Fusion Strategies for Unequal State Vectors in Distributed Kalman Filtering

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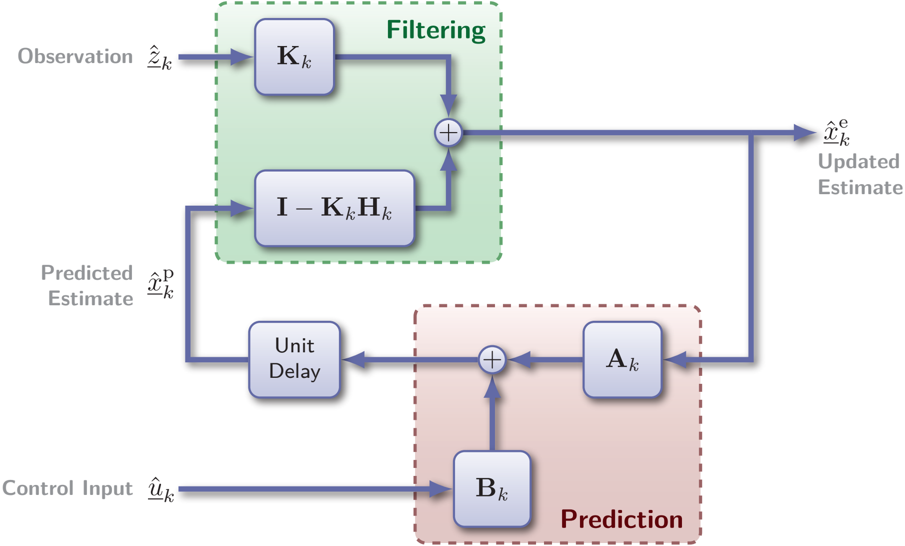
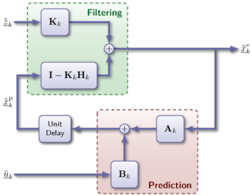
Intelligent Sensor-Actuator-Systems


Filtering
Observation $\underline{\hat{z}}_k$
$\mathbf{K}_k$
$\mathbf{I} - \mathbf{K}_k\mathbf{H}_k$
$\underline{\hat{x}}_k^{\mathrm{e}}$
Updated Estimate
Predicted Estimate $\underline{\hat{x}}_k^{\mathrm{p}}$
Unit Delay
$\mathbf{A}_k$
Control Input $\underline{\hat{u}}_k$
$\mathbf{B}_k$
Prediction

Kalman Filtering for Large-scale Systems



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State Estimate

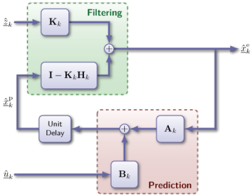
$$\hat{\underline{x}}_k = \begin{bmatrix} \hat{\underline{x}}_A \\ \hat{\underline{x}}_B \\ \hat{\underline{x}}_C \\ \vdots \end{bmatrix}$$

+

Covariance Matrix

$$\mathbf{C}_k = \begin{bmatrix} \mathbf{C}_A & \mathbf{C}_{AB} & \mathbf{C}_{AC} & \\ \mathbf{C}_{BA} & \mathbf{C}_B & \mathbf{C}_{BC} & \cdots \\ \mathbf{C}_{CA} & \mathbf{C}_{CB} & \mathbf{C}_C & \\ & \vdots & & \ddots \end{bmatrix}$$

Kalman Filtering for Large-scale Systems



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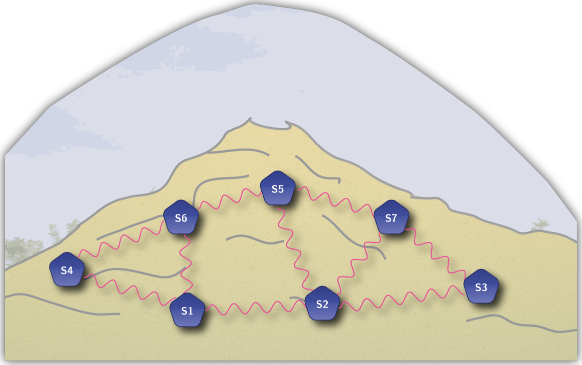
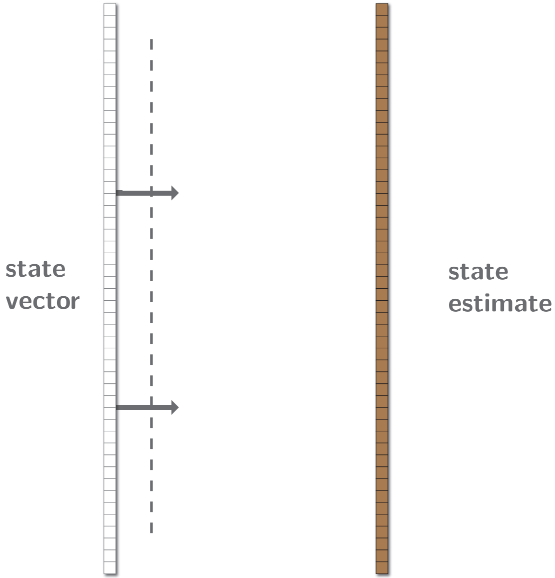


