Two-compartment models of Main Hawaiian Islands Yellowfin Tuna Population

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Introducetion

The Yellowfin Tuna (YFT) population in main Hawaiian Islands (MHI) is embedded in a larger pan-Pacific stock. Nevertheless, local fishermen believe that the MHI supports a "resident" yellowfin population, and some scientific observations are consistent with this belief. Recent tagging, tracking and studies show that the rate of exchange between the MHI population and the larger stock is low (Itano and Holland 2000). Analysis of YFT otoliths sampled from throughout the Pacific conclude that approximately 90% of the MHI population was reared in the MHI (Wells et al 2012).

The Hawaii based deep- and shallow-set longline, inshore- and offshore-

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troll and handline components of the fisheries combined catch approximately 5000 mt of YFT annually (ref). Management of these fisheries is an important issue deserving of scientific support.

This paper explores some potential models that might be used to analyze options for the management of fisheries for YFT in the MHI.

Models

The principle assumptions for modeling the MHI YFT population are:

1. The Pacific yellowfin population consists of two components: those that live in the MHI (region 1) and those that live elsewhere (region 2).

2. Fish immigrate from region 2 to region 1, mix thoroughly, and interact with the "resident" population according to shared population dynamics.

3. Fish emigrate from region 1 to region 2, but emigrant fish have no effect on region 2 population dynamics.

4. Immigrant fish are indistinguishable from "resident" fish (i.e. both groups of fish have the same population dynamics) and are caught by the same gear.

The general modeling approach to be applied will be similar to the state-space model utilized by Nielsen and Berg (2014). State-space models separate variability in the biological processes in the system (transition model) from errors in observing features of interest in the system (measurement model). This separation provides some statistical advantages in estimating model parameters. The models discussed here are potential candidates for the transition equation in a state-space model.

Logistic population dynamics (Schaefer Model)

Let $N_{1,1}$ equal the population size of fish originating in region 1 and residing in region 1 and $N_{2,1}$ equal the population size of fish originating in region 2 but residing in region 1. The total population size of fish residing in region 1 is thus $N_{1,1} + N_{2,1}$, and the dynamics of the population in region 1 is represented as

$$\frac{d}{dt}\big(N_{1,1} + N_{2,1}\big) = \big(N_{1,1} + N_{2,1}\big)\Big[r\Big(1 - \frac{N_{1,1} + N_{2,1}}{K}\Big) - F - T_{12}\Big] + T_{21} \quad (1)$$

where r is the per capita logistic growth rate per year (yr^{-1}), K is the logistic "carrying capacity" measured in the same units as $N_{1,1}$ and $N_{2,1}$, F is the fishing mortality (yr^{-1}) in region 1, and T_{12} is the emigration rate (yr^{-1}) from region 1 to region 2. T_{21} is the annual rate of immigration of fish from region 2 to region 1 measured in the same units as K per year.

Equivalent differential equations could be devised for the dynamics of fish residing in region 2 (i.e., $\frac{dN_{2,2}}{dt}$ and $\frac{dN_{1,2}}{dt}$) but the dynamics of the fish population in region 2 is external to this model. T_{21} can be considered to be form of population forcing from the larger stock in which the MHI population is embedded, perhaps estimated from other models, such as MULTIFAN-CL or SEAPODYM, or perhaps represented by an autocorrelated stochastic process. The appearance of $N_{1,1} + N_{2,1}$ in the numerator of the logistic term reflects the assumption that the population dynamics fish immigrating into region 1 depend on the population dynamics in region 1. This assumption leads to an important non-linearity in the model that permits overwhelming of a local stock by a more numerous immigrant stock as is often characteristics of mixed-stock fisheries.

