Quantum Manipulation of Mechanical Resonators with Electromagnetic Interactions

Ben Safvati

Featuring:

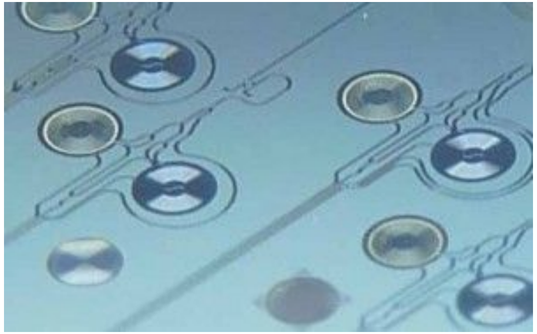
Chan, J., Alegre, T., Safavi-Naeini, A. et al. **Laser cooling of a nanomechanical oscillator into its quantum ground state**. Nature 478, 89–92 (2011).

Motivation

Precision Sensing Over Wide Scales

Quantum-limited measurements (or even better)

≈ 1 μm

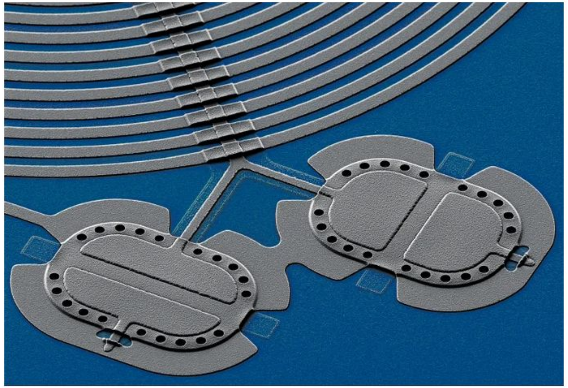


Source: Interuniversity Microelectronics

≈ 1 km

Classical-quantum crossover

Mitigating macroscale decoherence



Kotler et al. *Science*, vol. 372, no. 6542, 2021

- ❖ **Oscillators are universal in science and technology - how can quantum interactions with light fields improve measurements, advance physics?**

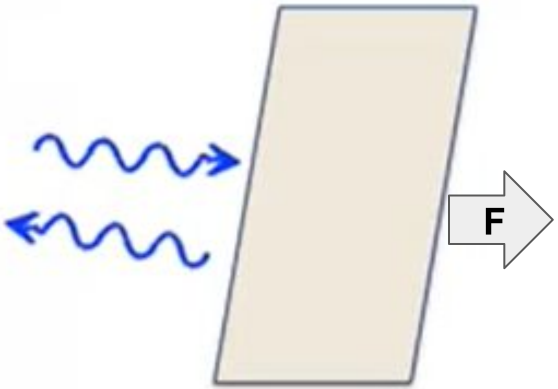
Outline

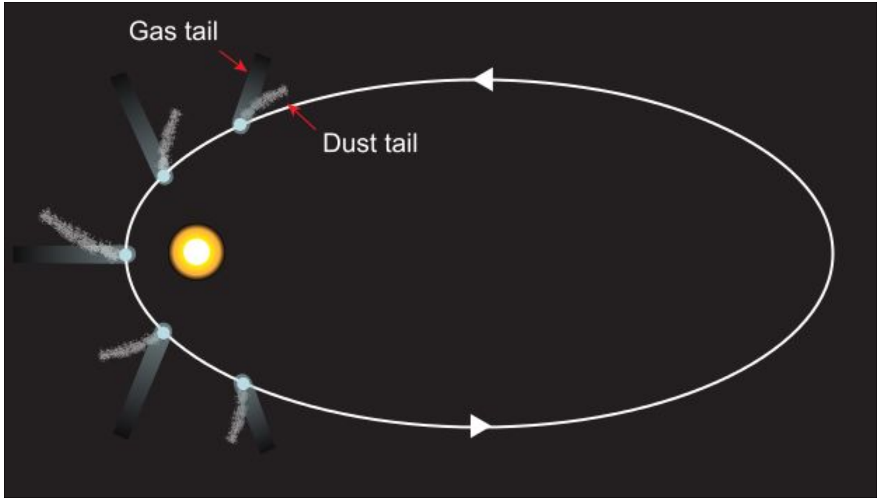
1. **Introduction: the mechanical effects of light**
2. Consequences of quantum fluctuations
3. Physical Description of Experiment
4. Experimental results
5. Discussion and relevance

Mechanical Effects of Light

Reflection

$$\Delta p = \frac{2\Delta U}{c}$$



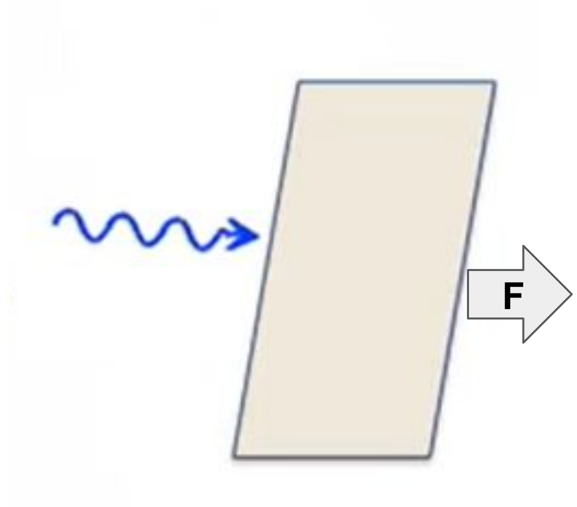


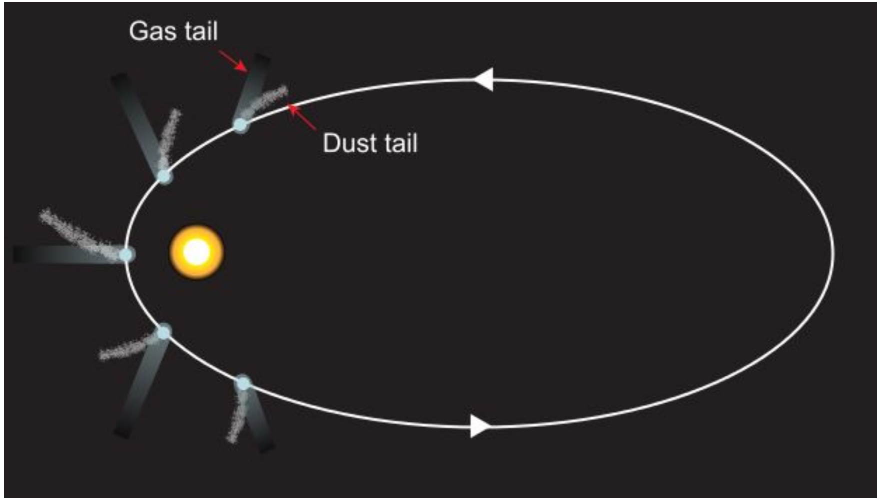
(1619) Kepler - Observation of solar radiation pressure on comet tails.

Mechanical Effects of Light

Absorption

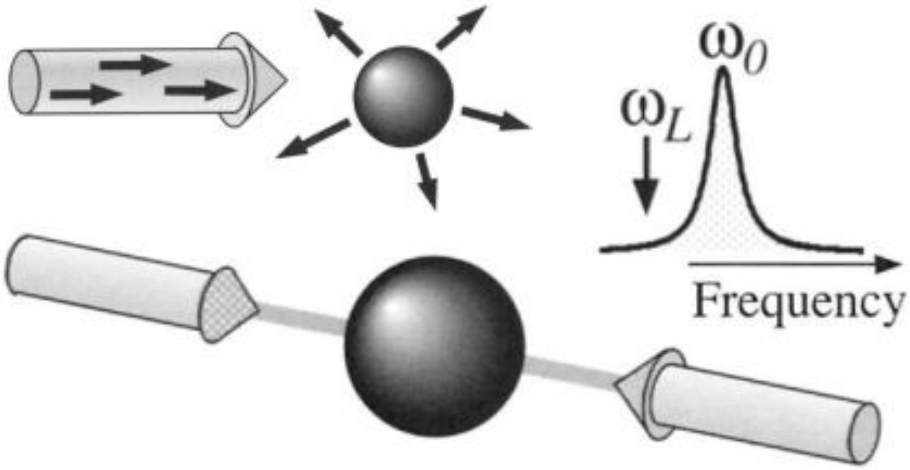
$$\Delta p = \frac{\Delta U}{c}$$





(1619) Kepler - Observation of solar radiation pressure on comet tails.

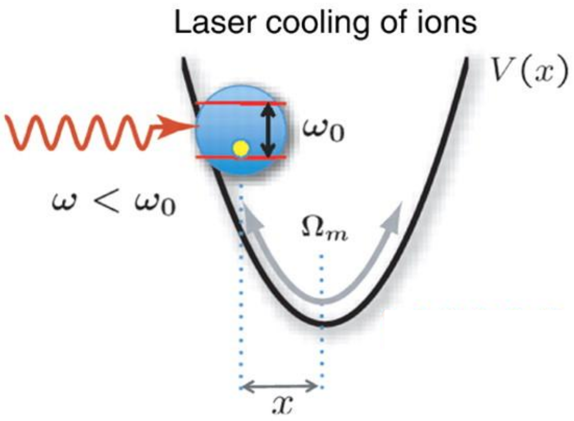
Mechanical Effects of Light



Doppler Cooling - mechanical damping by resonant absorption of blue-shifted light.

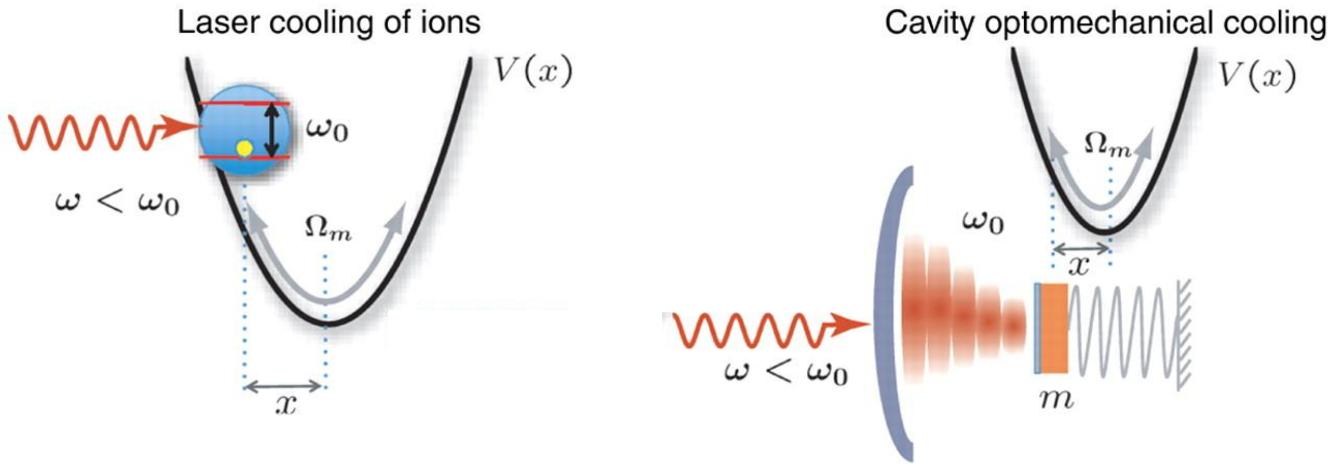
-Hänsch, T.W., and A.L. Schawlow. "Cooling of Gases by Laser Radiation." Optics Communications, vol. 13, no. 1, (1975)
-Suter, Dieter. The Physics of Laser-Atom Interactions. Cambridge University Press, (1997)

From Atomic Resonances...



Kippenberg, T. J., and K. J. Vahala. "Cavity Optomechanics: Back-Action at the Mesoscale." Science, vol. 321, no. 5893, 2008

From Atomic Resonances to Cavity Modes

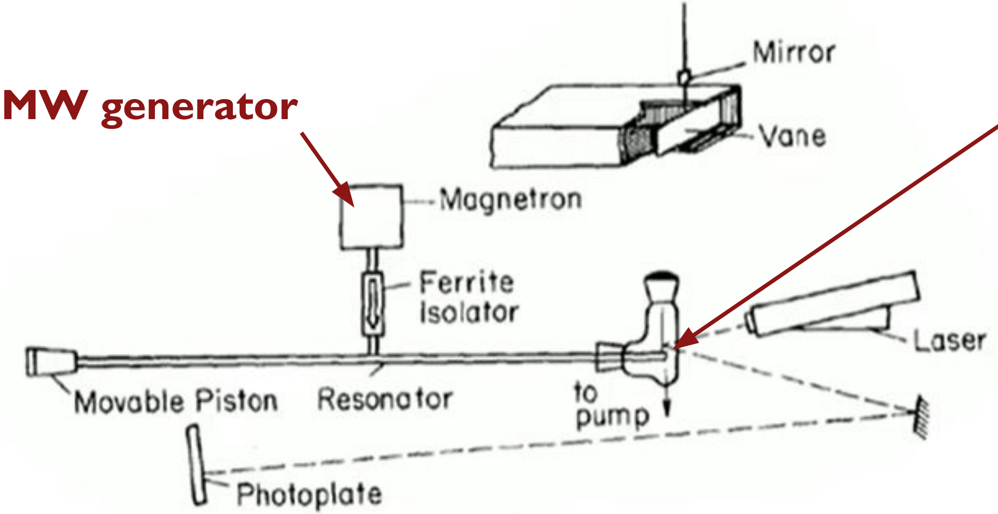


- **Apply atomic laser control methods to mechanical modes of macroscopic (~10^{14} atoms) objects.**

Kippenberg, T. J., and K. J. Vahala. "Cavity Optomechanics: Back-Action at the Mesoscale." Science, vol. 321, no. 5893, 2008

Radiative Effects on a Mechanical Resonator

MW generator



Movable end mirror couples MW field to mechanics

(1970) Braginsky - Mechanical dissipation with a microwave resonator.

<u>Observations</u>

- Increased (decreased) mechanical damping when radiation red- (blue-) shifted relative to cavity resonance
- Implications for sensitivity of precision measurements...

