THE BINOMIAL DISTRIBUTION

The Binomial Distribution

Suppose $X =$ the number of blues which result from spinning this spinner **once.**

X is a Bernoulli random variable with probability of success of $p = \frac{3}{4}$

If the spinner is spun n times and the number of times a blue occurs is counted then a **binomial experiment** has been performed.

Consider an experiment for which there are only two possible results: **success** if some event occurs, or **failure** if the event does not occur.

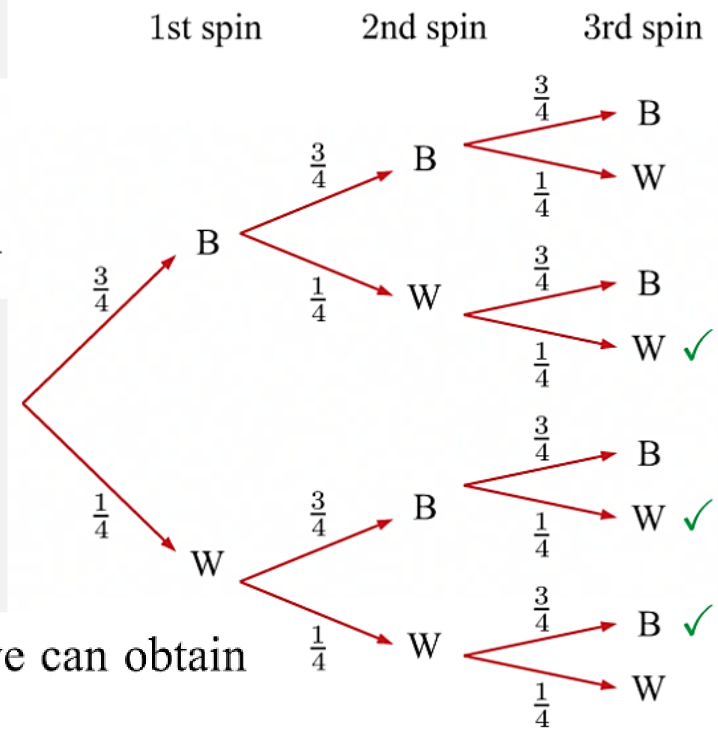
In any trial of the experiment, the probability of success is p, and the probability of failure is $1 - p$.

If we repeat this experiment in a number of **independent trials**, we call it a **binomial experiment**.

The **binomial random variable** X is the total number of successes in n trials.

For the blue and white spinner above, let $X =$ the number of blues which result from spinning the spinner three times. X is a binomial random variable which can take the values 0, 1, 2, or 3.

Let the random variable X be the number of "successes" or blue results, so $X = 0, 1, 2,$ or 3.



$$\begin{aligned} P(X=0) &= P(\text{none are blue}) \\ &= \tfrac{1}{4} \times \tfrac{1}{4} \times \tfrac{1}{4} \\ &= \left(\tfrac{1}{4}\right)^3 \end{aligned}$$

$$\begin{aligned} P(X=1) &= P(1 \text{ blue and } 2 \text{ white}) \\ &= P(BWW \text{ or } WBW \text{ or } WWB) \\ &= \left(\tfrac{3}{4}\right)\left(\tfrac{1}{4}\right)^2 \times 3 \quad \{\text{the 3 branches} \checkmark\} \end{aligned}$$

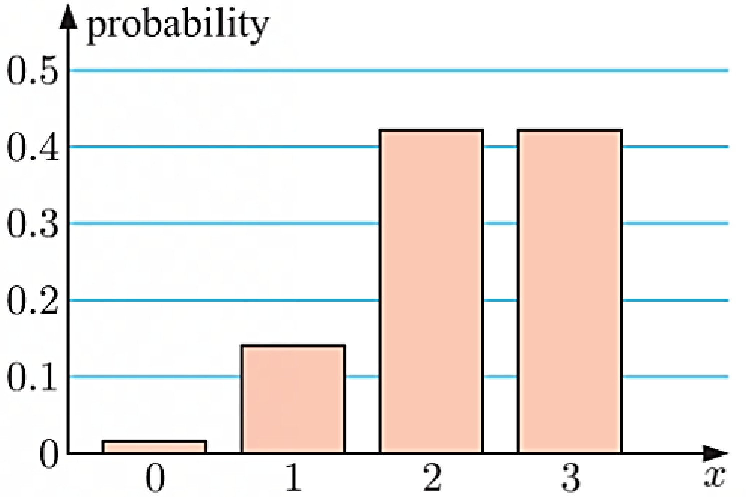
$$\begin{aligned} P(X=2) &= P(2 \text{ blue and } 1 \text{ white}) \\ &= P(BBW \text{ or } BWB \text{ or } WBB) \\ &= \left(\tfrac{3}{4}\right)^2\left(\tfrac{1}{4}\right) \times 3 \end{aligned}$$

$$\begin{aligned} P(X=3) &= P(3 \text{ blues}) \\ &= \left(\tfrac{3}{4}\right)^3 \end{aligned}$$

