University of Toronto
Department of Electrical and Computer Engineering
ECE311 Dynamic Systems and Control
Homework 3

Time response and control specifications

1. Consider the closed loop system in Figure 1. K and p are parameters you'll design in part 2 of this problem.

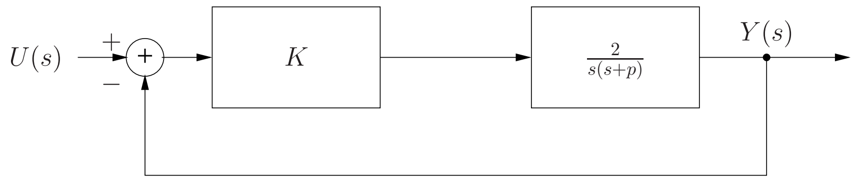
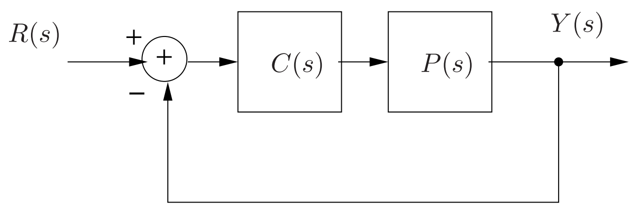


Figure 1: System block diagram

   (a) Using standard formulas for overshoot and settling time, sketch the region in the complex plane where the poles of the closed-loop system should lie in order for the following specifications to be met:

   $$\text{Settling time } T_s \leq 0.8 \text{ sec}, \qquad \text{Percent overshoot } \%OS \leq 1\% = 0.01.$$

   (b) Choose K and p so that the poles of the closed-loop system lie in the region you found in part 1.



2. You are given an unstable plant with a transfer function $P(s) = \dfrac{1}{(s+7)(s-1)}$. You are to design a proportional controller, $C(s) = K$, such that the closed-loop system is BIBO stable and meets the following performance specifications:

   (i) Rise time $T_r < 0.5$ seconds (where $T_r = \frac{1.8}{w_n}$)
   (ii) Percent overshoot $\%OS < 50\%$.

   Do the following:

   (a) Sketch the region in the complex plane where you would like the poles of the closed-loop system to be.
   (b) Choose a gain K that will meet the performance specifications.

3. Consider an LTI system with transfer function $G(s) = \frac{s+2}{(s+1)(s^2+2s+26)}$. Without computing inverse Laplace transforms, describe the qualitative properties of the output signal when the input signal is $u(t) = \mathbf{1}(t)$. Repeat with $u(t) = \sin(t)$. Extract as much information as you can from the transfer function and the input signal.

4. Consider the LTI system with transfer function $G_1(s) = 1/[(s+2)^2+1]$, subject to a step input $u(t) = \mathbf{1}(t)$. Let $y_1(t)$ be its output signal. Provide an estimate of the settling time, rise time, and percent overshoot of $y_1(t)$. Now consider a second system with an extra pole

$$G_2(s) = \frac{20}{(s+20)[(s+2)^2+1]}, \quad p > 0,$$

also subject to a step input $u(t) = \mathbf{1}(t)$. Let $y_2(t)$ be its output signal. We have argued in class that the outputs of the two systems should be very similar. Determine $y_1(t) - y_2(t)$ and find an upper bound for $|y_1(t) - y_2(t)|$, $t \geq 0$.

You can use the Matlab command `ilaplace` to aid in your computations.

5. Consider an RLC circuit with a voltage source u. Let the output be the capacitor voltage. Suppose $C = 10\mu F$. Find R and L such that, when $u(t) = \mathbf{1}(t)$, the overshoot is 15% and the settling time is 2 ms.

Stability

1. Given the unity feedback system of Figure 2 with

$$G(s) = \frac{Ks(s+2)}{(s^2-4s+8)(s+3)},$$

find the range of K such that the closed-loop system is BIBO stable.

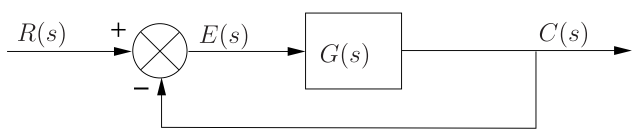


Figure 2: Unity feedback system

2. Consider again the unity feedback system of Figure 2 with

$$G(s) = \frac{K}{s(s+1)(s+2)(s+5)}.$$

Find the range of K such that the closed-loop system is BIBO stable.

3. Consider the model of a one degree-of-freedom robot

$$\ddot{\theta} = -\frac{mgl}{I}\sin\theta + \frac{u}{I},$$

with state $x = [\theta, \dot{\theta}]^\top$ and output $y = \theta$. We have seen in class that setting $u^\star = 0$ and $x^\star = [0 \;\; 0]^\top$, the pair $(x^\star, u^\star)$ is an equilibrium condition. Linearize the system at $(x^\star, u^\star)$ and find the transfer function. Is the linearized system BIBO stable or BIBO unstable?

4. Consider a state model of an RLC circuit (for instance, choose $x_1 = v_C$, $x_2 = i_L$). Show that the system is asymptotically stable for any $R, L, C > 0$.