UNIFORM DISCRETE RANDOM VARIABLES

Uniform discrete random variables.

If the possible values $x_1, x_2, ..., x_n$ of a discrete random variable X all have the same probability $\frac{1}{n}$ of occurring, then X is a **uniform discrete random variable**.

An example of a uniform discrete random variable is the result X when a die is rolled. The possible values of X are 1, 2, 3, 4, 5, and 6, and each value has probability $\frac{1}{6}$ of occurring.

By contrast, if two dice are rolled, the sum of the resulting numbers Y is *not* a uniform discrete random variable.

A magazine store recorded the number of magazines purchased by its customers in one week. 23% purchased one magazine, 38% purchased two, 21% purchased three, 13% purchased four, and 5% purchased five. Let X be the number of magazines sold to a randomly selected customer.

a State the possible values of X.

b Construct a probability table for X.

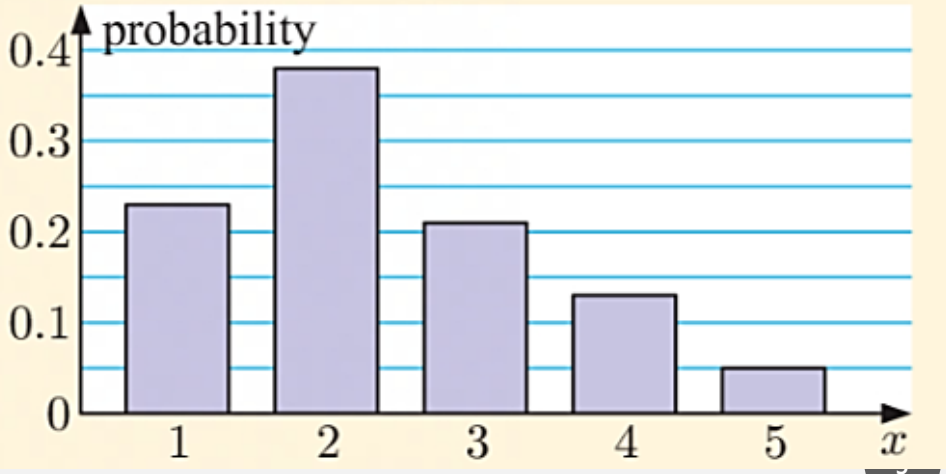
c Graph the probability distribution.

a $X = 1, 2, 3, 4,$ or 5

b

x	1	2	3	4	5
$P(X = x)$	0.23	0.38	0.21	0.13	0.05

c



A probability distribution of a random variable can be expressed as a **probability distribution function** $P(x) = P(X = x)$.

The domain is the set of possible values of the variable.

The range is the set of values in probability distribution.

Show that $P(x) = \frac{x^2 + 1}{34}$, $x = 1, 2, 3, 4$ is a valid probability distribution function.

$P(1) = \frac{2}{34}$, $P(2) = \frac{5}{34}$, $P(3) = \frac{10}{34}$, $P(4) = \frac{17}{34}$

All of these obey $0 \leqslant P(x_i) \leqslant 1$, $\sum_{i=1}^{n} P(x_i) = \frac{2}{34} + \frac{5}{34} + \frac{10}{34} + \frac{17}{34} = 1$

$\therefore$ $P(x)$ is a valid probability distribution function.

1 **a** State whether each of the following is a valid probability distribution:

i

x	1	2	3	4
$\mathrm{P}(X = x)$	0.2	0.4	0.15	0.25

ii

x	0	1	2	3
$\mathrm{P}(X = x)$	0.2	0.3	0.4	0.2

iii

x	0	1	2	3	4
$\mathrm{P}(X = x)$	0.2	0.2	0.2	0.2	0.2

iv

x	2	3	4	5
$\mathrm{P}(X = x)$	0.3	0.4	0.5	-0.2

b For which of the probability distributions in **a** is X a uniform random variable?

2 Find k in each of these probability distributions:

a

x	0	1	2
$\mathrm{P}(X = x)$	0.3	k	0.5

b

x	0	1	2	3
$\mathrm{P}(X = x)$	k	$2k$	$3k$	k

1 **a** **i** yes **ii** no **iii** yes **iv** no

b For **a iii**, X is a uniform random variable.

