Lab Report: Josephson Junctions

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Abstract

This experiment demonstrates the DC and RF-biased AC Josephson effects across a superconducting Josephson junction. The junction is a point-contact ensemble composed of niobium with an oxide layer between the metals acting as the thin insulating layer. A probe has been assembled for immersing the junction in liquid helium to trigger superconductivity, while maintaining tight control of the junction separation distance and electrical connection for data acquisition. This probe also mediates the flow of radio frequency electrical radiation into the junction for observation of the AC Josephson effect. This effect has a periodic nature that allows for the measurement of the fundamental ratio $2e/h$, which we found experimentally to be $478 \pm 2 \;\; MHz/\mu V$. This result is within 2% of the closest recorded value to date, demonstrating the Josephson junction's macroscopic realization of fundamental quantities at the quantum scale.

1 Introduction

The Josephson junction, theoretically predicted [1] by Brian Josephson in 1962, has become a critical component of superconducting quantum circuits for high-precision magnetic interferometry and coherent quantum computation [2]. Understanding the mechanism behind these devices requires a quantum-mechanical framework that accounts for the electron-lattice interactions within the superconductor. Superconducting materials, though initially creating interest for their lack of electrical resistivity and rejection of magnetic fields, are now also appreciated for their macroscopic demonstration of quantum states of matter, which we will explore both theoretically (in section 2) using the language of Quantum Mechanics and experimentally (in section 4) with a primitive point-contact superconducting junction. A basic explanation of the mechanism behind superconductivity is that one electron in the metal can attract another through interactions with nuclei in the lattice structure. Although this attraction is small, each metal has a critical temperature at which electrons will preferentially form bound electron pairs, known as Cooper pairs, due to this interaction. This process accounts for superconductivity's unique electromagnetic properties, and Josephson junctions in particular take advantage of these low energy paired states to exhibit quantum behavior–in particular quantum tunneling–at a scale capable of being viewed on an oscilloscope.

The effects studied in this paper concern the voltage-current characteristics of the Josephson junction due to the tunneling action of the Cooper pairs between two superconductors through a thin insulator. The junction we have fashioned is composed of a niobium point making contact with a niobium base, with the oxide layer of the metal acting as the insulating barrier where the tunneling process occurs. This ensemble is fashioned into a metallic probe, then dunked into a liquid helium dewar to bring the Niobium into a superconducting state. When no voltage is driven along the junction, a non-zero current can develop across the insulator as a result of this tunneling, with the magnitude of the current determined by the phase difference between the quantum state of each superconductor; This instantaneous current jump is known as the DC Josephson effect. When the junction is irradiated with a certain high-frequency AC voltage, we observe a stepwise I-V relationship that holds in its periodic behavior a precise measurement of the ratio of fundamental constants e/h. Theoretical predictions of these Josephson effects, detailed below, show that the coupling of the superconducting states on each side of the insulator account for these phenomena, with the AC effect behaving similar to a resonance effect in the supercurrent. With our analysis of these effects we are able to extract information about fundamental quantum constants on a macroscopic scale, bridging the size gap between the Classical and Quantum realm.

2 Theoretical Background

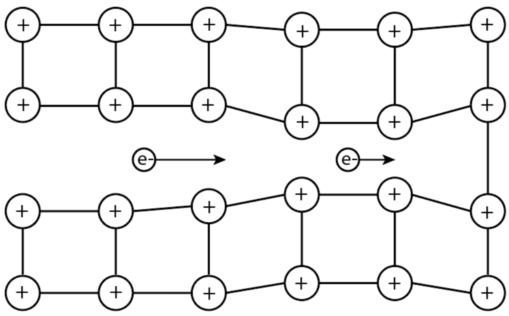

Figure 1: The electron-phonon interaction within a lattice. An electron's passage through the conductor can deform the crystal lattice structure of the nuclei and create an attractive force for other electrons. This binding mechanism, though easily understood classically, is a quantum process that is best described by a coupling Hamiltonian describing the many-body electron interactions.