The ratio $\frac{N_{1,1}}{N_{1,1}+N_{2,1}}$ is of potential interest, so equation 1 needs to be expanded and rearranged leading to a set of simultaneous differential equation for the components of the population inhabiting region 1.

$$\begin{aligned}
\frac{dN_{1,1}}{dt} &= N_{1,1}\left[r\left(1-\frac{N_{1,1}}{K}\right)-F-T_{12}\right]-\frac{r}{K}T_{12}N_{1,1}N_{2,1} \\
\frac{dN_{2,1}}{dt} &= N_{2,1}\left[r\left(1-\frac{N_{2,1}}{K}\right)-F-T_{12}\right]-\frac{r}{K}T_{12}N_{1,1}N_{2,1}+T_{21} \qquad (2)
\end{aligned}$$

The non-linear term from equation 1 appears the term $N_{1,1}N_{2,1}$ above equations.

The equilibrium of this system can be found by setting $\frac{dN_{1,1}}{dt} = 0 = \frac{dN_{2,1}}{dt}$. After some simplification the result is $\frac{T_{21}}{N_{2,1}} = 0$ In other words, there is no equilibrium in this coupled system, other than the degenerate case of no immigration ($T_{21} = 0$) into the MHI. Thus, the notion of using equilibrium-based reference points such as MSY to manage fisheries for MHI yellowfin is ill advised.

Example populations trajectories from equationis 2 can be seen in figures 1 through 5. The lack of a steady state caused immigration can be clearly seen in figure 4. The simulation in figure 5 was generated with correlated normally distributed random variation in both populations with standard deviation equal to 5% of the intrinsic growth rate and correlation coefficient of

-0.5. That is, small inversely correlation random fluctuations were imposed on both populations. The ratio of emigration to immigration was about 0.1. This result suggests that random variability may stabilize the population so that a persistent "local stock" is maintained.

Equations 2 are solved by finite difference approximations using explicit time stepping.

$$\begin{aligned}
\frac{dN_{1,1}}{dt} \approx \frac{N_{1,1_t} - N_{1,1_{t-\Delta t}}}{\Delta t} &= N_{1,1_{t-\Delta t}}\Big[r\Big(1 - \frac{N_{1,1_{t-\Delta t}}}{K}\Big) - F - T_{12}\Big] \\
&- \frac{r}{K}T_{12}N_{1,1_{t-\Delta t}}N_{2,1_{t-\Delta t}} \qquad (3) \\
\frac{dN_{2,1}}{dt} \approx \frac{N_{2,1_t} - N_{2,1_{t-\Delta t}}}{\Delta t} &= N_{2,1_{t-\Delta t}}\Big[r\Big(1 - \frac{N_{2,1_{t-\Delta t}}}{K}\Big) - F - T_{12}\Big] \\
&- \frac{r}{K}T_{12}N_{1,1_{t-\Delta t}}N_{2,1_{t-\Delta t}} + T_{21} \qquad (4)
\end{aligned}$$

Rearrangement and addition of process error terms ($\eta_{r,t}$) yields a form suitable for use as the transition equation in a state-space model.

$$\begin{aligned}
N_{1,1_t} = \Big[N_{1,1_{t-\Delta t}} &+ \Delta t N_{1,1_{t-\Delta t}}\Big[r\Big(1 - \frac{N_{1,1_{t-\Delta t}}}{K}\Big) - F - T_{12}\Big] \\
&- \frac{r}{K}T_{12}N_{1,1_{t-\Delta t}}N_{2,1_{t-\Delta t}}\Big]e^{\eta_{1,t}} \qquad (5) \\
N_{2,1_t} = \Big[N_{2,1_{t-\Delta t}} &+ \Delta t N_{2,1_{t-\Delta t}}\Big[r\Big(1 - \frac{N_{2,1_{t-\Delta t}}}{K}\Big) - F - T_{12}\Big] \\
&- \frac{r}{K}T_{12}N_{1,1_{t-\Delta t}}N_{2,1_{t-\Delta t}} + T_{21}\Big]e^{\eta_{2,t}} \qquad (6)
\end{aligned}$$

Experience shows that these approximations are sufficiently stable with reasonably small time steps. *Is it appropriate to use different process error time series ($\eta_{1,t}$ and $\eta_{2,t}$) for both segments of the population under the assumption of identical population dynamics for both segments?*

Measurement equation needs to be developed.