monitoring of large-scale phenomenon

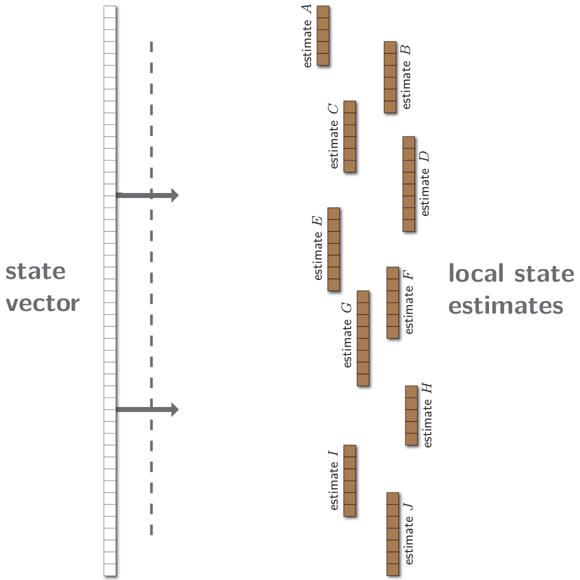
Considered Problem

state
vector

Considered Problem



Considered Problem



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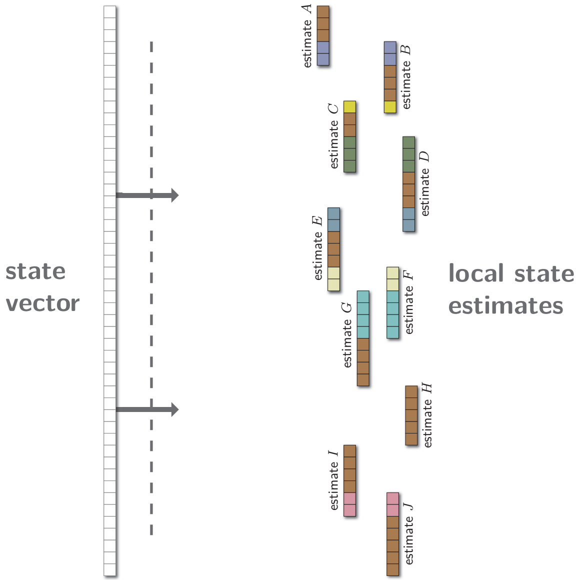
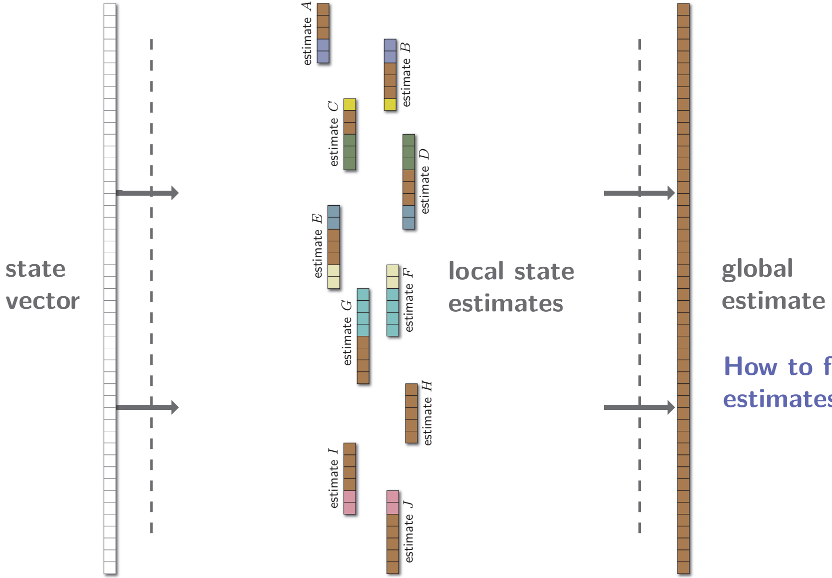
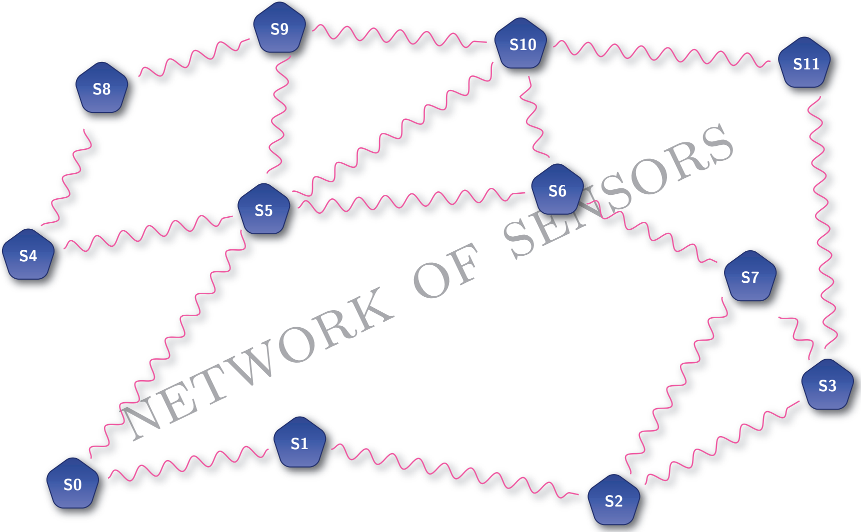


Considered Problem



How to fuse estimates?

Review: Multi-sensor Data Fusion



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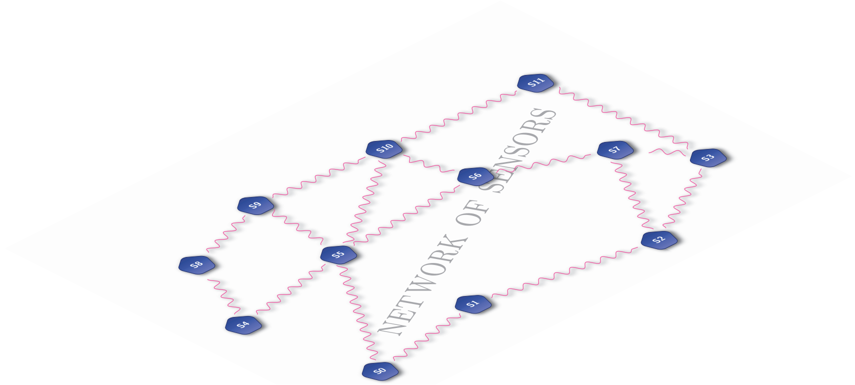
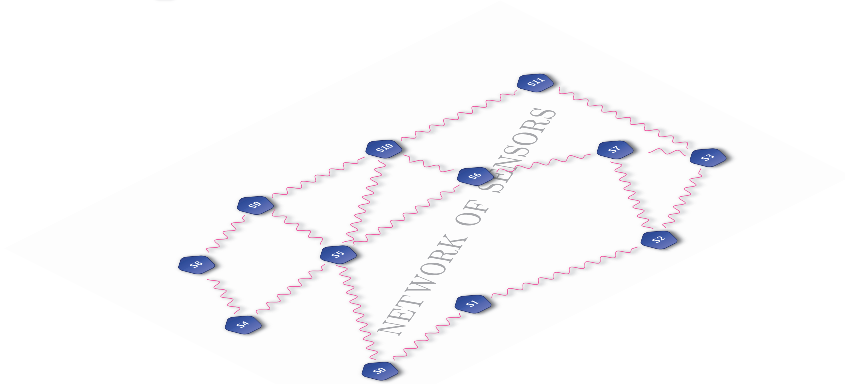