A comprehensive theory of superconductivity [3], first described in 1957 by Bardeen, Cooper, and Schrieffer, has established that the action of superconducting materials is mediated by quantum-mechanical principles. Because electrons have half-integer spin, the Pauli Exclusion Principle dictates that electrons in a metal at room temperature cannot occupy the same ground state energy level, and so the electrons occupy a band of energy levels that characterize the conductive properties of the metal. Any system naturally moves towards its state of lowest energy, and in a conductor this band of energy levels characterizes the lowest states that the electrons can occupy. But once the material is brought below some critical temperature, a lower energy state becomes possible for the electrons as bound pairs. This electron binding is guided by the interactions of an electron with positively-charged nuclei within the lattice as shown above. This classical description is sufficient for developing intuition, but in reality this binding mechanism is a quantum phenomenon with a vast sea of electrons in the superconductor constantly switching in and out of the energetically preferred bound state. At high temperatures thermal energy in the metal creates sufficient agitation to prevent Cooper Pairs from forming. Although a lone electron behaves as a fermion, the pairing of two half-integer spin particles produces a quasi-bosonic pair with significantly different properties within the metal. Unlike fermions, a large set of bosons at low temperature prefer to occupy the same coherent low energy state, and so at low temperatures the Cooper Pairs in a superconductor will condense fairly uniformly into their ground state and thus they can be described by a single wave function once we approach the ideal limit T = 0 K.

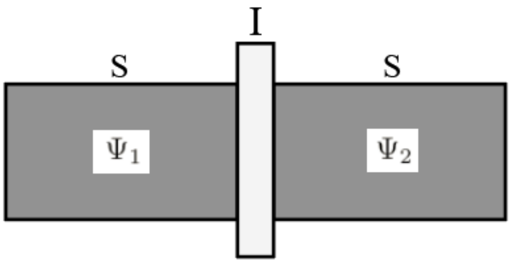

Figure 2: Theoretical model of a Josephson junction. The superconducting state of Cooper pairs on each side of the junction rest in coherent ground states of lowest energy.

Now we consider the Josephson junction shown above, composed of two superconductors separated by a thin layer of insulating material. When the insulator is sufficiently small, coupling between the superconducting states leads to an evolution of the systems as described by the coupled equations [4]

$$i\hbar \frac{d\psi_1}{dt} = E_1\psi_1 + K\psi_2$$

$$i\hbar\frac{d\psi_2}{dt} = E_2\psi_2 + K\psi_1$$

Where ψ_1 and ψ_2 are the states of each superconductor with ground state energies E_1 and E_2, respectively. The constant K is a measure of the interaction strength between each superconductor and depends on the physical characteristics of the junction. If the insulator is removed and the superconductors are connected, then $K = 0$ and the equations represent the standard time evolution of the lowest energy ground state. Now if we apply some non-zero voltage V across this junction, then the energies of the separate superconductors will vary so that $E_1 - E_2 = 2eV$. If we then assume that the energy across the insulator scales linearly (so that zero energy is at the center) then we can simplify our equations to

$$i\hbar\frac{d\psi_1}{dt} = eV\psi_1 + K\psi_2$$

$$i\hbar\frac{d\psi_2}{dt} = -eV\psi_2 + K\psi_1$$

Now we must find a suitable model for ψ_1 and ψ_2 that describes the state of the electron pairs within the superconductors. Because the magnitude squared of the wave function is meant to describe a probability of observing a pair of electrons, we can use the charge densities ρ_1 and ρ_2 of each superconductor as a description of the macroscopic system of Cooper pairs. Then the wave functions can be written as

$$\psi_1 = \sqrt{\rho_1}e^{i\phi_1}$$

$$\psi_2 = \sqrt{\rho_2}e^{i\phi_2}$$

This wave function model for the bosonic state of Cooper pairs, based on early work on superconductivity by Ginzburg and Landau [5], includes a phase term ϕ that is, like electric potential, set at a reference and only important relative to one another. We thus introduce $\delta = \phi_2 - \phi_1$ to account for the phase difference between the superconductors. If we now solve the coupled equations by substituting in these wave functions representing the electron pair states, we arrive at four equations describing the time evolution of the charge densities and the phases:

$$\dot{\rho_1} = \frac{2K}{\hbar}\sqrt{\rho_1\rho_2}\sin\delta \qquad \dot{\rho_2} = -\frac{2K}{\hbar}\sqrt{\rho_1\rho_2}\sin\delta$$

$$\dot{\phi_1} = -\frac{eV}{\hbar} - \frac{K}{\hbar}\sqrt{\frac{\rho_2}{\rho_1}}\cos\delta \qquad \dot{\phi_2} = \frac{eV}{\hbar} - \frac{K}{\hbar}\sqrt{\frac{\rho_1}{\rho_2}}\cos\delta$$

The first clear result is that $\dot{\rho_1} = -\dot{\rho 2}$, the rate of change of charge density for the left superconductor is opposite the change in the right superconductor. Of course we know that charge is conserved, so we can interpret this result to imply how the density of Cooper pairs would begin to change as a result of an applied voltage, and so we can interpret the current as

$$I = \frac{2K}{\hbar}\sqrt{\rho_1\rho_2}\sin\delta \equiv I_0\sin\delta$$

This remarkable result shows that the current of Cooper pairs, or supercurrent, across a Josephson junction is a function of the relative phase between the superconducting states. Now with our second set of time derivative equations we can find how this relative phase responds to an applied voltage.

$$\dot{\delta} = \dot{\phi_2} - \dot{\phi_1} = \frac{2eV}{\hbar}$$
$$\delta = \delta_0 + \frac{2e}{\hbar}\int V(t)dt$$

We have thus arrived at the primary equations for predicting the Josephson effects we plan to experimentally observe. When no voltage is applied across the junction, $\delta = \delta_0$ and so $I = I_0 \sin\delta_0$ does not necessarily equal zero. This nonlinearity appears as a discrete jump in current when no voltage is driven, with magnitude proportional to the critical current I_0 that is determined by the properties of the junction.

Now consider applying a high-frequency, low amplitude AC voltage in addition to the initial DC voltage across the junction so that $V = V_0 + u\cos\omega t$. Then we can solve for the relative phase and thus the current that arises as a result of this applied voltage. The full derivation, detailed in Feynman's Lectures on Physics, shows that the expression for the current becomes

$$I = I_0[\sin{(\delta_0 + \frac{2e}{\hbar}V_0t)} + \frac{2eu}{\hbar\omega}\sin\omega t\cos{(\delta_0 + \frac{2e}{\hbar}V_0t)}]$$

This lengthy equation can be simplified by noting that the first term in brackets averages over time to zero due to its high frequency. The second term describes the resonance effect that triggers the emergence of the AC Josephson Effect. When the frequency of the driven voltage $\omega = 2eV_0/\hbar$, the second term time averages to a non-zero value, appearing as stepwise bumps along a graph of current versus voltage. These steps [8], once appropriately amplified and tuned, will allow for an experimental measurement of the fundamental constant $2e/\hbar$.

3 Apparatus

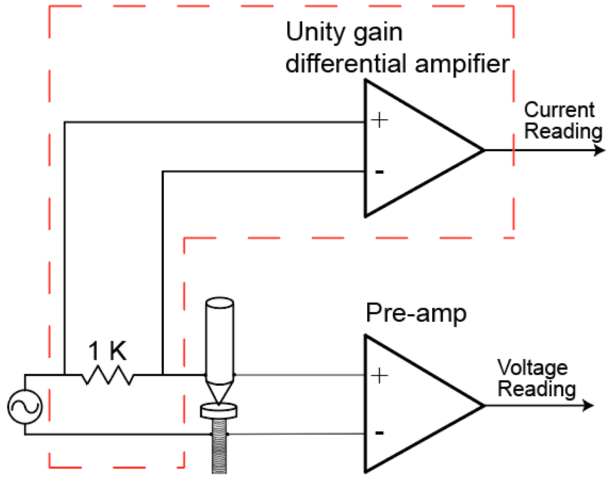

Figure 3: The four-wire measurement circuit connected to the Niobium point-contact Josephson junction. The voltage across the junction is recorded and measured through a Pre-amp, whereas the current is measured as the voltage across a resistor and interpreted to be a mA current reading.

