Lab Report: Quantum Interference and Entanglement

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Abstract

This experiment demonstrates a violation of Bell's Inequality to observe quantum entanglement's non-local properties in nature. Preparation of a Bell State of photons is done using beta Barium Borate (BBO) crystals that down-convert violet laser light into pairs of infrared photons with entangled polarization states. Correlations between entangled photons are measured by comparing the rate of coincident photon detection at different measurement bases determined by half-wave plates in front of the photon detectors. This setup allows us to test the CHSH variant of Bell's Inequality, finding $S = 2.189 \pm .065$ in clear violation of the inequality and providing significant evidence against local hidden variable theories as a valid description of reality.

Introduction

In formulating a physical theory it seems natural to keep the theory in accordance with reality as we all experience it. Einstein, Podolsky, and Rosen gave this call to physical intuition as evidence of quantum mechanics' incompleteness in mirroring reality, constructing the first notion of quantum entanglement as a contradiction of the principle of locality. Their thought experiment [1] consisted of two systems that are made to interact for a period of time, after which one could describe the joint wavefunction of the two systems at subsequent times. Quantum mechanics tells us that measuring some physical quantity of one of the systems will collapse the wavefunction into an eigenstate corresponding to the eigenvalue that our measurement produces. But when we consider the state of the system that we didn't measure, we realize that two different measurements on the first system will result in different wavefunctions describing the state of the second system long after the systems have stopped interacting, even if this distance was light-years away. This leaves only two options: either quantum mechanics is not a sufficient description of reality, or some nonlocal action beyond our current understanding is allowing the first system to influence the other.

The work of Einstein et al. led the way to extensive research on alternative models to account for this apparent paradox, and it was John S. Bell who managed to show [2] with just the assumptions of local realism that no local hidden variable theory could agree with all theoretical predictions of quantum mechanics. By constructing a hidden variable theory to model a similar scenario as the thought experiment presented above, Bell showed that any attempts at including a hidden variable in quantum theory will necessarily exhibit nonlocal behavior, breaking certain inequalities related to correlations between measurements that bound local realistic hidden variable theories. This result implies that experimental evidence of nature behaving quantum mechanically will rule out any local hidden variable theory from being a valid description of reality. Many experiments have been done since Bell's result that show violations of Bell's inequality in nature, providing support for quantum mechanics' validity but also raising ongoing debate on the underlying physical laws behind these results.

In our experiment the quantum-mechanical system under study is the polarization state vector of entangled photons, and the measurement bases we vary are determined by wave plates in front of photon detectors, which are used to measure the correlations between photons at different wave plate settings. The actual quantity that we will calculate is based on a variation of Bell's original result called the CHSH Inequality, first violated by Aspect [3] in 1982. With a successful violation of this inequality, we can rule out local hidden variable theories from accurately describing reality.

Theory

The central object of this experiment is the bipartite entangled Bell state of photons, which can be described as $|\Psi\rangle = \frac{1}{\sqrt{2}}(|hh\rangle + |vv\rangle)$, where $|h\rangle$ and $|v\rangle$ form an orthonormal basis describing the horizontal and vertical polarization orientations. Notice that it is not possible to separate the joint state as the tensor product of individual photon states, and so just as imagined in the EPR paradox a measurement of one of the photons will result in a collapse of the wavefunction into either the $|hh\rangle$ or $|vv\rangle$ states. The important fact to note is that regardless of any measurement done on one of the photons, there is no way for any change to be noticed in the other photon by measuring it. We can quantify this uncertainty in the second photon's state By taking the partial trace of the density matrix formed by a Bell state. To ease in calculation we will denote each photon's individual state component as either a or b.

$$\rho_a = \mathrm{Tr}_b\left(\frac{1}{2}(|h_a h_b\rangle \langle h_a h_b| + |h_a h_b\rangle \langle v_a v_b| + |v_a v_b\rangle \langle h_a h_b| + |v_a v_b\rangle \langle v_a v_b|)\right)$$

$$\rho_a = \sum_{i\in\{h_b, v_b\}} \frac{1}{2} \langle i| (|h_a h_b\rangle \langle h_a h_b| + |h_a h_b\rangle \langle v_a v_b| + |v_a v_b\rangle \langle h_a h_b| + |v_a v_b\rangle \langle v_a v_b|) |i\rangle$$

$$\rho_a = \frac{1}{2}(|h_a\rangle \langle h_a| + |v_a\rangle \langle v_a|)$$

The reduced density matrix of photon a shows that the photon is in a maximally mixed state, with equal probability of being either horizontally or vertically polarized. If this photon were measured first with the Bell state intact, the probability of each result would be identical, showing that although the state collapses nonlocally, no information can be communicated by measurements alone.