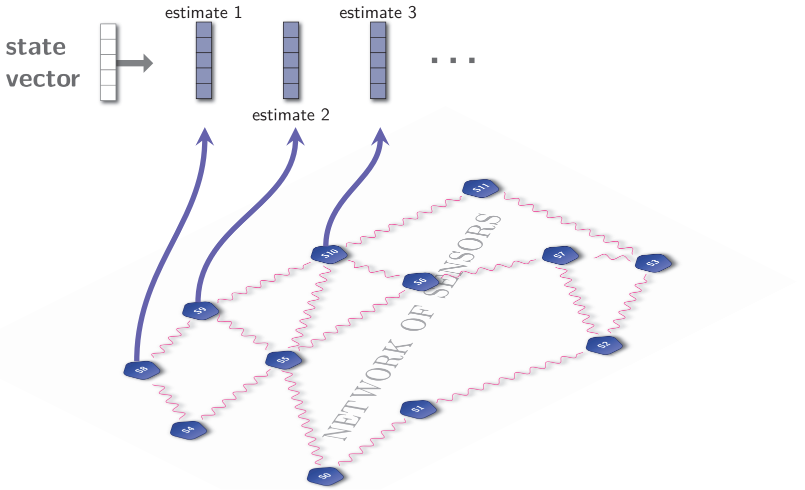
Review: Multi-sensor Data Fusion

state
vector



Review: Multi-sensor Data Fusion

estimates on same state vector (fully overlapping)



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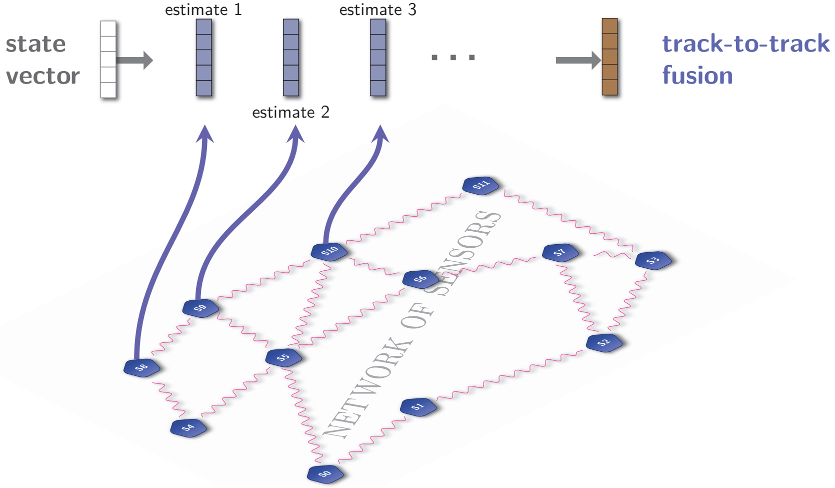


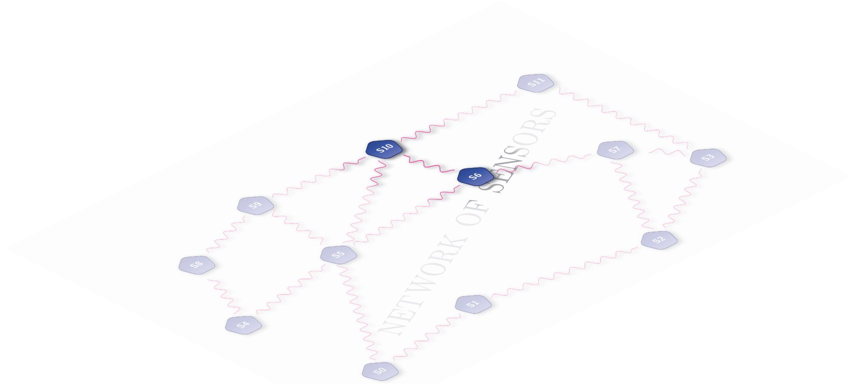
Optimal (Track-to-track) Fusion

A **Estimate A**

$(\hat{\underline{x}}_A, \mathbf{C}_A)$

B **Estimate B**

$(\hat{\underline{x}}_B, \mathbf{C}_B)$



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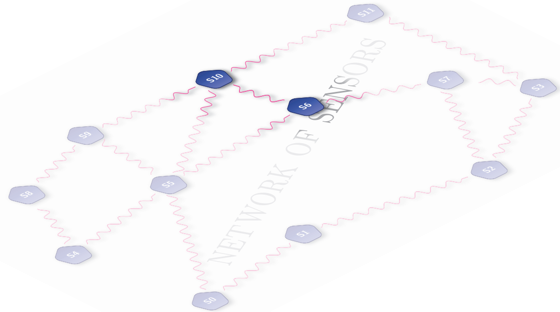
$$\hat{\underline{x}}_{\text{fus}} = (\mathbf{I} - \mathbf{K}_{\text{fus}})\,\hat{\underline{x}}_A + \mathbf{K}_{\text{fus}}\,\hat{\underline{x}}_B$$



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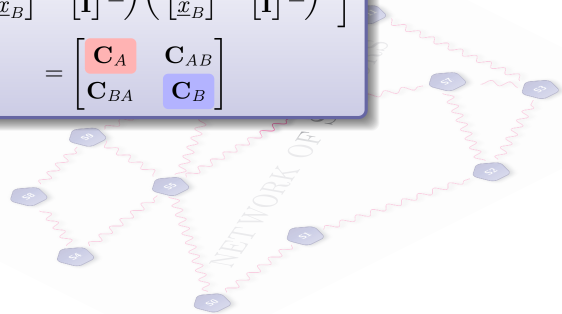
Track-to-track Fusion

$$\underline{\hat{x}}_{\mathrm{fus}} = (\mathbf{I} - \mathbf{K}_{\mathrm{fus}})\, \underline{\hat{x}}_A + \mathbf{K}_{\mathrm{fus}}\, \underline{\hat{x}}_B$$

Joint Covariance Matrix

$$\mathrm{E}\left[\left(\begin{bmatrix}\underline{\hat{x}}_A \\ \underline{\hat{x}}_B\end{bmatrix} - \begin{bmatrix}\mathbf{I} \\ \mathbf{I}\end{bmatrix}\underline{\boldsymbol{x}}\right)\left(\begin{bmatrix}\underline{\hat{x}}_A \\ \underline{\hat{x}}_B\end{bmatrix} - \begin{bmatrix}\mathbf{I} \\ \mathbf{I}\end{bmatrix}\underline{\boldsymbol{x}}\right)^{\mathrm{T}}\right] = \begin{bmatrix}\mathbf{C}_A & \mathbf{C}_{AB} \\ \mathbf{C}_{BA} & \mathbf{C}_B\end{bmatrix}$$



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Bar-Shalom/Campo Formula

Combination with gain

$$\mathbf{K}_{\text{fus}} = (\mathbf{C}_A - \mathbf{C}_{AB}) \cdot (\mathbf{C}_A + \mathbf{C}_B - \mathbf{C}_{AB} - \mathbf{C}_{BA})^{-1}$$

Estimates as Measurements

$$\begin{bmatrix} \underline{\hat{x}}_A \\ \underline{\hat{x}}_B \end{bmatrix}$$

Fusion as Weighted Least-squares Problem

Estimates as Measurements

$$\begin{bmatrix} \underline{\hat{x}}_A \\ \underline{\hat{x}}_B \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{x}} \\ \underline{\boldsymbol{x}} \end{bmatrix} + \begin{bmatrix} \underline{\hat{x}}_A - \underline{\boldsymbol{x}} \\ \underline{\hat{x}}_B - \underline{\boldsymbol{x}} \end{bmatrix}$$

