# BASIC PROBABILITY

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<u>Learning Intentions:</u>

- Review the basic concept of probability

# BASIC PROBABILITY

The probability of an event $A$ in a sample space $S$ is written as $P(A)$ and is a real number between 0 and 1. The probability of the sample space, $P(S) = 1$ and the probability of a union of disjoint events is the sum of their probabilities.

For a finite sample space whose elements have equal probabilities,

$$P(A) = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}} = \frac{n(A)}{n(S)}$$

If $P(A) = 1$, then event $A$ is certain to occur.

If $P(A) = 0$, then event $A$ is impossible.

The complement of event $A$ is represented as $A'$ or $\bar{A}$. $A'$ means 'not $A$', so $P(A')$ is the probability that $A$ will not occur.

$P(A) + P(A') = 1$

$P(A') = 1 - P(A)$

The **addition rule** of probability states that:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$P(A \cup B)$ is also written as $P(A \text{ or } B)$.

$P(A \cap B)$ is also written as $P(A \text{ and } B)$.

**Mutually exclusive** events cannot occur simultaneously, so $P(A \cap B) = 0$. For mutually exclusive events: $P(A \cup B) = P(A) + P(B)$

The **conditional probability** of event $A$ given event $B$ is written as $P(A|B)$ or 'the probability of $A$ given $B$' and defined as $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$.

The **multiplication rule** for any events $A$ and $B$ is $P(A \cap B) = P(A|B) \times P(B)$.

Events are **independent** if $P(A|B) = P(A)$. The outcome of one does not affect the probability of the other, so $P(A \cap B) = P(A) \times P(B)$.

If one card is chosen at random from a well-shuffled deck of 52 cards, what is the probability that the card is:

**a** an ace **b** not a heart **c** an ace or a heart **d** either a king or an ace?

**a** Let $A$ be the event 'the card drawn is an ace'. A standard deck of cards contains four aces, so

$$\Pr(A) = \frac{4}{52} = \frac{1}{13}$$

**b** Let $H$ be the event 'the card drawn is a heart'. There are 13 cards in each suit, so

$$\Pr(H) = \frac{13}{52} = \frac{1}{4}$$

and therefore

$$\Pr(H') = 1 - \Pr(H) = 1 - \frac{1}{4} = \frac{3}{4}$$

**c** Using the addition rule:

$$\Pr(A \cup H) = \Pr(A) + \Pr(H) - \Pr(A \cap H)$$

Now $\Pr(A \cap H) = \frac{1}{52}$, since the event $A \cap H$ corresponds to drawing the ace of hearts. Therefore

$$\Pr(A \cup H) = \frac{1}{13} + \frac{1}{4} - \frac{1}{52} = \frac{4}{13}$$

**d** Let $K$ be the event 'the card drawn is a king'. We observe that $K \cap A = \varnothing$. That is, the events $K$ and $A$ are mutually exclusive. Hence

$$\Pr(K \cup A) = \Pr(K) + \Pr(A) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

500 people were questioned and classified according to age and whether or not they regularly use social media. The results are shown in the table.

Do you regularly use social media?

| | Age $< 25$ | Age $\geq 25$ | Total |
|---|---|---|---|
| Yes | 200 | 100 | 300 |
| No | 40 | 160 | 200 |
| Total | 240 | 260 | 500 |

One person is selected from these 500. Find the probability that:

**a** the person regularly uses social media

**b** the person is less than 25 years of age

**c** the person is less than 25 years of age and does not regularly use social media.

**Solution**

**a** $\Pr(\text{Yes}) = \frac{300}{500} = \frac{3}{5}$

**b** $\Pr(\text{Age} < 25) = \frac{240}{500} = \frac{12}{25}$

**c** $\Pr(\text{No} \cap \text{Age} < 25) = \frac{40}{500} = \frac{2}{25}$

Suppose that a die is tossed 1000 times and the following outcomes observed:

| Outcome | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| Frequency | 135 | 159 | 280 | 199 | 133 | 97 |

**a** Use this information to estimate the probability of observing a 6 when this die is rolled.

**b** What outcome would you predict to be most likely the next time the die is rolled?

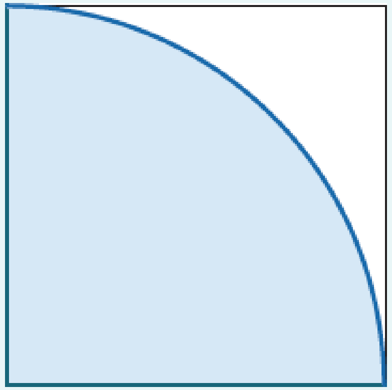
**Solution**

**a** $\Pr(6) \approx \dfrac{97}{1000} = 0.097$

**b** The most likely outcome is 3, since it has the highest relative frequency.

A dartboard consists of a square of side length 2 metres containing a blue one-quarter of a circular disc centred at the bottom-left vertex of the square, as shown.

If a dart thrown at the square is equally likely to hit any part of the square, and it hits the square every time, find the probability of it hitting the blue region.



**Solution**

Area of blue region $= \dfrac{1}{4}\pi r^2 = \dfrac{1}{4}\pi \times 4 = \pi \text{ m}^2$

Area of dartboard $= 2 \times 2 = 4 \text{ m}^2$

$$\Pr(\text{hitting blue region}) = \frac{\text{area of blue region}}{\text{area of dartboard}} = \frac{\pi}{4}$$

In a certain town, the probability that it rains on any Monday is 0.21. If it rains on Monday, then the probability that it rains on Tuesday is 0.83. If it does not rain on Monday, then the probability of rain on Tuesday is 0.3. For a given week, find the probability that it rains:

**a** on both Monday and Tuesday

**b** on Tuesday.

**a** The probability that it rains on both Monday and Tuesday is given by

$$\begin{aligned} \Pr(T \cap M) &= 0.21 \times 0.83 \\ &= 0.1743 \end{aligned}$$

**b** The probability that it rains on Tuesday is given by

$$\begin{aligned} \Pr(T) &= \Pr(T \cap M) + \Pr(T \cap M') \\ &= 0.1743 + 0.237 \\ &= 0.4113 \end{aligned}$$

As part of an evaluation of the school canteen, all students at a Senior Secondary College (Years 10–12) were asked to rate the canteen as poor, good or excellent. The results are shown in the table.

| | Year | | | |
|---|---|---|---|---|
| Rating | 10 | 11 | 12 | Total |
| Poor | 30 | 20 | 10 | 60 |
| Good | 80 | 65 | 35 | 180 |
| Excellent | 60 | 65 | 35 | 160 |
| Total | 170 | 150 | 80 | 400 |

What is the probability that a student chosen at random from this college:

**a** is in Year 12

**b** is in Year 12 and rates the canteen as excellent

**c** is in Year 12, given that they rate the canteen as excellent

**d** rates the canteen as excellent, given that they are in Year 12?

**a** $\Pr(T) = \frac{80}{400} = \frac{1}{5}$

**b** $\Pr(T \cap E) = \frac{35}{400} = \frac{7}{80}$

**c** $\Pr(T \mid E) = \frac{35}{160} = \frac{7}{32}$

**d** $\Pr(E \mid T) = \frac{35}{80} = \frac{7}{16}$

# EXERCISE 2.01 (PAGE 54)

- Question 4, 5, 6, 9, 11, 12, 13