Now we generalize the bipartite state to the form $|\psi\rangle = \cos\theta_l |hh\rangle + e^{i\phi} \sin\theta_l |vv\rangle$, where the parameters θ_l and ϕ will ultimately be adjusted by optical elements to get a close estimate of a Bell state. The measurement bases of each photon detector is set using half-wave plates with the optical axes at an angle α and β respectively from the horizontal direction. Because the entangled photons have an equal probability of being in either basis state, the probability of detecting a coincident pair of entangled photons is maximized when the wave plate angles are set equal to each other. For orthogonal measurement bases the probability of detecting a coincidence goes to zero, which agrees with the fact that the entangled photons share identical polarizations, so detection of one photon would imply complete attenuation of the other. If we write the horizontal basis vectors of each detector as $|h_a\rangle = \cos\alpha |h\rangle + \sin\alpha |v\rangle$ and $|h_b\rangle = \cos\beta |h\rangle + \sin\beta |v\rangle$, then the probability of detecting two entangled photons collapsing into horizontally polarized light can be found as

$$P_{hh} = |\langle h_a| \langle h_b| |\psi\rangle|^2$$

Which in the general case is a fairly large expression, (the general derivation can be found in Dehlinger and Mitchell) but for the Bell state with $\theta_l = 45°$ and $\phi = 0$ can be simplified to

$$P_{hh} = \frac{1}{2}\cos^2(\beta - \alpha)$$

Notice that the relevant parameter in this formula is the relative angle between the two wave plate settings, with maximum probability when $\alpha = \beta$ and minimum when the angles are orthogonal, as expected. Similar calculations [5] show that $P_{vv} = P_{hh}$ and $P_{vh} = P_{hv} = \frac{1}{2}\sin^2(\beta - \alpha)$. During the course of this experiment we will measure these values experimentally as a means of quantifying the polarization correlation and breaking the Bell inequality.

The CHSH inequality is formally stated as $|S| \leq 2$, where $S = E(a,b) - E(a,b') + E(a',b) + E(a',b')$ and $E(a,b)$ is a measure of the correlation between photons at each detector with wave plate angles set to a and b, respectively. This function is defined as

$$E(\alpha,\beta) = P_{vv} + P_{hh} - P_{vh} - P_{hv} = \cos^2(\beta - \alpha) - \sin^2(\beta - \alpha)$$

$$E(\alpha,\beta) = \cos 2(\beta - \alpha)$$

Because the goal of this experiment is to violate Bell's Inequality by finding $S > 2$, our choice of angles a, b, a', b' should be made to maximize S,

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b') = \cos 2(b-a) - \cos 2(b'-a) + \cos 2(b-a') + \cos 2(b'-a')$$

$$S = \cos 2(b-a) - \cos 2(b'-a) + \cos 2(b-a') + \cos 2(b'-a')$$

Not all angle values will produce an S value violating the CHSH inequality, but by choosing $a = -45°$, $a' = 0$, $b = 22.5°$, and $b' = 67.5°$ we get a theoretical maximum $S = 2\sqrt{2} \approx 2.828$.

Experimental Setup

This experiment is physically divided into two sections: the first for generation and calibration of the Bell state and the second for photon detection and setting of the measurement bases. The 405 nm diode laser