2 **a** $k = 0.2$ **b** $k = \frac{1}{7}$

3 Consider the probability distribution alongside.

x	0	1	2	3
$\mathrm{P}(X = x)$	0.1	0.25	0.45	a

a Find the value of a.

b Is X a uniform discrete random variable? Explain your answer.

d Find $\mathrm{P}(X \geqslant 2)$.

4 The probability distribution for Jason scoring X home runs in each game during his baseball career is given in the following table:

x	0	1	2	3	4	5
$P(x)$	a	0.3333	0.1088	0.0084	0.0007	0.0000

a State the value of $P(2)$.

b Find the value of a. Explain what this number means.

c Find the value of $P(1) + P(2) + P(3) + P(4) + P(5)$. Explain what this means.

d Draw a graph of $P(x)$ against x.

3 **a** $a = 0.2$

b No, as the probabilities of each outcome are not all equal.

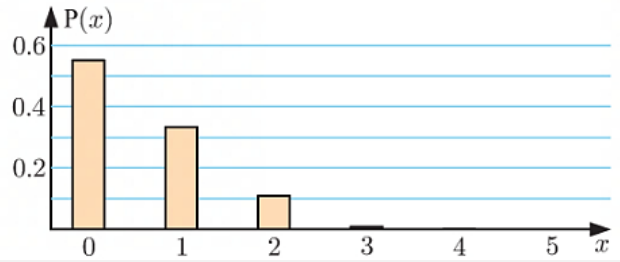
d $\mathrm{P}(X \geqslant 2) = 0.65$

4 **a** $P(2) = 0.1088$

b $a = 0.5488$ is the probability that Jason does not hit a home run in a game.

c $P(1) + P(2) + P(3) + P(4) + P(5) = 0.4512$ and is the probability that Jason will hit one or more home runs in a game.

d



9 Show that the following are valid probability distribution functions:

a $P(x) = \dfrac{x+1}{10}$ for $x = 0, 1, 2, 3$

b $P(x) = \dfrac{6}{11x}$ for $x = 1, 2, 3.$

10 Find k for the following probability distribution functions:

a $P(x) = k(x+2)$ for $x = 1, 2, 3$

b $P(x) = \dfrac{k}{x+1}$ for $x = 0, 1, 2, 3.$

9 **a** $P(0) = \frac{1}{10}$, $P(1) = \frac{2}{10}$, $P(2) = \frac{3}{10}$, $P(3) = \frac{4}{10}$

$0 \leqslant P(x_i) \leqslant 1$ in each case, and

$$\sum_{i=1}^{n} P(x_i) = \frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = 1$$

$\therefore$ $P(x)$ is a valid probability distribution function.

b $P(1) = \frac{6}{11}$, $P(2) = \frac{6}{22}$, $P(3) = \frac{6}{33}$

$0 \leqslant P(x_i) \leqslant 1$ in each case, and

$$\sum_{i=1}^{n} P(x_i) = \frac{6}{11} + \frac{6}{22} + \frac{6}{33} = 1$$

$\therefore$ $P(x)$ is a valid probability distribution function.

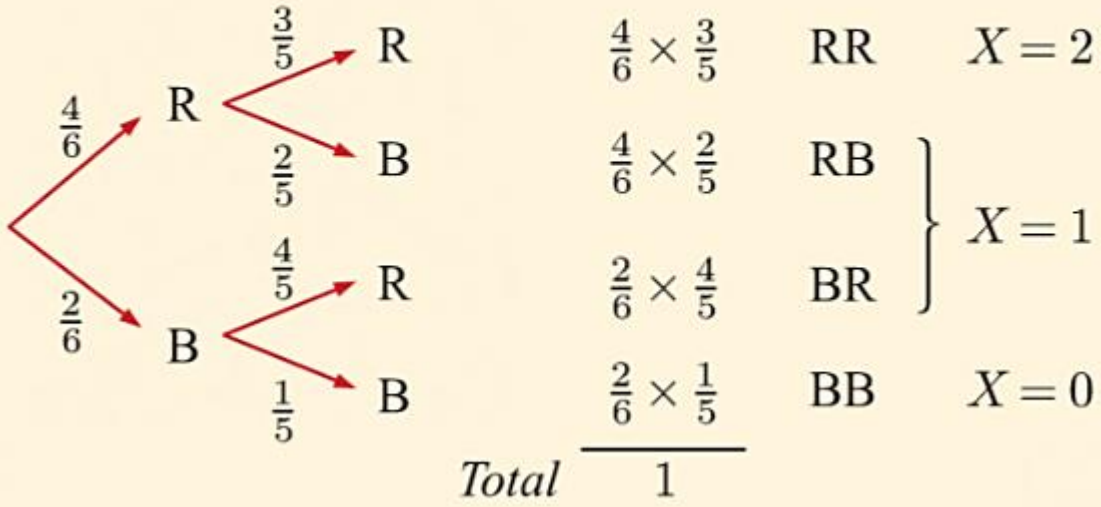
10 **a** $k = \frac{1}{12}$ **b** $k = \frac{12}{25}$