Braginsky, V. B., A. B. Manukin, and M. Y. Tikhonov, Sov. Phys. JETP 31, 829, (1970)

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Standard Quantum Limits on Interferometry

- **Recombination of light beams in both arms allows measurement of optical path length differences as relative phase**

Light frequency

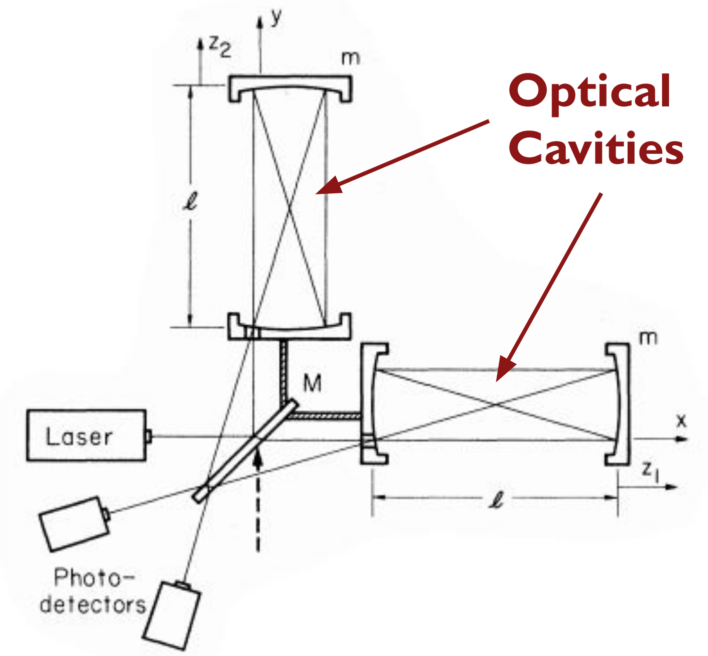
$$\delta\Phi = 2b\omega z/c$$

Phase difference

# of reflections at each mirror

Cavity length difference

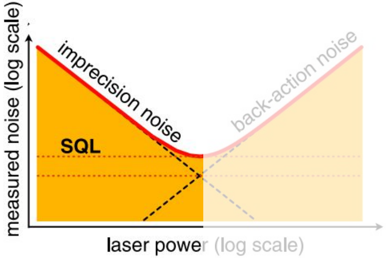
Optical Cavities

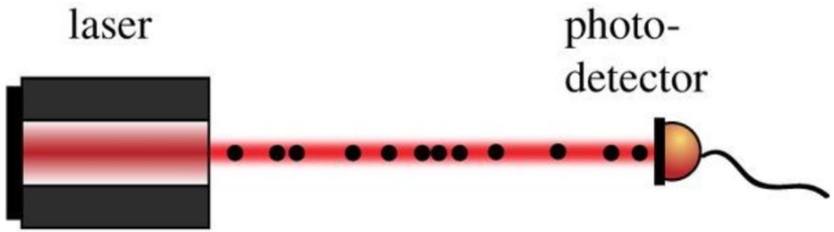


Caves, Carlton M, Physical Review Letters, vol. 45, no. 2, (1980)

Standard Quantum Limits on Interferometry

- **Mean photon number:** $N = P\tau/\hbar\omega$
- **At low laser power, sensitivity is limited by Poisson shot noise:** $\Delta N \sim N^{1/2}$
- **Increasing power initially improves precision relative to noise:** $\mathrm{SNR} \sim N^{-1/2}$





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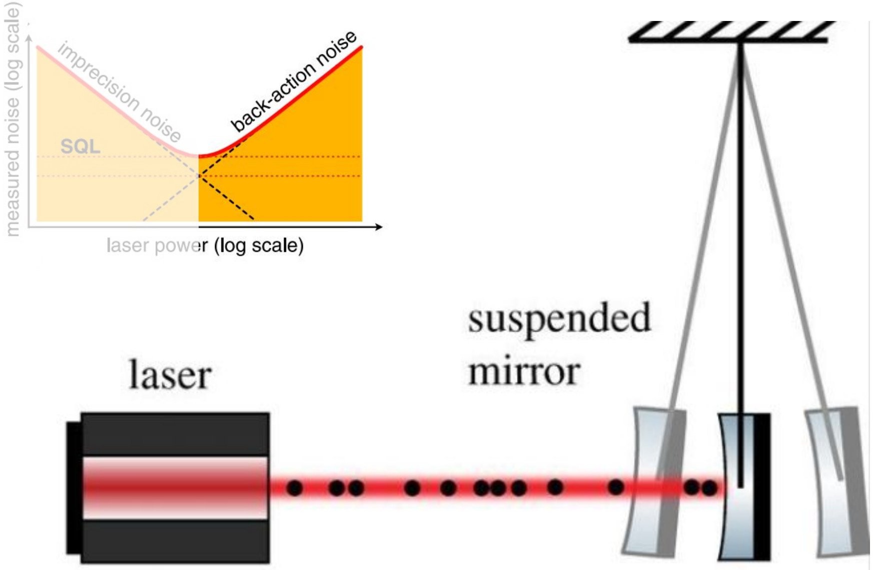
Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Heurs, M. Philosophical Transactions of the Royal Society A vol. 376, no. 2120, (2018)

Standard Quantum Limits on Interferometry

- **Fluctuations of the quantum field drive the mirror stochastically**
- **Mirror back-action due to radiation pressure creates new noise source**
- **Photon number fluctuations depend on shot noise ⇒** $\mathrm{SNR} \sim \sqrt{N}$



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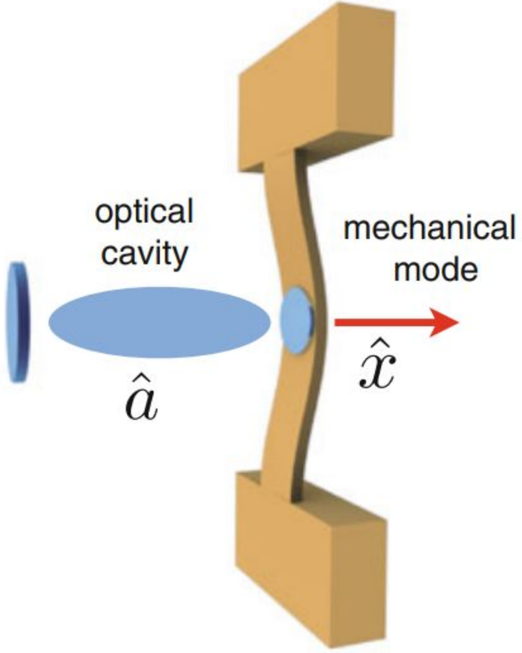
Canonical Optomechanical System

parametrically coupled quantum oscillators

$$\hat{H} = \hbar\omega_C(\hat{x})\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b}$$



For small, adiabatic displacements of mirror

$$\omega_C(\hat{x}) \approx \omega_C + \hat{x}\frac{\partial\omega_C}{\partial x} \equiv \omega_C - G\hat{x}$$

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)

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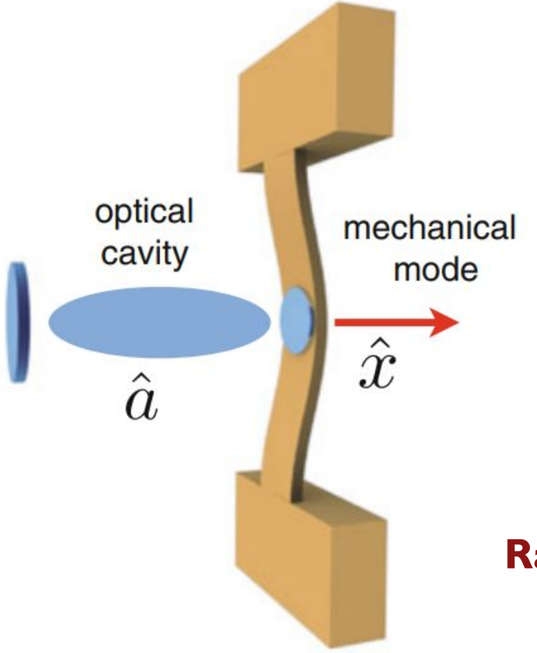

Define the **optomechanical interaction**

Vacuum coupling

$$g_0 \equiv Gx_{zpf}$$

$$\hat{H}_{int} = -\hbar G\hat{a}^\dagger\hat{a}\hat{x} = -\hbar G x_{zpf}\hat{a}^\dagger\hat{a}(\hat{b} + \hat{b}^\dagger)$$

Zero-point fluctuations $x_{zpf} = \sqrt{\dfrac{\hbar}{2m\omega_M}}$

Radiation pressure force $F_{RP} = \dfrac{dH_{int}}{d\hat{x}} = -\hbar G\hat{a}^\dagger\hat{a} \propto \hat{n}_{\text{cav}}$

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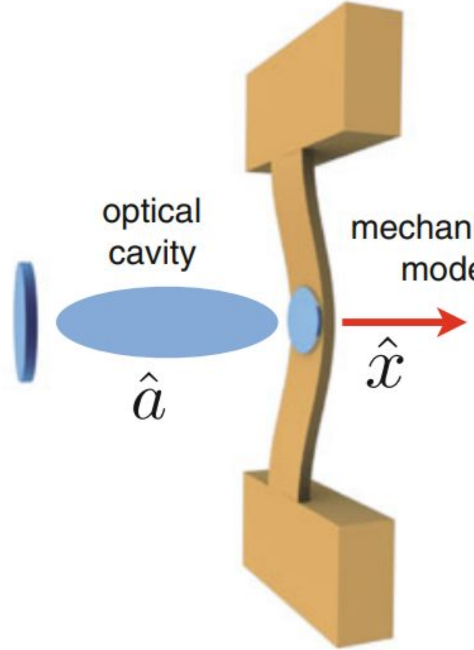
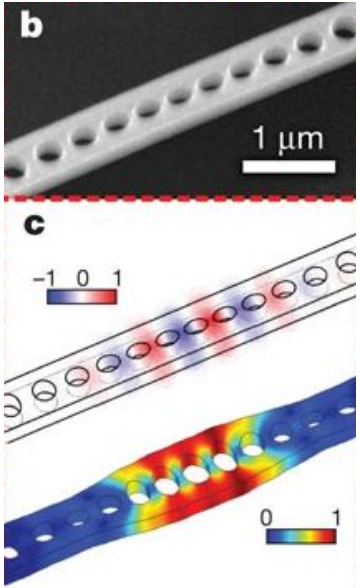


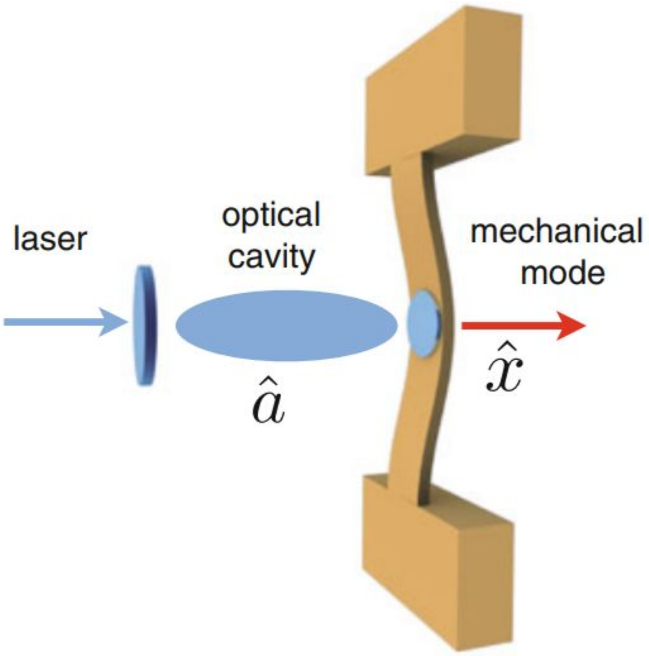
- **Nanobeam patterning creates colocalized optical and mechanical modes**
- **Radiation pressure distorts nanobeam surface, couples displacement to EM field.**



Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Chan et al. Nature 478, 89–92 (2011)

Canonical Optomechanical System

Detuning $\Delta = \omega_L - \omega_C$



Need to include effect of **laser driving** (rotating frame transformation)

$$\hat{H} = -\hbar\Delta\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b} - \underbrace{\hbar g_0\hat{a}^\dagger\hat{a}(\hat{b} + \hat{b}^\dagger)}$$

Nonlinear Hamiltonian produces complicated dynamics

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)

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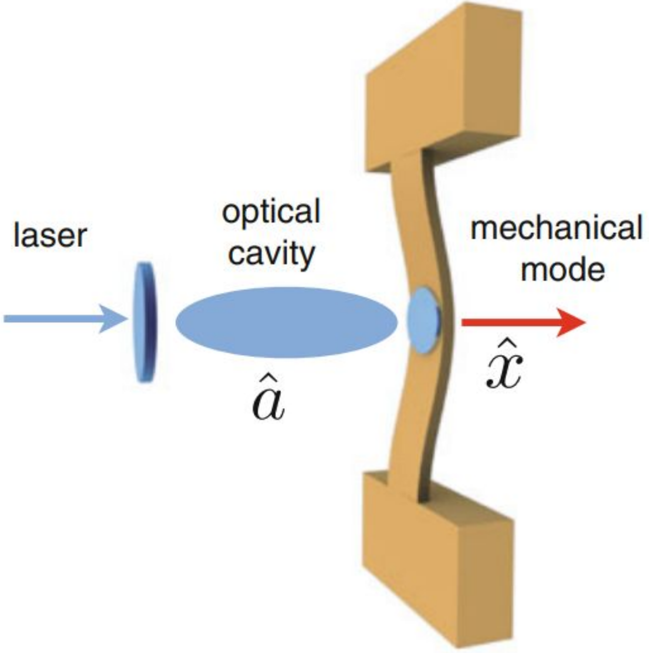

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Nonlinear Hamiltonian
produces complicated dynamics

Approximate cavity field with classical
steady-state + excitations

$$\hat{a} = \bar{\alpha} + \delta\hat{a}$$

$$\Rightarrow \quad \hat{H}^{(lin)}_{int} = -\hbar g_0|\bar{\alpha}|(\delta\hat{a}^\dagger + \delta\hat{a})(\hat{b}+\hat{b}^\dagger)$$

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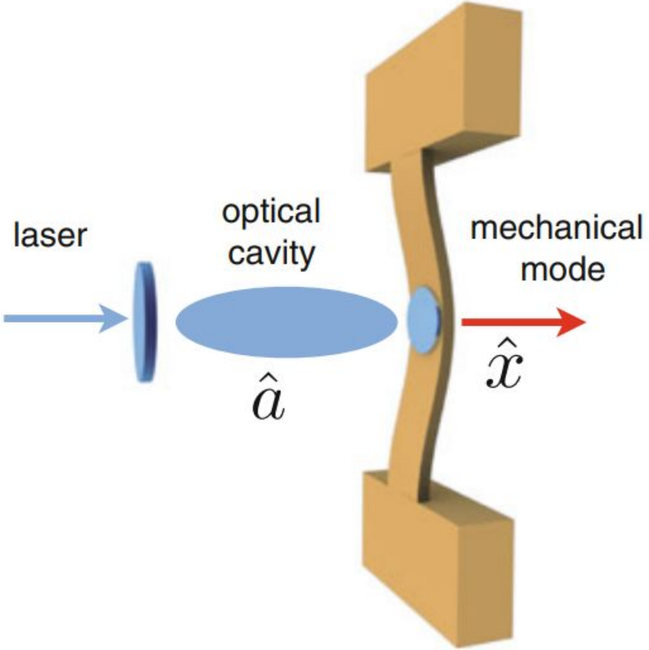

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Approximate cavity field with classical steady-state + excitations $\hat{a} = \bar{\alpha} + \delta\hat{a}$

$$\Rightarrow \quad \hat{H}_{int}^{(lin)} = -\hbar g_0\underbrace{|\bar{\alpha}|}\underbrace{(\delta\hat{a}^\dagger + \delta\hat{a})(\hat{b}+\hat{b}^\dagger)}$$

Tunable OM coupling! $g = g_0|\bar{\alpha}|$

Quadratic Interactions

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)

Canonical Optomechanical System

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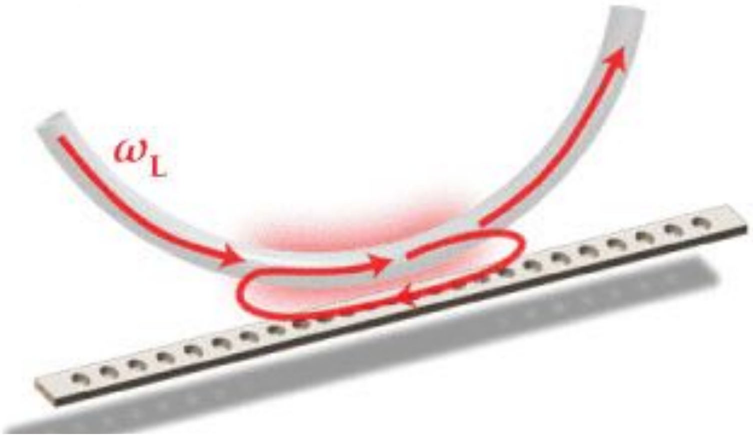

