Pitfalls and Prospects in Multisensor Data Fusion

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Karlsruhe Institute of Technology

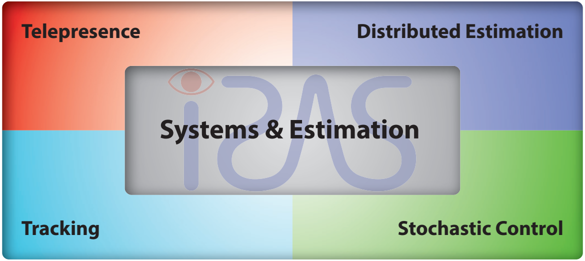
Intelligent Sensor-Actuator-Systems

TU Delft, May 31, 2017

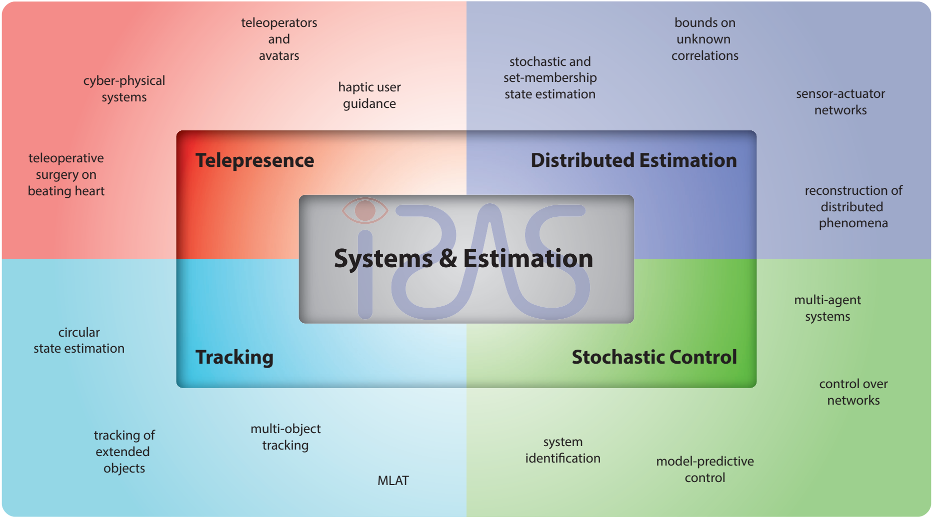
Intelligent Sensor-Actuator-Systems Laboratory

Systems & Estimation

Intelligent Sensor-Actuator-Systems Laboratory



Intelligent Sensor-Actuator-Systems Laboratory



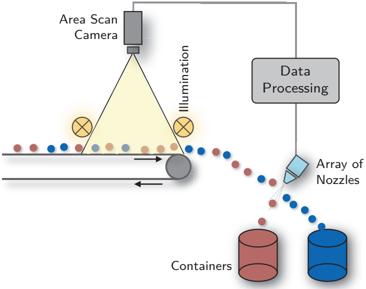
Projects at ISAS

Augmented Reality

Projects at ISAS

Augmented Reality

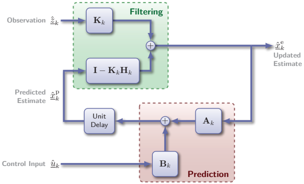
Multiobject-Tracking



side

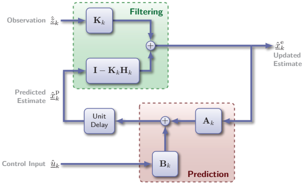
below

My Research at ISAS



How to treat uncertainties?

My Research at ISAS



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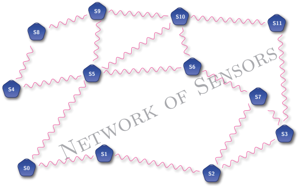
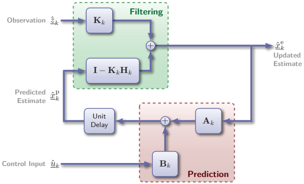


How to compute an estimate in a networked system?

My Research at ISAS



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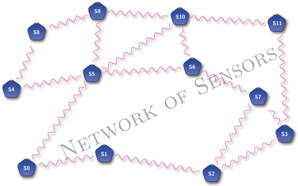

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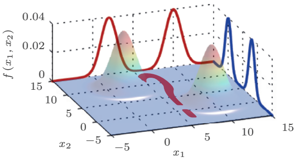
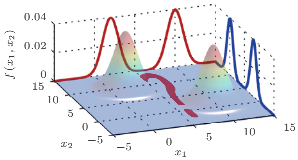
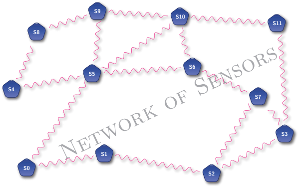


How to do decentralized nonlinear state estimation?

My Research at ISAS



How to do decentralized nonlinear state estimation?



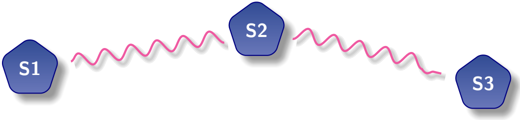
How to compute an estimate in a networked system?

Multisensor

Data

Fusion

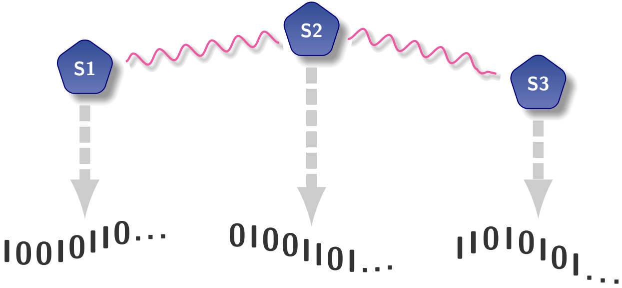
Multisensor



Data

Fusion

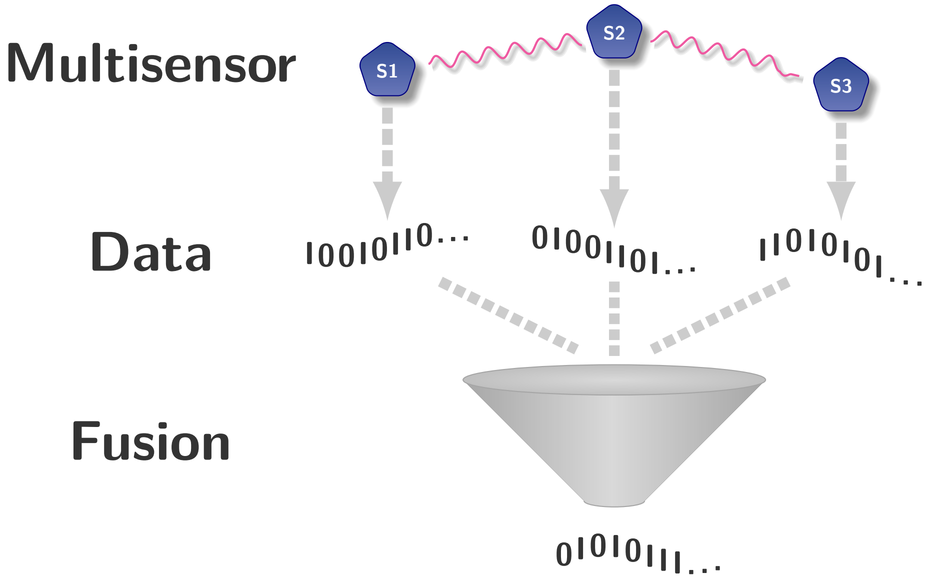
Multisensor



Data

Fusion

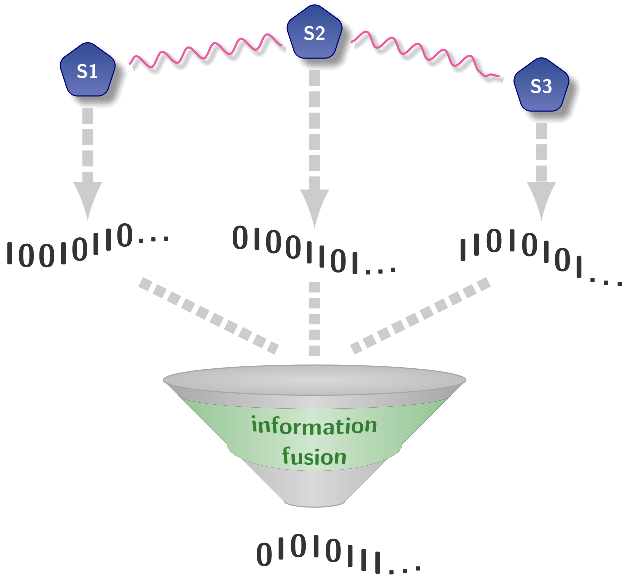
Multisensor Data Fusion



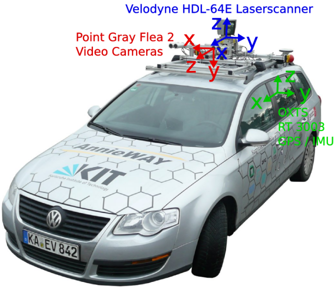
Multisensor

Data

Fusion



Why Multisensor Data Fusion?

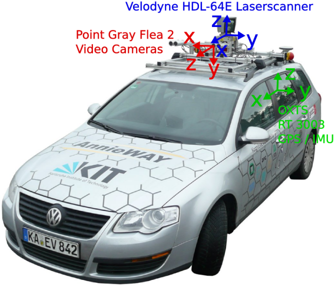


Autonomous Driving

Tracking / Telepresence

Robotics

Why Multisensor Data Fusion?

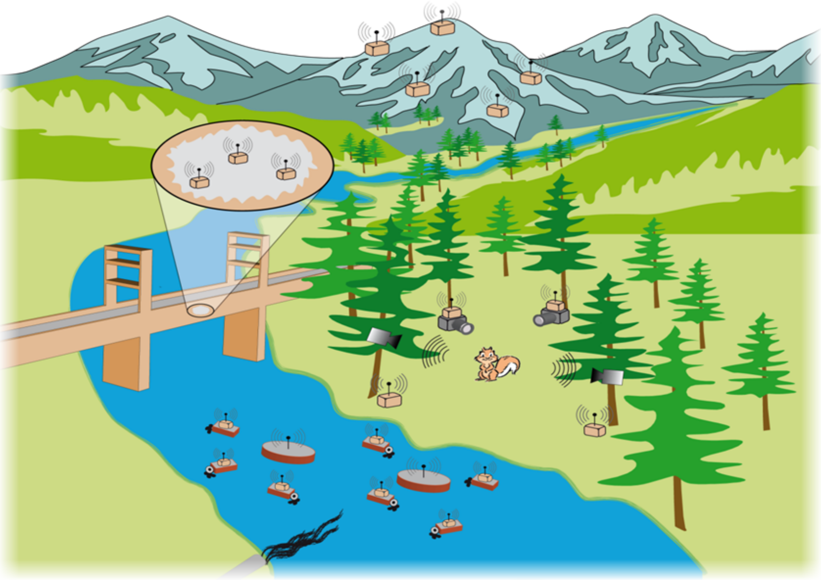


Autonomous Driving

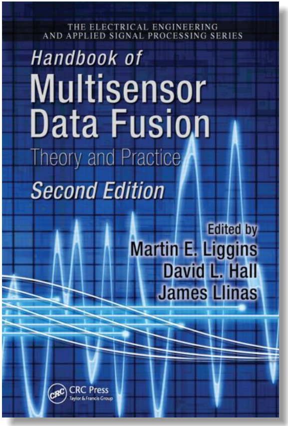
Tracking / Telepresence

Robotics

Sensor Networks

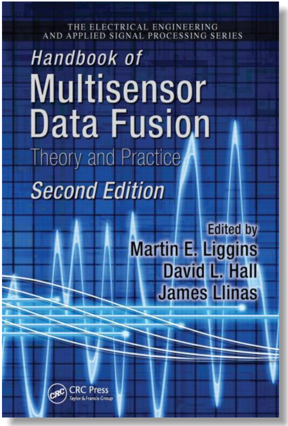


Dirty Secrets of Data Fusion



"There is still no substitute for a good sensor."

Dirty Secrets of Data Fusion



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"Downstream processing cannot make up for errors in upstream processing."

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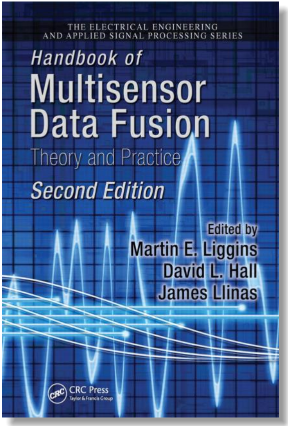

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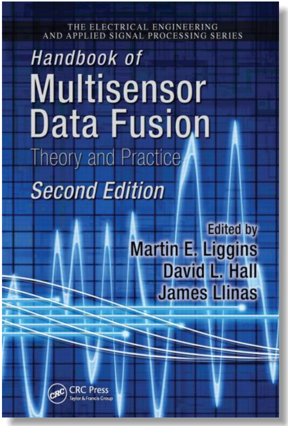

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"There are no magic algorithms."

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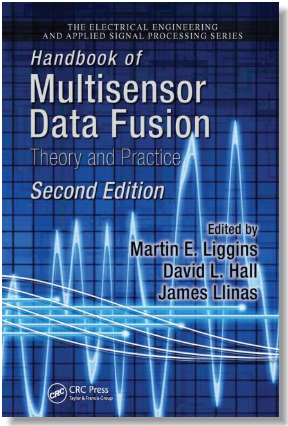


"There is still no substitute for a good sensor."

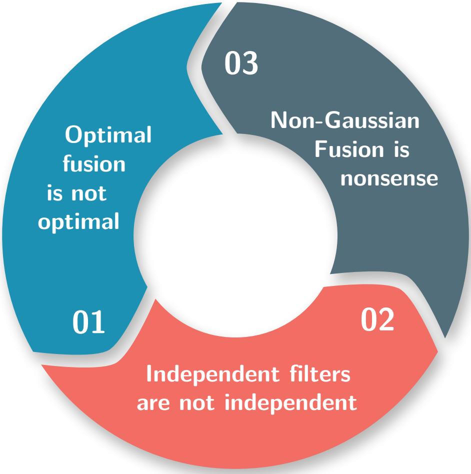
"Downstream processing cannot make up for errors in upstream processing."

"The fused answer may be worse than the best sensor."

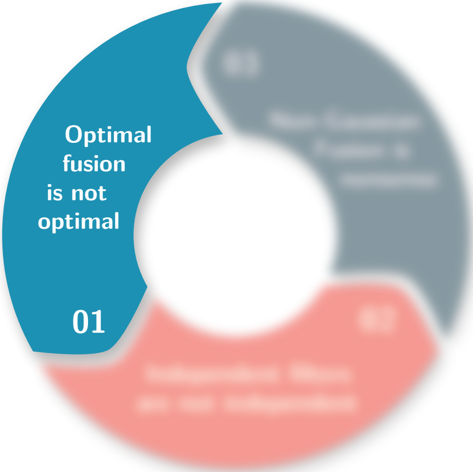
"There are no magic algorithms."

"Fusion is not a static process."

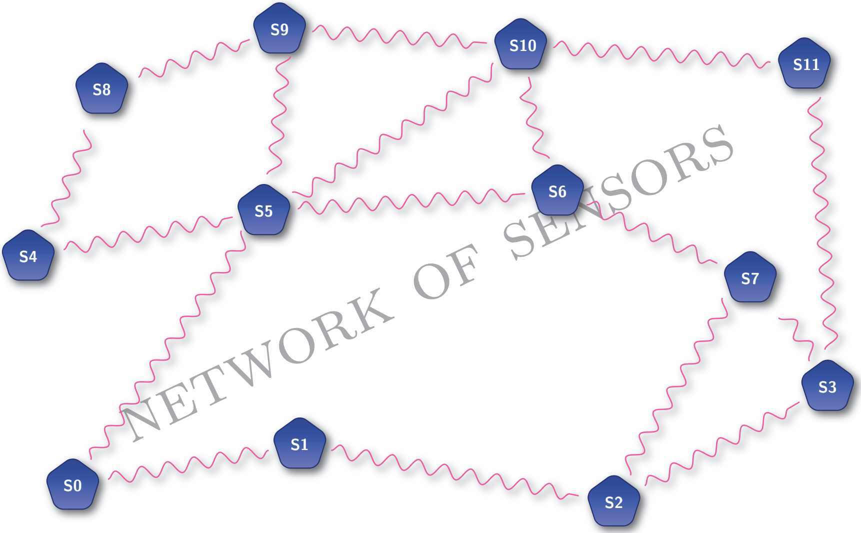
Even Dirtier Secrets of Data Fusion



Even Dirtier Secrets of Data Fusion



Decentralized Kalman Filtering



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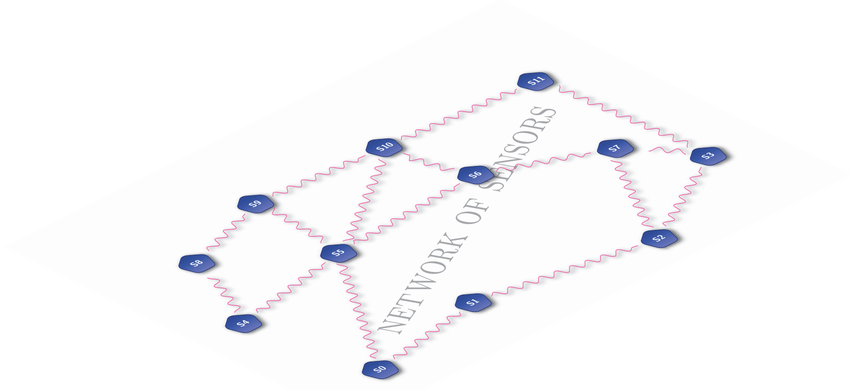

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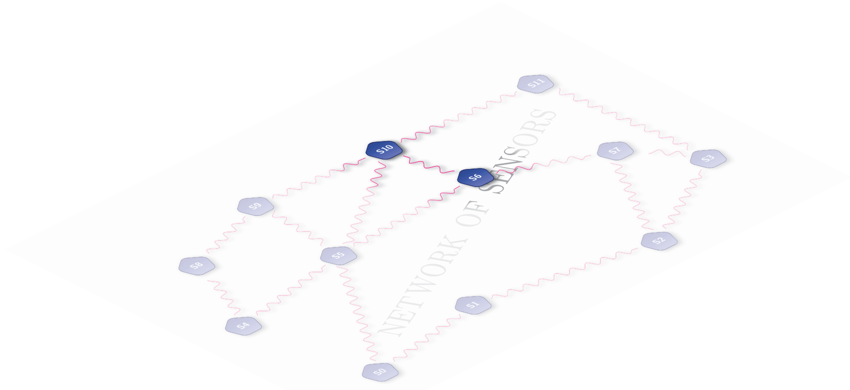

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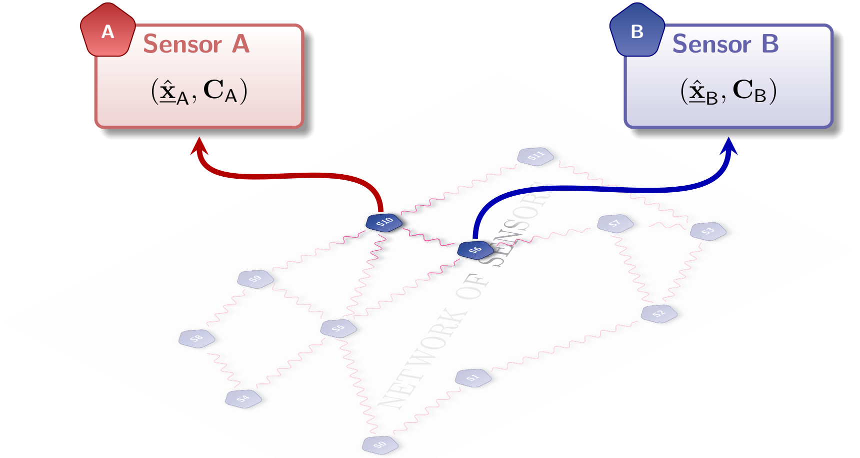