Estimates as Measurements

$$\begin{bmatrix} \hat{\underline{x}}_A \\ \hat{\underline{x}}_B \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{x}} \\ \underline{\boldsymbol{x}} \end{bmatrix} + \begin{bmatrix} \hat{\underline{x}}_A - \underline{\boldsymbol{x}} \\ \hat{\underline{x}}_B - \underline{\boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\boldsymbol{x}} + \underline{\tilde{\boldsymbol{x}}}$$

Fusion as Weighted Least-squares Problem

Estimates as Measurements

$$\begin{bmatrix} \underline{\hat{x}}_A \\ \underline{\hat{x}}_B \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{x}} \\ \underline{\boldsymbol{x}} \end{bmatrix} + \begin{bmatrix} \underline{\hat{x}}_A - \underline{\boldsymbol{x}} \\ \underline{\hat{x}}_B - \underline{\boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\boldsymbol{x}} + \underline{\tilde{\boldsymbol{x}}}$$

Measurement Matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix}$$

Fusion as Weighted Least-squares Problem

Estimates as Measurements

$$\begin{bmatrix} \underline{\hat{x}}_A \\ \underline{\hat{x}}_B \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{x}} \\ \underline{\boldsymbol{x}} \end{bmatrix} + \begin{bmatrix} \underline{\hat{x}}_A - \underline{\boldsymbol{x}} \\ \underline{\hat{x}}_B - \underline{\boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\boldsymbol{x}} + \underline{\tilde{\boldsymbol{x}}}$$

Measurement Matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix}$$

Measurement Error

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_A & \mathbf{C}_{AB} \\ \mathbf{C}_{BA} & \mathbf{C}_B \end{bmatrix}$$

Fusion as Weighted Least-squares Problem

Estimates as Measurements

$$\begin{bmatrix} \underline{\hat{x}}_A \\ \underline{\hat{x}}_B \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{x}} \\ \underline{\boldsymbol{x}} \end{bmatrix} + \begin{bmatrix} \underline{\hat{x}}_A - \underline{\boldsymbol{x}} \\ \underline{\hat{x}}_B - \underline{\boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\boldsymbol{x}} + \underline{\tilde{\boldsymbol{x}}}$$

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Weighted Least-squares Fusion

$$\underline{\hat{x}}_{\text{fus}} = \left(\mathbf{H}^{\mathrm{T}} (\widetilde{\mathbf{C}})^{-1} \mathbf{H}\right)^{-1} \mathbf{H}^{\mathrm{T}} (\widetilde{\mathbf{C}})^{-1} \begin{bmatrix} \underline{\hat{x}}_A \\ \underline{\hat{x}}_B \end{bmatrix}$$

Fusion as Weighted Least-squares Problem

Estimates as Measurements

$$\begin{bmatrix} \underline{\hat{x}}_A \\ \underline{\hat{x}}_B \end{bmatrix} = \begin{bmatrix} \underline{\boldsymbol{x}} \\ \underline{\boldsymbol{x}} \end{bmatrix} + \begin{bmatrix} \underline{\hat{x}}_A - \underline{\boldsymbol{x}} \\ \underline{\hat{x}}_B - \underline{\boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{I} \end{bmatrix} \underline{\boldsymbol{x}} + \underline{\tilde{\boldsymbol{x}}}$$

Measurement Matrix

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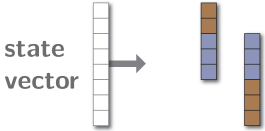
same result as Bar-Shalom/Campo fusion

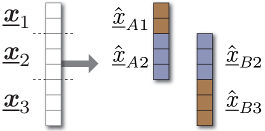
Fusion of Unequal State Vectors

state
vector

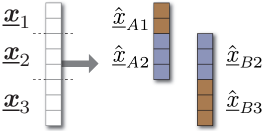
Fusion of Unequal State Vectors



Fusion of Unequal State Vectors



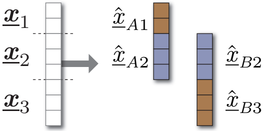
Fusion of Unequal State Vectors



Measurement Equation

$$\begin{array}{l} \hat{\underline{x}}_A \left\{ \vphantom{\begin{matrix} a \\ b \end{matrix}} \right. \\ \hat{\underline{x}}_B \left\{ \vphantom{\begin{matrix} a \\ b \end{matrix}} \right. \end{array} \begin{bmatrix} \hat{\underline{x}}_{A1} \\ \hat{\underline{x}}_{A2} \\ \hat{\underline{x}}_{B2} \\ \hat{\underline{x}}_{B3} \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}}_{=\mathbf{H}} \begin{bmatrix} \underline{\boldsymbol{x}}_1 \\ \underline{\boldsymbol{x}}_2 \\ \underline{\boldsymbol{x}}_3 \end{bmatrix} + \tilde{\underline{\boldsymbol{x}}}$$

Fusion of Unequal State Vectors



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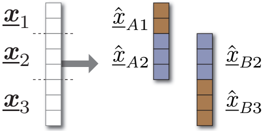
Fusion of Unequal State Vectors

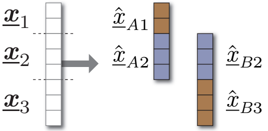


Measurement Equation

$$
\begin{array}{l}
\underline{\hat{x}}_A \left\{ \vphantom{\begin{matrix} a \\ b \end{matrix}} \right. \\
\underline{\hat{x}}_B \left\{ \vphantom{\begin{matrix} a \\ b \end{matrix}} \right.
\end{array}
\begin{bmatrix} \underline{\hat{x}}_{A1} \\ \underline{\hat{x}}_{A2} \\ \underline{\hat{x}}_{B2} \\ \underline{\hat{x}}_{B3} \end{bmatrix}
= \underbrace{\begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix}}_{=\mathbf{H}}
\begin{bmatrix} \underline{\boldsymbol{x}}_1 \\ \underline{\boldsymbol{x}}_2 \\ \underline{\boldsymbol{x}}_3 \end{bmatrix} + \underline{\tilde{\boldsymbol{x}}}
$$

Weighted Least-squares Fusion

$$
\begin{bmatrix} \underline{\hat{x}}_1 \\ \underline{\hat{x}}_2 \\ \underline{\hat{x}}_3 \end{bmatrix}
= \left(\mathbf{H}^{\mathrm{T}} (\widetilde{\mathbf{C}})^{-1} \mathbf{H}\right)^{-1} \mathbf{H}^{\mathrm{T}} (\widetilde{\mathbf{C}})^{-1}
\begin{bmatrix} \underline{\hat{x}}_{A1} \\ \underline{\hat{x}}_{A2} \\ \underline{\hat{x}}_{B2} \\ \underline{\hat{x}}_{B3} \end{bmatrix}
$$