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$$g = g_0|\bar{\alpha}|$$

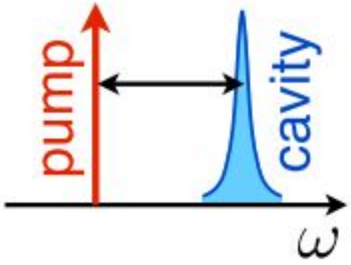
Quadratic Interactions

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)

Effect of Laser Detuning

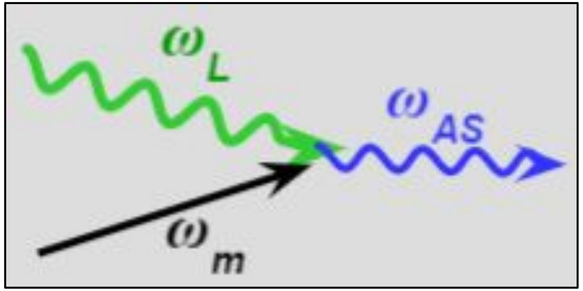
Use laser frequency to drive resonant transitions

$$\hat{H}_{int}^{(lin)} = -\hbar g_0 |\bar{\alpha}| (\delta\hat{a}^\dagger + \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$



$$\Delta = -\Omega_\mathrm{m}$$

- **Quantum coherent exchange of mechanical and driven cavity states.**
- **Laser sideband cooling!**



beam-splitter (cooling)

$$\delta\hat{a}^\dagger\hat{b} + \hat{b}^\dagger\delta\hat{a}$$

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)

Resolved-sideband Laser Cooling

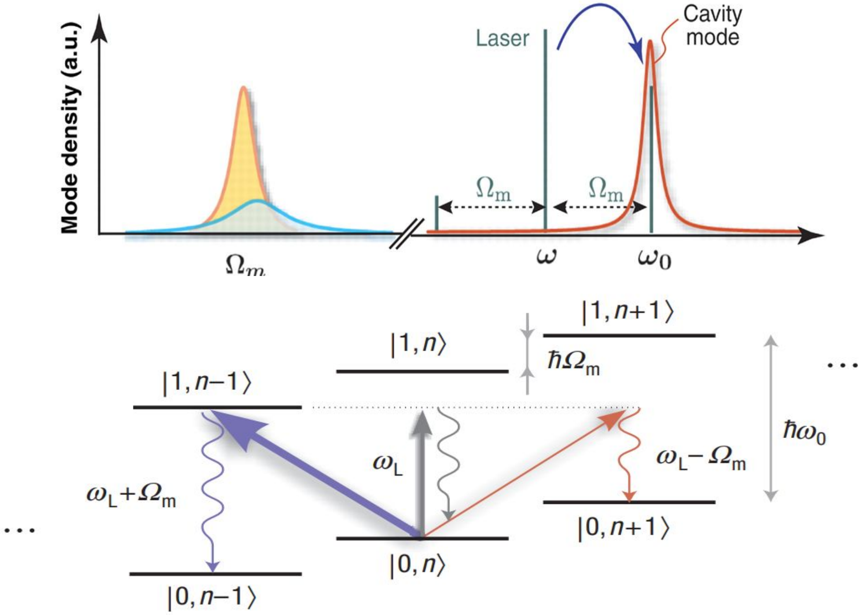
Doppler Limit

$$\Gamma_{\mathrm{opt}}|_{\kappa \gg \Omega_m} = 8\frac{g^2}{\kappa^2}\Omega_M$$

$$\bar{n}_{\mathrm{min}} = \frac{\kappa}{4\Omega_m} \gg 1$$

Steady-state phonon number after cooling

$$\bar{n}_f = \frac{\Gamma_{\mathrm{opt}}\bar{n}_{\mathrm{min}} + \Gamma_m \bar{n}_{\mathrm{th}}}{\Gamma_{\mathrm{opt}} + \Gamma_m}$$



- **Red detuning creates resonant absorption of mechanical energy**
- **Dissipative scattering process**

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)

Resolved-sideband Laser Cooling

Resolved-sideband limit

$$\Gamma_{\text{opt}}\big|_{\kappa \ll \Omega_m} = 4\bar{n}_{\text{cav}} \frac{g_0^2}{\kappa} = \frac{4g^2}{\kappa}$$

OM coupling

$$\bar{n}_{\min} = \left(\frac{\kappa}{4\Omega_m}\right)^2 < 1$$

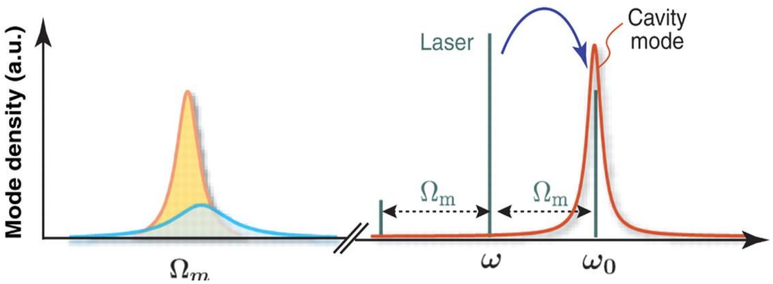
Sideband resolution

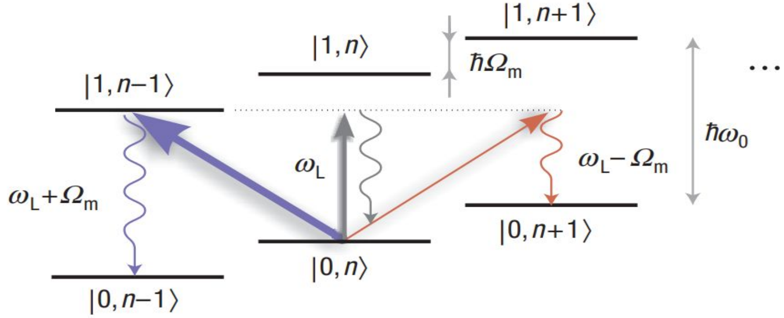
thermal contact

…

Steady-state phonon number after cooling

$$\bar{n}_f = \frac{\Gamma_{\text{opt}}\bar{n}_{\min} + \Gamma_m \bar{n}_{\text{th}}}{\Gamma_{\text{opt}} + \Gamma_m}$$





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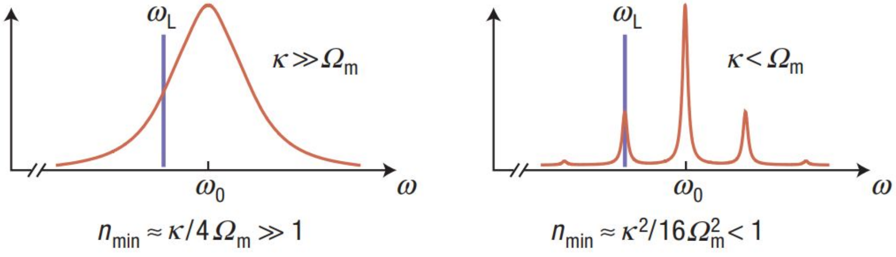
Parameter Ranges for Quantum Protocols

Ground State Cooling

- Laser cooling efficiency limited by sideband resolution: ω_m/κ



Strong Optomechanical Interactions:

- Requires detectable shift in cavity resonance due to single phonon: g_0/κ
- Measure of "photon granularity," how well we can resolve single-photon interaction with mechanics

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)

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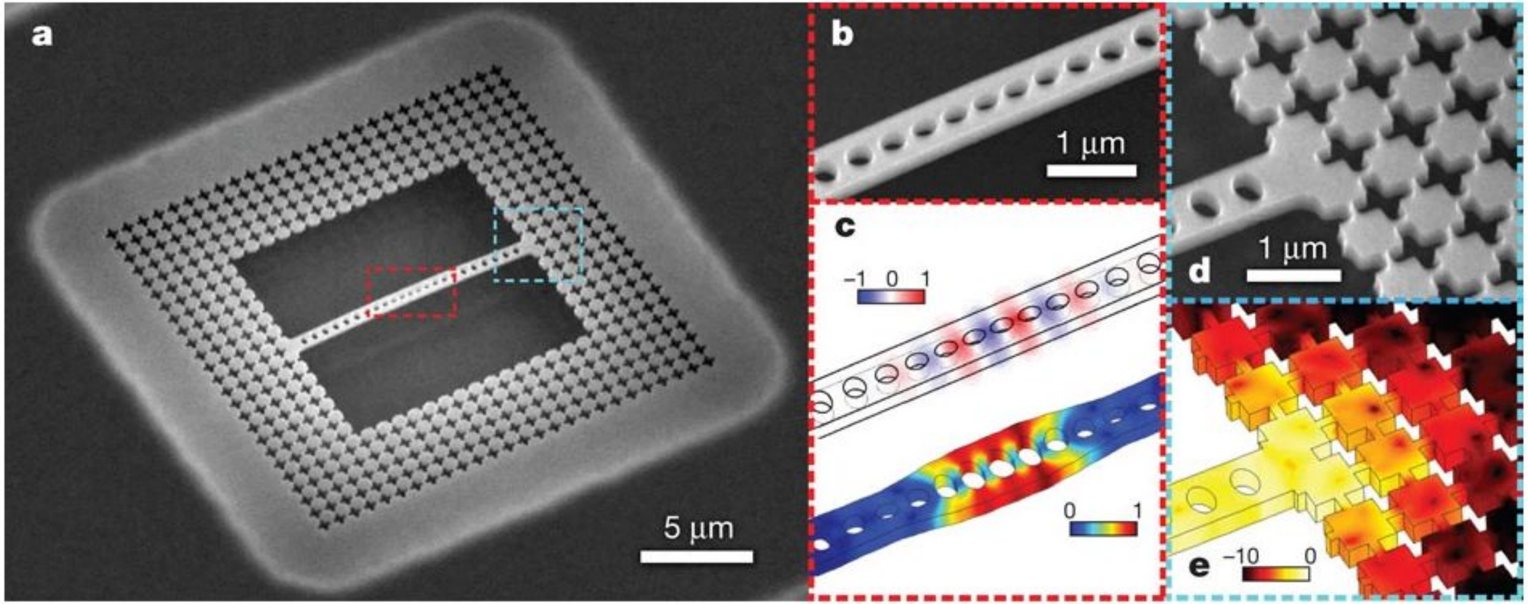
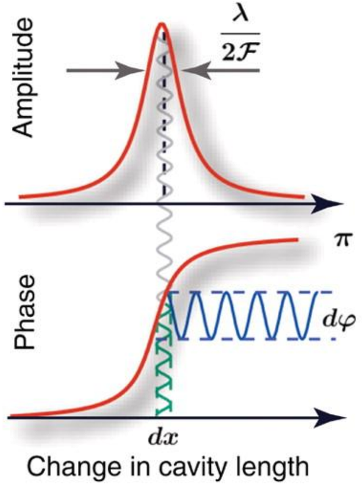
Ground-State Laser Cooling of a Nanoscale Resonator

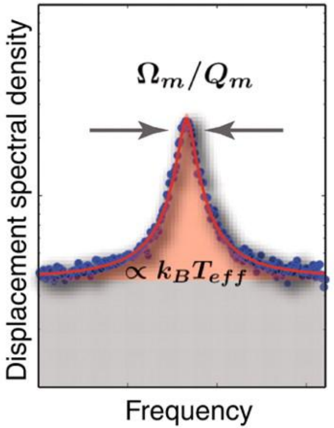
- 3.5 GHz mechanical mode, pre-cooled to 20 K with cryogenics
- Bulk pattern is acoustic insulator, decreases damping effect of thermal bath



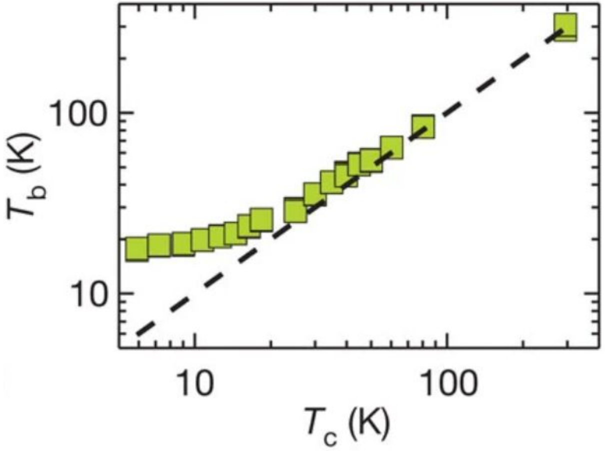
Optical Thermometry of Mechanical Mode

Noise power spectral density (PSD) of transmitted light measures mechanical noise spectra





Integrated PSD allows for system thermometry



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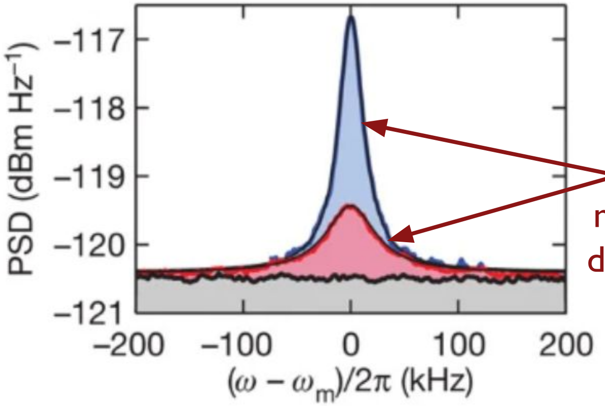
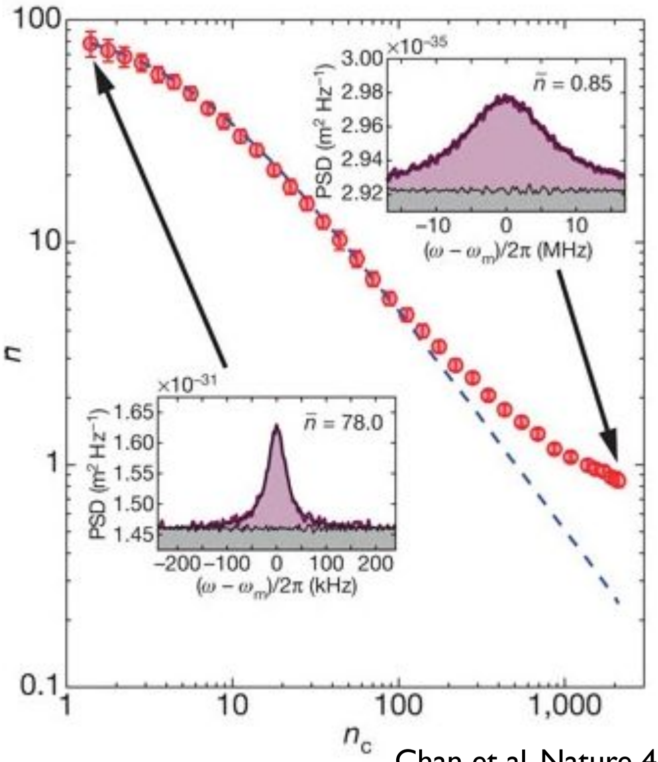
Optical Thermometry of Mechanical Mode

Thermometry at sideband detuning shows the cooling (heating) effect



Linewidths measure mechanical damping (red detuning) or amplification (blue detuning).