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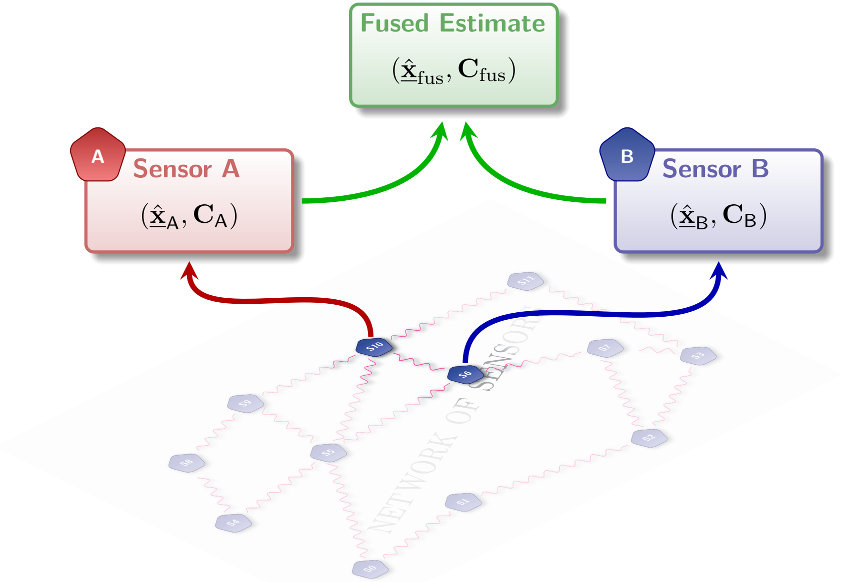
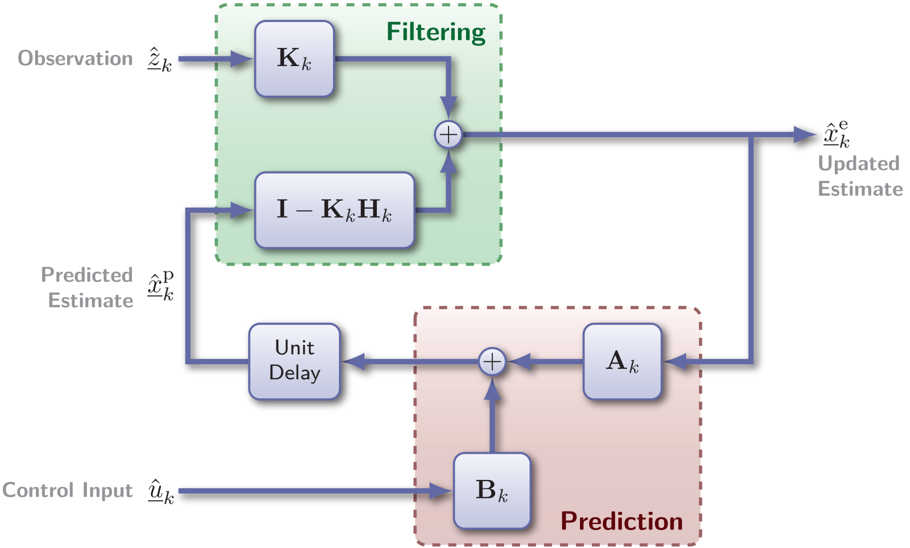


Fusion of Estimates



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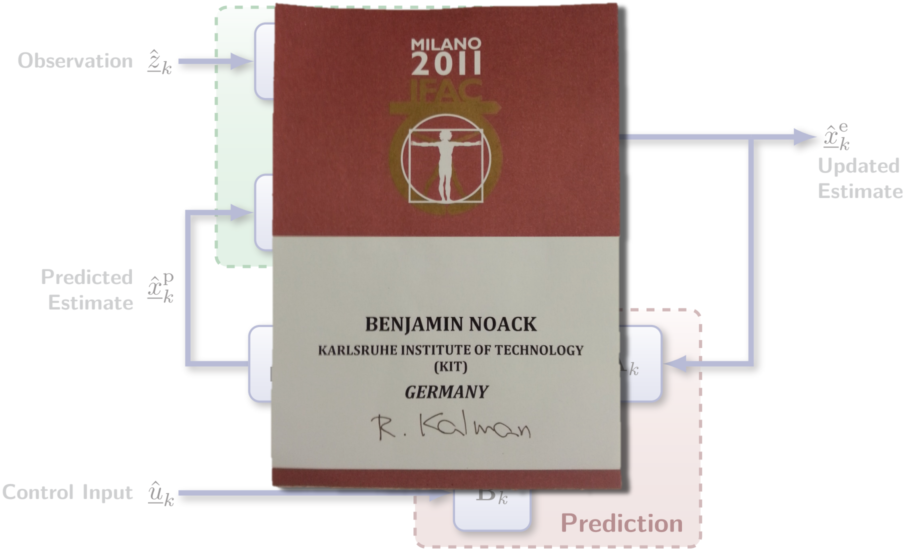

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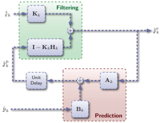

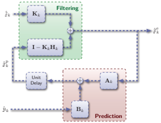

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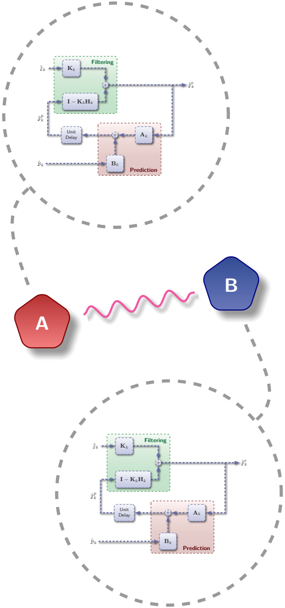

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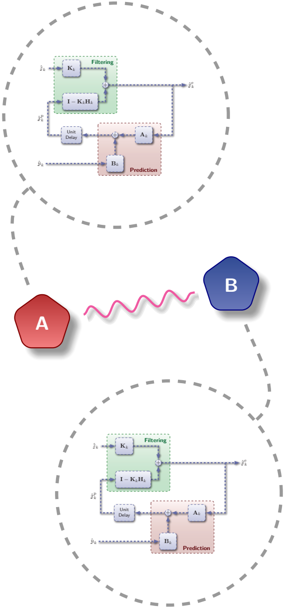


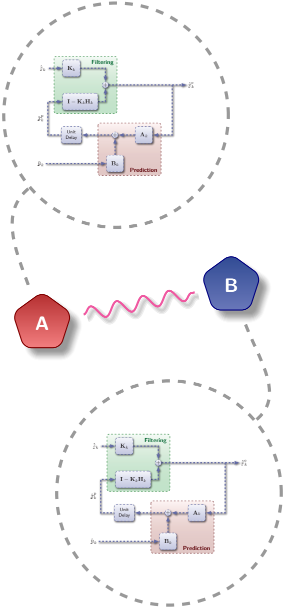
Sensor A

$(\underline{\hat{\mathbf{x}}}_{\mathsf{A}}, \mathbf{C}_{\mathsf{A}})$

Sensor B

$(\underline{\hat{\mathbf{x}}}_{\mathsf{B}}, \mathbf{C}_{\mathsf{B}})$

Fusion of Estimates



Sensor A

$(\underline{\hat{\mathbf{x}}}_{\mathsf{A}}, \mathbf{C}_{\mathsf{A}})$

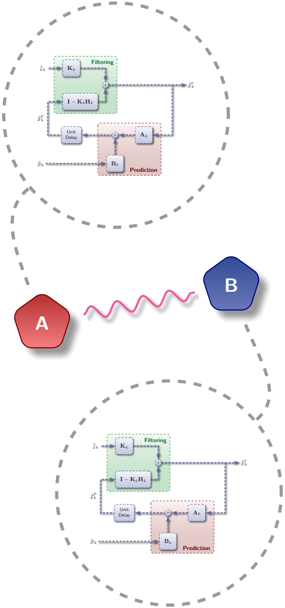
Sensor B

$(\underline{\hat{\mathbf{x}}}_{\mathsf{B}}, \mathbf{C}_{\mathsf{B}})$

Linear Fusion

$$
\begin{aligned}
\underline{\hat{\mathbf{x}}}_{\mathrm{fus}} &= \mathbf{K}\,\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{L}\,\underline{\hat{\mathbf{x}}}_{\mathsf{B}} \\
\mathbf{C}_{\mathrm{fus}} &= \mathbf{K}\,\mathbf{C}_{\mathsf{A}}\,\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_{\mathsf{AB}}\mathbf{L}^{\mathrm{T}} \\
&\quad + \mathbf{L}\mathbf{C}_{\mathsf{BA}}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\,\mathbf{C}_{\mathsf{B}}\,\mathbf{L}^{\mathrm{T}}
\end{aligned}
$$

Fusion of Estimates



A **Sensor A**

$(\hat{\underline{\mathbf{x}}}_\mathsf{A}, \mathbf{C}_\mathsf{A})$

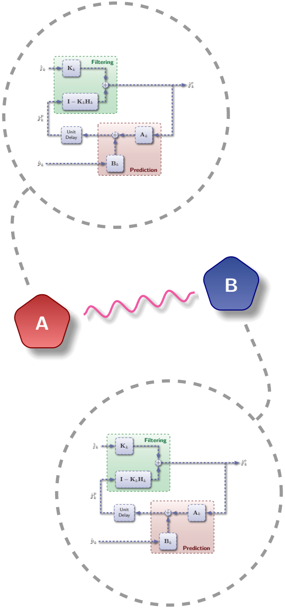
B **Sensor B**

$(\hat{\underline{\mathbf{x}}}_\mathsf{B}, \mathbf{C}_\mathsf{B})$

Linear Fusion

$$
\begin{aligned}
\hat{\underline{\mathbf{x}}}_{\mathrm{fus}} &= \mathbf{K}\,\hat{\underline{\mathbf{x}}}_\mathsf{A} + \mathbf{L}\,\hat{\underline{\mathbf{x}}}_\mathsf{B} \\
\mathbf{C}_{\mathrm{fus}} &= \mathbf{K}\mathbf{C}_\mathsf{A}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_\mathsf{AB}\mathbf{L}^{\mathrm{T}} \\
&\quad + \mathbf{L}\mathbf{C}_\mathsf{BA}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_\mathsf{B}\mathbf{L}^{\mathrm{T}}
\end{aligned}
$$

Fusion of Estimates



Sensor A

$(\underline{\hat{\mathbf{x}}}_{\mathsf{A}}, \mathbf{C}_{\mathsf{A}})$

Sensor B

$(\underline{\hat{\mathbf{x}}}_{\mathsf{B}}, \mathbf{C}_{\mathsf{B}})$

Linear Fusion

$$\underline{\hat{\mathbf{x}}}_{\text{fus}} = \mathbf{K}\,\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{L}\,\underline{\hat{\mathbf{x}}}_{\mathsf{B}}$$

$$\mathbf{C}_{\text{fus}} = \mathbf{K}\mathbf{C}_{\mathsf{A}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_{\mathsf{AB}}\mathbf{L}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{BA}}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{B}}\mathbf{L}^{\mathrm{T}}$$

Optimal Fusion

Combination with Bar-Shalom/Campo gains

$$\mathbf{K}_{\mathrm{B/C}} = \left(\mathbf{C}_{\mathsf{B}} - \mathbf{C}_{\mathsf{BA}}\right) \cdot \left(\mathbf{C}_{\mathsf{A}} + \mathbf{C}_{\mathsf{B}} - \mathbf{C}_{\mathsf{AB}} - \mathbf{C}_{\mathsf{BA}}\right)^{-1},$$

$$\mathbf{L}_{\mathrm{B/C}} = \mathbf{I} - \mathbf{K}_{\mathrm{B/C}}$$

Fusion of Estimates

Linear Fusion

$$\underline{\hat{\mathbf{x}}}_{\text{fus}} = \mathbf{K}\,\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{L}\,\underline{\hat{\mathbf{x}}}_{\mathsf{B}}$$

$$\begin{aligned}\mathbf{C}_{\text{fus}} &= \mathbf{K}\mathbf{C}_{\mathsf{A}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_{\mathsf{AB}}\mathbf{L}^{\mathrm{T}} \\ &\quad + \mathbf{L}\mathbf{C}_{\mathsf{BA}}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{B}}\mathbf{L}^{\mathrm{T}}\end{aligned}$$

Cross-covariance Matrix

$$\mathbf{C}_{\mathsf{AB}} = \mathbf{C}_{\mathsf{BA}}^{\mathrm{T}} = \mathrm{E}\left[(\underline{\hat{\mathbf{x}}}_{\mathsf{A}} - \underline{\mathbf{x}})(\underline{\hat{\mathbf{x}}}_{\mathsf{B}} - \underline{\mathbf{x}})^{\mathrm{T}}\right]$$

Fusion of Estimates

Optimal Fusion

Linear Fusion

$$\hat{\underline{\mathbf{x}}}_{\text{fus}} = \mathbf{K}\,\hat{\underline{\mathbf{x}}}_{\mathsf{A}} + \mathbf{L}\,\hat{\underline{\mathbf{x}}}_{\mathsf{B}}$$

$$\begin{aligned}\mathbf{C}_{\text{fus}} &= \mathbf{K}\mathbf{C}_{\mathsf{A}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_{\mathsf{AB}}\mathbf{L}^{\mathrm{T}} \\ &\quad + \mathbf{L}\mathbf{C}_{\mathsf{BA}}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{B}}\mathbf{L}^{\mathrm{T}}\end{aligned}$$

Cross-covariance Matrix

$$\mathbf{C}_{\mathsf{AB}} = \mathbf{C}_{\mathsf{BA}}^{\mathrm{T}} = \mathrm{E}\left[(\hat{\underline{\mathbf{x}}}_{\mathsf{A}} - \underline{\mathbf{x}})(\hat{\underline{\mathbf{x}}}_{\mathsf{B}} - \underline{\mathbf{x}})^{\mathrm{T}}\right]$$

Fusion of Estimates

Optimal Fusion

Linear Fusion

$$\underline{\hat{\mathbf{x}}}_{\mathrm{fus}} = \mathbf{K}\,\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{L}\,\underline{\hat{\mathbf{x}}}_{\mathsf{B}}$$

$$\mathbf{C}_{\mathrm{fus}} = \mathbf{K}\mathbf{C}_{\mathsf{A}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_{\mathsf{AB}}\mathbf{L}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{BA}}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{B}}\mathbf{L}^{\mathrm{T}}$$

Cross-covariance Matrix

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?

Optimal Fusion

Linear Fusion

$$\underline{\hat{\mathbf{x}}}_{\text{fus}} = \mathbf{K}\,\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{L}\,\underline{\hat{\mathbf{x}}}_{\mathsf{B}}$$

$$\begin{aligned}\mathbf{C}_{\text{fus}} &= \mathbf{K}\mathbf{C}_{\mathsf{A}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_{\mathsf{AB}}\mathbf{L}^{\mathrm{T}} \\ &\quad + \mathbf{L}\mathbf{C}_{\mathsf{BA}}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{B}}\mathbf{L}^{\mathrm{T}}\end{aligned}$$

Cross-covariance Matrix

$$\mathbf{C}_{\mathsf{AB}} = \mathbf{C}_{\mathsf{BA}}^{\mathrm{T}} = \mathrm{E}\left[(\underline{\hat{\mathbf{x}}}_{\mathsf{A}} - \underline{\mathbf{x}})(\underline{\hat{\mathbf{x}}}_{\mathsf{B}} - \underline{\mathbf{x}})^{\mathrm{T}}\right]$$

?

Suboptimal Fusion

Fusion of Estimates

Optimal Fusion

Linear Fusion

$$\underline{\hat{\mathbf{x}}}_{\text{fus}} = \mathbf{K}\,\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{L}\,\underline{\hat{\mathbf{x}}}_{\mathsf{B}}$$

$$\begin{aligned}\mathbf{C}_{\text{fus}} &= \mathbf{K}\mathbf{C}_{\mathsf{A}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_{\mathsf{AB}}\mathbf{L}^{\mathrm{T}} \\ &\quad + \mathbf{L}\mathbf{C}_{\mathsf{BA}}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{B}}\mathbf{L}^{\mathrm{T}}\end{aligned}$$

Cross-covariance Matrix

$$\mathbf{C}_{\mathsf{AB}} = \mathbf{C}_{\mathsf{BA}}^{\mathrm{T}} = \mathrm{E}\left[(\underline{\hat{\mathbf{x}}}_{\mathsf{A}} - \underline{\mathbf{x}})(\underline{\hat{\mathbf{x}}}_{\mathsf{B}} - \underline{\mathbf{x}})^{\mathrm{T}}\right]$$

?

Suboptimal Fusion

Naïve Fusion

$$\underline{\hat{\mathbf{x}}}_{\mathrm{n}} = \mathbf{C}_{\mathrm{n}}\left(\mathbf{C}_{\mathsf{A}}^{-1}\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{C}_{\mathsf{B}}^{-1}\underline{\hat{\mathbf{x}}}_{\mathsf{B}}\right)$$

$$\mathbf{C}_{\mathrm{n}} = \left(\mathbf{C}_{\mathsf{A}}^{-1} + \mathbf{C}_{\mathsf{B}}^{-1}\right)^{-1}$$

Fusion of Estimates

Optimal Fusion

Linear Fusion

$$\hat{\underline{\mathbf{x}}}_{\text{fus}} = \mathbf{K}\,\hat{\underline{\mathbf{x}}}_{\mathsf{A}} + \mathbf{L}\,\hat{\underline{\mathbf{x}}}_{\mathsf{B}}$$

$$\mathbf{C}_{\text{fus}} = \mathbf{K}\mathbf{C}_{\mathsf{A}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_{\mathsf{AB}}\mathbf{L}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{BA}}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{B}}\mathbf{L}^{\mathrm{T}}$$

Cross-covariance Matrix

?

$$\mathbf{C}_{\mathsf{AB}} = \mathbf{C}_{\mathsf{BA}}^{\mathrm{T}} = \mathrm{E}\left[(\hat{\underline{\mathbf{x}}}_{\mathsf{A}} - \underline{\mathbf{x}})(\hat{\underline{\mathbf{x}}}_{\mathsf{B}} - \underline{\mathbf{x}})^{\mathrm{T}}\right]$$

Suboptimal Fusion

Naïve Fusion

$$\hat{\underline{\mathbf{x}}}_{\mathrm{n}} = \mathbf{C}_{\mathrm{n}}\left(\mathbf{C}_{\mathsf{A}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{A}} + \mathbf{C}_{\mathsf{B}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{B}}\right)$$

$$\mathbf{C}_{\mathrm{n}} = \left(\mathbf{C}_{\mathsf{A}}^{-1} + \mathbf{C}_{\mathsf{B}}^{-1}\right)^{-1}$$

Covariance Intersection

$$\hat{\underline{\mathbf{x}}}_{\mathrm{CI}} = \mathbf{C}_{\mathrm{CI}}\left(\omega\mathbf{C}_{\mathsf{A}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{A}} + (1-\omega)\mathbf{C}_{\mathsf{B}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{B}}\right)$$

$$\mathbf{C}_{\mathrm{CI}} = \left(\omega\mathbf{C}_{\mathsf{A}}^{-1} + (1-\omega)\mathbf{C}_{\mathsf{B}}^{-1}\right)^{-1},\ \omega \in [0,1]$$

Fusion of Estimates

Optimal Fusion

Linear Fusion

$$\hat{\underline{\mathbf{x}}}_{\text{fus}} = \mathbf{K}\,\hat{\underline{\mathbf{x}}}_{\mathsf{A}} + \mathbf{L}\,\hat{\underline{\mathbf{x}}}_{\mathsf{B}}$$

$$\mathbf{C}_{\text{fus}} = \mathbf{K}\mathbf{C}_{\mathsf{A}}\mathbf{K}^{\mathrm{T}} + \mathbf{K}\mathbf{C}_{\mathsf{AB}}\mathbf{L}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{BA}}\mathbf{K}^{\mathrm{T}} + \mathbf{L}\mathbf{C}_{\mathsf{B}}\mathbf{L}^{\mathrm{T}}$$

Cross-covariance Matrix

$$\mathbf{C}_{\mathsf{AB}} = \mathbf{C}_{\mathsf{BA}}^{\mathrm{T}} = \mathrm{E}\left[(\hat{\underline{\mathbf{x}}}_{\mathsf{A}} - \underline{\mathbf{x}})(\hat{\underline{\mathbf{x}}}_{\mathsf{B}} - \underline{\mathbf{x}})^{\mathrm{T}}\right]$$

?