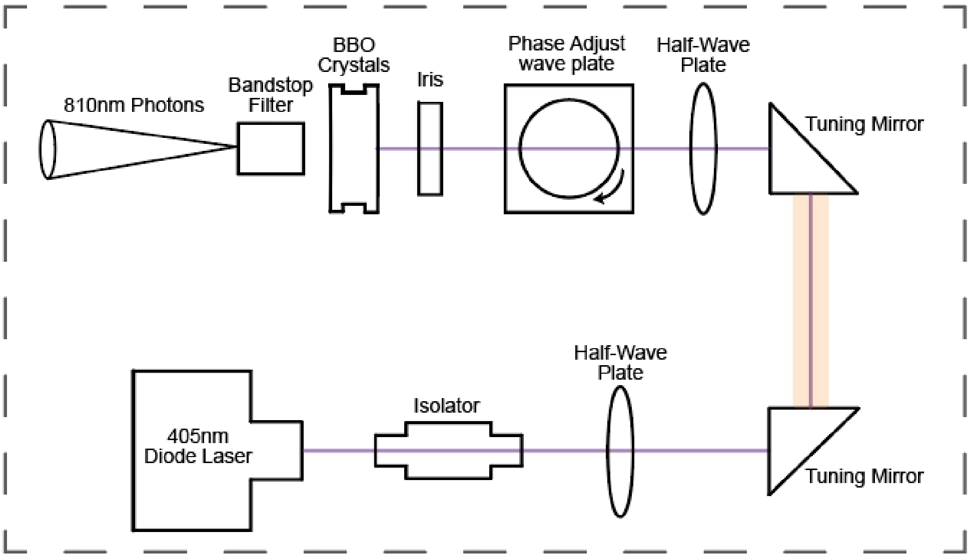

Figure 1: Optical apparatus for Bell State Preparation

acts as our photon source, with an adjustable current level that controls the Laser power. The supply current is limited to 100 mA where the laser power is measured past the isolator at 68 mW. Once we had started recording coincidences we found the laser power could be reduced to approximately 50 mW while still generating a reasonable amount of photon counts. To reduce the intensity of the violet beam after Bell state generation we placed a band-stop filter in front of the BBO crystal aperture, reducing the laser power to the mW range. A plot of the laser power as a function of supply current is provided in the appendix.

Understanding of the polarization state of the laser at different stages of the experiment is crucial to effectively tuning the Bell state parameters. The state of the emergent laser light is highly coherent and horizontally polarized, and the optical isolator works to prevent reflections of the laser from shining back into the laser diode. The isolator is made up of two linear polarizers at each end, oriented horizontally on the side closer to the laser and oriented 45 degrees off from horizontal on the outgoing end. The rotation mechanism is directionally dependent, so any light reflected back into the isolator will turn into vertically polarized light and attenuate completely after passing through the second polarizer. Any vertical component of the polarization will pick up a phase after passing through the tuning mirrors, and so we add a half-wave plate after the isolator to correct the rotation caused by the isolator. The tuning mirrors are used for aligning the laser path so that the beam is centered between the photons detectors and passes through the BBO crystal aperture.

The BBO crystals create entangled photons through a nonlinear optical process known as spontaneous parametric down-conversion (SPDC), where an incident photon is converted into two photons with double the wavelength and thus half the energy. This process also conserves momentum so the outgoing photon pairs will exit from the crystal at the same small angle. These photons will be in the infrared range so they are not visible, but also because the efficiency of the SPDC process is so low, the majority of the photons will remain as part of the violet laser beam that passes through the crystals. The small number of photons that do achieve down-conversion emerge as a cone of infrared light centered on the violet laser. Each detector arm is connected to the optical bench on a railing with an adjustable angle that must be calibrated to have the entangled photon exit angle aligned to the detectors without having stray photons from the violet laser disrupting the measurement counts.

The BBO crystal does have some limitations that need to be addressed. For one they are limited in that only one polarization of light can be down-converted for each crystal, with the outgoing photons each having the same polarization as the incident light. Our experiment thus utilizes two tightly connected BBO crystals and oriented perpendicularly in the horizontal and vertical basis so that the effect of the crystals on an arbitrary polarization state is

$$\cos\theta_l \left|h\right\rangle + e^{i\phi}\sin\theta_l \left|v\right\rangle \Rightarrow \cos\theta_l \left|hh\right\rangle + e^{i\phi}\sin\theta_l \left|vv\right\rangle$$

Another imperfection in the crystals is the presence of a slight relative phase that develops between the horizontally and vertically polarized eigenstates due to slight variations in the birefringent properties of the BBO crystal. This phase shift will decrease our expected coincidence count [4], deviating our results from the theoretical maximum of a pure entangled Bell state.

Creating the Bell state involves careful preparation of the violet laser polarization just before reaching the BBOs in an equal superposition of the horizontal and vertical states, $\theta_l = 45°$. This was done with a half-wave plate placed after the tuning mirrors. To remove any phase that may have developed in the laser up to this point and to cancel out phase shifts that arise from the BBO crystals, we used a piece of birefringent crystal at an angle with the light path that adds a phase to passing light. The necessary angle setting needed to remove any phase was done by trial and error, counting coincidences and varying the angle until a maximum was found.