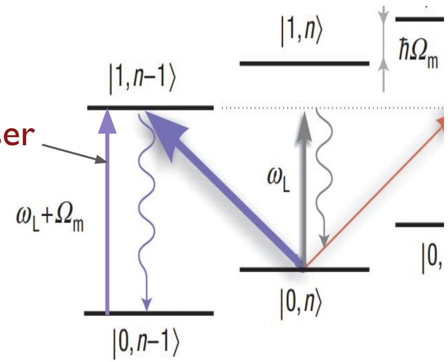


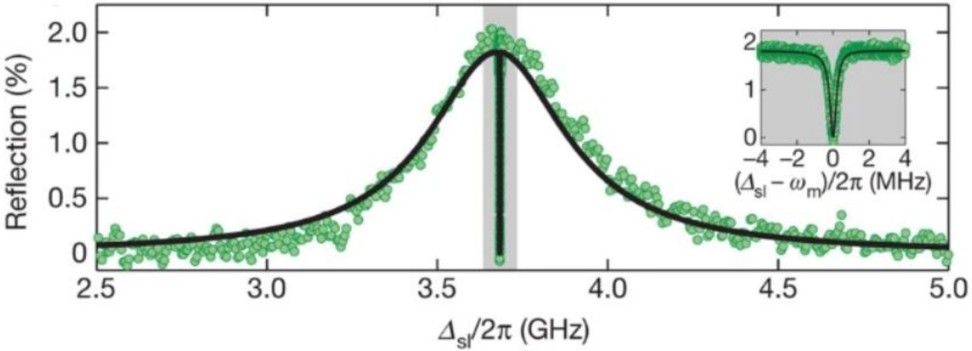
Chan et al. Nature 478, 89–92 (2011)

Optomechanically Induced Transparency

- Strong control field set to lower motional sideband (red-detuning)
- Dominant scattering effect receives energy from mechanics.

Probe laser





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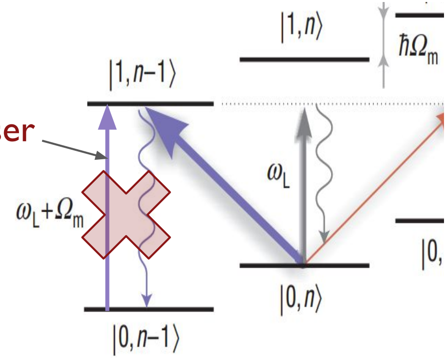

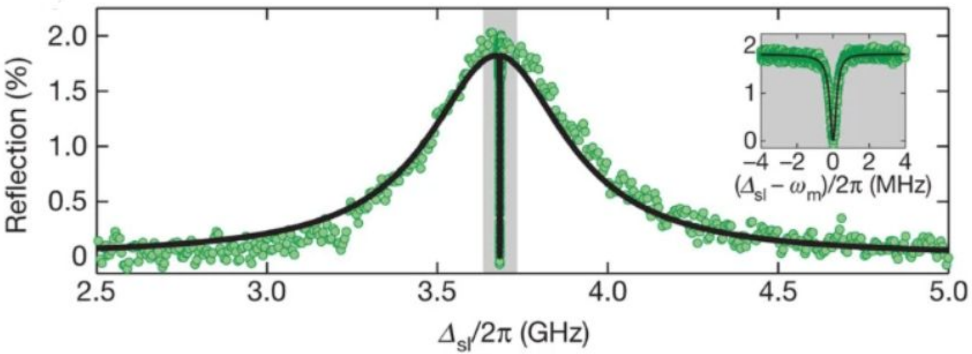

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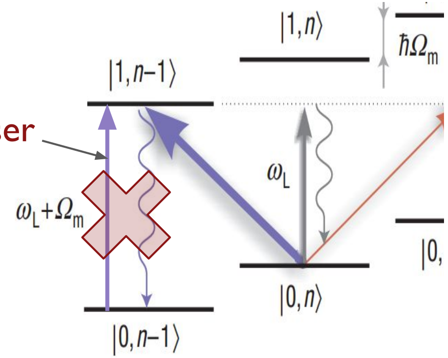


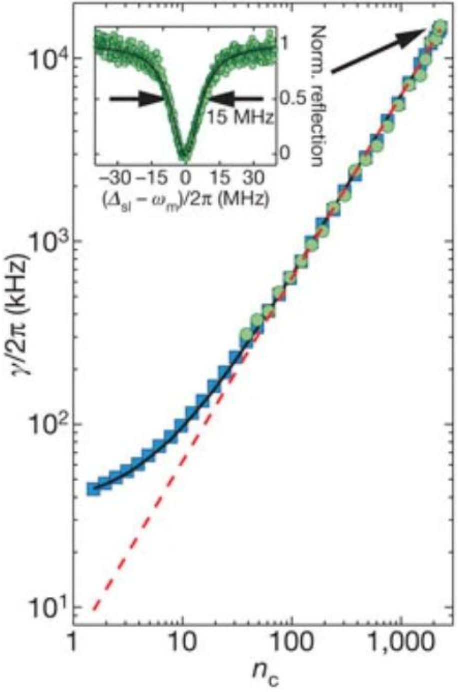
Allows characterization of mechanical damping and optomechanical coupling

$$g_0 = 910 \text{ kHz}$$

Much less than photon linewidth (inverse lifetime)

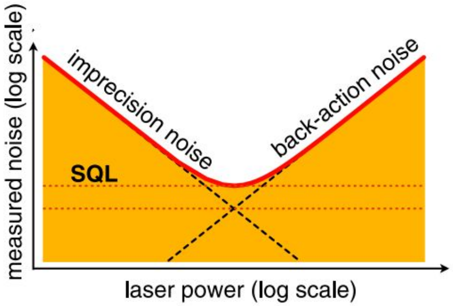
$$\kappa = 500 \text{ MHz}$$

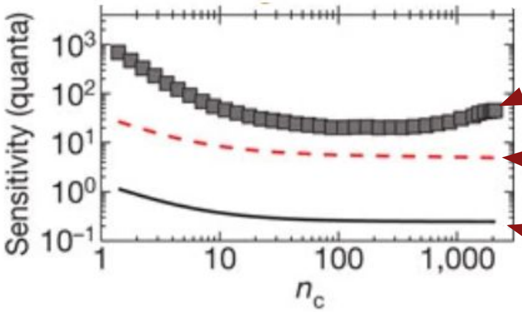
∴ No single photon-phonon interactions observable



Chan et al. Nature 478, 89–92 (2011)

Characterizing the Optomechanical Interaction

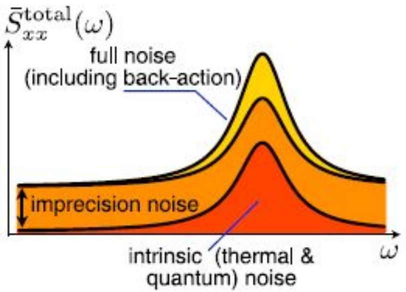




Experiment

Intrinsic + Equipment

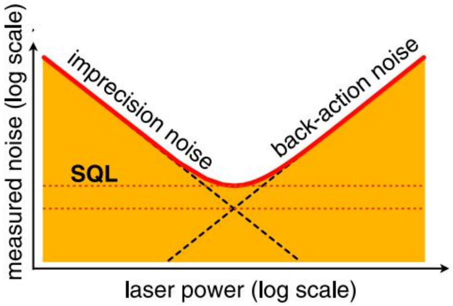
Intrinsic (quantum-limited)

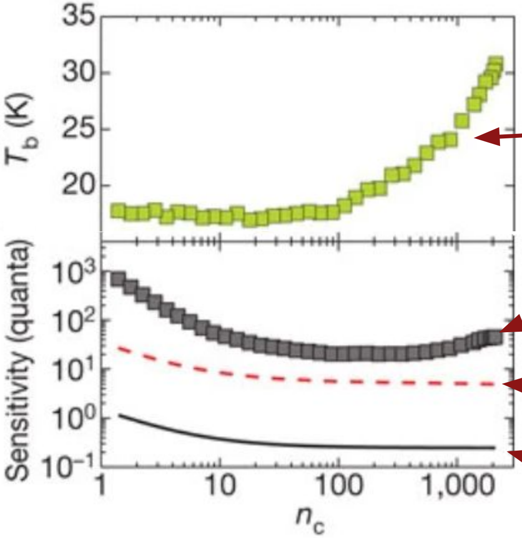


Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)

Characterizing the Optomechanical Interaction

Not yet at SQL



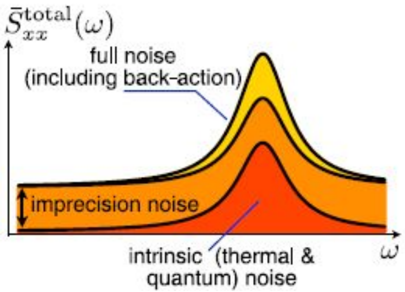


At high power, thermal noise due to absorption dominates before radiation pressure noise becomes observable

Experiment

Intrinsic + Equipment

Intrinsic (quantum-limited)

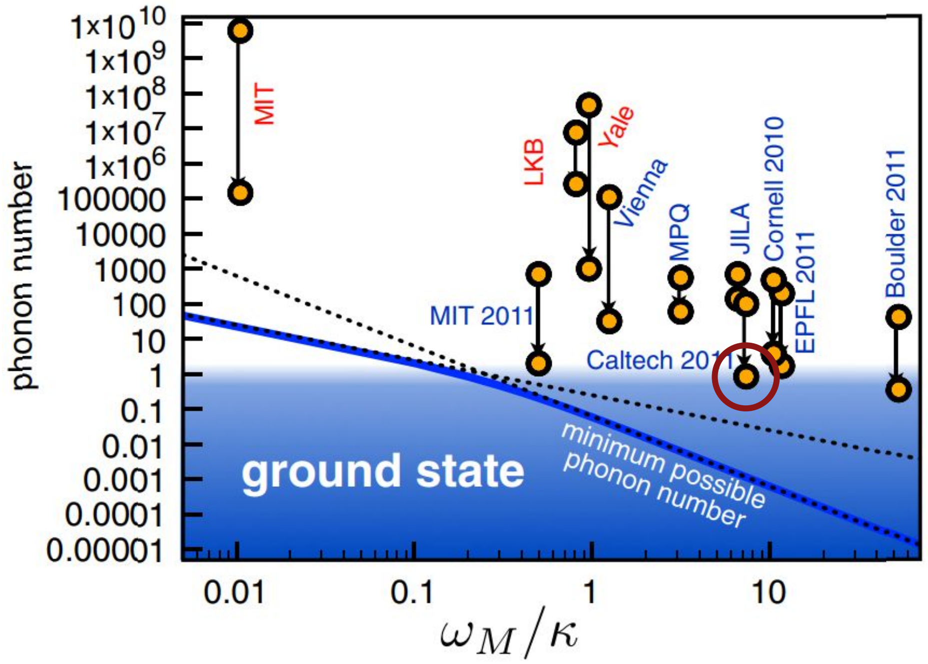


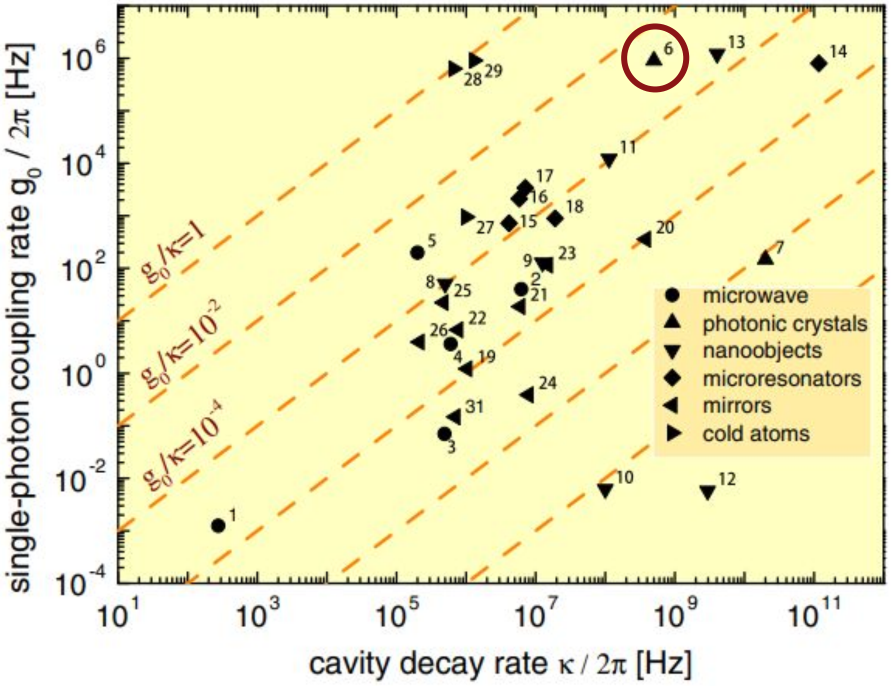
Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)

Outline

1. Introduction: the mechanical effects of light
2. Consequences of quantum fluctuations
3. Physical Description of Experiment
4. Experimental results
5. **Discussion and relevance**

Characterizing the Optomechanical Interaction

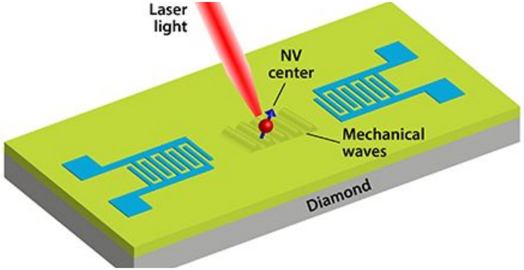




Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)

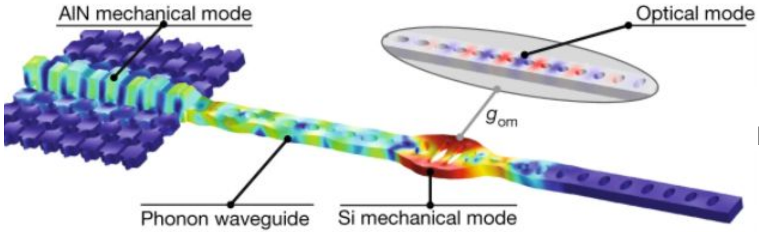
Looking Ahead: Quantum Hybrid Systems

Coupling to Diverse Systems



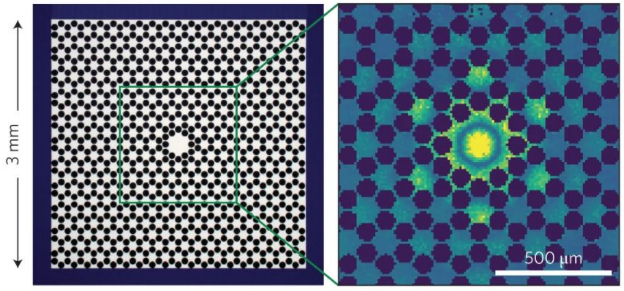
Golter et al. Phys. Rev. Lett. **116**, 143602 (2016)

Coherent Quantum Transduction



Jiang et al. Nat Commun 11, 1166 (2020)

Integrated Quantum Memories

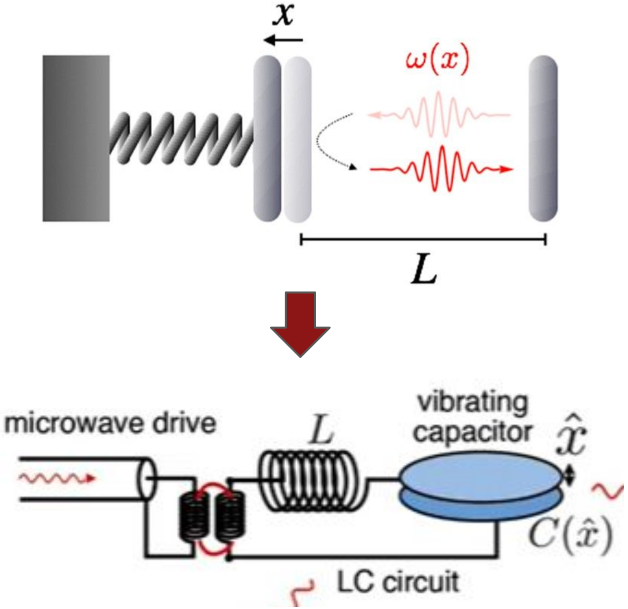


Tsaturyan et al. Nature Nanotech 12, 776–783 (2017)

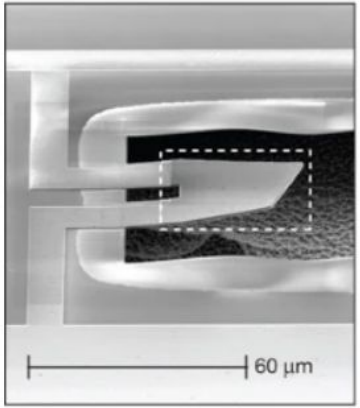
Thank you!