Suboptimal Fusion

Naïve Fusion

$$\hat{\underline{\mathbf{x}}}_{\mathrm{n}} = \mathbf{C}_{\mathrm{n}}\left(\mathbf{C}_{\mathsf{A}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{A}} + \mathbf{C}_{\mathsf{B}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{B}}\right)$$

$$\mathbf{C}_{\mathrm{n}} = \left(\mathbf{C}_{\mathsf{A}}^{-1} + \mathbf{C}_{\mathsf{B}}^{-1}\right)^{-1}$$

inconsistent

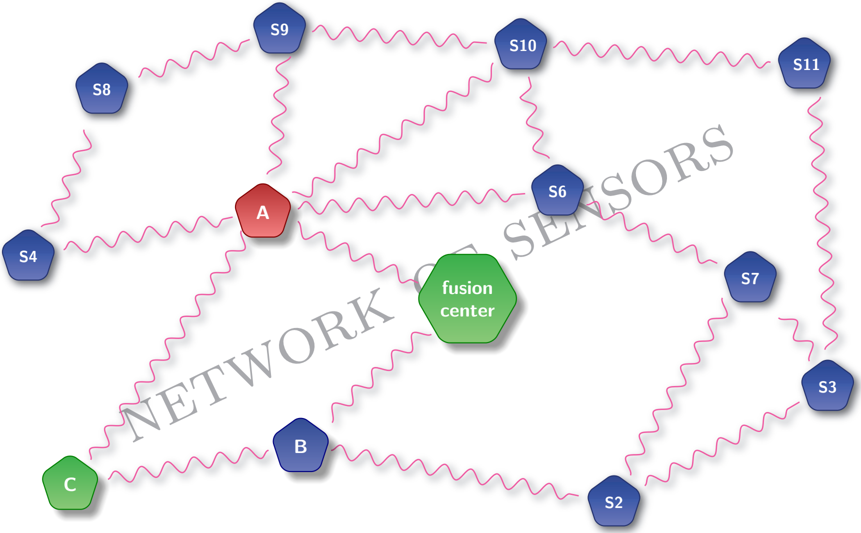
Covariance Intersection

$$\hat{\underline{\mathbf{x}}}_{\mathrm{CI}} = \mathbf{C}_{\mathrm{CI}}\left(\omega\mathbf{C}_{\mathsf{A}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{A}} + (1-\omega)\mathbf{C}_{\mathsf{B}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{B}}\right)$$

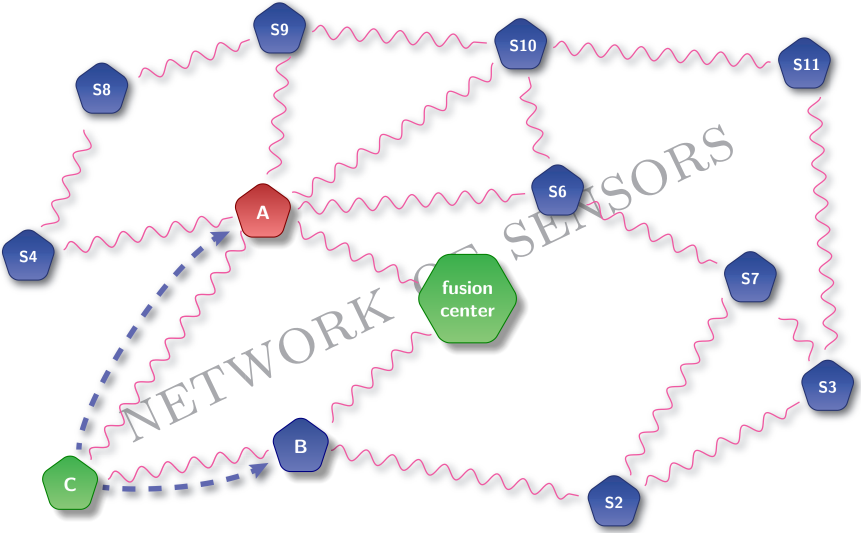
$$\mathbf{C}_{\mathrm{CI}} = \left(\omega\mathbf{C}_{\mathsf{A}}^{-1} + (1-\omega)\mathbf{C}_{\mathsf{B}}^{-1}\right)^{-1},\ \omega\in[0,1]$$

consistent but too conservative

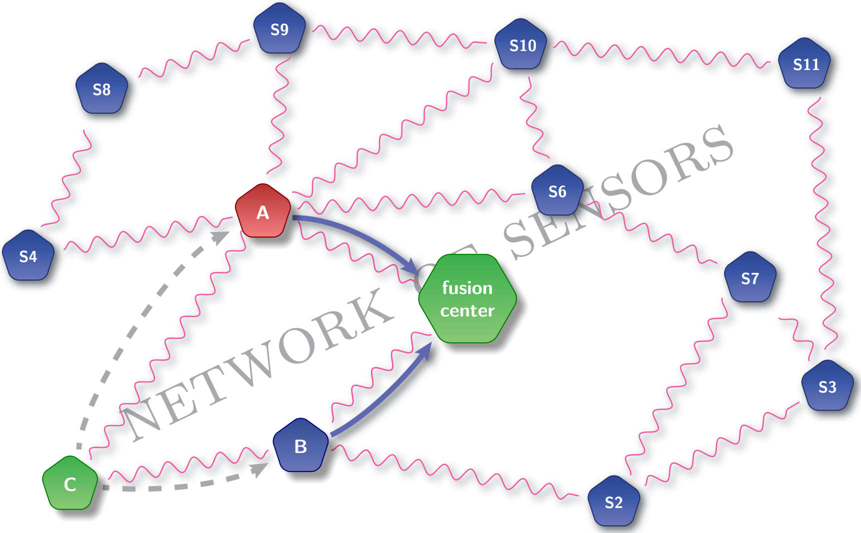
Problem of Common Information



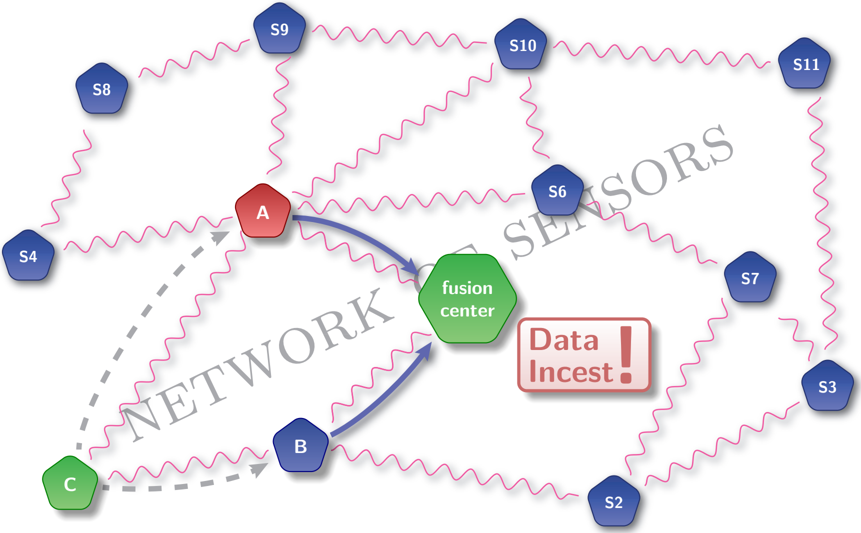
Problem of Common Information



Problem of Common Information



Problem of Common Information



Fusion Under Common Information

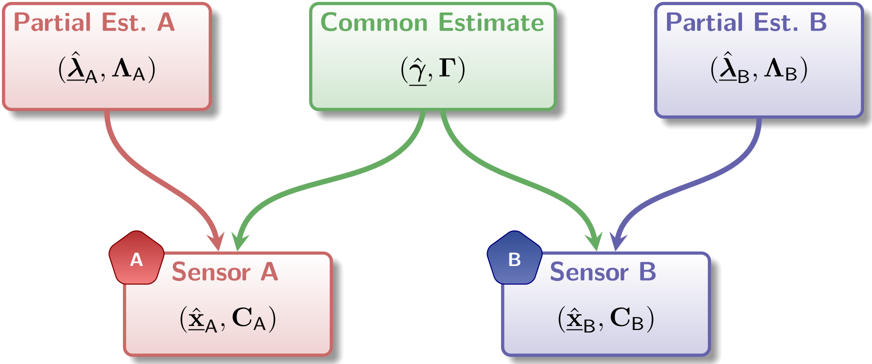
A
Sensor A

$(\hat{\underline{\mathbf{x}}}_{\mathsf{A}}, \mathbf{C}_{\mathsf{A}})$

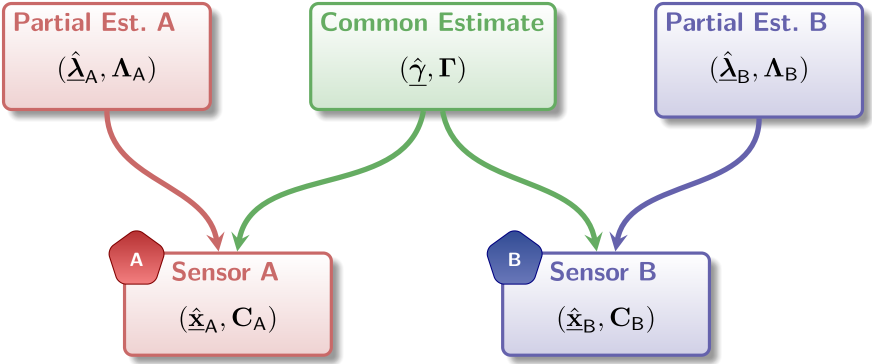
B
Sensor B

$(\hat{\underline{\mathbf{x}}}_{\mathsf{B}}, \mathbf{C}_{\mathsf{B}})$

Fusion Under Common Information



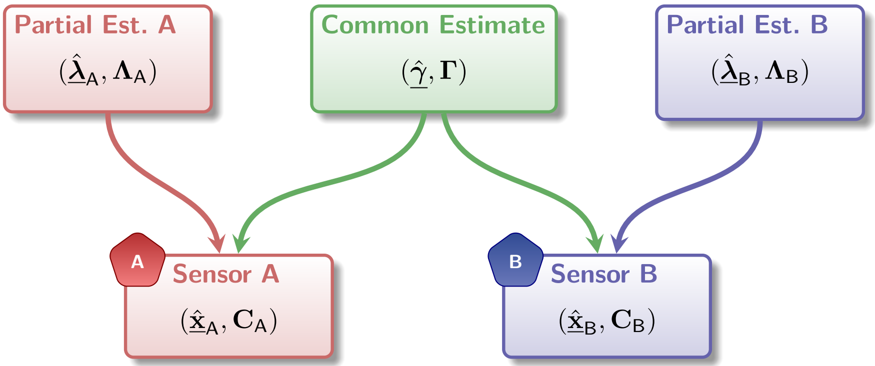
Fusion Under Common Information



$$\hat{\underline{\mathbf{x}}}_A = \mathbf{C}_A \left(\mathbf{\Lambda}_A^{-1} \hat{\underline{\boldsymbol{\lambda}}}_A + \mathbf{\Gamma}^{-1} \hat{\underline{\gamma}} \right) \qquad \hat{\underline{\mathbf{x}}}_B = \mathbf{C}_B \left(\mathbf{\Lambda}_B^{-1} \hat{\underline{\boldsymbol{\lambda}}}_B + \mathbf{\Gamma}^{-1} \hat{\underline{\gamma}} \right)$$

$$\mathbf{C}_A = \left(\mathbf{\Lambda}_A^{-1} + \mathbf{\Gamma}^{-1} \right)^{-1} \qquad \mathbf{C}_B = \left(\mathbf{\Lambda}_B^{-1} + \mathbf{\Gamma}^{-1} \right)^{-1}$$

Fusion Under Common Information



$$\underline{\hat{\mathbf{x}}}_{\mathsf{A}} = \mathbf{C}_{\mathsf{A}}\left(\boldsymbol{\Lambda}_{\mathsf{A}}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_{\mathsf{A}} + \boldsymbol{\Gamma}^{-1}\underline{\hat{\gamma}}\right) \qquad \underline{\hat{\mathbf{x}}}_{\mathsf{B}} = \mathbf{C}_{\mathsf{B}}\left(\boldsymbol{\Lambda}_{\mathsf{B}}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_{\mathsf{B}} + \boldsymbol{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_{\mathsf{A}} = \left(\boldsymbol{\Lambda}_{\mathsf{A}}^{-1} + \boldsymbol{\Gamma}^{-1}\right)^{-1} \qquad \mathbf{C}_{\mathsf{B}} = \left(\boldsymbol{\Lambda}_{\mathsf{B}}^{-1} + \boldsymbol{\Gamma}^{-1}\right)^{-1}$$

Optimal Combination

Sensor A

$$\underline{\hat{\mathbf{x}}}_\mathsf{A} = \mathbf{C}_\mathsf{A}\left(\mathbf{\Lambda}_\mathsf{A}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_\mathsf{A} + \mathbf{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_\mathsf{A} = \left(\mathbf{\Lambda}_\mathsf{A}^{-1} + \mathbf{\Gamma}^{-1}\right)^{-1}$$

Sensor B

$$\underline{\hat{\mathbf{x}}}_\mathsf{B} = \mathbf{C}_\mathsf{B}\left(\mathbf{\Lambda}_\mathsf{B}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_\mathsf{B} + \mathbf{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_\mathsf{B} = \left(\mathbf{\Lambda}_\mathsf{B}^{-1} + \mathbf{\Gamma}^{-1}\right)^{-1}$$

Optimal Combination

Sensor A

$$\underline{\hat{\mathbf{x}}}_{\mathsf{A}} = \mathbf{C}_{\mathsf{A}}\left(\mathbf{\Lambda}_{\mathsf{A}}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_{\mathsf{A}} + \mathbf{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_{\mathsf{A}} = \left(\mathbf{\Lambda}_{\mathsf{A}}^{-1} + \mathbf{\Gamma}^{-1}\right)^{-1}$$

Sensor B

$$\underline{\hat{\mathbf{x}}}_{\mathsf{B}} = \mathbf{C}_{\mathsf{B}}\left(\mathbf{\Lambda}_{\mathsf{B}}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_{\mathsf{B}} + \mathbf{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_{\mathsf{B}} = \left(\mathbf{\Lambda}_{\mathsf{B}}^{-1} + \mathbf{\Gamma}^{-1}\right)^{-1}$$

$$\underline{\hat{\mathbf{x}}}_{\mathrm{rem}} = \mathbf{C}_{\mathrm{rem}}\left(\mathbf{C}_{\mathsf{A}}^{-1}\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{C}_{\mathsf{B}}^{-1}\underline{\hat{\mathbf{x}}}_{\mathsf{B}} - \mathbf{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_{\mathrm{rem}} = \left(\mathbf{C}_{\mathsf{A}}^{-1} + \mathbf{C}_{\mathsf{B}}^{-1} - \mathbf{\Gamma}^{-1}\right)^{-1}$$

Optimal Combination

Sensor A

$$\underline{\hat{\mathbf{x}}}_{\mathsf{A}} = \mathbf{C}_{\mathsf{A}}\left(\mathbf{\Lambda}_{\mathsf{A}}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_{\mathsf{A}} + \mathbf{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_{\mathsf{A}} = \left(\mathbf{\Lambda}_{\mathsf{A}}^{-1} + \mathbf{\Gamma}^{-1}\right)^{-1}$$

Sensor B

$$\underline{\hat{\mathbf{x}}}_{\mathsf{B}} = \mathbf{C}_{\mathsf{B}}\left(\mathbf{\Lambda}_{\mathsf{B}}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_{\mathsf{B}} + \mathbf{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_{\mathsf{B}} = \left(\mathbf{\Lambda}_{\mathsf{B}}^{-1} + \mathbf{\Gamma}^{-1}\right)^{-1}$$

$$\underline{\hat{\mathbf{x}}}_{\mathrm{rem}} = \mathbf{C}_{\mathrm{rem}}\Bigg(\overbrace{\mathbf{C}_{\mathsf{A}}^{-1}\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{C}_{\mathsf{B}}^{-1}\underline{\hat{\mathbf{x}}}_{\mathsf{B}}}^{\text{naïve fusion}} - \mathbf{\Gamma}^{-1}\underline{\hat{\gamma}}\Bigg)$$

$$\mathbf{C}_{\mathrm{rem}} = \Big(\underbrace{\mathbf{C}_{\mathsf{A}}^{-1} + \mathbf{C}_{\mathsf{B}}^{-1}}_{\text{naïve fusion}} - \mathbf{\Gamma}^{-1}\Big)^{-1}$$

Optimal Combination

Sensor A

$$\hat{\underline{\mathbf{x}}}_{\mathsf{A}} = \mathbf{C}_{\mathsf{A}}\left(\boldsymbol{\Lambda}_{\mathsf{A}}^{-1}\hat{\underline{\boldsymbol{\lambda}}}_{\mathsf{A}} + \boldsymbol{\Gamma}^{-1}\hat{\underline{\gamma}}\right)$$

$$\mathbf{C}_{\mathsf{A}} = \left(\boldsymbol{\Lambda}_{\mathsf{A}}^{-1} + \boldsymbol{\Gamma}^{-1}\right)^{-1}$$

Sensor B

$$\hat{\underline{\mathbf{x}}}_{\mathsf{B}} = \mathbf{C}_{\mathsf{B}}\left(\boldsymbol{\Lambda}_{\mathsf{B}}^{-1}\hat{\underline{\boldsymbol{\lambda}}}_{\mathsf{B}} + \boldsymbol{\Gamma}^{-1}\hat{\underline{\gamma}}\right)$$

$$\mathbf{C}_{\mathsf{B}} = \left(\boldsymbol{\Lambda}_{\mathsf{B}}^{-1} + \boldsymbol{\Gamma}^{-1}\right)^{-1}$$

$$\hat{\underline{\mathbf{x}}}_{\mathrm{rem}} = \mathbf{C}_{\mathrm{rem}}\left(\overbrace{\mathbf{C}_{\mathsf{A}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{A}} + \mathbf{C}_{\mathsf{B}}^{-1}\hat{\underline{\mathbf{x}}}_{\mathsf{B}}}^{\text{naïve fusion}} - \overbrace{\boldsymbol{\Gamma}^{-1}\hat{\underline{\gamma}}}^{\text{removal of common information}}\right)$$

$$\mathbf{C}_{\mathrm{rem}} = \left(\underbrace{\mathbf{C}_{\mathsf{A}}^{-1} + \mathbf{C}_{\mathsf{B}}^{-1}}_{\text{naïve fusion}} - \underbrace{\boldsymbol{\Gamma}^{-1}}_{\text{removal of common information}}\right)^{-1}$$