The apparatus for this experiment allows us to immerse a superconducting junction within liquid helium at around 4 K with electrical connections across the junction for data acquisition of the current-voltage characteristics. The electrical values involved must be amplified to be accurately observed, and so a four wire measurement [6] circuit has been designed (as shown below) to transmit the voltage and current across the junction into a NI DAQ for data acquisition and an oscilloscope. The voltage across the junction is measured through a Pre-amp with calibrated gain of $512.57 \pm .02$ to to make the signal clear on an oscilloscope trace. By sweeping the junction with a low frequency AC current driven through a 1 kΩ resistor, we are able to trace a clear I-V pattern across the junction [7]. The junction apparatus, which we place into a 4 foot steel probe for insertion into a liquid helium dewar, is designed with tight fiberglass housings to keep both ends of the superconductor stable but also precisely movable while being immersed in liquid helium. The superconductors used in this setup are made of Niobium, which has critical temperature of ≈ 9 K, and the insulator is the oxide layer that develops on the surface of the metal when exposed to air overnight. A differential screw is placed against the upper end of the probe as a means of controlling the distance between the superconductors and thus the strength of the Cooper pair couplings across the junction. During the experiment, tuning of the junction contact is done with an extended hex key that travels down from the top of the probe and connects to the differential screw at the top of the junction assembly.

The accuracy of our experimental results is heavily reliant on the quality of the niobium junction and the secure placement of the niobium pieces into the probe. We have inserted a niobium screw within fiberglass housing to lock it within the assembly, and a thin (≈ 5 mm) niobium wire of length roughly 1.5 cm is tightened with screws into a separate fiberglass housing above the screw. The wire should be filed to a point on the contact end beforehand so that an oxide layer can be applied to the tip by exposure to air. The niobium superconductors should be in contact without excessive force; tightening of the junction could scrape off the oxide layer on the niobium and compromise the insulating layer of the junction. If this occurs, then lone electrons will pass through the junction, adding resistivity that appears as a diagonal current-voltage relationship on the I-V curve. In the case of a direct contact between the superconductors, the oscilloscope will read an electrical short in the form of a vertical line, and so the junction should be loosened. If the contact is disconnected then we would see a horizontal line on the scope since no tunneling of Cooper pairs or electrons can occur over that large of a separation. Finding the correct contact pressure is done by balancing between these two extremes, until the Josephson effects can be observed. The DC Josephson effect can be verified as an actual tunneling effect rather than a short between the superconductors by bringing a magnet close to the junction, which should create fluctuations in the supercurrent jump at zero applied voltage due to the superconductor's efforts to reject the magnetic fields.

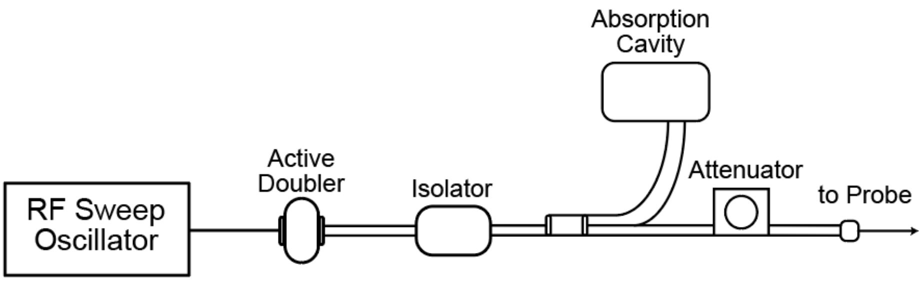

Figure 4: The microwave irradiation apparatus for observation of the AC Josephson effect. An RF Oscillator generates the initial AC voltage at around $9-13$ GHz and max output power of 20 dbm, and an active frequency doubler increases the power output by about 25 dbm and doubles the frequency. This voltage is sent through an isolator for electrical protection and enters the probe, with an attenuator attached for power control and an absorption cavity for accurate frequency measurements to one part in 10^7.