Optomechanics → Electromechanics

- Simple analogy with optomechanics (capacitive coupling)



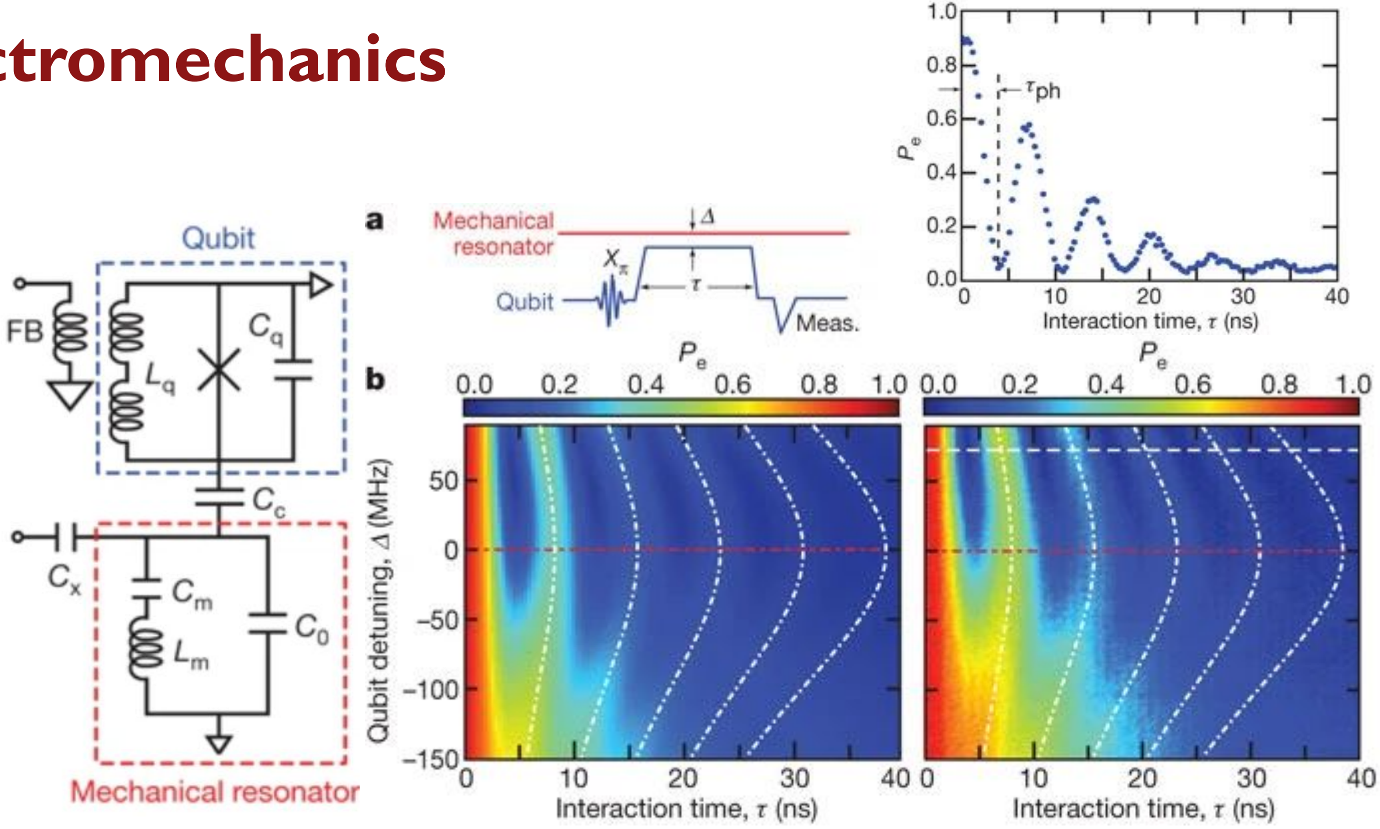
- Possibility of *cryogenic* ground-state cooling (without lasers)
  - 6 GHz piezoelectric acoustic resonator
  - mean phonon number n ~ 0.07 at 20 mK



Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)

Quantum Electromechanics

- Nonlinear energy levels of qubit allow simple single-photon excitation
- Photon in qubit can be swapped with mechanical resonator
- Operation has short (~ns) lifetime due to dissipation in resonator (low Q)



Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)

Outline

1. Introduction: the mechanical effects of light
    a. Radiation pressure
    b. Particle trapping and cooling
2. Consequences of quantum fluctuations
    a. Interferometry precision limits
3. General Optomechanical Model
    a. Hamiltonian form of interaction
    b. Mean-field approximation
    c. Laser cooling efficiency
4. Experimental Results
    a. First paper: Laser cooling, thermometry, analysis of parameters
    b. Compare and contrast optomechanics with electromechanics
    c. Second paper: Characterizing qubit-resonator coupling, mechanical thermometry with qubit, single excitation rabi oscillations
5. Discussion and Relevance
    a. Critique of experiment claims (might go with paper directly)
    b. Integration of mechanical devices in quantum technology

Mechanical Effects of Light

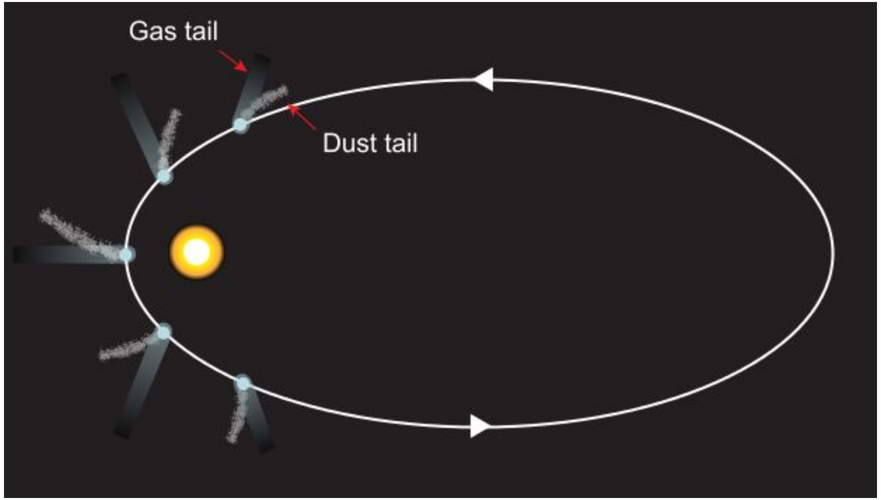
Hence in a medium in which waves are propagated there is a pressure in the direction normal to the waves, and numerically equal to the energy in unit of volume.

It is probable that a much greater energy of radiation might be obtained by means of the concentrated rays from an electric lamp. Such rays falling on a thin metallic disc, delicately suspended in a vacuum, might perhaps produce an observable mechanical effect.

(1873) Maxwell - Theoretical prediction of pressure due to light beam absorption or reflection .

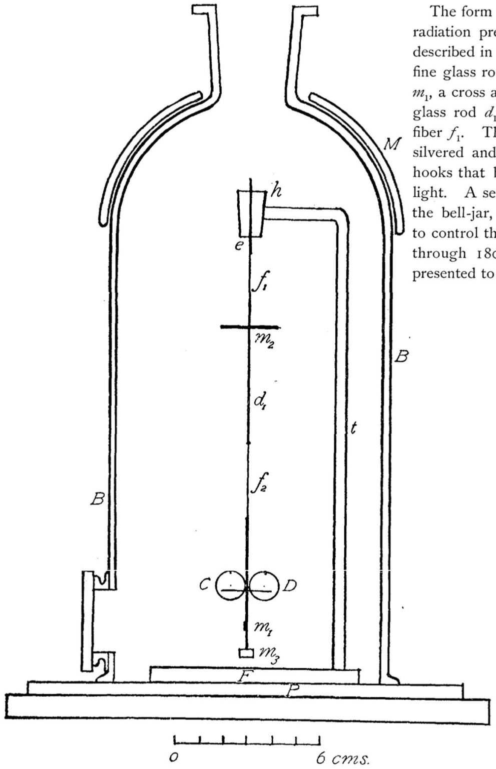
J. C. Maxwell, A Treatise on Electricity and Magnetism (1st Edition), II. 391. Oxford, (1873)

Mechanical Effects of Light



(1619) Kepler - Observation of solar radiation pressure on comet tails.

The form of suspension of the torsion balance, used to measure radiation pressure in the present study, is seen in Fig. 2 and is described in detail in our preliminary paper. Its essential parts are a fine glass rod *ab* to which was attached a weight m_3, a scale mirror m_1, a cross arm *c* holding the mirrors *C* and *D*; a quartz fiber f_2; a glass rod d_1 to which was attached a small magnet m_2 and a silk fiber f_1. The cover glasses *C* and *D* which served as vanes were silvered and brilliantly polished and were so hung on the small hooks that both silver or both glass faces were presented to the light. A semicircular magnet *M*, fitted to the vertical curvature of the bell-jar, was used to direct the suspended magnet m_2 and thus to control the zero position of the torsion balance. By turning *M* through 180°, the opposite faces of the vanes *C* and *D* could be presented to the light.



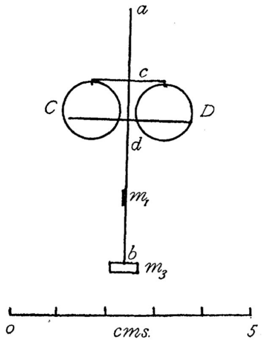
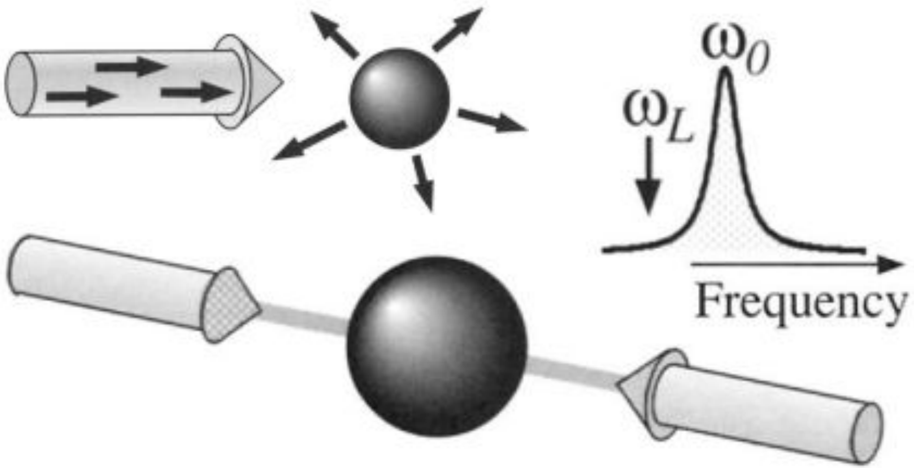


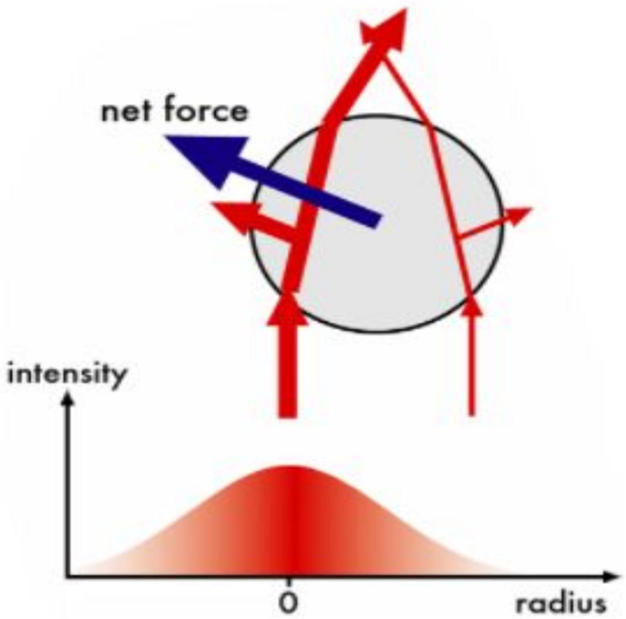
Fig. 2.

(1903) Nichols and Hull - semi-quantitative measurement of radiation pressure

E.F. Nichols and G.F. Hull, The Pressure due to Radiation, The Astrophysical Journal, Vol. 17 No.5, p. 315-351 (1903)

Mechanical Effects of Light





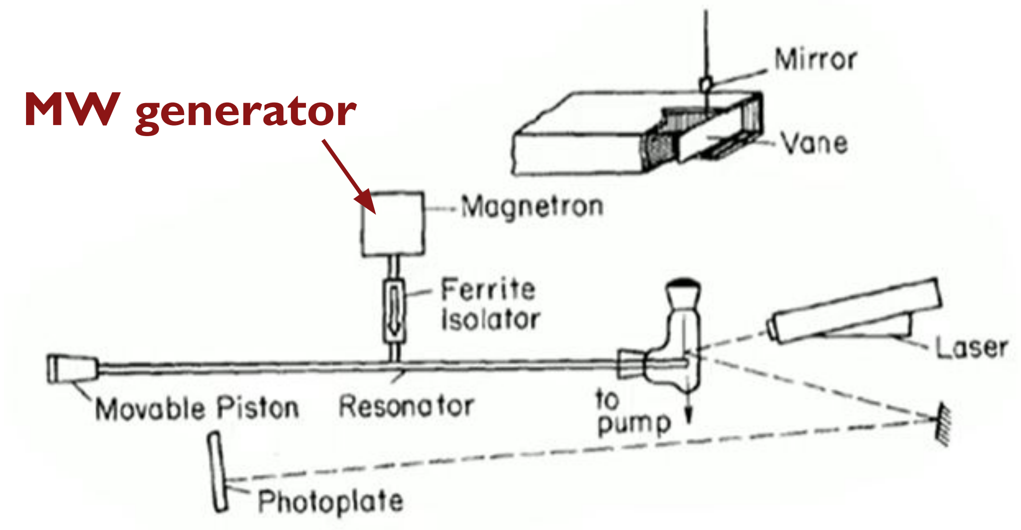
Doppler Cooling - mechanical damping by resonant absorption of blue-shifted light.

Optical Tweezers - refractive effect that traps particles on beam center.

-A. Ashkin: Acceleration and trapping of particles by radiation pressure, Phys. Rev. Lett. 24:156-159 (1970)
-Hänsch, T.W., and A.L. Schawlow. "Cooling of Gases by Laser Radiation." Optics Communications, vol. 13, no. 1, (1975)
-Suter, Dieter. The Physics of Laser-Atom Interactions. Cambridge University Press, (1997)

Radiative Effects on a Mechanical Resonator

MW generator



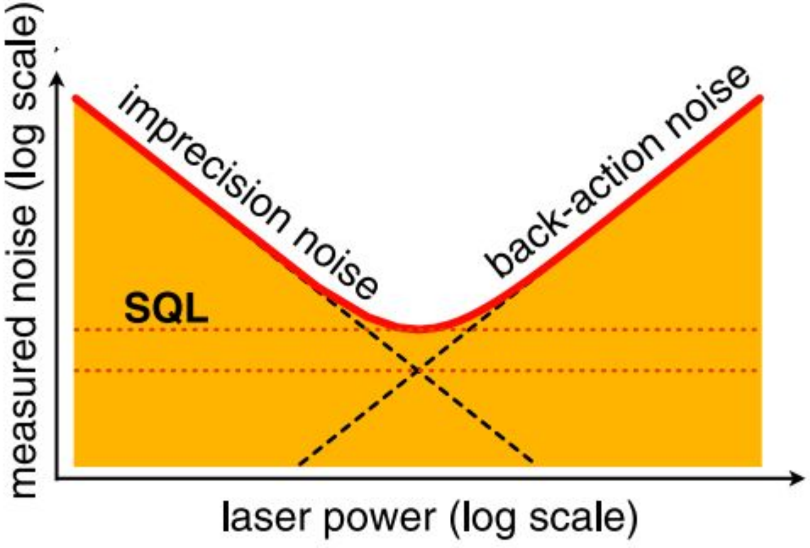
by coupling to electromagnetic resonator.