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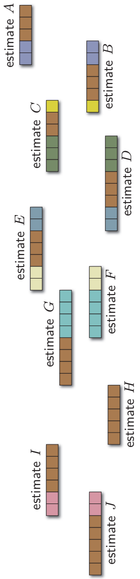
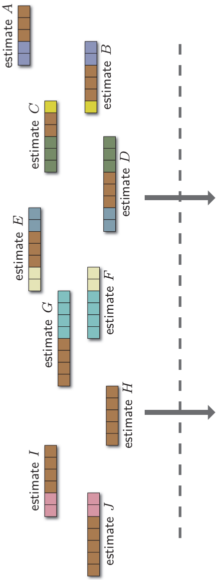
Error Covariance Matrix

$$\mathbf{C}_{\mathrm{fus}} = \left(\mathbf{H}^{\mathrm{T}}(\widetilde{\mathbf{C}})^{-1}\mathbf{H}\right)^{-1}$$

Optimal Fusion

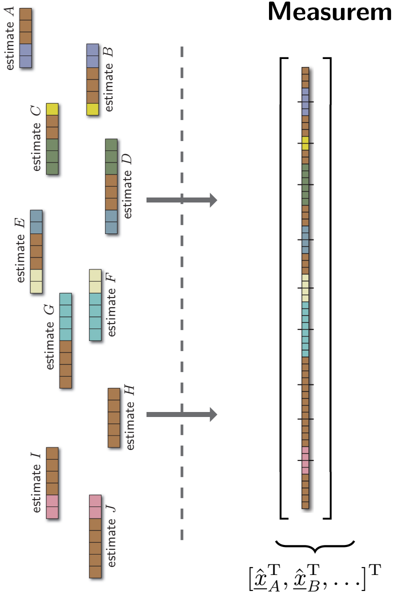


Optimal Fusion

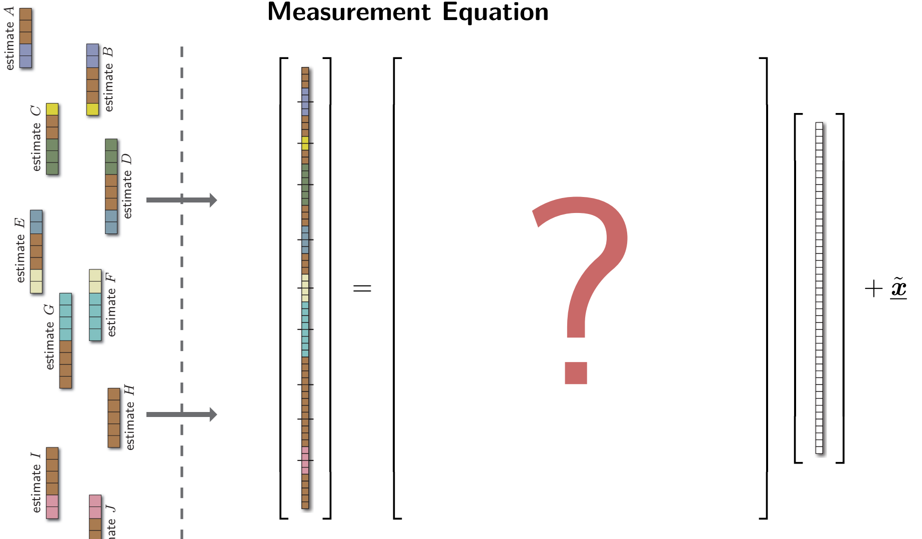


How to set up measurement mapping?

Measurement Equation

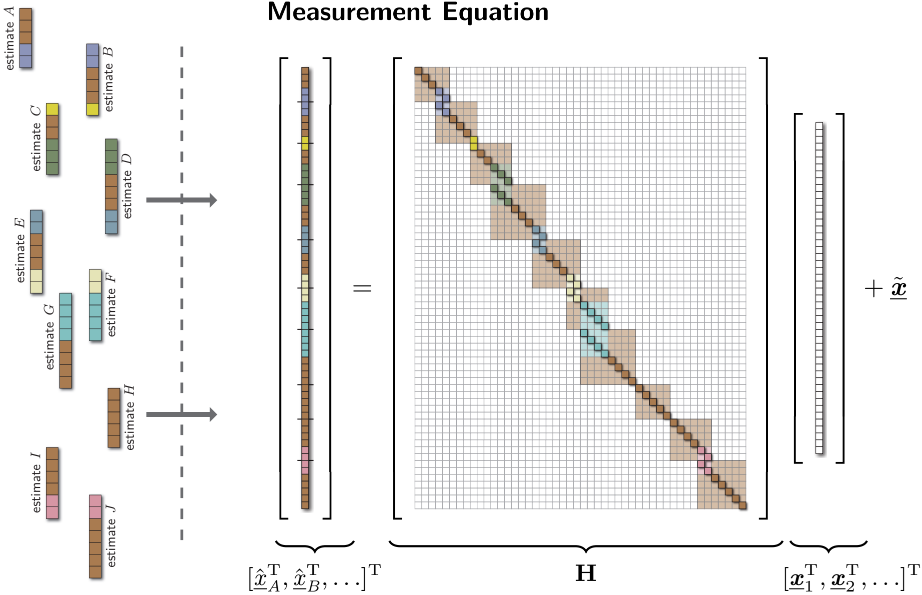


Measurement Equation



$$\underbrace{}_{[\hat{\underline{x}}_A^{\mathrm{T}}, \hat{\underline{x}}_B^{\mathrm{T}}, \ldots]^{\mathrm{T}}} = \underbrace{}_{\mathbf{H}} \underbrace{}_{[\underline{\boldsymbol{x}}_1^{\mathrm{T}}, \underline{\boldsymbol{x}}_2^{\mathrm{T}}, \ldots]^{\mathrm{T}}} + \tilde{\underline{\boldsymbol{x}}}$$