For observation of the AC Josephson Effect in the form of resonant Shapiro steps, we require an AC voltage source capable of driving the junction at high frequencies in the range 18-26 GHz. To accomplish this, we use the electrical setup shown above to irradiate the steel probe with a high power AC voltage. As shown in our calculations in the theory section, the steps emerge when the voltage is a multiple of $\omega\hbar/2e$, which for the applied voltage should be around the range of frequencies listed. There are significant imperfections in this method of AC voltage driving, including dissipation and noise from the steel tubing electrical medium, and so the accuracy of the effect is limited. An RF Oscillator drives the AC voltage at about half the listed frequency, and an active doubler amplifies the given output as well as doubles its frequency. This voltage passes through a waveguide and isolator for protection and an attenuator is placed before attaching to the probe for more precise control of the power output of the equipment. By attaching a power splitter to an absorption cavity, we can more accurately measure the output frequency to arrive at a precise calculation of the fundamental constants involved. Our RF voltage circuit had a measured power output between 7.8 and 112 mW, a suitable range for sending the AC voltage through the probe and across the junction.

4 Junction Analysis

Figure 5: Oscilloscope traces of the DC Josephson Effect (left) and Shapiro steps (right) on a 0.1 V/div by 0.1 mA/div scale. These measurements were also sent through an NI DAQ for precise data acquisition.

With our apparatus assembled the Josephson junction can be immersed in a liquid helium dewar with continuous current and voltage readings across the junction and driven with a high frequency AC voltage. The DC Josephson effect requires no added voltage source, and so observation of the effect relies on tuning the length of the insulating junction so that the superconducting states can couple without allowing lone electrons to cross. Our experimental data of the DC supercurrent at zero voltage is not a purely discrete vertical jump and is at a slight diagonal, which we attribute to imperfections in the oxide layer allowing a negligible amount of lone electrons to flow that are subject to Ohm's Law and thus introduce resistivity. Overall the DC effect was observed fairly easily, consistent with previous results on the subject.

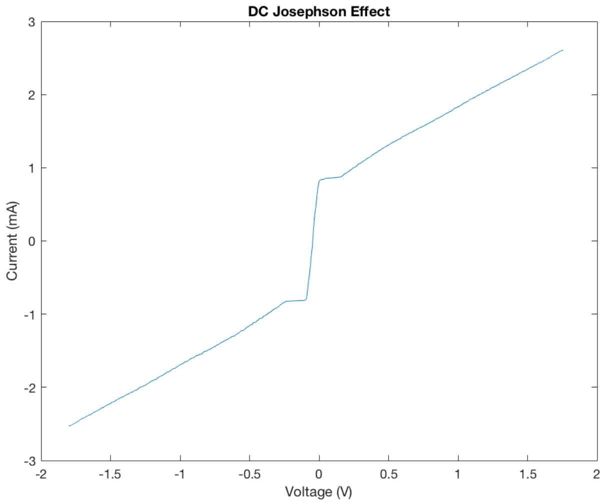

Figure 6: The DC effect as recorded by a pre-made Labview program with a sampling rate of 4.88E-6 sec/sample. The slight angle at which the DC effect occurs is a result of minimal electron flow across the junction creating resistivity in our measurements.