Recall from last year that $C_k^n = \binom{n}{k} = \dfrac{n!}{k!(n-k)!}$ is the number of *ways* in which we can obtain k successes from n trials.

Notice that:

$$\begin{aligned} P(X=0) &= \left(\tfrac{1}{4}\right)^3 &&= \binom{3}{0}\left(\tfrac{3}{4}\right)^0\left(\tfrac{1}{4}\right)^3 \approx 0.0156 \\ P(X=1) &= 3\left(\tfrac{1}{4}\right)^2\left(\tfrac{3}{4}\right)^1 &&= \binom{3}{1}\left(\tfrac{3}{4}\right)^1\left(\tfrac{1}{4}\right)^2 \approx 0.1406 \\ P(X=2) &= 3\left(\tfrac{1}{4}\right)^1\left(\tfrac{3}{4}\right)^2 &&= \binom{3}{2}\left(\tfrac{3}{4}\right)^2\left(\tfrac{1}{4}\right)^1 \approx 0.4219 \\ P(X=3) &= \left(\tfrac{3}{4}\right)^3 &&= \binom{3}{3}\left(\tfrac{3}{4}\right)^3\left(\tfrac{1}{4}\right)^0 \approx 0.4219 \end{aligned}$$



So, $P(X=x) = \binom{3}{x}\left(\tfrac{3}{4}\right)^x\left(\tfrac{1}{4}\right)^{3-x}$ where $x = 0, 1, 2, 3$.

Consider a binomial experiment for which p is the probability of a *success* and $1-p$ is the probability of a *failure*.

If there are n independent trials then the probability that there are k *successes* and $n-k$ *failures* is

$$\mathrm{P}(X=k) = \underbrace{\binom{n}{k}}_{\substack{\text{number of ways the } k \text{ successes} \\ \text{can be ordered amongst the } n \text{ trials}}} \underbrace{p^k(1-p)^{n-k}}_{\substack{\text{probability of obtaining } k \text{ successes} \\ \text{and } n-k \text{ failures in a particular order}}} \quad \text{where} \quad k = 0, 1, 2, 3, 4, \ldots., n.$$

$\mathrm{P}(X=k)$ is the **binomial probability distribution function**.

If X is the random variable of a binomial experiment with parameters n and p, then we write $\boldsymbol{X \sim \mathrm{B}(n, p)}$ where $\sim$ reads "*is distributed as*".

a Expand $\left(\frac{9}{10}+\frac{1}{10}\right)^5$.

b An archer has a 90% chance of hitting a target with each arrow. If 5 arrows are fired, determine the chance of hitting the target:

i twice only **ii** at most 3 times.

a $\left(\frac{9}{10}+\frac{1}{10}\right)^5$ $= \sum_{k=0}^{5} \binom{5}{k} \left(\frac{9}{10}\right)^k \left(\frac{1}{10}\right)^{5-k}$

$$= \left(\frac{1}{10}\right)^5 + 5\left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^4 + 10\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^3 + 10\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^2 + 5\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right) + \left(\frac{9}{10}\right)^5$$

b The probability of success with each arrow is $p=\frac{9}{10}$.

Let X be the number of arrows that hit the target.

$$\underbrace{\left(\frac{1}{10}\right)^5}_{\substack{\mathrm{P}(X=0)\\ \text{5 misses}}} + \underbrace{5\left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^4}_{\substack{\mathrm{P}(X=1)\\ \text{1 hit}\\ \text{4 misses}}} + \underbrace{10\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^3}_{\substack{\mathrm{P}(X=2)\\ \text{2 hits}\\ \text{3 misses}}} + \underbrace{10\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^2}_{\substack{\mathrm{P}(X=3)\\ \text{3 hits}\\ \text{2 misses}}} + \underbrace{5\left(\frac{9}{10}\right)^4\left(\frac{1}{10}\right)}_{\substack{\mathrm{P}(X=4)\\ \text{4 hits}\\ \text{1 miss}}} + \underbrace{\left(\frac{9}{10}\right)^5}_{\substack{\mathrm{P}(X=5)\\ \text{5 hits}}}$$

