated with the model.
- **mvdc**A list of objects of class **mvdc**. Each element of the list corresponds to a mixture component.
- **transformation**A character value indicating the transformation when identifying starting values. The value is **NULL** unless **lambda=0**. See [initialize.ecm](#).
- **marginal\_param**A list containing the marginal parameters of the model as estimated by ECM. Each element corresponds to a mixture component.
- **copula\_param**A list containing the copula parameters of the model as estimated by ECM. Each element corresponds to a mixture component.
- **copula\_param\_angles**A list containing the copula parameters of the model re-expressed as angles.
- **silhouette**See information regarding silhouette package. Add reference here.

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|                     |   |
|---------------------|---|
| extract_copula_pars | <i>Extraction of copula parameters from a model</i> |
|---------------------|---|

---

## Description

A function for extracting the copula parameters parameterizing the mixture components of a regularized copula-based mixture model from a list of **mvdc** objects. The copula parameter for each mixture component can be extracted in terms of a correlation matrix or a matrix of angles.

## Usage

```
extract_copula_pars(mvdc, as_angles = F)
```

## Arguments

|                  |  |
|------------------|--|
| <b>mvdc</b>      | A list of objects of class <b>mvdc</b> . Each element of the list corresponds to a mixture component and contains the estimates for the component distribution's marginal and copula parameters. |
| <b>as_angles</b> | A logical value indicating whether the copula parameter should be returned as a matrix of angles instead of a correlation matrix.  |

## Value

A list of matrices corresponding to the copula parameter of each mixture component contained in **mvdc**.

## See Also

[extract.marginal.pars](#)

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extract\_marginal\_pars *Extraction of marginal parameters from a model*

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### Description

A function for extracting the marginal parameters parameterizing the mixture components of a regularized copula-based mixture model.

### Usage

```
extract_marginal_pars(mvdc)
```

### Arguments

|      |  |
|------|--|
| mvdc | A list of objects of class mvdc. Each element of the list corresponds to a mixture component and contains the estimates for the component distribution's marginal and copula parameters. |
|------|--|

### Value

A list of lists corresponding to the marginal parameters of each mixture component contained in mvdc. Each sub-list in the list has a length corresponding to the number of marginal distributions defined by the model.

### See Also

[extract.copula.pars](#)

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|        |                         |
|--------|-------------------------|
| e_step | <i>Expectation step</i> |
|--------|-------------------------|

---

### Description

Implements the expectation step of the ECM algorithm for regularized copula-based mixture model.

### Usage

```
e_step(x, K, mixing_probs, mvdc, margins)
```

### Arguments

|              |  |
|--------------|--|
| x            | A numeric matrix or data frame of observations. Rows correspond to observations and columns correspond to variables.   |
| K            | The number of mixture components.  |
| mixing_probs | A numeric vector indicating the mixing proportions of the mixture model.   |
| mvdc         | A list of objects of class mvdc. Each element of the list corresponds to a mixture component and contains the current estimates for the component distribution's marginal and copula parameters.                       |
| margins      | A character vector specifying the marginal distributions of the components in the mixture. The vector must have a length equal to the number of columns in x. Each element must be equal to "norm", "beta" or "gamma". |

**Value**

A numeric matrix representing the posterior probabilities of membership of the observations after performing a single expectation step of the ECM algorithm. Columns are associated with a mixture component and rows are associated with observations.

**See Also**

[ecm](#)

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fit\_rcbmm

*Model selection*

---

**Description**

The function selects the most appropriate model from a family of regularized copula-based mixture models arising from a varying number of components and a differing shrinkage parameter.

**Usage**

```
fit_rcbmm(
  x,
  lambda_grid,
  K = seq.int(2, 9),
  margins,
  maxit = 1000,
  epsilon = 1e-06,
  transform = TRUE,
  trace = FALSE
)
```

**Arguments**

|             |  |
|-------------|--|
| x           | A numeric matrix or data frame of observations. Rows correspond to observations and columns correspond to variables.   |
| lambda_grid | An integer vector specifying the values of the shrinkage parameter for which a regularized copula-based mixture model should be fitted. The default is <code>lambda_grid = 0</code> . If the vector parsed does not contain 0, then 0 is appended as starting values can only be obtained in the case no regularization is applied to the model. |
| K           | An integer vector specifying the number of components for which a regularized copula-based mixture model should be fitted. The default is <code>K=2:9</code> .   |
| margins     | A character vector specifying the marginal distributions of the components in the mixture. The vector must have a length equal to the number of columns in x. Each element must be equal to "norm", "beta" or "gamma".   |
| maxit       | A numeric value specifying the maximum number of iterations the ECM algorithm should run before being halted.  |
| epsilon     | A numeric value indicating the tolerance for convergence.  |
| transform   | A logical value indicating whether or not starting values for the case <code>lambda = 0</code> should be obtained using the transformations SPH, PCS, PCR and SVD. The default is TRUE.  |
| trace       | A logical value indicating if an update regarding the step's progress should be displayed.   |



**Value**

- BICA matrix demonstrating the BIC values achieved by each model in the family. Each column corresponds to a given value of lambda and each row corresponds to a given number of components.
- SILA matrix demonstrating the average silhouette width achieved by each model in the family. Each column corresponds to a given value of lambda and each row corresponds to a given number of components.
- all\_models A list of lists with each element containing information about a specific model fitted.
- selected\_models A list containing the information about the optimal model for each number of mixture components in the family. The optimal model for each number of components is selected by picking the lambda that results in the largest average silhouette width. See the help file for silhouette for details.
- final\_model The model contained in selected\_models that maximized BIC.