General production model (Pella-Tomlinson Model)

Probably not worth the effort.

Age structured model

The issue of regulating MHI YFT fisheries by imposing minimum size limits on some components of the fishery has been raised yet again. Size- or age-structured models are obviously required to address catch-at-size issues. The following set of equations are a potential formulation of the transition equation in a state-space model and are based on the state-space stock assessment model of Nielsen and Bert (2014). The notation has been shortened to so that $N_{r,a,t}$ now represents the fish living in the MHI but originating in in region $r; r = 1, 2$ of age $a; a = 1, 2, \ldots, A$ in time interval t.

$$
\begin{aligned}
N_{1,1,t} &= \exp\Big(R\big(\sum_a p_{a,t-1}(N_{1,a,t-1} + N_{2,a,t-1})\big) + \eta_{1,1,t}\Big) && (7)\\
N_{2,1,t} &= T_{2,1,1,t}e^{\eta_{2,1,t}} && (8)
\end{aligned}
$$

$$
\begin{aligned}
N_{1,a,t} &= N_{1,a-1,t-1}\exp(Z_{a-1,t-1} + \eta_{1,a,t}); \qquad 1 < a < A && (9)\\
N_{2,a,t} &= N_{2,a-1,t-1}e^{Z_{a-1,t-1}} + T_{2,1,a,t}e^{\eta_{2,a,t}} && (10)
\end{aligned}
$$

$$
\begin{aligned}
N_{1,A,t} &= \Big(N_{1,A-1,t-1}e^{Z_{A-1,t-1}} + N_{1,A,t-1}e^{Z_{A,t-1}}\Big)e^{\eta_{1,A,t}} && (11)\\
N_{2,A,t} &= N_{2,A-1,t-1}e^{Z_{A-1,t-1}} + N_{2,A,t-1}e^{Z_{A,t-1}} + T_{2,1,A,t}e^{\eta_{2,A,t}} && (12)
\end{aligned}
$$

where $Z_{a,t} = -\sum_g F_{g,a,t} - M_a - T_{1,2}$ is a negative quantity representing the total mortality at fish of age a in time interval t and M_a is the natural mortality at age a, assumed to be constant over time. $F_{g,a,y}$ is the fishing mortality exerted by gear g, on fish of age a during time period t. Fishing mortality is usually represented as the product of several parameters and variables, for example, catchability coefficient, gear selectivity, and fishing effort. The parameters of this relationship are often confounded and difficult to estimate. Furthermore, in the case of MHI yellowfin fishery, effort estimates are known to be inconsistent, requiring adjustments to catchability and gear selectivity. Eliminating fishing effort would simply the model somewhat. Fishing mortality is assumed to vary seasonally (by quarter), to increase or decrease over time, and to depend on gear selectivity.

$$F_{g,a,t} = e^{-\alpha_{g,Q(t)}} \cdot e^{-\beta_{g,Y(t)}} \cdot S_{g,a,t} \tag{13}$$

α is a vector of quarterly deviations from the mean fishing mortality that sum to zero. β is a vector of autocorrelated annual deviations from the mean that reflect secular trends in fishing mortality. $S_{g,a,t}$ represents the selectivity of gear g on age class a fish during time period t. *At this point, is is not clear whether seculare trends in gear efficiency would be best parameterized by β or by S.*

The yield or catch in the MHI under this model would be

$$C_{g,a,t} = \frac{F_{g,a,t}}{Z_{a,t}}\left(1 - e^{Z_{a,t}}\right)\left(N_{1,a,t} + N_{2,a,t}\right)e^{\varepsilon_{g,a,t}} \tag{14}$$

using the standard Baranov approximation.