i P(hits twice only) $= \mathrm{P}(X=2)$

$= 10\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^3$

$= 0.0081$

ii P(hits at most 3 times) $= \mathrm{P}(X \leqslant 3)$

$= \mathrm{P}(X=0)+\mathrm{P}(X=1)+\mathrm{P}(X=2)+\mathrm{P}(X=3)$

$= \left(\frac{1}{10}\right)^5 + 5\left(\frac{9}{10}\right)\left(\frac{1}{10}\right)^4 + 10\left(\frac{9}{10}\right)^2\left(\frac{1}{10}\right)^3 + 10\left(\frac{9}{10}\right)^3\left(\frac{1}{10}\right)^2$ ≈ 0.0815

1 For which of these probability experiments does the binomial distribution apply? Explain your answers.

a A coin is thrown 100 times. The variable is the number of heads.

b One hundred coins are each thrown once. The variable is the number of heads.

c A box contains 5 blue and 3 red marbles. I draw out 5 marbles one at a time, replacing the marble before the next is drawn. The variable is the number of red marbles drawn.

d A box contains 5 blue and 3 red marbles. I draw out 5 marbles without replacement. The variable is the number of red marbles drawn.

e A large bin contains ten thousand bolts, 1% of which are faulty. I draw a sample of 10 bolts from the bin. The variable is the number of faulty bolts.

1 **a** The binomial distribution applies, as tossing a coin has two possible outcomes (a head or a tail) and each toss is independent of every other toss.

b The binomial distribution applies, as this is equivalent to tossing one coin 100 times.

c The binomial distribution applies as we can draw out a red or a blue marble with the same chances each time.

d The binomial distribution does not apply as the result of each draw is dependent upon the results of previous draws.

e The binomial distribution does not apply, assuming that ten bolts are drawn without replacement, as we do not have a repetition of independent trials. However, since there is such a large number of bolts in the bin, the trials are approximately independent, so the distribution is approximately binomial.

2 **a** Expand $(p+q)^4$.

b If a coin is tossed *four* times, what is the probability of getting 3 heads?

3 **a** Expand $(p+q)^5$.

b If *five* coins are tossed simultaneously, what is the probability of getting:

i 4 heads and 1 tail in any order **ii** 2 heads and 3 tails in any order

iii 4 heads and then 1 tail?

2 **a** $(p+q)^4 = p^4 + 4p^3q + 6p^2q^2 + 4pq^3 + q^4$

b $P(3 \text{ heads}) = 4p^3q$

$= 4\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right) \qquad \{\text{as } p = q = \frac{1}{2}\}$

$= \frac{1}{4}$

3 **a** $(p+q)^5 = p^5 + 5p^4q + 10p^3q^2 + 10p^2q^3 + 5pq^4 + q^5$

b **i** P(4H and 1T)

$= 5p^4q$

$= 5\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)$

$= \frac{5}{32}$

ii P(2H and 3T)

$= 10p^2q^3$

$= 10\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^3$

$= \frac{10}{32}$

$= \frac{5}{16}$

iii P(HHHHT)

$= \left(\frac{1}{2}\right)^4 \times \frac{1}{2}$

$= \frac{1}{32}$

4 **a** Expand $\left(\frac{2}{3}+\frac{1}{3}\right)^4$.

b A box of chocolates contains strawberry creams and almond centres in the ratio $2:1$. Four chocolates are selected at random, with replacement. Find the probability of getting:

i all strawberry creams

ii two of each type

iii at least 2 strawberry creams.

4 **a** $\left(\frac{2}{3}+\frac{1}{3}\right)^4 = \left(\frac{2}{3}\right)^4 + 4\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right) + 6\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 + 4\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4$

b $P(S) = \frac{2}{3}, \quad P(S') = \frac{1}{3}$

i $$\begin{aligned} P(\text{all } S) &= \left(\frac{2}{3}\right)^4 \\ &= \frac{16}{81} \end{aligned}$$

ii $$\begin{aligned} P(\text{two of each}) &= 6\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 \\ &= \frac{8}{27} \end{aligned}$$

iii $$\begin{aligned} P(\text{at least 2 strawberry creams}) &= 1 - P(\text{less than 2 strawberry creams}) \\ &= 1 - \left(4\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3 + \left(\frac{1}{3}\right)^4\right) \\ &= 1 - \frac{8}{81} - \frac{1}{81} \\ &= \frac{72}{81} \\ &= \frac{8}{9} \end{aligned}$$

How to find Binomial distributions using a calculator.
https://www.youtube.com/watch?v=RnuggcyZMSA

$P(x = \#)$ use binompdf

$P(x \leq \#)$ use binomcdf

$P(x \geq \#)$ use 1 - binomcdf

1 5% of electric light bulbs are defective at manufacture. 6 bulbs are randomly tested, with each one being replaced before the next is chosen. Determine the probability that:

a two are defective **b** at least one is defective.