**See Also**

[ecm initialize.ecm](#)

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|                    |                                |
|--------------------|--------------------------------|
| initialize.ecm.old | <i>Starting values for ECM</i> |
|--------------------|--------------------------------|

---

**Description**

Identifies optimal starting values for ECM using the results of MBHAC applied to the data. The data undergoes transformations to enhance separation amongst groups prior to performing MBHAC.

**Usage**

```
initialize.ecm.old(
  x,
  K,
  margins,
  transform = FALSE,
  hc_pairs = NULL,
  classification = NULL
)
```

**Arguments**

|           |  |
|-----------|--|
| x         | A numeric matrix or data frame of observations. Rows correspond to observations and columns correspond to variables.   |
| K         | The number of mixture components.  |
| margins   | A character vector specifying the marginal distributions of the components in the mixture. The vector must have a length equal to the number of columns in x. Each element must be equal to "norm", "beta" or "gamma". |
| transform | A logical value indicating whether or not starting values should be obtained using the transformations SPH, PCS, PCR and SVD. The default is TRUE.   |

|                |   |
|----------------|---|
| hc_pairs       | The results from MBHAC obtained from the function call <code>hc()</code> using the <code>mclust</code> package. If <code>NULL</code> , the function obtains the results by calling <code>hc()</code> .                  |
| classification | A numeric vector representing a partitioning of the data <code>x</code> . If not <code>NULL</code> , the clustering identified by the vector takes preference over the clustering identified by <code>hc_pairs</code> . |

### Value

- `mixing_probs` A numeric vector indicating the starting values of the mixing proportions.
- `mvdc` A list of objects of class `mvdc`. Each element of the list corresponds to a mixture component and contains the starting values for the component distribution's marginal and copula parameters.
- `transformation` A character string indicating the transformation applied prior to performing MBHAC.
- `loglik` The log-likelihood corresponding of the model given the starting values of the parameters and the data.

### See Also

[hc](#)

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|                             |                                |
|-----------------------------|--------------------------------|
| <code>initialize_ecm</code> | <i>Starting values for ECM</i> |
|-----------------------------|--------------------------------|

---

### Description

Identifies optimal starting values for ECM using the results of MBHAC applied to the data. The data undergoes transformations to enhance separation amongst groups prior to performing MBHAC.

### Usage

```
initialize_ecm(
  x,
  K,
  margins,
  transform = FALSE,
  hc_pairs = NULL,
  classification = NULL,
  trace = TRUE
)
```

### Arguments

|                      |  |
|----------------------|--|
| <code>x</code>       | A numeric matrix or data frame of observations. Rows correspond to observations and columns correspond to variables.   |
| <code>K</code>       | The number of mixture componets.   |
| <code>margins</code> | A character vector specifying the marginal distributions of the components in the mixture. The vector must have a length equal to the number of columns in <code>x</code> . Each element must be equal to "norm", "beta" or "gamma". |

|                |   |
|----------------|---|
| transform      | A logical value indicating whether or not starting values should be obtained using the transformations SPH, PCS, PCR and SVD. The default is TRUE.  |
| hc_pairs       | The results from MBHAC obtained from the function call <code>hc()</code> using the <code>mclust</code> package. If NULL, the function obtains the results by calling <code>hc()</code> .                  |
| classification | A numeric vector representing a partitioning of the data <code>x</code> . If not NULL, the clustering identified by the vector takes preference over the clustering identified by <code>hc_pairs</code> . |
| trace          | A logical value indicating if an update regarding the initialization procedure's progress should be displayed.  |

**Value**

- `mixing_probs` A numeric vector indicating the starting values of the mixing proportions.
- `mvdc` A list of objects of class `mvdc`. Each element of the list corresponds to a mixture component and contains the starting values for the component distribution's marginal and copula parameters.
- `transformation` A character string indicating the transformation applied prior to performing MBHAC.
- `loglik` The log-likelihood corresponding of the model given the starting values of the parameters and the data.

**See Also**

[hc](#)

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|            |                                 |
|------------|---------------------------------|
| P2p_angles | <i>Tool for handling angles</i> |
|------------|---------------------------------|

---

**Description**

A tool for alternating between a matrix of angles and a vector of angles

**Usage**

```
P2p_angles(theta)
```

```
p2P_angles(angles)
```

**Arguments**

|        |                    |
|--------|--------------------|
| theta  | A matrix of angles |
| angles | A vector of angles |

**Value**

A vector of angles

A matrix of angles

---

`rho2angles`*Matrix conversion tool*

---

**Description**

A tool for converting a correlation matrix to a matrix of angles using Cholesky factor

**Usage**

```
rho2angles(rho_mat)
```

**Arguments**

`rho_mat`            A  $p$  times  $p$  correlation matrix to be converted to a matrix of angles.

**Value**

A  $(p - 1)$  times  $(p - 1)$  matrix of angles.

**See Also**

[angles2rho](#)

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