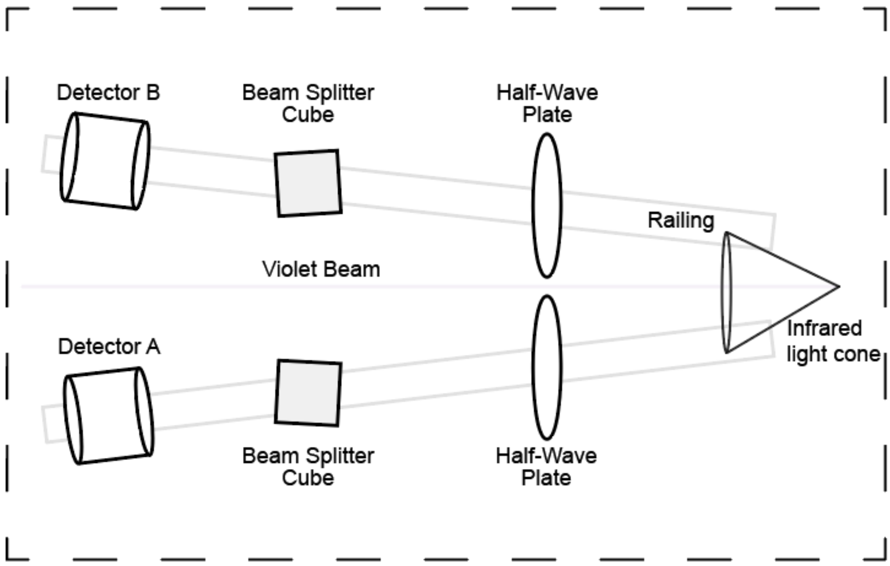

Figure 2: Photon Detection Apparatus. The entangled photons emerge as a cone of infrared light, and the detector arms should be aligned to the same angle as the cone without receiving false photon counts from the violet beam.

Performing the necessary measurements needed in the CHSH Inequality requires a photon detection apparatus that can set measurement bases at varying angles and measure instances of coincident photons hitting both detectors within a nanosecond time window. The detectors record all polarizations of light equally, so we employed a beam splitter cube in front of each detector that reflects vertically polarized light and allows horizontally polarized light to pass through. Now the probability of a detector recording an incident photon with state $|\psi\rangle$ is $P_h = |\langle h| |\psi\rangle|^2$. In the case of the bell state $P_h = 1/2$, and for vertically polarized light no photons should be measured, aside from ambient background light intruding on the experiment. Calibration of the wave plate used to to set the violet beam polarization was done with the beam splitter cubes installed, varying the angle and denoting the maximum photon counts as the zero angle that leaves the pump beam light horizontally polarized. By adding half-wave plates set for 810 nm light in front of each detector and beam splitter cube, we can now set the measurement basis for each detector by rotating the incoming light by some angle, changing what polarization of light will pass and what will reflect away from the detector. For orthogonal angle settings we expect to see no coincidences as predicted by our formulas for the probability of coincidence detection

$$P_{vv} = \frac{1}{2}\cos^2(90^\circ) = 0$$

. The detector angle settings needed for the CHSH experiment can now be set by varying these wave plates and recording the coincidence counts to obtain an experimentally measured value of S as described in the CHSH inequality.

The photon detectors themselves are fiber coupler lenses connected through an optical fiber to avalanche photodiodes. These allow us to record each photon's passage onto the detector as voltages that can be processed by an FPGA Board. Our Labview Program displays the photon counts measured on the two detectors as well as the number of photon detections that occur within a nanosecond time window that we set to 10 ns. This value, denoted by $N(\alpha, \beta)$ tells us the number of coincident photon detections, and by measuring $N(\alpha, \beta)$ we can calculate the correlation $E(\alpha, \beta)$ between detectors A and B as

$$E(\alpha, \beta) = P_{vv} + P_{hh} - P_{vh} - P_{hv} = \frac{N(\alpha, \beta) + N(\alpha_\perp, \beta_\perp) - N(\alpha_\perp, \beta) - N(\alpha, \beta_\perp)}{N_{total}}$$