2 Records show that 6% of the items assembled on a production line are faulty. A random sample of 12 items is selected with replacement. Find the probability that:

a none will be faulty **b** at most one will be faulty

c at least two will be faulty **d** less than four will be faulty.

3 The local bus service does not have a good reputation. The 8 am bus will run late on average two days out of every five. For any week of the year taken at random, find the probability of the 8 am bus being on time:

a all 7 days **b** only on Monday

c on any 6 days **d** on at least 4 days.

4 In a multiple choice test there are 10 questions. Each question has 5 choices, one of which is correct. Raj knows absolutely nothing about the subject, and guesses each answer at random. Given that the pass mark is 70%, determine the probability that he will pass.

5 An infectious flu virus is spreading through a school. The probability of a randomly selected student having the flu next week is 0.3. Mr C has a class of 25 students.

a Calculate the probability that 2 or more students from Mr C's class will have the flu next week.

b If more than 20% of the students have the flu next week, a class test will have to be cancelled. What is the probability that the test will be cancelled?

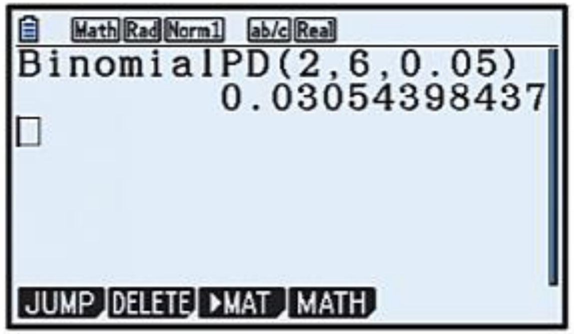
1	**a**	≈ 0.0305	**b**	≈ 0.265				
2	**a**	≈ 0.476	**b**	≈ 0.840	**c**	≈ 0.160	**d**	≈ 0.996
3	**a**	≈ 0.0280	**b**	$\approx 0.002\,46$	**c**	≈ 0.131	**d**	≈ 0.710
4		$\approx 0.000\,864$	**5**	**a** ≈ 0.998	**b**	≈ 0.807		

1 Let X be the number of defective light bulbs.

$n = 6$, so $X = 0, 1, 2, 3, 4, 5$, or 6, and $p = 5\% = 0.05$

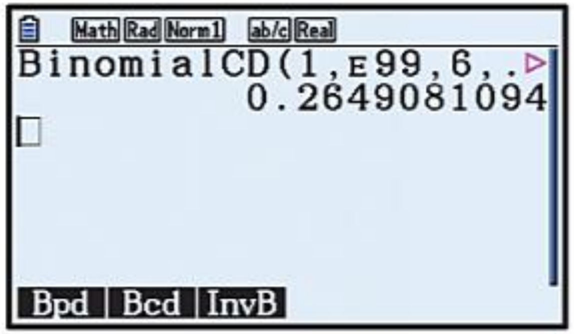
$\therefore \quad X \sim \mathrm{B}(6, 0.05)$

a



$\mathrm{P}(X = 2) \approx 0.0305$

b



$\mathrm{P}(X \geqslant 1) \approx 0.265$

2 Let X be the number of faulty items.

$n = 12$, so $X = 0, 1, 2, 3, \ldots\ldots,$ or 12, and $p = 6\% = 0.06$

$\therefore \quad X \sim \mathrm{B}(12, 0.06)$

a P(none will be faulty)

$= \mathrm{P}(X = 0)$

$= \binom{12}{0}(0.06)^0(0.94)^{12}$

≈ 0.476

b P(at most one is faulty)