Optimal Combination

Sensor A

$$\underline{\hat{\mathbf{x}}}_{\mathsf{A}} = \mathbf{C}_{\mathsf{A}}\left(\boldsymbol{\Lambda}_{\mathsf{A}}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_{\mathsf{A}} + \boldsymbol{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_{\mathsf{A}} = \left(\boldsymbol{\Lambda}_{\mathsf{A}}^{-1} + \boldsymbol{\Gamma}^{-1}\right)^{-1}$$

Sensor B

$$\underline{\hat{\mathbf{x}}}_{\mathsf{B}} = \mathbf{C}_{\mathsf{B}}\left(\boldsymbol{\Lambda}_{\mathsf{B}}^{-1}\underline{\hat{\boldsymbol{\lambda}}}_{\mathsf{B}} + \boldsymbol{\Gamma}^{-1}\underline{\hat{\gamma}}\right)$$

$$\mathbf{C}_{\mathsf{B}} = \left(\boldsymbol{\Lambda}_{\mathsf{B}}^{-1} + \boldsymbol{\Gamma}^{-1}\right)^{-1}$$

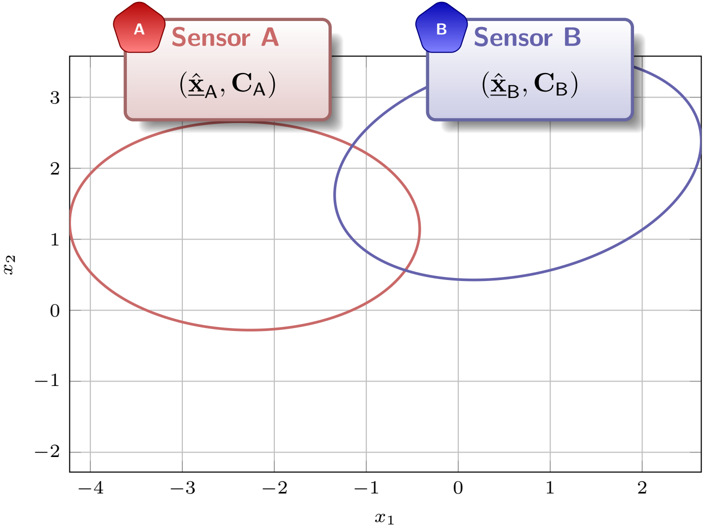
$$\underline{\hat{\mathbf{x}}}_{\mathrm{rem}} = \mathbf{C}_{\mathrm{rem}}\left(\overbrace{\mathbf{C}_{\mathsf{A}}^{-1}\underline{\hat{\mathbf{x}}}_{\mathsf{A}} + \mathbf{C}_{\mathsf{B}}^{-1}\underline{\hat{\mathbf{x}}}_{\mathsf{B}}}^{\text{naïve fusion}} - \overbrace{\boldsymbol{\Gamma}^{-1}\underline{\hat{\gamma}}}^{\text{removal of common information}}\right)$$

$$\mathbf{C}_{\mathrm{rem}} = \left(\underbrace{\mathbf{C}_{\mathsf{A}}^{-1} + \mathbf{C}_{\mathsf{B}}^{-1}}_{\text{naïve fusion}} - \underbrace{\boldsymbol{\Gamma}^{-1}}_{\text{removal of common information}}\right)^{-1}$$

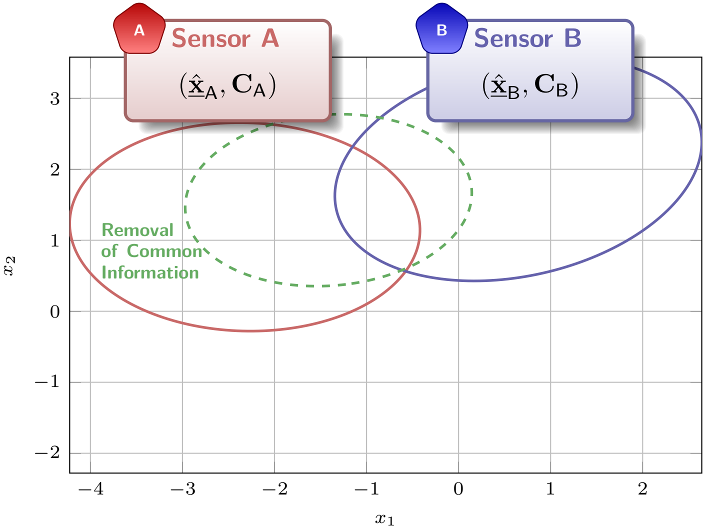
How does this compare to the B/C fusion rule?

(cross-covariance matrix required)

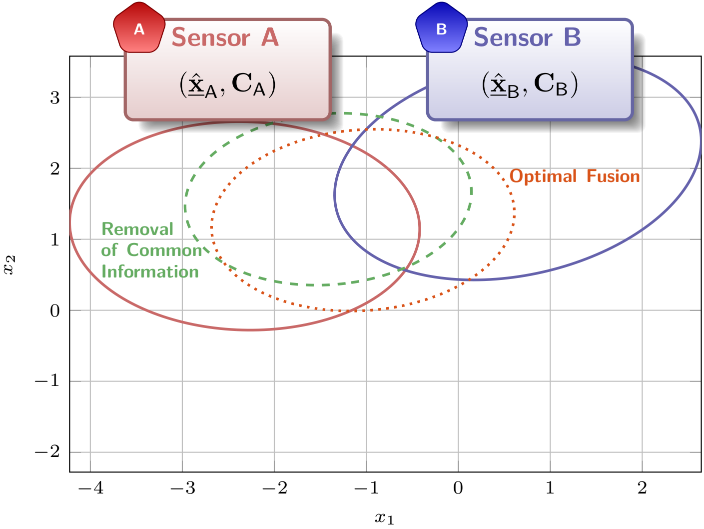
Removal of Common Information vs. Optimal Fusion



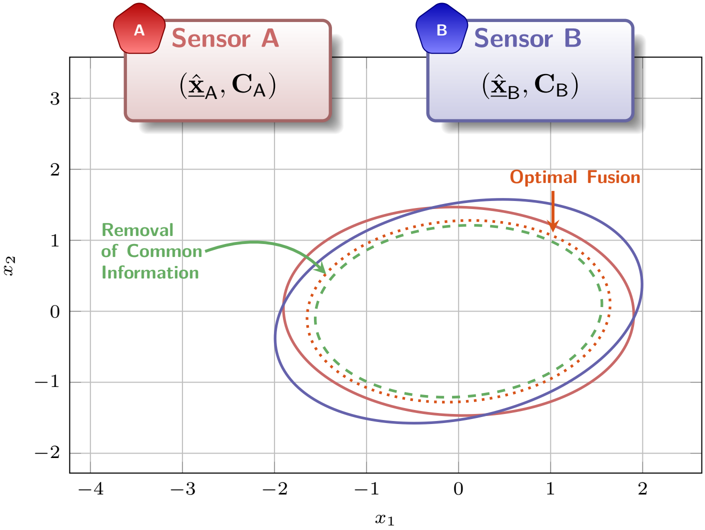
Removal of Common Information vs. Optimal Fusion



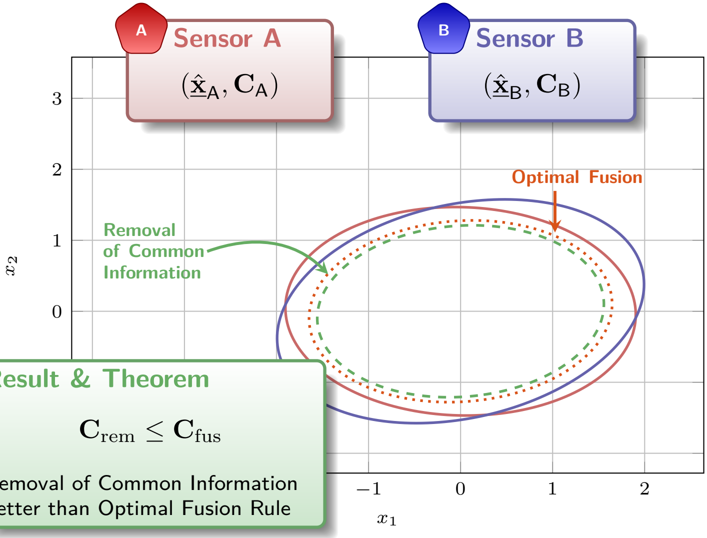
Removal of Common Information vs. Optimal Fusion



Removal of Common Information vs. Optimal Fusion



Removal of Common Information vs. Optimal Fusion

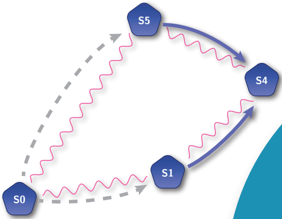


Result & Theorem

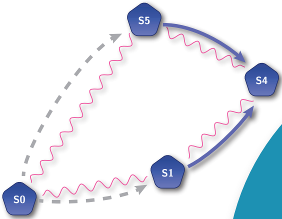
$$\mathbf{C}_{\text{rem}} \leq \mathbf{C}_{\text{fus}}$$

Removal of Common Information better than Optimal Fusion Rule

Overview

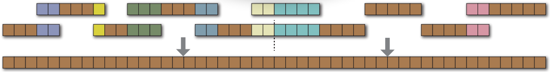


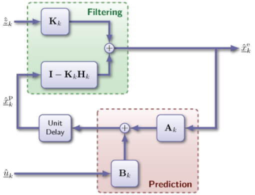
Overview



S5
S4
S1
S0

Optimal fusion is not optimal
01
02
Independent filters are not independent





:

State Estimate

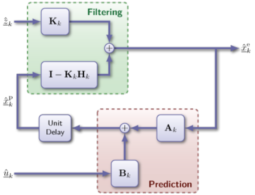
$$\hat{\underline{\mathbf{x}}}_k = \begin{bmatrix} \hat{\underline{\mathbf{x}}}_{\mathsf{A}} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{C}} \\ \vdots \end{bmatrix}$$

,

Covariance Matrix

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{C}_{\mathsf{A}} & \mathbf{C}_{\mathsf{AB}} & \mathbf{C}_{\mathsf{AC}} & \\ \mathbf{C}_{\mathsf{BA}} & \mathbf{C}_{\mathsf{B}} & \mathbf{C}_{\mathsf{BC}} & \cdots \\ \mathbf{C}_{\mathsf{CA}} & \mathbf{C}_{\mathsf{CB}} & \mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix}$$

Kalman Filtering for Large-Scale Systems



:

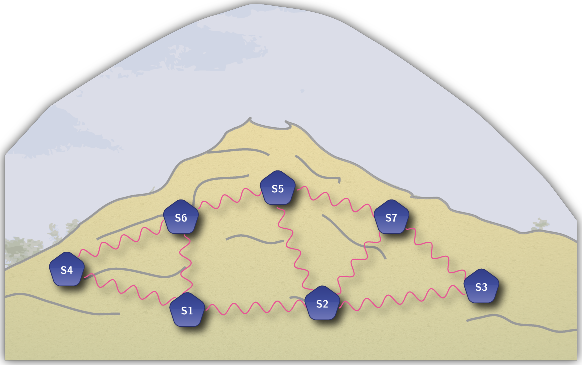
State Estimate

$$\underline{\hat{\mathbf{x}}}_k = \begin{bmatrix} \underline{\hat{\mathbf{x}}}_A \\ \underline{\hat{\mathbf{x}}}_B \\ \underline{\hat{\mathbf{x}}}_C \\ \vdots \end{bmatrix}$$

,

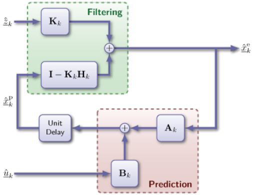
Covariance Matrix

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{C}_A & \mathbf{C}_{AB} & \mathbf{C}_{AC} & \\ \mathbf{C}_{BA} & \mathbf{C}_B & \mathbf{C}_{BC} & \cdots \\ \mathbf{C}_{CA} & \mathbf{C}_{CB} & \mathbf{C}_C & \\ & \vdots & & \ddots \end{bmatrix}$$



monitoring
of large-scale
phenomenon

Kalman Filtering for Large-Scale Systems



:

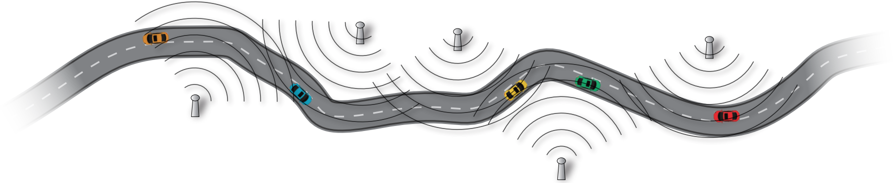
State Estimate

$$\hat{\underline{\mathbf{x}}}_k = \begin{bmatrix} \hat{\underline{\mathbf{x}}}_{\mathsf{A}} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{C}} \\ \vdots \end{bmatrix}$$

,

Covariance Matrix

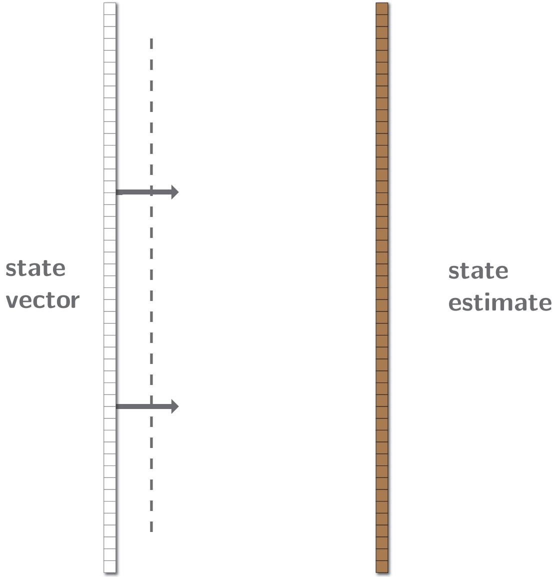
$$\mathbf{C}_k = \begin{bmatrix} \mathbf{C}_{\mathsf{A}} & \mathbf{C}_{\mathsf{AB}} & \mathbf{C}_{\mathsf{AC}} & \\ \mathbf{C}_{\mathsf{BA}} & \mathbf{C}_{\mathsf{B}} & \mathbf{C}_{\mathsf{BC}} & \cdots \\ \mathbf{C}_{\mathsf{CA}} & \mathbf{C}_{\mathsf{CB}} & \mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix}$$



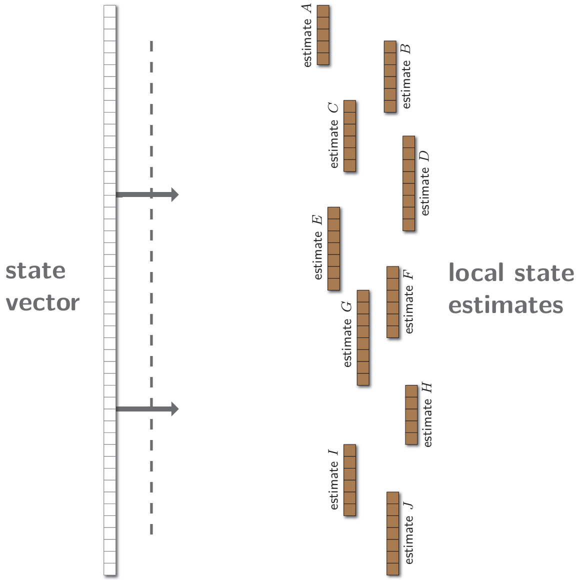
Considered Problem

state
vector

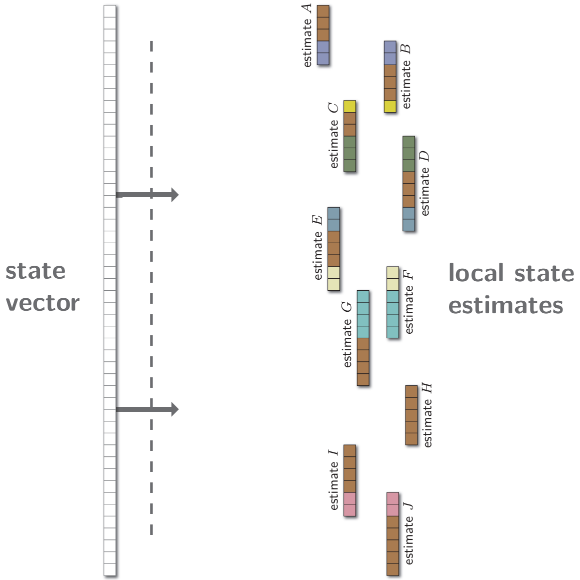
Considered Problem



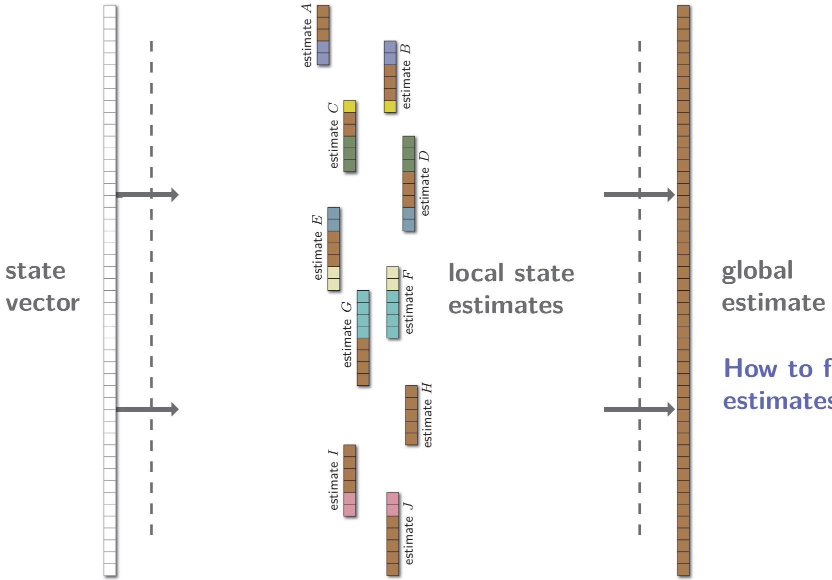
Considered Problem



Considered Problem



Considered Problem



How to fuse estimates?