- Rectangular waveguide cavity
- Movable mirror on one end of cavity parametrically couples MW field to mechanical oscillations.
- Observed increased (decreased) oscillator damping when radiation tuned to left (right) of cavity resonance.

Braginsky, V. B., A. B. Manukin, and M. Y. Tikhonov, Sov. Phys. JETP 31, 829, (1970)

Standard Quantum Limits on Interferometry

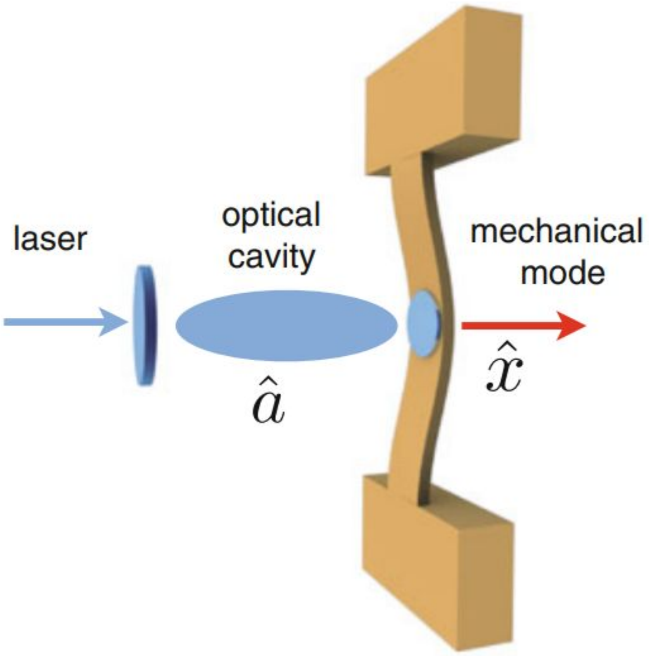
Optimal sensitivity when both noise sources are equal - highest precision allowed by Heisenberg uncertainty.*



Caves, Carlton M, Physical Review Letters, vol. 45, no. 2, (1980)
Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Heurs, M. Philosophical Transactions of the Royal Society A vol. 376, no. 2120, (2018)

* (without using squeezed states)

Canonical Optomechanical System



$$\hat{H} = \hbar\omega_C \hat{a}^\dagger \hat{a} + \hbar\omega_M \hat{b}^\dagger \hat{b} - \hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)$$

Need to include effect of **laser driving.**

$$\hat{H}_{drive} = i\hbar\sqrt{\kappa_{ex}}\alpha_{in}(\hat{a}e^{i\omega_L t} - \hat{a}^\dagger e^{-i\omega_L t})$$

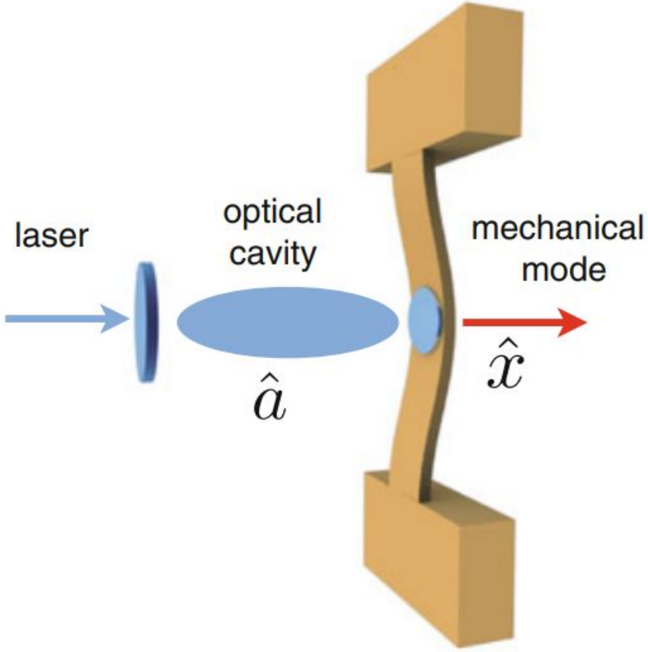
$$\hat{H} = -\hbar\Delta\hat{a}^\dagger\hat{a} + \hbar + \hbar\omega_M \hat{b}^\dagger \hat{b} - \hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) + \hat{H}_{drive}$$

Cavity resonance is expressed in terms of laser detuning.

Where $\Delta = \omega_L - \omega_C$, $\hat{H}_{drive} = i\hbar\sqrt{\kappa_{ex}}\alpha_{in}(\hat{a}^\dagger - \hat{a})$

Markus Aspelmeyer et al., Rev. Mod. Phys.,Vol. 86, No. 4, (2014)

Optical mean-field Linearization



$$\hat{H}_{int} = -\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)$$

Strong laser driving allows us to split radiation into average cavity amplitude plus fluctuations

$$\hat{a} = \bar{\alpha} + \delta\hat{a}$$

$$\hat{H}_{\text{int}} = -\hbar g_0(\bar{\alpha} + \delta\hat{a})^\dagger(\bar{\alpha} + \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$

Average force due to radiation pressure

$$-\hbar g_0|\bar{\alpha}|^2(\hat{b} + \hat{b}^\dagger) = -\bar{F}\hat{x}$$

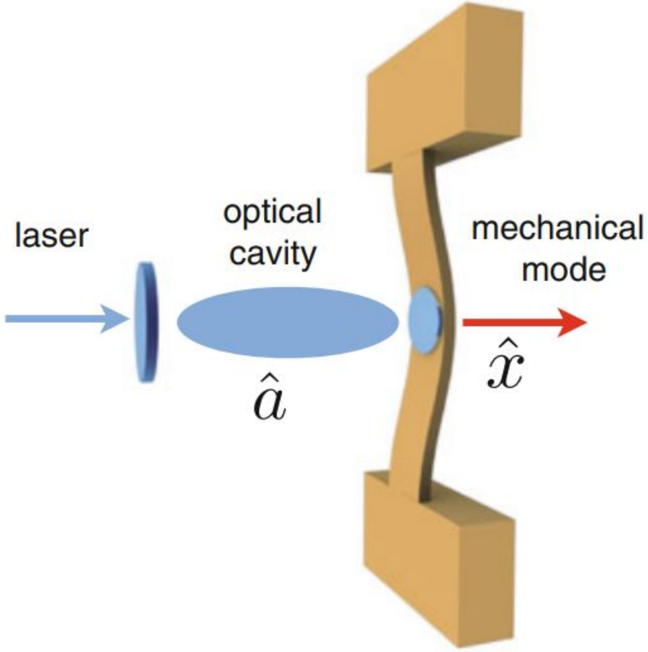
Leads to static shift in detuning and mechanical equilibrium.

$$\Delta_{\text{new}} \equiv \Delta_{\text{old}} + G\delta\bar{x}$$

$$\delta\bar{x} = \bar{F}/m_{eff}\omega_M^2$$

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)

Optical mean-field Linearization



$$\hat{H}_{int} = -\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)$$

Strong laser driving allows us to split radiation into a classical average cavity amplitude plus fluctuations

$$\hat{a} = \bar{\alpha} + \delta\hat{a}$$

$$\hat{H}_{\text{int}} = -\hbar g_0 (\bar{\alpha} + \delta\hat{a})^\dagger (\bar{\alpha} + \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$

Terms sublinear in photon amplitude are omitted

$$-\hbar g_0 \delta\hat{a}^\dagger \delta\hat{a}$$

$$\Rightarrow \hat{H}^{(lin)}_{int} = -\hbar g_0 |\bar{\alpha}| (\delta\hat{a}^\dagger + \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$

Remaining interaction is quadratic, **linear coupling** between oscillators

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
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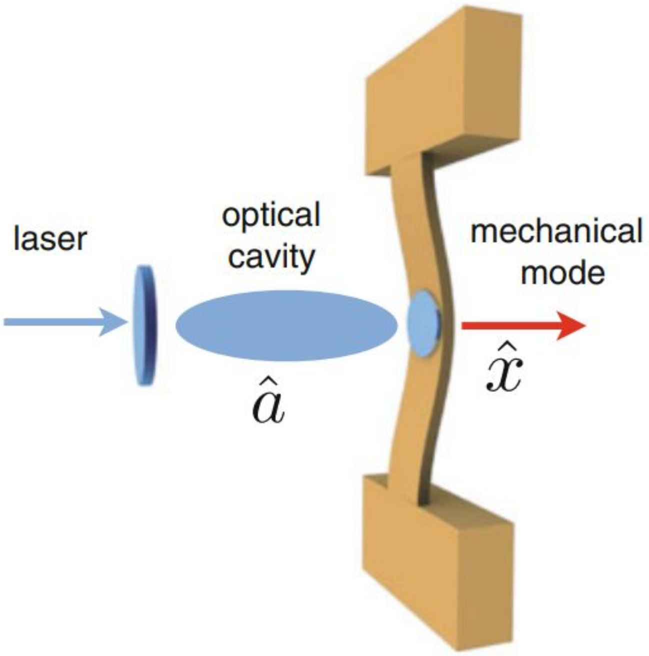
Canonical Optomechanical System

Detuning $\Delta = \omega_L - \omega_C$



$$\hat{H} = \hbar\omega_C \hat{a}^\dagger \hat{a} + \hbar\omega_M \hat{b}^\dagger \hat{b} - \hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)$$

Need to include effect of **laser driving** (rotating frame transformation)

$$\hat{H} = -\hbar\Delta \hat{a}^\dagger \hat{a} + \hbar\omega_M \hat{b}^\dagger \hat{b} \underbrace{- \hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger)}$$

Nonlinear Hamiltonian produces complicated dynamics

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)

Canonical Optomechanical System

Assume cavity dynamics can be described by classical steady-state field + fluctuations

$$\hat{a} = \bar{\alpha} + \delta\hat{a}$$

$$\hat{H}_{int} = -\hbar g_0 \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) \Rightarrow \hat{H}_{\mathrm{int}} = -\hbar g_0 (\bar{\alpha} + \delta\hat{a})^\dagger (\bar{\alpha} + \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$

Only keep operators that are linear in $\bar{\alpha}$

$$\Rightarrow \hat{H}_{int}^{(lin)} = -\hbar g_0 |\bar{\alpha}| (\delta\hat{a}^\dagger + \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$

Remaining interaction is quadratic, **linear coupling** between oscillators

Tunable optomechanical coupling with increased laser power! $g = g_0|\bar{\alpha}|$

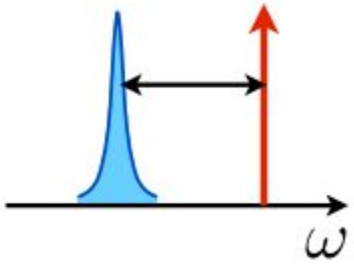
Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)

Quantum Optomechanical Regimes

$$\hat{H}_{int}^{(lin)} = -\hbar g_0 |\bar{\alpha}| (\delta\hat{a}^\dagger + \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$

Depending on detuning, can use rotating-wave approximation to select resonant terms.

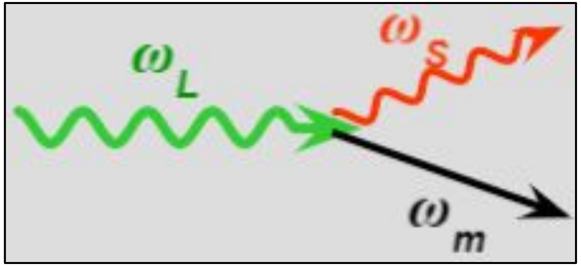
- **Joint excitation of optical and mechanical modes.**
- **Strong correlations between photon and phonon number.**



$$\Delta = +\Omega_{\mathrm{m}}$$

squeezer
(entanglement)

$$\delta\hat{a}^\dagger\hat{b}^\dagger + \hat{b}\delta\hat{a}$$



Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
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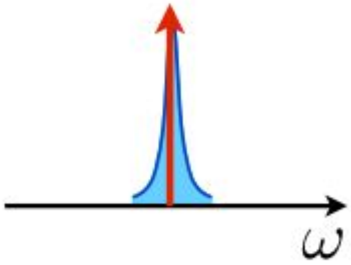
Quantum Optomechanical Regimes

$$\hat{H}_{int}^{(lin)} = -\hbar g_0 |\bar{\alpha}| (\delta\hat{a}^\dagger + \delta\hat{a})(\hat{b} + \hat{b}^\dagger)$$

Depending on detuning, can use rotating-wave approximation to select resonant terms.



$$\Delta = 0$$

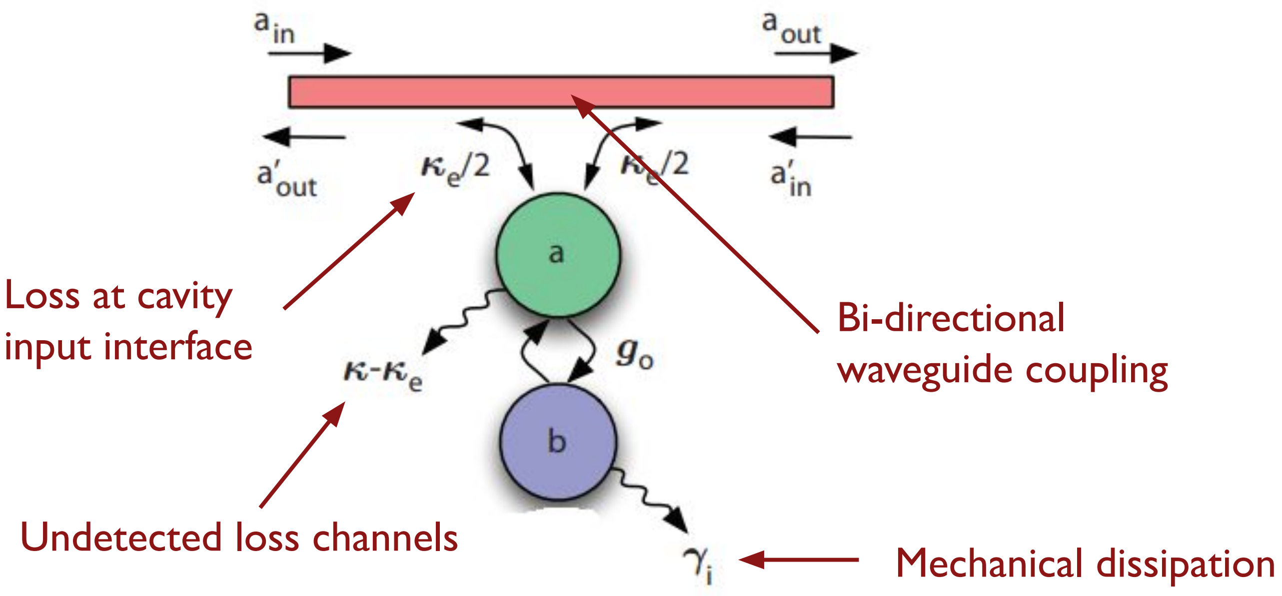
QND

$$\hat{x}_a \hat{x}_b$$

- **Precision Interferometry of mechanical motion.**
- **Potential for back-action evading mechanical measurements, beating SQL.**