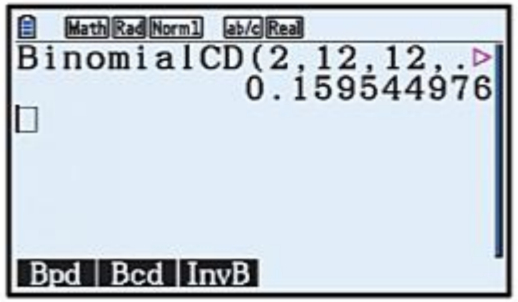
$= \mathrm{P}(X \leqslant 1)$

$= \mathrm{P}(X = 0) + \mathrm{P}(X = 1)$

$\approx 0.476 + \binom{12}{1}(0.06)^1(0.94)^{11}$

≈ 0.840

c



P(at least two are faulty) $= \mathrm{P}(X \geqslant 2)$

≈ 0.160

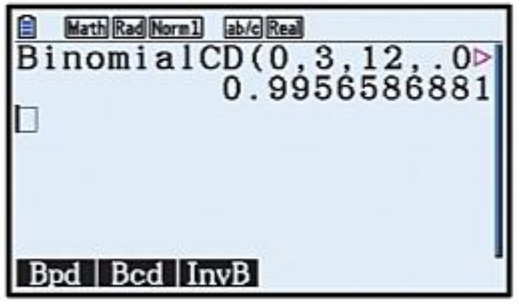
or P(at least two are faulty)

$= 1 - $ P(at most one is faulty)

$\approx 1 - 0.840$ {from **b**}

≈ 0.160

d



P(less than four are faulty) $= \mathrm{P}(X < 4)$

$= \mathrm{P}(X \leqslant 3)$

≈ 0.996

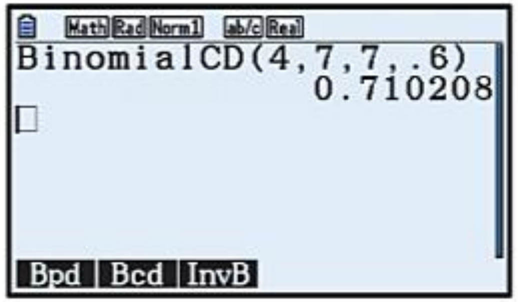
3 X is the random variable for the number of times in a week that the bus is on time.
Since it is late 2 in every 5 days, and on time 3 in every 5 days, $p = \frac{3}{5} = 0.6$.
$X = 0, 1, 2, 3, 4, 5, 6, \text{ or } 7$ and $X \sim \text{B}(7, 0.6)$.

a $\text{P}(X = 7) = \binom{7}{7}(0.6)^7(0.4)^0$
≈ 0.0280

b $\text{P(on time only on Monday)} = 0.6 \times (0.4)^6$
$\approx 0.002\,46$

c $\text{P}(X = 6) = \binom{7}{6}(0.6)^6(0.4)$
≈ 0.131

d



$\text{P}(X \geqslant 4) \approx 0.710$

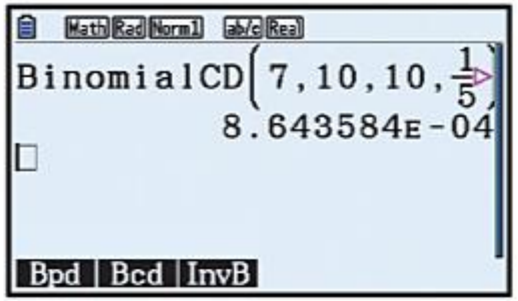
4 Let X denote the number of questions Raj answers correctly.

$n = 10$, so $X = 0, 1, 2, \ldots, \text{ or } 10$ and $p = \frac{1}{5}$

$\therefore \quad X \sim \mathrm{B}(10, \frac{1}{5})$

$\mathrm{P}(\text{Raj passes}) = \mathrm{P}(X \geqslant 7)$

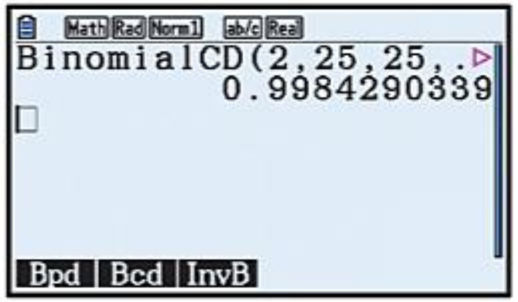
$\approx 0.000\,864$ {or about 9 in 10 000}