Data Analysis

Using the photon detection setup, we were able to carefully monitor the parameters of our entangled state of photons and the amount of parasitic ambient light creating false coincidences. Keeping one of the detector wave plates at a constant angle, we vary the other detector angle and keep track of the coincidence counts throughout. Using the formulas derived previously we can predict the expected number of coincidences for any two wave plate settings, and by observing how our measured coincidences vary from expected values we can identify imperfections in the experimental apparatus. The graphs below show our predicted and measured coincidence values for four different fixed angle settings on detector wave plate A, and we see that our experimental values roughly follow the sinusoidal pattern that we predicted. The relative phase between each set of measurements arises from the varying fixed angle α shifting where the maximum and minimum of coincidences will arise. There is a slight phase deviation between our predicted results and the measured ones, which we deduced was a result of imperfect placement of detector

B's arm angle. Trying to center detector B's railing by changing this angle did not improve our results significantly, and so we believe that this imperfection is due to the violet beam alignment creating a small angle in the emergent entangled photon light cone. Rather than repeating our alignment process we decided to leave the slight phase in the interest of preserving the fairly accurate results we already produced.

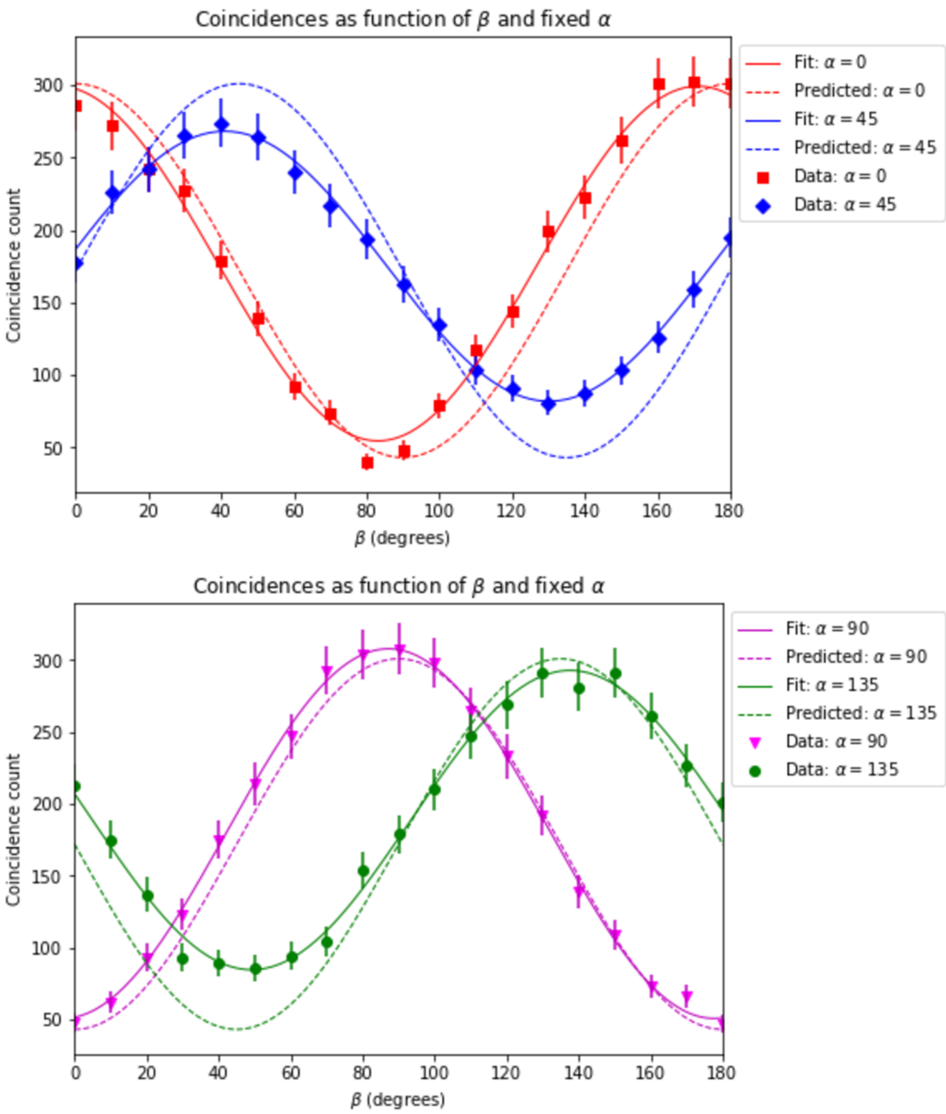

Figure 3: coincidence counts for varied detector B angle β and fixed detector A angle α. The error bars are made by modeling the coincidence counts as a Poisson distribution, in which case the uncertainty $\sigma_N = \sqrt{N}$

