APPLICATIONS OF DISCRETE RANDOM VARIABLES

A dice game involves rolling a pair of dice and finding the total of the numbers on the upper faces. If the total is 11 or 12, you win $10. If the total is 2, 3 or 4, you win $4. For any other total, you lose $2. Is the game fair?

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(11 \text{ or } 12) = \frac{3}{36} = \frac{1}{12}$$

$$P(2, 3 \text{ or } 4) = \frac{6}{36} = \frac{1}{6}$$

$$P(\text{others}) = \frac{27}{36} = \frac{3}{4}$$

Let X be the amount won on a roll.

x	10	4	-2
$P(x)$	$\dfrac{1}{12}$	$\dfrac{1}{6}$	$\dfrac{3}{4}$

$$E(X) = 10 \times \frac{1}{12} + 4 \times \frac{1}{6} + (-2) \times \frac{3}{4} = 0$$

The game is fair because you win $0 (and lose $0) in the long run, so you 'break even'.

A game consists of a wheel divided into twelve equal sectors numbered 1 through 12. You may place a $2 bet on any number. If your number comes up, you get $18 back. That is, you win $16 and get your $2 back.

a What is the house percentage?

b How much should you win for the game to be fair?

Let X be the amount won on a bet.

x	+16	−2
$P(x)$	$\dfrac{1}{12}$	$\dfrac{11}{12}$

$E(X) = 16 \times \frac{1}{12} + (-2) \times \frac{11}{12}$

$\quad = -0.5$

You would expect to lose 50c on each bet of $2.

House percentage $= \dfrac{0.50}{2.00}$

$\quad = 25\%$

Let $a be the amount won on a bet.

x	+a	−2
$P(x)$	$\dfrac{1}{12}$	$\dfrac{11}{12}$

$E(X) = a \times \frac{1}{12} + -2 \times \frac{11}{12}$

To be fair, $E(X) = 0$

$a \times \dfrac{1}{12} + (-2) \times \dfrac{11}{12} = 0$

$a = 22$

For the game to be fair, you should win $22, so you should get back $24.

An insurance company offers a policy that pays out $50 000 on death. The company wishes to make a profit of $225 on this type of policy. It is known that the probability of a male aged 38 dying is 0.009. What premium will the insurance company need to charge a 38-year-old male in order to achieve the desired profit?

Expected payout $= 50000 \times 0.009 = \$450$

Premium = expected payout + profit

$$= 450 + 225$$
$$= \$675$$

The insurance company need to charge $675.

EXERCISE 2.08 (PAGE 87)

- Question 2, 4, 6, 9, 11, 15, 16, 17, 19