Measurement Equation



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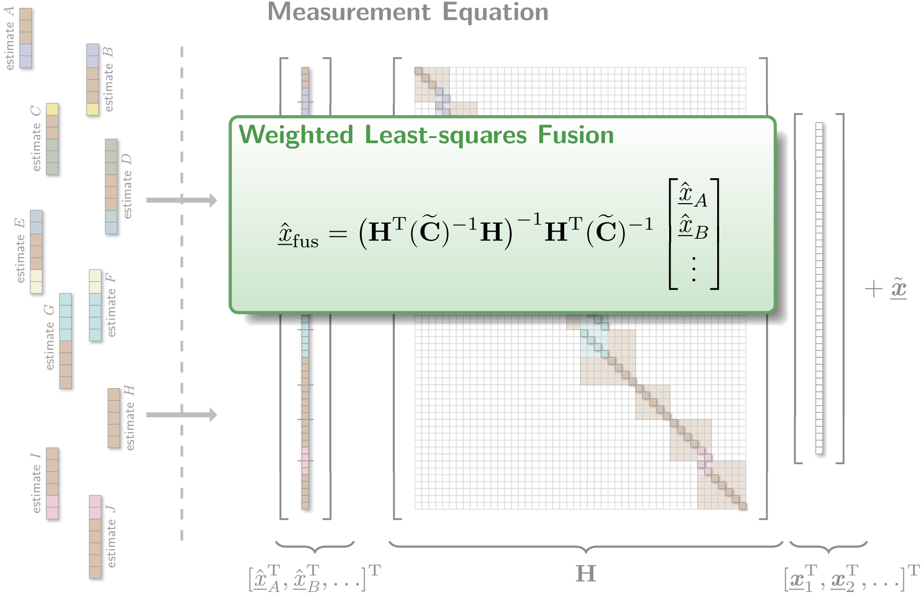
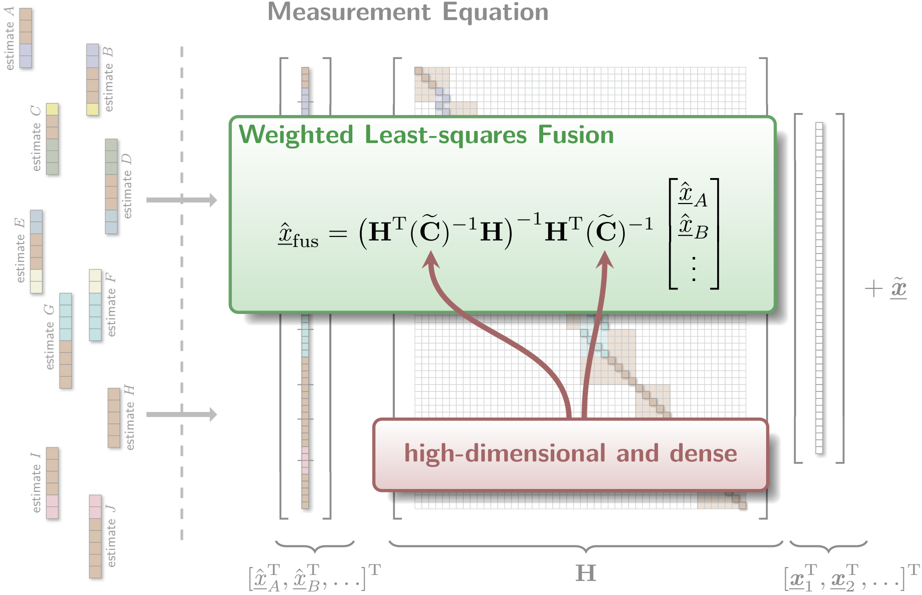


Weighted Least-squares Fusion

$$\underline{\hat{x}}_{\mathrm{fus}} = \left(\mathbf{H}^{\mathrm{T}}(\widetilde{\mathbf{C}})^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}(\widetilde{\mathbf{C}})^{-1}\begin{bmatrix}\underline{\hat{x}}_A \\ \underline{\hat{x}}_B \\ \vdots\end{bmatrix}$$

Measurement Equation



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high-dimensional and dense

Joint Cross-covariance Matrix

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_A & \mathbf{C}_{AB} & \mathbf{C}_{AC} & \\ \mathbf{C}_{BA} & \mathbf{C}_B & \mathbf{C}_{BC} & \cdots \\ \mathbf{C}_{CA} & \mathbf{C}_{CB} & \mathbf{C}_C & \\ & \vdots & & \ddots \end{bmatrix}$$

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Suboptimal Fusion

Joint Cross-covariance Matrix

$$\widetilde{\mathbf{C}} = \begin{bmatrix} \mathbf{C}_A & \mathbf{C}_{AB} & \mathbf{C}_{AC} & \\ \mathbf{C}_{BA} & \mathbf{C}_B & \mathbf{C}_{BC} & \dots \\ \mathbf{C}_{CA} & \mathbf{C}_{CB} & \mathbf{C}_C & \\ & \vdots & & \ddots \end{bmatrix}$$

Difficulties

- bookkeeping
- storing
- processing

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- bookkeeping
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Solution: Covariance Inflation

- $\sum \omega_X = 1,\ \omega_X > 0$
- upper bound
- suboptimal

Suboptimal Fusion

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Difficulties

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Solution: Covariance Inflation

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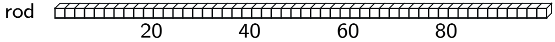
Conservative Fusion

$$\underline{\hat{x}}_{\mathrm{fus}} = \left(\mathbf{H}^{\mathrm{T}}(\mathbf{C}_{\mathrm{CB}})^{-1}\mathbf{H}\right)^{-1}\mathbf{H}^{\mathrm{T}}(\mathbf{C}_{\mathrm{CB}})^{-1}\begin{bmatrix} \underline{\hat{x}}_A \\ \underline{\hat{x}}_B \\ \vdots \end{bmatrix}$$

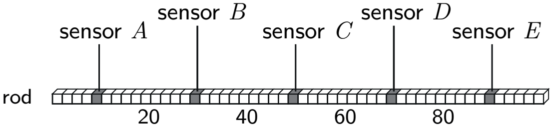
Simulation Setup

Heat Conduction



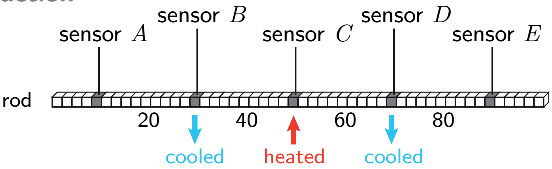
Simulation Setup

Heat Conduction

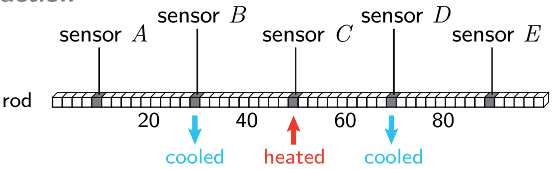


Simulation Setup

Heat Conduction



Heat Conduction



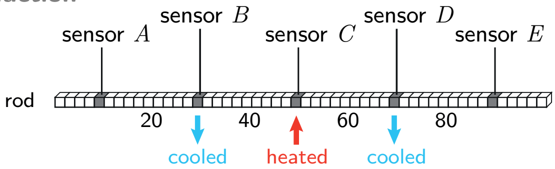
Estimation

Process Model

$$\boldsymbol{x}_{k+1,n} = 0.17\boldsymbol{x}_{k,n-1} + 0.66\boldsymbol{x}_{k,n} + 0.17\boldsymbol{x}_{k,n+1} + \boldsymbol{w}_{k,n}$$