Two marbles are randomly selected without replacement from a bag containing 4 red and 2 blue marbles. Let X denote the number of red marbles selected.

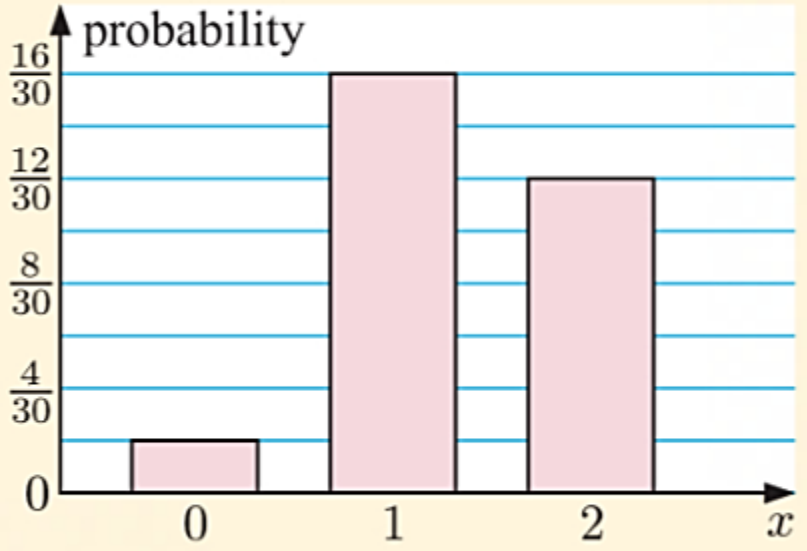
a Find the probability distribution of X.

b Illustrate the probability distribution using a graph.

a



b



x	0	1	2
$\mathrm{P}(X = x)$	$\frac{2}{30}$	$\frac{16}{30}$	$\frac{12}{30}$

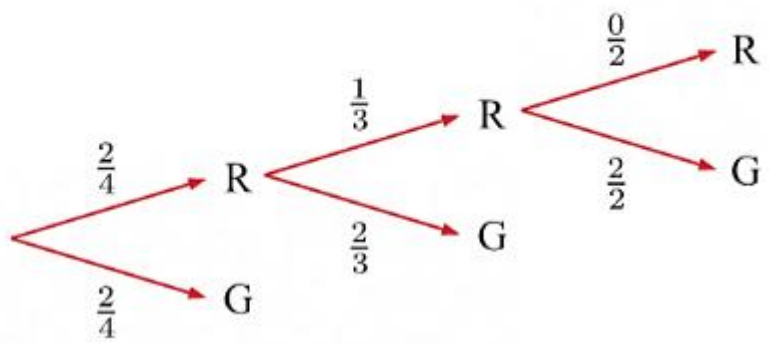
13 A hat contains 2 red balls and 2 green balls. Balls are randomly selected without replacement until a green ball is selected. Let X denote the total number of balls selected.

a State the possible values of X.

b Find the probability distribution of X.

13 **a** $X = 1$, 2, or 3

b 1st selection 2nd selection 3rd selection



$\frac{2}{4} \times \frac{1}{3} \times 1$	RRG	$X = 3$
$\frac{2}{4} \times \frac{2}{3}$	RG	$X = 2$
$\frac{2}{4}$	G	$X = 1$
Total 1		

x	1	2	3
$P(X = x)$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{6}$

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