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)

Input-Output Analysis of Damped Fields



Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)
Safavi-Naeini, Amir H, et al. New Journal of Physics, vol. 15, no. 3, (2013)

Input-Output Analysis of Damped Fields

$$\dot{a}(t) = -\Big(i\Delta + \overbrace{\frac{\kappa}{2}}^{\text{Cavity decay rate}}\Big)\hat{a} - \underbrace{ig\hat{a}(\hat{b}^\dagger + \hat{b})}_{\text{Position dependent detuning}} - \sqrt{\kappa_e/2}\,\hat{a}_{\mathrm{in}}(t) - \overbrace{\sqrt{\kappa'}\hat{a}_{\mathrm{in,i}}(t)}^{\text{Intrinsic optical loss channel}}$$

Coupled differential equations

Noise input channels

$$\dot{\hat{b}}(t) = -\Big(i\omega_{m0} + \underbrace{\frac{\gamma_i}{2}}_{\text{Resonator decay rate}}\Big)\hat{b} - \underbrace{ig\hat{a}^\dagger\hat{a}}_{\text{Amplitude dependent detuning}} - \sqrt{\gamma_i}\hat{b}_{\mathrm{in}}(t)$$

$$\Delta = \omega_o - \omega_L$$

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)
Safavi-Naeini, Amir H, et al. New Journal of Physics, vol. 15, no. 3, (2013)

Input-Output Analysis of Damped Fields

- **Linearize equations of motion about mean-field amplitude** $\hat{a} = \bar{\alpha} + \delta\hat{a}$
- **Take Fourier transform of operators to solve coupled dynamics** $\hat{A}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\ e^{i\omega t} \hat{A}(t)$

$$\hat{a}(\omega) = \frac{-\sqrt{\kappa_e/2}\hat{a}_{\mathrm{in}}(\omega) - \sqrt{\kappa'}\hat{a}_{\mathrm{in,i}} - iG(\hat{b}(\omega) + \hat{b}^\dagger(\omega))}{i(\Delta - \omega) + \kappa/2}$$

$$\hat{b}(\omega) = \frac{-\sqrt{\gamma_i}\hat{b}_{\mathrm{in}}(\omega)}{i(\omega_{m0} - \omega) + \gamma_i/2} - \frac{iG(\hat{a}(\omega) + \hat{a}^\dagger(\omega))}{i(\omega_{m0} - \omega) + \gamma_i/2}$$

$$\Delta = \omega_o - \omega_L$$

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)
Safavi-Naeini, Amir H, et al. New Journal of Physics, vol. 15, no. 3, (2013)

Input-Output Analysis of Damped Fields

❖ **Combine equations to find operator describing all mechanical fluctuations.**

$$\hat{b}(\omega) = \frac{-\sqrt{\gamma_i}\hat{b}_{\text{in}}(\omega)}{i(\omega_m - \omega) + \gamma/2} + \dots$$

$$\frac{iG}{i(\Delta - \omega) + \kappa/2} \frac{\sqrt{\kappa_e/2}\hat{a}_{\text{in}}(\omega) + \sqrt{\kappa'}\hat{a}_{\text{in},1}(\omega)}{i(\omega_m - \omega) + \gamma/2} + \dots$$

$$\frac{iG}{-i(\Delta + \omega) + \kappa/2} \frac{\sqrt{\kappa_e/2}\hat{a}^\dagger_{\text{in}}(\omega) + \sqrt{\kappa'}\hat{a}^\dagger_{\text{in},i}(\omega)}{i(\omega_m - \omega) + \gamma/2},$$

$$\Delta = \omega_o - \omega_L$$

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. “Optomechanics and Quantum Measurement.” (2020)
Safavi-Naeini, Amir H, et al. New Journal of Physics, vol. 15, no. 3, (2013)

Input-Output Analysis of Damped Fields

- **Combine equations to find operator describing all mechanical fluctuations.**

$$\delta\omega_m = |G|^2 \mathrm{Im}\left[\frac{1}{i(\Delta - \omega_m) + \kappa/2} - \frac{1}{-i(\Delta + \omega_m) + \kappa/2}\right]$$

$$\hat{b}(\omega) = \frac{-\sqrt{\gamma_i}\hat{b}_{\mathrm{in}}(\omega)}{i(\omega_m - \omega) + \gamma/2} + \dots$$

$$\frac{iG}{i(\Delta - \omega) + \kappa/2} \frac{\sqrt{\kappa_e/2}\hat{a}_{\mathrm{in}}(\omega) + \sqrt{\kappa'}\hat{a}_{\mathrm{in},1}(\omega)}{i(\omega_m - \omega) + \gamma/2} + \dots$$

$$\frac{iG}{-i(\Delta + \omega) + \kappa/2} \frac{\sqrt{\kappa_e/2}\hat{a}^{\dagger}_{\mathrm{in}}(\omega) + \sqrt{\kappa'}\hat{a}^{\dagger}_{\mathrm{in},i}(\omega)}{i(\omega_m - \omega) + \gamma/2},$$

$$\omega_m = \omega_{m0} + \delta\omega_m$$

"Optical Spring" frequency shift based on detuning, drive amplitude.

$$\Delta = \omega_o - \omega_L$$

Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)
Safavi-Naeini, Amir H, et al. New Journal of Physics, vol. 15, no. 3, (2013)

Input-Output Analysis of Damped Fields

- **Combine equations to find operator describing all mechanical fluctuations.**

$$\gamma_{\rm OM} = 2|G|^2 \mathrm{Re}\left[\frac{1}{i(\Delta - \omega_m) + \kappa/2} - \frac{1}{-i(\Delta + \omega_m) + \kappa/2}\right]$$

$$\hat{b}(\omega) = \frac{-\sqrt{\gamma_i}\hat{b}_{\rm in}(\omega)}{i(\omega_m - \omega) + \gamma/2} + \dots$$
$$\frac{iG}{i(\Delta - \omega) + \kappa/2}\,\frac{\sqrt{\kappa_e/2}\hat{a}_{\rm in}(\omega) + \sqrt{\kappa'}\hat{a}_{{\rm in},1}(\omega)}{i(\omega_m - \omega) + \gamma/2} + \dots$$
$$\frac{iG}{-i(\Delta + \omega) + \kappa/2}\,\frac{\sqrt{\kappa_e/2}\hat{a}^\dagger_{\rm in}(\omega) + \sqrt{\kappa'}\hat{a}^\dagger_{{\rm in},i}(\omega)}{i(\omega_m - \omega) + \gamma/2},$$

$$\gamma = \gamma_i + \gamma_{\rm OM}$$

"Optomechanical damping"

$$\Delta = \omega_o - \omega_L$$

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Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)
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Resolved-sideband Laser Cooling

- **Optomechanical damping set by net transition rate to lower sideband.**

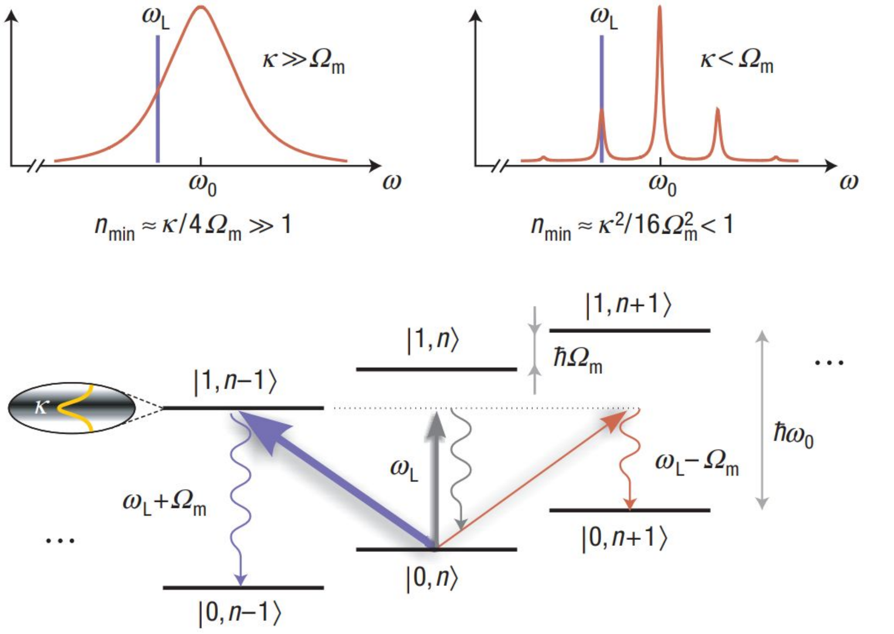
$$\Gamma_{\text{opt}} = A^- - A^+$$

- **Fourier Analysis of Quantum Langevin equations leads to optimal rate when** $\omega_m/\kappa > 1$

$$\Gamma_{\text{opt}}\big|_{\kappa \ll \Omega_m} = 4\bar{n}_{\text{cav}}\frac{g_0^2}{\kappa} = \frac{4g^2}{\kappa}$$

- **Steady state phonon number limited by thermal bath, sideband resolution, and OM coupling.**

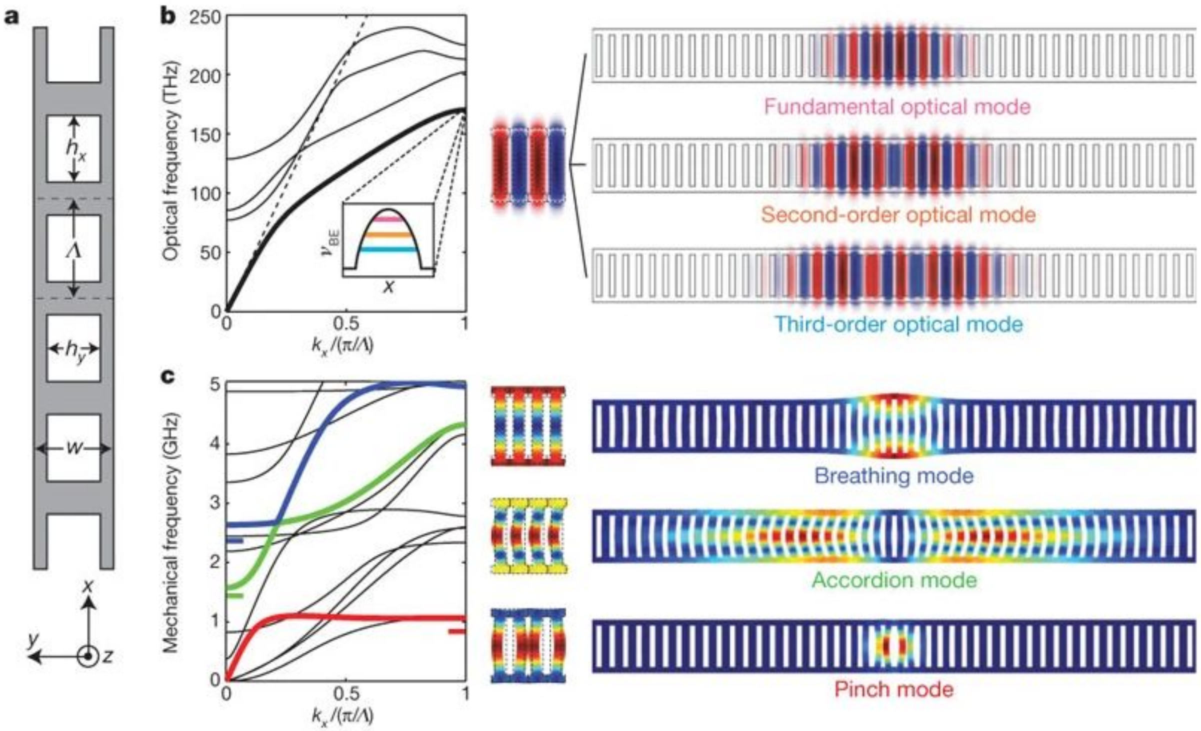
$$\bar{n}_{\min} = \left(\frac{\kappa}{4\Omega_m}\right)^2 \qquad \bar{n}_f = \frac{\Gamma_{\text{opt}}\bar{n}_{\min} + \Gamma_m\bar{n}_{\text{th}}}{\Gamma_{\text{opt}} + \Gamma_m}$$



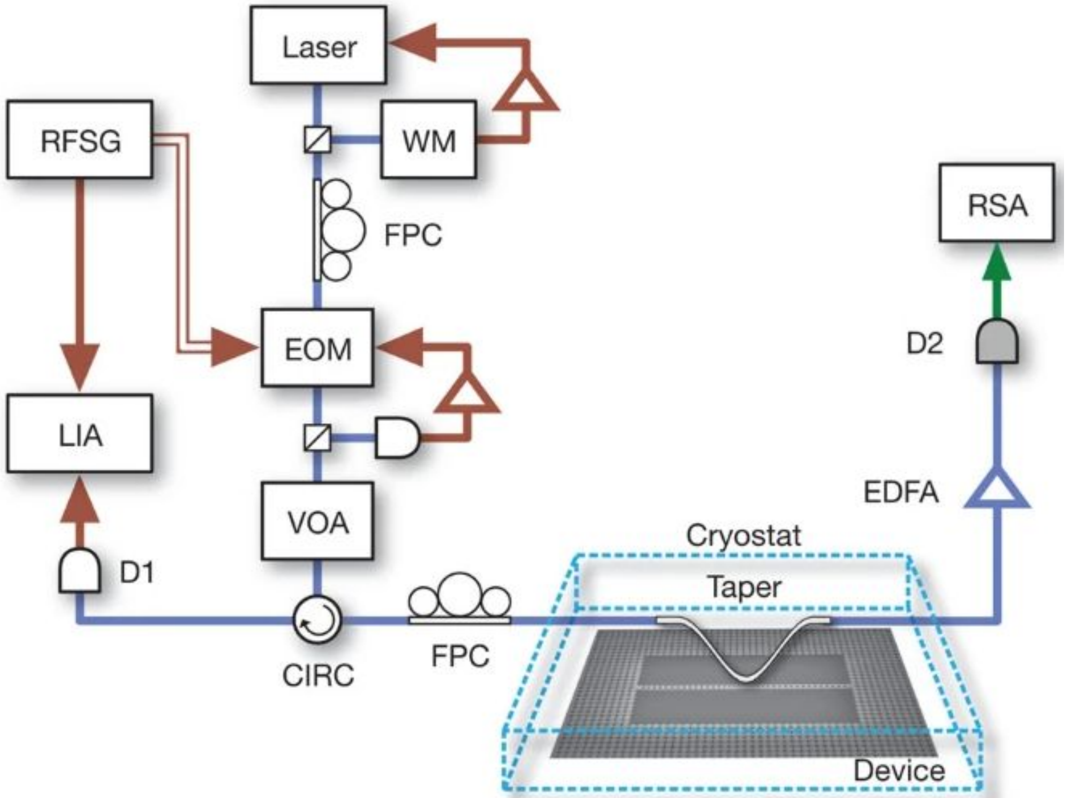
Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
Clerk, Aashish A. "Optomechanics and Quantum Measurement." (2020)

Optomechanical Crystal Nanobeam Design

- **Co-localized optical and mechanical modes** due to quadratic "chirp" defect
- Mechanical modes are chosen such that **induced displacement strongly couples to optical resonance**
- Effective mass set by mechanical mode volume, ~100 fg

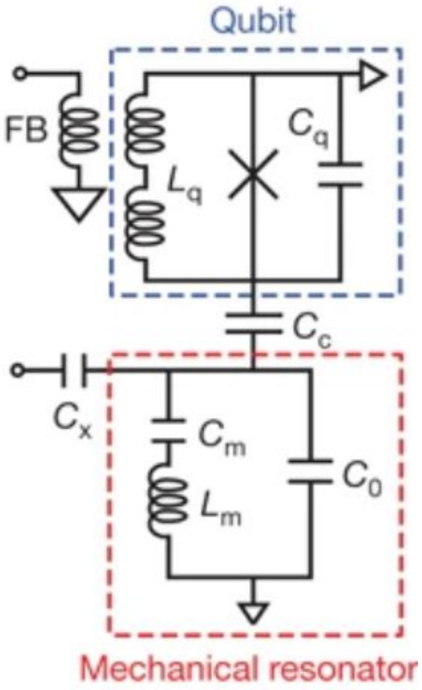


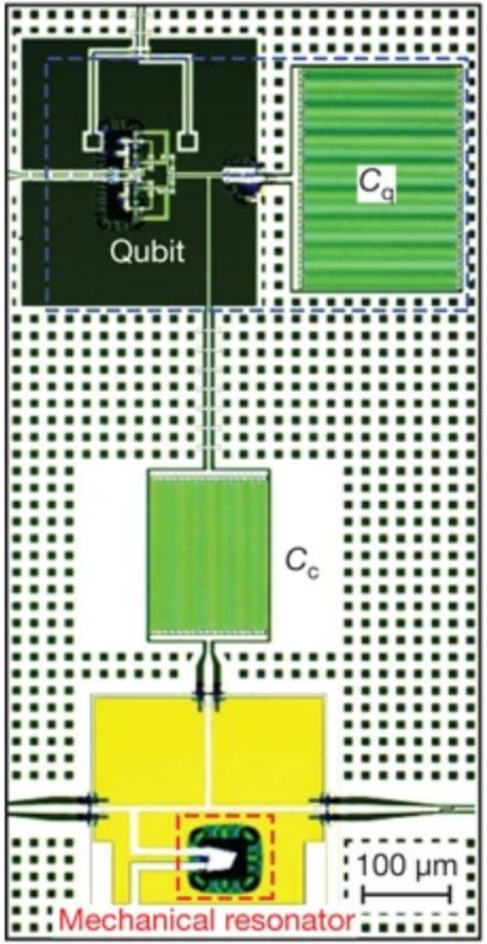
Chan et al. Experimental Design



O'Connell et al. Experimental Design

Depending on detuning, can use rotating-wave approximation to select resonant terms.