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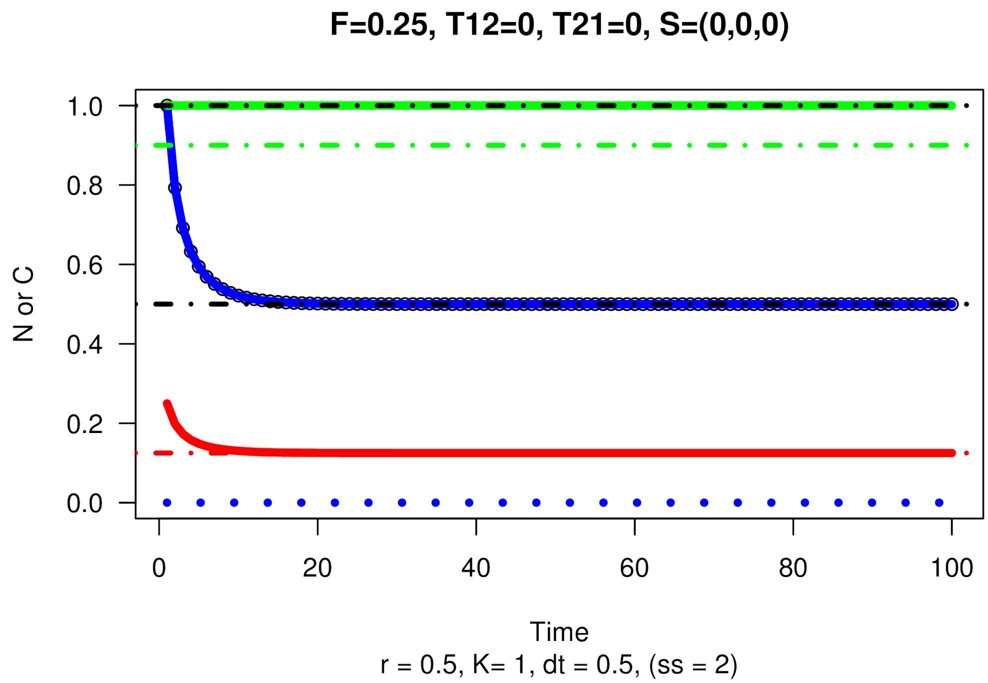

Figure 1: Simple Schaefer model dynamics with no emigration or immigration ($T_{12} = 0 = T_{21}$). The heavy blue line represents the size of the population ($N_{1,1} + N_{2,1}$) moving from its initial size of 1.0 to its equilibrium size of 0.5 The dashed blue line (in some figures) represents the size of the $N_{1,1}$ population. The dotted blue line represents the size of the $N_{2,1}$ population. The heavy red line represents the yield to the fishery dropping to MSY (red dash dot line). The solid green line represents the proportion of "local" fish in the population ($\frac{N_{1,1}}{N_{1,1}+N_{2,1}}$). The green dash dot line represents the value of the "proportion local" estimated from tagging and ololith analysis.