The accuracy of the AC Josephson effect we observed was limited by the poor contact between the stainless steel tubing of the probe and the junction itself, creating noise in the applied voltage and thus imperfections in the periodic current jumps. The plot of current vs. voltage for this effect does not have a clear step pattern, so we performed extensive data averaging to remove extraneous fluctuations in the data. Our averaging process performs a moving average over the data set with a five point sampling window. The resulting curve, plotted below, has well-defined steps, although not every step distance is equivalent due to fluctuations that are not smoothed precisely in the averaging process. The plot below shows the result of our averaging on the data within a ± 0.25 V window. This data was taken with an RF output frequency of $\omega = 24.600 \pm .005$ GHz as measured by the absorption cavity which allows for a more accurate measurement than through the RF oscillator.

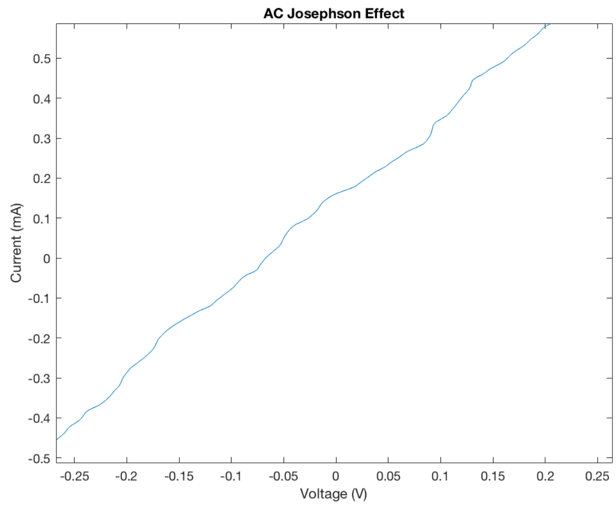

Figure 7: The AC Josephson Effect after averaging with a moving linearly weighted average over a 5 data point sliding window. Notice clear step patterns around -0.2 V, -0.05 V, and 0.1 V that are consistent with the theoretical spacing as predicted by Shapiro.

To find the voltage distance V_{step} between each Shapiro step and thus an estimate of $2e/h$, we analyzed the spectrum of periodic voltage behavior using the Fast Fourier Transform from the scipy python library. A rigorous examination of the AC effect by Clarke has shown that the characteristic transfer function of the junction can be described by Bessel functions up to a desired order; this construction hints to the periodic structure of the data and thus naturally allows for a description of its spectrum. We thus require a high bandwidth in the current to accurately capture the higher order harmonics, whereas the voltage is driven by the sweep oscillator and thus does not need the same bandwidth. The FFT measures the contribution of a given periodic voltage step in the overall collected data, and we expect that the step distance created by the application of microwaves into the junction will have the largest amplitude in the FFT. Based on this analysis and the FFT plot below, we see that the most prominent voltage step is $\Delta V = .0264$ V. This result is consistent with less accurate measurements by hand of the individual steps, and the centering of the FFT data along this point is a good sign of the method's effectiveness. Calculating the error in our voltage step measurement based on this analysis procedure is unclear. To resolve this, we consider the accuracy of the NI DAQ which we used for data acquisition as a measure of the error. The voltage error is based on the absolute accuracy of the DAQ that depends on the input gain error, ambient temperature, and the range of voltages that were acquired [7]. These values are available to us using the standard deviation of our recorded data, the DAQ datasheet, and the settings chosen for acquisition. The effect of averaging allows us to disregard the input noise, and because the voltage gain was calibrated to $G = 512.57 \pm 0.02$, we find a final voltage step with error of $\Delta V = .0264 \pm .0006$ V.

