DISCRETE RANDOM VARIABLES

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Learning Intentions:

- To define discrete random variables.
- To define the probability distribution of a discrete random variable.

The outcomes of probability events can be listed as word descriptions or numbers. In this topic only numbers will be investigated.

A **random variable** uses numbers to describe the possible outcomes which could result from a random experiment.

We often use capital letters such as X to represent random variables.

Random variables can be either **discrete** or **continuous**.

A **discrete random variable** X has a set of distinct possible values.

For example, X could be:

- the number of wickets a bowler takes in an innings of cricket, so X could take the values $0, 1, 2, \ldots., 10$
- the number of defective light bulbs in a purchase order of 50, so X could take the values $0, 1, 2, \ldots., 50$.

To determine the value of a discrete random variable, we need to **count**.

A **continuous random variable** X can take any value within some interval on the number line.

For example, X could be:

- the heights of men, which would all lie in the interval $50 \text{ cm} < X < 250 \text{ cm}$
- the volume of water in a rainwater tank, which could lie in the interval $0 \text{ m}^3 < X < 100 \text{ m}^3$.

To determine the value of a continuous random variable, we need to **measure**.

This topic will only consider **discrete** random variables with a finite number of outcomes.

The outcomes are labelled $x_1, x_2, x_3 \dots\dots\dots, x_n$.

A supermarket has three checkouts A, B, and C. A government inspector checks the weighing scales for accuracy at each checkout. The random variable X is the number of accurate weighing scales at the supermarket.

a List the possible outcomes and the corresponding values of X.

b What value(s) of X correspond to there being:

i one accurate scale

ii at least one accurate scale?

a Possible outcomes:

A	B	C	X
✗	✗	✗	0
✓	✗	✗	1
✗	✓	✗	1
✗	✗	✓	1
✗	✓	✓	2
✓	✗	✓	2
✓	✓	✗	2
✓	✓	✓	3

b **i** $X = 1$

ii $X = 1$, 2, or 3

For each scenario:

- **i** Identify the random variable being considered.
- **ii** State whether the variable is continuous or discrete.
- **iii** Give possible values for the random variable.

a To measure the rainfall over a 24-hour period in Singapore, water is collected in a rain gauge.

b To investigate the stopping distance for a tyre with a new tread pattern, a braking experiment is carried out.

c To check the reliability of a new type of light switch, switches are repeatedly turned off and on until they fail.

2 **a** **i** $X =$ the height of water in the rain gauge
ii continuous **iii** $0 \leqslant X \leqslant 400$ mm

b **i** $X =$ stopping distance **ii** continuous
iii $0 \leqslant X \leqslant 50$ m

c **i** number of switches until failure
ii discrete **iii** any integer $\geqslant 1$

5 A supermarket has four checkouts A, B, C, and D. Management checks the weighing devices at each checkout. The random variable X is the number of weighing devices which are accurate.

a What values can X have?

b List the possible outcomes and the corresponding values of X.

c What value(s) of X correspond to:

i exactly two devices being accurate **ii** at least two devices being accurate?

6 Suppose three coins are tossed simultaneously. The random variable X is the number of heads that result.

a State the possible values of X.

b List the possible outcomes and the corresponding values of X.

c Are the possible values of X equally likely to occur? Explain your answer.

5 **a** $X = 0, 1, 2, 3,$ or 4

b

✓✓✓✓	✓✓✓✗	✓✓✗✗	✗✗✗✓	✗✗✗✗
	✓✓✗✓	✓✗✓✗	✗✗✓✗	
	✓✗✓✓	✓✗✗✓	✗✓✗✗	
	✗✓✓✓	✗✗✓✓	✓✗✗✗	
		✗✓✗✓		
		✗✓✓✗		
$(X = 4)$	$(X = 3)$	$(X = 2)$	$(X = 1)$	$(X = 0)$

c **i** $X = 2$ **ii** $X = 2, 3,$ or 4

6 **a** $X = 0, 1, 2,$ or 3

b

HHH	HHT	TTH	TTT
	HTH	THT	
	THH	HTT	
$(X = 3)$	$(X = 2)$	$(X = 1)$	$(X = 0)$

c No, for example there is probability $\frac{1}{8}$ that $X = 3$, and probability $\frac{3}{8}$ that $X = 2$.

Discrete probability distrubutions.

For any random variable, there is a corresponding **probability distribution** which describes the probability that the variable will take a particular value.

The probability that the variable X takes value x is denoted $\mathrm{P}(X = x)$.

If X is a random variable with possible values $\{x_1, x_2, x_3, ..., x_n\}$ and corresponding probabilities $\{p_1, p_2, p_3, ..., p_n\}$ such that $\mathrm{P}(X = x_i) = p_i, \ i = 1, ..., n,$ then:

- $0 \leqslant p_i \leqslant 1$ for all $i = 1, ..., n$
- $\sum_{i=1}^{n} p_i = p_1 + p_2 + p_3 + ... + p_n = 1$
- $\{p_1, ..., p_n\}$ describes the **probability distribution** of X.