5 X is the random variable for the number of students with the flu.

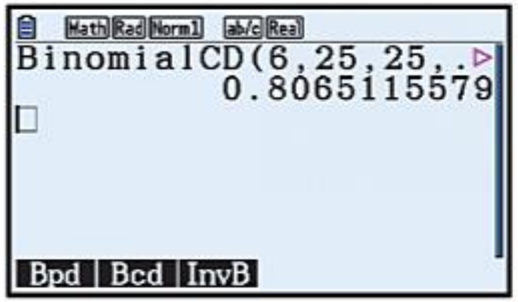
$\therefore \quad X = 0, 1, 2, 3, \ldots, \text{ or } 25$ and $X \sim \mathrm{B}(25, 0.3)$.

a



$\mathrm{P}(X \geqslant 2) \approx 0.998$

b



20% of $25 = 5$

$\therefore \quad \mathrm{P}(\text{test cancelled}) = \mathrm{P}(X \geqslant 6)$

≈ 0.807

At a particular telemarketing company, it is known that the probability that a call to a potential client (cold calling) results in a sale is 0.05. What is the least number of calls that must be made to ensure that the probability of making at least 2 sales is 90%?

There are 2 outcomes (sale or not), p is fixed and the variable is the number of "successes". The calls are independent. Determining the number of calls will make it binomial.

$$P(X \geq 2) > 0.9$$

$$1 - P(X < 2) > 0.9$$

$$P(X < 2) < 0.1$$

$$P(X < 2) = P(X = 0) + P(X = 1)$$

$$= \binom{n}{0}(0.05)^0(0.95)^n + \binom{n}{1}(0.05)^1(0.95)^{n-1}$$

$$= 0.95^n + n \times 0.05(0.95)^{n-1}$$

$$0.95^n + n \times 0.05(0.95)^{n-1} < 0.1$$

$$0.95^n + n \times 0.05(0.95)^{n-1} = 0.1$$

$$n = 76.336\ldots$$

At least 77 calls must be made to ensure the probability of making at least 2 sales is 90%.

The probability of Jacqui hitting a target is $\frac{1}{4}$.

a If she fires 7 times, what is the probability of her hitting the target at least twice?

b How many times must she fire so that the probability of hitting the target at least once is greater than $\frac{2}{3}$?

a $n = 7, P(X \geq 2) = 1 - P(X \leq 1) = 0.555$

b $n = ?, P(X \geq 1) > \dfrac{2}{3}$

$P(X \geq 1) = 1 - P(x = 0) > \dfrac{2}{3}$

i.e. $P(x = 0) < \dfrac{1}{3}$

$\Rightarrow \binom{n}{0}\left(\dfrac{1}{4}\right)^0\left(\dfrac{3}{4}\right)^n < \dfrac{1}{3}$

$\Rightarrow \left(\dfrac{3}{4}\right)^n < \dfrac{1}{3}$

$n = 4$

The mean and standard deviation of a binomial distribution

Suppose X is a binomial random variable with parameters n and p, so $X \sim \mathrm{B}(n, p)$.

- The **mean** of X is $\boldsymbol{\mu = np}$.
- The **standard deviation** of X is $\boldsymbol{\sigma = \sqrt{np(1-p)}}$.
- The **variance** of X is $\boldsymbol{\sigma^2 = np(1-p)}$.

A fair die is rolled twelve times, and X is the number of sixes that result. Find the mean and standard deviation of X.

This is a binomial distribution with $n = 12$ and $p = \frac{1}{6}$, so $X \sim \mathrm{B}(12, \frac{1}{6})$.

$$\begin{aligned} \mu &= np \\ &= 12 \times \tfrac{1}{6} \\ &= 2 \end{aligned} \qquad \text{and} \qquad \begin{aligned} \sigma &= \sqrt{np(1-p)} \\ &= \sqrt{12 \times \tfrac{1}{6} \times \tfrac{5}{6}} \\ &\approx 1.291 \end{aligned}$$

We expect a six to be rolled 2 times, with standard deviation 1.291 .

1 Suppose $X \sim \mathrm{B}(6, p)$. For each of the following cases:

i Find the mean and standard deviation of X.

ii Graph the distribution using a column graph.