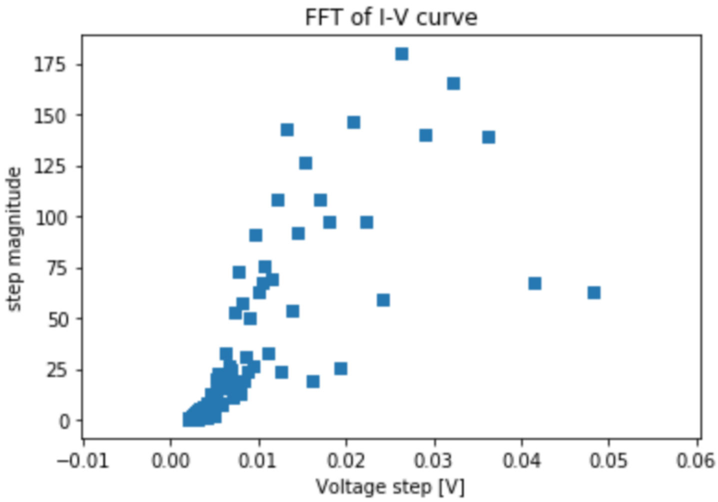

Figure 8: The FFT of the AC Josephson effect I-V curve within a ± 0.2 V window. The x axis of this plot describes the many periodic voltage steps that characterize the transfer function, with the y axis measuring the magnitude of each step in the complete plot. the cluster of small steps in the FFT are a result of small fluctuation that we interpret as small fluctuations in the system due to averaging.

As stated in the theory section, when a microwave frequency AC voltage is supplied across the junction we can observe current jumps at voltages that are multiples of $\omega\hbar/2e$ when ω is in radians/sec and $\omega h/2e$ for ω in Hertz. Because we now have a measure of this voltage step distance, we can work backwards to find an experimental measurement of the quantum constant ratio $2e/h$. This ratio is the inverse of the quantization of magnetic flux across a superconducting loop, and its appearance as a constant in the Josephson junction demonstrates how quantum phenomena emerge as a result of the condensed superconducting state. Because the actual voltage step is smaller as a result of pre-amp amplification, we must account for the gain $G = 512.57$ that we calibrated to high precision using a voltage box and a digital multimeter. Finally, we arrive at a measured ratio

$$\frac{2e}{h} = \frac{\omega G}{\Delta V} = (4.78 \pm .02) * 10^{14} Hz/V = 478 \pm 2 \;\; MHz/\mu V$$

Compared to the experimental value of $483.5912 \pm .0012$ MHz/μ V that was obtained by Parker, Taylor, and Langenberg, our result is within 1.16% of their value, although our error is significantly greater. We can attribute this larger error to the limited precision of our data acquisition device and the short amount of time we had to perfect our apparatus and procedure. The microwave irradiation process in which we blast high power AC voltage across the probe is a significant limiting factor in the accuracy of our results, adding temperature and noise to the junction that cannot be controlled or measured. Our

FFT plot of the current-voltage characteristics also exhibits a large amount of noise, illustrated by the spread of points clustered at lower voltage steps, and so the actual step value may be shifted due to noise obstructing current steps in the data. Overall, this calculation of $2e/h$ is a relative success for an undergraduate laboratory, demonstrating a determination of fundamental constants by exploiting the macroscopic quantum state of Cooper pairs within the superconductor.

5 Conclusions

The Josephson Junction's simple structure gives rise to extraordinary quantum behavior that can be explored with devices within the reach of an undergraduate laboratory. Although our junction was a simple niobium point contact separated by an oxide layer, our results clearly demonstrated the tunneling of Cooper pairs across the junction in the DC Josephson effect. With a radio-frequency voltage source connected to our probe, we can observe a stepwise current-voltage relationship that emerges due to coupling of the superconducting states in the AC Josephson effect. We see that the Cooper pairs' quasi-bosonic properties allow for a macroscopic quantum description in the same manner as photons in laser light. This collective quantum coherence among the electron pairs is at the heart of the Josephson junction's versatility in quantum technologies and precision in controlling quantum systems.

I would like to thank our professors and Don Orlando for their support and guidance, as well as the GSIs that helped us throughout the lab. Although the apparatus is very simple and practical, the microwave irradiation process is inefficient and makes observation of the AC effect difficult. Adding a more targeted RF radiation source will make this lab more accessible. Due to our difficulty in observing the AC effect, I would not recommend this lab for anyone not extremely interested in the workings of superconducting materials to avoid frustration.

6 References

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