Fusion as Weighted Least-Squares Problem

Estimates of the same state vector

$$\begin{bmatrix} \hat{\underline{\mathbf{x}}}_{\mathsf{A}} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}} \end{bmatrix}$$

Fusion as Weighted Least-Squares Problem

Estimates of the same state vector

$$\begin{bmatrix} \hat{\underline{\mathbf{x}}}_{\mathsf{A}} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{x}} \\ \underline{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \hat{\underline{\mathbf{x}}}_{\mathsf{A}} - \underline{\mathbf{x}} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}} - \underline{\mathbf{x}} \end{bmatrix}$$

Fusion as Weighted Least-Squares Problem

Estimates of the same state vector

$$\begin{bmatrix} \underline{\hat{\mathbf{x}}}_{\mathrm{A}} \\ \underline{\hat{\mathbf{x}}}_{\mathrm{B}} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{x}} \\ \underline{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \underline{\hat{\mathbf{x}}}_{\mathrm{A}} - \underline{\mathbf{x}} \\ \underline{\hat{\mathbf{x}}}_{\mathrm{B}} - \underline{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\mathbf{x}} + \underline{\tilde{\mathbf{x}}}$$

Estimates of the same state vector

$$\begin{bmatrix} \underline{\hat{\mathbf{x}}}_{\mathsf{A}} \\ \underline{\hat{\mathbf{x}}}_{\mathsf{B}} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{x}} \\ \underline{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \underline{\hat{\mathbf{x}}}_{\mathsf{A}} - \underline{\mathbf{x}} \\ \underline{\hat{\mathbf{x}}}_{\mathsf{B}} - \underline{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\mathbf{x}} + \underline{\tilde{\mathbf{x}}}$$

Measurement Matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix}$$

Estimates of the same state vector

$$\begin{bmatrix} \underline{\hat{\mathbf{x}}}_\mathsf{A} \\ \underline{\hat{\mathbf{x}}}_\mathsf{B} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{x}} \\ \underline{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \underline{\hat{\mathbf{x}}}_\mathsf{A} - \underline{\mathbf{x}} \\ \underline{\hat{\mathbf{x}}}_\mathsf{B} - \underline{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\mathbf{x}} + \underline{\tilde{\mathbf{x}}}$$

Measurement Matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix}$$

Measurement Error

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_\mathsf{A} & \mathbf{C}_\mathsf{AB} \\ \mathbf{C}_\mathsf{BA} & \mathbf{C}_\mathsf{B} \end{bmatrix}$$

Fusion as Weighted Least-Squares Problem

Estimates of the same state vector

$$\begin{bmatrix} \hat{\underline{\mathbf{x}}}_\mathsf{A} \\ \hat{\underline{\mathbf{x}}}_\mathsf{B} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{x}} \\ \underline{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \hat{\underline{\mathbf{x}}}_\mathsf{A} - \underline{\mathbf{x}} \\ \hat{\underline{\mathbf{x}}}_\mathsf{B} - \underline{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\mathbf{x}} + \tilde{\underline{\mathbf{x}}}$$

Measurement Matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix}$$

Measurement Error

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_\mathsf{A} & \mathbf{C}_\mathsf{AB} \\ \mathbf{C}_\mathsf{BA} & \mathbf{C}_\mathsf{B} \end{bmatrix}$$

Weighted Least-Squares Fusion

$$\hat{\underline{\mathbf{x}}}_{\text{fus}} = \left(\mathbf{H}^\mathrm{T} (\widetilde{\mathbf{C}})^{-1} \mathbf{H}\right)^{-1} \mathbf{H}^\mathrm{T} (\widetilde{\mathbf{C}})^{-1} \begin{bmatrix} \hat{\underline{\mathbf{x}}}_\mathsf{A} \\ \hat{\underline{\mathbf{x}}}_\mathsf{B} \end{bmatrix}$$

Fusion as Weighted Least-Squares Problem

Estimates of the same state vector

$$\begin{bmatrix} \underline{\hat{\mathbf{x}}}_\mathsf{A} \\ \underline{\hat{\mathbf{x}}}_\mathsf{B} \end{bmatrix} = \begin{bmatrix} \underline{\mathbf{x}} \\ \underline{\mathbf{x}} \end{bmatrix} + \begin{bmatrix} \underline{\hat{\mathbf{x}}}_\mathsf{A} - \underline{\mathbf{x}} \\ \underline{\hat{\mathbf{x}}}_\mathsf{B} - \underline{\mathbf{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\mathbf{x}} + \underline{\tilde{\mathbf{x}}}$$

Measurement Matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix}$$

Measurement Error

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_\mathsf{A} & \mathbf{C}_\mathsf{AB} \\ \mathbf{C}_\mathsf{BA} & \mathbf{C}_\mathsf{B} \end{bmatrix}$$

Weighted Least-Squares Fusion

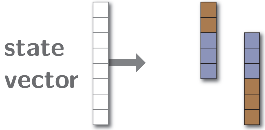
$$\underline{\hat{\mathbf{x}}}_{\text{fus}} = \left(\mathbf{H}^\mathrm{T}(\widetilde{\mathbf{C}})^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^\mathrm{T}(\widetilde{\mathbf{C}})^{-1} \begin{bmatrix} \underline{\hat{\mathbf{x}}}_\mathsf{A} \\ \underline{\hat{\mathbf{x}}}_\mathsf{B} \end{bmatrix}$$

same result as Bar-Shalom/Campo fusion

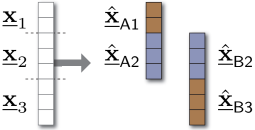
Fusion of Unequal State Vectors

state
vector

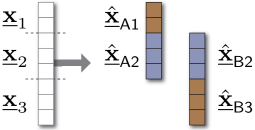
Fusion of Unequal State Vectors



Fusion of Unequal State Vectors

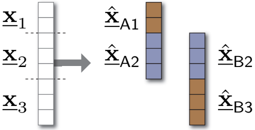


Fusion of Unequal State Vectors



Measurement Equation

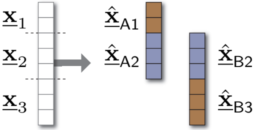
$$
\begin{matrix} \underline{\hat{\mathbf{x}}}_{\mathsf{A}} \left\{ \vphantom{\begin{matrix} a \\ b \end{matrix}} \right. \\ \underline{\hat{\mathbf{x}}}_{\mathsf{B}} \left\{ \vphantom{\begin{matrix} a \\ b \end{matrix}} \right. \end{matrix}
\begin{bmatrix} \underline{\hat{\mathbf{x}}}_{\mathsf{A1}} \\ \underline{\hat{\mathbf{x}}}_{\mathsf{A2}} \\ \underline{\hat{\mathbf{x}}}_{\mathsf{B2}} \\ \underline{\hat{\mathbf{x}}}_{\mathsf{B3}} \end{bmatrix}
= \underbrace{\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}}_{=\mathbf{H}}
\begin{bmatrix} \underline{\mathbf{x}}_1 \\ \underline{\mathbf{x}}_2 \\ \underline{\mathbf{x}}_3 \end{bmatrix} + \underline{\tilde{\mathbf{x}}}
$$



Measurement Equation

$$\begin{array}{l} \hat{\underline{\mathbf{x}}}_{\mathrm{A}} \left\{ \vphantom{\begin{matrix} a \\ b \end{matrix}} \right. \\ \hat{\underline{\mathbf{x}}}_{\mathrm{B}} \left\{ \vphantom{\begin{matrix} a \\ b \end{matrix}} \right. \end{array} \begin{bmatrix} \hat{\underline{\mathbf{x}}}_{\mathrm{A}1} \\ \hat{\underline{\mathbf{x}}}_{\mathrm{A}2} \\ \hat{\underline{\mathbf{x}}}_{\mathrm{B}2} \\ \hat{\underline{\mathbf{x}}}_{\mathrm{B}3} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}}_{=\mathbf{H}} \begin{bmatrix} \underline{\mathbf{x}}_1 \\ \underline{\mathbf{x}}_2 \\ \underline{\mathbf{x}}_3 \end{bmatrix} + \tilde{\underline{\mathbf{x}}}$$

Fusion of Unequal State Vectors



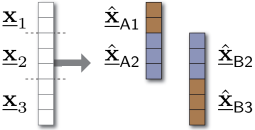
Measurement Equation

$$\begin{matrix} \hat{\underline{\mathbf{x}}}_{\mathsf{A}} \left\{ \begin{matrix} \\ \\ \end{matrix} \right. \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}} \left\{ \begin{matrix} \\ \\ \end{matrix} \right. \end{matrix} \begin{bmatrix} \hat{\underline{\mathbf{x}}}_{\mathsf{A}1} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{A}2} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}2} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}3} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}}_{=\mathbf{H}} \begin{bmatrix} \underline{\mathbf{x}}_1 \\ \underline{\mathbf{x}}_2 \\ \underline{\mathbf{x}}_3 \end{bmatrix} + \tilde{\underline{\mathbf{x}}}$$

Weighted Least-Squares Fusion

$$\begin{bmatrix} \hat{\underline{\mathbf{x}}}_1 \\ \hat{\underline{\mathbf{x}}}_2 \\ \hat{\underline{\mathbf{x}}}_3 \end{bmatrix} = \left(\mathbf{H}^{\mathrm{T}}(\widetilde{\mathbf{C}})^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}(\widetilde{\mathbf{C}})^{-1} \begin{bmatrix} \hat{\underline{\mathbf{x}}}_{\mathsf{A}1} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{A}2} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}2} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}3} \end{bmatrix}$$

Fusion of Unequal State Vectors



Measurement Equation

$$\begin{array}{c} \hat{\underline{\mathbf{x}}}_A \left\{ \begin{array}{c} \\ \\ \end{array} \right. \\ \hat{\underline{\mathbf{x}}}_B \left\{ \begin{array}{c} \\ \\ \end{array} \right. \end{array} \begin{bmatrix} \hat{\underline{\mathbf{x}}}_{A1} \\ \hat{\underline{\mathbf{x}}}_{A2} \\ \hat{\underline{\mathbf{x}}}_{B2} \\ \hat{\underline{\mathbf{x}}}_{B3} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{I} & 0 & 0 \\ 0 & \mathbf{I} & 0 \\ 0 & \mathbf{I} & 0 \\ 0 & 0 & \mathbf{I} \end{bmatrix}}_{=\mathbf{H}} \begin{bmatrix} \underline{\mathbf{x}}_1 \\ \underline{\mathbf{x}}_2 \\ \underline{\mathbf{x}}_3 \end{bmatrix} + \tilde{\underline{\mathbf{x}}}$$

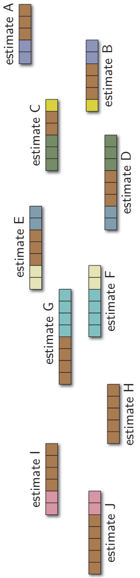
Weighted Least-Squares Fusion

$$\begin{bmatrix} \hat{\underline{\mathbf{x}}}_1 \\ \hat{\underline{\mathbf{x}}}_2 \\ \hat{\underline{\mathbf{x}}}_3 \end{bmatrix} = \left(\mathbf{H}^{\mathrm{T}} (\widetilde{\mathbf{C}})^{-1} \mathbf{H}\right)^{-1} \mathbf{H}^{\mathrm{T}} (\widetilde{\mathbf{C}})^{-1} \begin{bmatrix} \hat{\underline{\mathbf{x}}}_{A1} \\ \hat{\underline{\mathbf{x}}}_{A2} \\ \hat{\underline{\mathbf{x}}}_{B2} \\ \hat{\underline{\mathbf{x}}}_{B3} \end{bmatrix}$$

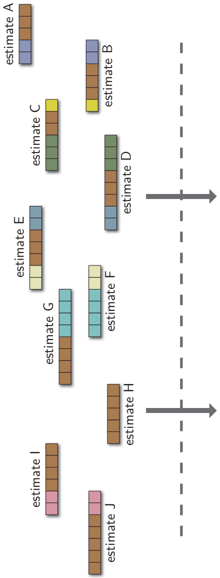
Error Covariance Matrix

$$\mathbf{C}_{\text{fus}} = \left(\mathbf{H}^{\mathrm{T}} (\widetilde{\mathbf{C}})^{-1} \mathbf{H}\right)^{-1}$$

Optimal Fusion

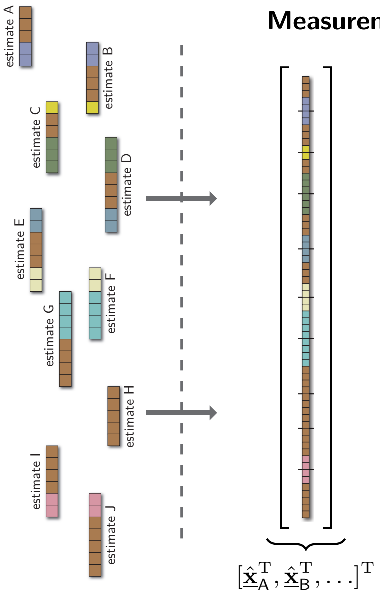


Optimal Fusion



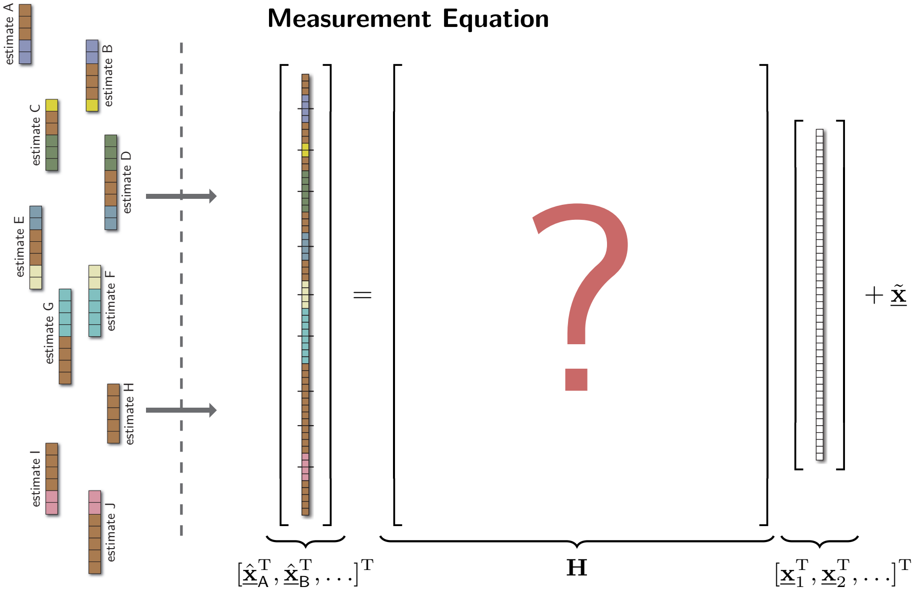
How to set up measurement mapping?

Measurement Equation



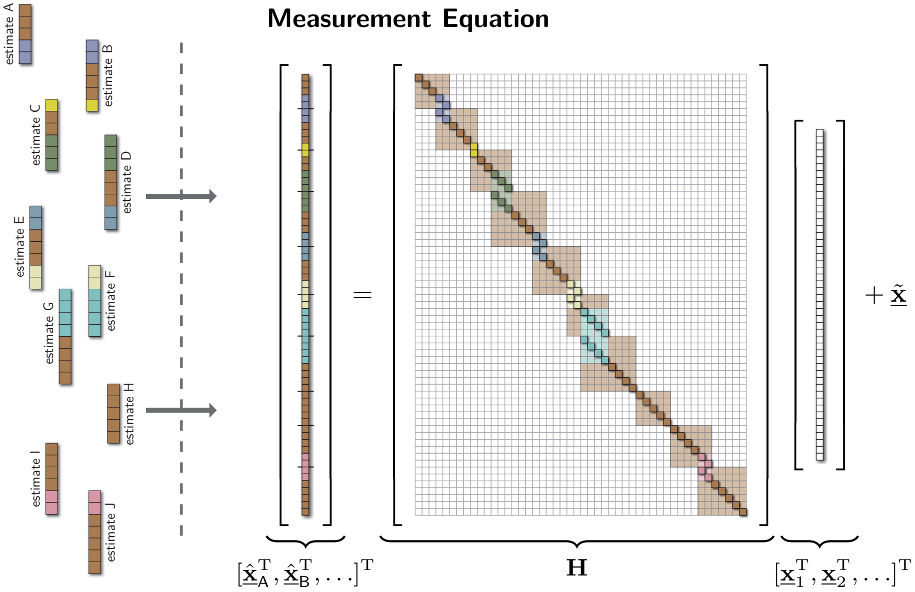
Optimal Fusion

Measurement Equation



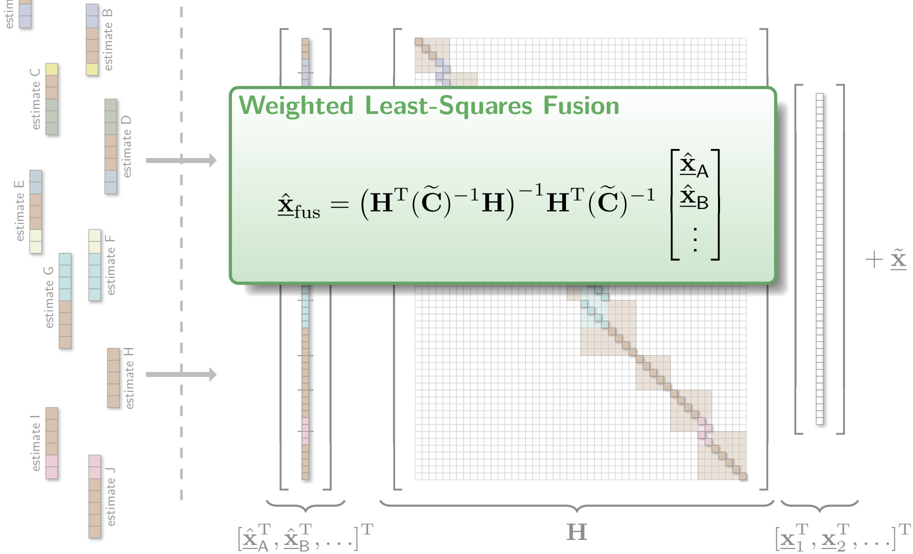
Optimal Fusion

Measurement Equation



Optimal Fusion

Measurement Equation

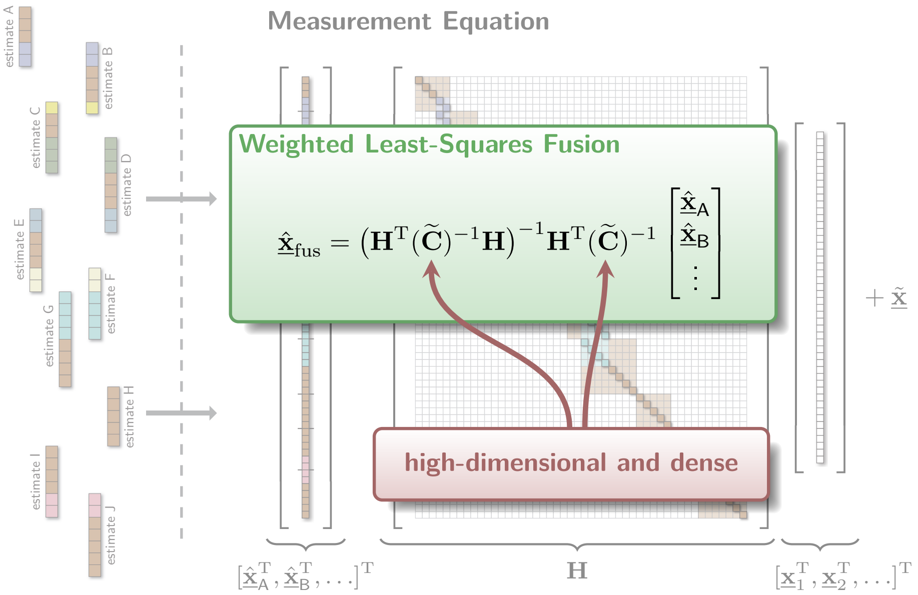


Weighted Least-Squares Fusion

$$\hat{\underline{\mathbf{x}}}_{\mathrm{fus}} = \left(\mathbf{H}^{\mathrm{T}}(\widetilde{\mathbf{C}})^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}(\widetilde{\mathbf{C}})^{-1}\begin{bmatrix}\hat{\underline{\mathbf{x}}}_{\mathrm{A}} \\ \hat{\underline{\mathbf{x}}}_{\mathrm{B}} \\ \vdots\end{bmatrix}$$