Markus Aspelmeyer et al., Rev. Mod. Phys., Vol. 86, No. 4, (2014)
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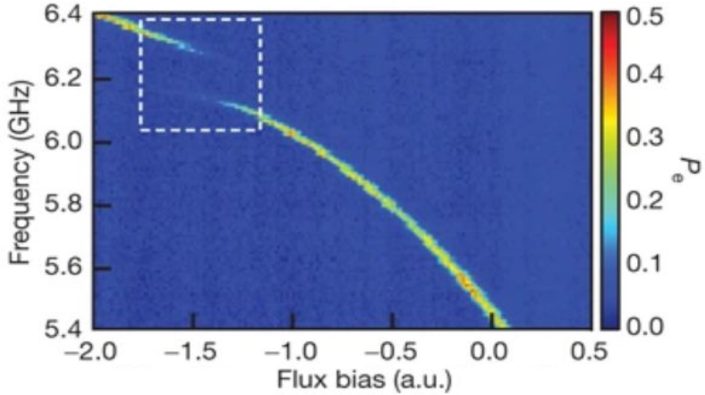
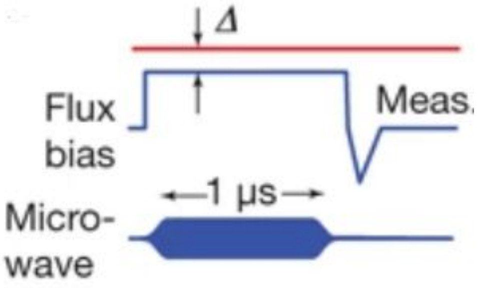
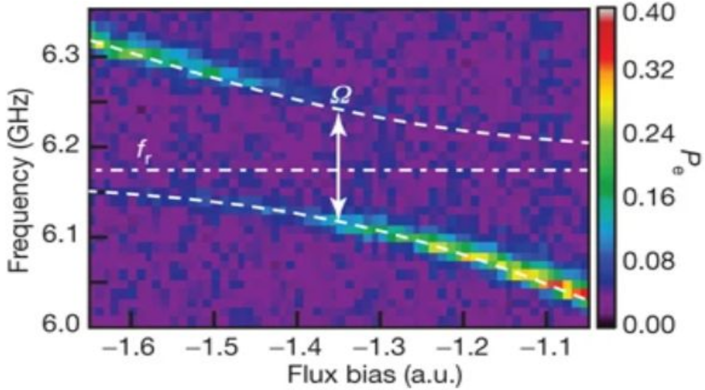
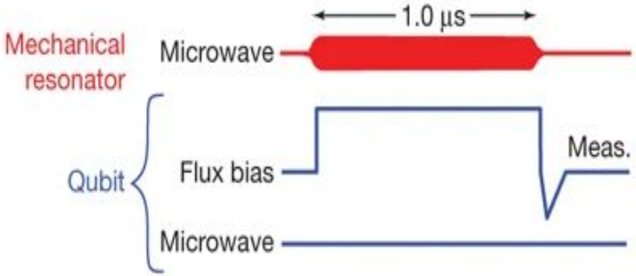
Coupled Qubit-Resonator Spectroscopy

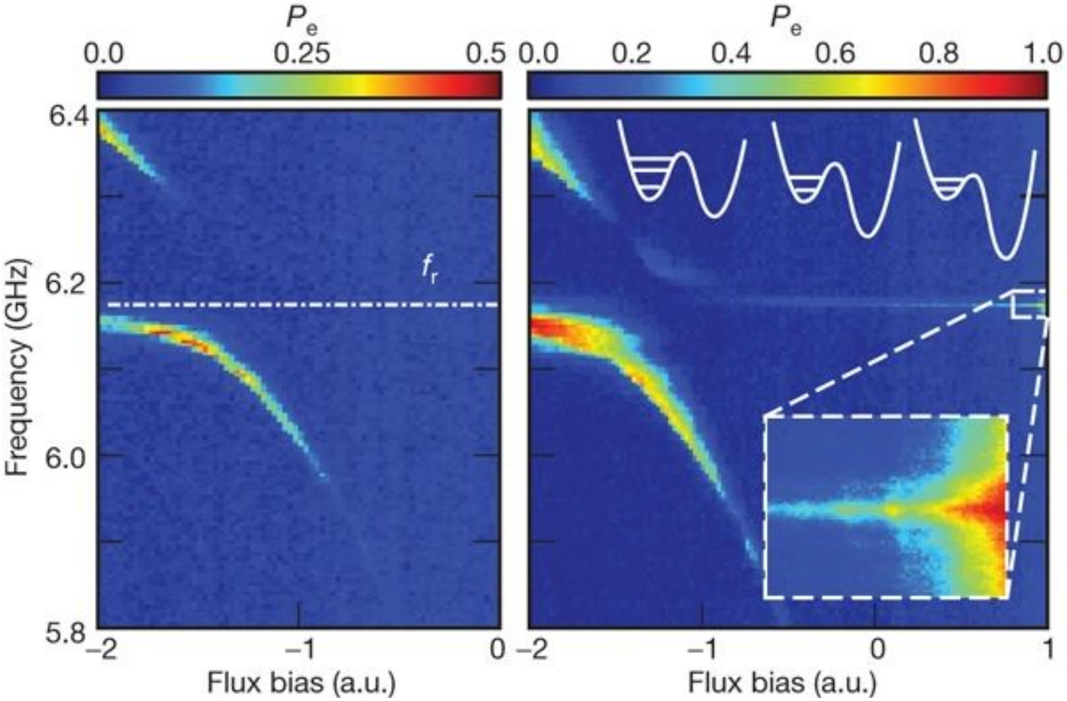






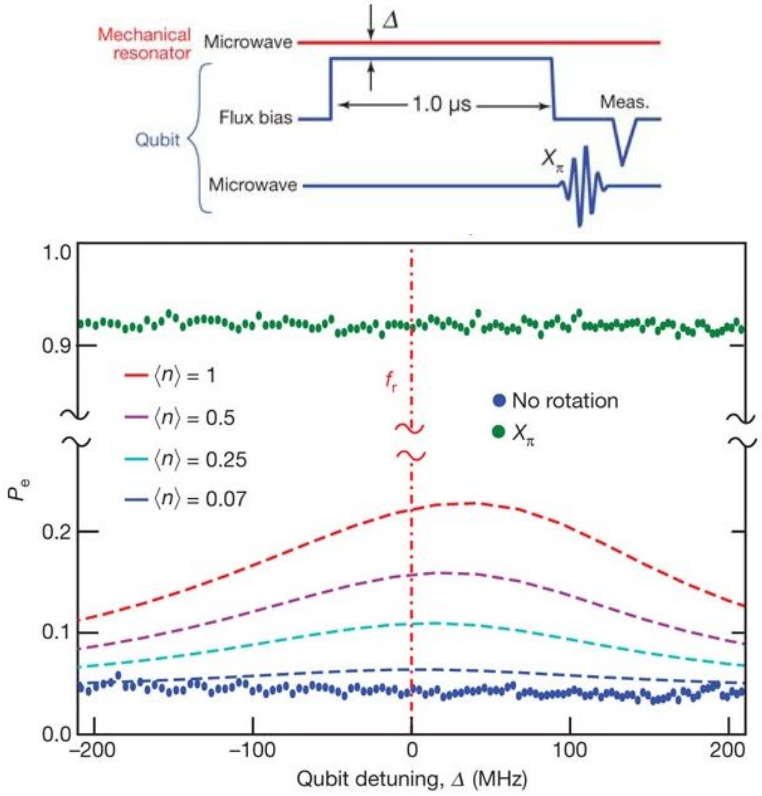
Coupled Qubit-Resonator Spectroscopy





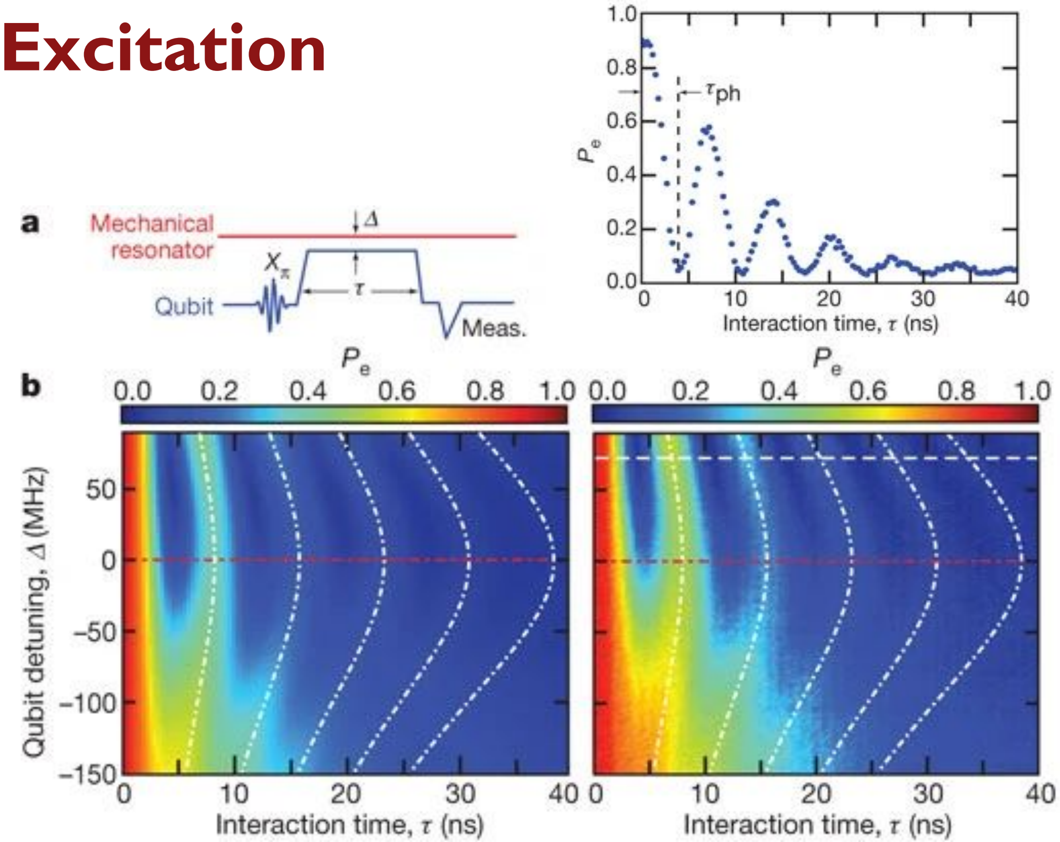
Mechanical Thermometry with a Qubit

Depending on detuning, can use rotating-wave approximation to select resonant terms.



Qubit-Resonator Single Excitation Rabi Oscillations

Depending on detuning, can use rotating-wave approximation to select resonant terms.



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Characterizing the Electromechanical Interaction

Depending on detuning, can use rotating-wave approximation to select resonant terms.

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Discussion on Experimental Results

Depending on detuning, can use rotating-wave approximation to select resonant terms.

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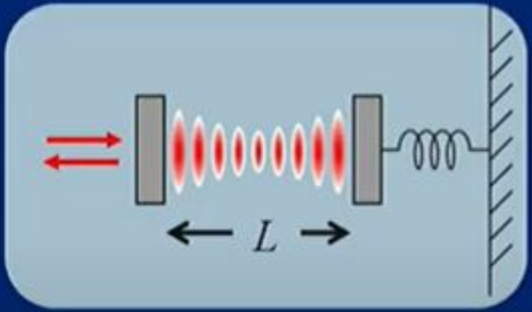
Opto-mechanics to Electro-mechanics

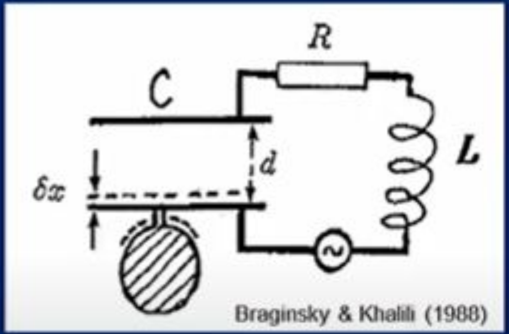
$$\hat{H}_I = \hbar g_0 a^\dagger a (b^\dagger + b)$$

Fabry-Perot Cavity

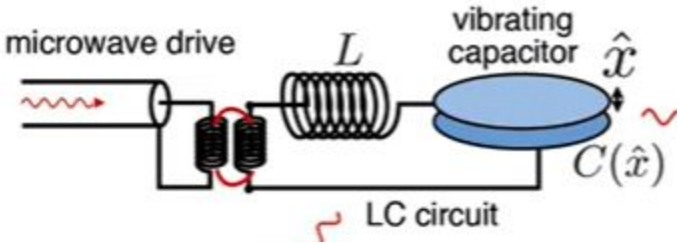


$$g_0 = \omega_c \frac{x_{zp}}{L}$$

LC Transducer Circuit



$$g_0 = \omega_c \frac{x_{zp}}{2d}\left(\frac{C_m}{C_{total}}\right)$$


microwave drive
L
vibrating
capacitor
$\hat{x}$
$C(\hat{x})$
LC circuit

Mechanical oscillators in technology

Quartz tuning forks, MEMS, atomic force microscope,

- piezomechanical oscillators used for filtering (high Q at room temperature)
    - Higher Q than electrical resonant LC circuits
- AFM and magnetic force detection using magnetic tip on cantilever

- Can be quantized to canonical QHO form, requires $kT < hW$
    - Can use radiation pressure to cool to low enough temperatures
- Quantum Control is challenging because
    - Occupation number $n \sim kT/hW >> 1$
    - Displacement sensitivity on order of zero point motion needed $x_{zpf} = \mathrm{sqrt}(h/(2mW))$
    -