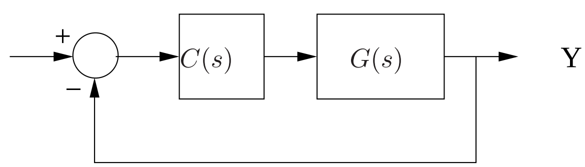
University of Toronto
Department of Electrical and Computer Engineering
ECE311 Dynamic Systems and Control
Homework 6

Solve at least one of the problems in Section 1, and two of the problems in Section 2.

1 Bode plots

Referring to the control system,



draw the Bode plots of the open-loop system, and find the gain and phase margins in each of the following cases.

1. $C(s) = \dfrac{500}{s(s+10)}$, $G(s) = \dfrac{1}{s+100}$.
2. $C(s) = \dfrac{100(s+1)}{s^2+2s+100}$, $G(s) = \dfrac{1}{s}$.
3. $C(s) = \dfrac{100(s+1)}{s^2+2s+100}$, $G(s) = \dfrac{s+10}{s}$.

2 Control design in the frequency domain

4. Consider a unity feedback system with open-loop transfer function

$$G(s) = \frac{10^8}{(s+10)(s+10^3)^2}.$$

   Design a controller $C(s)$ yielding the following properties:

   (i) $\lim_{t\to\infty} e(t) = 0$ for step reference signals.

   (ii) The phase margin is $> 45°$.

5. Given a second-order system with complex conjugate poles (or a higher-order system with two dominant poles), one can show that the following relationships hold:

$$\mathrm{PM} = \arctan \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1+4\zeta^4}}},$$

$$\omega_{BW} = \omega_n \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}},$$

   where PM is the phase margin of the open-loop system and ω_{BW} is the bandwidth of the transfer function $CG/(1+CG)$[1]. Using these formulas, design a lead-lag controller for a unity feedback system with open-loop transfer function $G(s) = \dfrac{(s+10)}{s(s+1)(s+100)}$ meeting the following specifications:

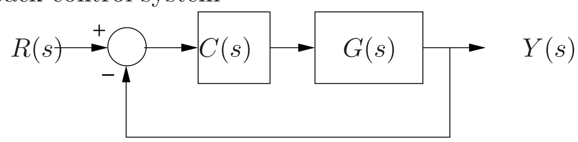
[1] Note that while you can determine PM, as usual, from the Bode plots of the open-loop system, you can't determine ω_{BW} from these plots! You can *estimate* it using the relationship $\omega_c \simeq 0.5 \cdot \omega_{BW}$ (ω_c is the crossover frequency).

(i) Transient performance: $\%OS = 1.44\%$, $T_s = 0.06$ seconds.

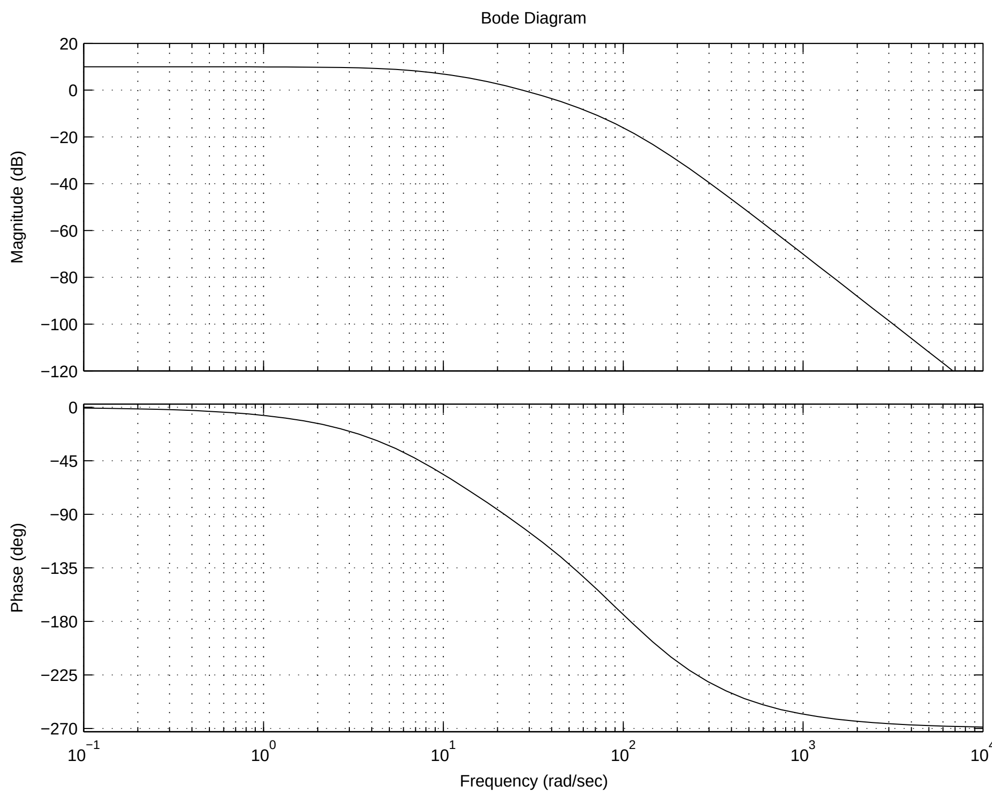
(ii) Steady-state tracking: when $r(t) = \mathbf{1}(t)$, asymptotic tracking is achieved. When $r(t) = t\mathbf{1}(t)$, the steady-state error is $e(\infty) = 10^{-3}$.

In solving the problem, neglect the fact that the system is not second-order.

6. Consider the unity feedback control system



The Bode plots of $G(s)$ are given below



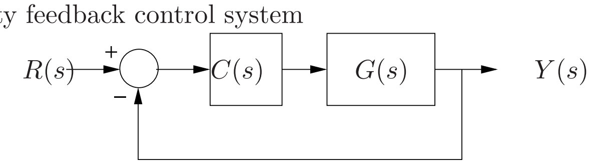
(a) Find the phase margin of the system.

(b) Design $C(s)$ to achieve

(i) Steady-state error $e(\infty) = \lim_{t\to\infty} e(t) < 0.01$ when $R(s) = \frac{1}{s}$.

(ii) Phase margin $PM > 45°$.

7. Consider again the unity feedback control system



where the plant $G(s)$ is given as

$$G(s) = \frac{50}{s^2 + 2s + 100}.$$

Design a controller $C(s)$ yielding

- Phase margin $PM \geq 60°$.
- DC gain $K_p \geq 10$.