Optimal Fusion

Measurement Equation



Weighted Least-Squares Fusion

$$\hat{\underline{\mathbf{x}}}_{\mathrm{fus}} = \left(\mathbf{H}^{\mathrm{T}}(\widetilde{\mathbf{C}})^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}(\widetilde{\mathbf{C}})^{-1}\begin{bmatrix}\hat{\underline{\mathbf{x}}}_{\mathrm{A}}\\ \hat{\underline{\mathbf{x}}}_{\mathrm{B}}\\ \vdots\end{bmatrix}$$

high-dimensional and dense

Joint Cross-Covariance Matrix

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_{\mathsf{A}} & \mathbf{C}_{\mathsf{AB}} & \mathbf{C}_{\mathsf{AC}} & \\ \mathbf{C}_{\mathsf{BA}} & \mathbf{C}_{\mathsf{B}} & \mathbf{C}_{\mathsf{BC}} & \cdots \\ \mathbf{C}_{\mathsf{CA}} & \mathbf{C}_{\mathsf{CB}} & \mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix}$$

Suboptimal Fusion

Joint Cross-Covariance Matrix

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_{\mathsf{A}} & \mathbf{C}_{\mathsf{AB}} & \mathbf{C}_{\mathsf{AC}} & \\ \mathbf{C}_{\mathsf{BA}} & \mathbf{C}_{\mathsf{B}} & \mathbf{C}_{\mathsf{BC}} & \cdots \\ \mathbf{C}_{\mathsf{CA}} & \mathbf{C}_{\mathsf{CB}} & \mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix}$$

Suboptimal Fusion

Joint Cross-Covariance Matrix

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_{\mathsf{A}} & \mathbf{C}_{\mathsf{AB}} & \mathbf{C}_{\mathsf{AC}} & \\ \mathbf{C}_{\mathsf{BA}} & \mathbf{C}_{\mathsf{B}} & \mathbf{C}_{\mathsf{BC}} & \cdots \\ \mathbf{C}_{\mathsf{CA}} & \mathbf{C}_{\mathsf{CB}} & \mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix}$$

Difficulties

- bookkeeping
- storing
- processing

Suboptimal Fusion

Joint Cross-Covariance Matrix

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_{\mathsf{A}} & \mathbf{C}_{\mathsf{AB}} & \mathbf{C}_{\mathsf{AC}} & \\ \mathbf{C}_{\mathsf{BA}} & \mathbf{C}_{\mathsf{B}} & \mathbf{C}_{\mathsf{BC}} & \cdots \\ \mathbf{C}_{\mathsf{CA}} & \mathbf{C}_{\mathsf{CB}} & \mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix} \leq \begin{bmatrix} \frac{1}{w_A}\mathbf{C}_{\mathsf{A}} & \mathbf{0} & \mathbf{0} & \\ \mathbf{0} & \frac{1}{w_B}\mathbf{C}_{\mathsf{B}} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \frac{1}{w_C}\mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix} =: \mathbf{C}_{\mathrm{CB}}$$

Difficulties

- bookkeeping
- storing
- processing

Suboptimal Fusion

Joint Cross-Covariance Matrix

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_{\mathsf{A}} & \mathbf{C}_{\mathsf{AB}} & \mathbf{C}_{\mathsf{AC}} & \\ \mathbf{C}_{\mathsf{BA}} & \mathbf{C}_{\mathsf{B}} & \mathbf{C}_{\mathsf{BC}} & \cdots \\ \mathbf{C}_{\mathsf{CA}} & \mathbf{C}_{\mathsf{CB}} & \mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix} \leq \begin{bmatrix} \frac{1}{w_A}\mathbf{C}_{\mathsf{A}} & \mathbf{0} & \mathbf{0} & \\ \mathbf{0} & \frac{1}{w_B}\mathbf{C}_{\mathsf{B}} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \frac{1}{w_C}\mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix} =: \mathbf{C}_{\mathrm{CB}}$$

Difficulties

- bookkeeping
- storing
- processing

Joint Cross-Covariance Matrix

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_{\mathsf{A}} & \mathbf{C}_{\mathsf{AB}} & \mathbf{C}_{\mathsf{AC}} & \\ \mathbf{C}_{\mathsf{BA}} & \mathbf{C}_{\mathsf{B}} & \mathbf{C}_{\mathsf{BC}} & \cdots \\ \mathbf{C}_{\mathsf{CA}} & \mathbf{C}_{\mathsf{CB}} & \mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix} \leq \begin{bmatrix} \frac{1}{w_A}\mathbf{C}_{\mathsf{A}} & \mathbf{0} & \mathbf{0} & \\ \mathbf{0} & \frac{1}{w_B}\mathbf{C}_{\mathsf{B}} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \frac{1}{w_C}\mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix} =: \mathbf{C}_{\mathrm{CB}}$$

Difficulties

- bookkeeping
- storing
- processing

Solution: Covariance Inflation

- $\sum \omega_X = 1,\ \omega_X > 0$
- upper bound
- suboptimal

Suboptimal Fusion

Joint Cross-Covariance Matrix

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_{\mathsf{A}} & \mathbf{C}_{\mathsf{AB}} & \mathbf{C}_{\mathsf{AC}} & \\ \mathbf{C}_{\mathsf{BA}} & \mathbf{C}_{\mathsf{B}} & \mathbf{C}_{\mathsf{BC}} & \cdots \\ \mathbf{C}_{\mathsf{CA}} & \mathbf{C}_{\mathsf{CB}} & \mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix} \leq \begin{bmatrix} \frac{1}{w_A}\mathbf{C}_{\mathsf{A}} & \mathbf{0} & \mathbf{0} & \\ \mathbf{0} & \frac{1}{w_B}\mathbf{C}_{\mathsf{B}} & \mathbf{0} & \cdots \\ \mathbf{0} & \mathbf{0} & \frac{1}{w_C}\mathbf{C}_{\mathsf{C}} & \\ & \vdots & & \ddots \end{bmatrix} =: \mathbf{C}_{\mathrm{CB}}$$

Difficulties

- bookkeeping
- storing
- processing

Solution: Covariance Inflation

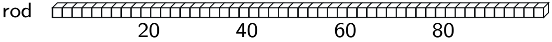
- $\sum \omega_X = 1,\ \omega_X > 0$
- upper bound
- suboptimal

Conservative Fusion

$$\hat{\underline{\mathbf{x}}}_{\mathrm{fus}} = \left(\mathbf{H}^{\mathrm{T}}(\mathbf{C}_{\mathrm{CB}})^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}(\mathbf{C}_{\mathrm{CB}})^{-1}\begin{bmatrix}\hat{\underline{\mathbf{x}}}_{\mathsf{A}} \\ \hat{\underline{\mathbf{x}}}_{\mathsf{B}} \\ \vdots\end{bmatrix}$$

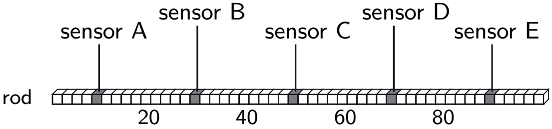
Simulation Setup

Heat Conduction



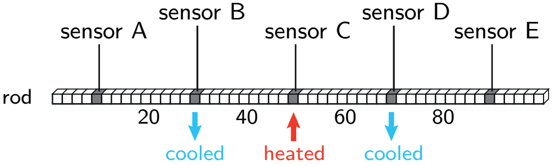
Simulation Setup

Heat Conduction



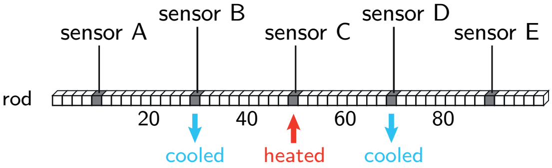
Simulation Setup

Heat Conduction



Simulation Setup

Heat Conduction



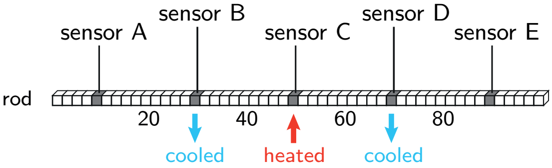
Estimation

Process Model

$$\mathbf{x}_{k+1,n} = 0.17\mathbf{x}_{k,n-1} + 0.66\mathbf{x}_{k,n} + 0.17\mathbf{x}_{k,n+1} + \mathbf{w}_{k,n}$$

Simulation Setup

Heat Conduction



Estimation

Process Model

$$\mathbf{x}_{k+1,n} = 0.17\mathbf{x}_{k,n-1} + 0.66\mathbf{x}_{k,n} + 0.17\mathbf{x}_{k,n+1} + \mathbf{w}_{k,n}$$

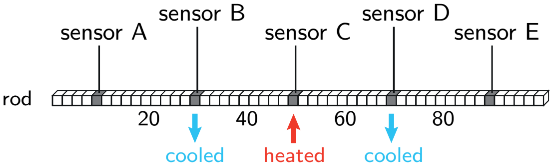
Sensor Models

$$\mathbf{z}_k^{\mathsf{A}} = \mathbf{x}_{k,10} + \mathbf{v}_k^{\mathsf{A}} \ , \quad \mathbf{z}_k^{\mathsf{B}} = \mathbf{x}_{k,30} + \mathbf{v}_k^{\mathsf{B}} \ , \quad \mathbf{z}_k^{\mathsf{C}} = \mathbf{x}_{k,50} + \mathbf{v}_k^{\mathsf{C}} \ ,$$

$$\mathbf{z}_k^{\mathsf{D}} = \mathbf{x}_{k,70} + \mathbf{v}_k^{\mathsf{D}} \ , \quad \mathbf{z}_k^{\mathsf{E}} = \mathbf{x}_{k,90} + \mathbf{v}_k^{\mathsf{E}} \ ,$$