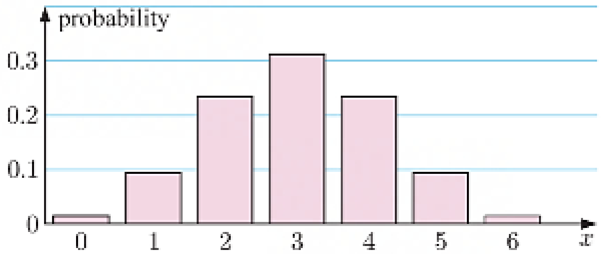
iii Comment on the shape of the distribution.

a $p = 0.5$ **b** $p = 0.2$ **c** $p = 0.8$

1 a i $\mu = np = 6 \times 0.5 = 3$ $\quad \sigma = \sqrt{np(1-p)} = \sqrt{6 \times 0.5 \times 0.5} \approx 1.22$

ii

x_i	0	1	2	3	4	5	6
$P(x_i)$	0.0156	0.0938	0.2344	0.3125	0.2344	0.0938	0.0156

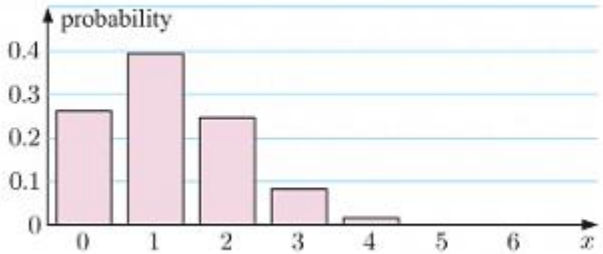


iii The distribution is symmetric.

b i $\mu = np = 6 \times 0.2 = 1.2$ $\quad \sigma = \sqrt{np(1-p)} = \sqrt{6 \times 0.2 \times 0.8} \approx 0.980$

ii

x_i	0	1	2	3	4	5	6
$P(x_i)$	0.2621	0.3932	0.2458	0.0819	0.0154	0.0015	0.0001



iii The distribution is positively skewed.

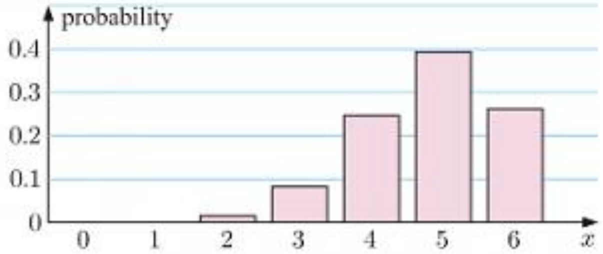
c **i** $\mu = np$ $\quad\quad$ $\sigma = \sqrt{np(1-p)}$

$= 6 \times 0.8$ $\quad\quad$ $= \sqrt{6 \times 0.8 \times 0.2}$

$= 4.8$ $\quad\quad$ ≈ 0.980

ii

x_i	0	1	2	3	4	5	6
$P(x_i)$	0.0001	0.0015	0.0154	0.0819	0.2458	0.3932	0.2621



iii The distribution is negatively skewed, and is the exact reflection of the distribution

2 A coin is tossed 10 times and X is the number of heads which occur. Find the mean and variance of X.

2 $X \sim \mathrm{B}(10,\ 0.5)$ mean $\mu = np = 10 \times \frac{1}{2} = 5$ and variance $\sigma^2 = np(1-p) = 10 \times \frac{1}{2} \times \frac{1}{2} = 2.5$

$$\begin{aligned} \mu &= np \\ &= 10 \times \tfrac{1}{2} \\ &= 5 \end{aligned} \qquad \begin{aligned} \sigma^2 &= np(1-p) \\ &= 10 \times \tfrac{1}{2} \times \tfrac{1}{2} \\ &= 2.5 \end{aligned}$$

3 Suppose $X \sim \mathrm{B}(2, p)$, so $P(x) = \binom{2}{x} p^x (1-p)^{2-x}$.

a Find $P(0)$, $P(1)$, and $P(2)$.

b Use $\mu = \sum p_i x_i$ to show that $\mu = 2p$.

c Use $\sigma = \sqrt{\sum x_i{}^2 p_i - \mu^2}$ to show that $\sigma = \sqrt{2p(1-p)}$.

3 $X \sim \mathrm{B}(2, p)$

a $P(0) = \binom{2}{0} p^0 (1-p)^2$
$= (1-p)^2$

$P(1) = \binom{2}{1} p^1 (1-p)^1$
$= 2p(1-p)$

$P(2) = \binom{2}{2} p^2 (1-p)^0$
$= p^2$

b
$$\begin{aligned}\mu &= \sum x_i p_i \\ &= 0(1-p)^2 + 1(2p(1-p)) + 2p^2 \\ &= 2p(1-p) + 2p^2 \\ &= 2p - 2p^2 + 2p^2 \\ &= 2p \quad \text{as required}\end{aligned}$$

c
$$\begin{aligned}\sigma &= \sqrt{\sum x_i{}^2 p_i - \mu^2} \\ &= \sqrt{0^2 \times (1-p)^2 + 1^2 \times (2p(1-p)) + 2^2 \times p^2 - (2p)^2} \\ &= \sqrt{2p(1-p) + 4p^2 - 4p^2} \\ &= \sqrt{2p(1-p)} \quad \text{as required}\end{aligned}$$