Simulation Setup

Heat Conduction



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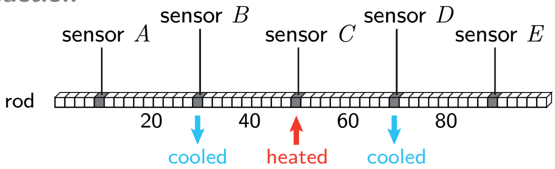
Sensor Models

$$\boldsymbol{z}_k^A = \boldsymbol{x}_{k,10} + \boldsymbol{v}_k^A \ , \quad \boldsymbol{z}_k^B = \boldsymbol{x}_{k,30} + \boldsymbol{v}_k^B \ , \quad \boldsymbol{z}_k^C = \boldsymbol{x}_{k,50} + \boldsymbol{v}_k^C \ ,$$

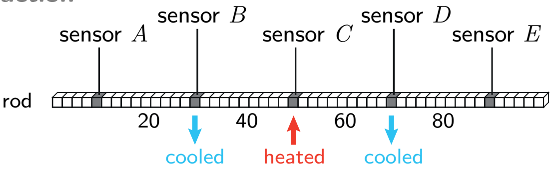
$$\boldsymbol{z}_k^D = \boldsymbol{x}_{k,70} + \boldsymbol{v}_k^D \ , \quad \boldsymbol{z}_k^E = \boldsymbol{x}_{k,90} + \boldsymbol{v}_k^E \ ,$$

Heat Conduction



Simulation: Centralized

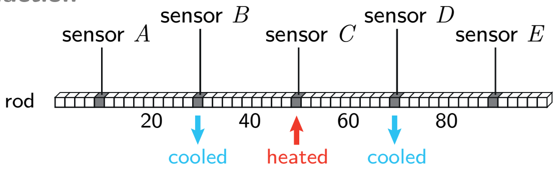
Heat Conduction



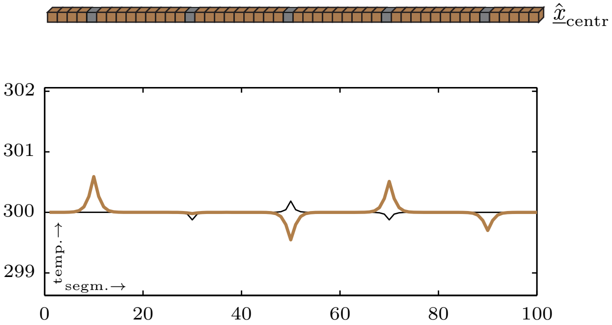
Estimation

Simulation: Centralized

Heat Conduction



Estimation



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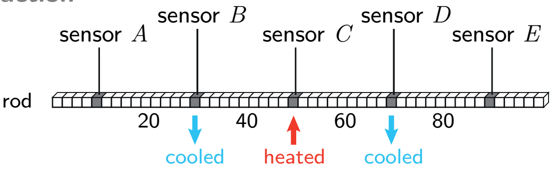

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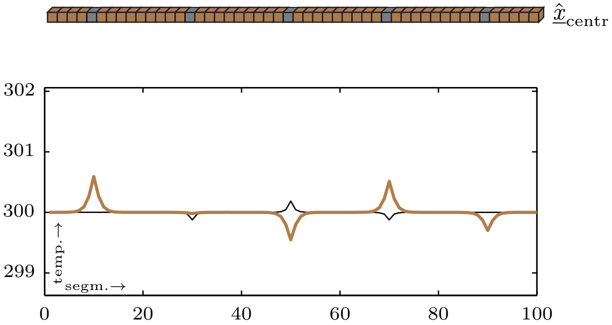

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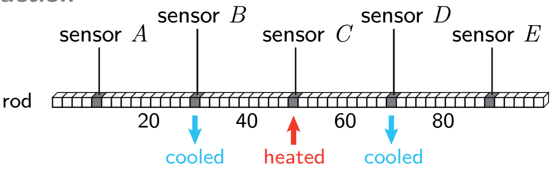

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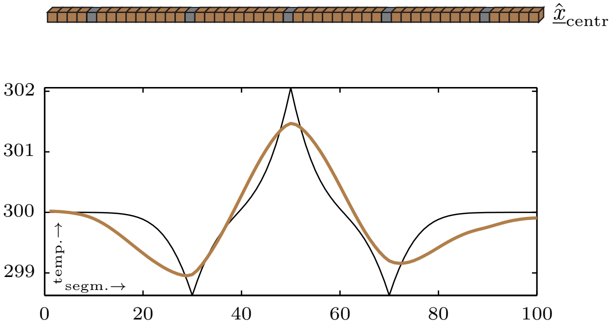

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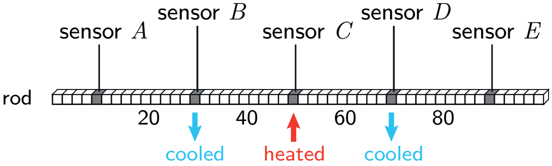
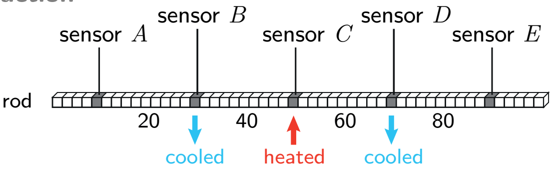
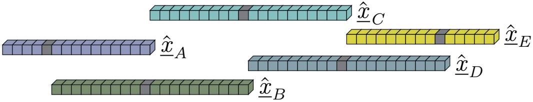


Simulation: Distributed

Heat Conduction

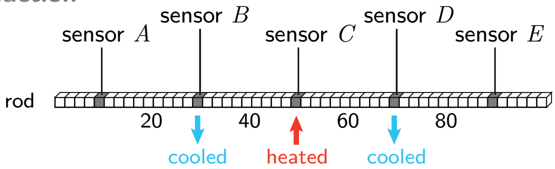


Estimation



Simulation: Distributed

Heat Conduction



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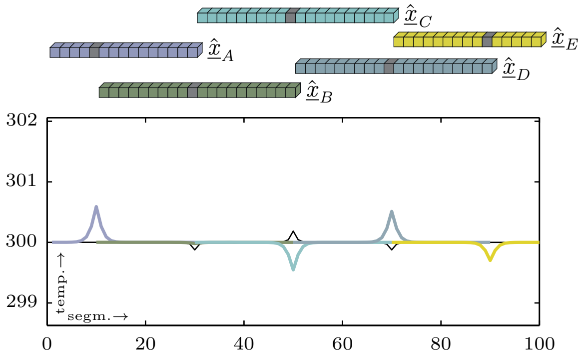