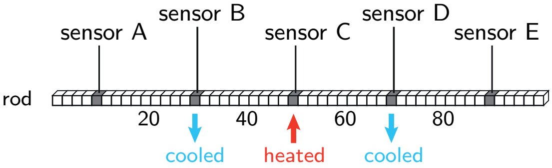
Simulation: Centralized

Heat Conduction



Simulation: Centralized

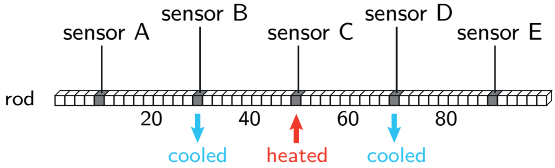
Heat Conduction



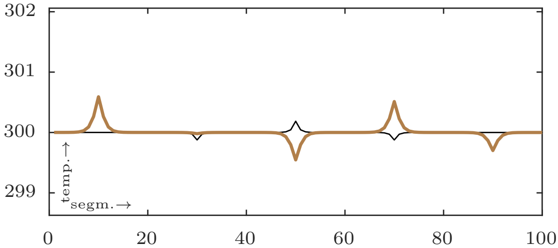
Estimation

Simulation: Centralized

Heat Conduction

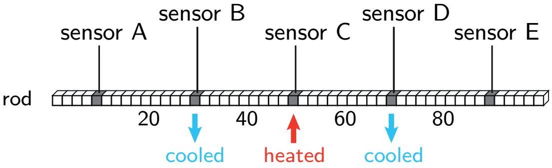


Estimation

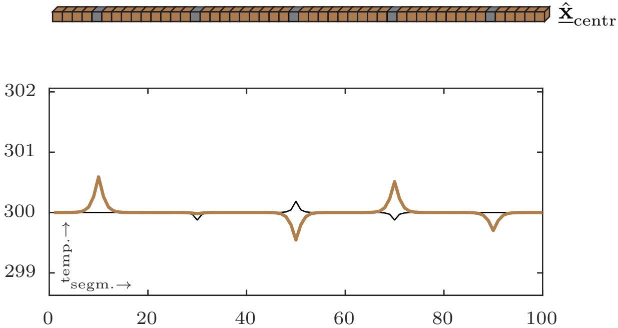


Simulation: Centralized

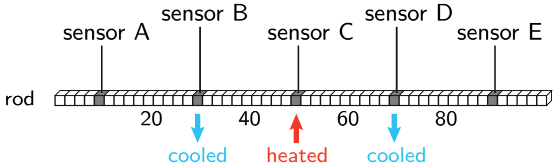
Heat Conduction



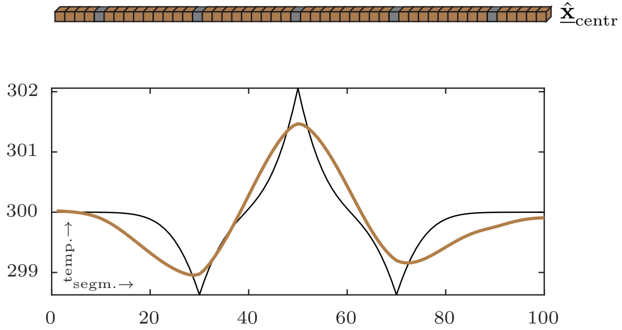
Estimation



Heat Conduction

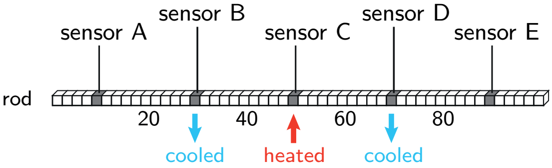


Estimation



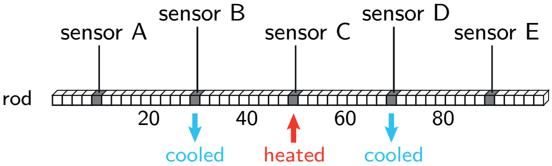
Simulation: Distributed

Heat Conduction

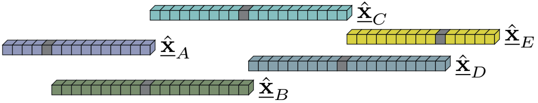


Simulation: Distributed

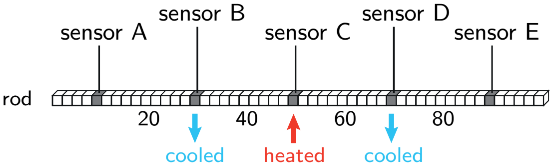
Heat Conduction



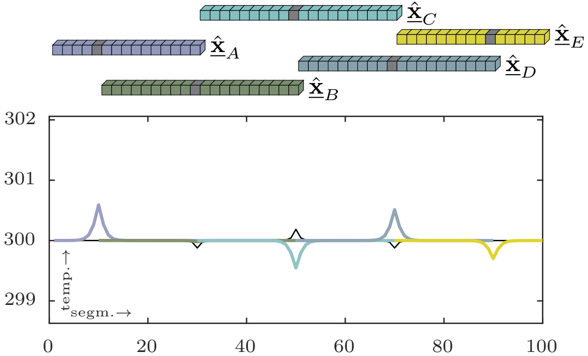
Estimation



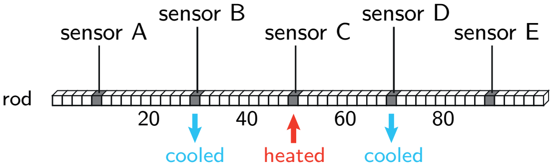
Heat Conduction



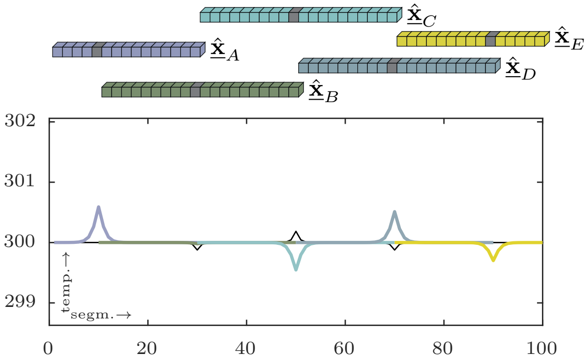
Estimation



Heat Conduction

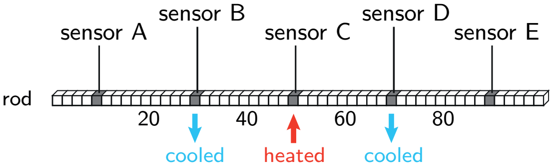


Estimation

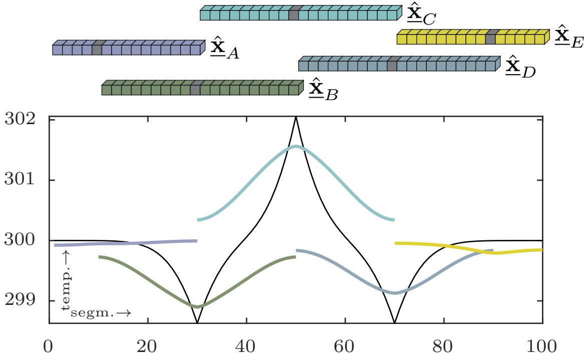


Simulation: Distributed

Heat Conduction

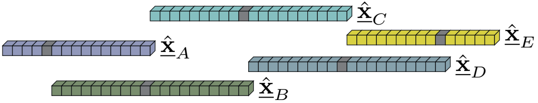


Estimation



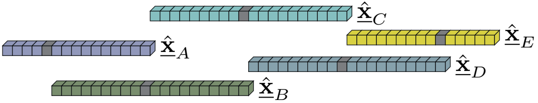
Simulation: Fusion

Local Estimates



Simulation: Fusion

Local Estimates

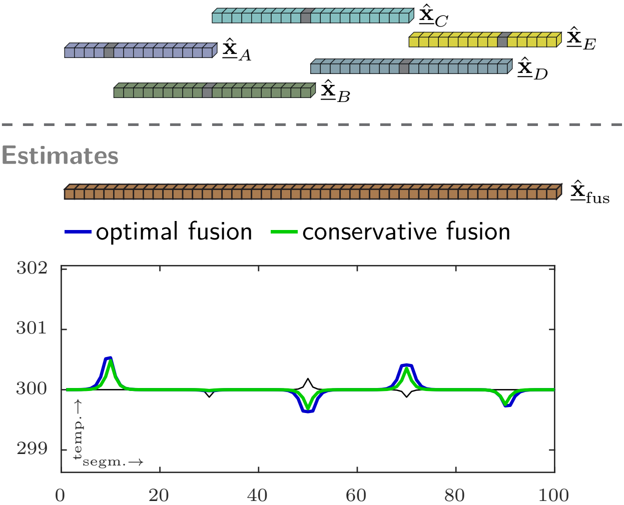


Fusion of Estimates

Simulation: Fusion

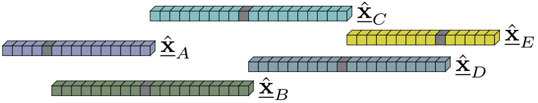
Local Estimates

Fusion of Estimates

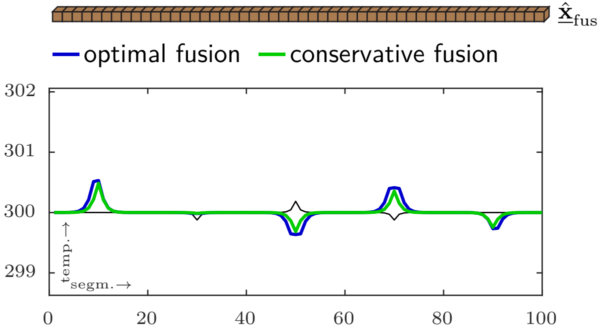


Simulation: Fusion

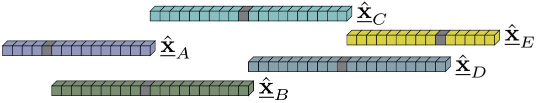
Local Estimates



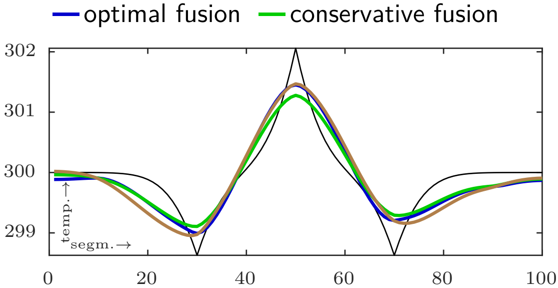
Fusion of Estimates



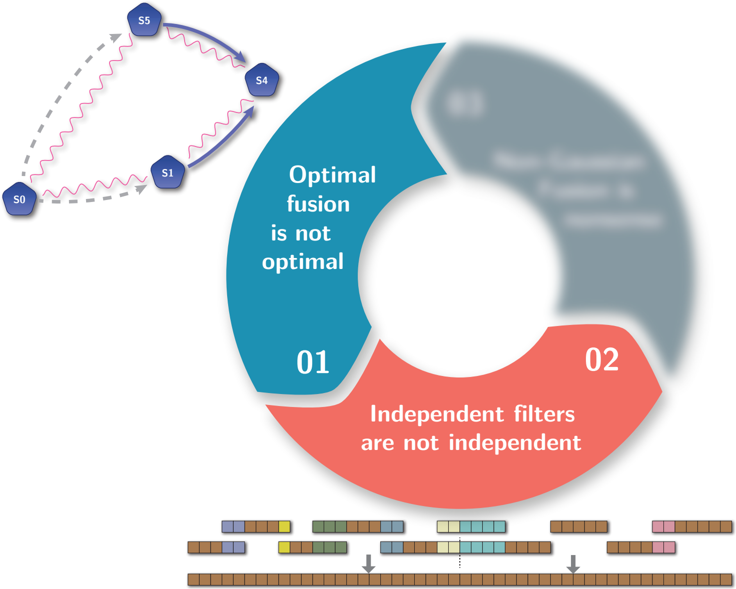
Local Estimates



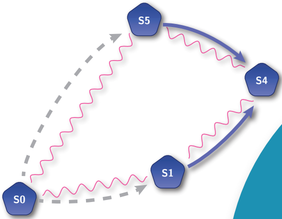
Fusion of Estimates

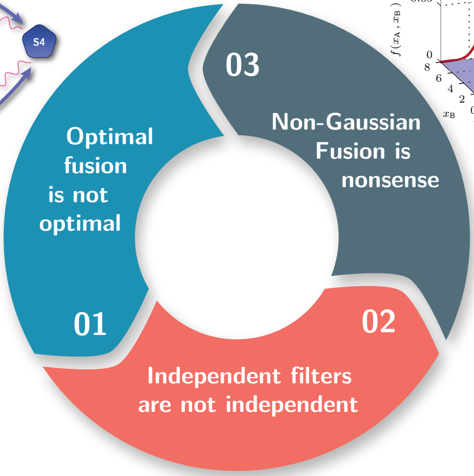


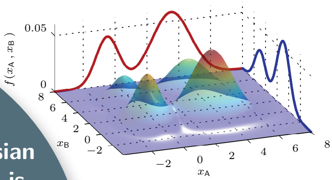
Overview

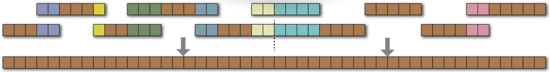

S5
S4
S1
S0
Optimal fusion is not optimal
01
02
Independent filters are not independent

Overview

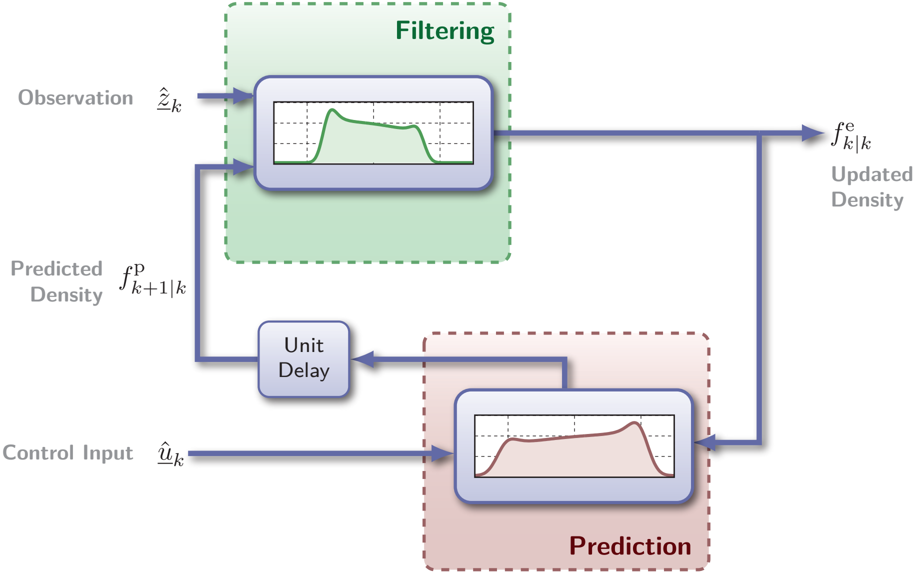

S5
S4
S1
S0


01
Optimal fusion is not optimal
02
Independent filters are not independent
03
Non-Gaussian Fusion is nonsense

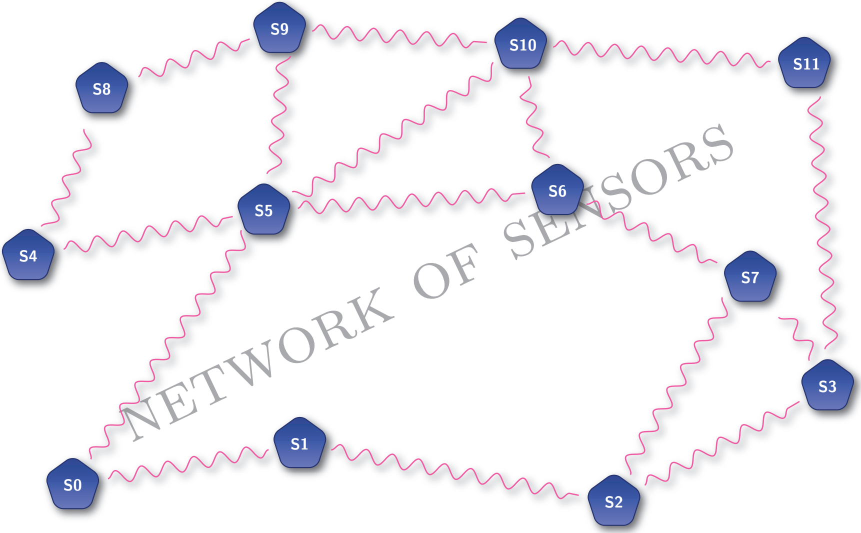

0.05
0
$f(x_A, x_B)$
x_B
x_A



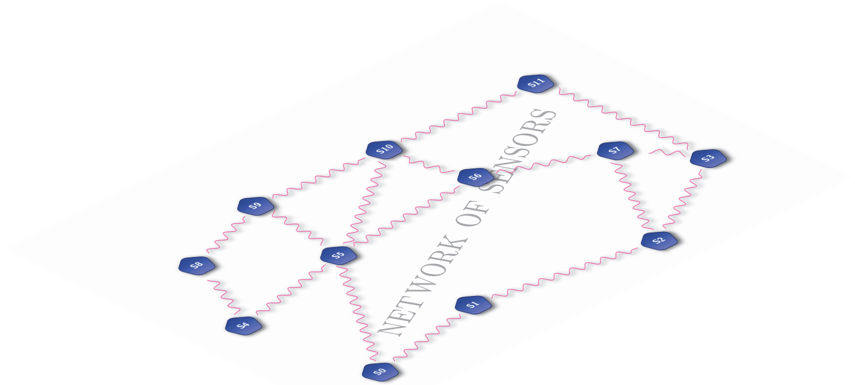
Nonlinear State Estimation



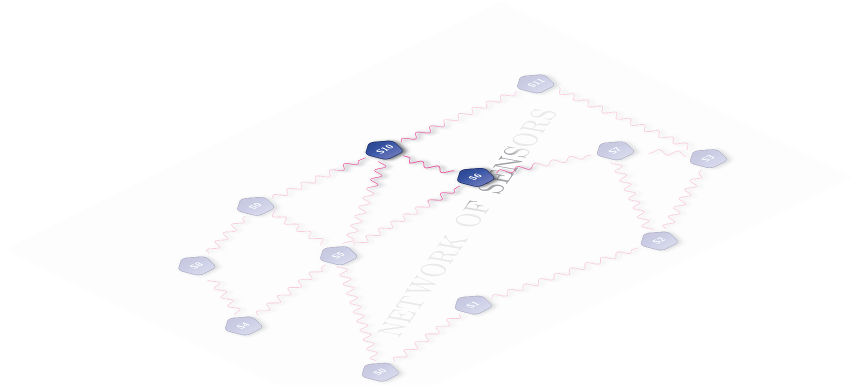
Nonlinear Non-Gaussian Fusion



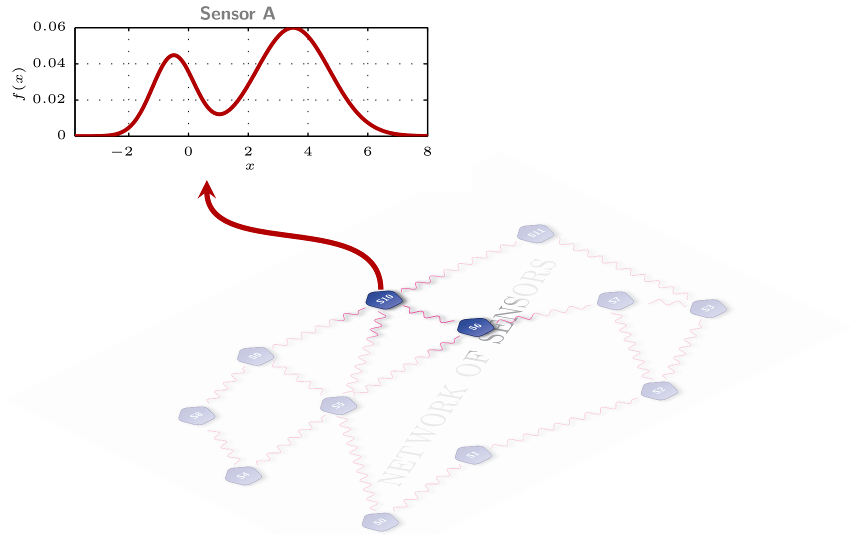
Nonlinear Non-Gaussian Fusion



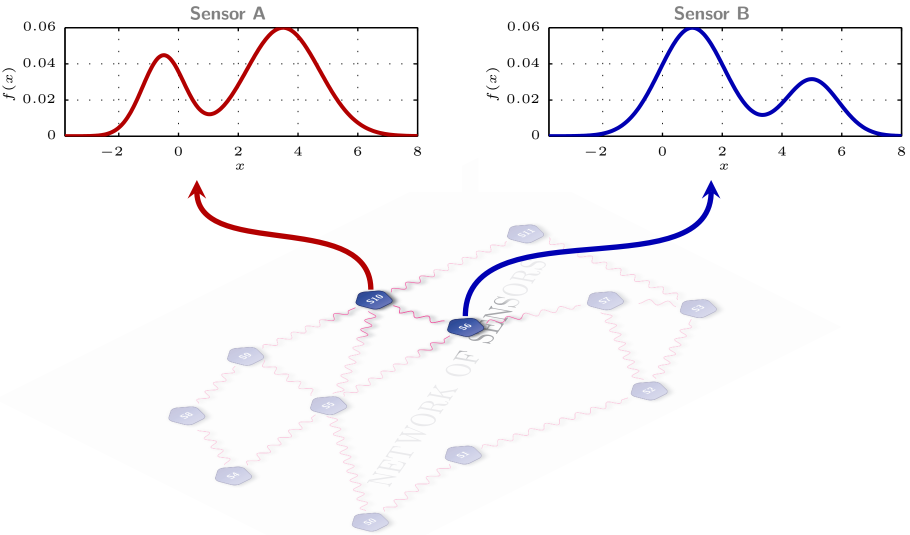
Nonlinear Non-Gaussian Fusion



Nonlinear Non-Gaussian Fusion

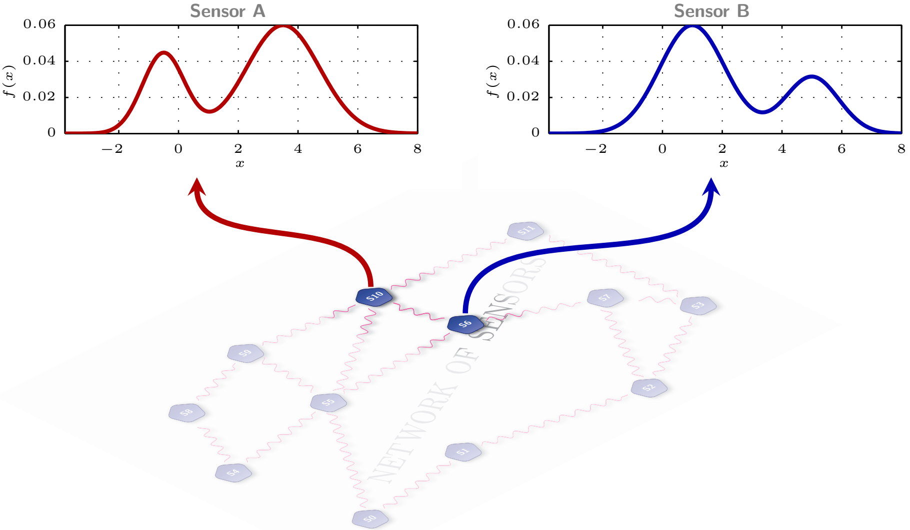


Nonlinear Non-Gaussian Fusion

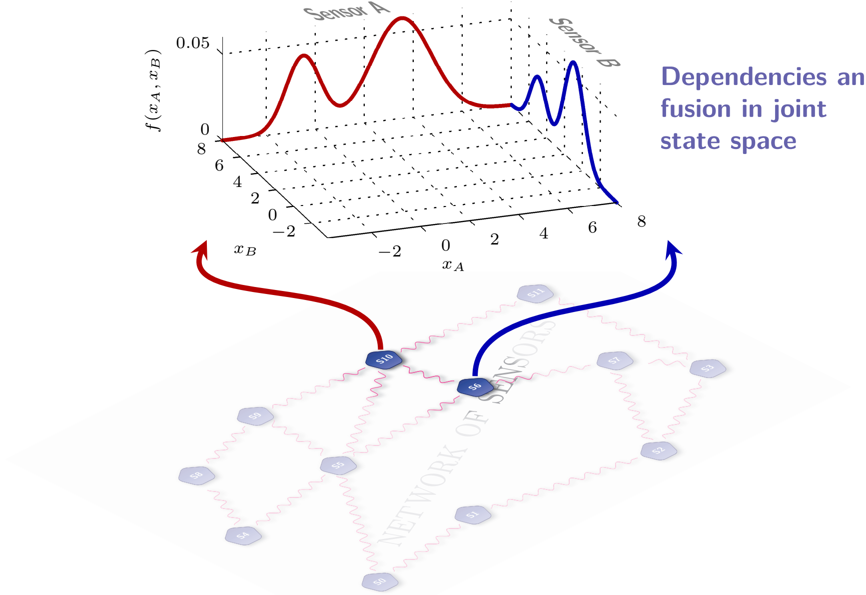


Nonlinear Non-Gaussian Fusion

How to fuse local estimates?

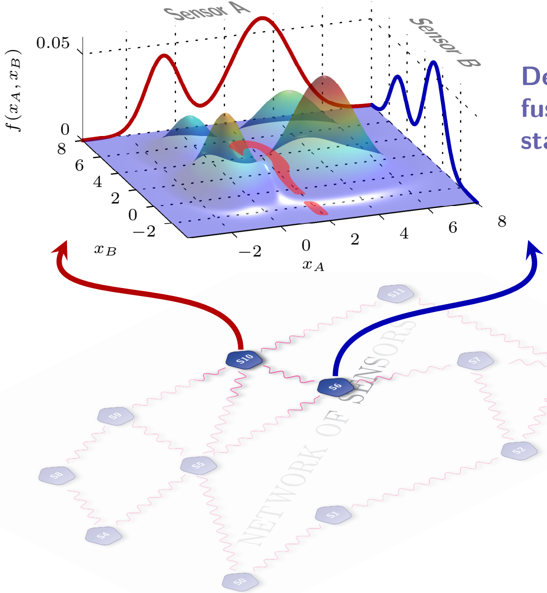


Nonlinear Non-Gaussian Fusion



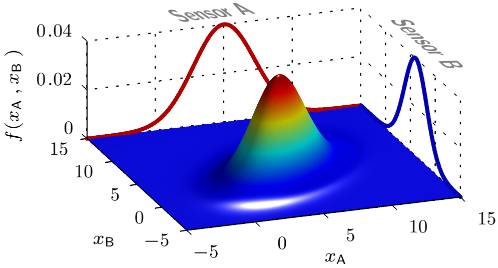
Dependencies and fusion in joint state space

Nonlinear Non-Gaussian Fusion



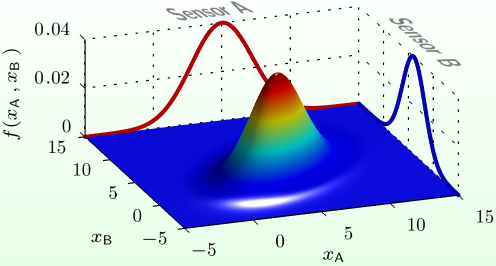
Dependencies and fusion in joint state space

Linear Models, Linear Dependencies...



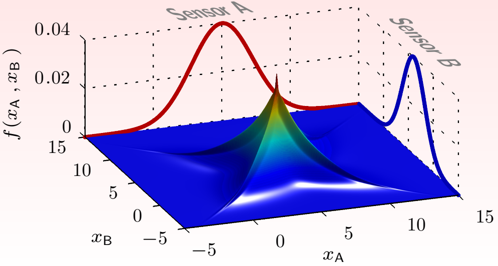
- **Linear models**
- **Gaussian densities**
- **joint Gaussian density**

Linear Models, Linear Dependencies...

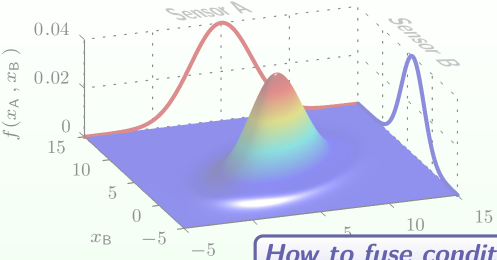


- Linear models
- Gaussian densities
- joint Gaussian density

- Nonlinear models
- Gaussian marginals ...
- ... but arbitrary joint



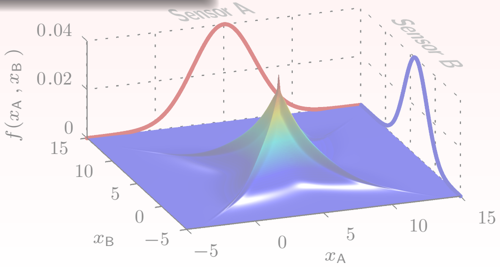
Linear Models, Linear Dependencies...