4 Bolts produced by a machine vary in quality. The probability that a given bolt is defective is 0.04. Random samples of 30 bolts are taken from the week's production.

a If X is the number of defective bolts in a sample, find the mean and standard deviation of X.

b If Y is the number of non-defective bolts in a sample, find the mean and standard deviation of Y.

4 **a** $X \sim \mathrm{B}(30, 0.04)$

$$\begin{aligned} \mu_X &= np \\ &= 30 \times 0.04 \\ &= 1.2 \\ \sigma_X &= \sqrt{np(1-p)} \\ &= \sqrt{30 \times 0.04 \times 0.96} \\ &\approx 1.07 \end{aligned}$$

b $Y \sim \mathrm{B}(30, 0.96)$

$$\begin{aligned} \mu_Y &= np \\ &= 30 \times 0.96 \\ &= 28.8 \\ \sigma_Y &= \sqrt{np(1-p)} \\ &= \sqrt{30 \times 0.96 \times 0.04} \\ &\approx 1.07 \end{aligned}$$

5 A city restaurant knows that 13% of reservations are not honoured, which means the group does not arrive. Suppose the restaurant receives 30 reservations. Let the random variable X be the number of groups that do not arrive. Find the mean and standard deviation of X.

5 $X \sim \mathrm{B}(30, 0.13)$

$$\begin{aligned} \mu &= np \\ &= 30 \times 0.13 \\ &= 3.9 \end{aligned}$$

$$\begin{aligned} \sigma &= \sqrt{np(1-p)} \\ &= \sqrt{30 \times 0.13 \times 0.87} \\ &\approx 1.84 \end{aligned}$$

6 A new drug has a 75% probability of curing a patient within one week. Suppose 38 patients are treated using this drug. Let X be the number of patients who are cured within a week.

a Find the mean μ and standard deviation σ of X.

b Find $\mathrm{P}(\mu - \sigma < X < \mu + \sigma)$.

6 $X \sim \mathrm{B}(38, 0.75)$

a
$$\begin{aligned} \mu &= np \\ &= 38 \times 0.75 \\ &= 28.5 \end{aligned}$$

$$\begin{aligned} \sigma &= \sqrt{np(1-p)} \\ &= \sqrt{38 \times 0.75 \times 0.25} \\ &\approx 2.67 \end{aligned}$$

b
$$\begin{aligned} \mu - \sigma &\approx 28.5 - 2.67 \\ &\approx 25.8 \end{aligned}$$

$$\begin{aligned} \mu + \sigma &\approx 28.5 + 2.67 \\ &\approx 31.2 \end{aligned}$$

$$\begin{aligned} \therefore\ \mathrm{P}(\mu - \sigma < X < \mu + \sigma) &= \mathrm{P}(26 \leqslant X \leqslant 31) \\ &\approx 0.740 \end{aligned}$$

7 Let X be the number of heads which occur when a coin is tossed 100 times, and Y be the number of ones which occur when a die is rolled 300 times.

a Show that the mean of both distributions is 50.

b Calculate the standard deviation of each distribution.

c Which variable do you think is more likely to lie between 45 and 55 (inclusive)? Explain your answer.

d Find: **i** $P(45 \leqslant X \leqslant 55)$ **ii** $P(45 \leqslant Y \leqslant 55)$

7 $X \sim B(100, \frac{1}{2})$, $Y \sim B(300, \frac{1}{6})$

a
$$\mu_X = np = 100 \times \tfrac{1}{2} = 50$$
$$\mu_Y = np = 300 \times \tfrac{1}{6} = 50$$

b
$$\sigma_X = \sqrt{np(1-p)} = \sqrt{100 \times \tfrac{1}{2} \times \tfrac{1}{2}} = \sqrt{25} = 5$$
$$\sigma_Y = \sqrt{np(1-p)} = \sqrt{300 \times \tfrac{1}{6} \times \tfrac{5}{6}} \approx 6.45$$

c X is more likely to lie between 45 and 55 inclusive because the standard deviation of X is lower than that of Y, which means there are more values of X which lie close to the mean.

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