There is one significant error in our measured results that arises for fixed angle settings $\alpha = 45°$ and $\alpha = 135°$, with a smaller span of coincidence counts than predicted. These compressed results imply that the preparation of our Bell state is uneven, creating non-zero amplitudes in the $|hv\rangle$ and $|vh\rangle$ states for certain rotation angles. This would account for decreased coincidences at the maximum angle when $\alpha = \beta$ because the amplitudes of the $|hh\rangle$ and $|vv\rangle$ states would have to be lower as a result. This would similarly produce increased coincidences for orthogonal α and β because of false coincidences created by the $|hv\rangle$ and $|vh\rangle$ states. This can also be seen by our calculations of the contrast for each angle setting in the table below, which shows lower results for $\alpha = 45°$ and $\alpha = 135°$ because the transformation induced by these rotations maximizes the populations of $|hv\rangle$ and $|vh\rangle$ states generated. The $\alpha = 0°$ rotation clearly doesn't affect the state and $\alpha = 90°$ swaps the horizontal and vertical states, so we know that we have corrected our Bell state if these contrasts all approach the contrast values at $\alpha = 0°$ and $\alpha = 90°$. An ideal Bell state is invariant under any transformation of its basis vectors, so we know that by tuning the pump beam polarization state we can achieve results closer to our predictions. Observation of this sinusoidal behavior and the narrowing of the coincidences at different angles confirms that the entangled state we are producing is truly a superposition rather than just a mix of $|hh\rangle$ and $|vv\rangle$ states, which would not display this behavior. Instead we would see a sawtooth pattern as we rotate the detector B measurement angle [4], but if this was the case we wouldn't be able to violate the CHSH Inequality.

α(deg)	contrast
0	.766
45	.544
90	.735
135	.544

Table 1: Measurement of contrast $= \frac{\text{max(coincidences) - min(coincidences)}}{\text{max(coincidences) + min(coincidences)}}$ for four fixed detector angles α

The table below shows the calibration of our entangled state from the initial tests (that produced the data above) to the final state we used for our calculation of S in the CHSH Inequality. We can measure the parameters of our Bell state by taking four measurements as described in Dehlinger and Mitchell and using our knowledge of the probability of coincidence detection to infer what these measurements tell us about the superposition state. Measuring the coincidences that we see for orthogonal angle settings $N(0, 90°)$ allows us to estimate an offset created by ambient light, and then adding two measurements $N(0, 0)$ and $N(90, 90)$ then subtracting twice the offset gives us a good estimate of the maximum photon coincidence count. Using these parameters we can measure the angle θ_l that we tuned as close to 45 degrees as possible using the pump beam half-wave plate, as well as the phase ϕ by using the equations found in Dehlinger and Mitchell.

The results for our initial entangled state confirm that there is an uneven population of $|hh\rangle$ and $|vv\rangle$ states, and so we varied the angle of the pump beam half-wave plate and repeated the measurements described above until we arrived at the final calibrated state with coefficients in the table below. In the process of tuning the Bell State we encountered a slightly large phase than we started with, and no rotation of the phase adjust plate improved the results significantly. This phase was still small enough that it did not affect our calculations for violating the CHSH Inequality.

attempt	A_1	A_2	ϕ
1st	.685	.729	.431
final	.712	.702	.456

Table 2: Measurement of Bell State purity before and after tuning for $|\Psi\rangle = A_1 |hh\rangle + e^{i\phi} A_2 |vv\rangle$

With the Bell state finely tuned and the detection apparatus ready, we are able to conduct the necessary measurements to calculate S. Without proper tuning of the Bell state these measurements would not violate the CHSH Inequality, but with the analysis of our entangled photon state above we were confident that we could find $S > 2$. The table below shows our measurements using the optimal angles for maximizing S that we found previously in the theory section. We will find the error in S by solving for the error in each $E(\alpha, \beta)$ expression that makes up S. Every measurement $N(\alpha, \beta)$ of the coincidences can be described by a Poisson distribution with parameter N equal to the number of coincidences measured within that time window, so the uncertainty of a given measurement is $\sqrt{N}$. The error of $E(\alpha, \beta)$ can then be found by summing through the 4 measurement errors making up the function:

$$\sigma_{E(\alpha,\beta)} = \sqrt{\sum_{i=1}^{4} N_i (\frac{dE}{dN_i})^2}$$

$$\sigma_{E(\alpha,\beta)} = \sqrt{(N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp))\frac{4(N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta))^2}{N_{tot}^4} + (N(\alpha_\perp,\beta) + N(\alpha,\beta_\perp))\frac{4(N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp))^2}{N_{tot}^4}}$$

$$\sigma_{E(\alpha,\beta)} = \frac{2}{N_{tot}^2}\sqrt{(N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp))(N(\alpha,\beta_\perp) + N(\alpha_\perp,\beta))^2 + (N(\alpha_\perp,\beta) + N(\alpha,\beta_\perp))(N(\alpha,\beta) + N(\alpha_\perp,\beta_\perp))^2}$$

Using this expression the uncertainty in S can be neatly written as

$$\sigma_S = \sqrt{\sigma^2_{E(a,b)} + \sigma^2_{E(a,b')} + \sigma^2_{E(a',b)} + \sigma^2_{E(a',b')}}$$

With this data and the formulas found previously we calculated $S = 2.189 \pm .065$, well over the bound for any local hidden variable theory $|S| < 2$ and consistent with the fact that the predictions of quantum mechanics occur to be true in nature. This value is far from the predicted maximum of $S_{max} = 2\sqrt{2}$, and we can attribute this lower value to slight imperfections in the experimental setup. As mentioned previously the produced Bell State has a non-zero phase and slight deviations from the ideal parameters, so this would affect our results for certain rotation angles. The half-wave plates used in every step of the experiment are accurate to 1 degree, and because the half-wave plates rotate the polarization state by twice the angle between the optical axis and the state vector, any imperfection in angle placement could affect our results. Additionally, the angle between each detector arm could have been optimized further, but we ran short on time and didn't want to risk affecting other parts of the apparatus by changing the angles again.

α(deg)	β(deg)	$N(\alpha, \beta)$
-45	-22.5	264
-45	22.5	86
-45	67.5	65
-45	112.5	226
0	-22.5	254
0	22.5	248
0	67.5	58
0	112.5	56
45	-22.5	77
45	22.5	247
45	67.5	257
45	112.5	93
90	-22.5	84
90	22.5	71
90	67.5	261
90	112.5	258

Table 3: Coincidence measurements as a function of the two half-wave plate measurement angles α and β over a coincidence window of 10 ns, with accidental counts subtracted from the true count. The Labview program calculates the rate of accidental coincidences based on the equations derived in the Pre-lab.

Conclusion

A violation of Bell's inequality as done in this experiment implies that our perception of the world, and the assumptions we make based on our experience, may not be accurate of the fundamental structure of reality. Using a violet laser beam, some optics, and a nonlinear BBO crystal, we generated an entangled state of photons matching the famous thought experiment of Einstein, Podolsky, and Rosen. Observation of this entangled behavior that was initially believed to be a contradiction of Quantum Physics tells us that the models we use to interpret the world need revision. The work of John S. Bell allowed us to inch closer towards a more accurate universal model by disproving the existence of local hidden variable theories in nature. This "no-go" result still leaves much unanswered, and it will take further study of the axioms of Quantum Mechanics before the nature of measurement and entanglement can be understood.

I would first like to thank my Lab partner Andrew Gleeson for working diligently to make this experiment a success, as well as all of the GSIs and professors that gave us advice on experimental procedure when we became stuck. I would also like to acknowledge Dehlinger and Mitchell's text on this laboratory for aiding in our procedure and analysis. This lab, though full of room for error and thus difficult to calibrate, provides a very solid introduction to experimental quantum physics and working with an optical apparatus.

References

[1] Einstein, A; B Podolsky; N Rosen (1935-05-15). "Can Quantum-Mechanical Description of Physical Reality be Considered Complete?" Physical Review. 47 (10): 777–780

[2] J.S. Bell, "On the Einstein Podolsky Roden paradox".

[3] Experimental Realization of Einstein-Podolsky-Rosen-Bohm Gedankenexperiment: A New Violation of Bell's Inequalities, A. Aspect, P. Grangier, and G. Roger, Physical Review Letters, Vol. 49, Iss. 2, pp. 91–94 (1982)

[4] D. Dehlinger and M.W. Mitchell, "Entangled photons, nonlocality, and Bell inequalities in the undergraduate laboratory".

[5] Svetlana G. Lukishova, "Quantum Optics and Quantum Information Teaching Laboratories at the Institute of Optics, University of Rochester".