- Linear models
- Gaussian densities
- joint Gaussian density

How to fuse conditional probability densities?

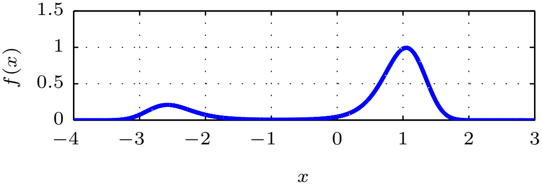
- Nonlinear models
- Gaussian marginals ...
- ... but arbitrary joint



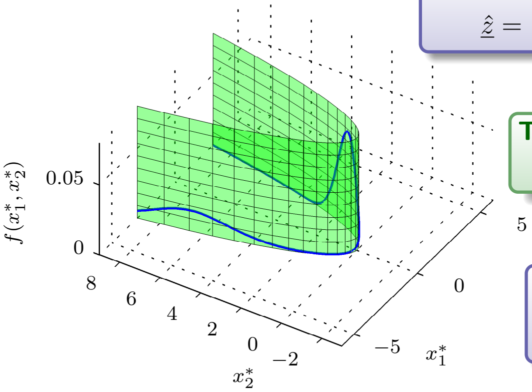
Pseudo Gaussian Densities

Sensor Model

$$\underline{\hat{z}} = (\mathbf{x} - P)^2 + \mathbf{v}$$



Pseudo Gaussian Densities



Sensor Model

$$\underline{\hat{z}} = (\mathbf{x} - P)^2 + \mathbf{v}$$

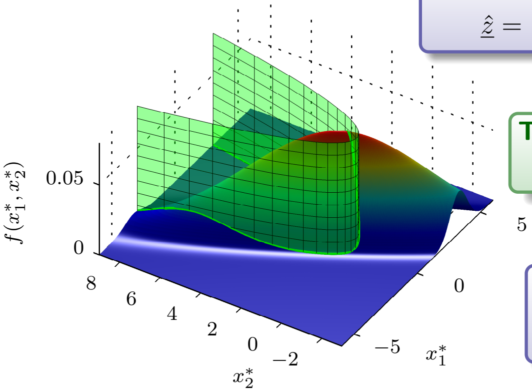
Transformation

$$\underline{\mathbf{x}}^* = [\mathbf{x}, \mathbf{x}^2]^{\mathrm{T}}$$

New Sensor Model

$$\underline{\hat{z}} = [-2\,P, 1] \cdot \underline{\mathbf{x}}^* + P^2 + \mathbf{v}$$

Pseudo Gaussian Densities



Pseudo Gaussian Density

$$f(\underline{x}^*) = \mathcal{N}\left(\underline{x}^*; \hat{\underline{x}}^*, \mathbf{C}^*\right)$$

Sensor Model

$$\hat{\underline{z}} = (\mathbf{x} - P)^2 + \mathbf{v}$$

Transformation

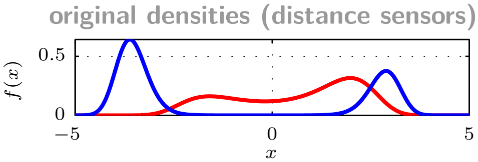
$$\underline{\mathbf{x}}^* = [\mathbf{x}, \mathbf{x}^2]^{\mathrm{T}}$$

New Sensor Model

$$\hat{\underline{z}} = [-2\,P, 1] \cdot \underline{\mathbf{x}}^* + P^2 + \mathbf{v}$$

Fusion in Transformed State Space

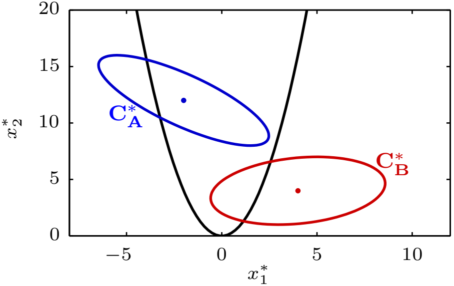
How to fuse estimate A and estimate B ?



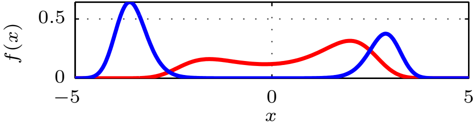
Fusion in Transformed State Space

state space transformation

contours of Pseudo Gaussian Densities



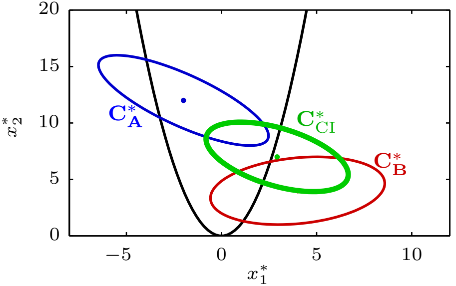
original densities (distance sensors)



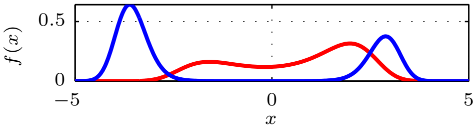
Fusion in Transformed State Space

state space transformation

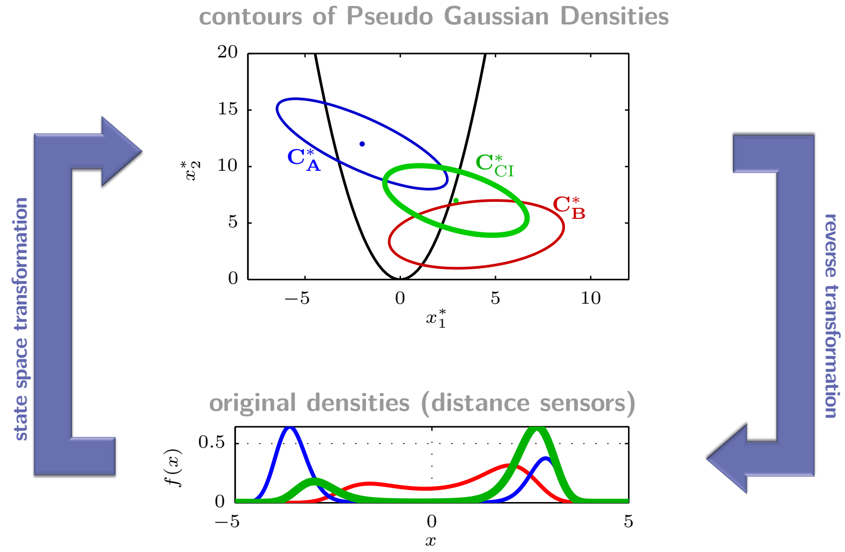
contours of Pseudo Gaussian Densities



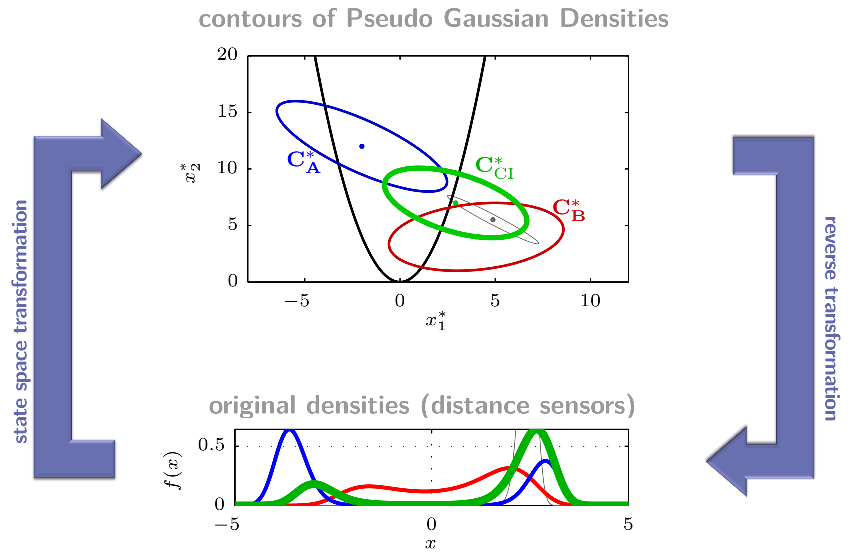
original densities (distance sensors)



Fusion in Transformed State Space



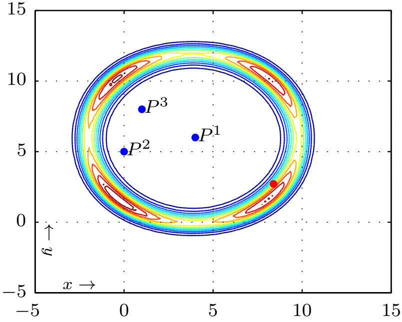
Fusion in Transformed State Space



Sensor Model

$$\underline{\hat{z}}_k^i = (\mathbf{x}_k - P_x^i)^2 + (\mathbf{y}_k - P_y^i)^2 + \mathbf{v}_k$$

Density Contours at Sensor P^1

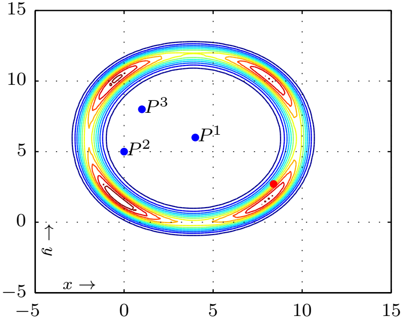


Simulation: Localization in Sensor Network I

Sensor Model

$$\underline{\hat{z}}_k^i = (\mathbf{x}_k - P_x^i)^2 + (\mathbf{y}_k - P_y^i)^2 + \mathbf{v}_k$$

Density Contours at Sensor P^1

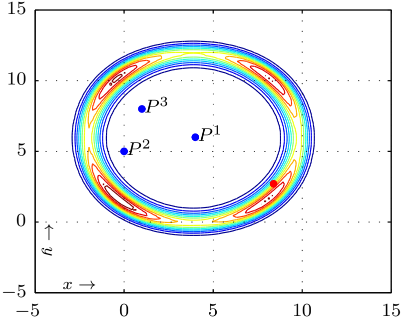


Fusion Method

Ignoring Dependencies

Simulation: Localization in Sensor Network I

Density Contours at Sensor P^1



Sensor Model

$$\underline{\hat{z}}_k^i = (\mathbf{x}_k - P_x^i)^2 + (\mathbf{y}_k - P_y^i)^2 + \mathbf{v}_k$$

Fusion Method

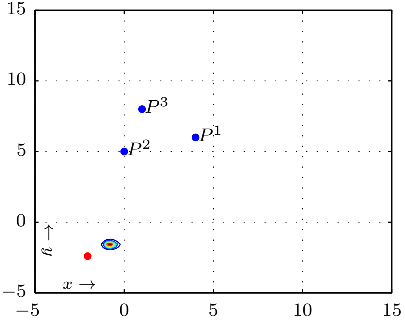
Ignoring Dependencies

Simulation: Localization in Sensor Network I

Sensor Model

$$\underline{\hat{z}}_k^i = (\mathbf{x}_k - P_x^i)^2 + (\mathbf{y}_k - P_y^i)^2 + \mathbf{v}_k$$

Density Contours at Sensor P^1



Fusion Method

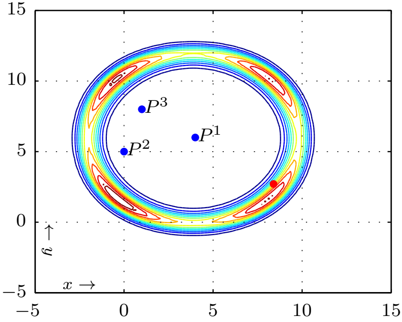
Ignoring Dependencies

Simulation: Localization in Sensor Network II

Sensor Model

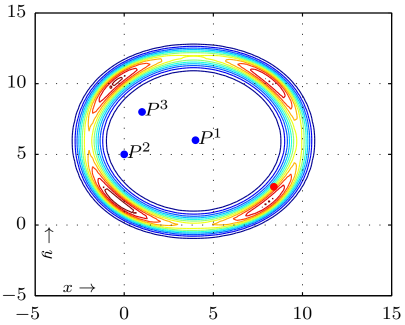
$$\underline{\hat{z}}_k^i = (\mathbf{x}_k - P_x^i)^2 + (\mathbf{y}_k - P_y^i)^2 + \mathbf{v}_k$$

Density Contours at Sensor P^1



Simulation: Localization in Sensor Network II

Density Contours at Sensor P^1



Sensor Model

$$\underline{\hat{z}}_k^i = (\mathbf{x}_k - P_x^i)^2 + (\mathbf{y}_k - P_y^i)^2 + \mathbf{v}_k$$

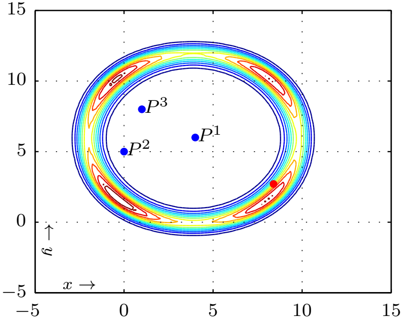
Transformation

$$\underline{\mathbf{x}}_k^* = [\mathbf{x}_k, \mathbf{x}_k^2, \mathbf{y}_k, \mathbf{y}_k^2]^{\mathrm{T}}$$

New Sensor Model

$$\underline{\hat{z}} = [-2P_x^i, 1, -2P_y^i, 1] \cdot \underline{\mathbf{x}}^* + c + \mathbf{v}$$

Density Contours at Sensor P^1



Sensor Model

$$\hat{\underline{z}}_k^i = (\mathbf{x}_k - P_x^i)^2 + (\mathbf{y}_k - P_y^i)^2 + \mathbf{v}_k$$

Transformation

$$\underline{\mathbf{x}}_k^* = [\mathbf{x}_k, \mathbf{x}_k^2, \mathbf{y}_k, \mathbf{y}_k^2]^\mathrm{T}$$

New Sensor Model

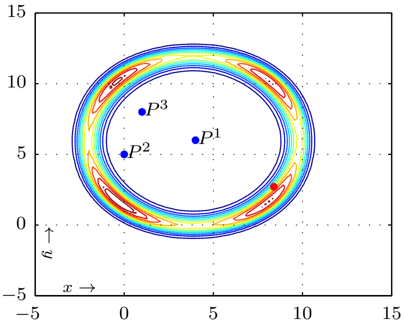
$$\hat{\underline{z}} = [-2P_x^i, 1, -2P_y^i, 1] \cdot \underline{\mathbf{x}}^* + c + \mathbf{v}$$

Fusion Method

Transformed State
Covariance Intersection

Simulation: Localization in Sensor Network II

Density Contours at Sensor P^1



Sensor Model

$$\hat{\underline{z}}_k^i = (\mathbf{x}_k - P_x^i)^2 + (\mathbf{y}_k - P_y^i)^2 + \mathbf{v}_k$$

Transformation

$$\underline{\mathbf{x}}_k^* = [\mathbf{x}_k, \mathbf{x}_k^2, \mathbf{y}_k, \mathbf{y}_k^2]^{\mathrm{T}}$$

New Sensor Model

$$\hat{\underline{z}} = [-2P_x^i, 1, -2P_y^i, 1] \cdot \underline{\mathbf{x}}^* + c + \mathbf{v}$$

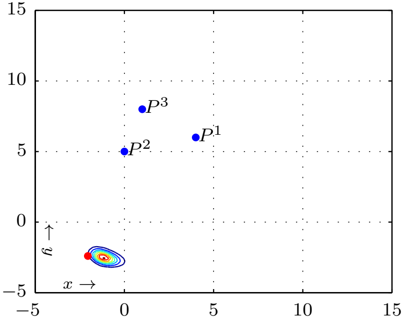
Fusion Method

Transformed State

Covariance Intersection

Simulation: Localization in Sensor Network II

Density Contours at Sensor P^1



Sensor Model

$$\underline{\hat{z}}_k^i = (\mathbf{x}_k - P_x^i)^2 + (\mathbf{y}_k - P_y^i)^2 + \mathbf{v}_k$$

Transformation

$$\underline{\mathbf{x}}_k^* = [\mathbf{x}_k, \mathbf{x}_k^2, \mathbf{y}_k, \mathbf{y}_k^2]^{\mathrm{T}}$$

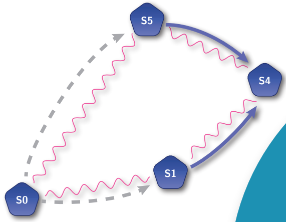
New Sensor Model

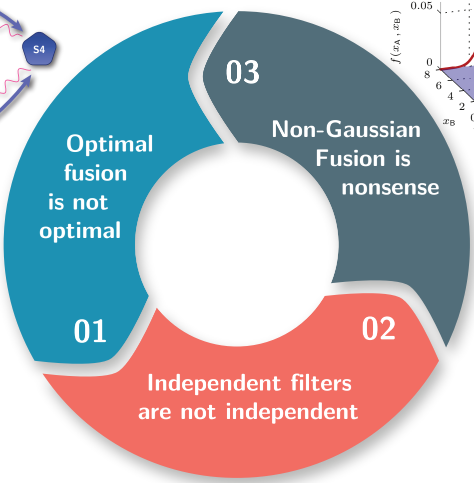
$$\underline{\hat{z}} = [-2P_x^i, 1, -2P_y^i, 1] \cdot \underline{\mathbf{x}}^* + c + \mathbf{v}$$

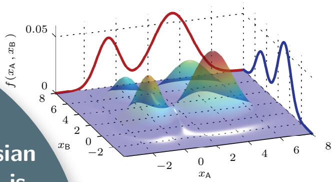
Fusion Method

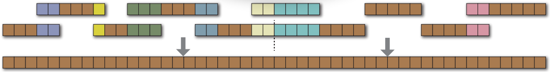
Transformed State
Covariance Intersection

Summary & Conclusions

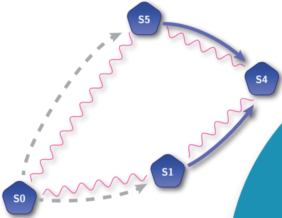


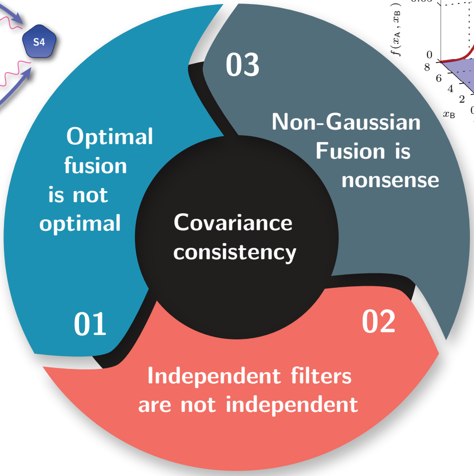


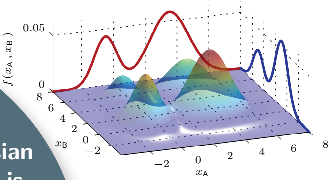


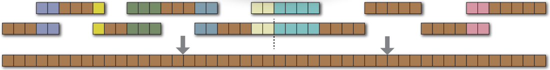


Summary & Conclusions









Thank you for your attention!

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Sensor-Actuator-Systems

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