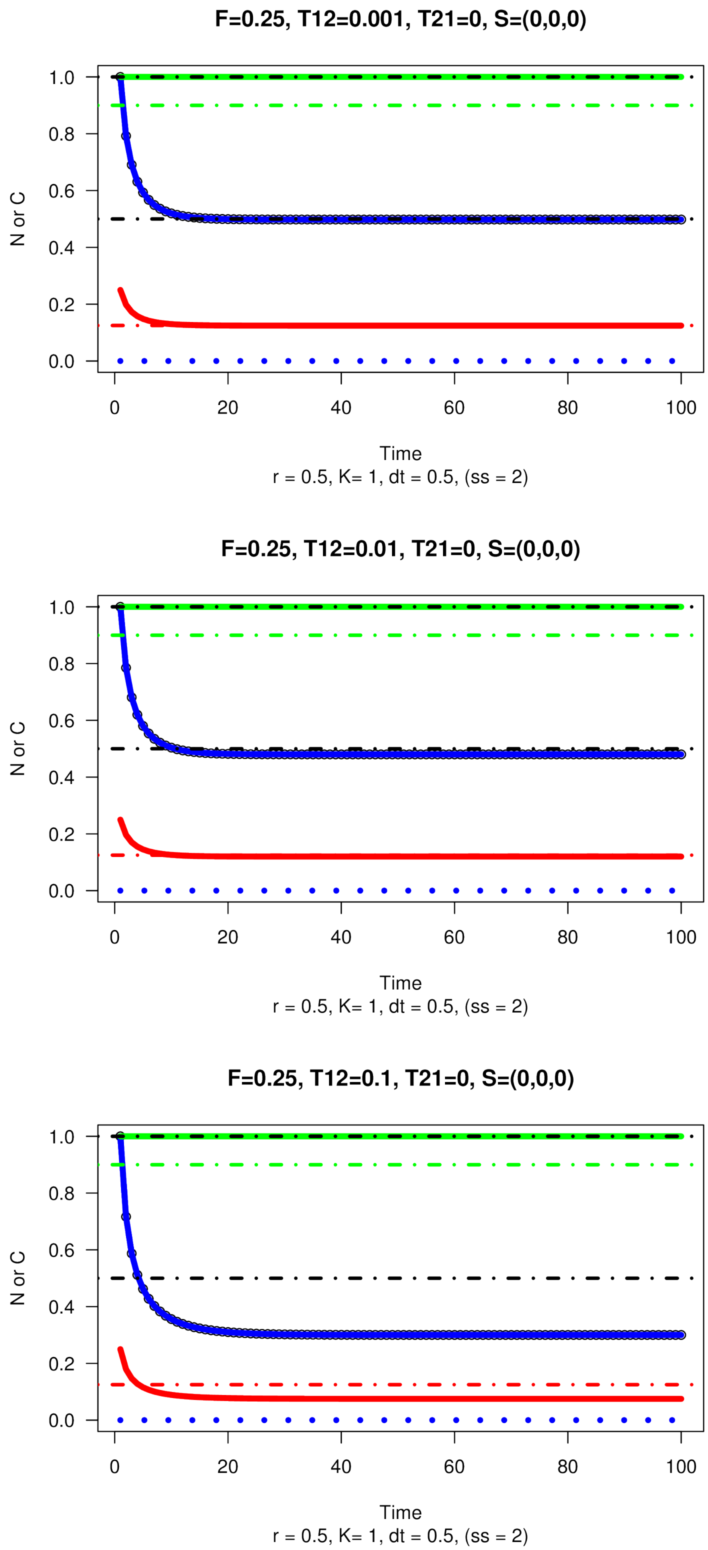

Figure 2: Effects if increasing levels of emigration,$T_{12} > 0; T_{21} = 0$. As emigration increases, the size of the $N_{1,1}$ population decreases, and yield falls below MSY. Since there is no immigration, the proportion local remains unchanged.

