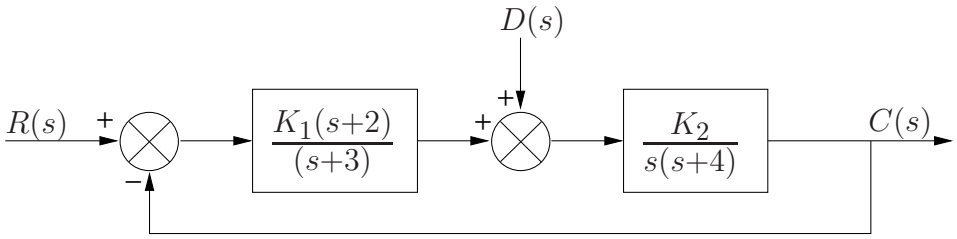
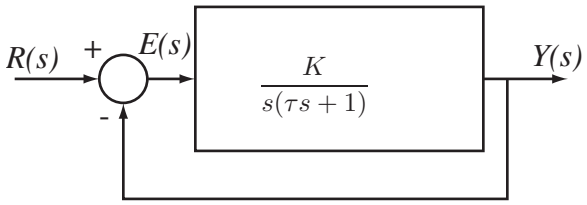
University of Toronto
Department of Electrical and Computer Engineering
ECE311 Dynamic Systems and Control
Homework 4

Steady-State Error

1. Design the values of K_1 and K_2 in the system below to meet the following specifications: steady-state error component due to a unit step disturbance is -0.000012; steady-state error component due to a unit ramp input is 0.003.



2. Consider the unity feedback control system shown below.



   Compute the steady-state tracking error due to a ramp input $r(t) = R_0\ t \cdot \mathbf{1}(t)$ (where $\mathbf{1}(t)$ denotes the unit step).

3. Consider the following system:

$$Y(s) = \frac{K(s+a)(s+b)}{s^2(s^2+Ks+Kb)+K(s+a)(s+b)}R(s)$$

   where it is assumed that $a > 0$, $b > 0$, $K > 0$.

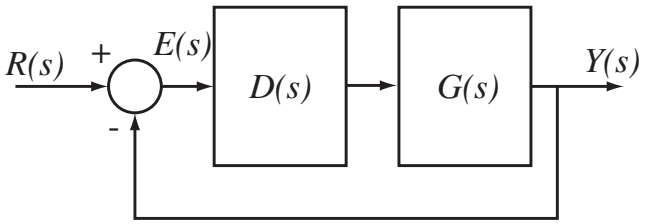
   (a) Find conditions on K, a, and b such that the system with input $R(s)$ and output $Y(s)$ is BIBO stable.

   (b) Letting $R(s) = \frac{1}{s}$ and assuming K is selected such that the closed-loop system is BIBO stable, find an expression for $y_{ss} := \lim_{t\to\infty} y(t)$.

4. A controller for a satellite attitude control with transfer function $G(s) = \frac{1}{s^2}$ has been designed with a unity feedback structure and has the transfer function

$$D(s) = \frac{K(s+2)}{s+5},$$

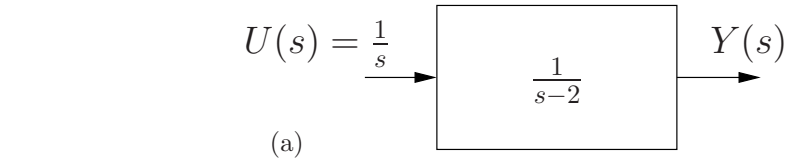
   where $K > 0$ is a parameter to be designed (see the Figure below).



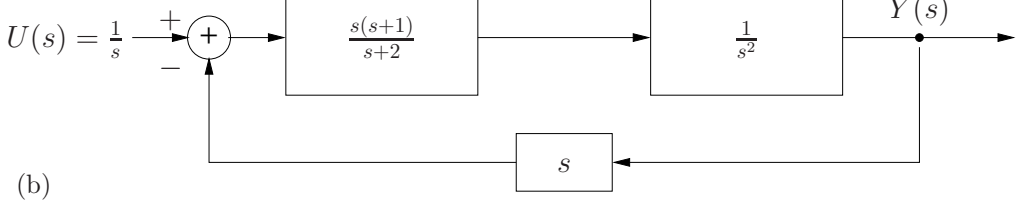
Assuming that the reference trajectory is given by $r(t) = \frac{t^2}{2}\mathbf{1}(t)$, calculate the value of K guaranteeing that the steady-state error is 0.01.

5. For each of the following block diagrams find the steady-state tracking error

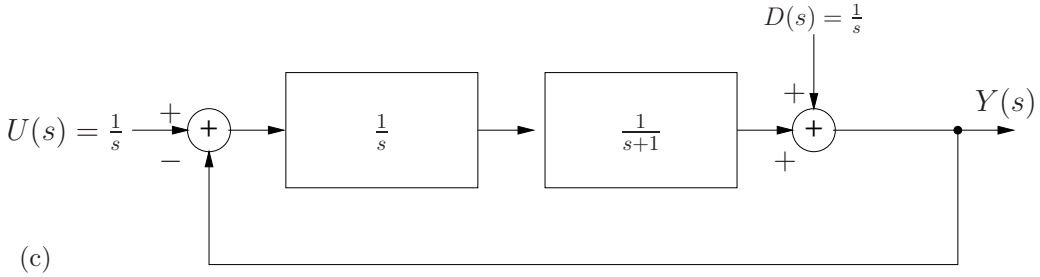
$$e(\infty) := \lim_{t\to\infty} u(t) - y(t).$$



(a)

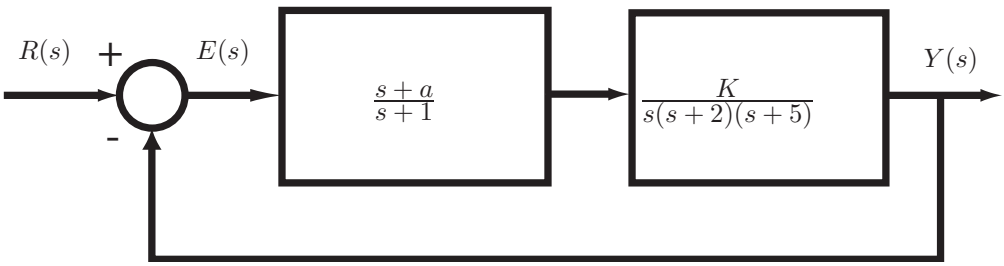


(b)



(c)

6. Consider the control system in the figure below.



(i) Find the most general conditions on the parameters $K > 0$ and $a > 0$ so that the closed-loop system with input $r(t)$ and output $y(t)$ is BIBO stable.

(ii) Let $r(t)$ be a ramp input given by $r(t) = R\,t\cdot\mathbf{1}(t)$, with $R > 0$. Assuming that $\lim_{t\to\infty} e(t)$ exists, find all values of the parameters $K > 0$ and $a > 0$ so that $\lim_{t\to\infty} e(t) \le 0.25\,R$.

(iii) Let $a = 2$. Find all values of $K > 0$ so that the conditions you found in (i) and (ii) both hold.