Suppose X is the number of heads obtained when two coins are tossed.

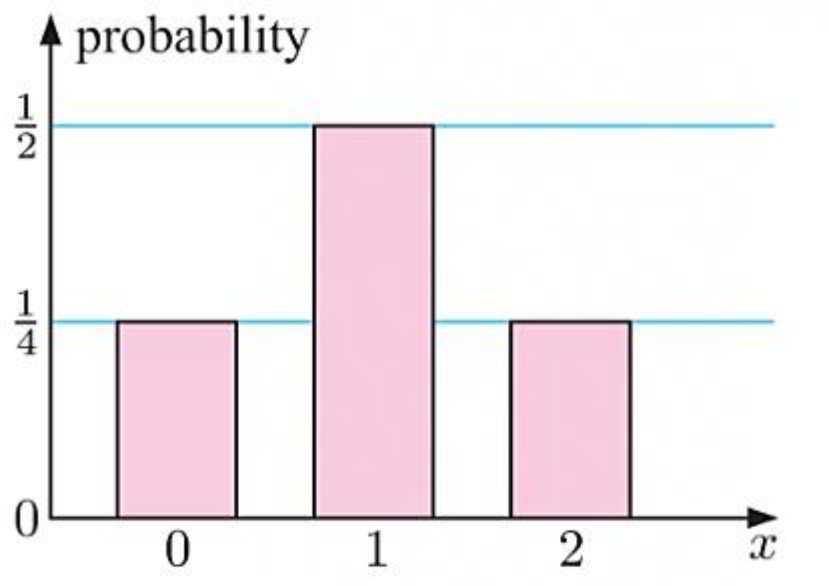
The possible values of X are $\{0, 1, 2\}$.

The corresponding probabilities are $\left\{\frac{1}{4}, \frac{1}{2}, \frac{1}{4}\right\}$.

Note that $0 \leq p_i \leq 1$ for each value of i and the probabilities sum to 1.

The probability distribution can be displayed in a table or graph.

x	0	1	2
$P(X = x)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$



Let X be the number of heads showing when a fair coin is tossed three times.

a Find the probability distribution of X and show that all the probabilities sum to 1.

b Find the probability that one or more heads show.

c Find the probability that more than one head shows.

$$p(0) = \Pr(X = 0) = \Pr(\{TTT\}) = \frac{1}{8}$$

$$p(1) = \Pr(X = 1) = \Pr(\{HTT, THT, TTH\}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

$$p(2) = \Pr(X = 2) = \Pr(\{HHT, HTH, THH\}) = \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}$$

$$p(3) = \Pr(X = 3) = \Pr(\{HHH\}) = \frac{1}{8}$$

Thus the probability distribution of X is:

x	0	1	2	3
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

b The probability that one or more heads shows is

$$\Pr(X \geq 1) = p(1) + p(2) + p(3) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$$

c The probability that more than one head shows is

$$\Pr(X > 1) = \Pr(X \geq 2) = p(2) + p(3) = \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

The random variable X represents the number of chocolate chips in a certain brand of biscuit, and is known to have the following probability distribution.

x	2	3	4	5	6	7
$p(x)$	0.01	0.25	0.40	0.30	0.02	0.02

Find:

a $\Pr(X \geq 4)$ **b** $\Pr(X \geq 4 \mid X > 2)$ **c** $\Pr(X < 5 \mid X > 2)$

a
$$\begin{aligned}\Pr(X \geq 4) &= \Pr(X = 4) + \Pr(X = 5) + \Pr(X = 6) + \Pr(X = 7)\\ &= 0.4 + 0.3 + 0.02 + 0.02\\ &= 0.74\end{aligned}$$

b
$$\begin{aligned}\Pr(X \geq 4 \mid X > 2) &= \frac{\Pr(X \geq 4)}{\Pr(X > 2)}\\ &= \frac{0.74}{0.99} \qquad \text{since } \Pr(X > 2) = 1 - 0.01 = 0.99\\ &= \frac{74}{99}\end{aligned}$$

c
$$\begin{aligned}\Pr(X < 5 \mid X > 2) &= \frac{\Pr(2 < X < 5)}{\Pr(X > 2)}\\ &= \frac{\Pr(X = 3) + \Pr(X = 4)}{\Pr(X > 2)}\\ &= \frac{0.65}{0.99}\\ &= \frac{65}{99}\end{aligned}$$

A discrete probability function is defined by the following table.

a Find the value of p.

b Hence calculate $P(X > 2)$.

x	0	1	2	3	4
$P(X = x)$	0.2	0.3	$2p$	p	0.05

$$0.2 + 0.3 + 2p + p + 0.05 = 1$$

$$3p + 0.55 = 1$$

$$p = 0.15$$

$$P(X > 2) = P(X = 3) + P(X = 4)$$

$$= 0.15 + 0.05$$

$$\mathrm{P}(X > 2) = 0.2$$

EXERCISE 2.02 (PAGE 58)

- Question 1-7