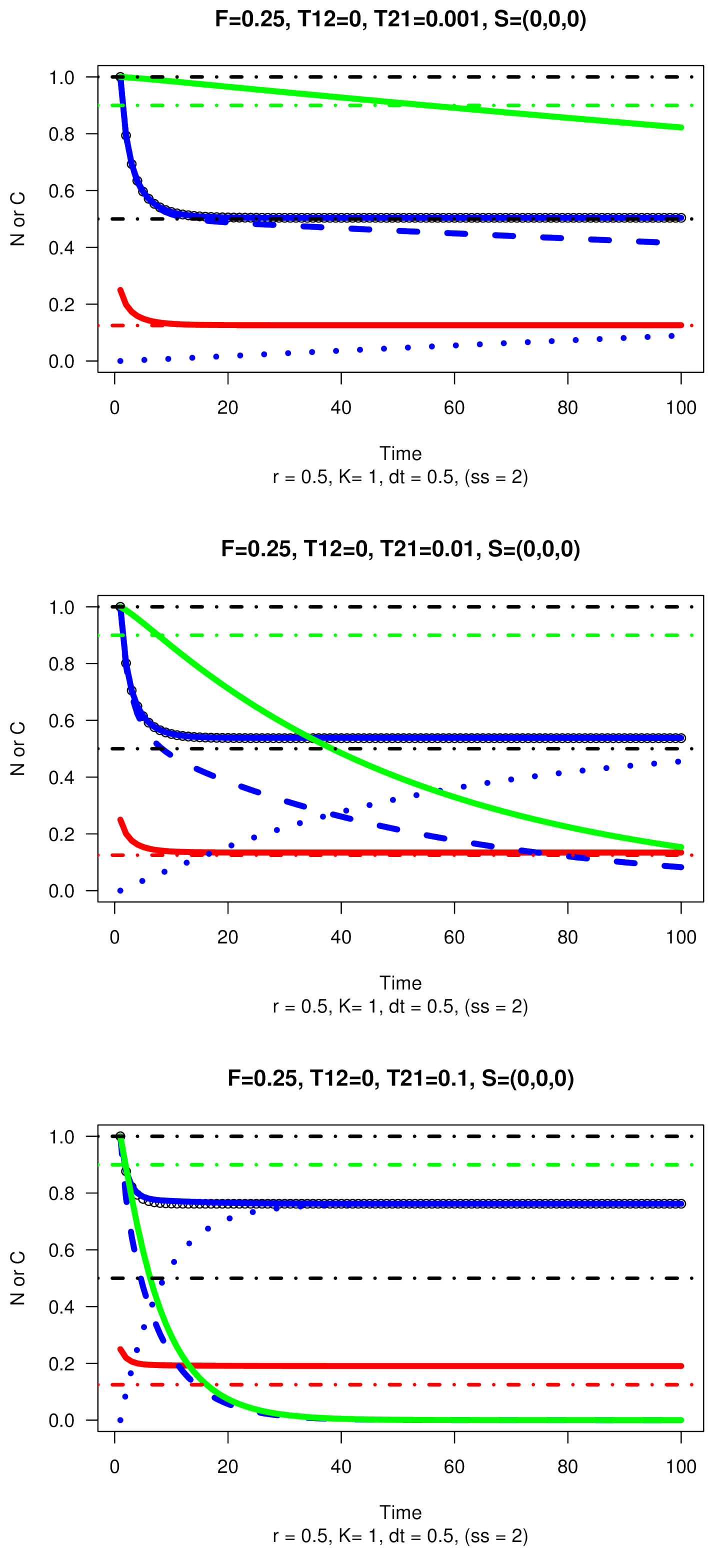

Figure 3: Effects if increasing levels of immigration,$T_{12} = 0; T_{21} > 0$. As immigration increases, the size of immigrant population, $N_{2,1}$, increases (dotted blue line) and the proportion local declines well below the 0.9 reference level. Yield exceeds MSY for the local stock.