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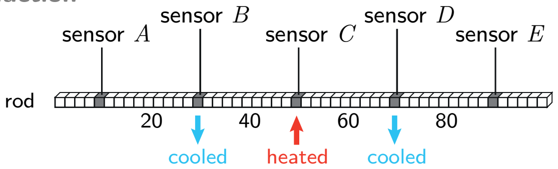

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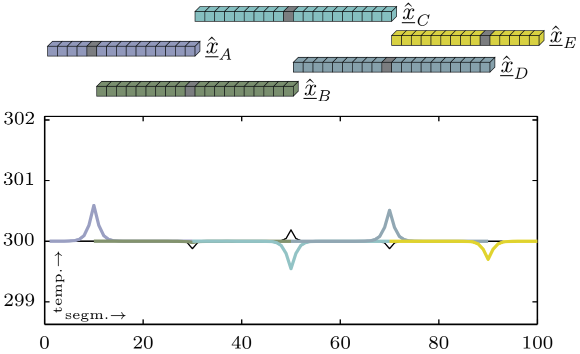

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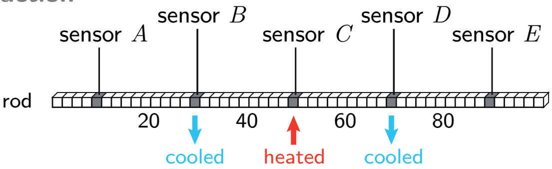

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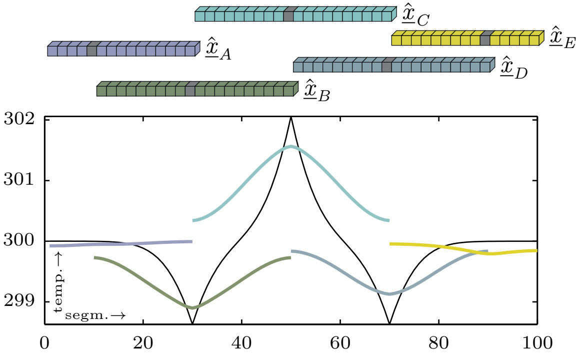
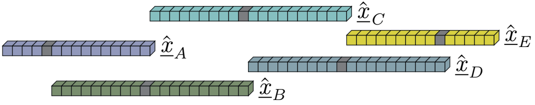


Simulation: Fusion

Local Estimates



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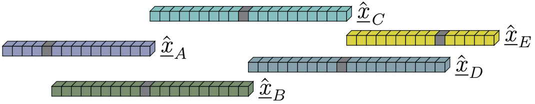
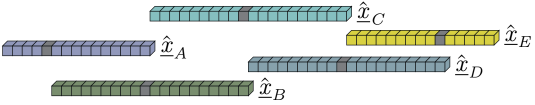


Fusion of Estimates

Simulation: Fusion

Local Estimates



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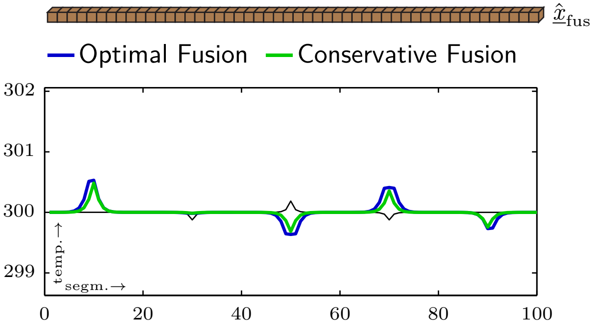

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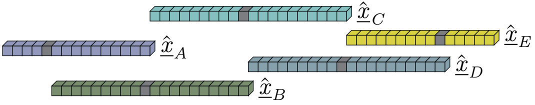

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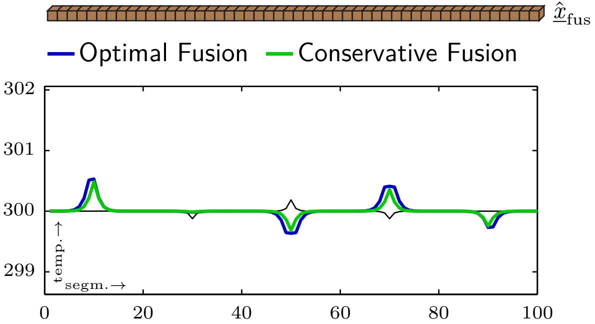

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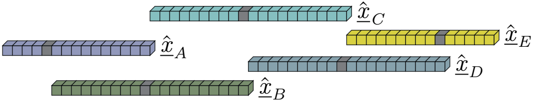

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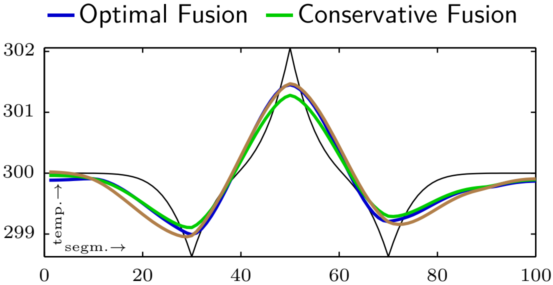
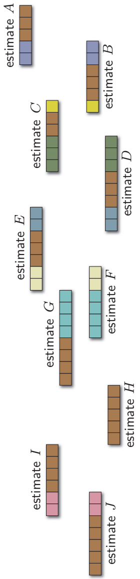


Conclusions

- **Fusion of Unequal State Vectors**
  - Local Kalman filter estimates for unequal components
  - Fusion reformulated to least-squares problem
  - Assessment of estimation error
- **Advantages**
  - Optimal incorporation of cross-correlations
  - Conservative fusion with covariance inflation
- **Outlook**
  - Applications in data assimilation
  - Applications in parallel Kalman filtering



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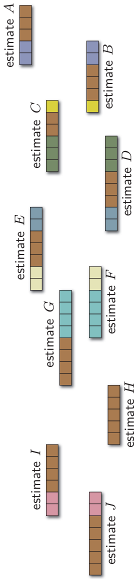

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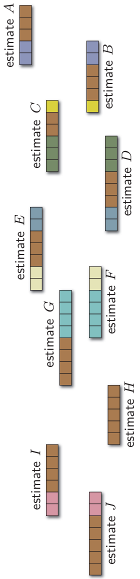

Thank you for your attention!

Intelligent

Sensor-Actuator-Systems