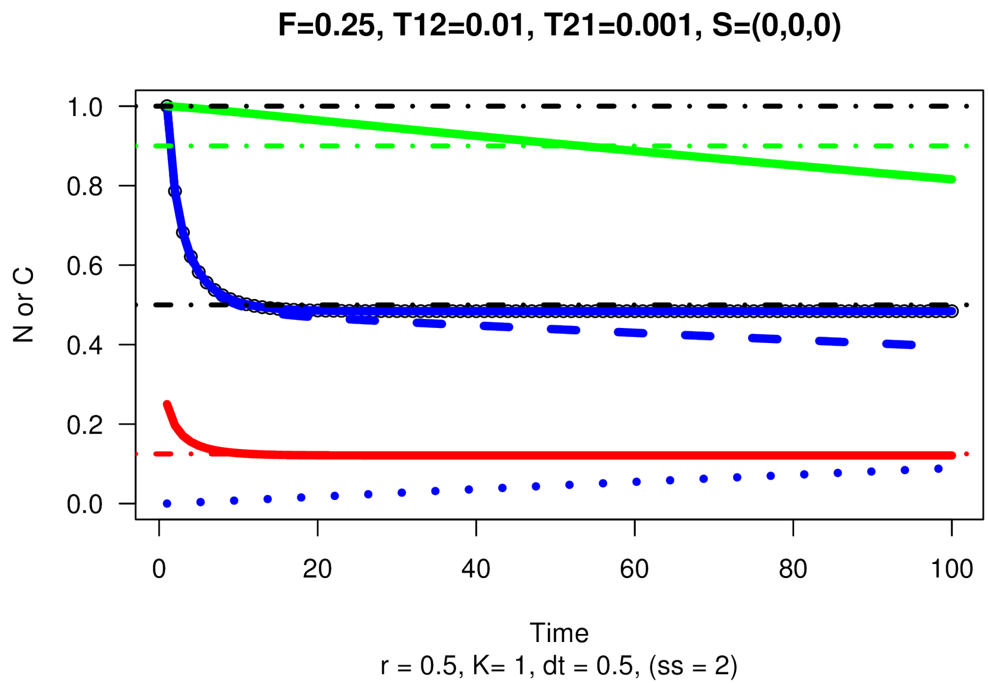

Figure 4: Effects of moderate levels of immigration and emigration. The local population, $N_{1,1}$, decreases and will eventually reach equilibrium at $N_{1,1} = 0$.

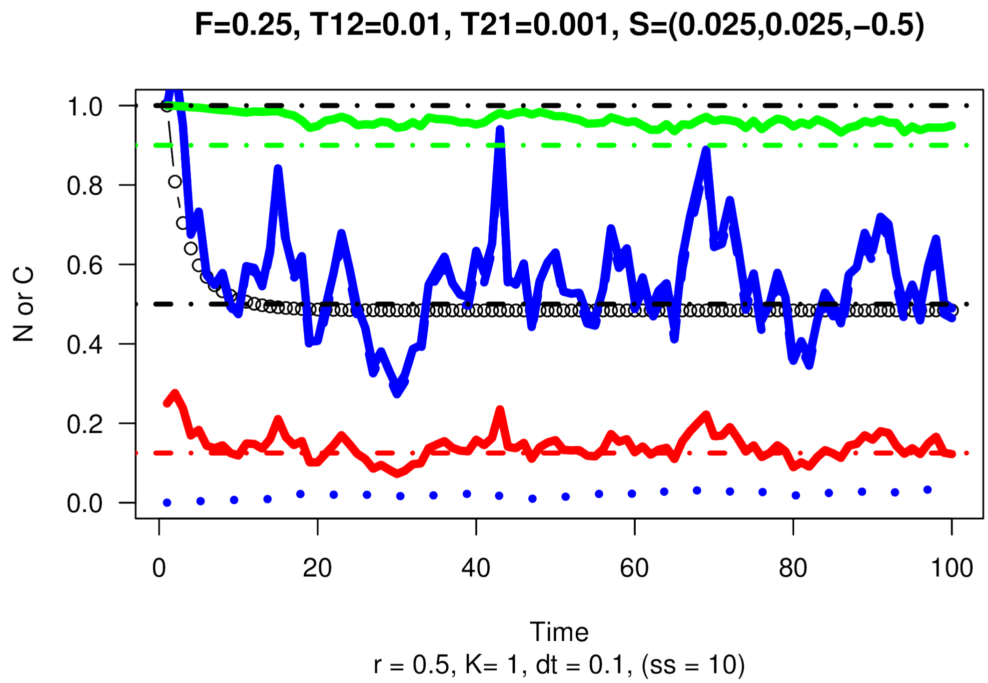

Figure 5: Effects of correlated random errors in the in $N_{1,1}$ and $N_{2,1}$ with moderate levels of immigration and emigration. Both populations appear to stabilize around some sort of steady state size and the yield fluctuates around MSY.